Universiteti i Tiranës
Fakulteti i Shkencave të Natyrës

1951 1991 UPT

Data Science, Particle and Astroparticle Physics School in Tirana 2022

Jun 14 – 18, 2022, Faculty of Natural Sciences

Lattice Space in LQCD with QCDLAB2

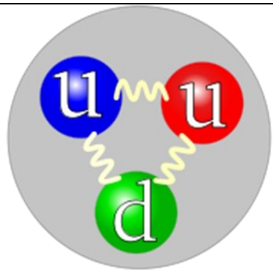
Prof. As. Dafina Xhako, *Polytechnic University of Tirana, Faculty of Mathematical Engineering and Physical Engineering, Department of Physics Engineering*, **Tirana, Albania**

Prof.As. Rudina Zeqirllari, *Department of Physics, University of Tirana,* **Tirana, Albania.**

In Memoriam of prof. Artan Borici who died from the Covid19 pandemic.

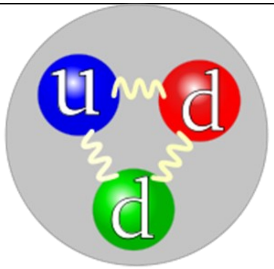
OUTLINE

- Motivation
- Introduction to Lattice QCD
- Interquark potential with QCDLAB2
- Lattice Space from Q-antiQ potential
- Conclusions



proton ≈938MeV

Motivation



neutron≈939 MeV

- The physics of strong interaction is based on *spontaneous chiral symmetry breaking*
- Theory of strong interactions can be study only with numerical simulations of the quantum theory of interacting quarks and gluons (Quantum Chromodynamics - QCD) formulated in an Euclidean 4-dimensional finite and regular lattice with periodic boundary conditions
- In lattice QCD simulations the formulation of the theory in lattice should be chiral in order that symmetry breaking happens dynamically from interactions
- This is the main motivation why we have to do lattice simulations with chiral fermions

Quantum Chromodynamics (QCD)

- Quantum field theory of strong interactions between quarks and gluons

Quarks (spinor fields)

$$(\psi_\alpha)^a_f \Rightarrow \begin{cases} \textbf{\textit{color}} & a = 1..3 \\ \textbf{\textit{spin}} & \alpha_{L,R} = 1/2, -1/2 \\ \textbf{\textit{flavor}} & f = 1...6 \end{cases}$$

Gluons (gauge fields)

$$A^a_\mu \Rightarrow \begin{cases} \textbf{\textit{color}} & a = 1...8 \\ \textbf{\textit{spin}} & e_\mu^{\pm} = 1 \end{cases}$$

- based on the SU(3) local (gauge) invariance group

- complex because of gluon-gluon interaction (non linear)

- characterized by two important features:

    1. *asymptotic freedom*
    2. *quark confinement*

Quantum Chromodynamics (QCD)

- **High energy scale (**exp. *QCD>10 GeV***)**:
  → weak coupling
  → perturbative methods can be applied
  → very good quantitative predictions
  → asymptotic freedom of quarks

$$\alpha_s(q^2 \to \infty) \to 0$$

asymptotic freedom

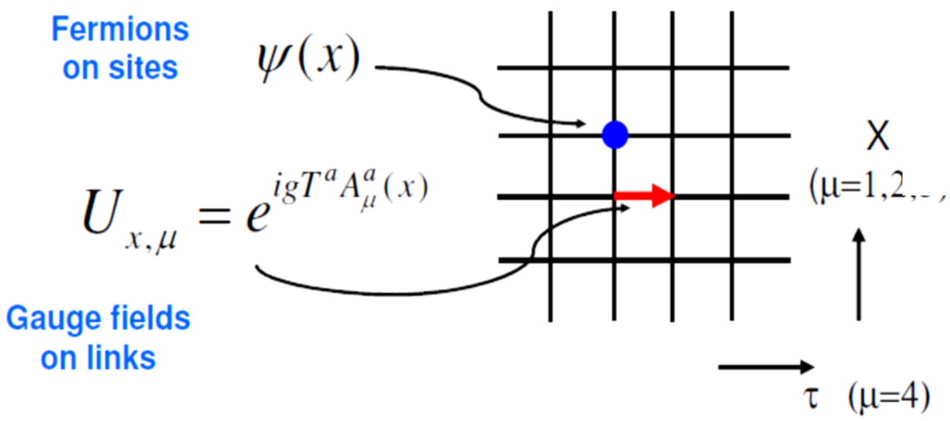
- **Low energy scale (Lattice** *QCD ~ 1GeV***)**:
  → strong coupling (perturbative methods can't be applied)
  → based on discretization of space-time in a hypercu
  lattice, with *N*- node per
  direction separated by a distance *a*.
  → fields representing quarks are defined at lattice sit
  the gluon fields are defined on the links connecting
  neighboring sites
  → quarks confinement
  → can be "solved" numerically with Monte-Carlo simulations

