# Impact of nuclear effects in the measurement of neutrino oscillation parameters

Davide Meloni

ROMA TRE — Dipartimento di Fisica "E. Amaldi"

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based on E. F. Martinez& DM, 1010.2329,PLB

**Main motivation of this exercise**

comparing Fermi gas (FG) and advanced nuclear model predictions for physically interesting neutrino observables

- this is relevant because many MonteCarlo codes, used to study the sensitivity to still *unknown* parameters at future $\nu$ facilities are based on FG
- impossible to discuss all recent nuclear models

$\Downarrow$

focus the attention on *three* different approaches in the *quasi-elastic* regime

# *Global 3 $\nu$ fit to the world neutrino data*

At $1\sigma$ $(3\sigma)$                    G.L. Fogli et al., arXiv:1106.6028

- well known parameters

$$\Delta m_{sol}^2 = 6.99 - 8.18 \times 10^{-5} \; eV^2 \, ,$$

$$\Delta m_{atm}^2 = 2.06 - 2.67 \times 10^{-3} \; eV^2$$

$$\sin^2 \theta_{23} = 0.34 - 0.64 \; \begin{pmatrix} +0.054 \\ -0.046 \end{pmatrix} \, ,$$

$$\sin^2 \theta_{12} = \begin{cases} 0.259 - 0.359 & \text{(old fluxes)} \, , \\ 0.265 - 0.364 & \text{(new fluxes)} \, , \end{cases}$$

- poor and unknown parameters

$$\sin^2 \theta_{13} = \begin{cases} 0.001 - 0.044 & \text{(old fluxes)} \, , \\ 0.005 - 0.05 & \text{(new fluxes)} \, , \end{cases}$$

$$\delta_{\text{CP}} \in [0, \; 360] \, (\text{unknown})$$

$$\text{sign}(\Delta m_{31}^2) \qquad \text{octant of } \theta_{23} \qquad \text{Majorana or Dirac Neutrinos?}$$

# *Great interest on $\theta_{13}$ and $\delta$*

The appearance neutrino oscillation probability ($\alpha \neq \beta$)

$$P_{\alpha\beta}^{\pm}(\theta_{13}, \delta) = A_{\pm} \sin^2 2\,\theta_{13} \;\pm\; B_{\pm} \sin\delta \sin 2\,\theta_{13} \;+\; Z$$

where

- $\pm$ referes to neutrino and antineutrino
- $A_{\pm}, B_{\pm}$ and $Z$ functions of "known" parameters

Many future experiments will look for a precise measurement of $\theta_{13}$.

Large $\theta_{13}$ means good chance to reveal the CP violation in the leptonic sector

One needs to control:

- flux composition
- detector response
- nuclear cross sections

## Elementary charged current neutrino-nucleon cross section

we focus on charged current interactions at neutrino energies around 1 GeV (CCQE)



$$\nu_l + n \rightarrow l^- + p$$
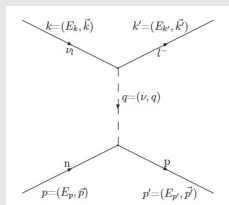
- effective lagrangian:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{\ell} \gamma^{\mu} (1 - \gamma_5) \nu \, J_{\mu}^{had}$$

- Feynman amplitude:

$$\mathcal{M} = \frac{G_F \, V_{ud}}{\sqrt{2}} \underbrace{\bar{\ell}(k^{'}) \gamma_{\mu} (1 - \gamma_5) \nu(k)}_{leptonic\,current} \underbrace{< N(p^{'})|J_{\mu}^{had}|N(p) >}_{hadronic\,current}.$$

## *Differential cross section*

- we need a parametrization of the hadronic matrix element

$$< N(p^{'})|J^{\mu}|N(p) > \quad =$$

$$\bar{u}(p') \left[ \underbrace{\gamma^{\mu} F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2M_N} F_2(Q^2)}_{hadronic\ vector\ current\ J_V^{\mu}} \quad + \quad \overbrace{\gamma^{\mu}\gamma^5 F_A(Q^2)}^{hadronic\ axial\ current\ -J_A^{\mu}} \right] u(p)$$

