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On-Shell Renormalization of Scalar Sectors

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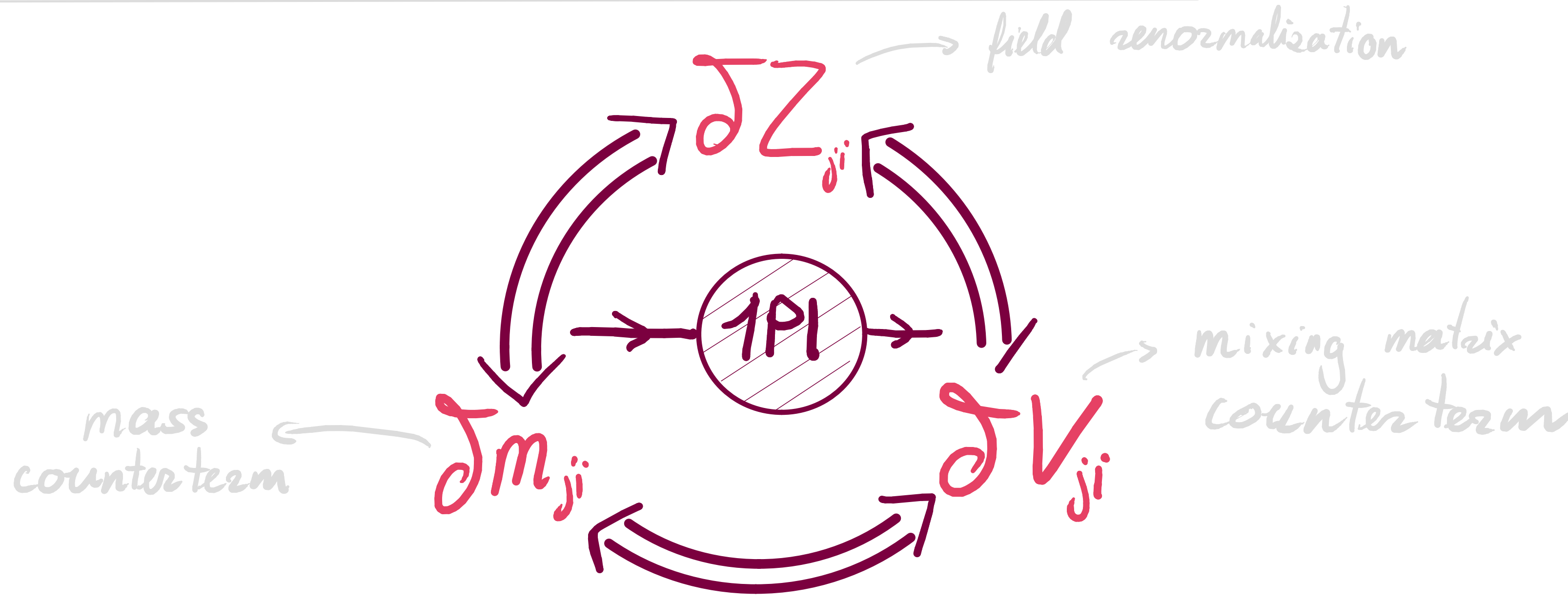
Fermions I



Renormalization and mixing:

$$\psi^0_j \longrightarrow Z_{jk}\,\psi_k$$

flavour indices

$$m^0_{ji} \longrightarrow m_i + \delta m_{ji}$$

Fermions II

On-Shell conditions:

spinor $\not p u_i = m_i$

$$\boxed{\Sigma^R_{ji}(\not p) u_i = 0} \xrightarrow{\text{1-loop}} \left[(m_i^2 - m_j^2)\,\delta Z_{ji} \boxed{- m_j \delta m_{ji} - \delta m^\dagger_{ji} m_i}\right] u_i = -(\not p + m_j)\Sigma_{ji}(\not p)\, u_i$$

$(m_i^2 - m_j^2)$?

self-energy

Includes tadpoles!

- δZ^A and δm are degenerate !
- δZ^H is as usual

$$(\not p + m_j)\Sigma_{ji}(\not p) u_i \Big|^{\text{Hermitian}}_{UV} \sim \frac{1}{\epsilon_{UV}} \times \left\{ m_i,\ m_j,\ 2 m_i m_j,\ m_i^2 + m_j^2 \right\}$$

from 1-loop PV functions

↳ associated with δm!