$$\alpha_s(q^2 \to 0) >> 1$$

confinement

Lattice QCD (LQCD)

- Discretize space-time



parameters of the lattice

select {
- L/a : sites per dimension
- m_q : light quark mass
- g : value of bare coupling

must take the limits :

- continuum limit $\alpha \to 0$
- infinite volume $L \to \infty$
- chiral limit $m_q \to 0$

Advantages:

- *Finite degree of freedom,* physics quantity can be calculated numerically with Monte Carlo simulations
- It can study features of QCD that are impossible in experimental QCD such is *quark confinement*
- Introduce a *"cut-off" effect* ~ 1/a (first Brillouin zone)
- The action is invariant under gauge transformations

Disadvantages:

- *Discretization errors* ~ a^2 , that can be reduced adding appropriate terms at the action maintaining invariant calibration
- *Finite size effects,* Physics results are taken from calculated lattice quantity in continuum limit
- Monte Carlo method introduce *statistical errors* ~ 1/sqrt(N)

QCDLAB2

- QCDLAB, a special package software which is a research tool for lattice QCD algorithms. (Boriçi, 2006), (Boriçi, 2007)
- It is a collection of MATLAB/OCTAVE functions, that is based on a “small-code” and a “minutes-run-time” algorithmic design philosophy
- In QCDLAB 1 we have developed fast algorithm in Quantum Electrodynamics U(1) gauge theory (in two dimension) which uses the Schwinger model on the lattice that shares many features with the algorithms of lattice QCD. (K. Melnikov, 2000).
- we have developed a fast inversion algorithm “preconditioned GMRESR” (Generalized Minimal Residual Method - Recursive) as part of QCDLAB 1 U(1) gauge theory
- From these results we bring now QCDLAB 2 and our calculation about interquark potential
- QCDLAB2 is a set of programs, written in GNU Octave, for lattice QCD computations.
- Version 2.0 includes the generation of configurations for the SU(3) theory, computation of rectangle Wilson loops as well as the low lying meson spectrum.

Interquark potential with QCDLAB2

- The quark-antiquark potential derive from Wilson loops by calculating effective potential

$$V(r)_{eff} = -\log \frac{W(r,t+1)}{W(r,t)}.$$

- for each r , we select effective potentials when for long time t is reached a plateau. Calculated quark-antiquark values in lattice are modeled as:
- $V(r) = V_0 + Kr + \frac{a}{r}$ or unitless $\hat{V}(R)_{eff} = \hat{V}_0 + \hat{K}\frac{R}{a} + \frac{a}{R}\hat{\alpha}.$ ★
- To setting the scale of theory we have used the new method from Sommer relation, with r_0=0.5fm: $r_0^2 F(r_0) = 1.65$
- To take physical quantity in continuum we repeat simulation for different lattice volume (taking physical length constant ~ *L=1.6fm*) and extrapolate in continuum limit a→0.

- We have calculated the string tension K repeating simulations for 100 different configurations, for 8^4, 12^4 and 16^4 lattice.

- The results taken are fitted with the equation★

- The R was chosen for fit it is between $R = 0.5$ and $R = 8$. The lattice physical volume (L^4) with length L per directions, it is equivalented to the lattice volume (N^4) with N point for direction from $L = aN$.

- We have done all simulations using Wilson action for 8^4, 12^4, 16^4 lattices with different background gauges fields, specifically for 100 gauges fields and testing for three different coupling constants, using QCDLAB2 soft.

