Yukawa couplings

in the Grimus-Neufeld model

Vytautas Dūdenas[a], Simonas Draukšas[a], Thomas Gajdosik[a],

Uladzimir Khasianevich[b], Wojciech Kotlarski[b], Dominik Stöckinger[b]

Vilnius University[a] and TU Dresden[b]

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Standard Model (SM) + one fermionic singlet + two Higgs doublets

- is not a new idea: [G-N] W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.

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outline of the talk

- the Grimus-Neufeld model (GNM) Lagrangian
- the Grimus-Lavoura approximation
  - allowing the analytic prediction of neutrino masses
- determining Lagrangian parameters
  - from masses and mixings
    - in the Grimus-Lavoura approximation !
- the tiny seesaw scenario
  - with a new parametrization of the Yukawas
  - and approximate symmetries
- summary, progress, and plans

The GNM Lagrangian

- Gauge sector $\mathcal{L}_{\mathsf{G}}$ and Fermion-Gauge sector of the SM:
  - gauge group $U(1)_{\mathsf{Y}} \otimes SU(2)_{\mathsf{L}} \otimes SU(3)_{\mathsf{color}}$
  - gauge covariant derivative $D_\mu \psi$
  - and the Lagrangian $\mathcal{L}_{\mathsf{G\text{-}F}} = \sum_\psi \bar{\psi}\, i \not{D}\, \psi$ (1)

- Gauge-Higgs sector with the gauge covariant derivative $D_\mu \phi_a$
  and the Lagrangian $\mathcal{L}_{\mathsf{G\text{-}H}} = (D^\mu \phi_a)^\dagger (D_\mu \phi_a) - V(\phi_a)$ (2)

- Higgs sector: two Higgs doublets ϕ_a in the Higgs potential $V(\phi_a)$
  [H-ON] H. E. Haber and D. O'Neil, Phys. Rev. D **83** (2011) 055017 [arXiv:1011.6188 [hep-ph]]

- Fermion-Higgs sector with the Yukawa couplings (ignoring quarks)

$$\mathcal{L}_{\mathsf{F\text{-}H}} = -\bar{\ell}^0_{Lj}\, \phi_a\, Y^{\bar{a}}_{Ljk}\, e^0_{Rk} - \bar{\ell}^0_{Lj}\, \tilde{\phi}_{\bar{a}}\, \tilde{Y}^a_{Lj} N^0 + h.c. \qquad (3)$$

with the adjoint Higgs doublet $\tilde{\phi}_{\bar{a}} = \epsilon \phi^*_a = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} (\phi^+_a)^* \\ (\phi^0_a)^* \end{pmatrix} =: \begin{pmatrix} \phi^{0*}_{\bar{a}} \\ -\phi^-_{\bar{a}} \end{pmatrix}$

- Majorana sector with the Majorana singlet N^0: $D_\mu N^0 = \partial_\mu N^0$

The bare GNM has additional parameters (compared to the "original" SM)

- the (complex) singlet Majorana mass term M_R
- parameters in the Higgs sector – like a general 2HDM see [H-ON]
  - we use the Higgs basis: it fixes where the vev sits
    - * distinguishes the neutrino couplings between seesaw / loop
- the neutrino Yukawa coupling of the first Higgs doublet

  $(Y_N^{(1)})_j \; := \tilde{Y}^1_{Lj} = \frac{\sqrt{2}}{v}(M_D)_j \; \ldots$ the "Dirac mass" term
  - is responsible for the seesaw: $y^2 = \sum_j |(Y_N^{(1)})_j|^2 = \frac{2 m_s m_4}{v^2}$ (4)
- the Yukawa couplings of the second Higgs doublet

  $(Y_N^{(2)})_j \; := \tilde{Y}^2_{Lj}$ to lepton doublets and neutral fermionic singlet N_R
  - is essential for the loop mass $\Rightarrow$ we have a general 2HDM

  $(Y_E^{(2)})_{jk} := Y^2_{Ljk}$ to lepton doublets and charged lepton singlets ℓ_{Rj}
  - is not restricted by neutrino data at one loop

