
Mu HFS measurement : RF cavity

Y. Matsuda, H.A. Torii, K. Tanaka
(Graduate School of Arts and Sciences, Univ. of Tokyo)



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Purpose of the talk

- ▶ **NOT** reporting the progress of the RF cavity design
- ▶ But **asking for help/advice** for the design of the cavity and the gas chamber.

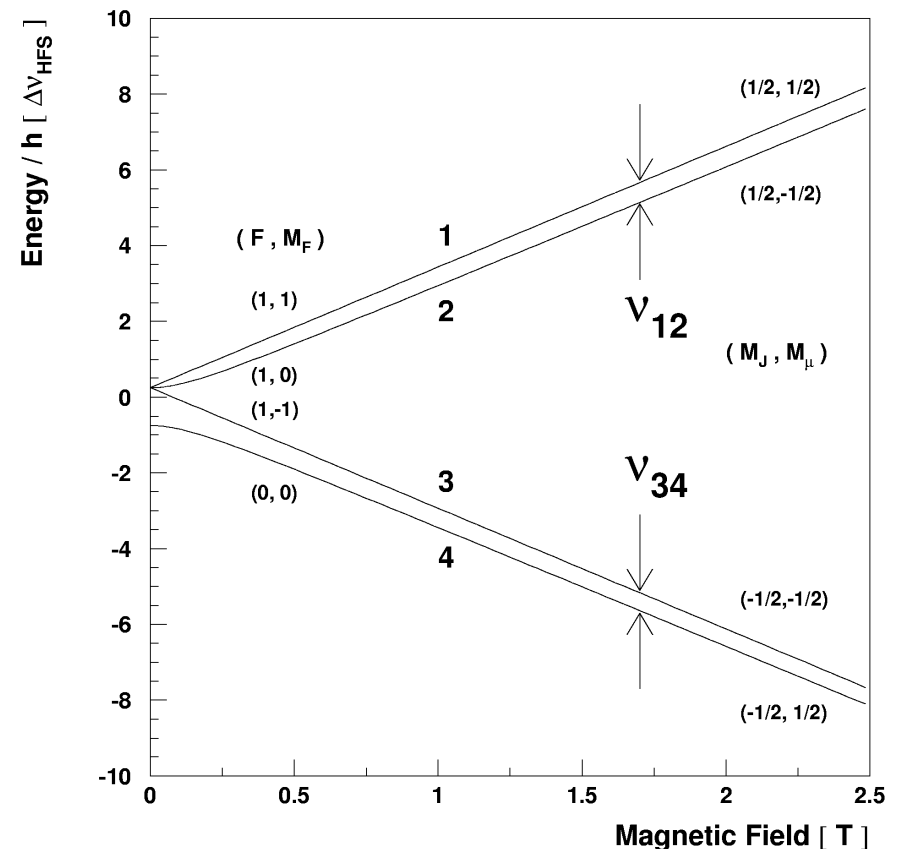


Mu HFS measurement

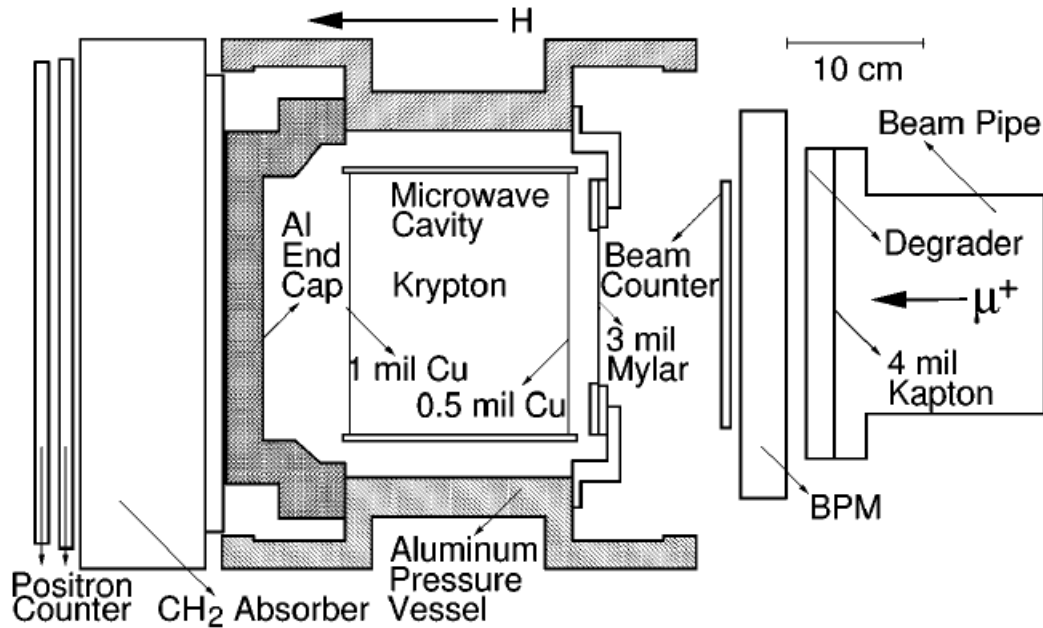
- ▶ Two frequencies (ν_{12} and ν_{34}) will be measured under the same magnetic field.

$$\rightarrow \nu_{\text{HFS}} = \nu_{12} + \nu_{34}$$

- ▶ Both transitions flip muon spin direction, which can be detected by change of positron angle distribution



LAMPF experiment - starting point

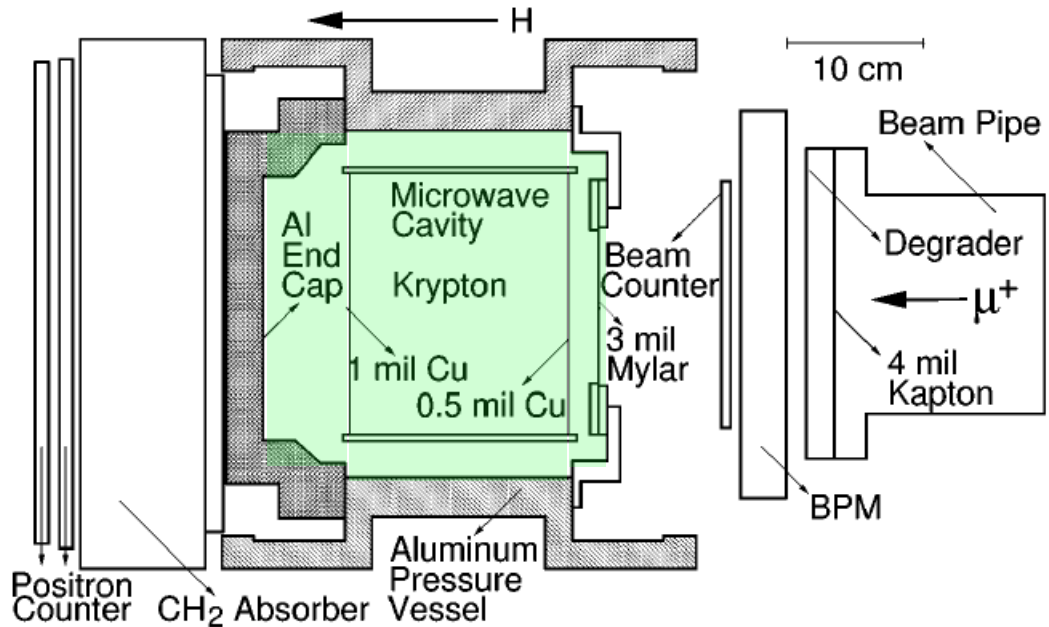


Chamber has a dual role

- ▶ Gas chamber : where muonium is formed
- ▶ RF cavity : where transition happens



Gas chamber



Chamber has a dual role

- ▶ **Gas chamber** : where muonium is formed
- ▶ RF cavity : where transition happens

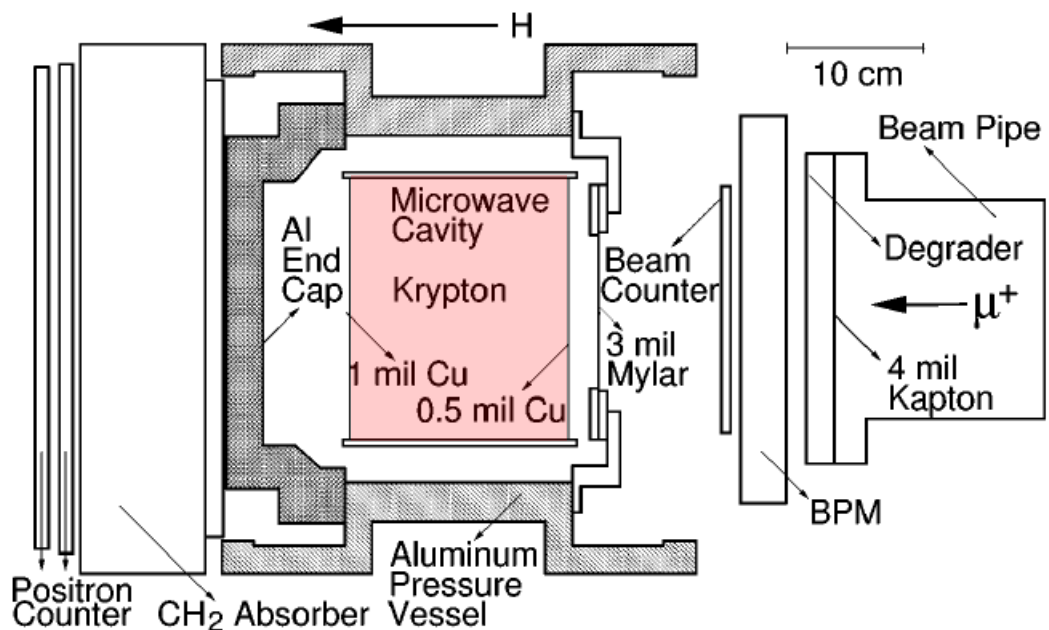
- ▶ Kr gas impurities less than 5ppm
pressure monitored with 3×10^{-4} atm accuracy.

