

Neutrino Masses in Grand Unified Theories



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Topics of interest

- Tiny masses and Mass hierarchy: $\Delta m_{31}^2 > or < 0$

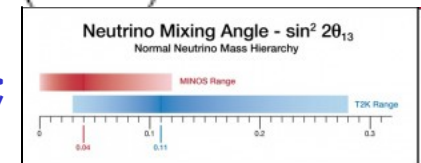
- Majorana, Dirac or neither

- Mixings: $U_{PMNS} = U_\ell^+ U_\nu$
 U_{PMNS} Strikingly close to:
$$= \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

 Tri-bi-maximal mixing(TBM)

(Harrison, Perkins, Scott; Xing;He, Zee; Pakvasa, Sugawara and Yamanaka; Wolfenstein)

- θ_{13} **T2K result** : $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$
MINOS best fit: $\sin^2 2\theta_{13} = 0.04(0.08)$;



- Also θ_{12} outside 90%c.l. (Fogli et al.arXiv:1102.6028)

➔ U_{PMNS} Surely not TBM but close

Could it be close to Bi-Maximal (BM)?

- Alternative possibility:

$$U_{PMNS} = U_{BM} + \delta U \quad U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- With δU such that $\theta_{12} = \frac{\pi}{4} - \theta_C; \theta_{23} = \frac{\pi}{4} - \theta_C^2 \equiv \frac{\pi}{4} - V_{cb}$

- Quark-lepton complementarity:** Smirnov; Raidal; Minakata, Smirnov

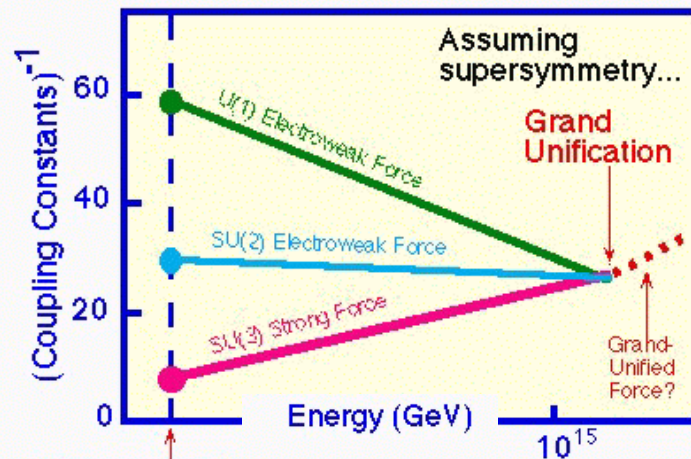


Other Topics of interest

- Leptonic CP phase
- Unitarity of PMNS
- Are there sterile neutrinos ?
- Do neutrinos have nonstandard interactions ?
- This talk:
 - What can grand unified theories tell us about any of these topics ?
 - What can more precise determination of neutrino mixings say about grand unification ?
 - Three issues: θ_{13} ; ~~CP~~ ; unitarity of PMNS

Grand Unification: the grand vision of Physics

★ $\alpha_i(M_Z)$ + SUSY at TeV \rightarrow couplings unify



$$M_U \approx 10^{16} \text{ GeV}$$

- Explains electric charge quantization
- Unifies all matter- **possible hints:**

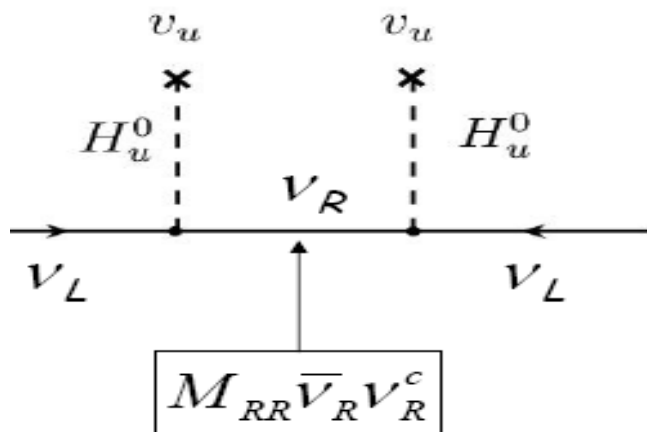
at GUT scale \rightarrow $m_b \approx m_\tau$ $m_\mu = -3m_s$

★ Are neutrinos leading us towards this grand vision ?

