

FORWARD DIJET PRODUCTION IN SATURATION FORMALISM

PIOTR KOTKO

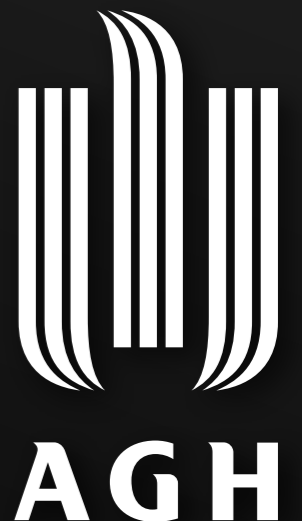
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SUPPORTED BY:

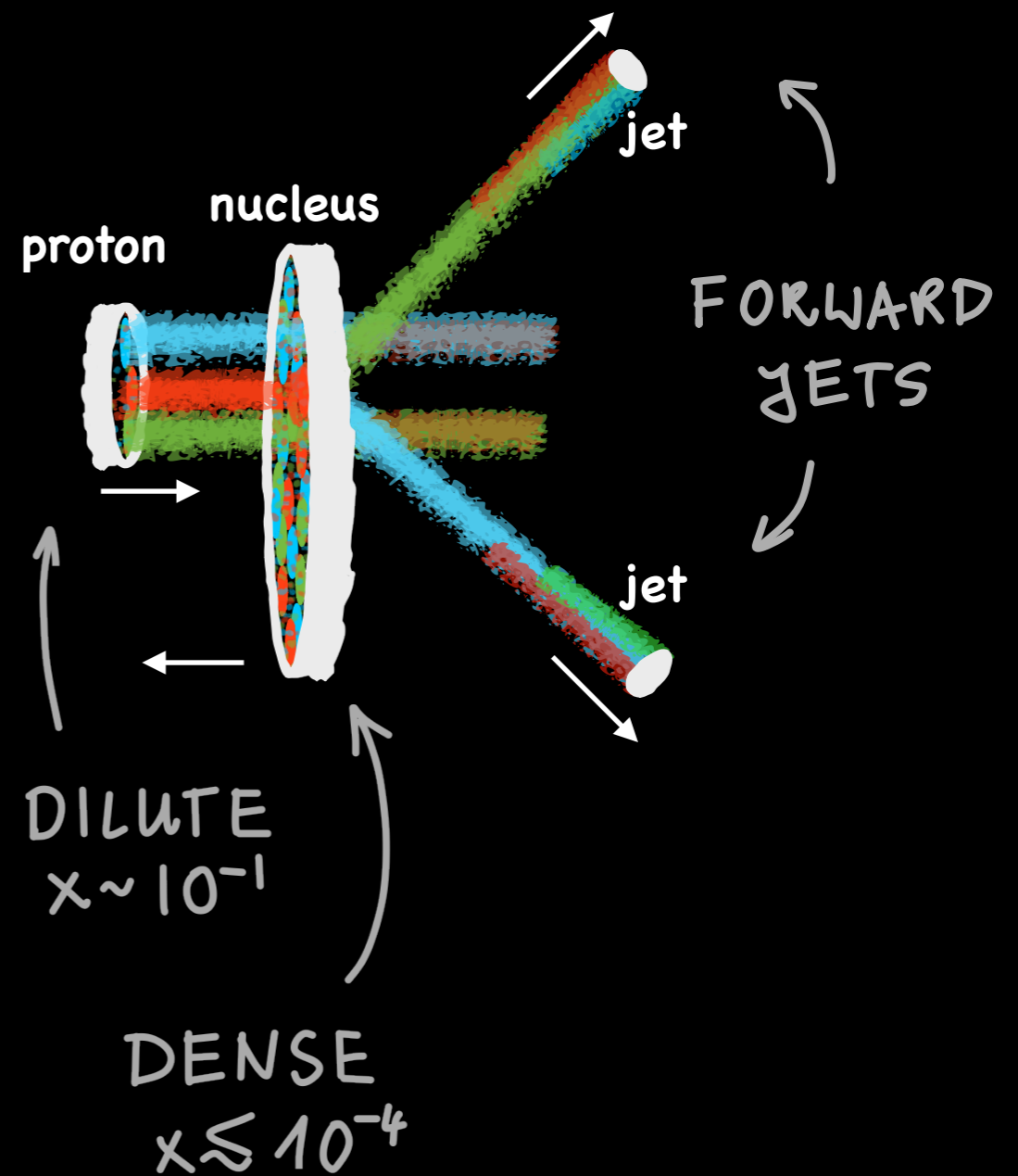
NCN GRANT DEC-2018/31/D/ST2/02731



MOTIVATION

Forward dijets in dilute-dense collisions

- probing small- x regime of gluon distributions
- sensitive to nuclear effects
- sensitive to internal k_T of gluons
- sensitive to interplay between k_T of gluons and p_T of jets
- non-universality of TMD gluon distributions



PLAN

1. Framework

- A. Small-x Improved TMD factorization (ITMD) for pA
- B. Dilute-dense collisions in Color Glass Condensate (CGC)
- C. TMD gluon distributions at small x
- D. Sudakov resummation

2. Phenomenology for ATLAS and FoCal kinematics

- A. Azimuthal dijet correlations at parton level for p-p and p-Pb
- B. Attempts to estimate hadron-level corrections

3. Summary and Outlook

FRAMEWORK Small-x Improved TMD Factorization (ITMD)

Factorization formula for forward dijets in p-p and p-A

[PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, 2015]

$$\frac{d\sigma_{pA \rightarrow 2j+X}}{dy_1 dy_2 d^2p_{T1} d^2p_{T2}} \sim \sum_{a,c,d} f_{a/p}(x_1, \mu) \sum_{i=1,2} K_{ag \rightarrow cd}^{(i)}(k_T) \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_T)$$

RAPIDITY

TRANSVERSE
MOMENTA

COLLINEAR
PROTON PDF

GAUGE
INVARIANT
OFF-SHELL
HARD FACTORS

TMD GLUON
DISTRIBUTIONS
AT SMALL-X

$$x_2 \ll x_1 \quad |\vec{p}_{T1} + \vec{p}_{T2}| = k_T$$

ITMD factorization formula has been proven from the Color Glass Condensate (CGC) theory.

⇒ RESUMMATION OF KINEMATIC TWISTS AND NEGLECTING GENUINE TWISTS.

[T. Altinoluk, R. Boussarie, PK, 2019]

TWO PER CHANNEL
($g^*q \rightarrow qq, q^*q \rightarrow qq, g^*q \rightarrow q\bar{q}$)

easy to implement in
Monte Carlo

FRAMEWORK Limiting cases of Color Glass Condensate (CGC)

CGC
dilute - dense

three scales:

$Q_s \gg \Lambda_{\text{QCD}}$ — saturation scale

k_T — jet transverse momentum imbalance

P_T — jet average transverse momentum

$$P_T \gg k_T \sim Q_s$$



TMD
GENERALIZED FACTORIZATION

leading twist

- [F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]
- [C. Marquet, E. Petreska, C. Roiesnel, 2016]
- [C. Marquet, C. Roiesnel, P. Taels, 2018]
- [T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2019, 2020]
- [P. Taels, T. Altinoluk, G. Beuf, C. Marquet, 2022]

$$P_T \sim k_T \gg Q_s$$

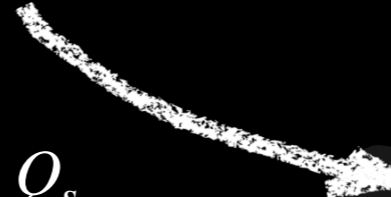


DILUTE

k_T -FACTORIZATION
BFKL dynamics

- [S. Catani, M. Ciafaloni, F. Hautmann, 1991]
- [M. Deak, F. Hautmann, H. Jung, K. Kutak, 2009]
- [E. Iancu, J. Leidet, 2013]

$$P_T \gg Q_s$$



ITMD
"IMPROVED"
TMD factorization
all kinematic twists

- [PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, 2015]
- [A. van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, 2016]
- [T. Altinoluk, R. Boussarie, PK, 2019]
- [H. Fujii, C. Marquet, K. Wanatabe, 2020]
- [T. Altinoluk, C. Marquet, P. Taels, 2021]