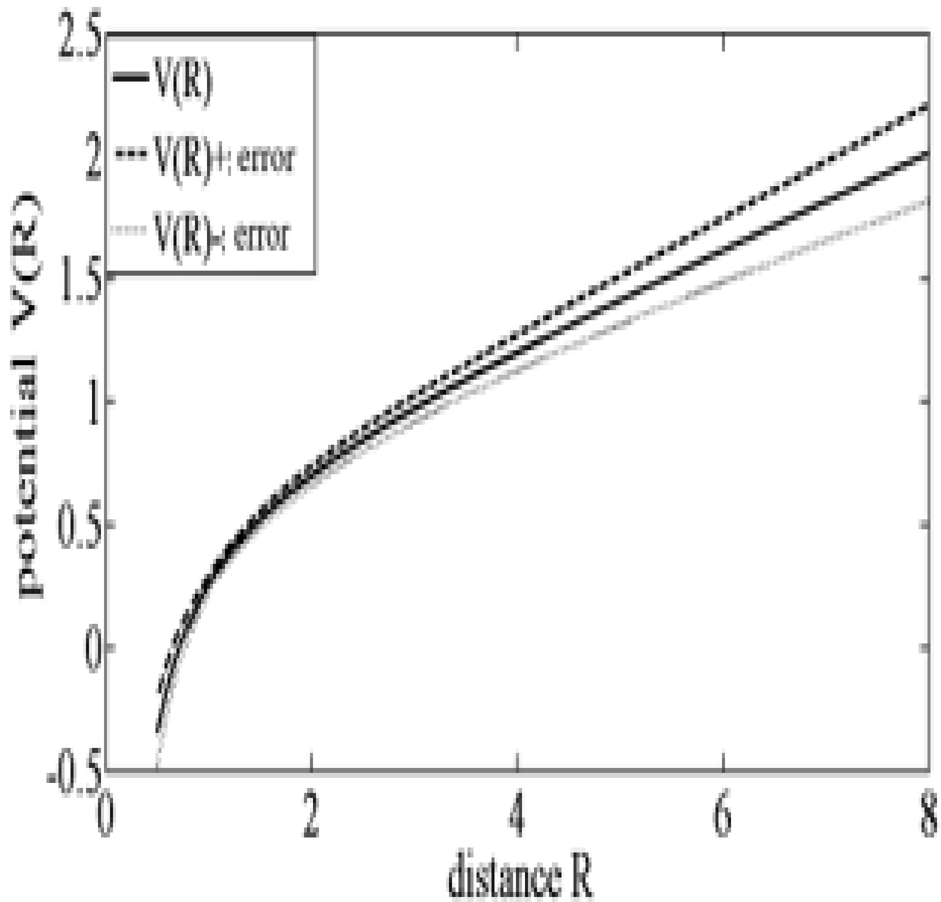


Figure 1. Graphical representation of potential between two static quarks in 8^4 lattice, in gauge field background with 5.7 coupling constant and dimensionless unit