Why neutrino masses are so small - $m_\nu \ll m_{q,l}$?

■ Seesaw paradigm:

- Add right handed neutrinos N_R to SM with Majorana mass:



$$m_\nu \cong - \frac{h_\nu^2 v_{wk}^2}{M_R} \quad (\text{Type I})$$

- neutrinos necessarily Majorana
- M_R large so that nu mass small

How Large ? **GUTs** → $h_{\nu,33} \approx h_t$; Δm_{atm}^2 → $M_R \approx 10^{14} \text{ GeV}$

- GUTs appealing and natural for seesaw !

Challenges for GUTs

- Why are PMNS and CKM so different ?

$$U_{PMNS} = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix} \text{ vs } U_{CKM} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$\lambda \cong 0.22$

i.e. Quark mass matrix hierarchical –

but neutrino mass matrix non-hierarchical ?

- Why is $\frac{m_{solar}}{m_{atm}} \approx \theta_C \gg \frac{m_{\mu}}{m_{\tau}}, \frac{m_s}{m_b}$?

SO(10) SUSY GUT for neutrinos

- SU(5) does not unify neutrinos with quarks !
- SO(10) unifies all fermions/family (including RH nu) in single rep.

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$$

- **Contains all the ingredients for seesaw mechanism:**
- **Minimal Renormalizable version predictive for nu masses and mixings:**

Seesaw: Type I or II ?

- **Type I** Seesaw formula:

$$m_\nu \cong \frac{h_\nu^2 v_{wk}^2}{M_R}$$

Flavor sensitive

Flavor sensitive

- GUTs relate $h_\nu \approx h_{up}$ - hence hierarchical
- So M_R must be “**doubly hierarchical**”-

Seesaw: Type I or II ?

- **Type I** Seesaw formula:

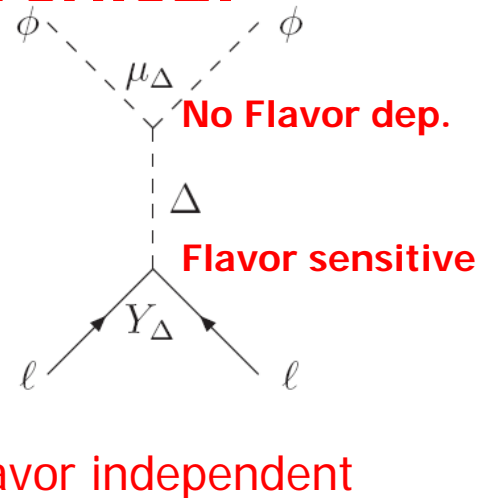
$$m_\nu \cong \frac{h_\nu^2 v_{wk}^2}{M_R}$$

Flavor sensitive (pointing to h_ν)

Flavor sensitive (pointing to M_R)

- GUTs relate $h_\nu \approx h_{up}$ - hence hierarchical
- So M_R must be **“doubly hierarchical”** -
- **Type II bypasses h_ν**
no such hierarchy issue!