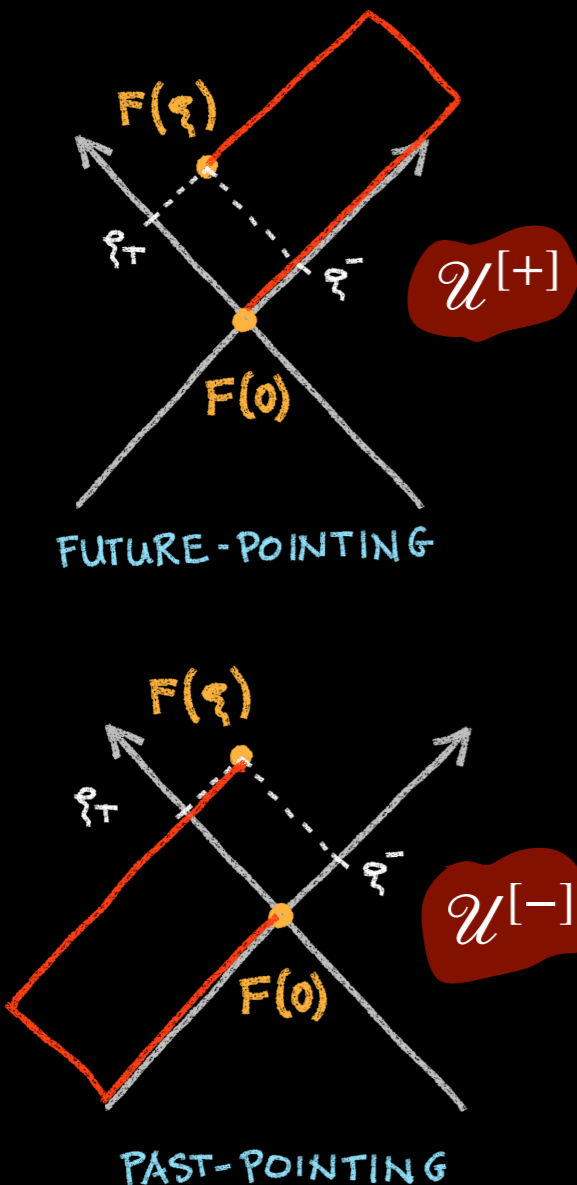


Generic operator definition

$$\mathcal{F}_g(x, k_T) = 2 \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3 P^-} e^{ixP^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi^+, \vec{\xi}_T, \xi^- = 0) \mathcal{U}_{C_1} \hat{F}^{i-}(0) \mathcal{U}_{C_2} \right] | P \rangle$$

↑ GLUON FIELD $\hat{F} = F_a t^a$
↑ GAUGE LINKS in fundamental color representation

(unpolarized)



Gauge links $\mathcal{U}_{C_1}, \mathcal{U}_{C_2}$ depend on the color structure of the hard process. They are built from two basic Wilson lines:

[C. Bomhof, P. Mulders, F. Pijlman, 2004]

$$\begin{aligned} \mathcal{U}^{[\pm]} &= [0, (\pm\infty, \vec{0}_T, 0)] \\ &\times [(\pm\infty, \vec{0}_T, 0), (\pm\infty, \vec{\xi}_T, 0)] \\ &\times [(\pm\infty, \vec{\xi}_T, 0), (\xi^+, \vec{\xi}_T, 0)] \end{aligned}$$

$$[x, y] = \mathcal{P} \exp \left\{ ig \int_{xy} dz_\mu A_a^\mu(z) t^a \right\}$$

STRAIGHT LINE SEGMENT

Light-cone basis:

$$v^\pm = v^\mu n_\mu^\pm, \quad n^\pm = (1, 0, 0, \mp 1)$$

$$v^\mu = \frac{1}{2} v^+ n^- + \frac{1}{2} v^- n^+ + v_T^\mu$$

FRAMEWORK

All possible operators

[M. Bury, PK, K. Kutak, 2018]

$$\mathcal{F}_{qg}^{(1)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

DIPOLE

$$\mathcal{F}_{qg}^{(2)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]}}{N_c} \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

↑
in the
small-x
limit
↓

$$\mathcal{F}_{qg}^{(3)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]\dagger}}{N_c} \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(2)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[\square]\dagger} \right] \text{Tr} \left[\hat{F}^{i-}(0) \mathcal{U}^{[\square]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(3)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

WEIZSACKER
-WILLIAMS

$$\mathcal{F}_{gg}^{(4)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[-]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle P | \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] | P \rangle$$

$$\mathcal{F}_{gg}^{(6)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]}}{N_c} \frac{\text{Tr} \mathcal{U}^{[\square]\dagger}}{N_c} \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

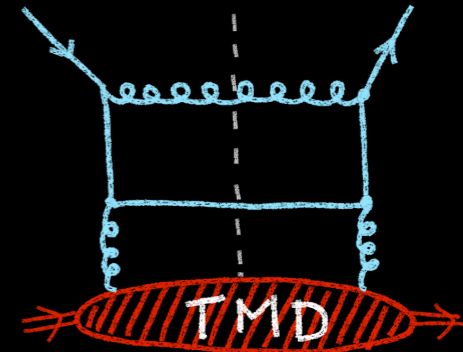
$$\mathcal{F}_{gg}^{(7)} \sim \langle P | \frac{\text{Tr} \mathcal{U}^{[\square]}}{N_c} \text{Tr} \left[\hat{F}^{i-}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i-}(0) \mathcal{U}^{[+]} \right] | P \rangle$$

WILSON LOOP → $\mathcal{U}^{[\square]} = \mathcal{U}^{[+]} \mathcal{U}^{[-]\dagger}$

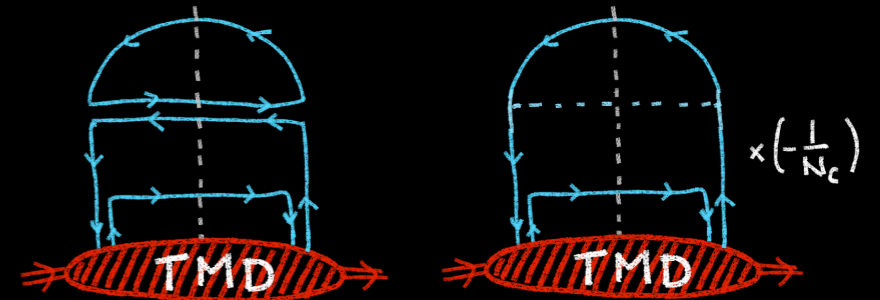
TMD gluon distributions

Example

TMD gluon distribution for the following process:



Two independent color flows:



$$\rightsquigarrow \frac{N_c}{2C_F} \mathcal{F}_{qg}^{(2)} - \frac{1}{2N_c C_F} \mathcal{F}_{qg}^{(1)}$$

Gluon TMD for any multiparticle process is given by a linear combination of these "basis" TMDs.

Data-driven dipole TMD

Balitsky-Kovchegov type equation with kinematic constraint,
DGLAP correction and running coupling:

[J. Kwieciński, A. Martin, A. Stasto, 1997]

[K. Kutak, J. Kwieciński, 2003]

$$\begin{aligned}
 \mathcal{F}_{qg}^{(1)}(x, k_T^2) = & \mathcal{F}_0(x, k_T^2) + \frac{\alpha_s N_c}{\pi} \int_x^1 \frac{dz}{z} \int_{k_{T0}^2}^{\infty} \frac{dq_T^2}{q_T^2} \left\{ \frac{q_T^2 \mathcal{F}\left(\frac{x}{z}, q_T^2\right) \theta\left(\frac{k_T^2}{z} - q_T^2\right) - k_T^2 \mathcal{F}\left(\frac{x}{z}, k_T^2\right)}{|q_T^2 - k_T^2|} + \frac{k_T^2 \mathcal{F}\left(\frac{x}{z}, k_T^2\right)}{\sqrt{4q_T^4 + k_T^4}} \right\} \\
 & + \frac{\alpha_s}{2\pi k_T^2} \int_x^1 dz \left\{ \left(P_{gg}(z) - \frac{2N_c}{z} \right) \int_{k_{T0}^2}^{k_T^2} dq_T^2 \mathcal{F}\left(\frac{x}{z}, q_T^2\right) + z P_{gq}(z) \Sigma\left(\frac{x}{z}, k_T^2\right) \right\} \\
 & - \frac{2\alpha_s^2}{R^2} \left\{ \left[\int_{k_T^2}^{\infty} \frac{dq_T^2}{q_T^2} \mathcal{F}(x, q_T^2) \right]^2 + \mathcal{F}(x, k_T^2) \int_{k_T^2}^{\infty} \frac{dq_T^2}{q_T^2} \ln\left(\frac{q_T^2}{k_T^2}\right) \mathcal{F}(x, q_T^2) \right\}
 \end{aligned}$$

for NUCLEUS: $R_A = A^{1/3} R_p$

fitted to
DIS HERA
data

[K. Kutak, S. Sapeta, 2012]

How to get various TMD distributions?