NO $m_i^2 - m_j^2$!

↳ $\dots \Big|^A_{UV} \sim \frac{1}{\epsilon_{UV}} \times \{ m_i^2 - m_j^2 \}$

Fermions III

Nielsen Identity:

$$\boxed{\partial_\xi \Sigma_{ji}(\not{p}) = \left[\Lambda\Sigma + \Sigma\bar{\Lambda}\right]_{ji}}$$

→ All orders

→ Analogous for Scalars

→ $\overset{(-)}{\Lambda} = 0$ @ tree-level

→ $\overset{(-)}{\Lambda} = \overset{(-)}{\Lambda}(\not{p})$

$\Downarrow$ 1-loop

$$\partial_\xi \Sigma_{ji}(\not{p}) = \left[\Lambda(\underbrace{\not{p}-m_i}_{=0}) + (\not{p}-m_j)\bar{\Lambda}\right]_{ji}$$

$\Downarrow$ OS

$$(\not{p}+m_j)\,\partial_\xi \Sigma_{ji}(\not{p})\,u_i = (m_i^2-m_j^2)\,\bar{\Lambda}\,u_i$$

Fermions IV

$$\delta Z^A_{ji}\, u_i \equiv -(\not{p}+m_j)\,\Sigma_{ji}\, u_i \text{ „} + H.c.\text{"}\Big|_{(m_i^2-m_j^2)}$$

$$\delta m = \delta m(\Sigma, \delta Z^A)$$

1-loop!

↳ Explicit computation @ 1-loop for SM and 2HDM fermions

- Can be shown that the same „logic" is valid to **ALL ORDERS** in perturbation theory!

Mixing Renormalization I

- What happens to δV_{ji}? (mixing matrix counterterm)

standard approach: $\delta V_{ji} \sim \left[\delta Z^A V - V \delta Z^A\right]_{ji}^{UV} \neq 0$

But now $\left[\delta Z^A_{ji}\right]^{UV} = 0$ and $\left[\delta m_{ji}\right]^{UV} \neq 0$

$$\boxed{\left[\delta V_{ji}\right]^{UV} = 0}$$

- UV divergences stay in the mass term

- Is it needed to renormalize mixing matrices?

Mixing Renormalization II

Finite rotation can give:

$$\cancel{V_{ji}} + \delta V_{ji}'$$

Standard:

$$\boxed{m^0_{ji}} \xrightarrow{\text{diag.}} \boxed{\begin{matrix} m^0_{ii} \\ V^0_{ji} \end{matrix}} \xrightarrow{\text{Renorm.}} \boxed{\begin{matrix} m_i + \delta m_i \\ V_{ji} + \delta V_{ji} \\ \delta Z^A_{ji} \end{matrix}}$$

Basis dependence!

UV

Proposed:

$$\boxed{m^0_{ji}} \xrightarrow{\text{Renorm.}} \boxed{\begin{matrix} m_{ji} + \delta m_{ji} \\ \cancel{V + \delta V} \\ \delta Z^A_{ji} \end{matrix}} \xrightarrow{\text{diag.}} \boxed{\begin{matrix} m_i + \delta m'_{ji} \\ V_{ji} \; \cancel{+ \delta V} \\ \delta Z^{A'}_{ji} \end{matrix}}$$

UV

NO counterterm!

- Renorm. and finite rotations commute only if:

$$\delta V_{ji} = 0 \quad \text{and} \quad \delta m_{\boxed{ji}} \; !$$

Can we apply this to Scalar Sectors?

Scalars I

Similar to fermions @ 1-loop

$$\Pi^R_{ji}(p^2) = \Pi_{ji}(p^2) + \delta Z^\dagger_{ji}(p^2 - m_i^2) + (p^2 - m_j^2)\delta Z_{ji} - \delta m^2_{ji}$$

↳ Self-energy

$$\Downarrow \; \partial_\xi$$

$$\partial_\xi \Pi_{ji}(p^2) = (\Lambda \Pi)_{ji} + (\Pi \Lambda^\dagger)_{ji} \xRightarrow{\text{1-loop}} \partial_\xi \Pi_{ji}(p^2) = \Lambda_{ji}(\underbrace{p^2 - m_i^2}_{=0}) + (p^2 - m_j^2)\Lambda^\dagger_{ji}$$

$$\xRightarrow{\text{OS}} \partial_\xi \Pi_{ji}(m_i^2) = (m_i^2 - m_j^2)\Lambda^\dagger_{ji}$$

$$\Rightarrow \boxed{\begin{aligned} \delta Z^A_{ji} &= -\tfrac{1}{2}\left(\Pi_{ji}(m_i^2) + \Pi_{ji}(m_j^2)\right)\Big|_{m_i^2 - m_j^2} \\ \delta m &= \delta m(\Pi, \delta Z^A) \end{aligned}}$$

→ Extend to all orders, but....