- ▶ Temperature controlled $\pm 1^\circ\text{C}$

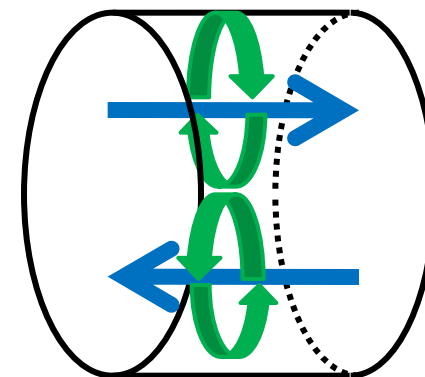
- ▶ NMR probe for monitoring field

- ▶ **We need to do same or better.**

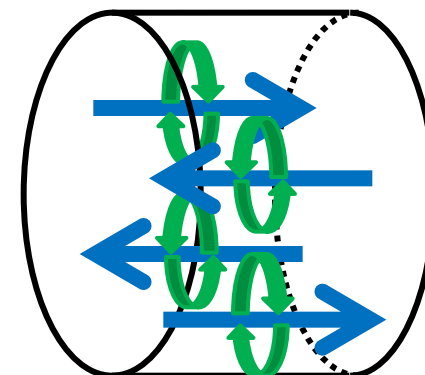
RF cavity



ν_{12} 1925.0MHz in TM₁₁₀



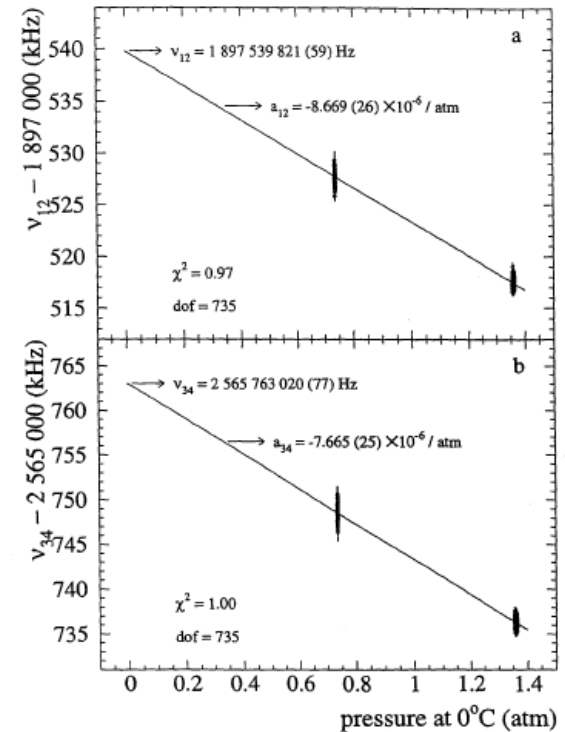
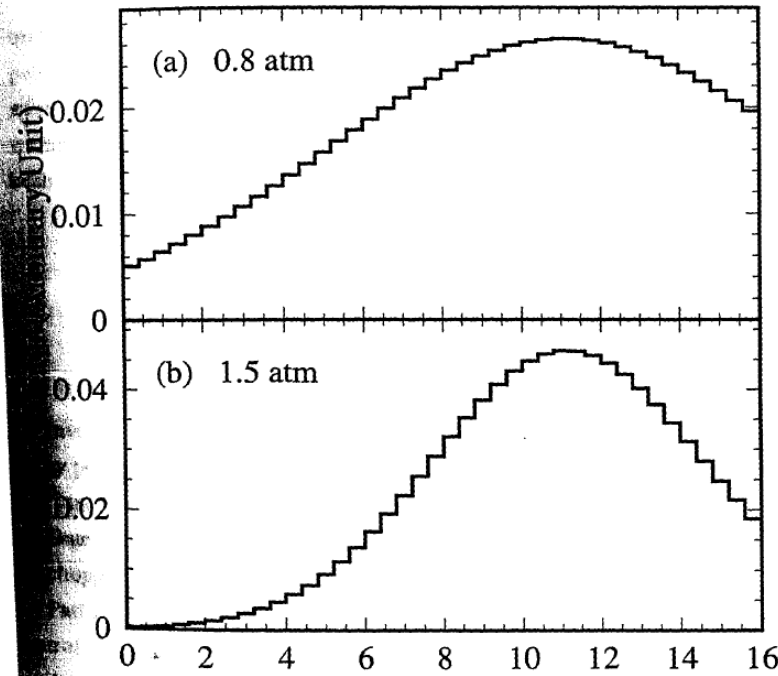
ν_{34} 2581.3MHz in TM₂₁₀



Chamber has a dual role

- ▶ Gas chamber : where muonium is formed
- ▶ **RF cavity : where transition happens**
 - ▶ **uniform field strength along z-axis**

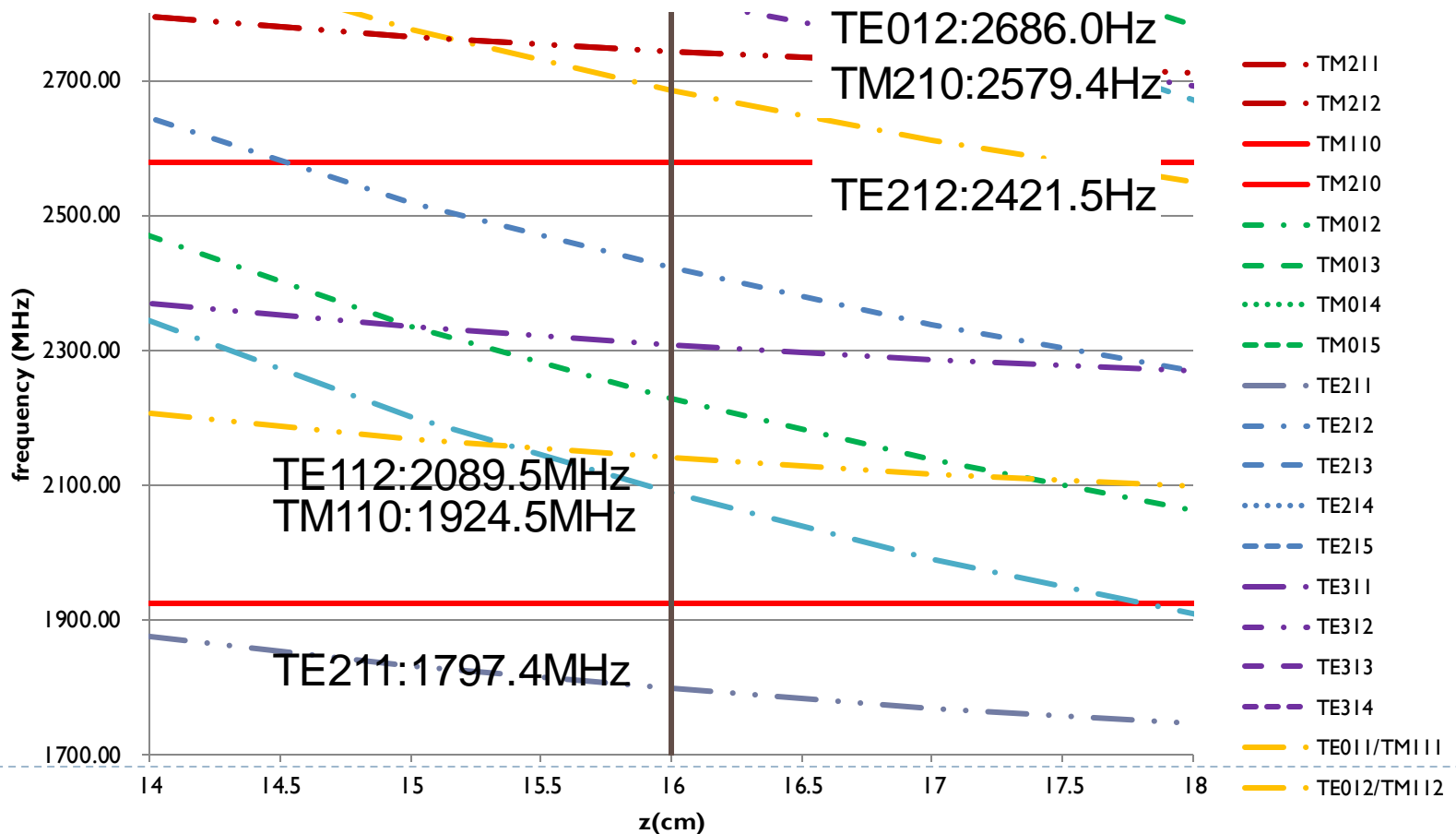
Uniform field along z-axis is important



- ▶ Uniform field strength along z-axis made it easy to compare the results with different gas pressure.
- ▶ **longer cavity** would be beneficial for better compensation of the pressure shift.