Using CGC theory one can derive a relation between the small-x TMDs using:

- (i) large N_c limit
- (ii) mean field (Gaussian) approximation.

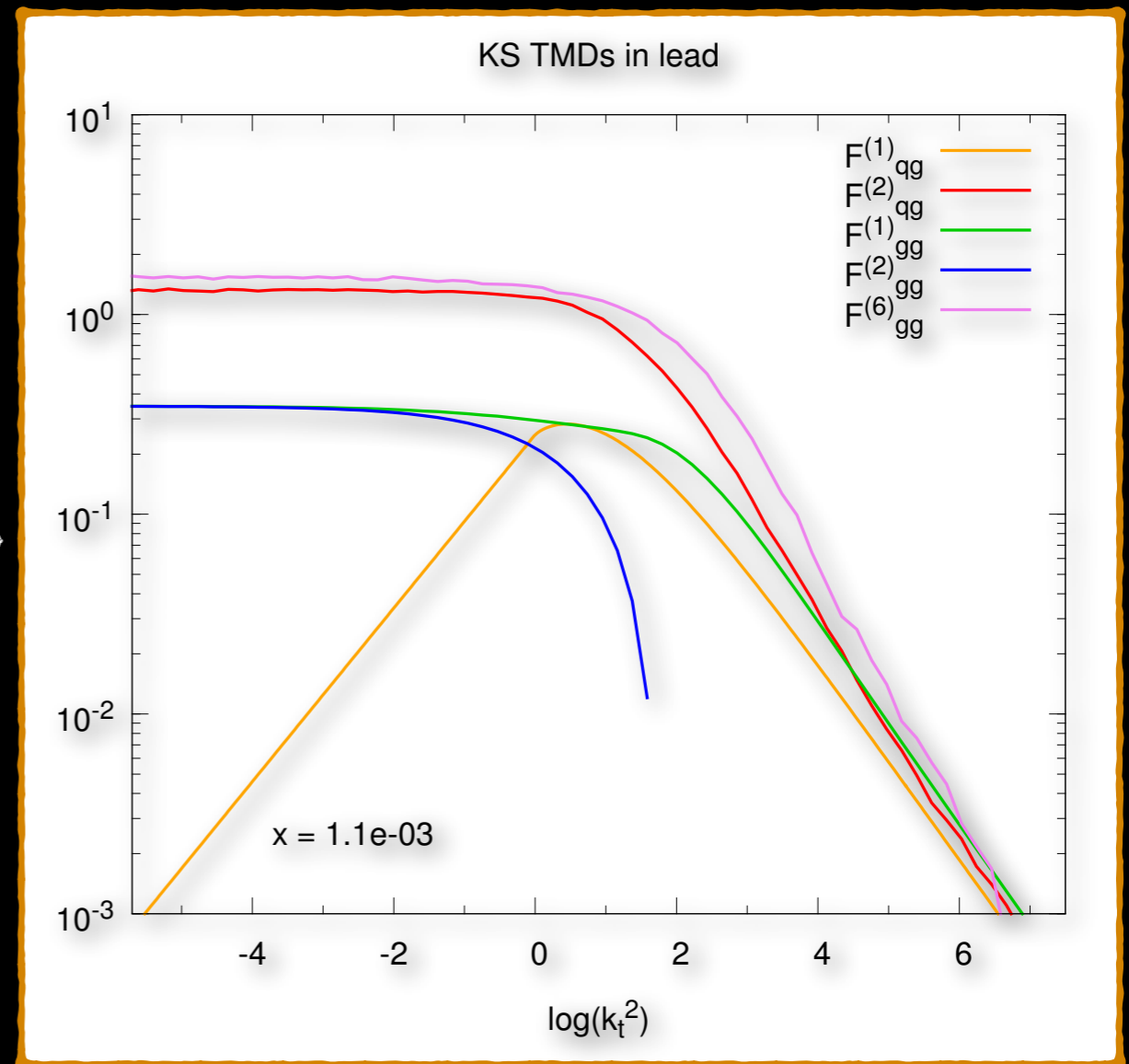
All TMDs needed for dijet production can be calculated from the dipole gluon distribution $\mathcal{F}_{qg}^{(1)}$.

It is possible to relax the assumptions (i) and (ii) using the JIMWLK equation.
Prove of concept:

[C. Marquet, E. Petreska, C. Roiesnel, 2016]

Improvements:

[S. Cali, K. Cichy, P. Korcyl, PK, K. Kutak, C. Marquet, 2021]



[A. Van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, 2016]

Hard scale in TMD gluon distributions

- Typical small- x evolution (BFKL, BK, JIMWLK) evolves only in energy at fixed hard scale:

$$\mathcal{F}_{ag}^{(i)}(x, k_T^2) = \mathcal{F}_{qg}^{(1)}(x, k_T^2, \mu = \mu_0), \quad \mu_0 \sim k_T$$

- Evolution in hard scale only (at fixed x) is the DGLAP evolution:

$$f_a(x, \mu^2) = S_a(\mu^2, \mu_0^2) f_a(x, \mu_0^2) + \int_{\mu_0}^{\mu^2} \frac{dp_T^2}{p_T^2} \frac{\alpha_s(p_T^2)}{2\pi} S_a(\mu^2, p_T^2) P_{ba}(z) \otimes f_b\left(\frac{x}{z}, p_T^2\right)$$

Sudakov
form factor

$$S_a(\mu^2, \mu_0^2) = \exp\left(-\int_{\mu_0^2}^{\mu^2} \frac{dp_T^2}{p_T^2} \frac{\alpha_s(p_T^2)}{2\pi} \sum_i \int_{\epsilon}^{1-\epsilon} P_{ia}(z)\right)$$

- Trying to mix both types of evolution has a long history...

- CCFM [M. Ciafaloni, S. Catani, F. Fiorani, G. Marchesini, 1990]
- KMR [M.A. Kimber, A.D. Martin, M.G. Ryskin, 2000]
- CASCADE [H. Jung, G. Salam, 2000]
- [K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek, 2012]
- [I. Balitsky, A. Tarasov, 2015]
- [A. van Hameren, PK, K. Kutak, S. Sapeta, 2014]
- ...

- Resummation of Sudakov logs can be also done in the impact-parameter space.
More general, but harder to implement in Monte Carlo.

b-space Sudakov resummation for dijets in ITMD framework

$$\frac{d\sigma_{pA \rightarrow 2j+X}}{dy_1 dy_2 d^2p_{T1} d^2p_{T2}} \sim \sum_{a,c,d} \sum_{i=1,2} K_{ag \rightarrow cd}^{(i)}(k_T, \mu) \int d^2b f_{a/p}(x_1, \mu_b) \widetilde{\Phi}_{ag \rightarrow cd}^{(i)}(x_2, \vec{b}_T) e^{-S^{ag \rightarrow cd}(\mu, \mu_b)}$$

where

$$\mu_b = 2e^{-\gamma_E}/b_*$$

$$b_* = b_T / \sqrt{1 + b_T^2/b_{T\max}^2}$$

↑
F.T. of TMD
gluon distributions

↑
Sudakov
factors

The perturbative Sudakov factors $S^{ag \rightarrow cd}(\mu, \mu_b)$ were calculated in [A.H. Mueller, B-W. Xiao, F. Yuan, 2013]

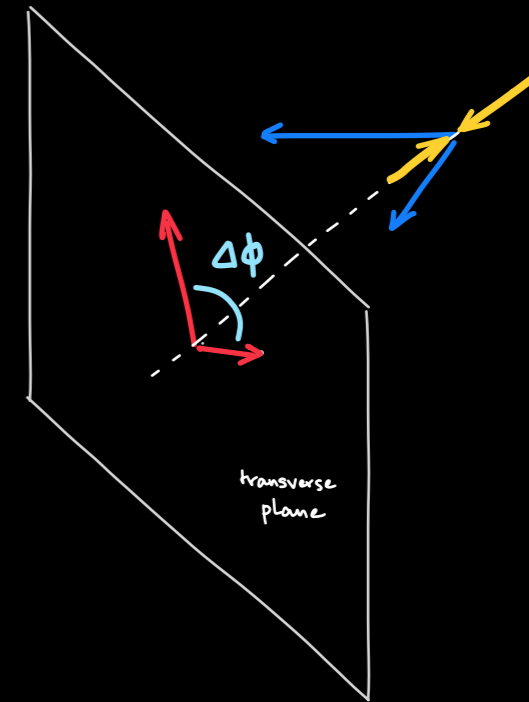
How to apply this in Monte Carlo?