- $F_{1,2}(Q^2)$ are *vector* form factors
- $F_A(Q^2)$ is the *axial* form factor, depending on a mass parameter: *the axial mass* $m_A$

in principle we should also consider an additional form factor $F_P(Q^2)$ but its contribution to the cross section is negligible for $e$ and $\mu$ (but not for $\tau$'s)
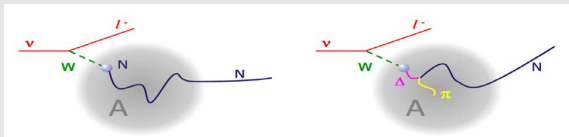
*Scattering on nuclei in the quasi-elastic (QE) regime*

$$\nu_l + A \to l^- + X$$

- the description of the interaction is more complicated because nucleons are bound

  I see two classes of problems:

1- other processes make difficult to experimentally measure "pure" CCQE



⇑ ⇑                                    ⇑ ⇑

*pure* CCQE event              *fake* CCQE if $\pi$ is not detected

2- theoretical calculations should rely on assumptions

## *The Relativistic Fermi Gas model (RFG)*

### Problem #2 (assuming experiments are measuring pure CCQE)

- simplest model to account for nuclear effects **R. A. Smith and E. J. Monitz, Nucl. Phys. B 43, 605 (1972)**
- target state as a superposition of non-interacting neutron (and proton) Fermi gases with momentum distribution
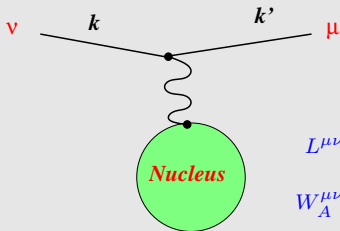
$$\frac{d^2\sigma}{d\Omega dE_l} = \frac{G_F^2 |V_{ud}|^2}{16\pi^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L_{\mu\nu} W_A^{\mu\nu}$$
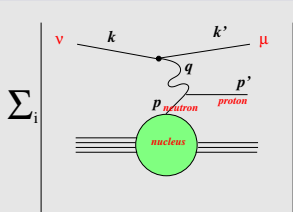
$$L^{\mu\nu} = 8\left[k'_\mu k_\nu + k'_\nu k_\mu - (k \cdot k') - i\varepsilon_{\mu\nu\alpha\beta} k'^\beta k^\alpha\right]$$

$$W_A^{\mu\nu} = \sum_X \langle 0|J_A^{\mu\dagger}|X\rangle\langle X|J_A^\nu|0\rangle\delta^{(4)}(p_0 + q - p_X)$$

- $W_A^{\mu\nu}$ is the *nuclear* hadronic tensor
- we need a set of simplifying assumptions to compute $W_A^{\mu\nu}$:

the Impulse Approximation

target nucleus seen as a collection of individual nucleons: $J_\mu \to \sum_i j_\mu^i$

## The Relativistic Fermi Gas model (RFG)



$W_A^{\mu\nu}$ same structure of the *free* one $W^{\mu\nu}$ in terms of form factors

$$\Downarrow$$

$$W_A^{\mu\nu} = \int d\mathbf{p}\, f(\mathbf{p}, \mathbf{q})\, W^{\mu\nu}$$

$$f(\mathbf{p}, \mathbf{q}) = v_{rel}^{-1} \cdot \underbrace{n_i(\mathbf{p})}_{momentum\ distribution} \cdot \underbrace{[1 - n_f(\mathbf{p} + \mathbf{q})]}_{unoccupation\ probability} \quad .$$

$$\begin{cases} v_{rel} & = f(E_{\mathbf{p}}) \quad \text{where} \quad E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m_n^2} - \epsilon_b \\ n_i(\mathbf{p}) & = \theta(\mathbf{p_F} - \mathbf{p}) \\ 1 - n_f(\mathbf{p} + \mathbf{q}) & = \theta(\mathbf{p} + \mathbf{q} - \mathbf{p_F}) \end{cases}$$
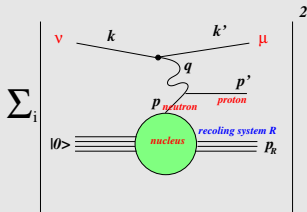
The model depends on two parameters:

- $\epsilon_b$ = effective binding energy
- $\mathbf{p_F}$ = Fermi momentum

| Nucleus | $p_F$ (MeV) | $\epsilon_b$ (MeV) |
|---------|-------------|--------------------|
| $^{12}C$ | 221 | 25.6 |
| $^{16}O$ | 225 | 26.6 |