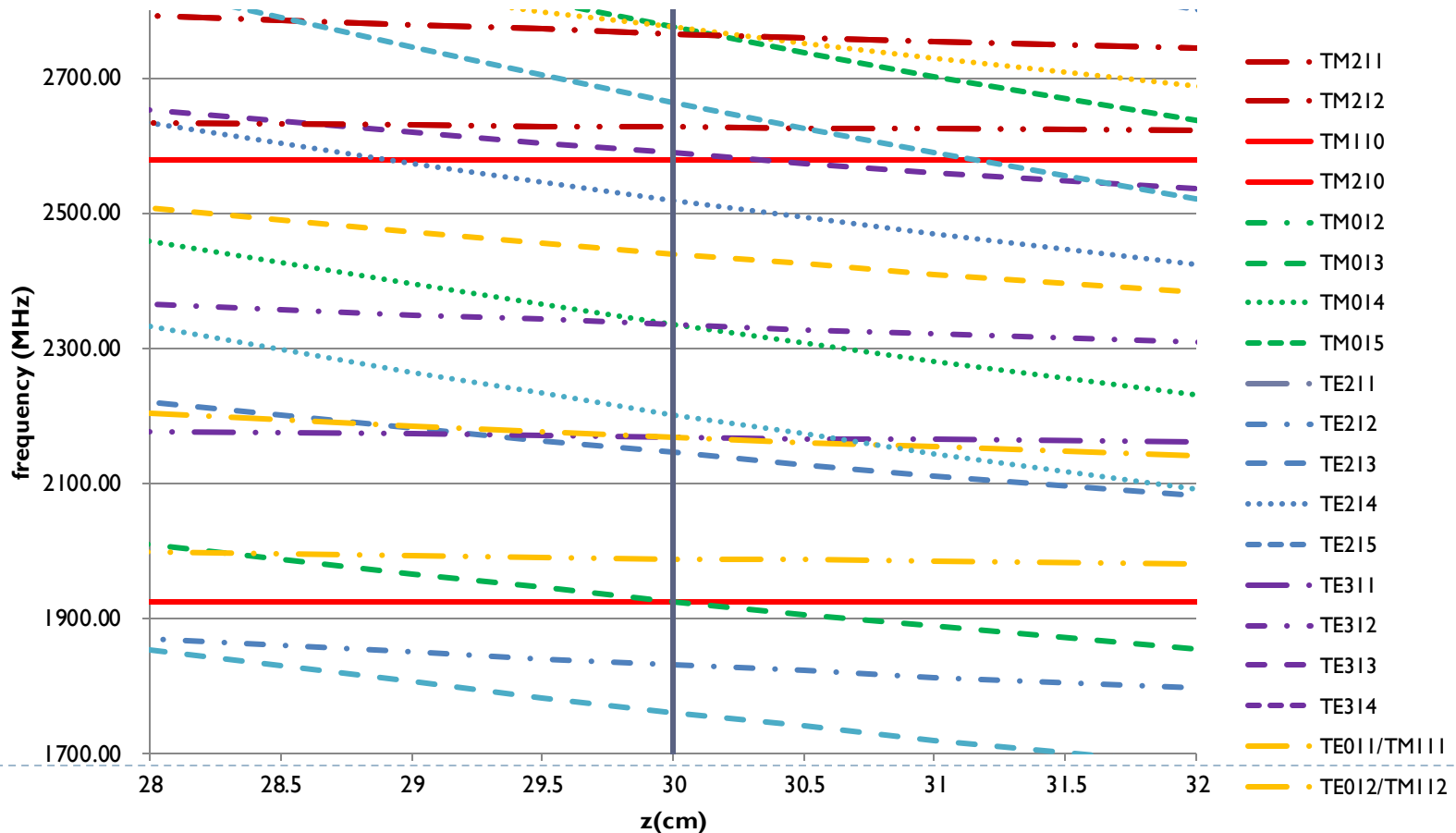
Longer cavity makes interference severe

- ▶ $\ell=16\text{cm}$, other modes are fairly separated from the operation modes (**TM110** and **TM210**)

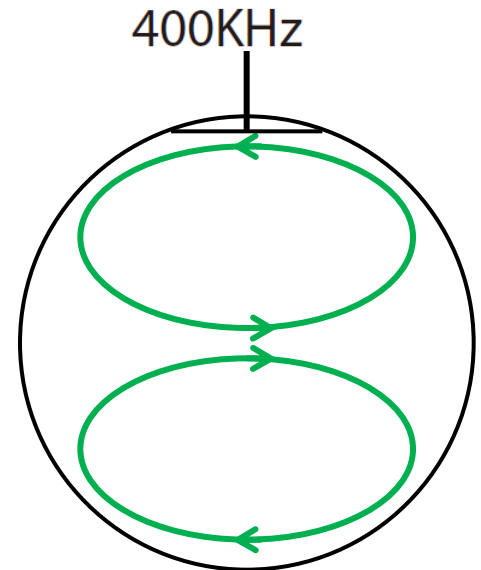
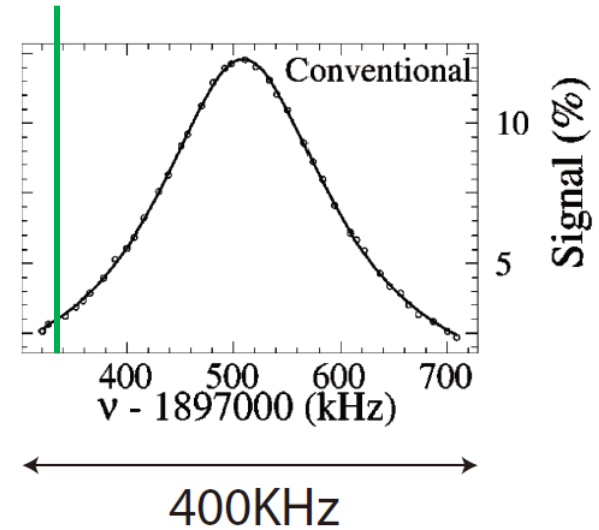
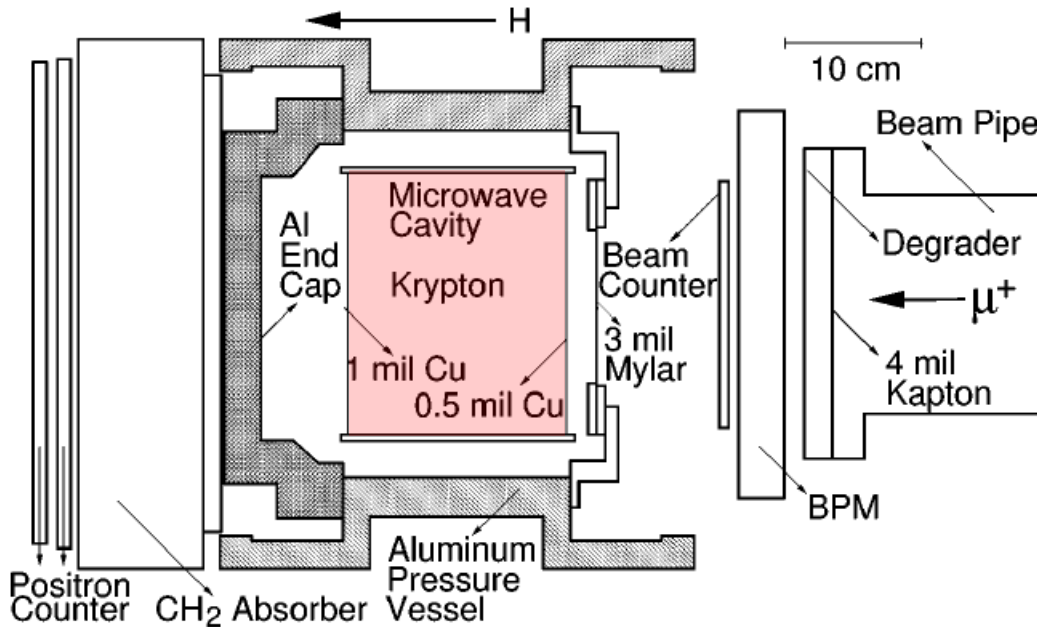


Longer cavity makes interference severe

- ▶ $\ell=30\text{cm}$, many modes appears nearby operation modes
 - ▶ TM110 : 1924.3MHz \Leftrightarrow TM013 : 1925.0MHz
 - ▶ TM210 : 2579.4MHz \Leftrightarrow TE314 : 2588.3MHz



Tuning rod is necessary for RF cavity



Chamber has a dual role

- ▶ Gas chamber : where muonium is formed
- ▶ **RF cavity : where transition happens**
 - ▶ uniform field strength along z-axis
 - ▶ resonance frequencies must be tunable

Chamber / RF cavity components

Gas chamber

- ▶ Gas inlet and outlet
 - ▶ Gas pressure monitor
 - ▶ Temperature monitor
- (total 4)
-
- ▶ Water pipes are attached for temperature control

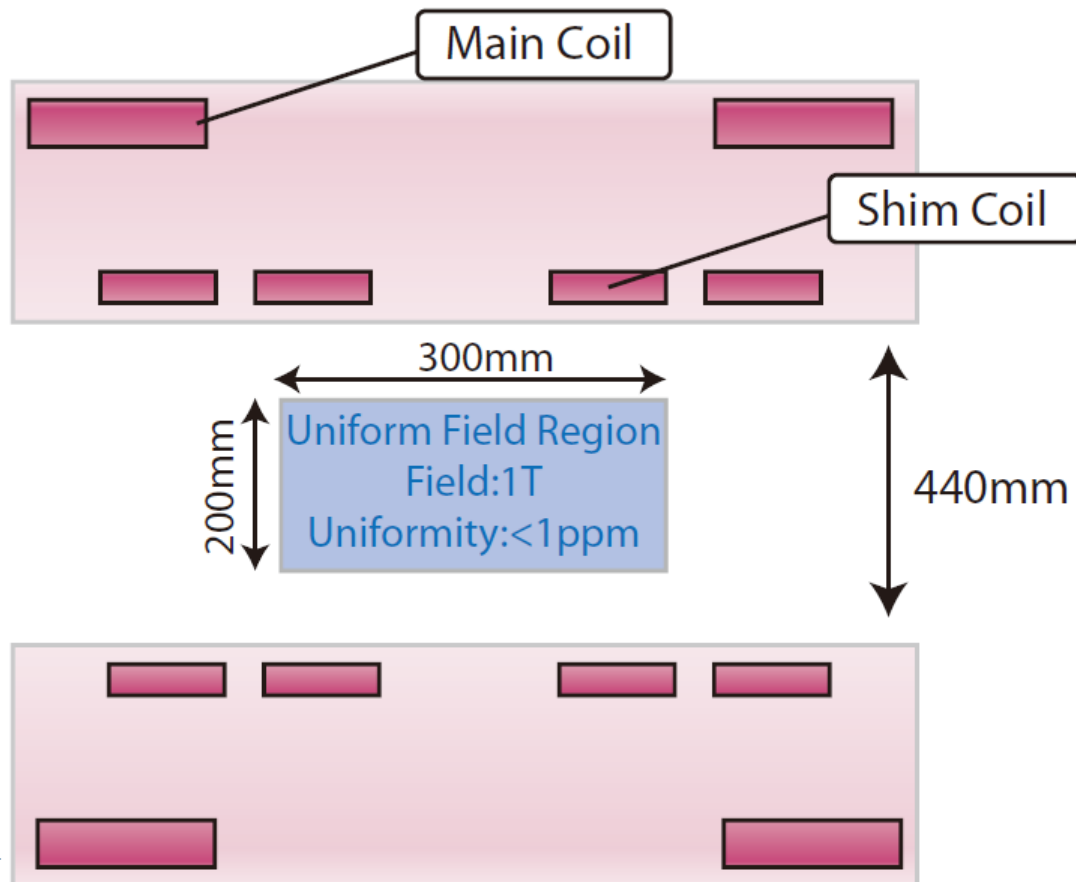
- ▶ RF Cavity
 - ▶ v_{12} inlet and v_{12} pickup
 - ▶ v_{34} inlet and v_{34} pickup
 - ▶ water pipe for temperature control
 - ▶ v_{12} tuning bar
 - ▶ v_{34} tuning bar
 - ▶ magnetic probe
- (total 8)