- Approach 1: ignore the b-dependence in the collinear PDF
 - the hard scale-dependent TMD distribution can be computed separately
 - missing threshold type logarithms
- Approach 2: compute the b-integral for MC generated phase space points
 - first compute the weights according to Approach 1
 - next, reweight the events using the full b-space luminosity

We test
both
approaches.

Overview of the computations

- azimuthal correlations between jets
- p-p and p-Pb cross sections in FoCal and ATLAS setup
- nuclear modification ratios
- ITMD framework with KS TMD gluon distributions using KaTie Monte Carlo
- both the full b-space Sudakov resummation and the approximate MC-convenient approach
- Pythia computations to estimate nonperturbative corrections



Kinematic cuts

- CM energy: $\sqrt{s} = 8.16 \text{ TeV}$ per nucleon
- jet radius: $\Delta R > 0.5$
- jet transverse momenta:
- rapidity:

$$p_{T1} > p_{T2} > 10 \text{ GeV}$$

$$3.8 < y_1^*, y_2^* < 5.1$$

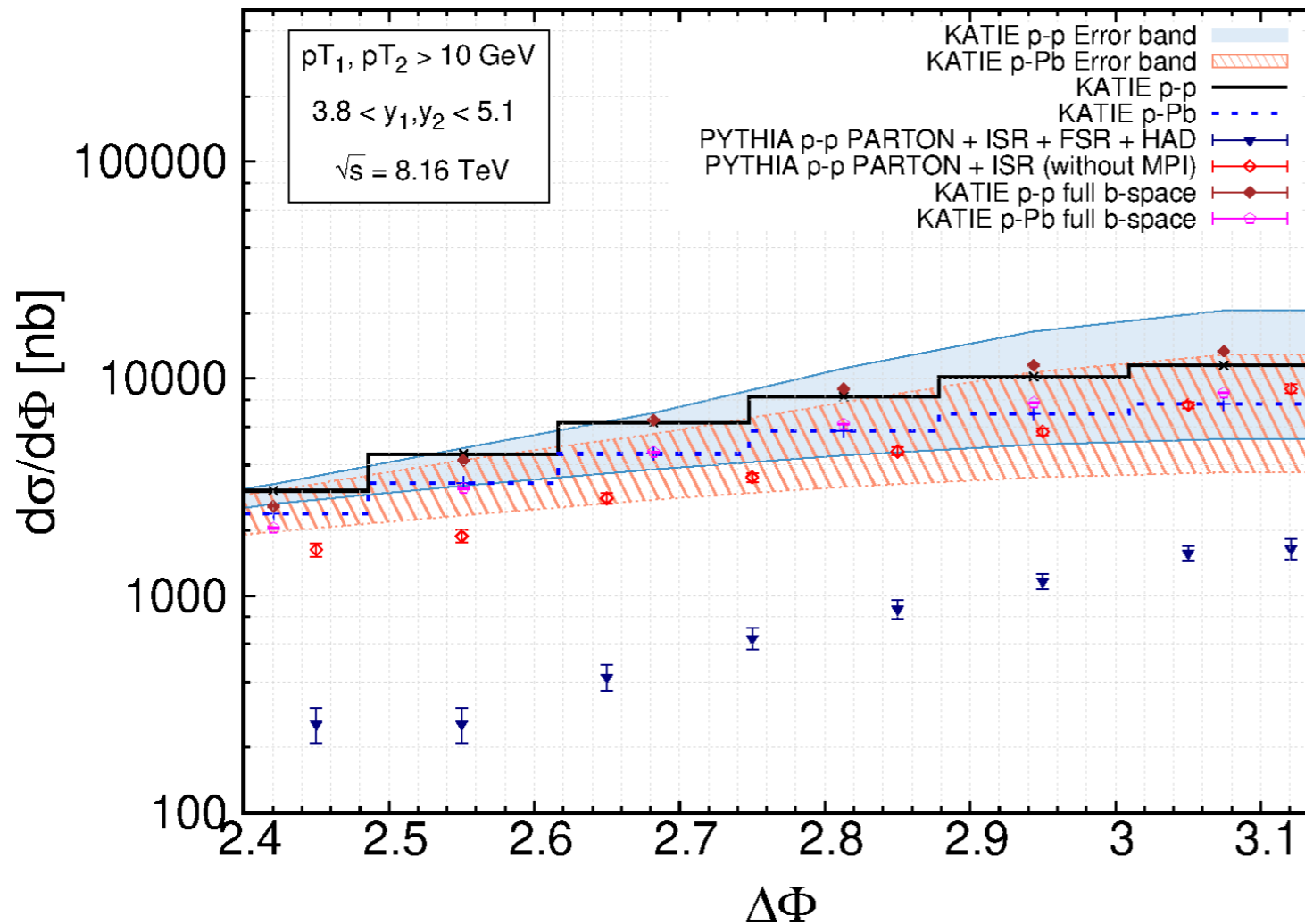