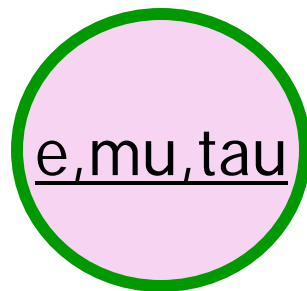
$$M_\nu \cong f_{126} \frac{v_{wk}^2}{M}$$



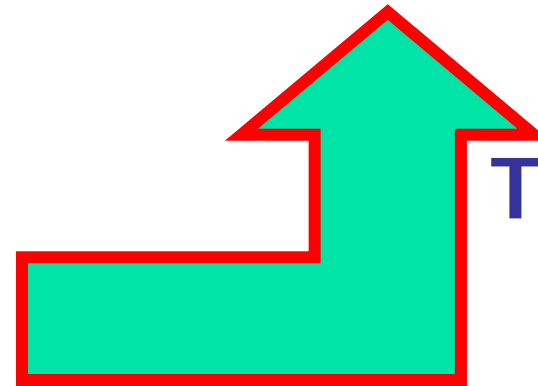
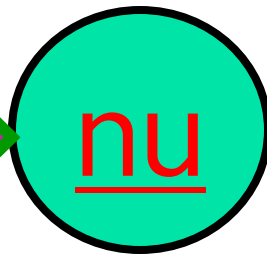
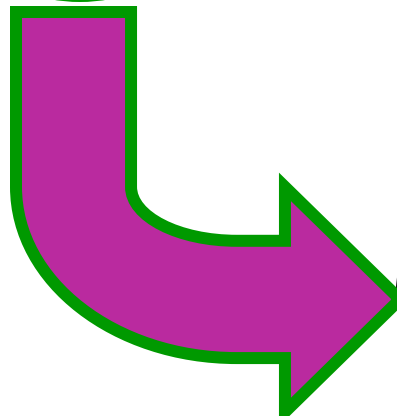
Intimate quark-lepton connection in $SO(10)$ -II

Leptons

Quarks



$SO(10)$



Type II


$$U_{PMNS} = U_{\ell}^+ U_{\nu}$$

Large mixings in minimal SO(10)

- **II seesaw** $M_\nu \cong c(M_d - M_l)$

- **GUT relation** $m_b \approx m_\tau (1 + \lambda^2) \rightarrow \lambda = \theta_C$

$$M_\nu \propto m_b \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 + \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} - m_\tau \begin{pmatrix} \lambda^6 & \sim \lambda^3 & \sim \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \sim \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$



$$\sim \lambda^2 \begin{pmatrix} \lambda^3 & \sim \lambda & \sim \lambda \\ \lambda & 1 + \lambda & -1 \\ \lambda & -1 & 1 \end{pmatrix}$$

non-hierarchical

(Bajc, Senjanovic, Vissani'02)

Predictions

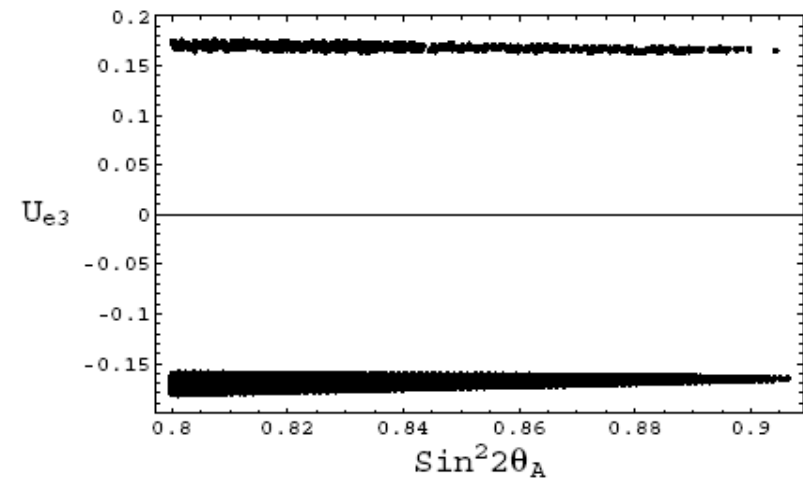
- Works qualitatively and quantitatively

- Predictions:

- ★ θ_{12}, θ_{23} large

- ★ $\theta_{13} \approx \lambda$ "large"

- ★ $\frac{m_{solar}}{m_{atmos}} \sim \lambda$



(Goh, RNM, Ng, PLB, 570 (2003))

no CKM CP

$$\theta_{13} \cong 0.15$$

Current Status

- Subsequent analysis of fermion masses with CKM:

- **Pros:** (Fukuyama, Okada; Babu, Macesanu; Bertolini, Malinsky, Schwetz;)

Fits to charged fermions →

$$\theta_{12} = 32^\circ \quad \theta_{23} = 37^\circ$$
$$\theta_{13} = 6^\circ \quad \delta_{PMNS} = 4^\circ$$

Bertolini, Malinsky fit

- **Cons:** -very limited parameter space;
-proton decay requires cancellations

- **Improved minimal model:**

- **Add 120 Higgs or extra 10;** broadens parameter space;
solves p-decay issues and still predictive.

Oshimo; Dutta, Mimura, RNM; Bertolini, Frigerio Malinsky; Yang, Wang; Grimus, Kuhboch; Aulakh,
Garg; Joshipura, Kodrani, Patel;

Quantitative analysis of improved SO(10)

- Two classes of SO(10) type II models with TBM by choice of basis/or symmetry

(i) SO(10)-type II with 120 (17 param) $\theta_{13} \geq 0.07$

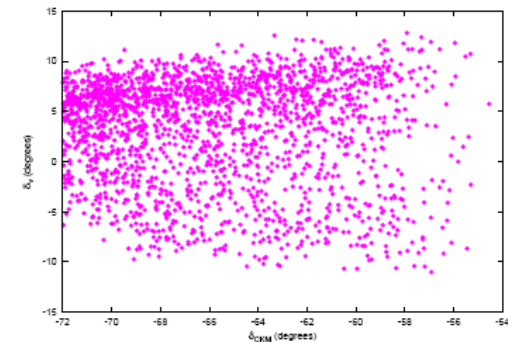
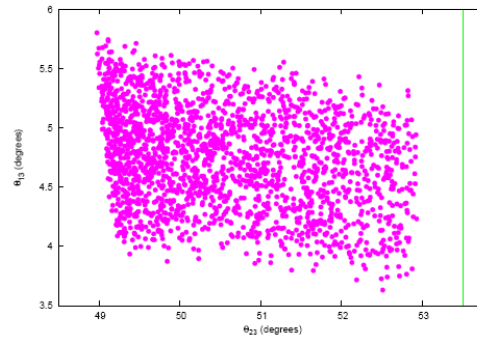
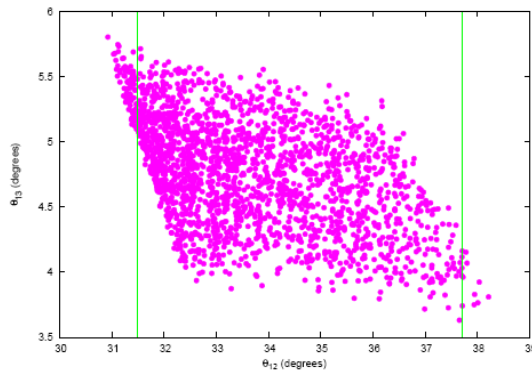
Altarelli, Blankenburg'10; Joshipura, Patel'11

$$\delta_D \approx 274^\circ$$

(ii) SO(10) type II + 10's: (13 param) $\theta_{13} \sim 4-5^\circ$

$$-10^\circ < \delta_D < 10^\circ$$

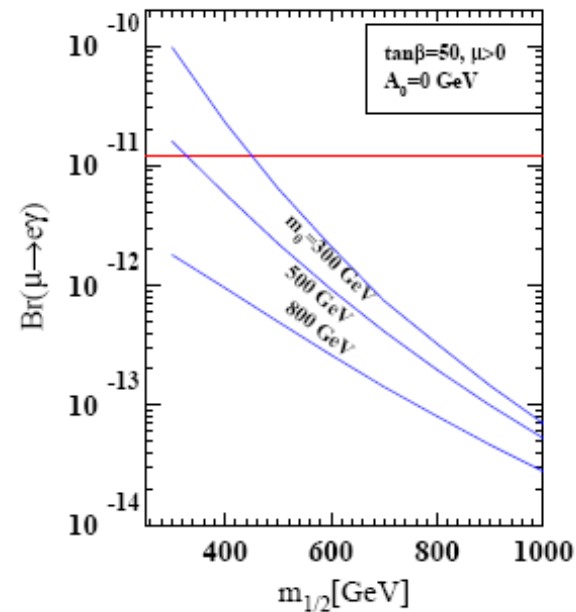
θ_{13}




(Dev, RNM, Severson arXiv:1107.2378)