↳ Explicit computation @ 1-loop in 2HDM

Scalars II

(preliminary)

- Physical states MIX with Goldstones and longitudinal modes
- NI is for the reduced functional

Reduced functional

$$\tilde{\Gamma} = \Gamma^R - \int dx\, \mathcal{L}^{GF}$$

Gauge-fixing (Faddeev-Popov)

Always renormalized („classical")

1-loop

$$\Pi^R_{HG} = \Pi_{HG} + \delta Z^T_{HG}(p^2 - m_H^2) + p^2 \delta Z_{HG} - \delta m^2_{HG}$$

„Higgs" „Goldstone"

from the „Higgs potential"

propagator is still $\frac{i}{p^2 - m_G^2}$

- Renormalize @ $p^2 = m_H^2$ and $p^2 = 0$
- $m_H^2 \, \delta Z^A_{HG} - \delta m^2_{HG} = \frac{1}{2}\left(\Pi_{HG}(0) + \Pi_{HG}(m_H^2)\right)$
- No $m_H^2 - m_G^2$!

Scalars III

(preliminary)

$$\partial_\xi \Pi^R_{HG} = \Lambda_{Hk} \tilde{\Pi}_{kG} + \tilde{\Pi}_{Hk} \Lambda^+_{kG} + \partial_\xi \Pi^{GF} \text{ @ tree-level}$$

"k" also includes longitudinal modes! $\Rightarrow p^\nu \tilde{\Pi}'_{ZG}$ (e.g. Z^μ_L)

not diagonal!

$\Downarrow$ 1-loop

$$\partial_\xi \Pi^R_{HG} = \Lambda_{HG} p^2 + p^2 \Lambda'_{HZ} \tilde{\Pi}'_{ZG} + (p^2 - m_H^2)\Lambda^+_{HG} + \cancel{p^2 \tilde{\Pi}_{HZ} \Lambda^+_{ZG}} \to 0$$

$\Downarrow$ OS

$$m_H^2 \partial_\xi \delta Z^A_{HG} - \partial_\xi \delta m^2_{HG} = -\frac{m_H^2}{2}\left(-\Lambda^+_{HG}(0) + \Lambda_{HG}(m_H^2) + \Lambda'_{HZ}(m_H^2)\tilde{\Pi}'_{ZG}\right)$$

m_Z

not very distinct...

+ terms from longitudinal modes

Scalars IV

(preliminary)

the real **PROBLEM!**

Higher orders depend on all previous orders!

exclude tree-level

(order)

$$\partial_\xi\left[(m_i^2 - m_j^2)\,\delta Z_{ji}^{(n)} - \delta m_{ji}^{(n)}\right] = -\partial_\xi\left[\Gamma(m_i^2) + \boxed{\Gamma^{(>0)}(m_i^2)\,\delta Z}\right]_{ji}^{(n)}$$

physical indices

short-hand notation; includes c.t.'s up to order n

- includes non-physical parts
  - $\delta Z_{ss'}$ only for scalars, but $\partial_\xi \Gamma$ also includes longitudinal modes!

↳ STI? Other nice cancelations?

Can $(m_i^2 - m_j^2)$ and $\partial_\xi \delta m_{ji}^2 = 0$ be preserved to all orders?

Summary / Outlook

- 1-loop physical states OK
  - δZ^A_{ji} as coeff. of $m_i^2 - m_j^2$; also UV finite
  - Solve for δm_{ji}
  - $\delta V_{ji} = 0$, $\partial_\xi \delta m_{ji} = 0$
  - Universal, process-independent...
- 1-loop non-physical : no $m_i^2 - m_j^2$ => How to select δZ^A?
  - δm^2 and δZ^H ensure OS and $\partial_\xi \delta m^2 \neq 0$?

! Not obvious n^{th} order extension
  - depends on all previous orders + longitudinal modes

$$\boxed{\text{OUTLOOK: ALL ORDER EXTENSION}}$$