The GNM tree level for the neutral fermions

- the Yukawa coupling $(Y_N^{(1)})_j$ mixes the neutral leptons ν_j with N_R
- the mixing gives a $(3+1) \times (3+1)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix} \quad \text{with} \quad \begin{aligned} M_L &= 0_{3\times 3} \\ M_D^\top &= \frac{v}{\sqrt{2}} Y_N^{(1)} \end{aligned} \tag{5}$$

  – M_ν has rank 2 $\Rightarrow$ only two masses are non-zero

- diagonalizing M_ν

$$U_{(\nu)}^\dagger M_\nu = \mathrm{diag}(m_{o=\text{"zero"}}, m_{t=\text{"third"}}, m_{s=\text{"seesaw"}}, m_4) U_{(\nu)}^\top =: \hat{m} U_{(\nu)}^\top \tag{6}$$

with $m_o = m_t = 0$ by the unitary matrix

$$U_{(\nu)} = \begin{pmatrix} u_{eo} & u_{et} & c u_{es} & -is u_{es} \\ u_{\mu o} & u_{\mu t} & c u_{\mu s} & -is u_{\mu s} \\ u_{\tau o} & u_{\tau t} & c u_{\tau s} & -is u_{\tau s} \\ 0 & 0 & -is & c \end{pmatrix} \quad \text{where} \quad \begin{aligned} c^2 &= \frac{m_4}{m_4 + m_s} \\ s^2 &= \frac{m_s}{m_4 + m_s} \end{aligned} \tag{7}$$

  – with $u_{k\alpha}$ being a unitary 3×3-matrix

The GNM tree level for the neutral fermions

- from ${U_{(\nu)}}^\dagger M_\nu = \hat{m} {U_{(\nu)}}^\top$ and $(Y_N^{(1)})_k = \frac{\sqrt{2}}{v}(M_D^\top)_k$ we get

$$u^*_{ko}(Y_N^{(1)})_k = u^*_{kt}(Y_N^{(1)})_k = 0 \qquad (8)$$

- the two tree level massless "neutrinos" $\nu'_{o,t}$ are degenerate
- use the second Higgs coupling $(Y_N^{(2)})_k$ to distinguish them:

$$u^*_{ko}(Y_N^{(2)})_k = 0 \qquad \text{and} \qquad u^*_{kt}(Y_N^{(2)})_k =: d \neq 0 \qquad (9)$$

⇒ parametrize the Yukawa couplings as

$$(Y_N^{(1)})_k = i\, y\, u_{ks} \qquad (Y_N^{(2)})_k := d\, u_{kt} + i\, d'\, u_{ks} \qquad (10)$$

⇒ we can choose a basis for the neutrinos with simple Yukawas

– where the neutrino ν'_o does not couple to Higgses

  * with a 3HDM we could not guarantee the last feature

At one loop the GNM generates a loop induced mass $m_t \propto d^2$

determining the parameters of the GNM at tree level

- we can use physical masses and couplings
  - for the Higgs sector
    - Higgs masses m_h, m_H, m_A, $m_{H^\pm}$ and Higgs-Gauge couplings
  - for the neutrino sector (i.e. m_4 and $(Y_N^{(a)})_k$)
    - neutrino mixing matrix U_{PMNS}
    - neutrino mass differences Δm^2_{atm} and Δm^2_{sol}

!! but we have only a single mass difference at tree level:

$$\Delta m^2_{so} - \Delta m^2_{st} = \Delta m^2_{to} = 0 \quad \text{since} \quad m_o = m_t = 0 \tag{11}$$

inconsistent !

⇒ we need the one-loop level to determine parameters

Including one-loop predictions:

- renormalizing a Lagrangian expressed in mass eigenstates
  - needs a counter term $\delta^{\text{ct}}m$ for each non vanishing mass m
    * we have $m_3 > 0$ already at tree level ...