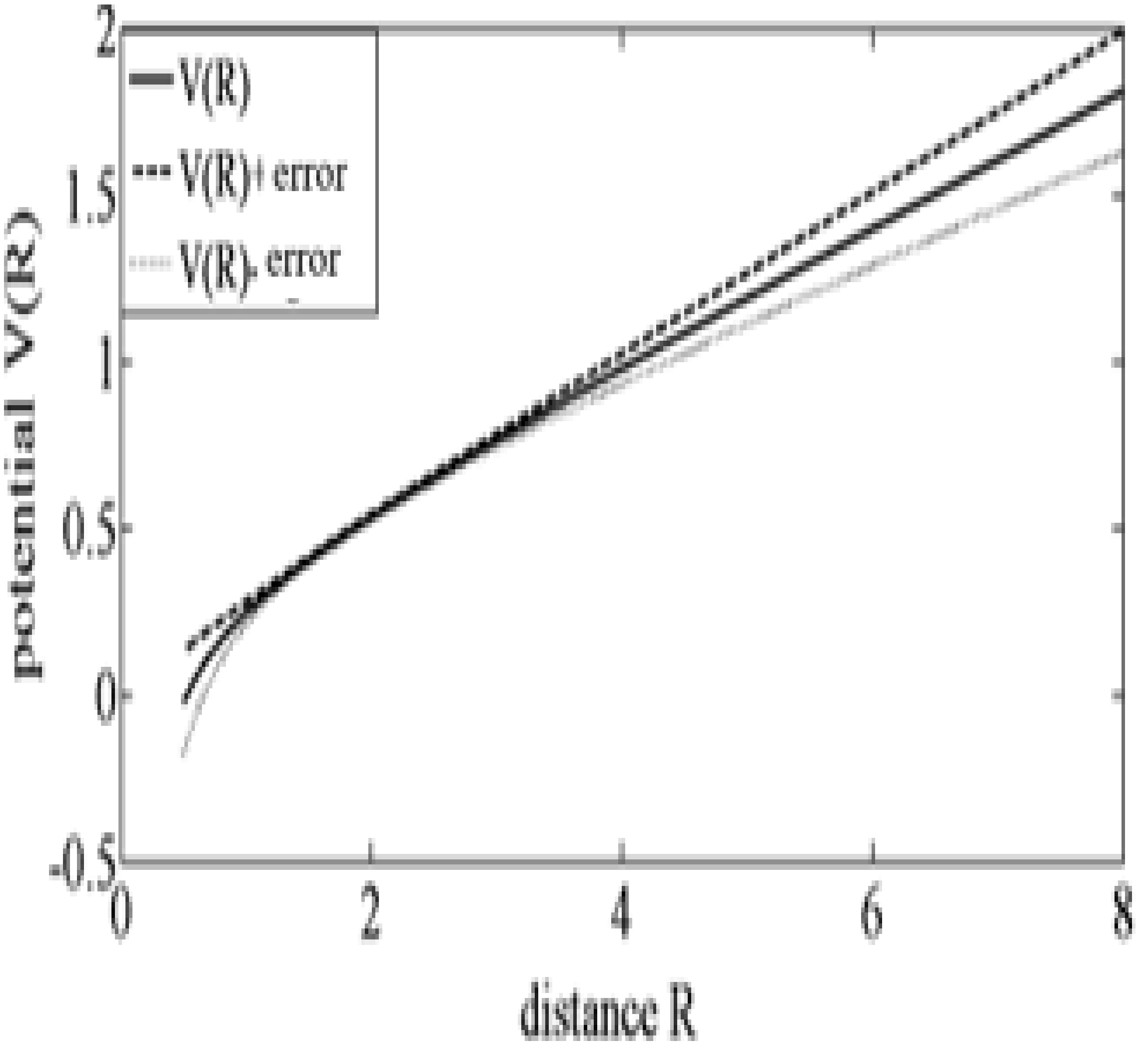


Figure 2. Graphical representation of potential between two static quarks in 12^4 lattice, in gauge field background with 5.8 coupling constant and dimensionless unit