Predictions for $\mu \rightarrow e + \gamma$

- $\mu \rightarrow e + \gamma$ for typical SO(10) theories with type II seesaw:



- $B(\mu \rightarrow e + \gamma) > 10^{-14}$ within range of MEG 
- No $\mu \rightarrow e + \gamma$ \rightarrow nonsusy GUT with seesaw !



Bottom line for SO(10)

Generic to all SO(10) with type I or II seesaw

- Naturally explains small neutrino masses;
- Neutrinos all Majorana:
- But mass hierarchy generally normal → No Signal in ongoing nu-less double beta decay searches !!
- No TeV scale signal at LHC
- Observable $[\mu \rightarrow e + \gamma]$ with SUSY

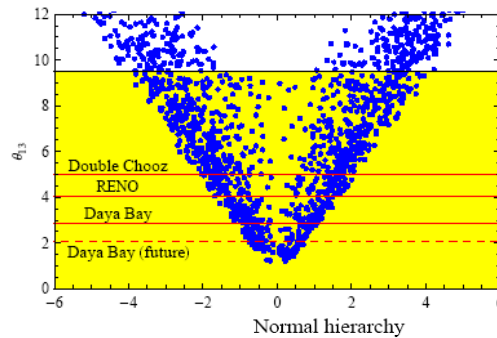
Specific to type II models

- Predicts $\theta_{13} > 0.07$; good news for ~~CP~~ searches.
- Bimaximal starting point does not work- **too large solar angle** → Expt telling us how to build theory !!

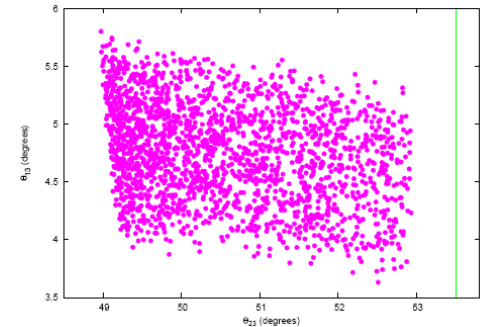
Discriminating between models

- Many models for θ_{13} - how to tell the difference ?
- θ_{13} - θ_{23} correlation –testable feature of models

Approx $\mu - \tau$

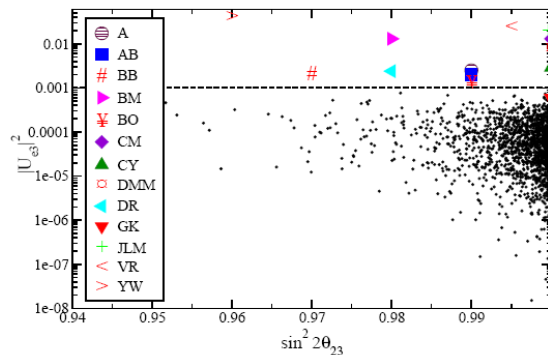


SO(10)xS4
(Dev, RNM, Severson)



(figure from He, Yin'11)

- GUTs vs \sim TBM
(Albright, Rodejohann)



SU(5)xT'
(Pakvasa, Bjorken, King, Chen, Mahanthappa,..)