"Trick" of Grimus and Lavoura

[G-L] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

- renormalize the Lagrangian expressed in interaction eigenstates

  $\Rightarrow$ the counter term structure is simpler

- reduce the problem to the "light" neutrinos
  - get an effective 1-loop improved 3×3-mass matrix as a function of the model parameters
    * since the matrix is singular, it can be further reduced to a 2×2 matrix $\hat{\Sigma}$
  - the singular values are the light neutrino masses
    * in general this involves solving a 4^{th} order equation

neutrino mass eigenstates from the Grimus-Lavoura approximation

- the "heavy" state $\nu''_4 \sim \nu'_4$ with mass m_4 was "integrated out"
- the massless state $\nu''_o = \nu'_o$ with mass $m_o = 0$ was left untouched
  - since it does not couple to any Higgs
- the tree level states $\nu'_{t,s}$ were mixed into one-loop mass eigenstates $\nu''_{2,3}$
  - the masses m_t and m_s can be determined from the measured mass differences

$$\Delta m^2_{\text{sol}} = \Delta m^2_{21} \quad \text{and} \quad \Delta m^2_{\text{atm}} \approx |\Delta m^2_{31}| \tag{12}$$

[SoNO2018] P. F. de Salas *et al.*, Phys. Lett. B **782** (2018) 633

  - one has to be careful with normal or inverted hierarchy: $m_t \overset{?}{<} m_s$

- the transformation chain: [DGKKS2022] V. Dūdėnas *et al.*, JHEP 09 (2022) 174

$$\begin{pmatrix} 0^{0\ell}_{3\times 3} & \frac{v}{\sqrt{2}} Y^{(1)} \\ \frac{v}{\sqrt{2}} Y^{(1)T} & M \end{pmatrix} \overset{\tilde{V}}{\to} \begin{pmatrix} 0^{1\ell} & 0^{1\ell} & 0^{1\ell} & 0^{1\ell} \\ 0^{1\ell} & 0^{0\ell} & 0^{0\ell} & 0^{0\ell} \\ 0^{1\ell} & 0^{0\ell} & 0^{0\ell} & i\frac{vy}{\sqrt{2}} \\ 0^{1\ell} & 0^{0\ell} & i\frac{vy}{\sqrt{2}} & M \end{pmatrix} \overset{\tilde{S}}{\to} \begin{pmatrix} 0^{1\ell} & 0^{1\ell} & 0^{1\ell} & 0^{1\ell} \\ 0^{1\ell} & & & 0^{1\ell} \\ 0^{1\ell} & & \hat{\Sigma} & 0^{1\ell} \\ 0^{1\ell} & 0^{1\ell} & 0^{1\ell} & m_4 + 0^{1\ell} \end{pmatrix} \overset{\tilde{R}}{\to} \hat{m}$$

$$\begin{array}{cccc} \| & & & \| \\ M^F_\nu & & & \tilde{U}^* M^F_\nu \tilde{U}^\dagger \\ \nu_\alpha := \{\nu_i, N\} & \nu'_\alpha & \approx \nu'_\alpha & \nu''_\alpha \\ Y^{(i)} & Y^{(i\prime)} & \approx Y^{(i\prime)} & Y^{(i\prime\prime)} \end{array}$$

values for the seesaw

- the physical light masses are determined (here in normal hierarchy)

$$m_o = m_1 = 0 \ , \quad m_2 = \sqrt{\Delta m^2_{\text{sol}}} \ , \quad m_3 = \sqrt{\Delta m^2_{\text{atm}}} \tag{13}$$

  – but m_4 is a free parameter

  – implementing this model in FlexibleSUSY exhibits an instability:

    * one loop Higgs masses are not consistent with tree-level mass values: for stable loop level Higgs masses we are limited to $m_4 < 10^6$GeV

- using the seesaw relation $y^2 = \sum_j |(Y_N^{(1)})_j|^2 = \frac{2 m_s m_4}{v^2}$ (4)

  – we see, that y becomes a small parameter !

⇒ motivates the definition of the tiny seesaw scenario $y \leq 10^{-7}$ (14)

sidestep: what happens when $y \to 0$ (i.e. $(Y_N^{(1)})_j \to 0$) ?