total 12 ports! + water pipes



Design restriction

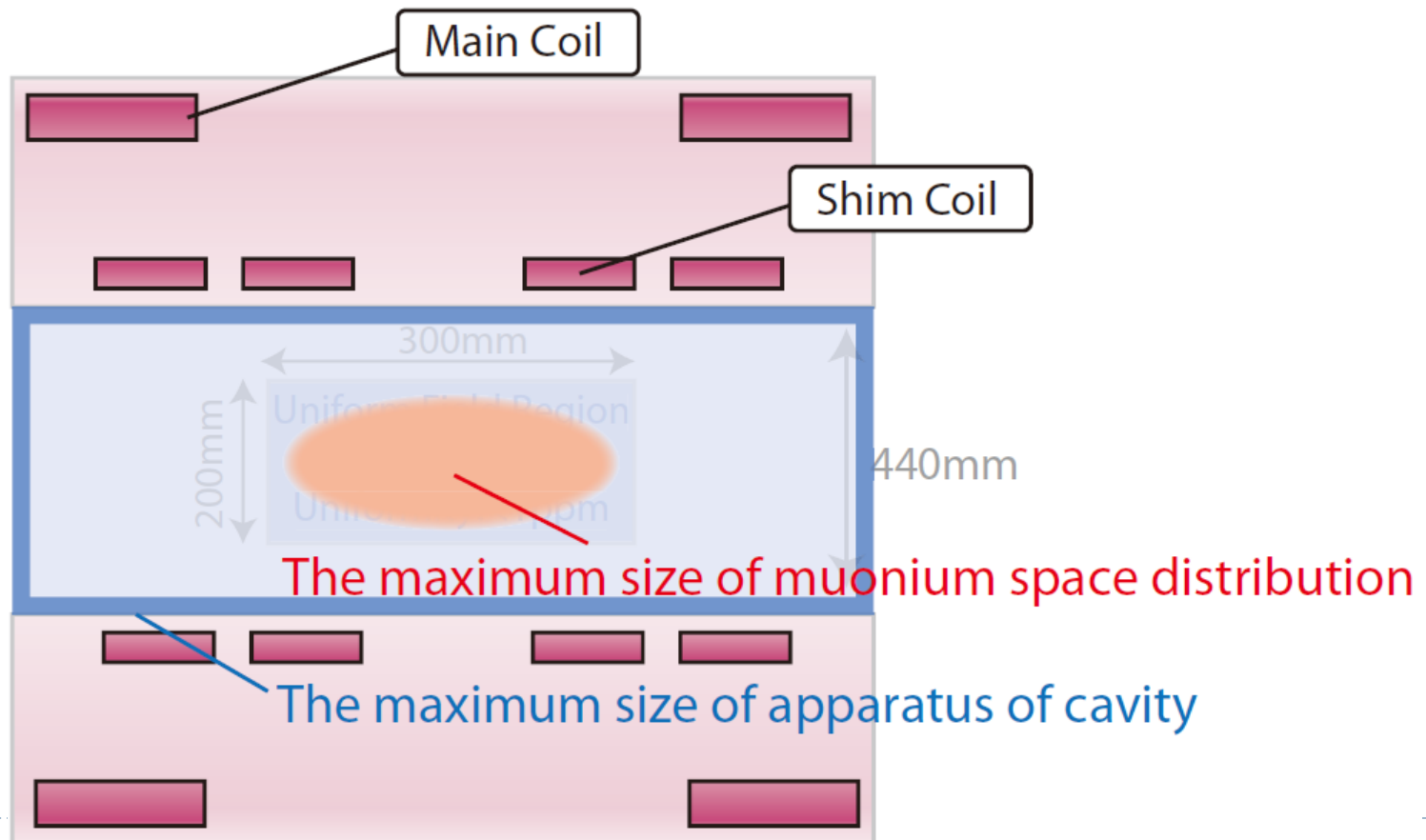
- ▶ The bore size of our magnet is not such large...



Design restriction

- ▶ The bore size of our magnet is not such large...

And all components needs to be non-magnetic



Coaxial cable or wave guide?

- ▶ LAMPF experiment used coaxial cables for transporting RF power to/from the cavity.
- ▶ Usage of wave guides was recommended by a colleague who works for positronium HFS measurement.

Good

- ▶ Wave guides gives much better reliability (reproducibility) when we dismount / reassemble components during the beam time.

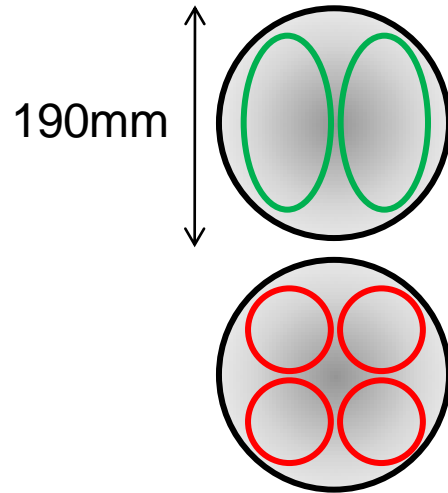
Bad

- ▶ Wave guides are bulky (w:109mm, h:13mm)
- ▶ Needs 3D CAD drawing to see if it is possible
- ▶ At this stage, it seems possible to use waveguides, but more detailed 3D models are needed.



Rectangular cavity?

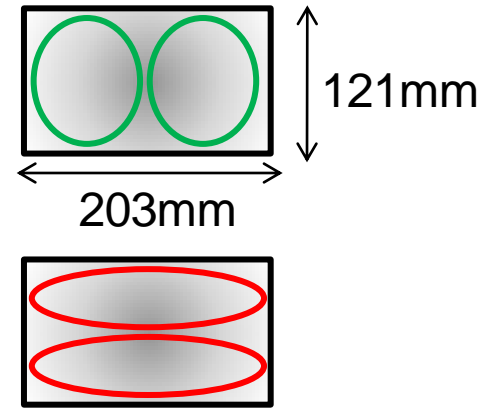
▶ cylindrical cavity



ν_{12} 1.925GHz

ν_{34} 2.581GHz

▶ rectangular cavity



Rectangular cavity

- ▶ gives similar overlap of muonium distribution and field strength for both frequencies.
- ▶ can perform measurement under ANY magnetic field
- ▶ number of ports we can place (without breaking symmetry) is limited.

Materials for RF cavity?

- ▶ non-magnetic, high-conductivity metal
 - ▶ OFC is the metal of choice.
 - ▶ Aluminium alloy can be used as well?
 - ▶ cheap, light, easy to manufacture
 - ▶ larger loss of RF power, possible oxidization on the surface
- ▶ Manufacturing process
 - ▶ RF Cavity should be manufactured from one piece of metal?
 - ▶ Assembling rectangular cavity with plain plates is acceptable?
 - ▶ Depends on loss of RF power, smoothness of the angle



Time Schedule

~ Sep 2011

- ▶ FEM calculation of the RF cavity
- ▶ CAD design of the RF cavity and the Gas chamber

~Nov 2011

- ▶ Manufacturing the cavity and chamber

~Dec 2011

- ▶ Measurement of the performance of the system
 - ▶ Frequency response
 - ▶ Resonance tunability
 - ▶ Temperature stability, air tightness, long term stability



Summary

- ▶ We are in process of designing and manufacturing a RF cavity and a gas chamber for muonium HFS measurement.
- ▶ We have identified functions required for both the RF cavity and the gas chamber.
- ▶ We have not come with detailed design yet
 - Need expertise of our friends!
- ▶ We sincerely welcome your comments, thoughts, help!

