The Final Measurement of the Muon Decay Parameters from the $\mathcal{TWIST}$ Experiment

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For the **T**RIUMF **W**eak **I**nteraction **S**ymmetry **T**est Collaboration

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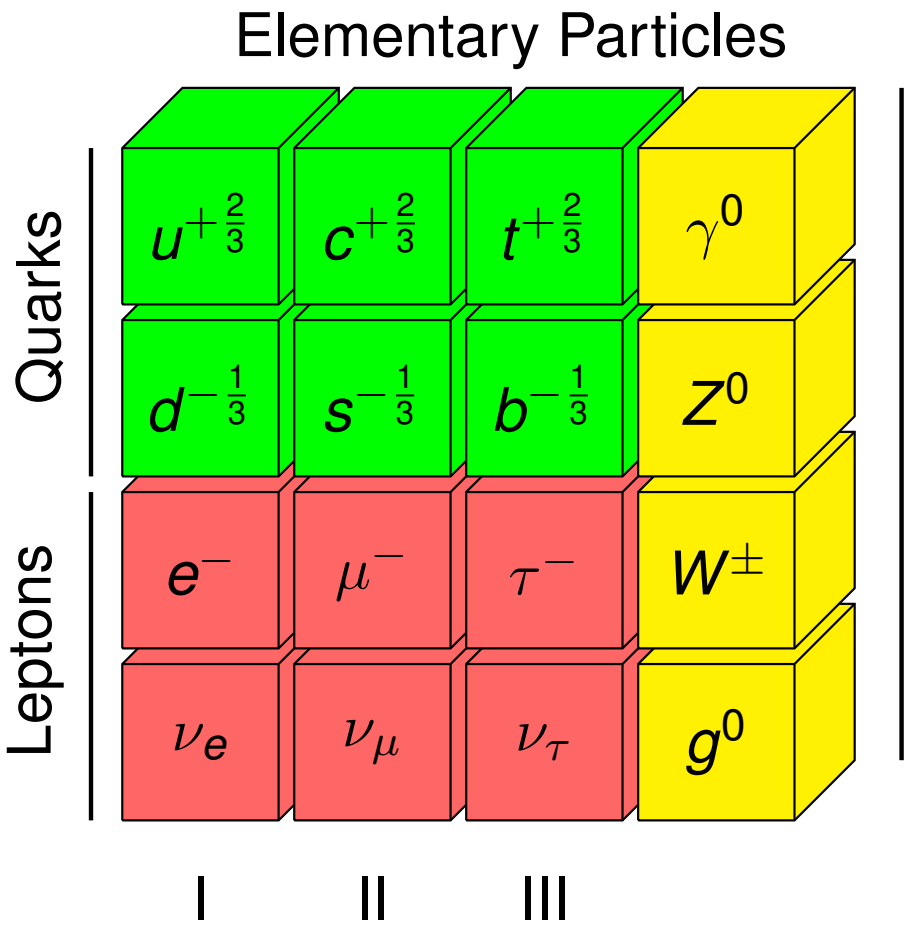
NUFACT 2011

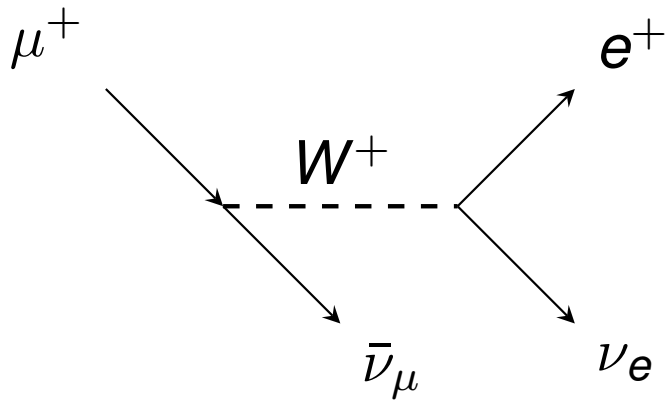
Outline

1. Introduction
2. TWIST Apparatus
3. Analysis
4. Systematics
5. Physics Results
6. Conclusions

Standard Model Weak Interactions



- Leptons appear in 3 flavour families
- Charged weak interactions moderated by $W^\pm$
  - Behave with a $V - A$ coupling



Muon Decay as a Probe for the Weak Interaction

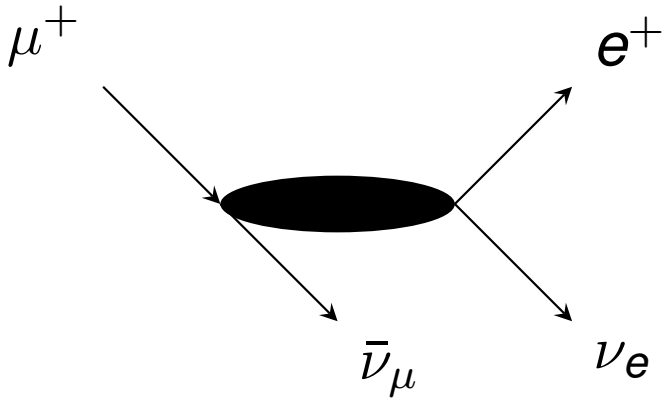
- General Lorentz invariant, derivative-free, interaction[1]

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \epsilon,\mu=R,L}} g^{\gamma}_{\epsilon\mu} \langle \bar{e}_{\epsilon} | \Gamma^{\gamma} | (\nu_e)_n \rangle \langle (\bar{\nu}_{\mu})_m | \Gamma_{\gamma} | \mu_{\mu} \rangle .$$

General Case

- 19 degrees of freedom:
  - 12 complex parameters
  - $g^T_{LL} \equiv 0$, $g^T_{RR} \equiv 0$
  - Required to be unitary
- In SM $g^V_{LL} = 1$, all others zero.

$$Q_{\epsilon\mu} = \frac{1}{4}|g^S_{\epsilon\mu}|^2 + |g^V_{\epsilon\mu}|^2 + 3(1-\delta_{\epsilon\mu})|g^T_{\epsilon\mu}|^2$$



[1] W. Fetscher, H.J. Gerber, and K.F. Johnson, *Phys. Lett.* **B173 (1986) 102**

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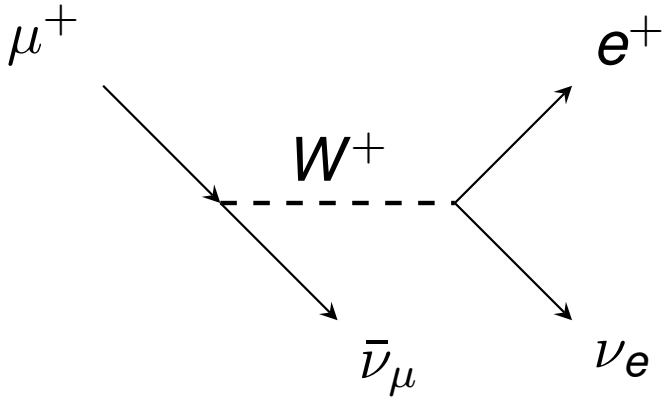

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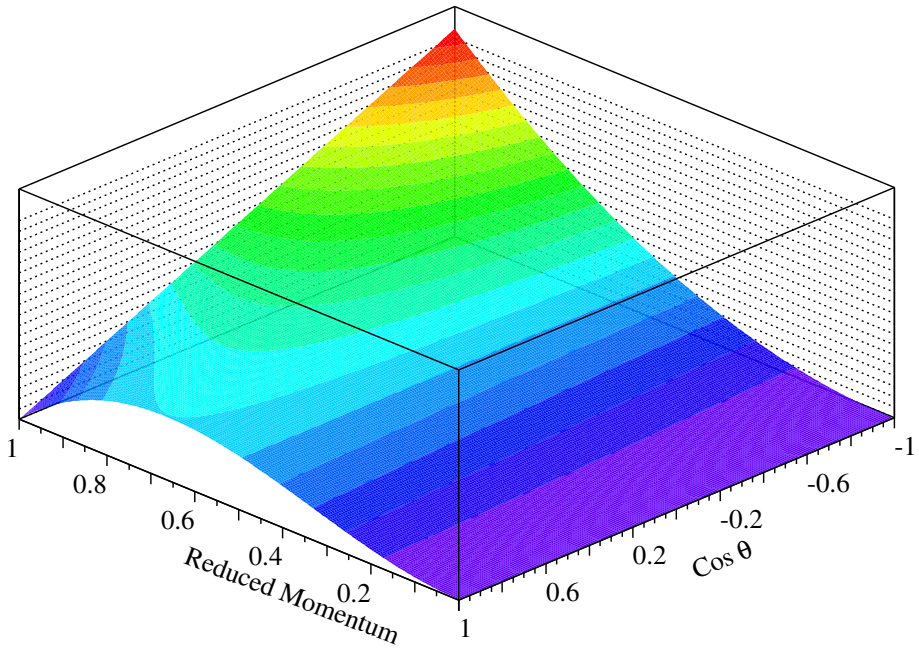
Decay Spectrum Parametrization

- Given in energy and angle as [2]

$$\frac{\partial^2\Gamma}{\partial x \partial \cos\theta} = \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \left(F(x) + |P_\mu| \cos\theta\, G(x)\right) + R.C.,$$



$$x = \frac{E_e}{W_{e\mu}}, \cos\theta = \vec{p}_e \cdot \vec{P}_\mu$$

In the Standard Model

ρ	=	0.75
η	=	0
δ	=	0.75
$P_\mu^\pi \xi$	=	1

[2] K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010)

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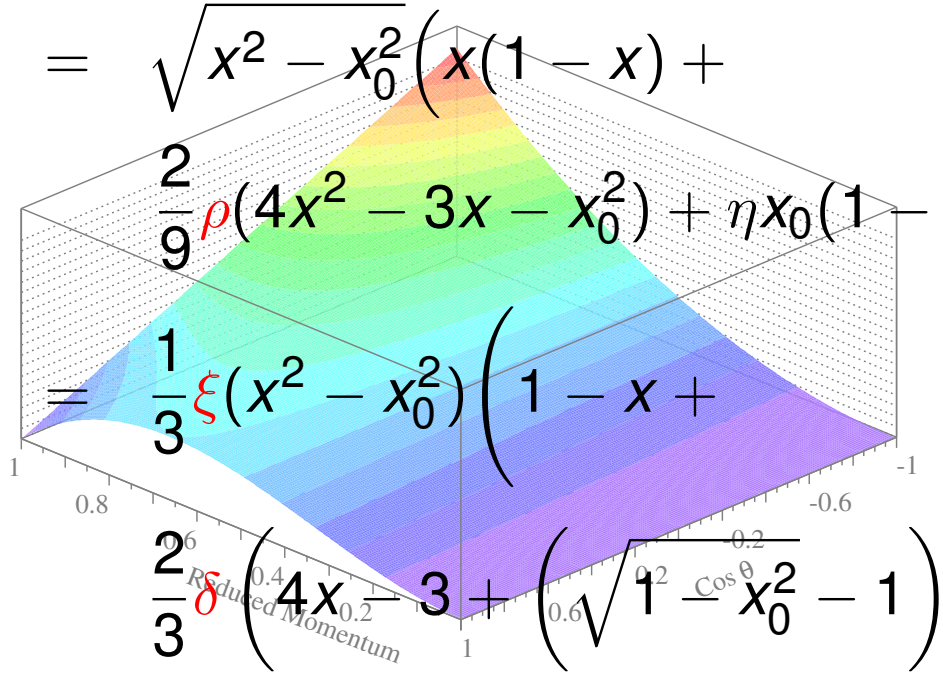
$$F(x) = \sqrt{x^2 - x_0^2}\left(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)\right)$$

$$G(x) = \frac{1}{3}\xi(x^2 - x_0^2)\left(1 - x + \frac{2}{3}\delta\left(4x - 3 + \left(\sqrt{1 - x_0^2} - 1\right)\right)\right)$$

$$x = \frac{E_e}{W_{e\mu}}, \cos\theta = \vec{p}_e \cdot \vec{P}_\mu$$



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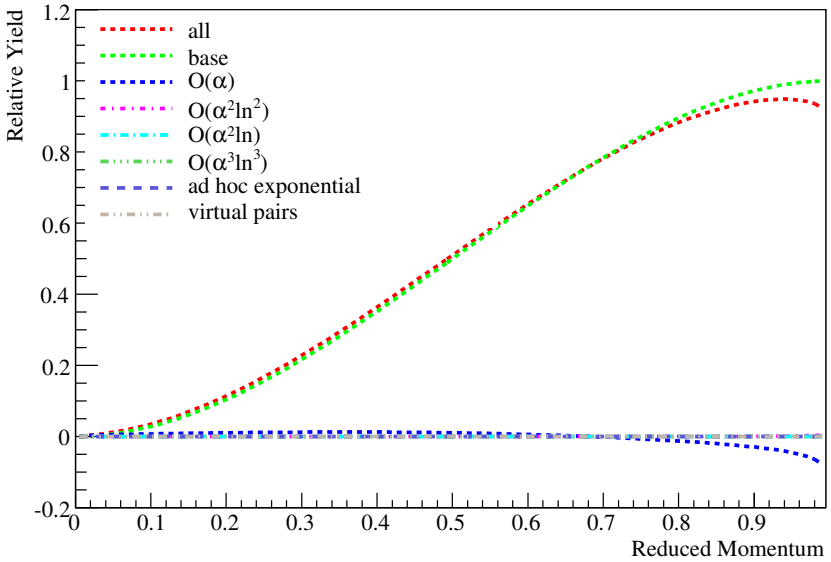
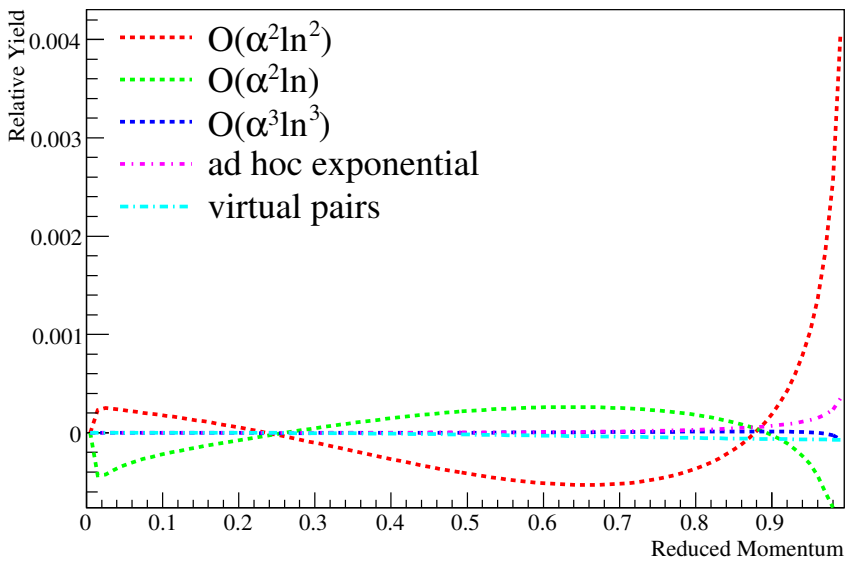
Radiative Corrections





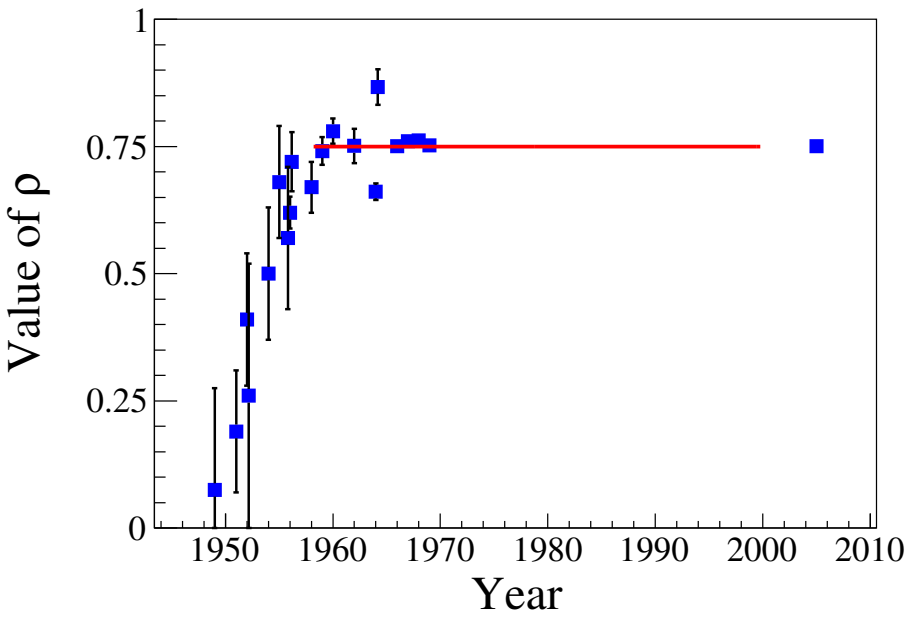
- Highest order correction contributes variations in spectrum at 10^{-5} level.
- Known second order leading logarithmic corrections make this measurement possible.[3]
- Contribution of higher order corrections represent systematic uncertainties.