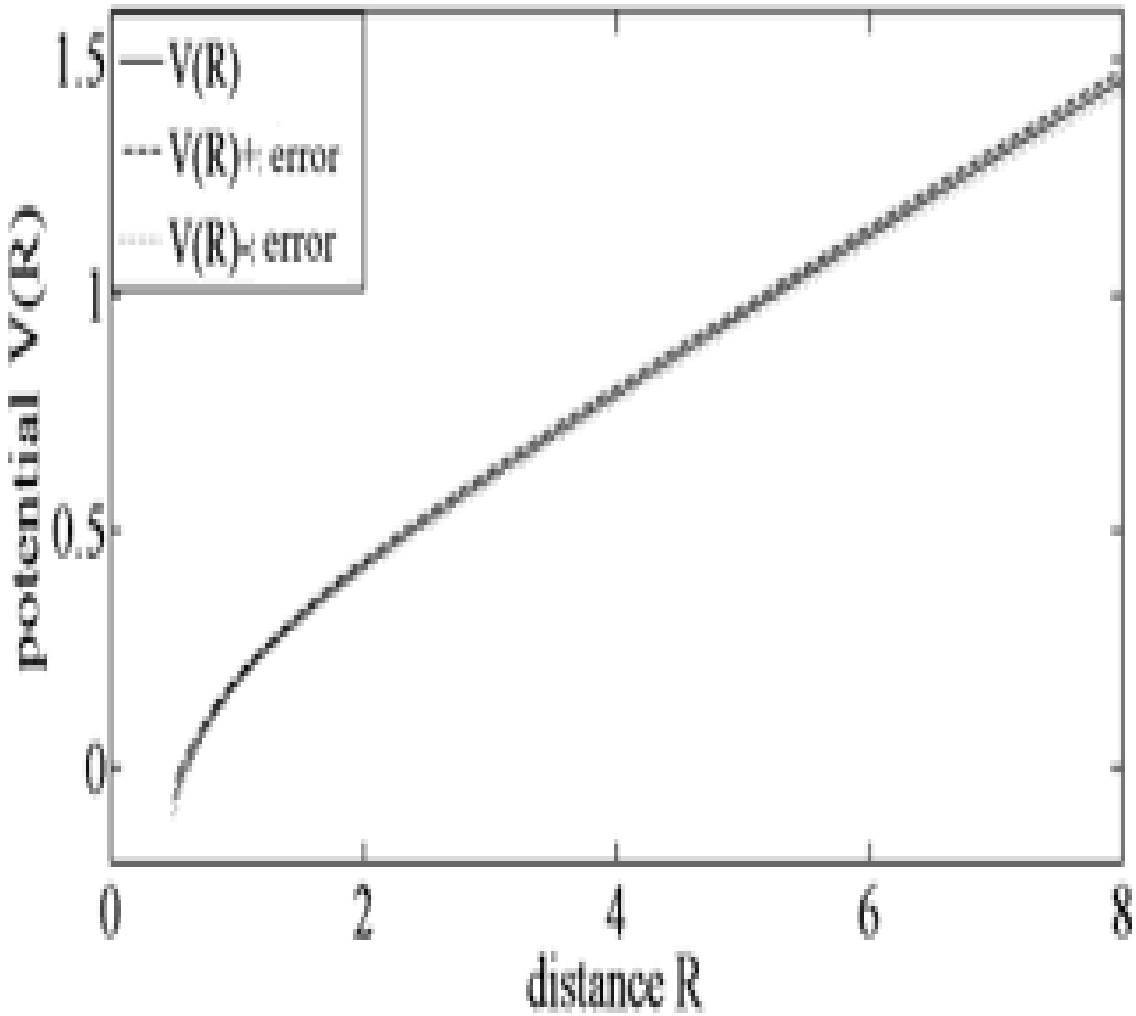


Figure 3. Graphical representation of potential between two static quarks in 16^4 lattice, in gauge field background with 6 coupling constant and dimensionless unit

Table 1. The calculated lattice space a, string tension $\widehat{K}$, and their respective statistical errors 8^4, 12^4, 16^4 lattices

Lattice	The coupling constant parameter	The lattice space parameter (femtometer- fm)	The string tension parameter (lattice unit)	The error of lattice space	The error of string tension
8^4	5.7	0.2011(15)	0.1670(79)	1.1003e-04	2.54-03
12^4	5.8	0.1630(07)	0.1592(15)	9.0097e-05	2.67e-03
16^4	6	0.1095(73)	0.1410(27)	1.0980e-05	2.84e-04

Referring numerical results, tabulated in Table 1, we have the values of the lattice space parameter and the values of the string tension parameter for 8^4, 12^4, 16^4 lattices. Also, in this table are represented the statistical errors of the lattice space parameter and the string tension parameter using Jackknife method. As we can see from these numerical results the values of lattice space and string tension are taken within the statistical error rate of calculated. Specifically, for lattice 8^4 the lattice space parameter is $a = [0.2011(15) \pm 1.1003 \times 10^{-4}]$ (fm) and the string tension is $K = [0.1670(79) \pm 2.67 \times 10^{-3}]$ (in lattice unit). The same analyze can be done for other lattice volumes.

Conclusions

I. Studying and calculating the interquark potential in the pure gauge theories serves to determine the scale of theory from which all the other calculated quantity in the lattice can then be converted into physical quantity.

II. The results of calculation of interquark potential with QCDLAB2 confirms that quark are confined into hadrons.

III. QCDLAB2 is a set of codes very efficient in lattice QCD simulations in SU(3) gauge field.

IV. Motivated by results of this work, in our future work we have to develop inversion algorithms for chiral fermions in SU(3) gauge theory using as environment QCDLAB2

ACKNOWLEDGMENTS

Acknowledgements goes to the serves in the parallel system of the Faculty of Information Technology and the Department of Engineering Physics in the Faculty of Engineering Mathematics and Engineering Physics (FIMIF), Polytechnic University of Tirana, Albania. This work is done to honor the memoriam of **prof. Artan Borici** who died from the Covid19 pandemic.