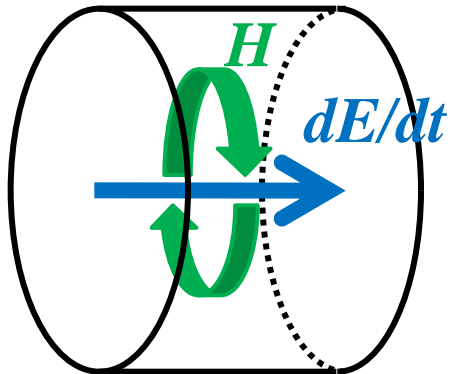


-- fin --

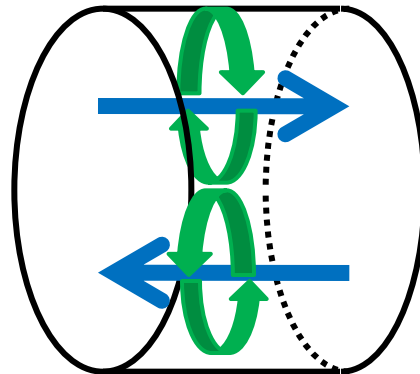


Resonance modes of cylindrical cavity

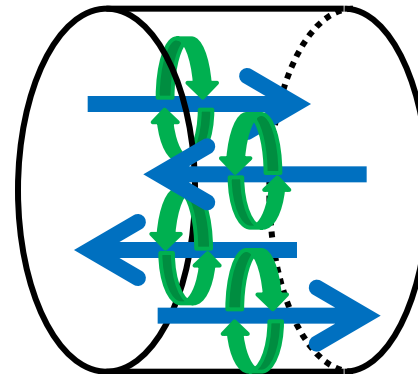
TM₀₁₀



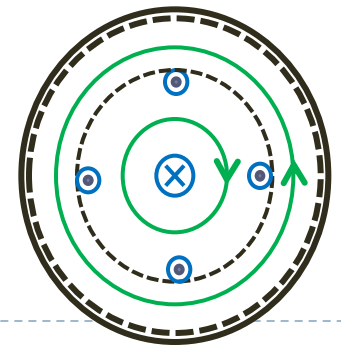
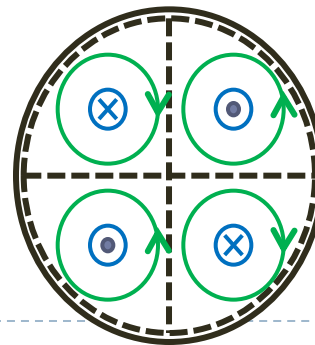
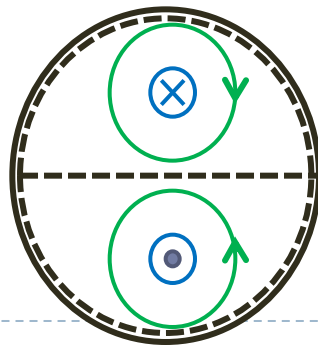
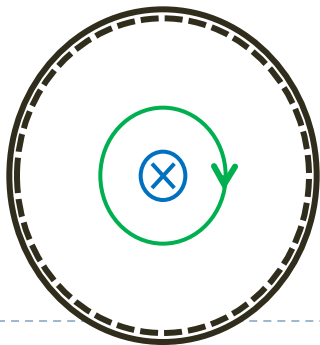
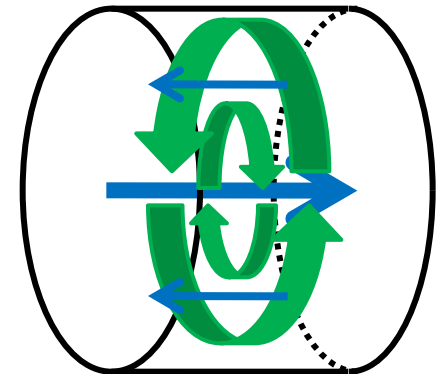
TM₁₁₀



TM₂₁₀

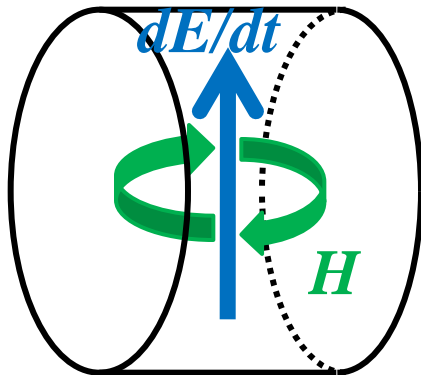


TM₀₂₀

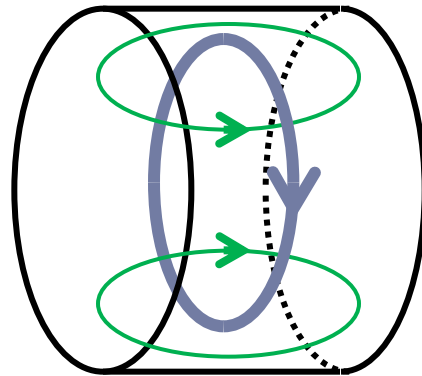


Resonance modes of cylindrical cavity

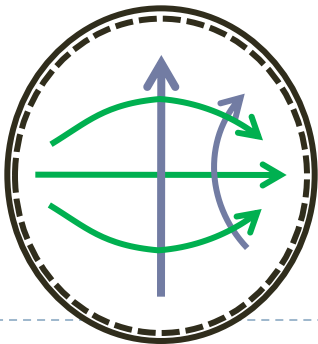
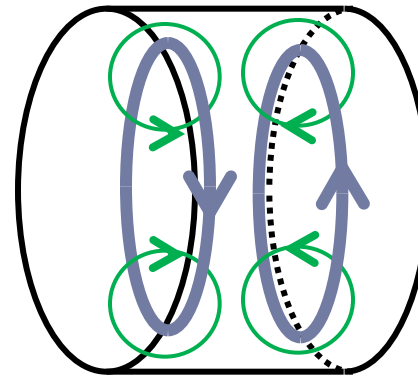
TE₁₁₁



TE₀₁₁



TE₀₁₂



Muonium HFS measurement at J-PARC MLF

Y. Matsuda (U. Tokyo)



Collaborators

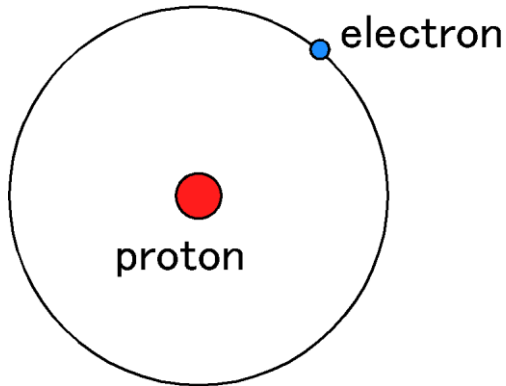
Masaharu Aoki (Osaka U.)
Yuya Fujiwara (RIKEN)
Hiromi Inuma (KEK)
Yutaka Ikedo (KEK)
Katsuhiko Ishida (RIKEN)
Masahiko Iwasaki (RIKEN)
Ryosuke Kadono (KEK)
Osamu Kamigaito (RIKEN)
Naritoshi Kawamura (KEK)
Kenji Kojima (KEK)
Yasuyuki Matsuda (U. Tokyo)
Tsutomu Mibe (KEK)
Yasuhiro Miyake (KEK)
Kusuo Nishiyama (KEK)

Toru Ogitsu (KEK)
Ryuji Ohkubo (KEK)
Naohito Saito (KEK)
Ken'ichi Sasaki (KEK)
Koichiro Shimomura (KEK)
Patrick Strassor (KEK)
Kenji Suda (RIKEN)
Michinaka Sugano (KEK)
Kaduo Tanaka (U. Tokyo)
Ken'ichi Tanaka (KEK)
Dai Tomono (RIKEN)
Hiroyuki Torii (U. Tokyo)
Akihisa Toyoda (KEK)
Akira Yamamoto (KEK)



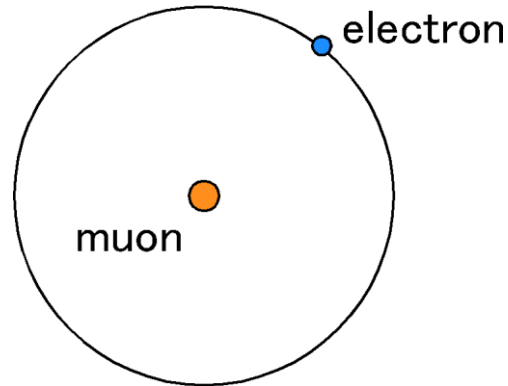
Muonium as an analog of hydrogen

Hydrogen



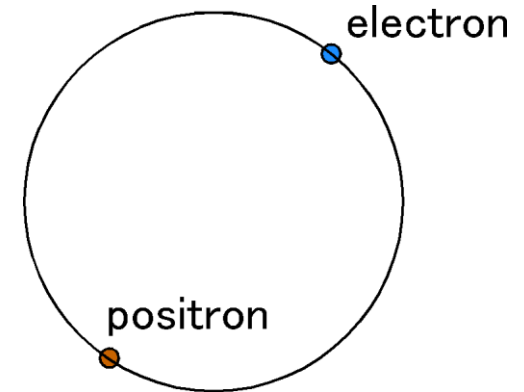
- ▶ lifetime = ∞
- ▶ calculation hindered by finite-size and structure of proton
- ▶ very precise spectroscopic data
- ▶ spectroscopy under way by MPI

Muonium



- ▶ lifetime = $2.2\mu\text{s}$
- ▶ pure-leptonic system
- ▶ 1s-2s spectroscopy: Meyer et al. PRL 84, 1136(2000)
- ▶ HFS spectroscopy: Liu et al. PRL 82, 711(1999)
- ▶ No experiment going on

Positronium

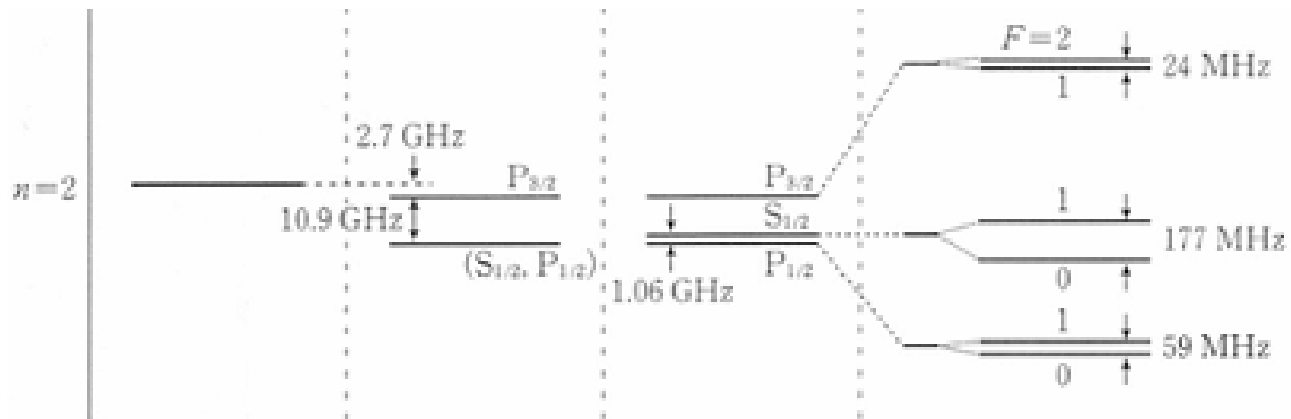


- ▶ lifetime = 125ps (p-Ps) / 142ns (o-Ps)
- ▶ pure-leptonic system
- ▶ HFS spectroscopy under way by U-Tokyo group