[3] Arbuzov et. al., PR**D65** (2002) 1130067

Measurements of Muon Decay Parameters

State before 2003

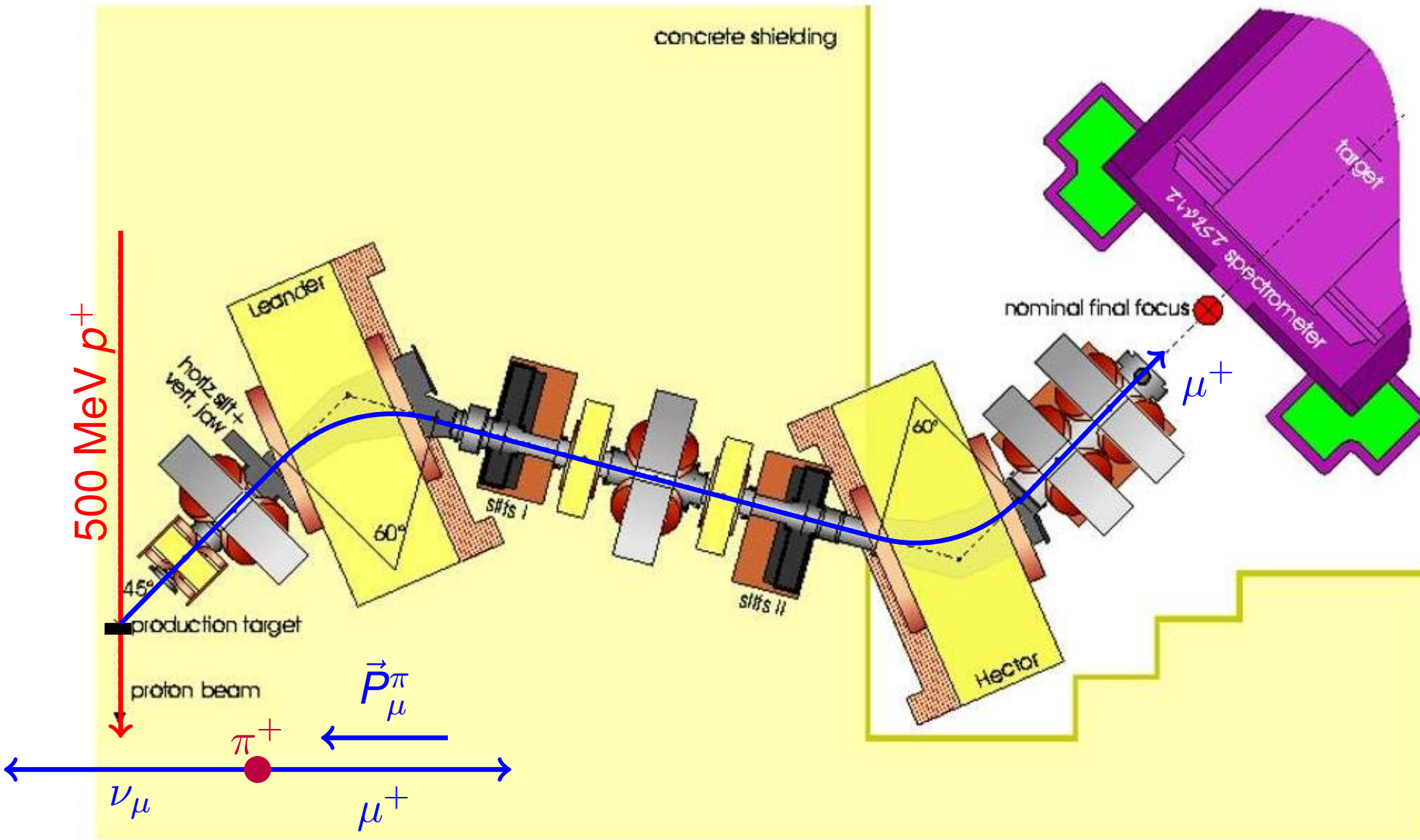
ρ	0.7518±0.0026	Derenzo, Phys. Rev. **181** (1969) 1854
δ	0.7486±0.0026±0.0026	Balke, PRD **37** (1988) 587
ξ	1.0027±0.0079±0.0030	Beltrami, Phys. Lett. **B194** (1987) 326



TWIST Purpose

- Order of magnitude improvement in precision
- Explicitly test weak model predictions
- Use the shape of the spectrum in p and cos θ to determine ρ, δ, and $P_\mu\xi$

TWIST Experiment



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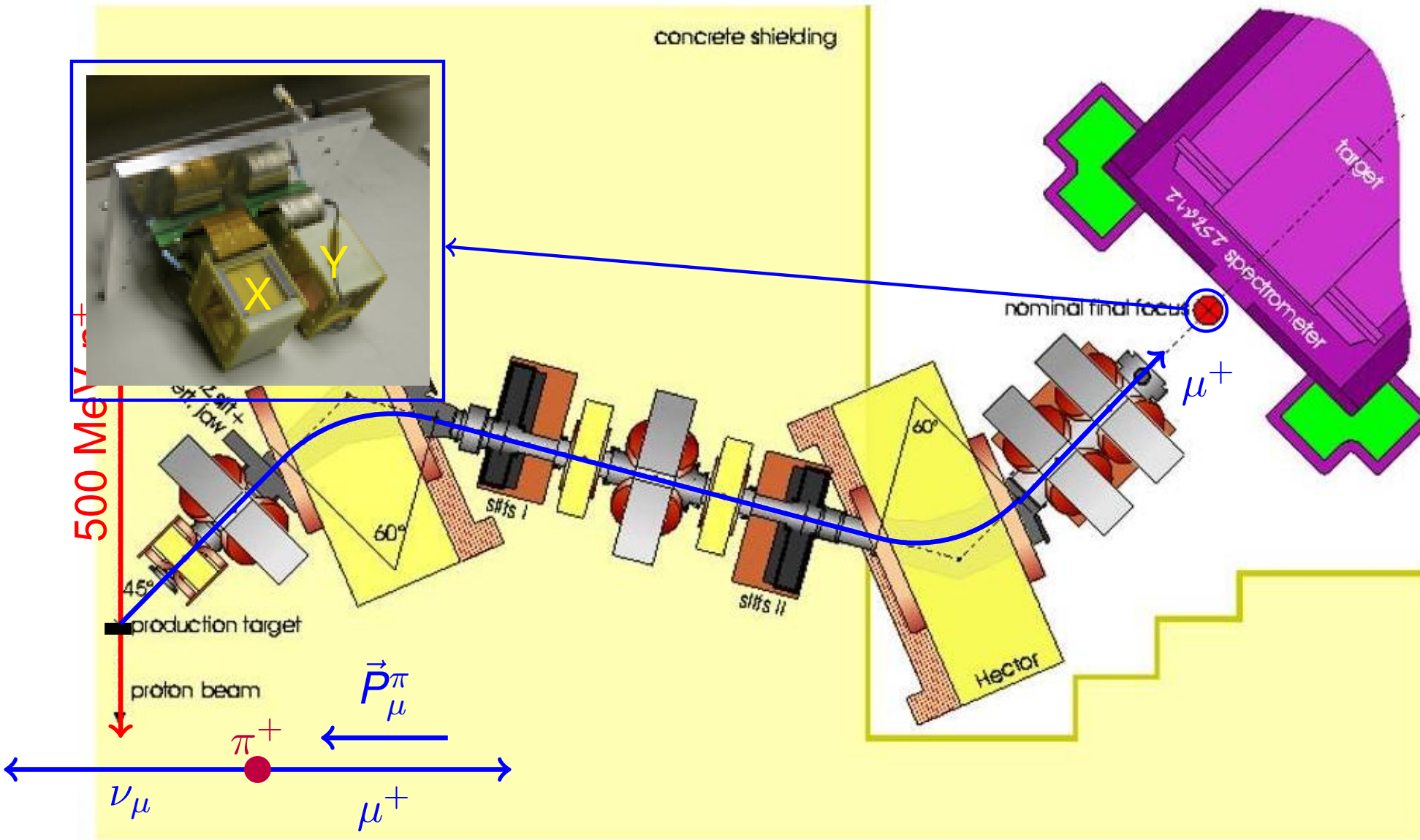

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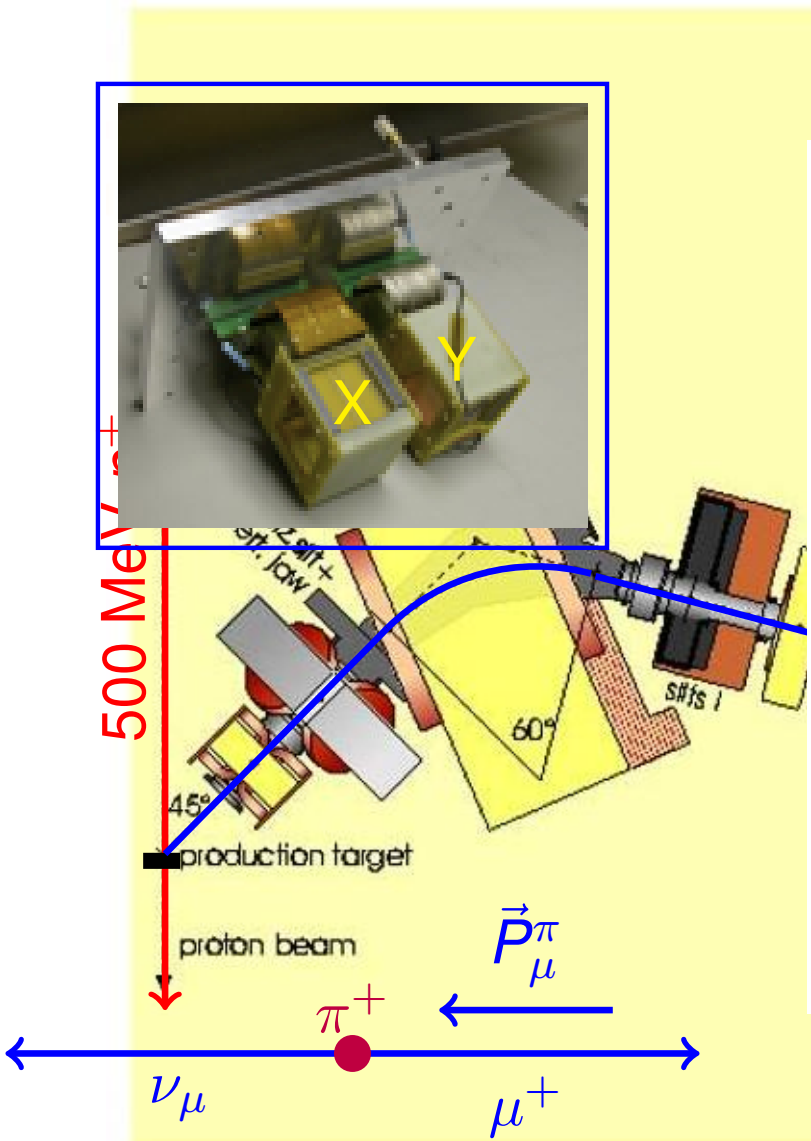
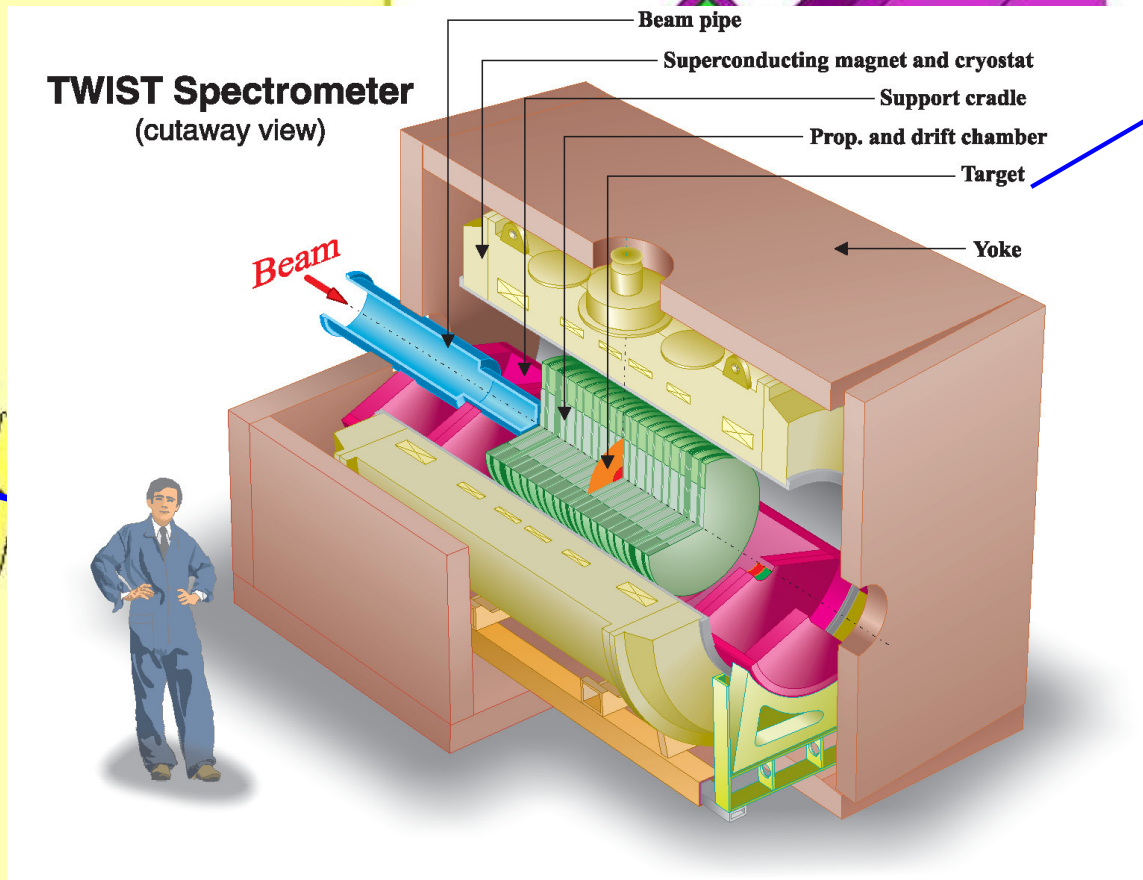
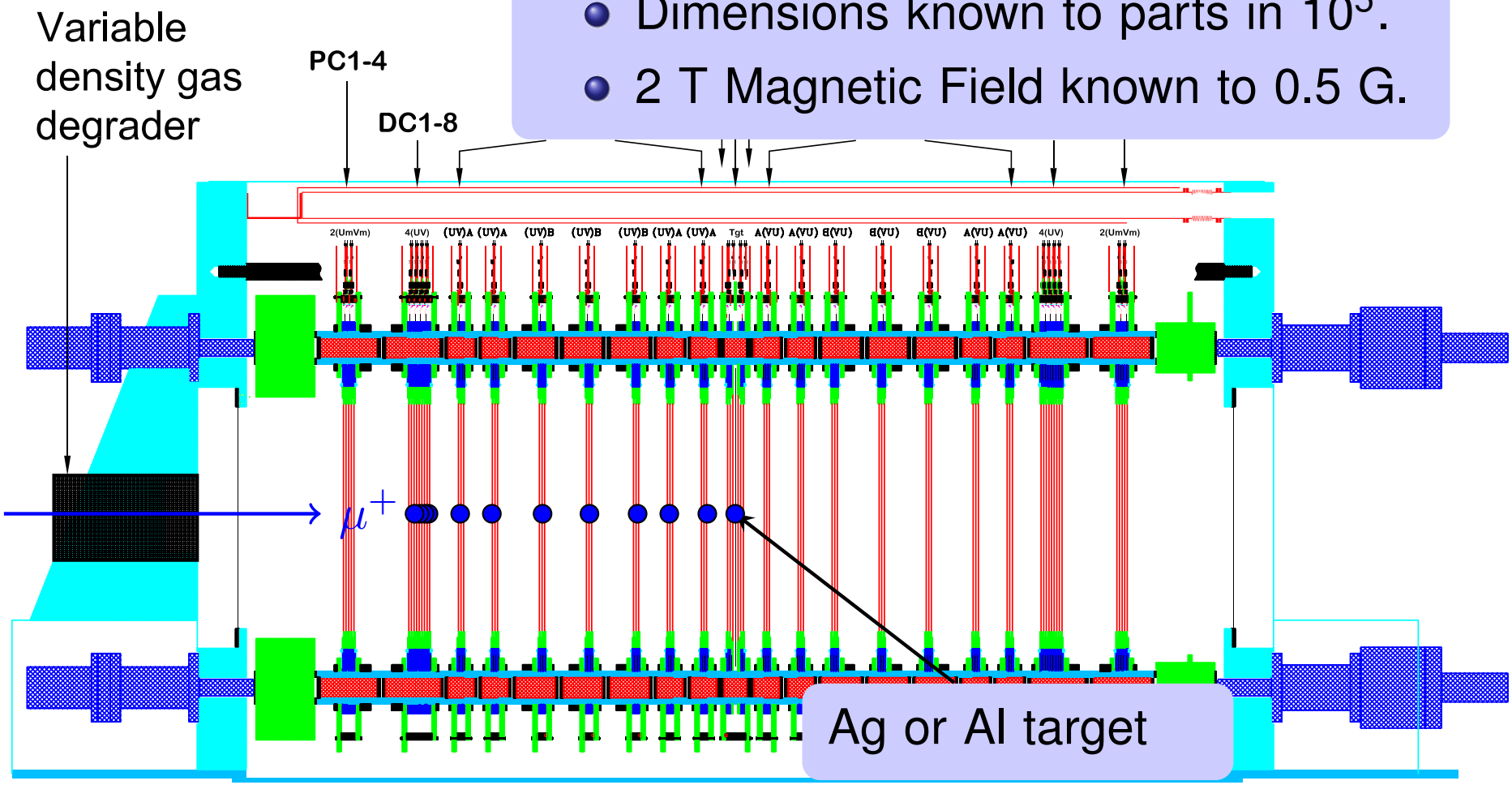