FoCal

$$45 \text{ GeV} > p_{T1} > p_{T2} > 28 \text{ GeV}$$

$$2.7 < y_1^*, y_2^* < 4.0$$

ATLAS

Azimuthal correlations for p-p and p-Pb



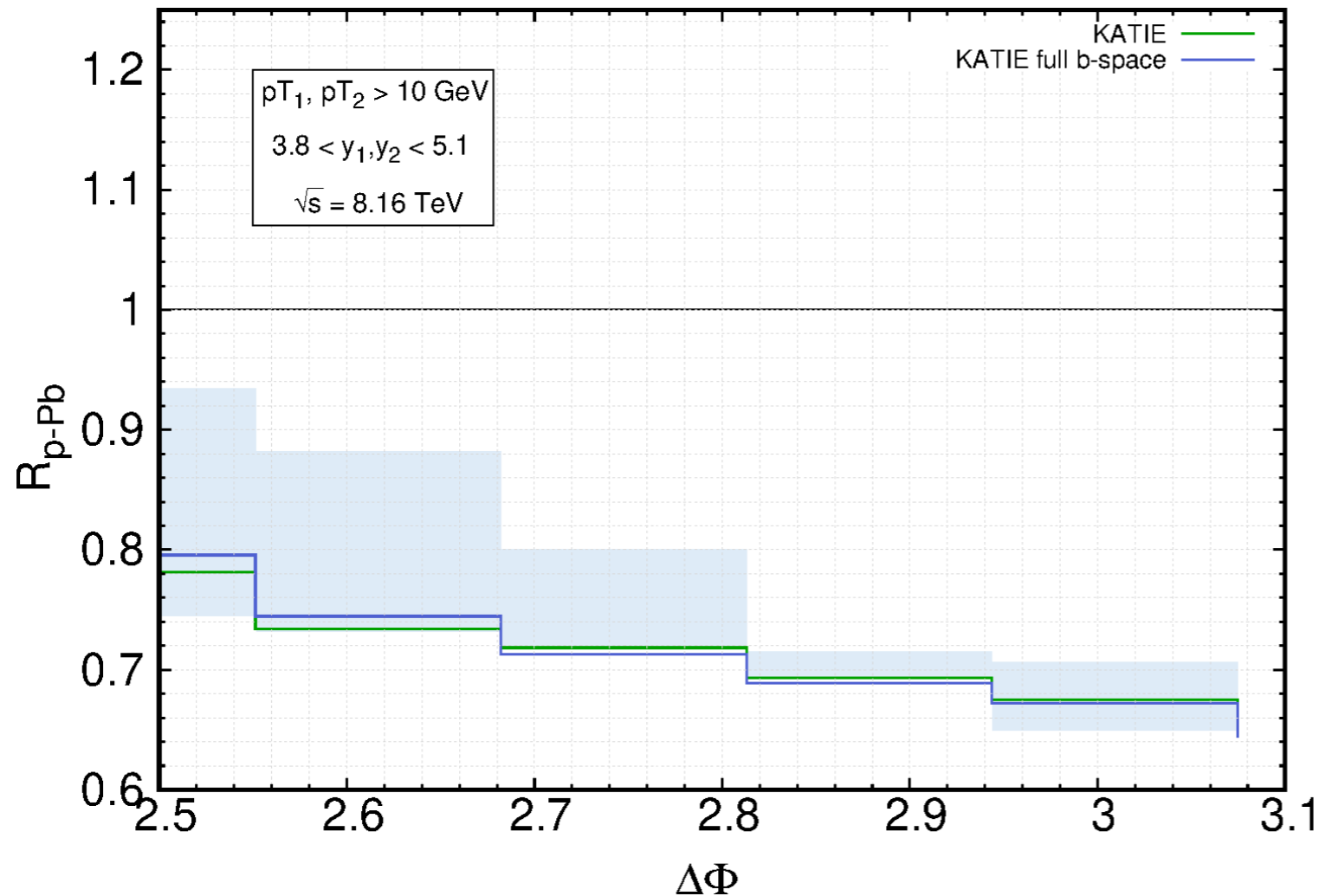
ITMD@KaTie

- large suppression of the p-Pb cross section compared to p-p
- the full b-space Sudakov resummation as well as the simplified approach are very similar
- the saturation effects do not go away when including the Sudakov resummation

lessons from Pythia:

- final state shower and nonperturbative corrections (MPI and hadronization) seem to significantly affect the spectrum
- too low p_T cut?
- can we extract nonperturbative "form factor"?

Azimuthal correlations - nuclear modification ratio



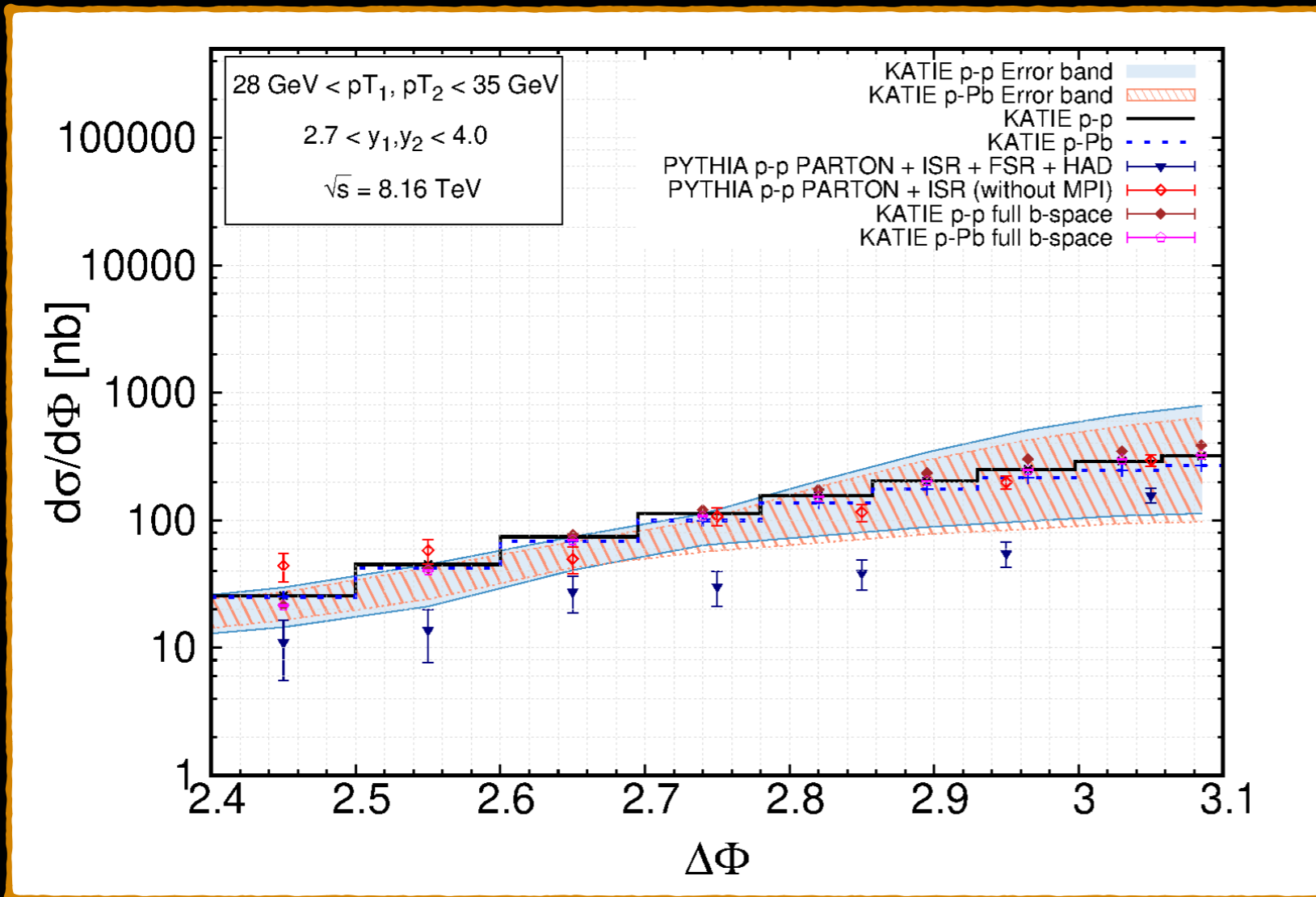
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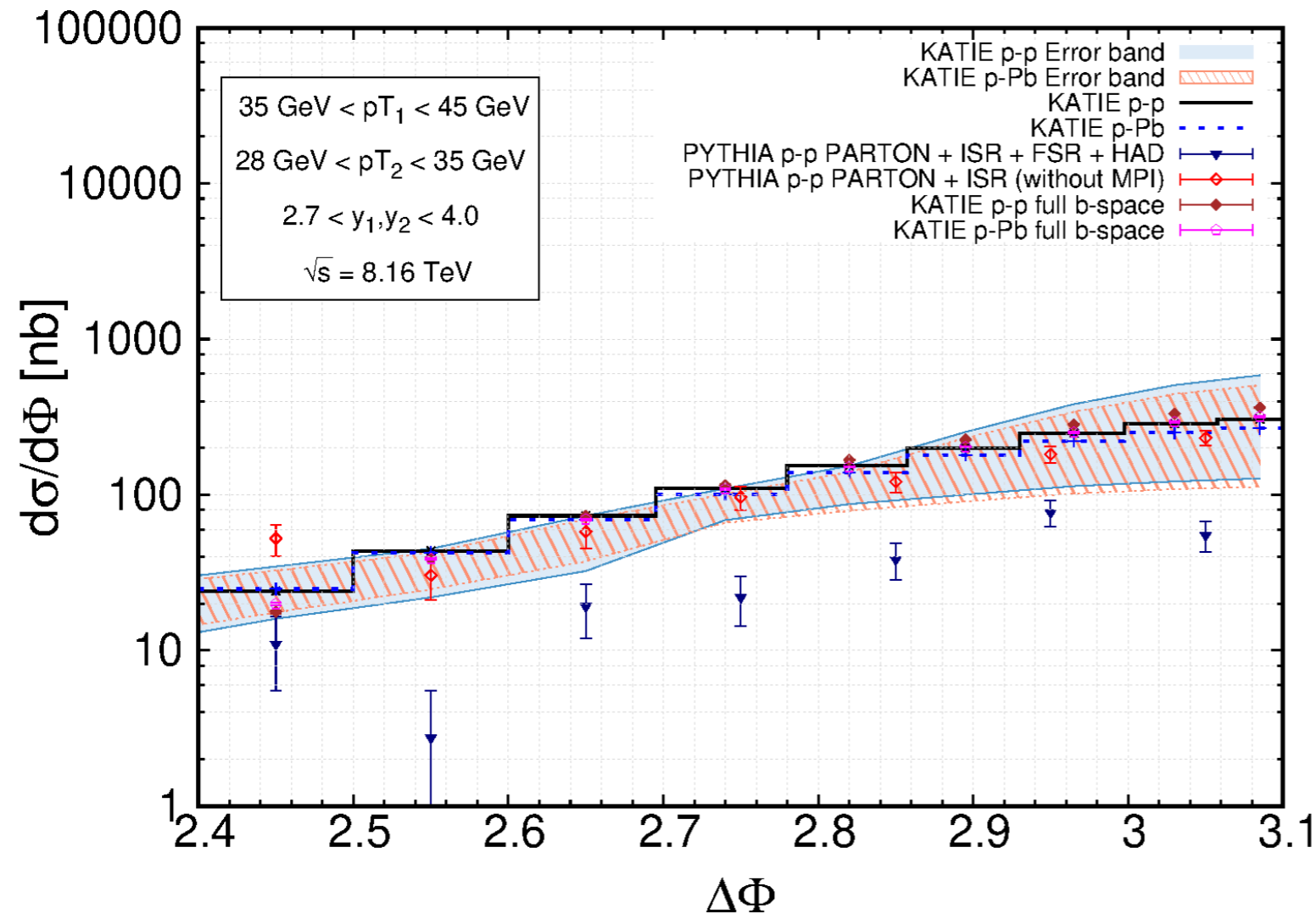
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- suppression up to 20% for the lowest p_T cut
- the Sudakov resummation has the same features as for the FoCal cuts