- $\mathcal{L}_{\text{GNM}}$ gains an additional Z_2 symmetry : $\phi_2 \leftrightarrow -\phi_2 \, , \quad N^0 \leftrightarrow -N^0$ (15)

⇒ the tiny seesaw scenario has an approximate Z_2 symmetry features of the tiny seesaw scenario

- the seesaw scale becomes smaller than the EW scale : $m_4 < v$ (16)
- the loop inducing couplings d and $|d'|$ become large
  - d is determined by the determinant of the 2×2 mass matrix $\hat{\Sigma}$

$$m_2 m_3 = m_t m_s = \det[\hat{\Sigma}] = d^2 m_3^{\text{tree}} \Lambda \tag{17}$$

with the loop function of the neutral Higgses

$$\Lambda = \frac{m_4}{32\pi^2}[B_0(0, m_4^2, m_A^2) - B_0(0, m_4^2, m_H^2)] \propto \frac{m_4}{32\pi^2}\lambda_5 \tag{18}$$

  - $|d'|$ is determined by a simple 2^{nd} order equation for $|\frac{d'}{d}|$
    - instead of the 4^{th} order equation in the general case
      [DG2021] V. Dūdėnas and T. Gajdosik, Acta Phys. Polon. Supp. **15** (2022) no.2, 1

- it allows a more convenient parametrization of the Yukawa couplings
  - determined by the elements of the 2×2 rotation matrix $\hat{R}$ that diagonalizes $\hat{\Sigma}$

Parametrizing the Yukawas with the rotation matrix $\hat{R}$

- using Murnaghan's parameterization

$$\hat{R} = \begin{pmatrix} R_{22} & -R_{32}^{*} e^{i\phi_R} \\ R_{32} & R_{22}^{*} e^{i\phi_R} \end{pmatrix} \text{, with} \quad \begin{array}{rcl} R_{22} & = & \cos r \; e^{i\omega_{22}} \\ R_{32} & = & \sin r \; e^{i\omega_{32}} \end{array} \tag{19}$$

  we parametrize the Yukawa couplings in mass eigenstates as

$$Y_N^{(1)} = \frac{i}{e^{i\phi_R}} \sqrt{\frac{2 m_3 m_4}{|z| v^2}} (0, -R_{32}, R_{22}) \tag{20}$$

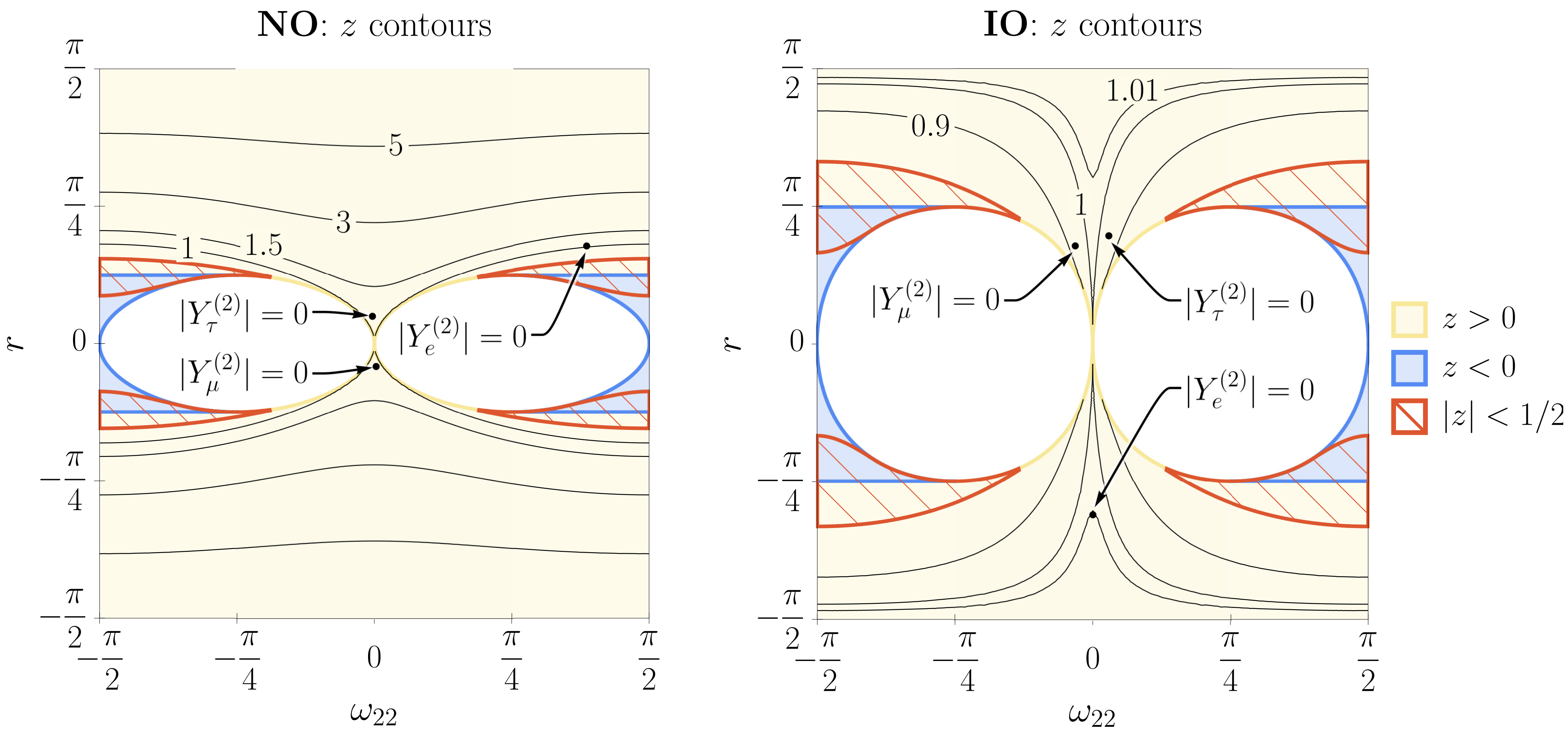
$$Y_N^{(2)} = \text{sign}(\Lambda) \sqrt{\frac{m_2}{|z\,\Lambda|}} (0, R_{22}, t_{32} R_{32}) \tag{21}$$