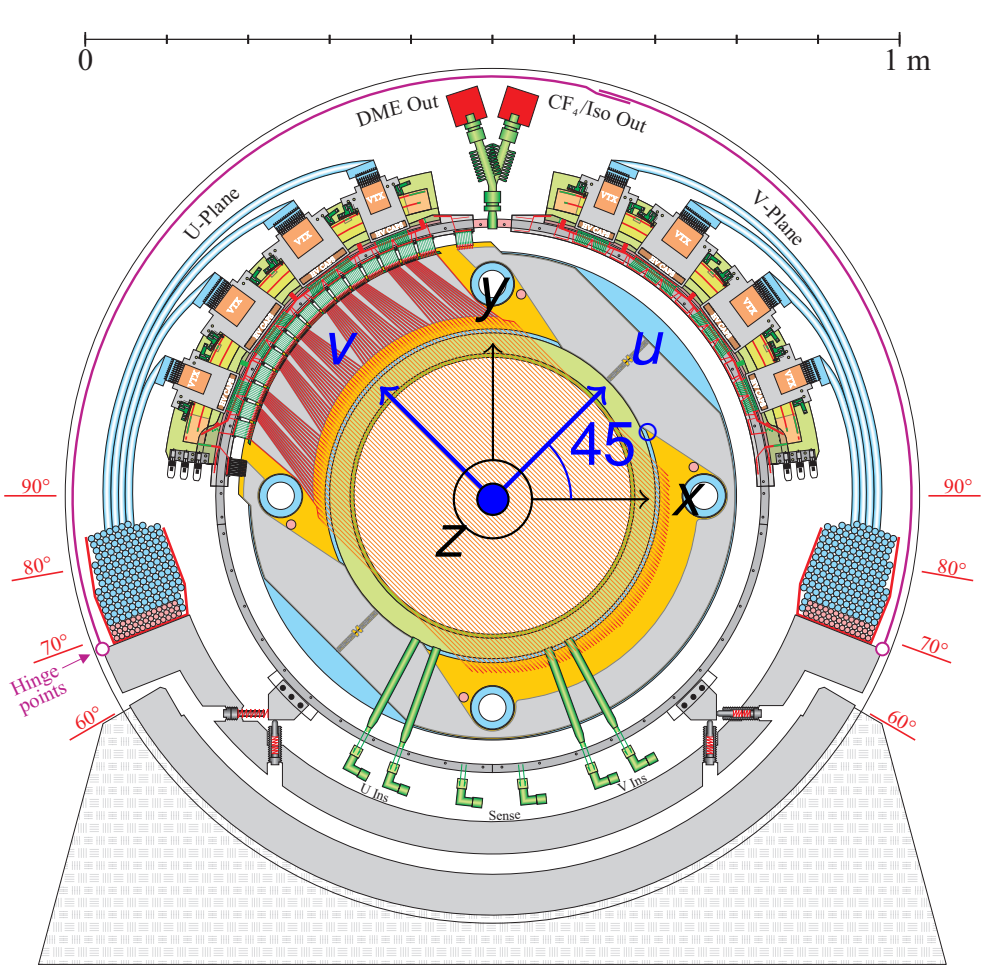


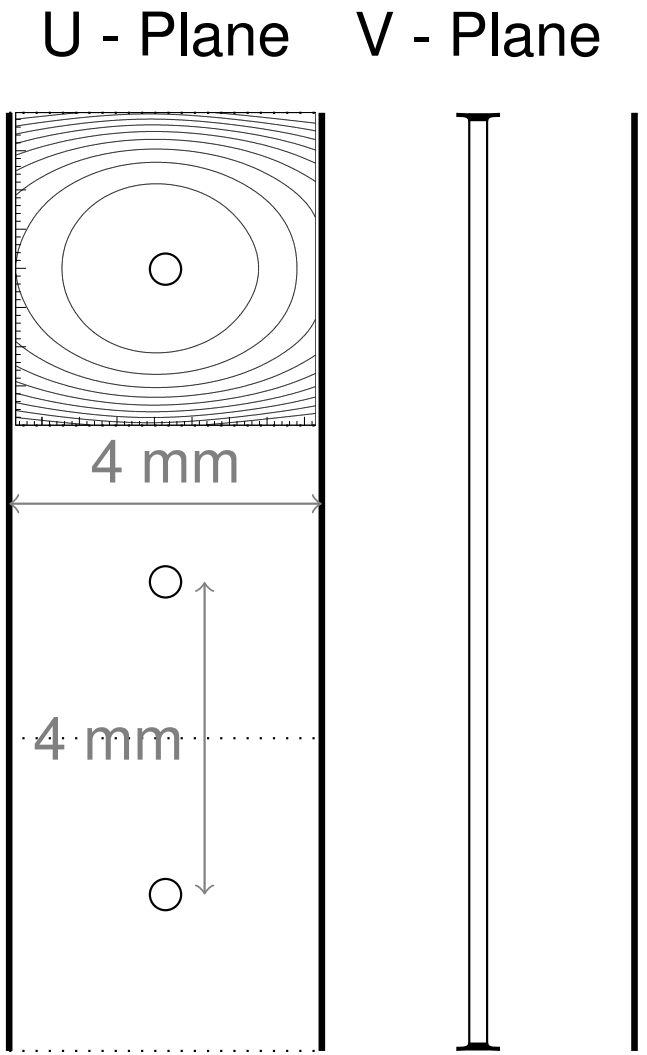
TWIST Spectrometer

- Dimensions known to parts in 10^5.
- 2 T Magnetic Field known to 0.5 G.



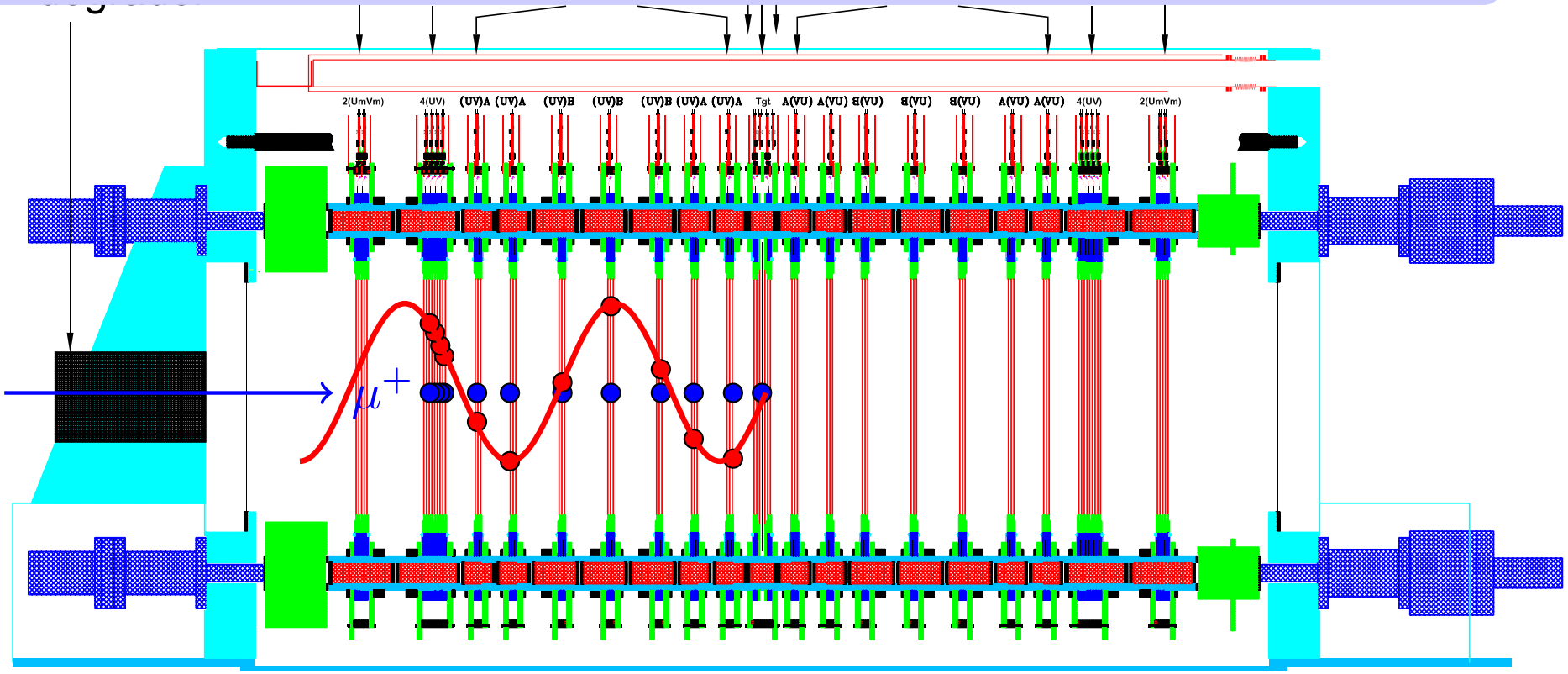
Drift Chambers





Reconstruction of Decay events

- Pattern recognition using PC times and DC wire centres
- Helix fits completed using least squares fit with drift distances



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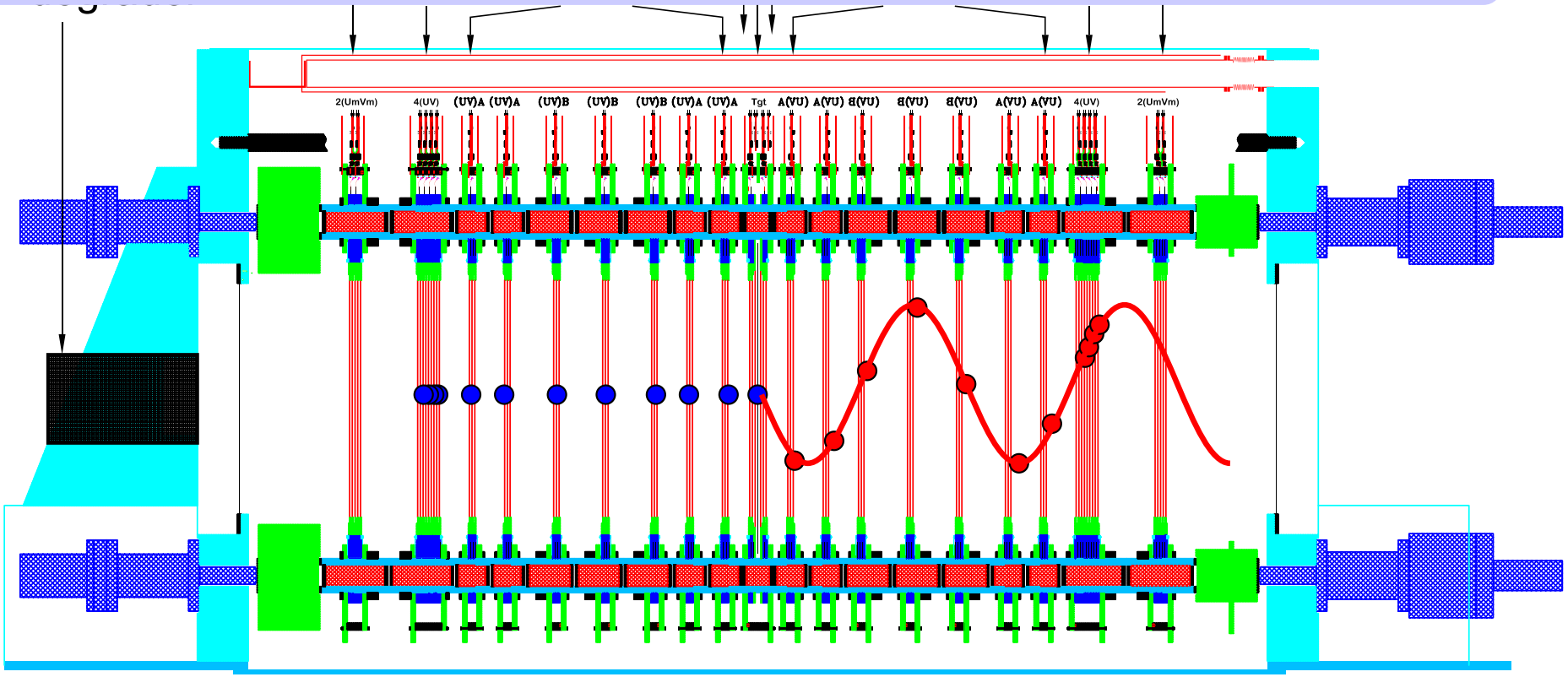