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- nonperturbative corrections (in particular hadronization) are milder, as one should expect due to larger p_T cut

Azimuthal correlations for p-p and p-Pb



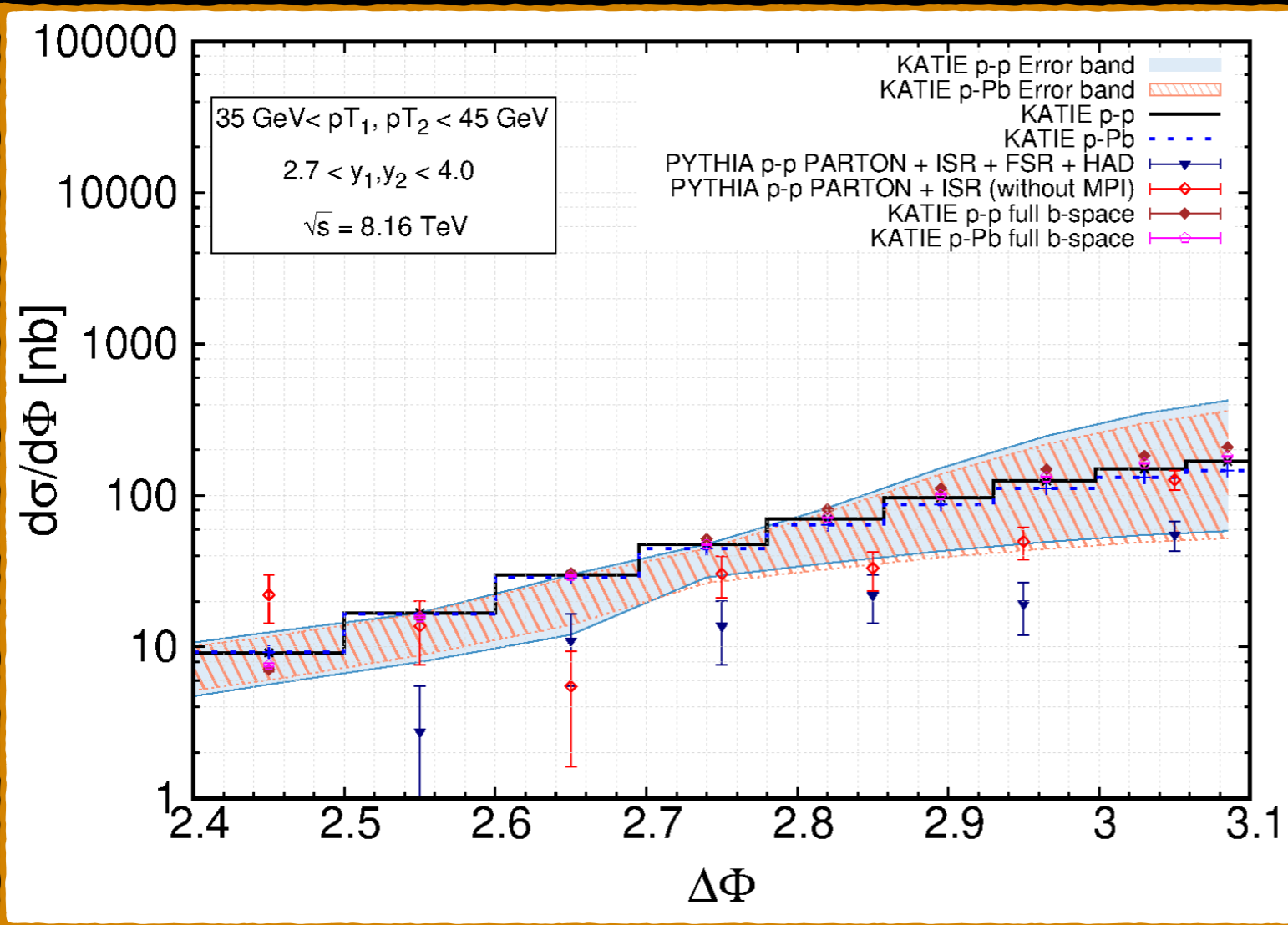
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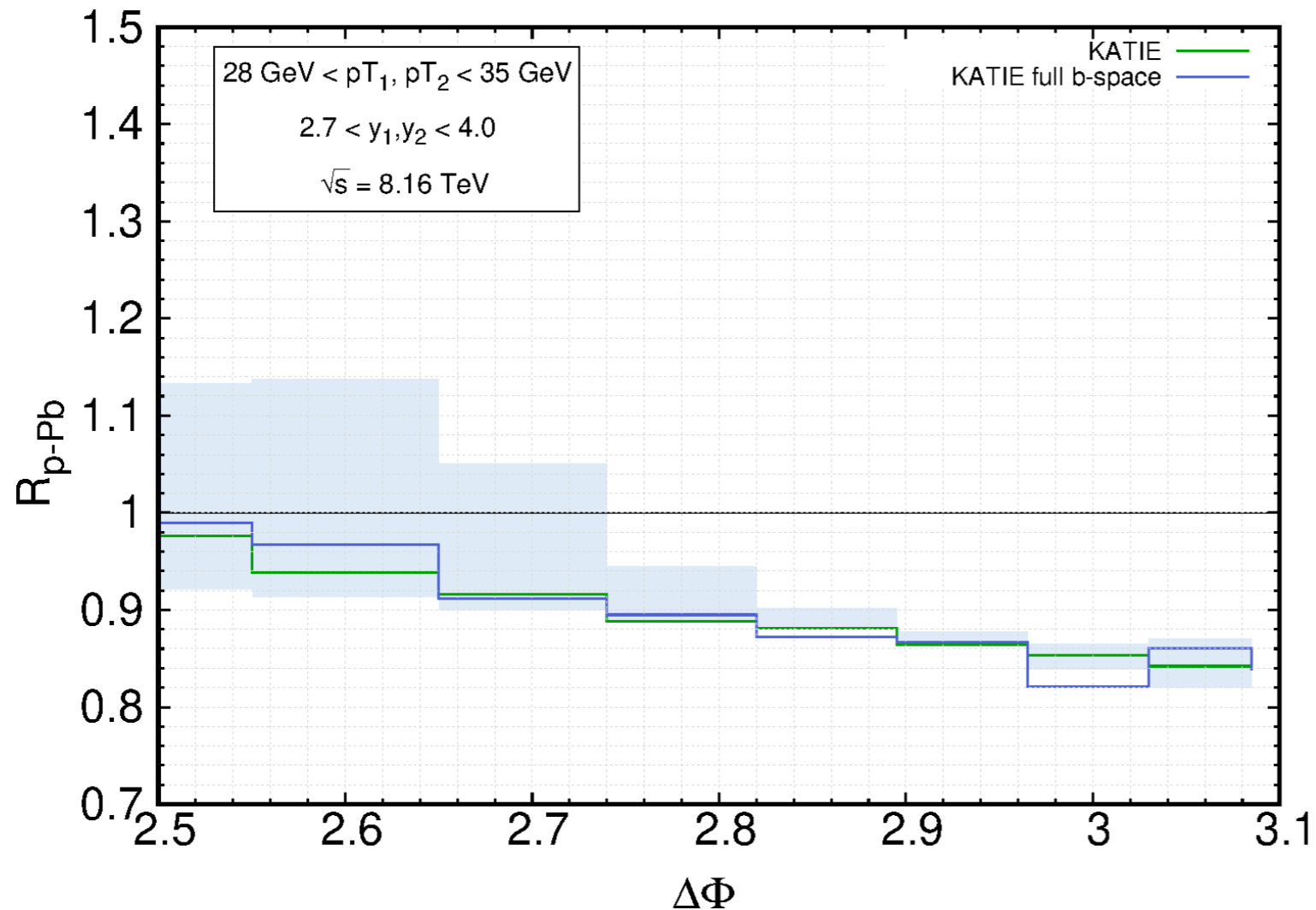
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Azimuthal correlations - nuclear modification ratio