$$\theta_{13} = \frac{\theta_c}{3\sqrt{2}} \sim 3^0$$

Non-unitarity of PMNS

- Is U_{PMNS} unitary?: Seesaw \rightarrow No !!
- **Type I case- tiny and unobservable!**
- With inverse seesaw can be significant:
$$\begin{pmatrix} 0 & hv_{wk} & 0 \\ hv_{wk} & 0 & M \\ 0 & M & \mu \end{pmatrix}$$
 (RNM, Valle'86)
- Parameterize $N = (1 + \eta)U$
- \rightarrow subset of possible non-standard interactions:
$$\mathcal{L}_{NSI} = -\varepsilon_{\alpha\beta}^{fP} 2\sqrt{2}G_F (\bar{\nu}_\alpha \gamma_\mu L \nu_\beta) (\bar{f} \gamma^\mu P f)$$
 with $\varepsilon_{\alpha\beta}^{fP} = \eta_{\alpha\beta}$

Non-unitarity in oscillations

- Lepton flavor violation, muon and pion decays:

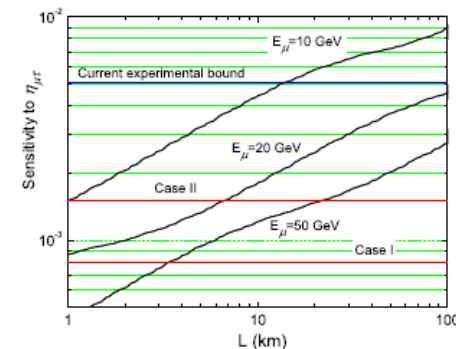
$$\Rightarrow |\eta| < \begin{pmatrix} 2.0 \times 10^{-3} & 6.0 \times 10^{-5} & 1.6 \times 10^{-3} \\ \sim & 8.0 \times 10^{-4} & 1.1 \times 10^{-3} \\ \sim & \sim & 2.7 \times 10^{-3} \end{pmatrix} .$$

~~CP~~

$$P_{\alpha\beta} = \sin^2(2\theta) \sin^2\left(\frac{\Delta L}{2}\right) - 4|\eta_{\alpha\beta}| \sin \delta_{\alpha\beta} \sin(2\theta) \sin\left(\frac{\Delta L}{2}\right) + 4|\eta_{\alpha\beta}|^2$$

Yasuda,
Gavela,
Martinez,
Pavon'07

- $\eta_{\mu\tau} < 10^{-4}$ Observable in oscillations in neutrino factories with OPERA type near detector.



Malinsky,
Ohlsson,
Zhang

[Antusch](#), [Biggio](#), [Fernandez-Martinez](#), [Gavela](#), [J. Lopez-Pavon](#)
[Goswami](#), [Ota](#); [Meloni](#); [Xing](#); [Kopp](#), [Lindner](#); [Winter](#); [de Gouvea](#) , ...

Inverse seesaw in SO(10) and non-unitarity

- Allows TeV scale seesaw inspite of GUT constraint:
(unlike type I)

- Inverse seesaw:
$$m_\nu \cong -m_D^T M^{-1} \mu M^{-1} m_D$$

μ breaks lepton number \rightarrow nu mass- so m_D can satisfy GUT rel.

- TeV M consistent with coupling unification (Dev, RNM'09) \rightarrow
Predicts large η_{ij} as well as new CP phase.

m_{N_1}	m_{N_2}	m_{N_3}	$ \eta_{e\mu} $	$ \eta_{e\tau} $	$ \eta_{\mu\tau} $
1100	1100	1100	3.7×10^{-7}	1.5×10^{-5}	6.5×10^{-5}
100	100	1100	7.9×10^{-7}	1.6×10^{-5}	8.9×10^{-5}
50	50	1200	2.5×10^{-6}	2.2×10^{-5}	1.6×10^{-4}
30	30	2100	6.7×10^{-6}	4.4×10^{-5}	3.2×10^{-4}