Data Used in Analysis

Silver Target

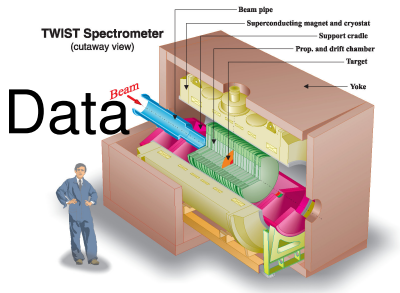
Set	Runs	Description
68	619	Nominal settings
70	855	B=1.96 T
71	771	B=2.04 T
72	979	TEC in data set
74	549	Nominal settings
75	838	Nominal settings
76	689	Mis-steered data set

Aluminum Target

Set	Runs	Description
83	974	Nominal with DS beam package
84	874	Nominal without DS beam package
86	119	Mis-steered
87	908	Nominal settings
91	241	Low Momentum
92	316	Low Momentum
93	533	Low Momentum

- data run $\approx$ 800000 events
- Total $\approx 10^{10}$ events

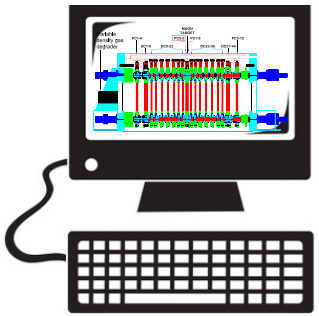
TWIST Analysis Overview



Data

ρ_0, δ_0, ξ_0

Simulation



e^+ Track Analysis

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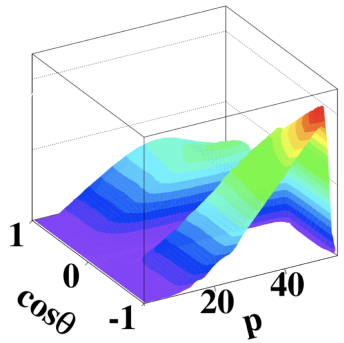
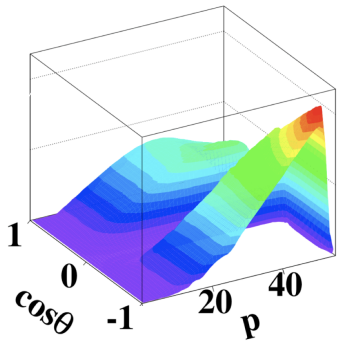


DATA - MC Fit

$\Delta\rho, \Delta\delta, \Delta\xi$



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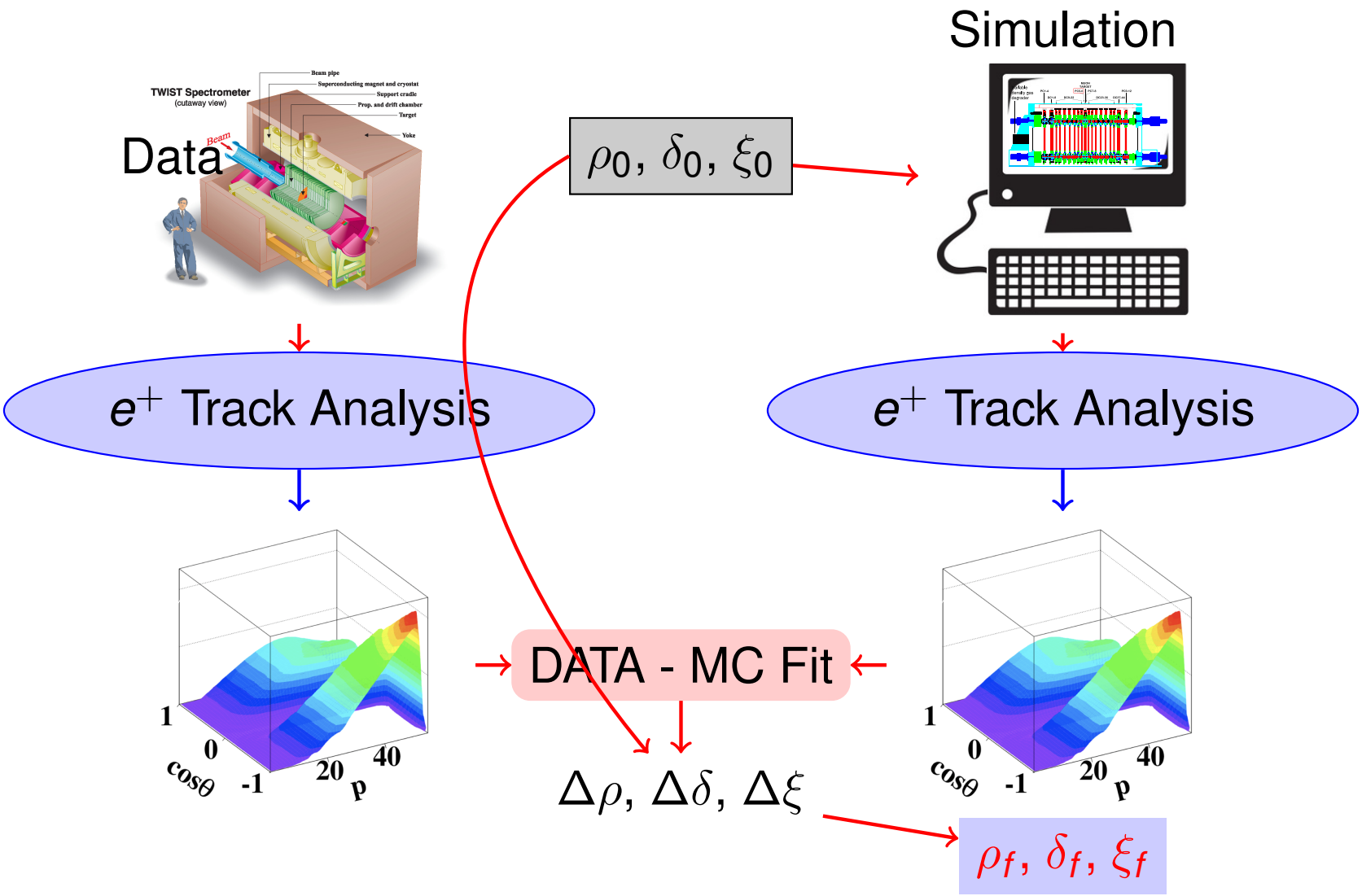
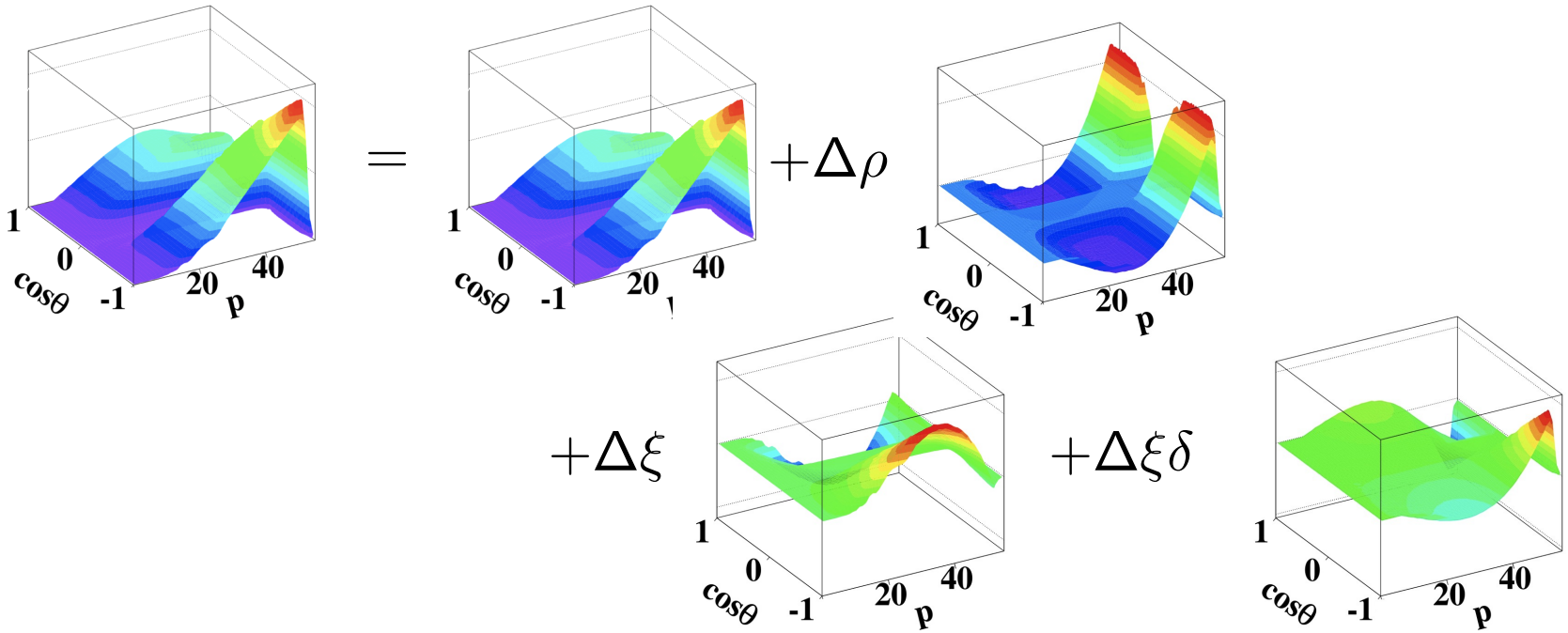


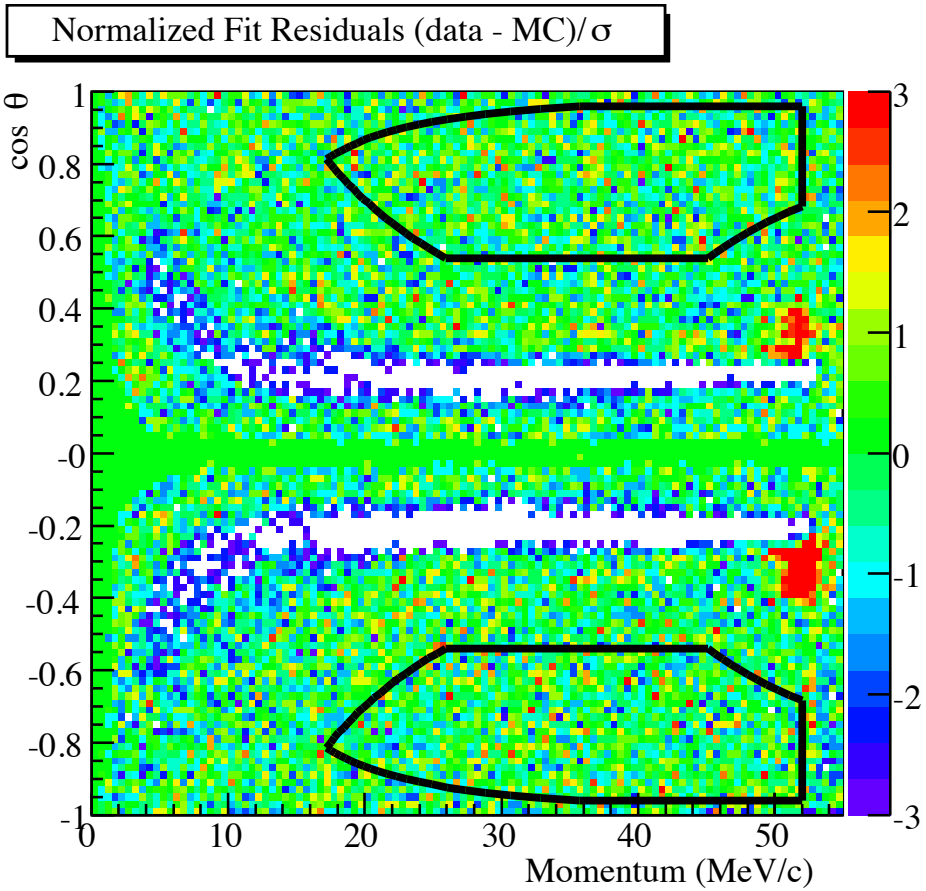
Spectrum Fits

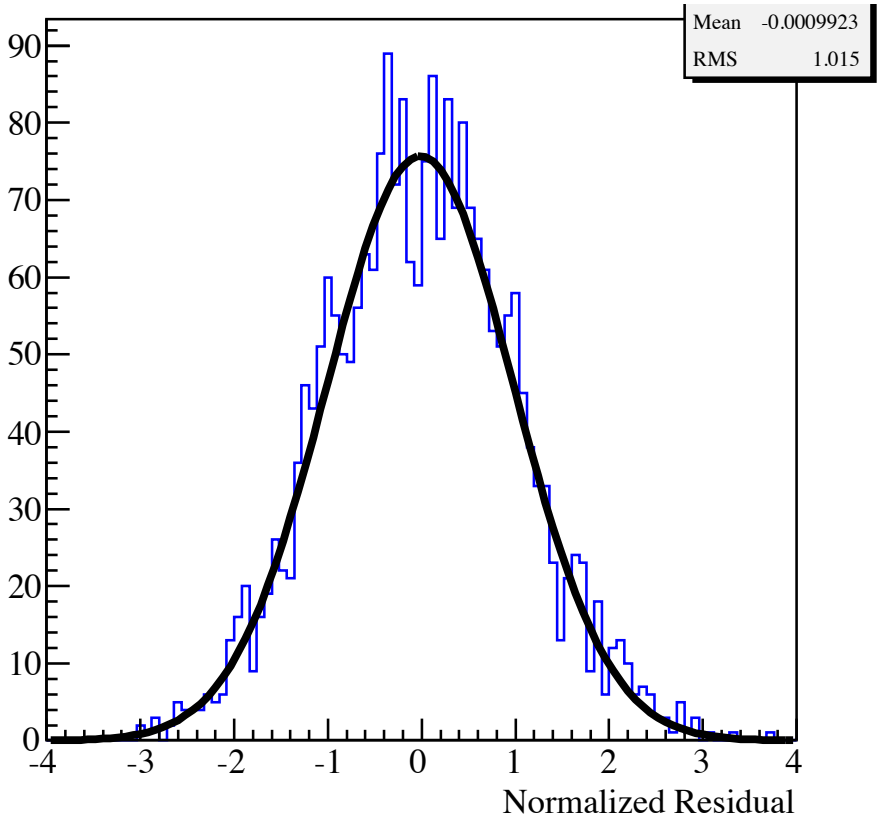
- Sum of simulated spectra used as fitting function



- Parameters minimized using a χ^2 statistic.