## The Spectral Function Approach

**Benhar et al., Phys.Rev.D72:053005,2005**



scattered nucleons and recoiling system $\mathcal{R}$ evolve independently of one another: $|X\rangle \to |i, p'\rangle \otimes |\mathcal{R}, p_{\mathcal{R}}\rangle$
(no Final State Interactions)

$$\langle 0|J^\mu|X\rangle \quad \sim \quad \langle 0|R, \mathbf{p}_R; N, -\mathbf{p}_R\rangle \sum_i \langle -\mathbf{p}_R, N|j_i^\mu|x, \mathbf{p}'\rangle$$

$$\frac{d^2\sigma_{IA}}{d\Omega dE_l} = \int d^3 p \, dE \, P(\mathbf{p}, E) \frac{d^2\sigma_{\text{elem}}}{d\Omega dE_l}$$

$$P(E, \mathbf{p}) \sim \sum_R |\langle 0|R, -\mathbf{p}; N, \mathbf{p}\rangle|^2$$

- $P(\mathbf{p}, E)$ is the $\boxed{\text{target spectral function}}$ : probability distribution of finding a nucleon with momentum $\mathbf{p}$ and removal energy $E$ in the target nucleus

**it encodes all the informations about the nuclear dynamics**

- overwhelming evidence from electron scattering that the energy-momentum distribution of nucleons in the nucleus is quite different from that predicted by Fermi gas
- the most important feature is the presence of strong nucleon-nucleon (NN) correlations (virtual scattering processes leading to the excitation of the participating nucleons to states of energy larger than the Fermi energy)

**spectral function extends to $|\mathbf{p}| \gg p_F$ and $E \gg \varepsilon$**
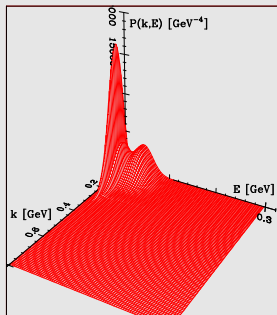


momentum distribution

$$n(\mathbf{p}) = \int dE \, P(\mathbf{p}, E) \Longrightarrow$$
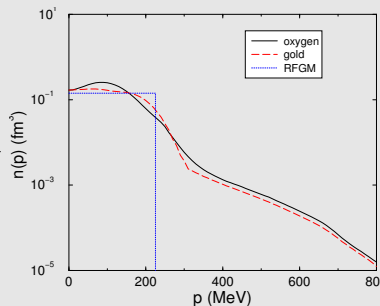
## The Random Phase Approximation (RPA)

**model based on**
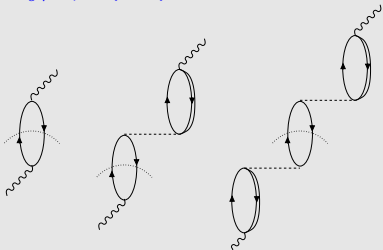**M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C81, 045502 (2010)**
**M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C80, 065501 (2009).**

$$\frac{d^2\sigma_{IA}}{d\Omega dE_l} \quad \propto \quad \sum_i K_i R_i$$

- $K_i$ = kinematical factors
- $R_i$ = response functions,

$$R(\omega, q) = -\frac{\mathcal{V}}{\pi} \operatorname{Im}[\Pi(\omega, q, q)].$$

To lowest order the QE cross section is given by the terms in $R^{NN}$ [$R_\tau^{NN}$ (isovector interaction), $R_{\sigma\tau}^{NN}$ (isospin spin-transverse interaction)]



**Lowest-order contribution from $R^{NN}$, $R^{N\Delta}$ and $R^{\Delta\Delta}$.**
**Wiggly lines represent the external probe,**
**solid lines correspond to the propagation of a nucleon (or a hole),**
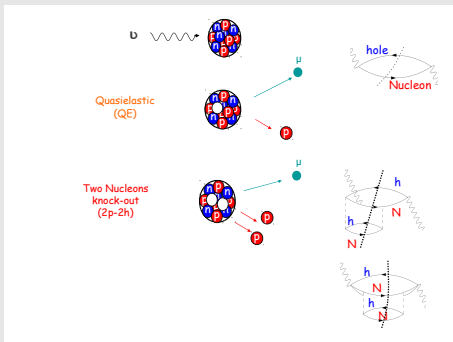**double lines to the propagation of a $\Delta$**
**and dashed lines to an effective interaction between nucleons and/or $\Delta$s.**

**Dotted lines show which particles are placed on-shell**

## The Random Phase Approximation (RPA 2p-2h)

- large contribution from two-body current?!