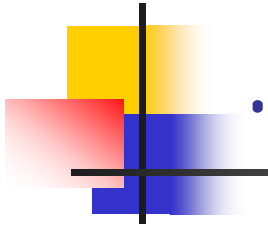
Summary

- **SO(10)-type II- “Big Picture Dream Scenario”** for neutrino masses !
- ‘Larger θ_{13} ’ fits very well into it !!
- **Further test** → Precise measurement of angles, mass hierarchy and Majorana nature !
- Non observation of $\mu \rightarrow e + \gamma$ could indicate **nonsusy GUT** seesaw → no susy at LHC !!
- Predictions for CP phase-model dependent –
- **True test of GUT- observation of proton decay.**



Summary: contd

- . Unitarity in SBL expts would be a sign of low seesaw scale as any signal of W_R, Z_{LR} at LHC
- Sterile nus would require significant change in GUT model bldg-



Extra slides

Could neutrinos be not Dirac nor Majorana (“*Schizophrenic*”)

- Possible if there are ν_R ‘s that form Dirac mass with one or two active mass eigenstates (ν_1, ν_2, ν_3)

$$L_m = m_1 \nu_1 \nu_R + m_2 \nu_2 \nu_2 + m_3 \nu_3 \nu_3$$

Dirac mass

Majorana

(Usual Majorana case: $L_m \equiv m_1 \nu_1 \nu_1 + m_2 \nu_2 \nu_2 + m_3 \nu_3 \nu_3$)

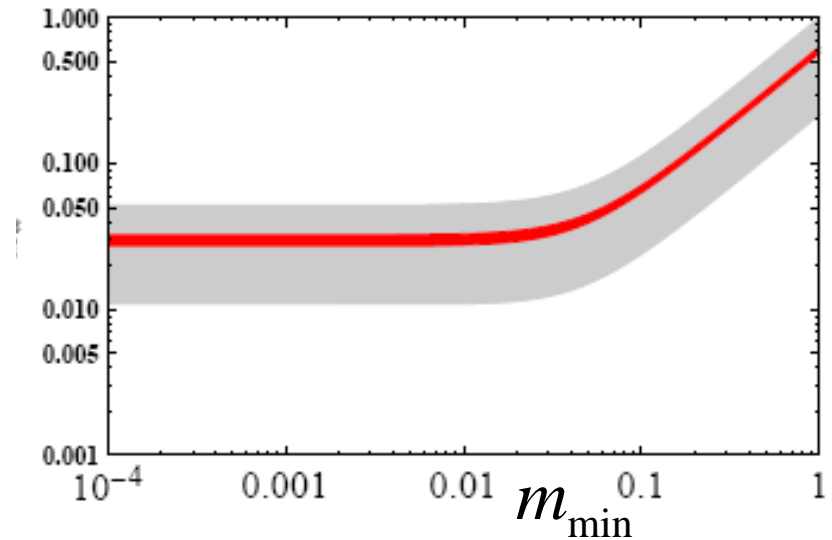
- Oscillation phenomenology, MSW effect in Sun etc remain unaffected !! (Allahverdi, Dutta, Mohapatra'10)
- **Different from Pseudo-Dirac**

Nu-less double beta decay- *Majorana vs Schizophrenic*

- Higher lower bound on M_{ee} for IH

(Allahverdi, Dutta, RNM)

m_{ee}



- **17 meV** (usual IH) vs **34 meV** (Schizo-nu IH) as lower bound: **schizo-nu can be ruled out if IH from LBL expts.**
- One loop Dirac \rightarrow pseudo-Dirac (Leung, Petcov'83) \rightarrow affects extragalactic neutrino flux in Km^3 detectors

(Barry, RNM, Rodejohann'10)

- Several particle physics models already constructed- (Pleitez, Machado; Haba, Shindou; Morisi, Peinado; Chen, Lin; Ghosh et al.)