Fit Quality

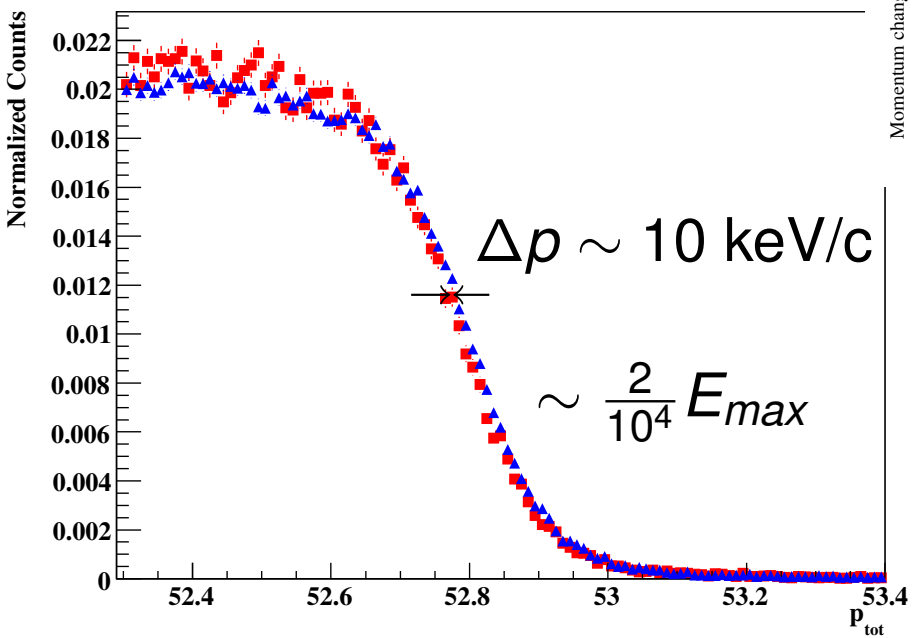


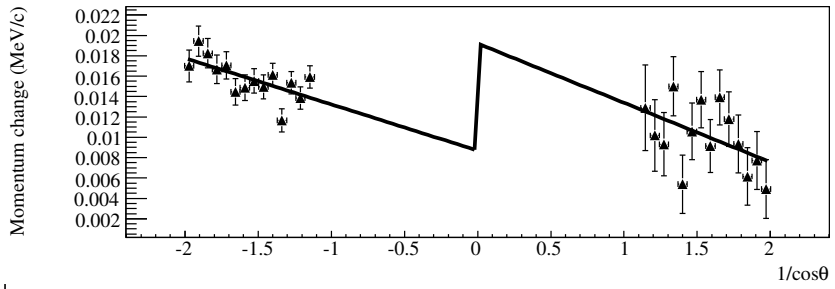


- All data sets: 0.5×10^9 events used in fits
- Simulation composed of 2.7 times data statistics

Endpoint Calibration

- Determine differences between data and sim endpoint spectra
- Calibration applied to correct for differences

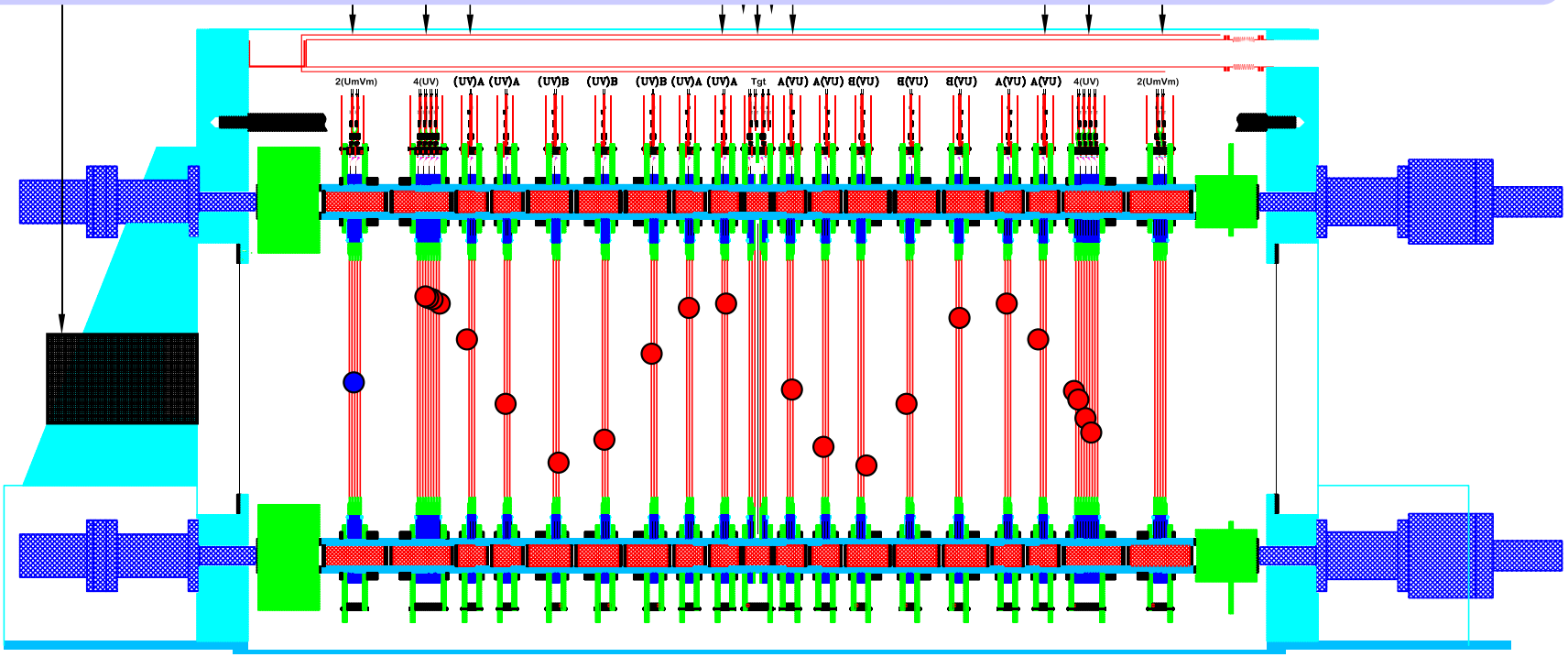




- Two different corrections used:
  1. Shift: Offset of Spectrum
  2. Scale: Deviation increases as p
- Final result is an average of these

Monte Carlo Validation: Upstream Stops

- Stop muon in upstream PCs
- Fit positron tracks upstream and downstream of target
- Physics independent characterization of detector system



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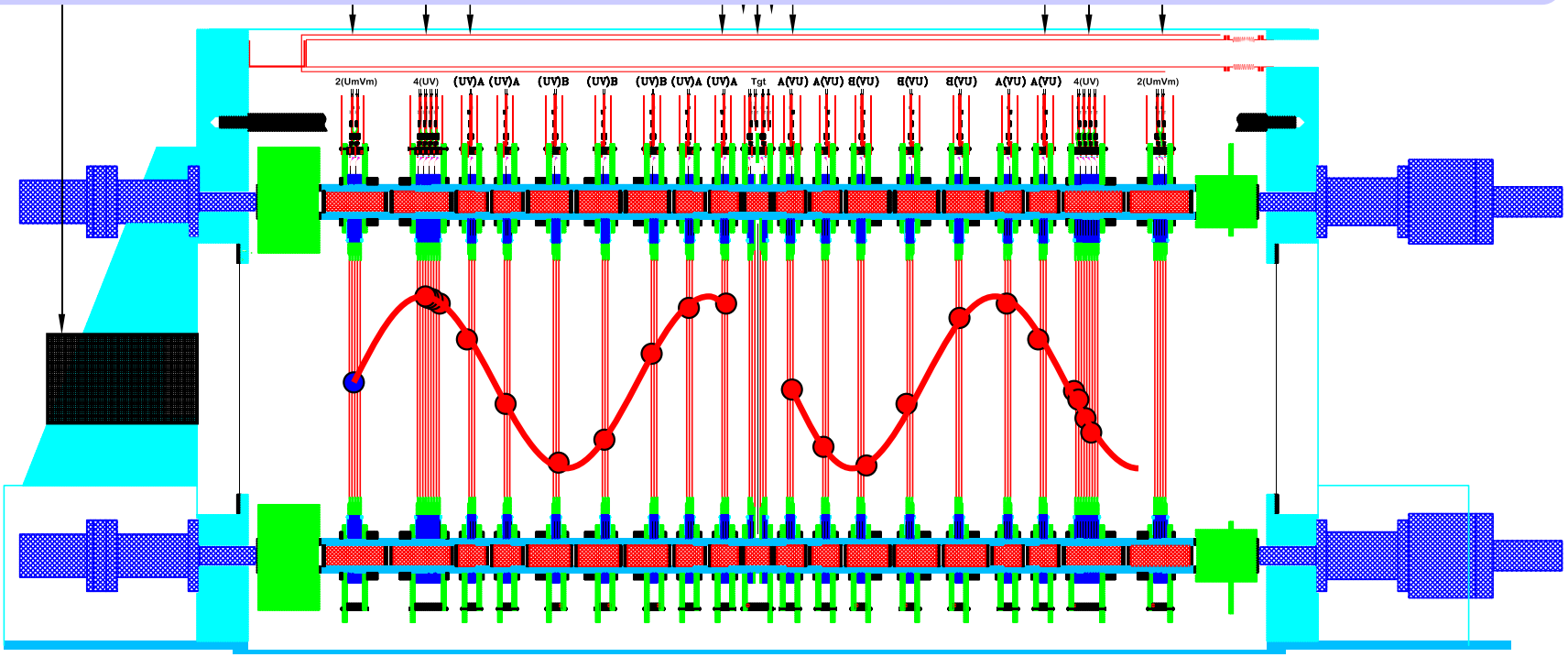
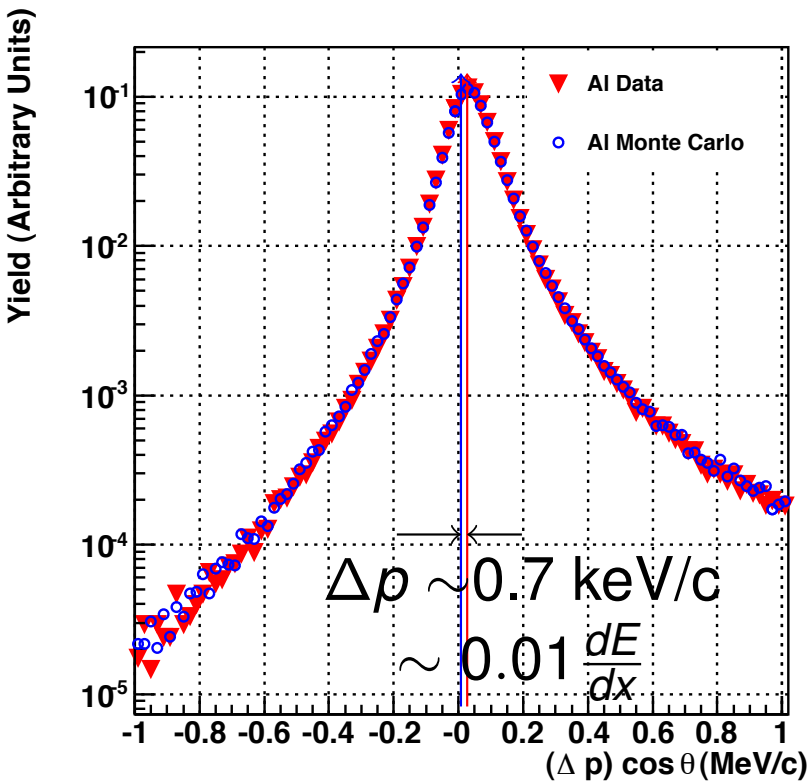
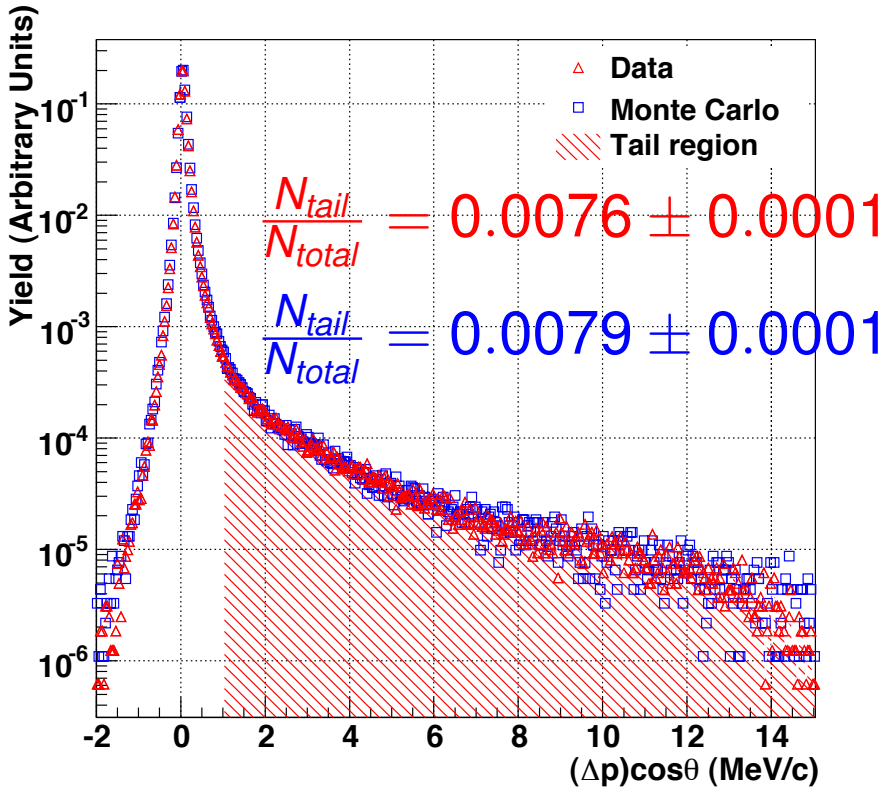


Upstream Stops: Momentum Response

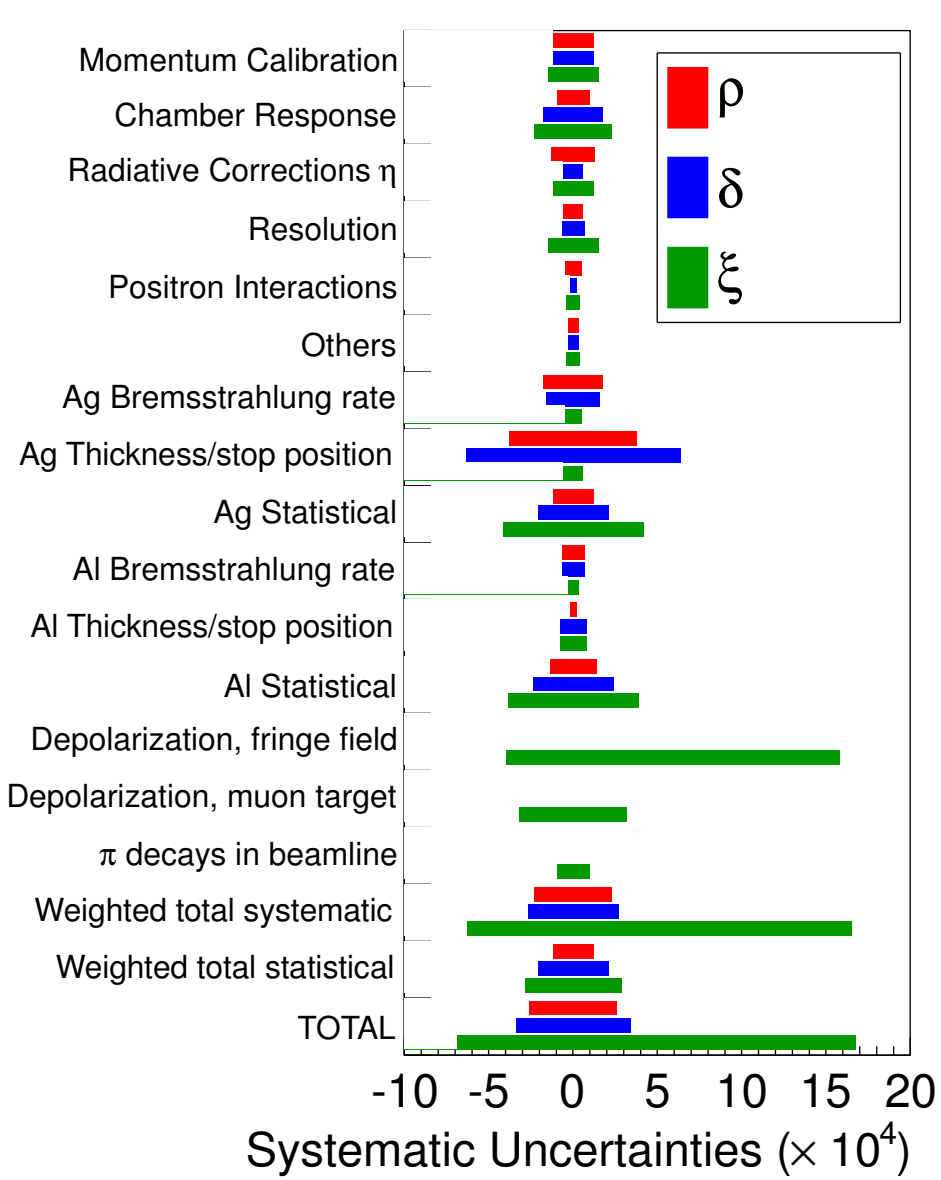
- Small differences in Peak Momentum Response