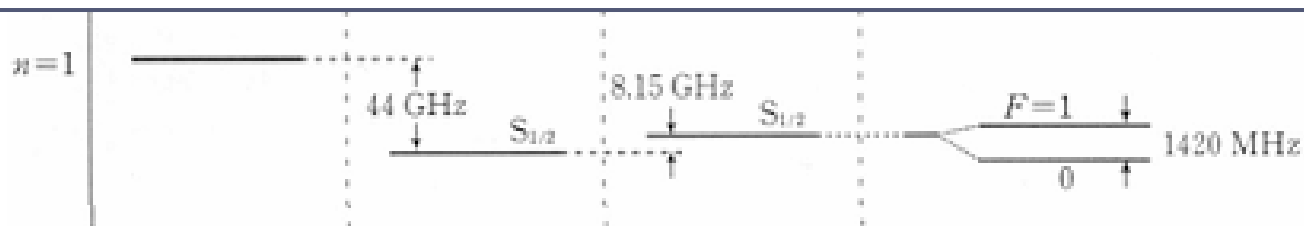


All I really need to know, I learned in Hydrogen

- ▶ Precise spectroscopy of Hydrogen atom has been driving the advance of physics in 20th century



“To understand hydrogen is to understand all of physics”
(Victor Weisskopf)



▶ Bohr's model Fine structure (Dirac equation) QED (Lamb shift) Hyperfine Structure

Precise spectroscopy of Hydrogen

- ▶ 1s-2s transition : 2 466 061 413 187 074(34) Hz (0.013ppt)
- ▶ 1S HFS transition : 1 420 405 751.768(1) Hz (0.7ppt)
 - ▶ Experimental precision far exceeds theoretical calculation, whose uncertainty is **dominated by the uncertainty of the proton's structure.**



Precise spectroscopy of Muonium

- ▶ 1s-2s transition : 2 455 528 941.0(9.8) MHz (4.0ppb)
 - ▶ Meyer et al. at ISIS (PRL84, 1136 (2000))
 - ▶ No uncertainty of proton structure. Theoretical uncertainty is mainly comes from the uncertainty of muon mass.
 - ▶ Given R_∞ and α , one can obtain muon's mass as
$$\frac{m_{\mu^+}}{m_{e^-}} = 206.76838(17) \text{ (820ppb)}$$
 - ▶ With the advance of laser technology, and high intensity at J-PARC, one can reasonably expect that we can improve the accuracy of the experiment by an order of magnitude.
 - ▶ Is it competitive over other experiments ?



Precise spectroscopy of Muonium

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Precise spectroscopy of Muonium

- ▶ HFS transition : $\Delta\nu=4\,463\,302\,765(53)\text{Hz}$ (12ppb)
 - ▶ Liu et al. at LAMPF (PRL82, 711 (1999))

- ▶ From this measurement alone

$$\frac{\mu_{\mu}}{\mu_p} = 3.18334513(39) \text{ (120ppb)}$$

$$\frac{m_{\mu^+}}{m_{e^-}} = 206.768277(24) \text{ (120ppb)}$$

- ▶ Using given R_{∞} and α (CODATA 2006)

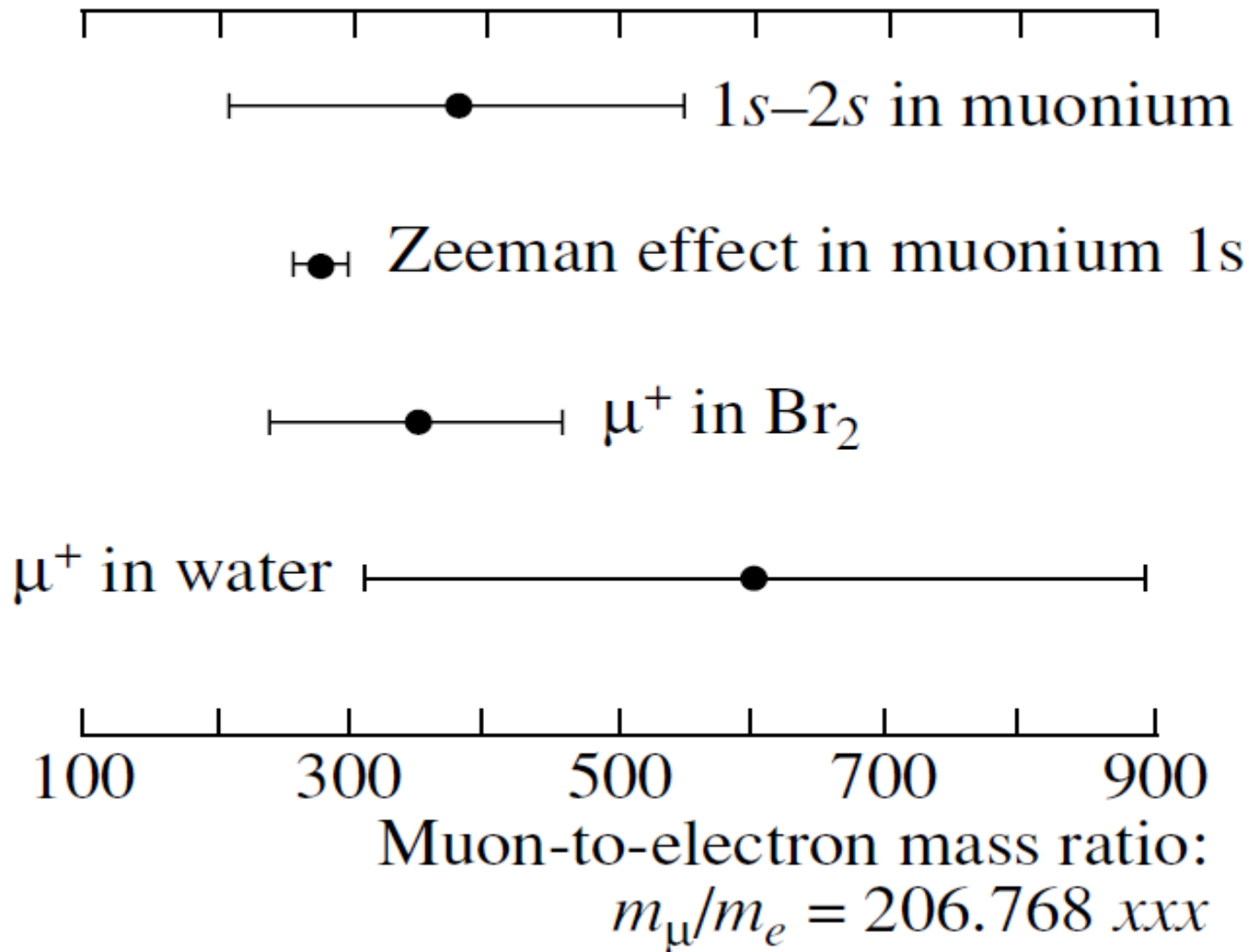
$$\frac{\mu_{\mu}}{\mu_p} = 3.183345137(85) \text{ (27ppb)}$$

$$\frac{m_{\mu^+}}{m_{e^-}} = 206.7682823(52) \text{ (25ppb)}$$

- ▶ Gives more stringent limit on muon mass and magnetic moment

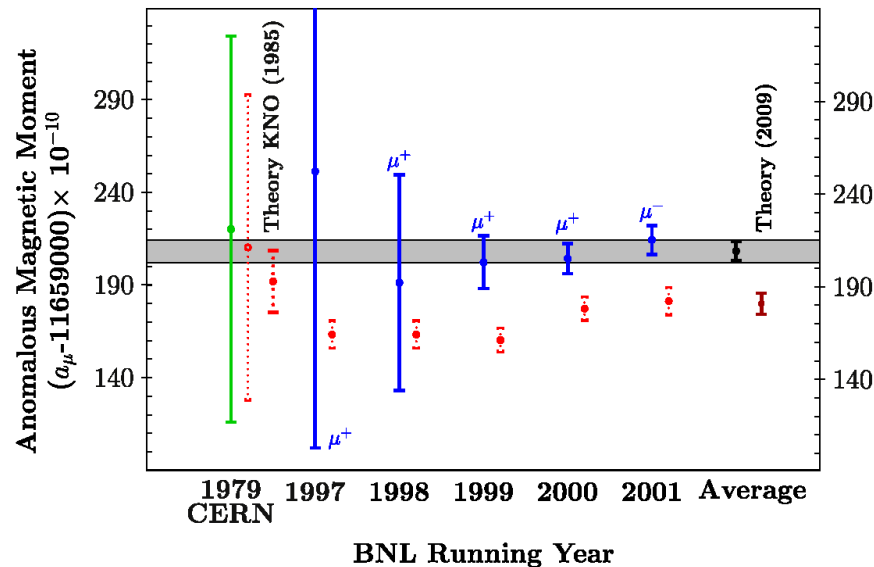


muon / electron mass ratio



muon g-2 experiment

- ▶ BNL E821 experiment reports 3σ deviation from SM



- ▶ To obtain g-2 from experimental data, we need to know $\frac{\mu_\mu}{\mu_p}$, which is given by **muonium HFS measurement**

other experiments

- ▶ Positronium 1S HFS measurement reports $>3\sigma$ deviation from the SM calculation.

$$\Delta\nu(\text{exp}) = 203.389\ 10(74)\ \text{GHz} : \text{Ritter et al. PRA 30, 1331 (1984)}$$

$$\Delta\nu(\text{th}) = 203.391\ 69(16)\ \text{GHz} : \text{Melnikov et al. PRL 84, 1498 (2001)}$$

- ▶ **A new experiment is on going at U. Tokyo**

- ▶ Hydrogen 1S HFS measurement reports $\sim 3\sigma$ deviation.

$$\Delta\nu(\text{exp}) = 1.420\ 405\ 751\ 766\ 7(9)\ \text{GHz} : \text{Nature 229,110 (1971)}$$

$$\Delta\nu(\text{th}) = 1.420\ 452\ 04(2)\ \text{GHz} : \text{Eide et al. Springer Tracts in Mod. Phys. 222 (2007)}$$