  M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C80, 065501 (2009) [arXiv:0910.2622 [nucl-th]].



- wiggly lines represent the external probe,
- solid lines correspond to the propagation of a nucleon (or a hole),
- double lines to the propagation of a $\Delta$
- dashed lines to an effective interaction between nucleons and/or $\Delta$s.
Dotted lines show which particles are placed on-shell

## The $\nu$-nucleus cross sections ($\nu A \rightarrow \mu X$)

- some of the *qualitative* impacts of several nuclear models on the $\nu$ observables can already be understood at the "cross section" level

- however the *quantitative* differences should be carefully evaluated
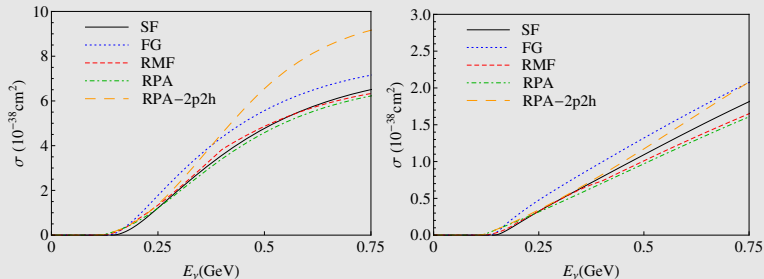


- FG overstimates the $\bar{\nu}$ xsection over the whole QE energy regime
- $m_A \sim 1$ GeV in all models
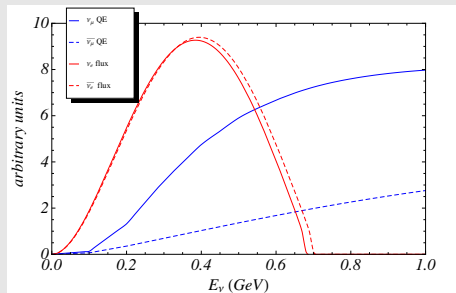
## *The $\beta$ Beam concept*

- the value of the Lorentz boost factor $\gamma$ and the source-detector distance $L$ determine the neutrino spectra
- interested in $\nu_e \rightarrow \nu_\mu$ oscillation
- leading terms in $P_{\nu_e \nu_\mu}$ depend on $\theta_{13}^2$ and $\theta_{13} \cdot \sin \delta$

- here we focus on
$(\gamma, L) = (100, 130 \, Km)$

-$(\nu \text{-} \bar{\nu})$ spectra very similar

- QE events

- very low backgrounds
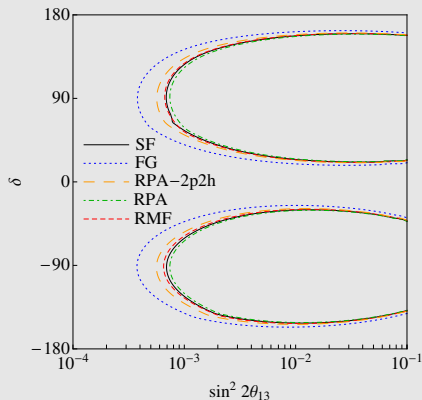  (from NC $1\pi^0$+atmospheric $\mu$)



- use this $\beta$ Beam as a prototype!

## *The CP discovery potential*

### Definition

for any $\theta_{13}$ is the ensemble of true values of $\delta_{CP}$ for which the $3\sigma$ CL do not touch $\delta_{CP} = 0, \pi, \pm\pi$
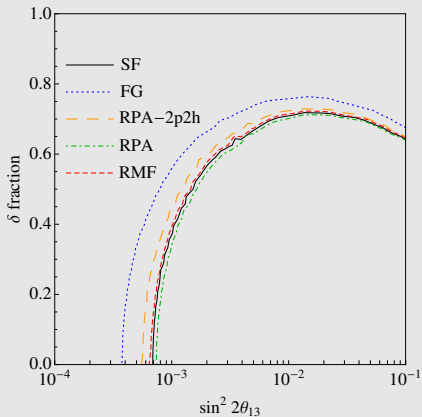


1- the FG performs *too* well compared with the other two models

2- at $\delta \sim \pm\pi/2$ the largest discrepancy: $\sim 2$ better than other computations!