- GEANT 3 simulation not tuned to produce matches

Systematics Summary



Systematics categorized as

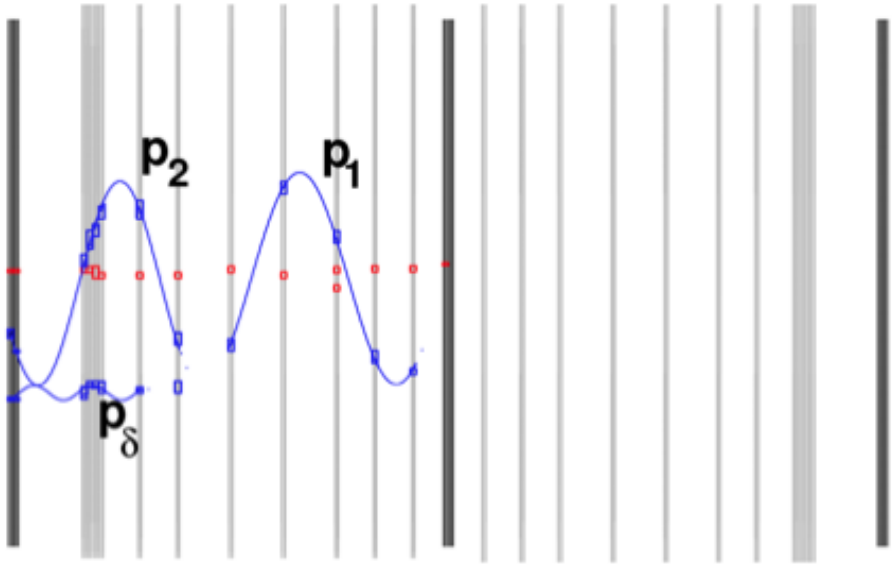
- Common
- Silver target only
- Aluminium target only
- P_μ^π specific

Measured Results

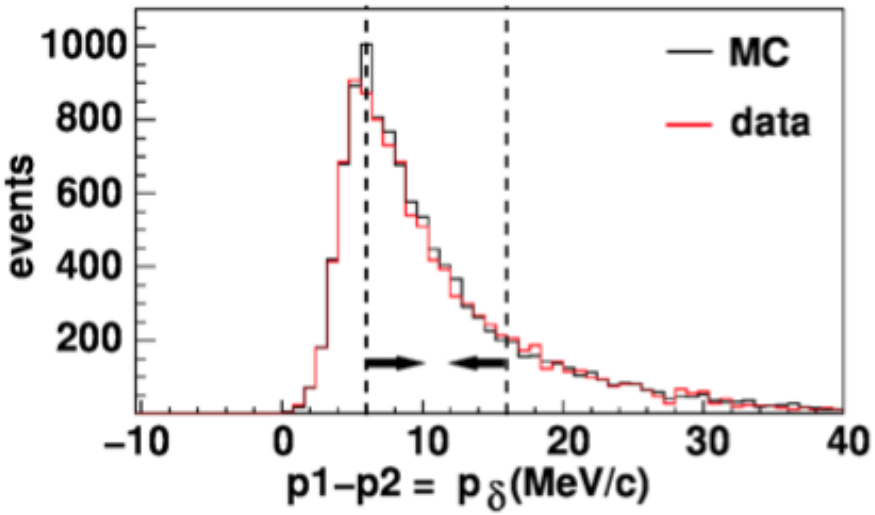
	Units of $\times 10^{-4}$		
	Ave. Diff.	Stat.	Sys
ρ	95.1	± 1.2	± 2.3
δ	51.3	± 2.1	± 2.7
ξ	80.3	± 2.9	$^{+16.5}_{-6.3}$

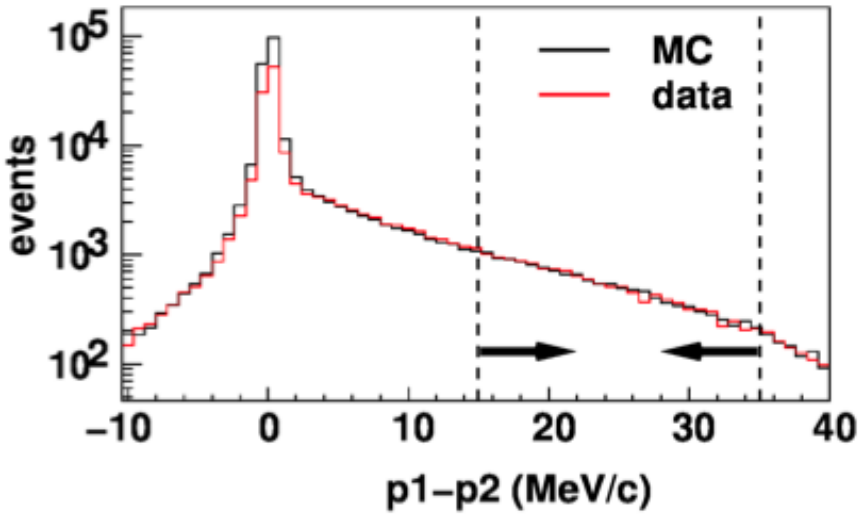
Positron Interactions in Detector

"Hard" momentum loss determined from broken tracks

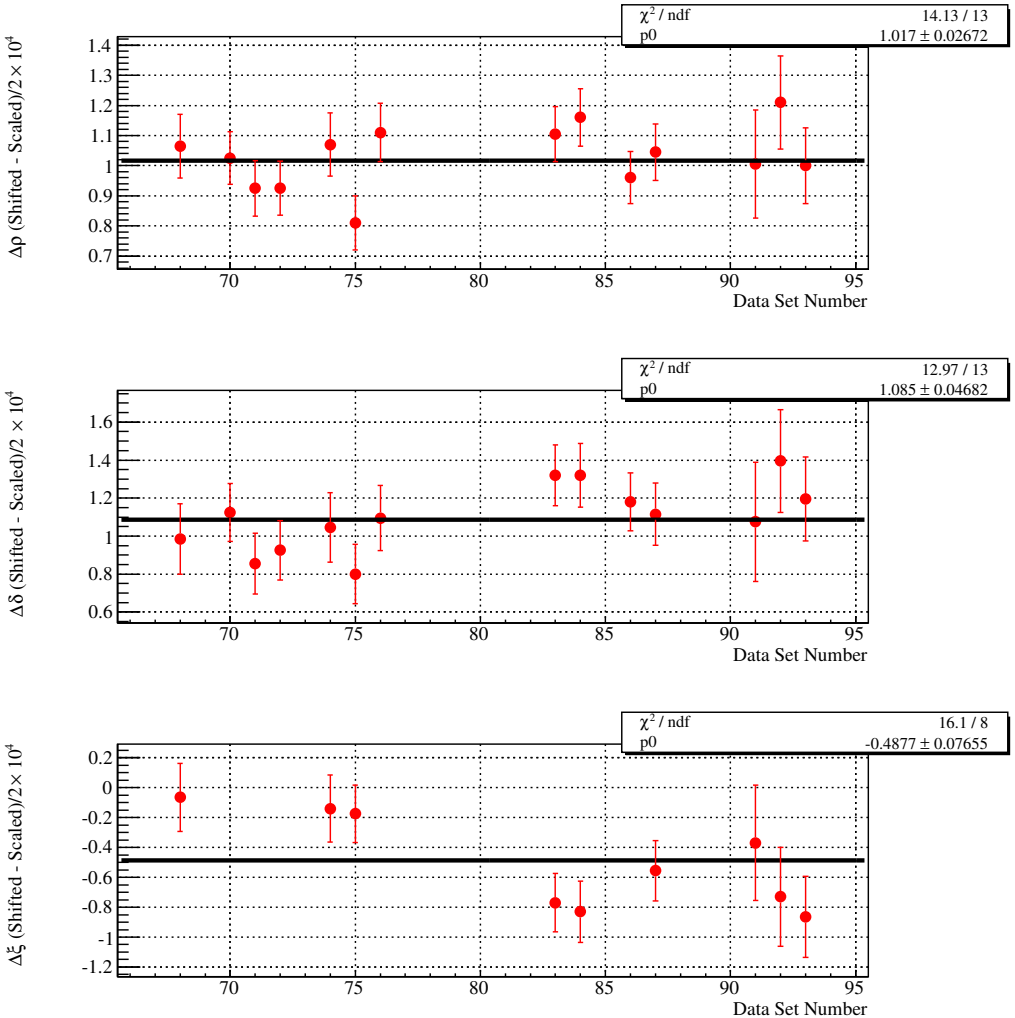


- δ-ray production:
  - 2 e^+ and 1 e^-
  - $<$ 1% difference between data and MC
- bremsstrahlung:
  - 2 e^+ tracks
  - 2.4% difference between data and MC





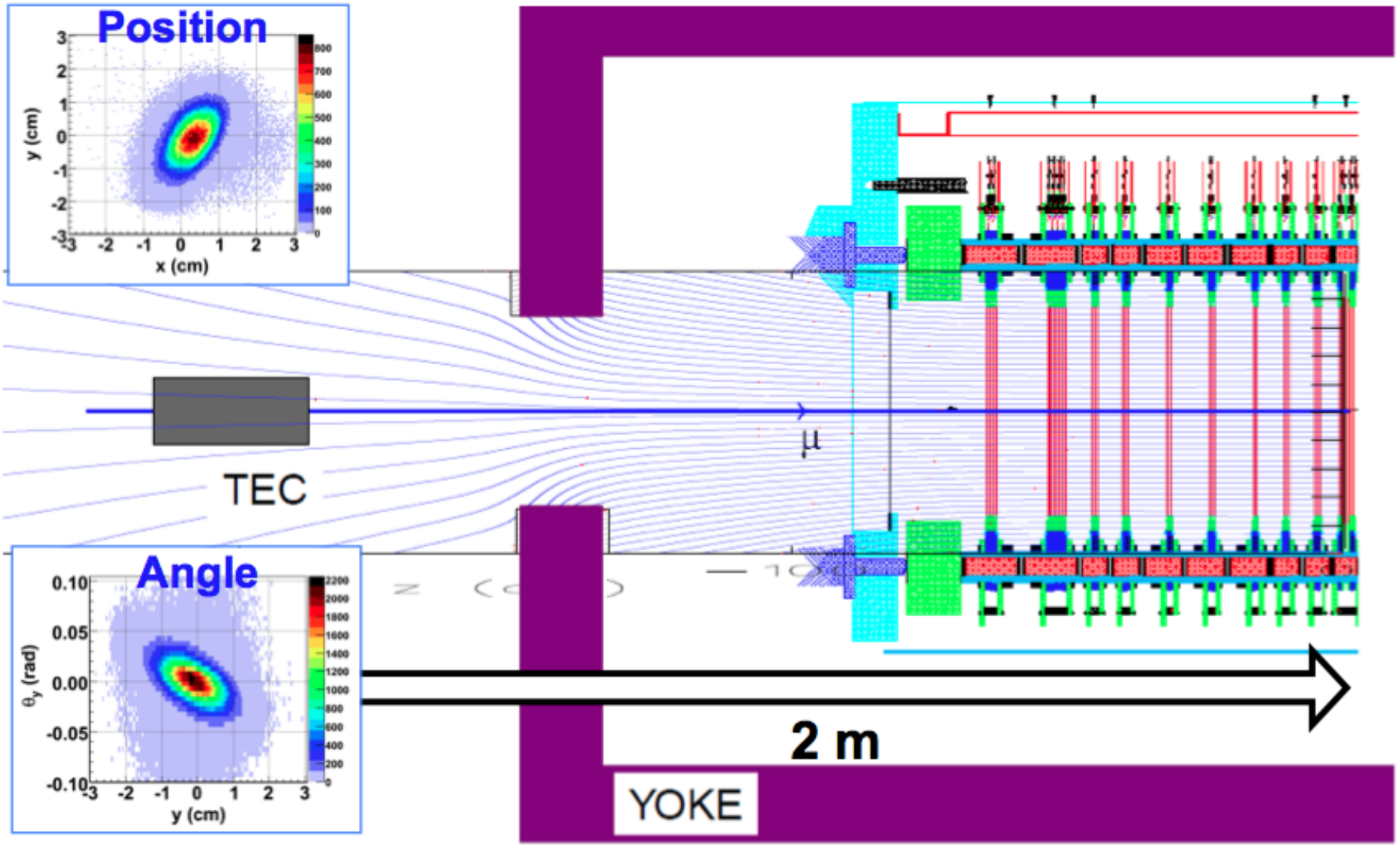
Sensitivity to Calibration Effects



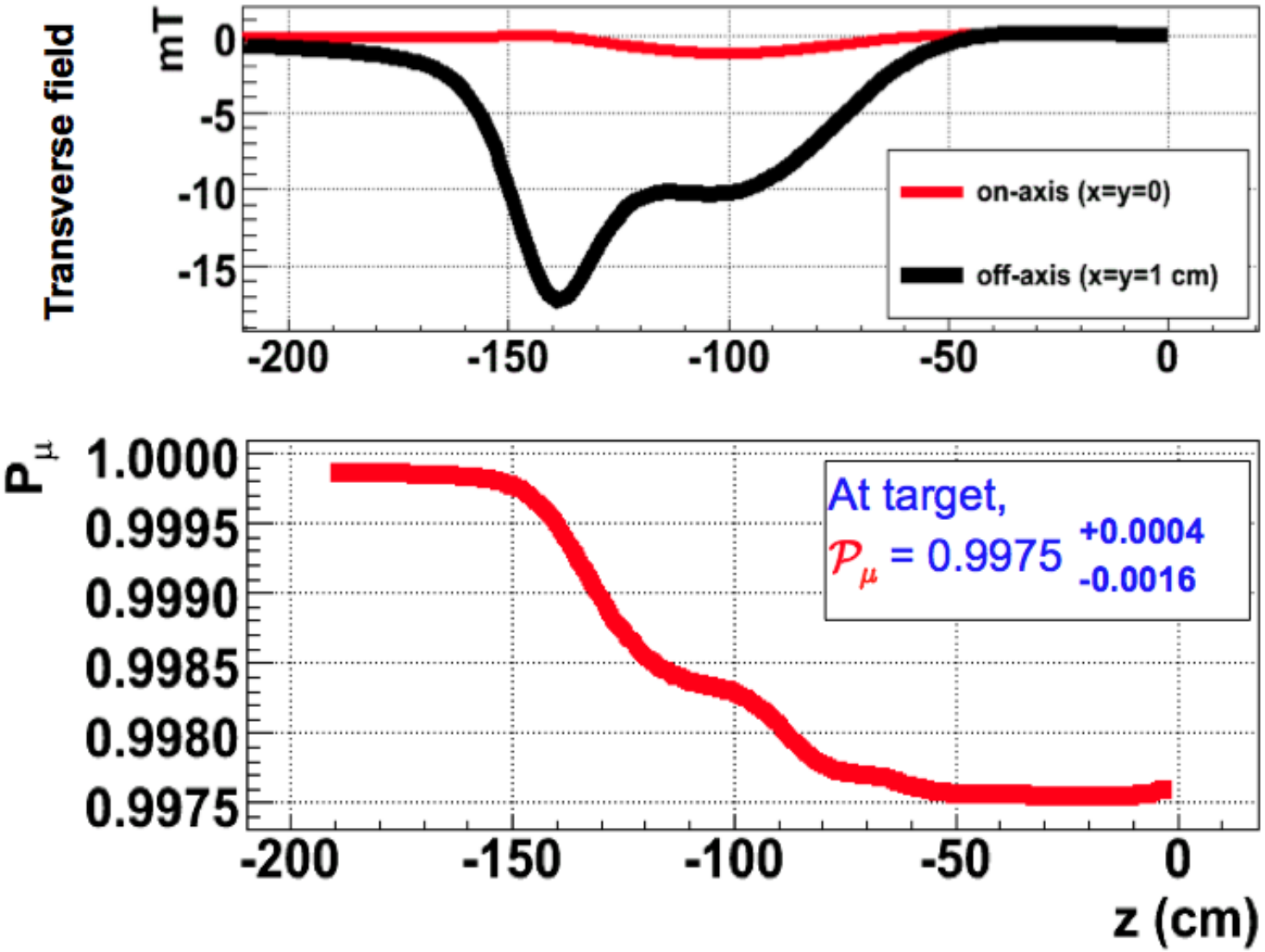
- Leading Contribution: Momentum Dependence
- All data calibrated with momentum dependent and independent methods
- Half of average difference used