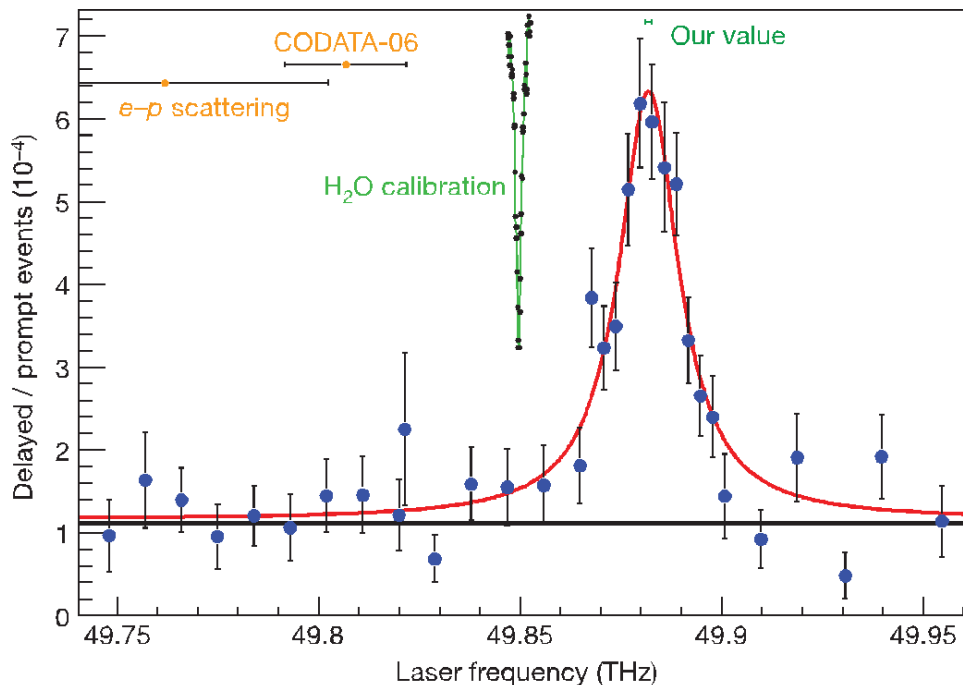
- ▶ **Long standing discrepancy, but not taken seriously.**

- ▶ 2s2p muonic Hydrogen experiment reports 5s deviation from theoretical calculation (Pohl et al. Nature 466, 213(2010))



Pohl et al. Nature 466, 213(2010)

- ▶ 2s2p spectroscopy of muonic hydrogen ($\mu\text{-p}$)
 - ▶ the radius of muon's orbit in the atom is 200 times closer to the nuclei. \rightarrow "proton structure" becomes significant.
 - ▶ Conflict with proton charge radius given by ep scattering
 - ▶ R_∞ to move by 5σ from CODATA 2006 !?



NATURE|Vol 466|8 July 2010

NEWS & VIEWS

QUANTUM ELECTRODYNAMICS

A chink in the armour?

Jeff Flowers

A measurement of the size of the proton, obtained using spectroscopy of an exotic atomic system, yields a result of unprecedented accuracy — but in disagreement with values obtained by previous methods.

Richard Feynman quipped: "There's a reason physicists are so successful with what they do, and that is they study the hydrogen atom and the helium ion and then they stop." On page 213 of this issue, Pohl and colleagues¹ revisit the hydrogen atom — or, more precisely, an exotic form of it — and come up with a surprise. They describe a measurement of the size of the proton that provides a rigorous test of quantum electrodynamics (QED), the quantum theory of how light and matter interact. QED boasts the most numerically accurate predictions of any physical theory, but is based on techniques that are still unproven more than 60 years since its foundation. The authors' measurement uses a novel method that is more sensitive than any of the earlier methods. But it gives a result that is significantly discrepant from that obtained by the next most accurate method, throwing doubt

on the QED calculations that underlie both methods.

Much of quantum theory was developed as a result of attempts to explain the spectral lines of the elements, in particular atomic hydrogen² — the bound state of a proton and an electron. Being a simple two-body system, hydrogen has a structure that, although it took many decades of work to describe by theory, is still significantly simpler than any multi-electron atom. High-precision hydrogen spectroscopy performed by Lamb and Retherford³ in 1947 showed that the existing theoretical description of the hydrogen atom was incomplete, and this led to the new theory of QED⁴. Among the predictions of this new theory was the existence of a small splitting between two of the atom's energy levels that were previously calculated to be the same as each other, and their energy difference measured in Lamb and Retherford's

other experiments

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- ▶ Long standing discrepancy, but not taken seriously.

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We should measure muonium HFS again!



Liu et al. PRL82, 711(1999) LAMPF experiment

- ▶ Largest uncertainties comes from the statistical error.
 - ▶ Liu et al. accumulated about 10^{13} muons for the experiment

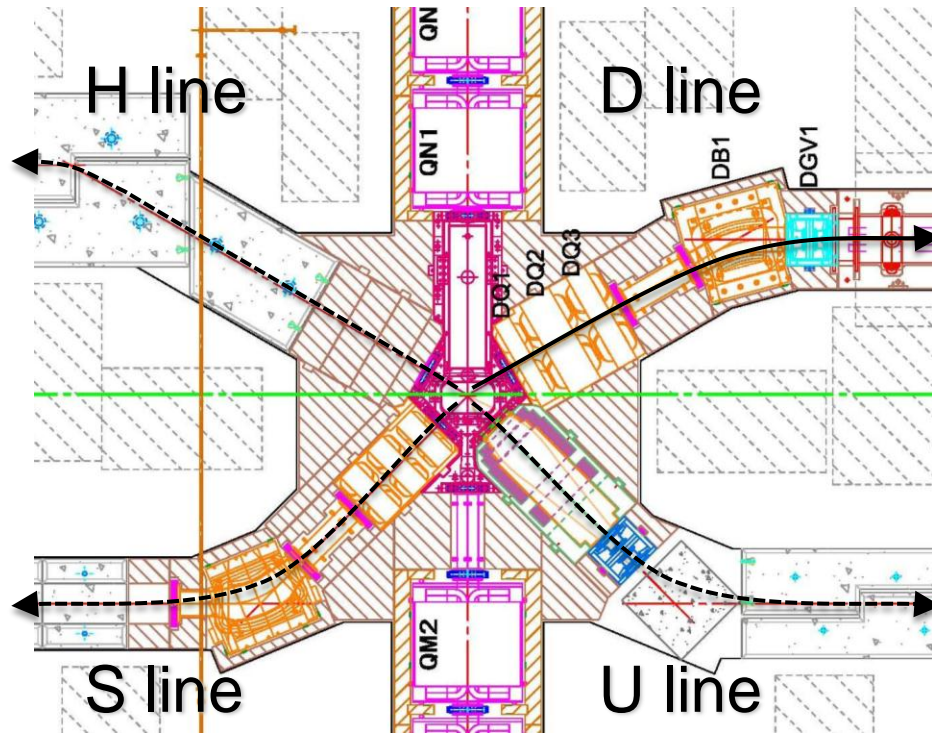
Run dependent uncertainties	$\delta \Delta \nu$ [Hz]		$\delta \Delta \nu$ [ppb]		$\delta(\mu_\mu / \mu_p)$ [ppb]	
	<i>H</i>	<i>ν</i>	<i>H</i>	<i>ν</i>	<i>H</i>	<i>ν</i>
Sweep mode						
Statistical error	89	60	20	13	191	129
Kr density fluctuations	2	2	0.4	0.4	0	0
Drift of Kr density calibration	22	11	4.9	2.5	0	0
Muon stopping distribution	8	5	1.8	1.2	19	17
Magnetic field distribution	0	0	0	0	67	54
Microwave power uncertainty	5	9	1.1	2.0	11	20
Subtotal	92	62	21	14	204	142
Uncertainty in combined results						
			51	12		117

At where shall we measure Mu HFS?

- ▶ The previous experiment was carried out at LAMPF.
 - ▶ The event rate is not limited by accelerator power, but the necessary interval between events
→ difficult to improve the statistics
- ▶ J-PARC will give more muons with pulsed beam structure, with excellent extinction ratio.
 - ▶ $3 \times 10^7 \mu^+/\text{sec}$ at D-line
 - ▶ $3 \times 10^8 \mu^+/\text{sec}$ (?) at H-line
→ Detailed talks by Dr. Kawamura & Dr. Toyoda (yesterday)
- ▶ Pulsed nature of the beam gives challenges, especially on the detector design



Which beam line?



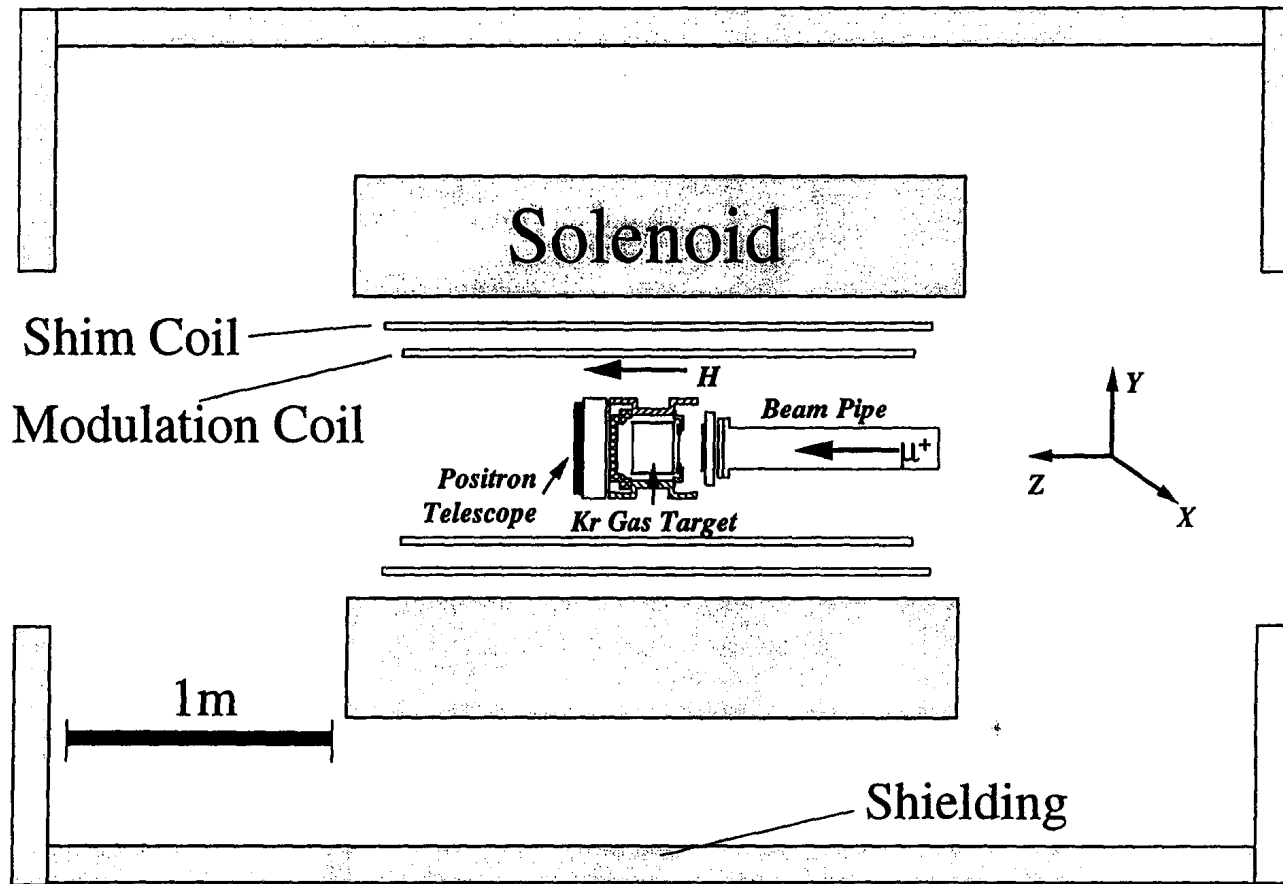
Which beam line?