3- notice that $RPA - 2p2h$ does not perform better than FG.

*The CP fraction*

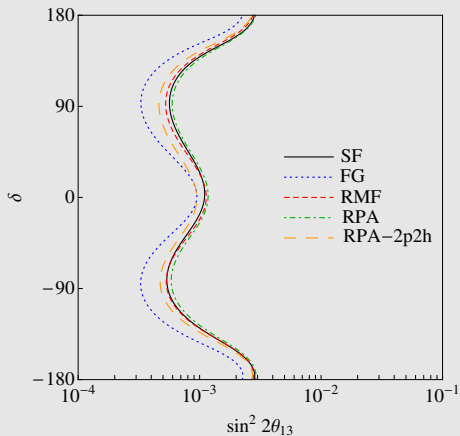Definition

just the total number of "good" $\delta$s divided by $2\pi$

## The sensitivity to $\theta_{13}$

- same analysis for $\theta_{13}$

### Definition

for any $\delta_{CP}$, true values of $\theta_{13}$ for which the $3\sigma$ CL do not touch $\theta_{13} = 0$



- a bit less evident than before: something of $\mathcal{O}(60)\%$

## A combined analysis

What about a simultaneous fit to $\theta_{13}$ and $\delta_{CP}$ ?

To see the impact of various models:

- we first fix some *true* value $(\theta_{13}, \delta_{CP}) = (0.9^o, 30^o)$
- then we study the capability of the facility to *measure* them

general problem of this kind of analysis:
presence of fake points $(\bar\theta_{13}, \bar\delta)$ where
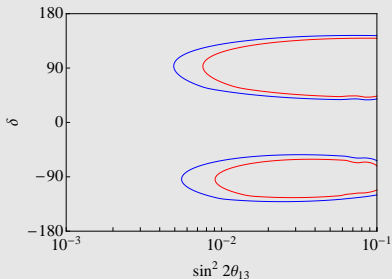$P_{e\mu}^{\pm}(\bar\theta_{13}, \bar\delta) = $ to the true transition
probability

- no fake solutions for FG
  This would point to a $\beta$-beam as a
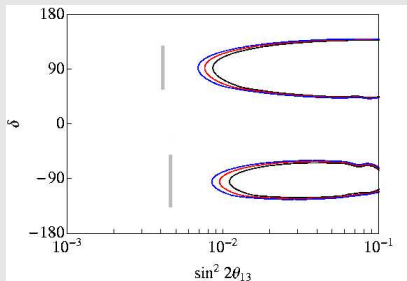  good facility to measure $\theta_{13}$ and $\delta$
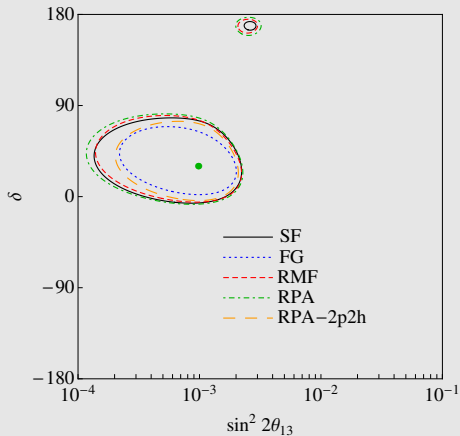
## Generalizing the previous results

- same effects with $^{56}Fe$ target
  blue: FG, red: SF

- mild dependence on the axial mass
  SF with blue: $m_A$=1.2 GeV, red: $m_A$=1.1 GeV
  black: $m_A$=1.0 GeV

*Summary and perspectives*

- *Summary*
  - *We studied the impact of nuclear effects on the determination of various neutrino parameters*
  - *In particular, we compare the FG results (widely adopted in MonteCarlo codes) with the SF and RPA approaches*
  - *The different behaviour of the cross sections translates into overstimated sensitivity to $\theta_{13}$ and $\delta_{CP}$*
  - *Although we focused on Oxygen, the same pattern is observed for other nuclear targets*