$\Delta\rho$	1.02×10^{-4}
$\Delta\delta$	1.08×10^{-4}
$\Delta P_\mu\xi$	-0.49×10^{-4}

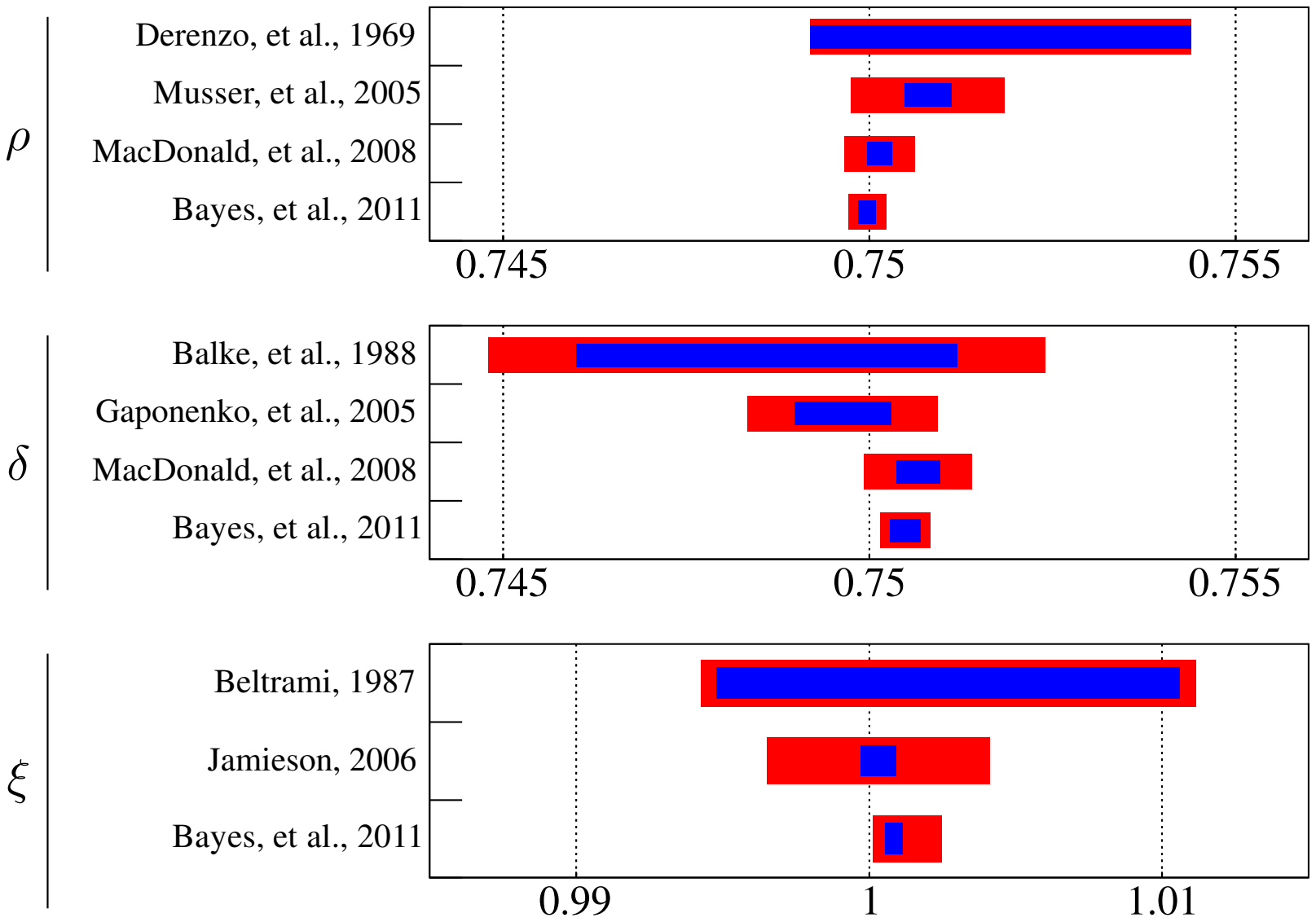
Fringe Field Depolarization



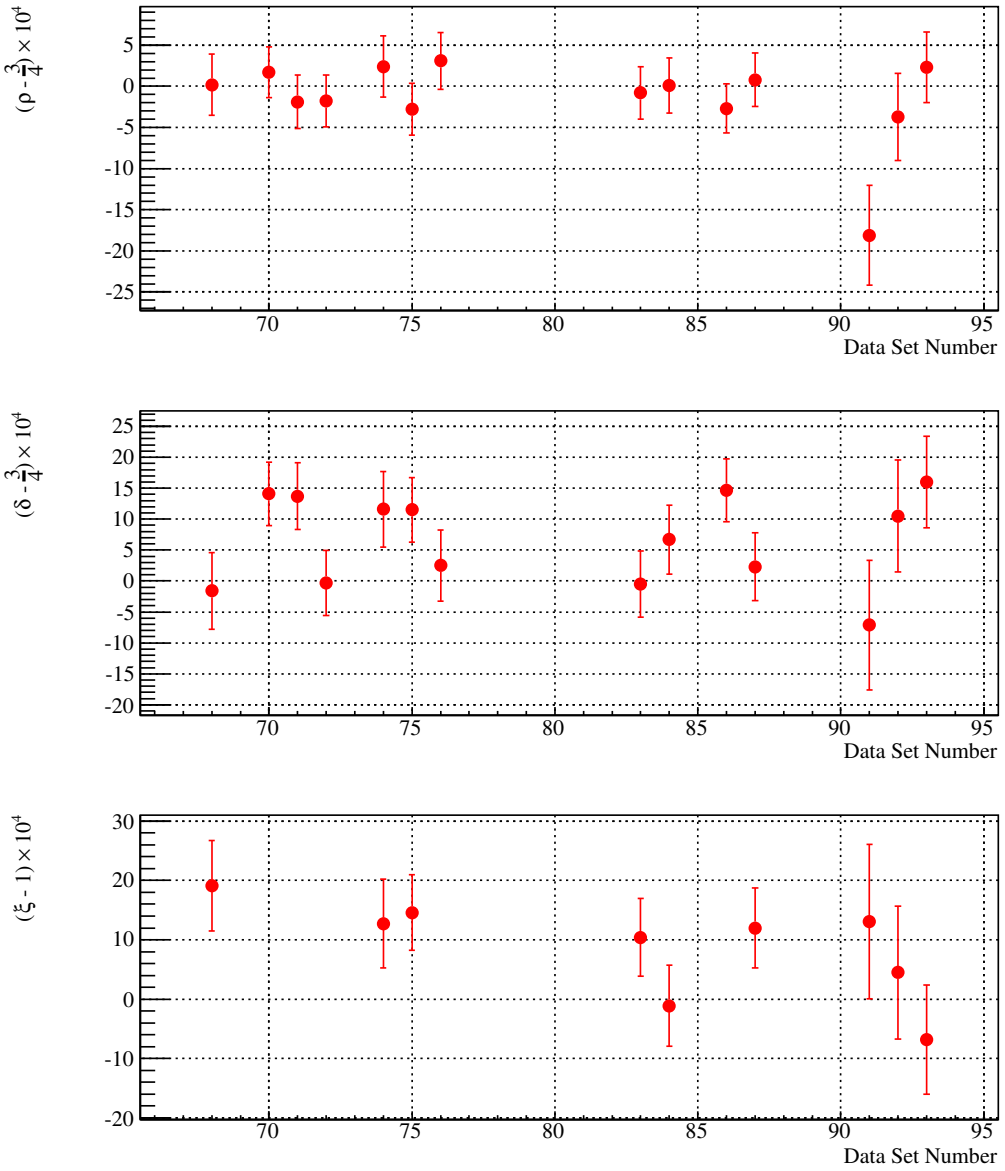
Asymmetric Polarization Uncertainty



Decay Parameters



Consistency of Results



- There is a strong internal consistency of the results.

	χ^2/ndf
ρ	16.5/13
δ	14.8/13
ξ	8.7/8

- Measured Values

$$\rho = 0.74997 \pm 0.00012 \pm 0.00023$$
$$\delta = 0.75049 \pm 0.00021 \pm 0.00027$$
$$P_\mu^\pi \xi = 1.00084 \pm 0.00029^{+0.00165}_{-0.00063}$$

Revision Due to $P_\mu^\pi \xi\delta/\rho$

Endpoint Anisotropy

$P_\mu^\pi \xi\delta/\rho = 1.00179^{+0.00156}_{-0.00063}$
> 0.99909 (90% C.L.)

- $P_\mu^\pi \xi\delta/\rho > 1$ by 2.9σ

- $P_\mu^\pi \xi\delta/\rho$ changed in Ag and Al targets by 3.9 σ

- Prompted review of systematics after black box opening

Changes in the Revised analysis

- Motivated categorization of systematics
- Corrected parameter weighting
- Identified systematics from mean stopping position

Change between blind and revised results

	Units of $\times 10^4$	
	Value	σ_{total}
ρ	-1.4	-0.3
δ	-2.3	+0.1
$P_\mu \xi$	0	-0.2

Global Analysis

e^+ spectrum measurements are a subset of muon decay parameters

Parameter	Value	Reference
Current TWIST decay parameters		
ρ	0.74997 ± 0.00028	
δ	0.75049 ± 0.00033	
ξ	$1.00084^{+16.9}_{-11.9}$	
Previous decay parameters		
ρ	0.7518 ± 0.0026	PDG average (2003)
δ	0.7486 ± 0.0038	Balke,1988
$P_\mu \xi$	1.0027 ± 0.0085	Beltrami,1987
$P_\mu \xi \delta / \rho$	0.99787 ± 0.00082	Jodidio,1986
Parameters from positron Polarization		
ξ'	1.00 ± 0.04	PDG average (2003)
ξ''	0.65 ± 0.36	Burkard,1985
$\bar{\eta}$	0.02 ± 0.08	PDG average (2003)
α / A	0.015 ± 0.052	Burkard,1985
β / A	0.002 ± 0.018	Burkard,1985
η	0.071 ± 0.037	Danneberg,2005
η''	0.105 ± 0.052	Danneberg,2005
α' / A	-0.047 ± 0.052	Burkard,1985
	-0.0034 ± 0.0219	Danneberg,2005
β' / B	0.017 ± 0.018	Burkard,1985
	-0.0005 ± 0.00080	Danneberg,2005

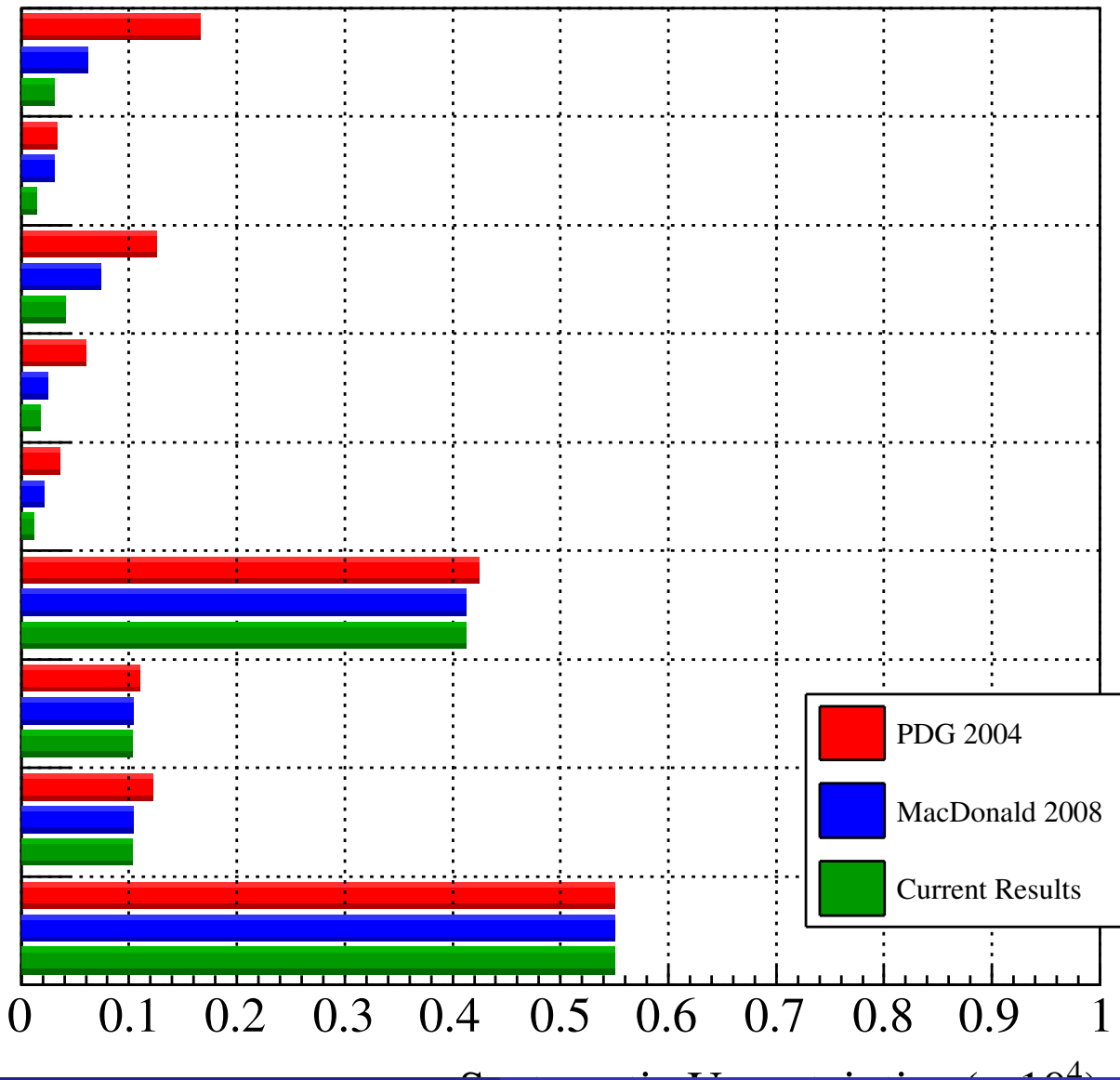
- Required for limits on interaction probabilities and coupling constants

Interaction Probabilities

	2008 ($\times 10^{-3}$)	2011 ($\times 10^{-3}$)
Q_{RR}	< 0.96	< 0.24
Q_{LR}	< 1.38	< 0.42
Q_{RL}	< 42	< 42
Q_{LL}	> 955	> 955

Coupling Constants

	2004	2008	2010
$\lvert g^S_{RR} \rvert$	0.166	0.062	0.031
$\lvert g^V_{RR} \rvert$	0.033	0.031	0.015
$\lvert g^S_{LR} \rvert$	0.125	0.074	0.041
$\lvert g^V_{LR} \rvert$	0.060	0.025	0.018
$\lvert g^T_{LR} \rvert$	0.036	0.021	0.012
$\lvert g^S_{RL} \rvert$	0.424	0.412	0.412
$\lvert g^V_{RL} \rvert$	0.110	0.104	0.103
$\lvert g^T_{RL} \rvert$	0.122	0.104	0.103
$\lvert g^S_{LL} \rvert$	0.550	0.550	0.550



Left - Right Symmetric Models

$$W_L = \cos\zeta W_1 + \sin\zeta W_2 \qquad W_R = e^{i\omega}(-\sin\zeta W_1 + \cos\zeta W_2)$$