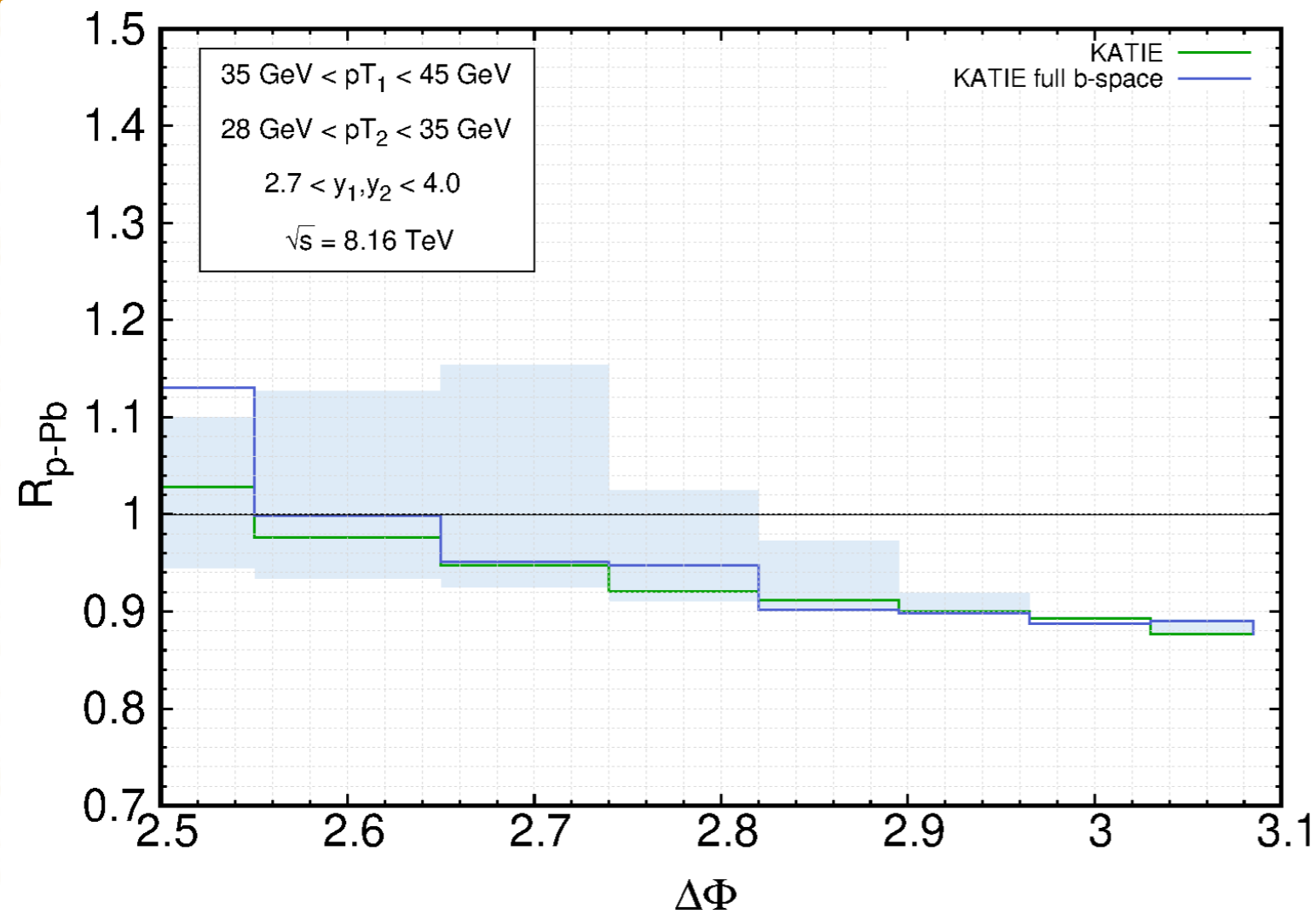
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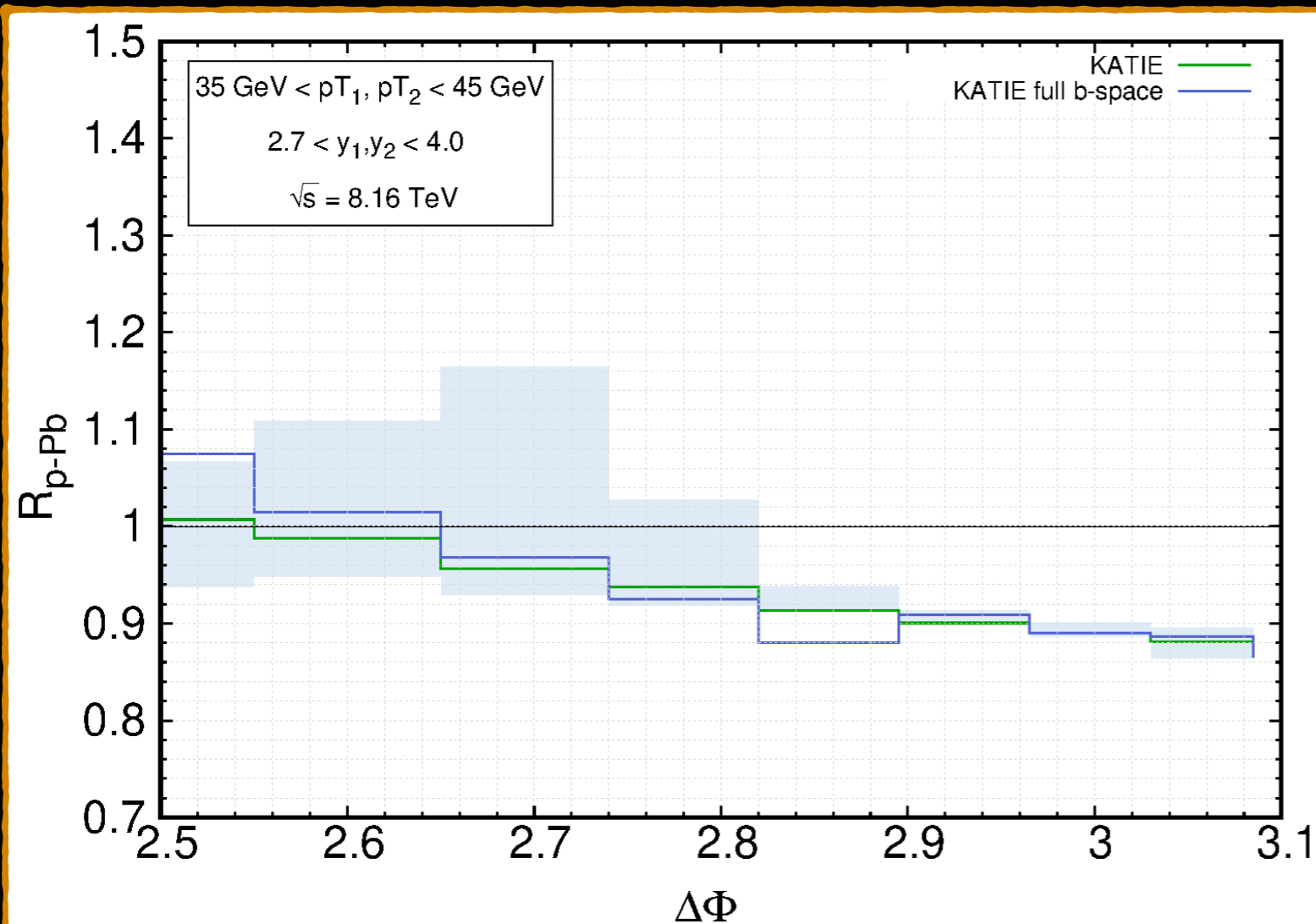
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SUMMARY