SO(10) with 120

- : (Dutta, Mimura, RNM,2005,06,07,10)
- Solving proton decay using flavor texture:

$$h_{10} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; h_{126} = \begin{pmatrix} 0 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix};$$
$$h_{120} = \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix};$$

The Neutrino Matrix:

Flavor of the Neutrino flavor research

Generic mass matrix (NH) $\varepsilon_i \approx \lambda_{Cabibbo} \ll 1$

θ_{23} NEAR MAXIMAL

θ_{23} maximal

$$\begin{pmatrix} \varepsilon_5^{n \geq 1} & \varepsilon_4 & \varepsilon_3 \\ \varepsilon_4 & 1 + \varepsilon_1 & -1 \\ \varepsilon_3 & -1 & 1 + \varepsilon_2 \end{pmatrix} \rightarrow \begin{pmatrix} \varepsilon_5^{n \geq 1} & \varepsilon_3 & \varepsilon_3 \\ \varepsilon_3 & 1 + \varepsilon_1 & -1 \\ \varepsilon_3 & -1 & 1 + \varepsilon_1 \end{pmatrix} \xrightarrow{\text{TBM}} \begin{pmatrix} \varepsilon_1 & \varepsilon_3 & \varepsilon_3 \\ \varepsilon_3 & 1 + \varepsilon_1 & -1 + \varepsilon_3 \\ \varepsilon_3 & -1 + \varepsilon_3 & 1 + \varepsilon_1 \end{pmatrix}$$

5 parameters

$\rightarrow \mu \leftrightarrow \tau$ sym.

3 param.

2 param.



Two incarnations of $SO(10)$

I

(16-Higgs)

- ❖ B-L breaks 1 unit
- ❖ no susy DM
- ❖ low representations:
(an advantage)
- ❖ String compatible

II

(126-Higgs)

- B-L breaks by 2 units
- **SUSY DM automatic**
- Minimal version: few parameters < 17 —
hence more predictive
- ??

Early papers:

Albright, Barr'99; Babu, Barr'98; Dermisek, Raby'00.....

Babu, Mohapatra'92

TBM vs BM in GUTs

What starting point

- $SO(10)$ + type II seesaw, neutrino mass matrix in TBM or BM form without sym:

- Since $U_{PMNS} = U_{\ell}^+ U_{TBM}$ and $m_{\mu} = -3m_s$ fitting charged fermions is expected to give:

- $\theta_{13} = \frac{\theta_c}{3\sqrt{2}} \sim 0.05$; $\delta_D \approx$ model dep.

(Bjorken, Pakvasa, King, Chen, Mahanthappa)

- $\sin \theta_{12} \sim \frac{1}{\sqrt{3}} + \frac{\theta_c}{3\sqrt{3}}$ TBM; $\sin \theta_{12} = \frac{1}{\sqrt{2}} + \frac{\theta_c}{6}$ BM

~BM in $SO(10)$ unlikely to fit solar;

New physics implied by Neutrino Mass

- Standard model- no $\nu_R \rightarrow m_\nu = 0$
- Two possibilities for m_ν
- (i) SM + $\nu_R \xrightarrow{\text{green arrow}} L_Y = h_\nu \bar{L} H \nu_R + h.c. \xrightarrow{\text{red arrow}} m_\nu = h_\nu v_{wk}$

Observations $\rightarrow h_\nu \cong 10^{-12}$ (neutrino Dirac)

★ Since in GUTs, all couplings are expected to be same order, **Dirac neutrino not "GUT friendly"**

(ii) Alternative: Mass from high scale physics: $\frac{LHLH}{M}$
(Neutrino Majorana)

Large mixings from symmetries vs dynamics ?

- many sym models with SM, MSSM, GUTs

- **Sym. candidates:**

$$S_{2(\mu-\tau)} \subset S_3, S_4, A_4, \Delta(3n^2), \dots$$

(Ishimori, Saga, Shimizu, Tanimoto; Altarelli, Feruglio, Hagedorn; Morisi, Picarello; King, Luhn; Chen, Mahanthappa; Hagedorn, Smirnov, Schmidt, Dutta, Mimura, R.N.M.; Bazzocchi et. Al.; Joshipura, Kodrani, Patel,.....)

- **Fitting quarks and CKM in GUTs “shatters” sym**
- **Generically predict much smaller θ_{13}**
- **Perhaps another lesson for model building !!**