- $W_{R(L)}$ mediate $V + A(V - A)$ currents[4]
- ζ is the mixing angle between W_1 and W_2
- ω CP violating phase

Decay Parameters in This Model

$$\rho \simeq \frac{3}{4}\left(1 - 2\left(\frac{g_R}{g_L}\right)^2\zeta^2\right), \delta \equiv \frac{3}{4}, \ \xi \simeq 1 - 2\left(\left(\frac{g_R m_1}{g_L m_2}\right)^4 + \left(\frac{g_R}{g_L}\right)^2\zeta^2\right)$$

$$1 - \frac{P_\mu^\pi \xi\delta}{\rho} \simeq 2\frac{g_R^4 m_1^4}{g_L^4 m_2^4}\left(1 + \frac{\cos^2\theta_1^R}{\cos^2\theta_1^L}\right) + 2\frac{g_R^2}{g_L^2}\zeta^2 + 4\frac{g_R^3 m_1^2 \cos\theta_1^R}{g_L^3 m_2^2 \cos\theta_1^L}\zeta\cos(\alpha+\omega)$$

[4] P. Herczeg, **PRD** 34,3449,(1986)

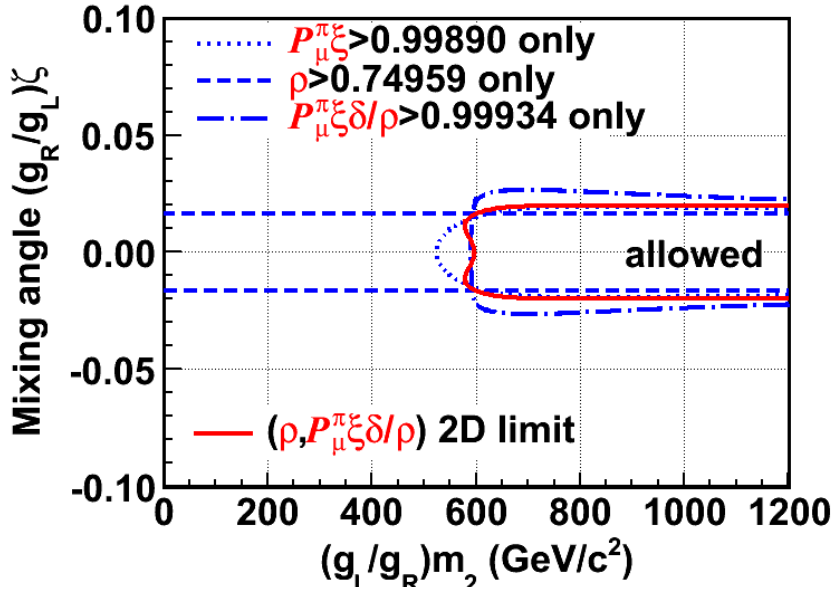
Left-Right Symmetric Models

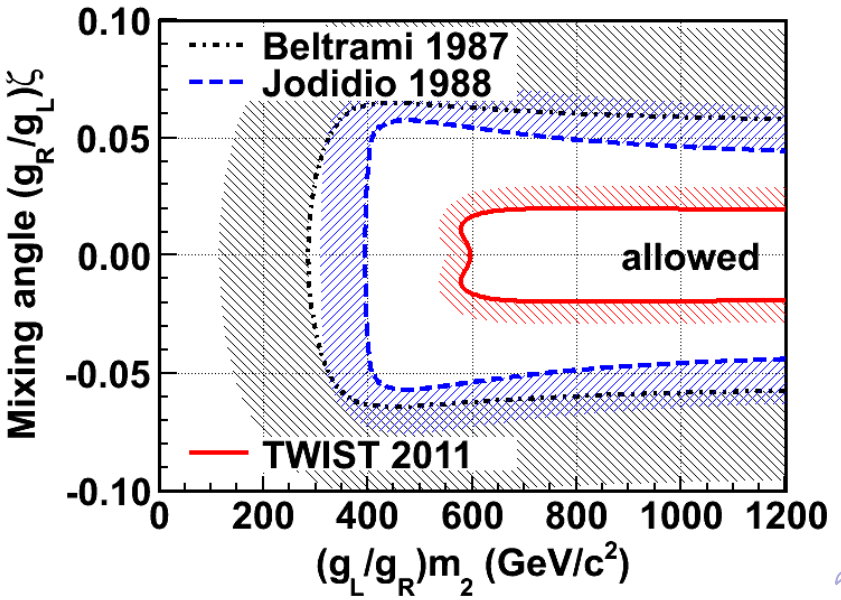
90% Confidence limits

- $|\frac{g_R}{g_L}\zeta| < 0.02$
- $|\frac{g_R}{g_L}|m_2 > 578$ GeV/c
- Set using a combination of 90% limits on ρ, and $P_\mu \xi\delta/\rho$

Generalized approach to model

- No assumption of model parameters
- W_2 Direct searches assume $g_R = g_L, \omega = 0$





Conclusions

- Order of magnitude improvement in precision of decay parameters has been completed by the TWIST experiment

$$
\begin{aligned}
\rho &= 0.74997 \pm 0.00012 \pm 0.00023 \\
\delta &= 0.75049 \pm 0.00021 \pm 0.00027 \\
P_\mu \xi &= 1.00084 \pm 0.00029 ^{+0.00165}_{-0.00063} \\
P_\mu \xi \delta / \rho &= 1.00179 ^{+0.00156}_{-0.00071} \\
&> 0.99909 \text{ (90\% C.L.)}
\end{aligned}
$$

- No deviation from the standard model has been detected
- $P_\mu \xi \delta / \rho > 1$ has been investigated; no problem with analysis has been identified

Thank you

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∗∗ Graduated
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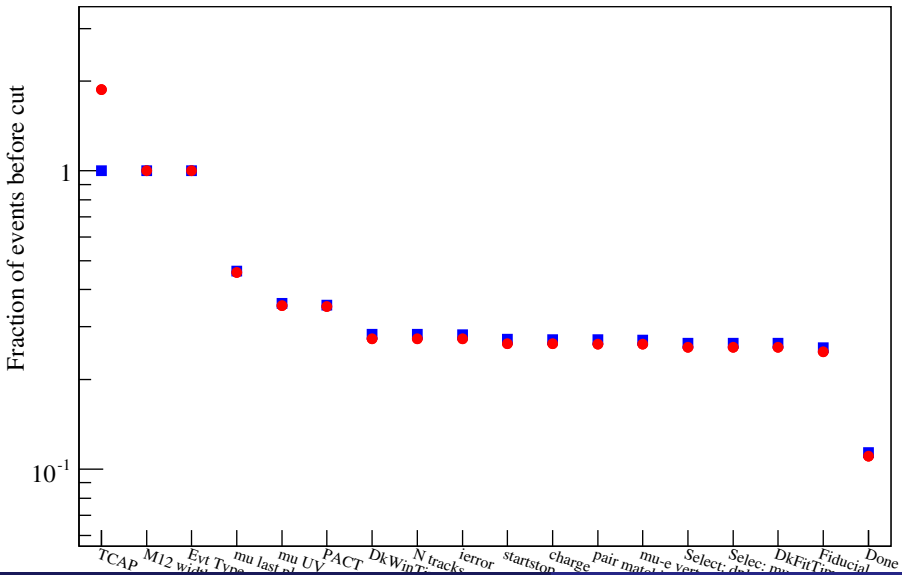
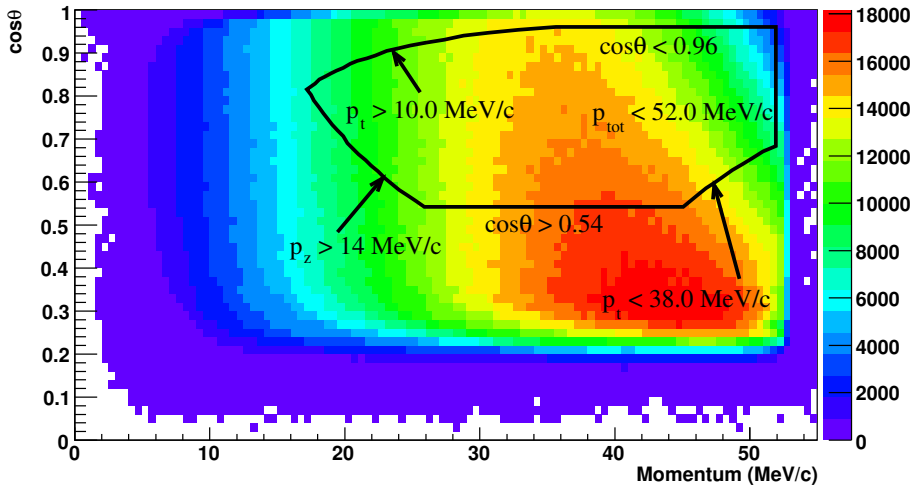
§ deceased

Cuts Imposed on the Analysis

- 17 cuts and selections imposed on events
- Based on geometric and physical constraints of detector system.
- leave just over 10% of events



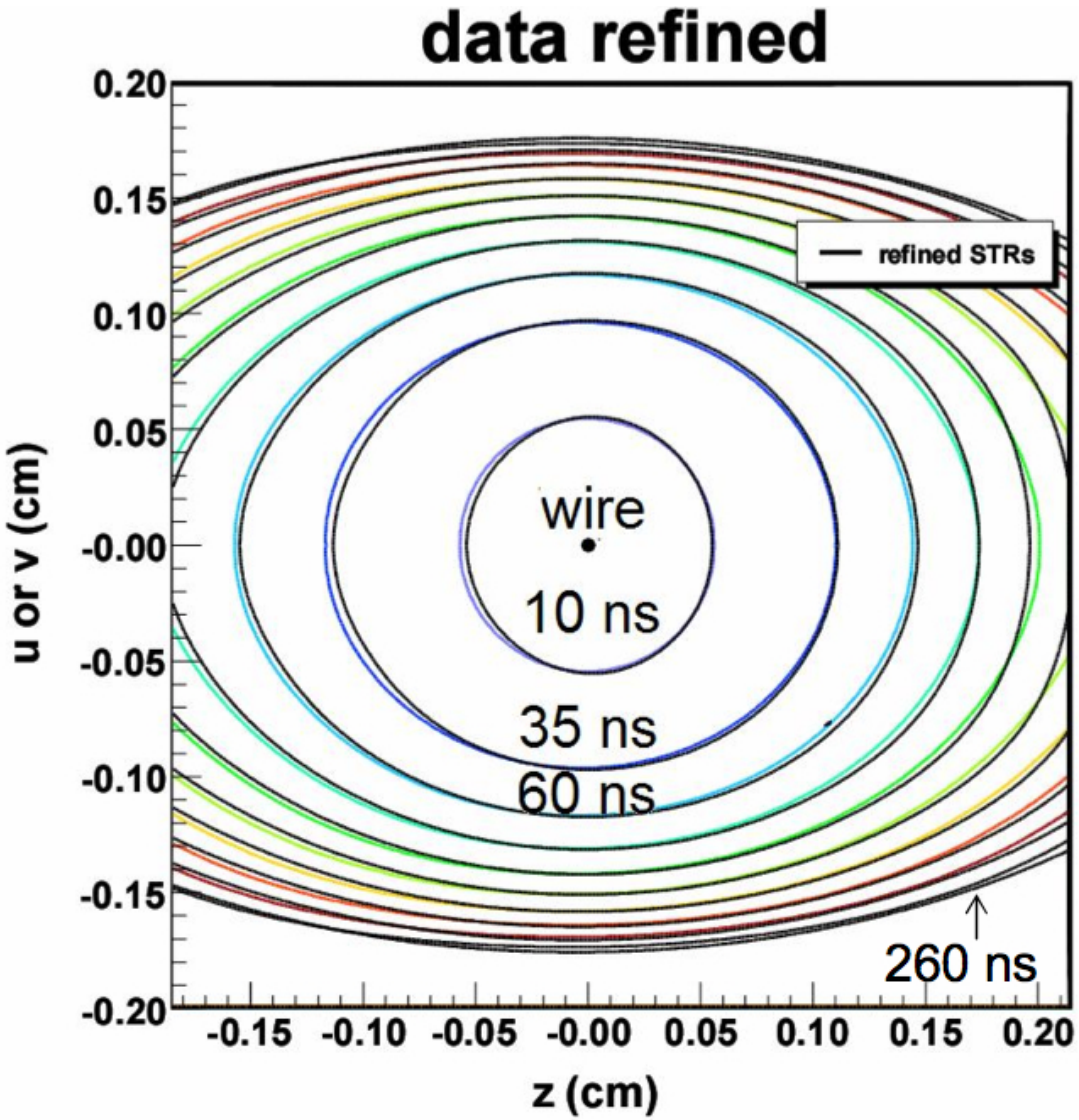
- Example: Kinematic cuts



- Events selected to avoid bias between data and simulation

Chamber Response

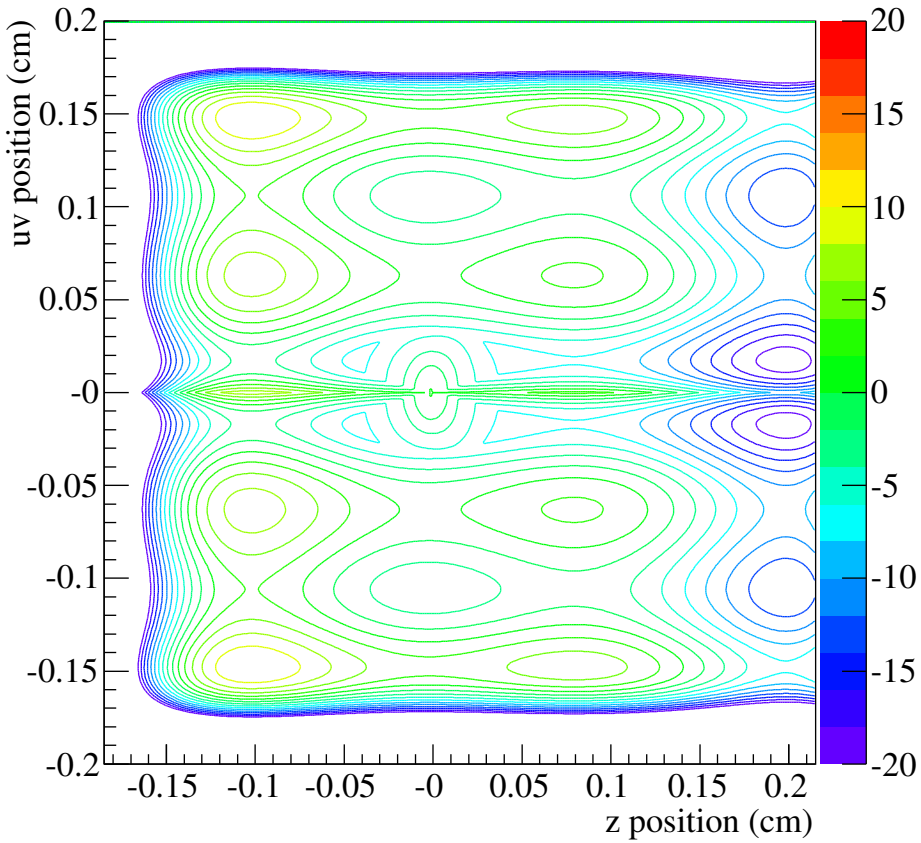
Vast improvement made using data corrected STRs



- Base space time relationships generated using GARFIELD simulation
- Chamber STRs corrected to minimize e^+ track fit time residuals
- Corrects for plane to plane construction differences
- Procedure completed for data and Monte Carlo

Systematic Effect From Chamber Response

Exaggerated - Standard STR



- Exaggerated time residuals between data and MC
- Constructed STRs with the difference multiplied by a factor of 10.

$\Delta\rho$	-0.31 $\pm$ 0.17
$\Delta\delta$	-1.03 $\pm$ 0.29
$\Delta\xi$	0.88 $\pm$ 0.36