- Improved small- x TMD factorization (ITMD) is an approximation to CGC which is suitable for jet production at LHC
- ITMD has been implemented in parton level Monte Carlo programs: KaTie and LxJet
- despite proliferation of TMD gluon distributions, it is possible to calculate them with the data-driven input
- we included the Sudakov resummation in the Monte Carlo computations, including the full b -space resummation
- the Sudakov resummation is essential for a proper description of jet production

- we computed dijet azimuthal correlations for FoCal and ATLAS kinematics
- there are significant saturation effects present and they are not destroyed by the Sudakov form factors
- for the lower cuts on the jet transverse momenta the nonperturbative effects (estimated using Pythia) are large; it is important to study their dependence on the target

BACKUP

Small-x limit of TMD gluon distributions

$$\int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3 P^-} e^{ixP^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left[\hat{F}^{i-} \left(\xi^+, \vec{\xi}_T, \xi^- = 0 \right) \mathcal{U}_{C_1} \hat{F}^{i-} (0) \mathcal{U}_{C_2} \right] | P \rangle$$

x → 0 limit.

Dependence on x is only via the small-x evolution equations:

- BFKL (Balitsky-Fadin-Kuraev-Lipatov).
- BK (Balitsky-Kovchegov) and modifications
- JIMWLK (Balitsky-Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner)

Correspondence to CGC

Example:

$$\mathcal{F}_{qg}^{(1)} \sim \int \frac{d^2x_T d^2y_T}{(2\pi)^4} k_T^2 e^{-i\vec{k}_T \cdot (\vec{x}_T - \vec{y}_T)} \langle \text{Tr} [U(\vec{x}_T) U^\dagger(\vec{y}_T)] \rangle_x$$

DIPOLE
GLUON DISTRIBUTION

WILSON LINES

AVERAGE OVER
CGC COLOR SOURCES

$$U(\vec{x}_T) = \mathcal{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ A_a^-(x^+, \vec{x}_T) t^a \right\}$$

$$\langle \dots \rangle_x \rightarrow \frac{\langle | \dots |^2 \rangle}{\langle | 1 |^2 \rangle}$$

Intensively studied:

[D. Kharzeev, Y. Kovchegov, K. Tuchin, 2003]
 [B. Xiao, F. Yuan, 2010]
 [F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]
 [A. Metz, J. Zhou, 2011]
 [E. Akcakaya, A. Schafer, J. Zhou, 2012]
 [C. Marquet, E. Petreska, C. Roiesnel, 2016]
 [I. Balitsky, A. Tarasov, 2015, 2016]
 [D. Boer, P. Mulders, J. Zhou, Y. Zhou, 2017]
 [C. Marquet, C. Roiesnel, P. Taels, 2018]
 [Y. Kovchegov, D. Pitonyak, M. Sievert,
 T. Altinoluk, R. Boussarie, 2019]
 [R. Boussarie, Y. Mehtar-Tani, 2020]

BACKUP

Dijet correlations in pA collisions

Measurement of dijet azimuthal correlations in p+p and p+Pb. [ATLAS, Phys. Rev. C100 (2019)]

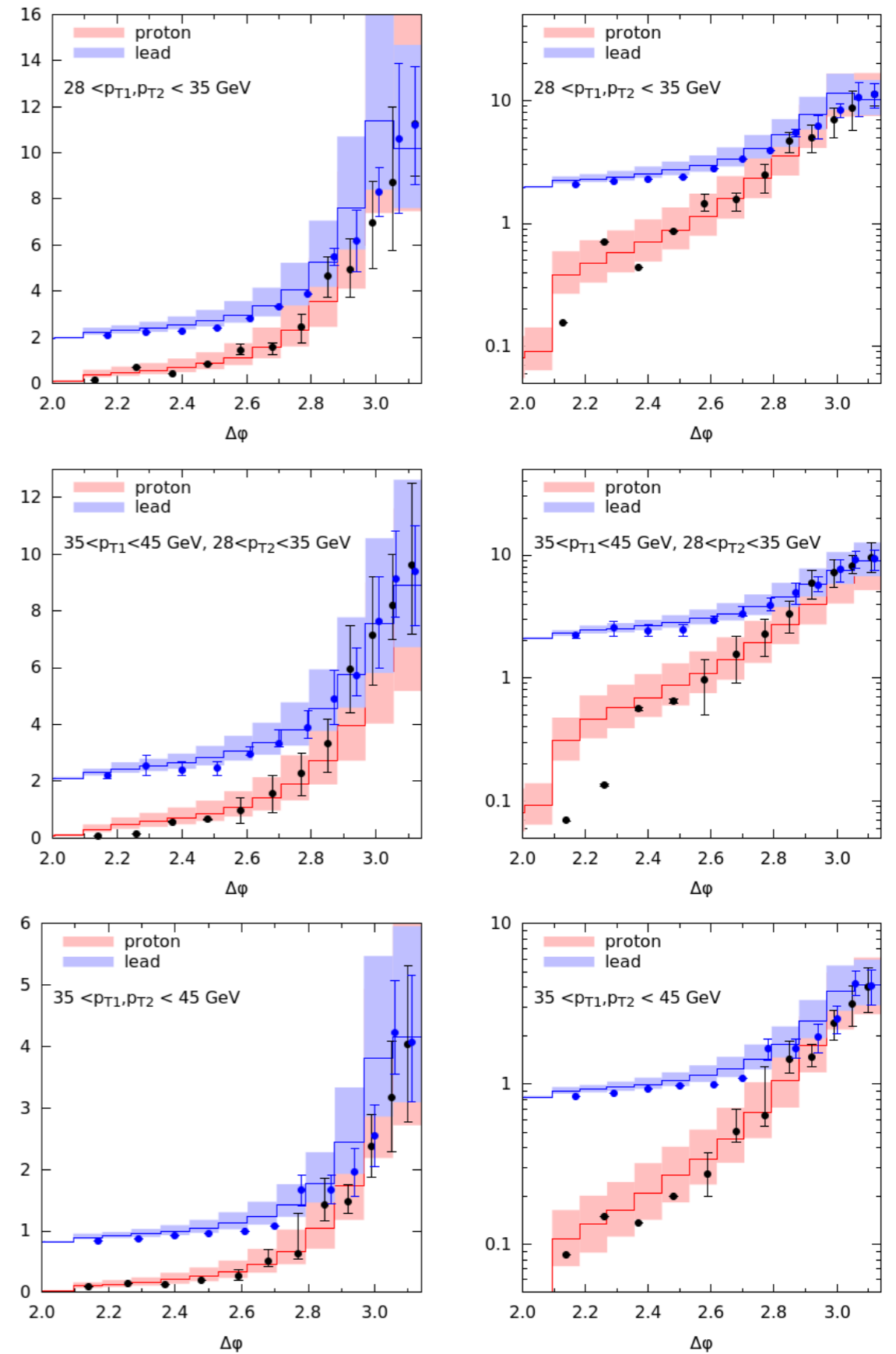
$\sqrt{s} = 5.02 \text{ TeV}$ rapidity: $2.7 < y_1, y_2 < 4.5$

$$C_{12} = \frac{1}{N_1} \frac{dN_{12}}{d\Delta\phi}$$

number of dijets (pointing to dN_{12})
azimuthal angle between jets (pointing to $d\Delta\phi$)
number of leading jets (pointing to N_1)

We study an interplay of saturation and Sudakov resummation vs the **shape** of C_{12} .

Good description of the broadening effects



A. Van Hameren, P. Kotko, K. Kutak, S. Sapeta, Phys. Lett. B795 (2019) 511

KATIE

<https://bitbucket.org/hameren/katie>

- parton level event generator, like ALPGEN, HELAC, MADGRAPH, etc.
- arbitrary processes within the standard model (including effective Higgs-gluon coupling) with several final-state particles.
- 0, 1, or 2 off-shell initial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids, or with TMDlib.
- a calculation is steered by a single input file.
- employs an optimization stage in which the pre-samplers for all channels are optimized.
- during the generation stage several event files can be created in parallel.
- event files can be processed further by parton-shower program like CASCADE.
- (evaluation of) matrix elements now separately available, including C++ interface.