  where $t_{32} = \frac{m_3}{m_2}$ and $z = \cos^2 r \; e^{2i\omega_{22}} + t_{32} \sin^2 r \; e^{2i\omega_{32}}$ (22)

  - z has to fulfill the constraint $|z| = \frac{m_3}{m_3^{\text{tree}}}$ and parametrises the relative loop correction for the heaviest light neutrino
  - we replace the previous free parameters by r and ω_{22}

- this rewriting simplifies the numerical input for FlexibleSUSY and gives a minimal parameter space for the model

parameter space for Lepton flavor violation



- in the white area the constraint for z, eq. (22), cannot be fulfilled
- points where the flavour Yukawa couplings vanish are shown:
  - in these points the corresponding charged lepton does not couple to $H^\pm$

Summary of the GNM

- the GNM extends the SM with a Higgs doublet and a Majorana singlet
  - the neutrinos become Majorana particles
    - the lightest neutrino stays mass less at one loop
  - neutrino oscillations determine the neutrino Yukawa coupling
    - allows predictions of Lepton Flavor violating processes
    - the other possible new Yukawa couplings stay free parameters
  - a large seesaw scale causes numerical problems in FlexibleSUSY
- An approximate Z_2 symmetry defines the tiny seesaw scenario
  - motivates the suppression of the undefined (free) new Yukawas
  - stabilizes the numerical renormalization in FlexibleSUSY
  - the explicit but small breaking parameters y and λ_5 interpolate between seesaw and radiative neutrino masses

$\Rightarrow$ the GNM can be seen as generalization of Dark matter models
  - in terms of predicting Lepton Flavor violating processes

Progress in the last four years

- implementation in FlexibleSUSY is stable regarding neutrinos
  - for the large seesaw a high precision package is needed
    - Higgses have to be taken at tree-level
  - tiny seesaw scenario solves also this problem
- the new definition for the Yukawa couplings
  - simplifies the presentation of the parameter space:
    - clear boundaries, numerically simple
    - no doubling of Yukawa coupling values by different parameters
    - no reverse engineering of input parameters
- Phenomenological analysis of Lepton Flavour violating processes
  - provides limits for the Higgs potential
    - see talk by V. Dūdenas

Plans

- Extending the Phenomenological analysis of Lepton Flavour violation
  - covering the "corners" in the Higgs potential

- fully renormalizing the model
  - see talk by S. Draukšas
    - some success already, but not finished

- Exploring the Cosmology connection
  - lifetime of the particles in the GNM
  - could there be a Dark Matter candidate ?
  - what about Leptogenesis ?
    - mostly for having themes for students ...

? What changes if we get a third Higgs doublet ?

Thank you

for discussion

and comments

and of course for the conference! ☺