## Summary and perspectives

- *Perspectives*



Low Energy QE Cross Section for $\nu_\mu {}^{12}C \rightarrow \mu^- p$

- *It could be necessary to implement more realistic nuclear effects in MC codes*
  (plot from http://regie2.phys.uregina.ca/neutrino/qel.html)
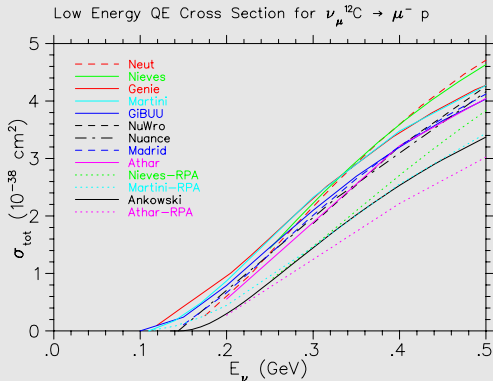
- *what happens for precision measurements at other facilities?*
- *what happens at much higher energies (Neutrino Factories) ?*

# Backup slides

$$\frac{d^2\sigma_{\text{elem}}}{d\Omega dE_l} = \frac{G_F^2\,V_{ud}^2}{32\,\pi^2}\,\frac{|k^{'}|}{|k|}\,\frac{1}{4\,E_{\mathbf{p}}\,E_{|\mathbf{p+q}|}}\,L_{\mu\nu}W^{\mu\nu}$$

- The hadronic tensor is decomposed in structure functions as usual

$$\begin{aligned} W^{\mu\nu} &= -g^{\mu\nu}\,W_1 + \tilde{p}^\mu\,\tilde{p}^\nu\,\frac{W_2}{m_N^2} + i\,\varepsilon_{\mu\nu\alpha\beta}\,\tilde{q}^\alpha\,\tilde{p}^\beta\,\frac{W_3}{m_N^2} + \tilde{q}^\mu\,\tilde{q}^\nu\,\frac{W_4}{m_N^2} + \\ &\quad (\tilde{p}^\mu\,\tilde{q}^\nu + \tilde{p}^\nu\,\tilde{q}^\mu)\,\frac{W_5}{m_N^2} \end{aligned}$$

- the formalism can be applied to **both** elastic and anelastic processes specifying the form of the structure functions $W_i$

- **details in the SF computation**
  - using the no-FSI approximation

  $$\sum_X |X\rangle\langle X| \quad \rightarrow \quad \sum_x \int d^3 p_x |x, \mathbf{p}_x\rangle\langle \mathbf{p}_x, x| \times \sum_{\mathcal{R}} d^3 p_{\mathcal{R}} |\mathcal{R}, \mathbf{p}_{\mathcal{R}}\rangle\langle \mathbf{p}_{\mathcal{R}}, \mathcal{R}|$$

  - complete set of free nucleon states, satisfying

  $$\int d^3 p \, |\mathrm{N}, \mathbf{p}\rangle\langle \mathbf{p}, \mathrm{N}| = I$$

  - hadronic matrix element

  $$\langle 0|J^\mu|X\rangle \quad = \quad \frac{m}{\sqrt{\mathbf{p}_{\mathcal{R}}^2 + m^2}} \langle 0|\mathcal{R}, \mathbf{p}_{\mathcal{R}}; \mathrm{N}, -\mathbf{p}_{\mathcal{R}}\rangle \times \sum_i \langle -\mathbf{p}_{\mathcal{R}}, \mathrm{N}|j_i^\mu|x, \mathbf{p}_x\rangle$$

# *The Spectral Function*

A. Ramos, A. Polls, W. H. Dickhoff, Nucl., Phys. A503, (1989) 1
O. Benhar, A. Fabrocini, S. Fantoni, Nucl., Phys. A505, (1989) 267
O. Benhar, A. Fabrocini, S. Fantoni and I. Sick, Nucl., Phys. A579, (1994) 493

- the calculation of $P(\mathbf{p}, E)$ for any A is a complicated task
- for nuclei from Carbon to Gold has been modeled using the Local Densitiy Approximation (LDA)

$$P_{LDA}(\mathbf{p}, E) = P_{MF}(\mathbf{p}, E) + P_{corr}(\mathbf{p}, E)$$

measured contribution corresponding to low momentum nucleons, occupying the shell model states

high momentum nucleons
calculable using the result of uniform nuclear matter "recomputed" for a finite nucleus of mass number A