- ▶ D-line : the only beam line currently operational.
 - ▶ $3 \times 10^7 \mu^+$ /sec is just enough to outdo the previous experiment.
 - ▶ Overcrowded with many material science applications (rightly so!)
- ▶ H-line : partly being constructed, **not fully funded**.
 - ▶ Higher intensity ($3 \times 10^8 \mu^+$ /sec(?)) is always desirable.
 - ▶ Two big physics proposals (g-2 and μ^-e^- conversion) are already there.
 - ▶ Mu HFS experiment can strengthen the case for the H-line, **if this makes the H-line produce the scientific output sequentially. (the schedule of the experiment does not conflict with others)**

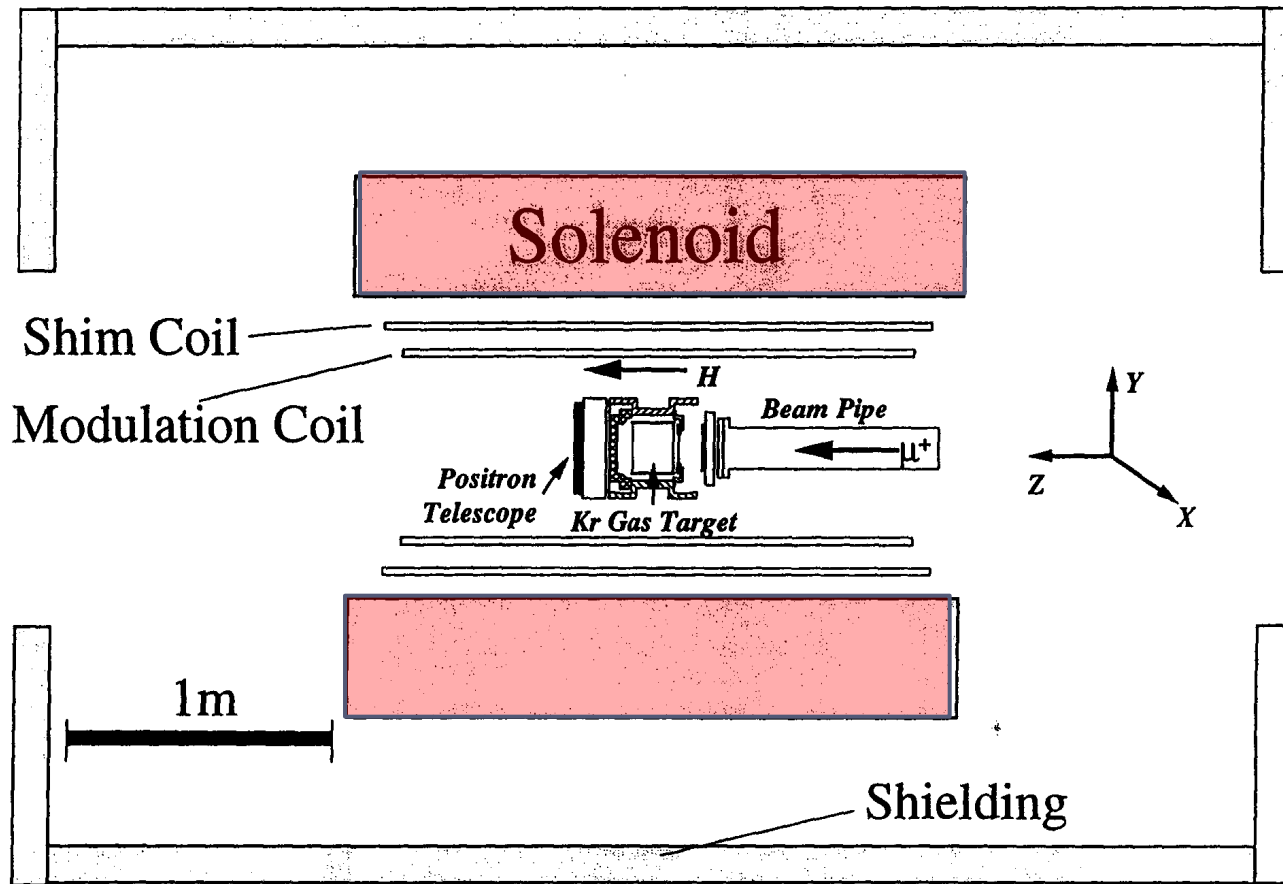
We believe we can do that → H-line as the first candidate



LAMPF experiment set up



LAMPF experiment set up



Magnet

Field must be stable

- ▶ Long term drift $<10^{-8}/\text{h}$, and short term drift $<10^{-7}/\text{h}$ at LAMPF experiment ; we need to be better than that.

Field must be homogeneous where muonium is formed.

- ▶ Larger (longer) homogeneous region makes us possible to use longer cavity (gas target).
→ This will reduce pressure-shifts of the frequency.

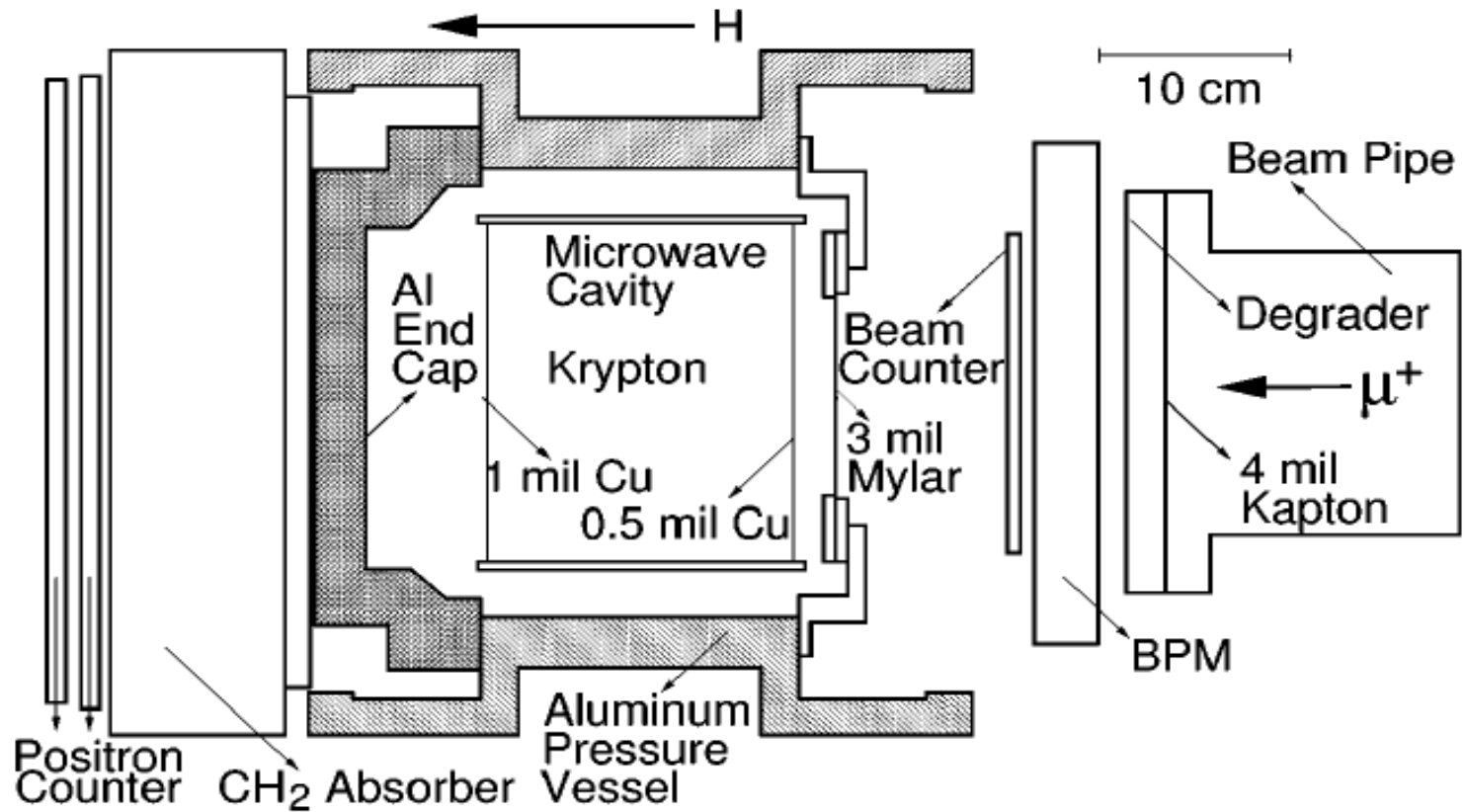
Field must be monitored accurately

- ▶ The NMR probe should be also used for g-2 experiment

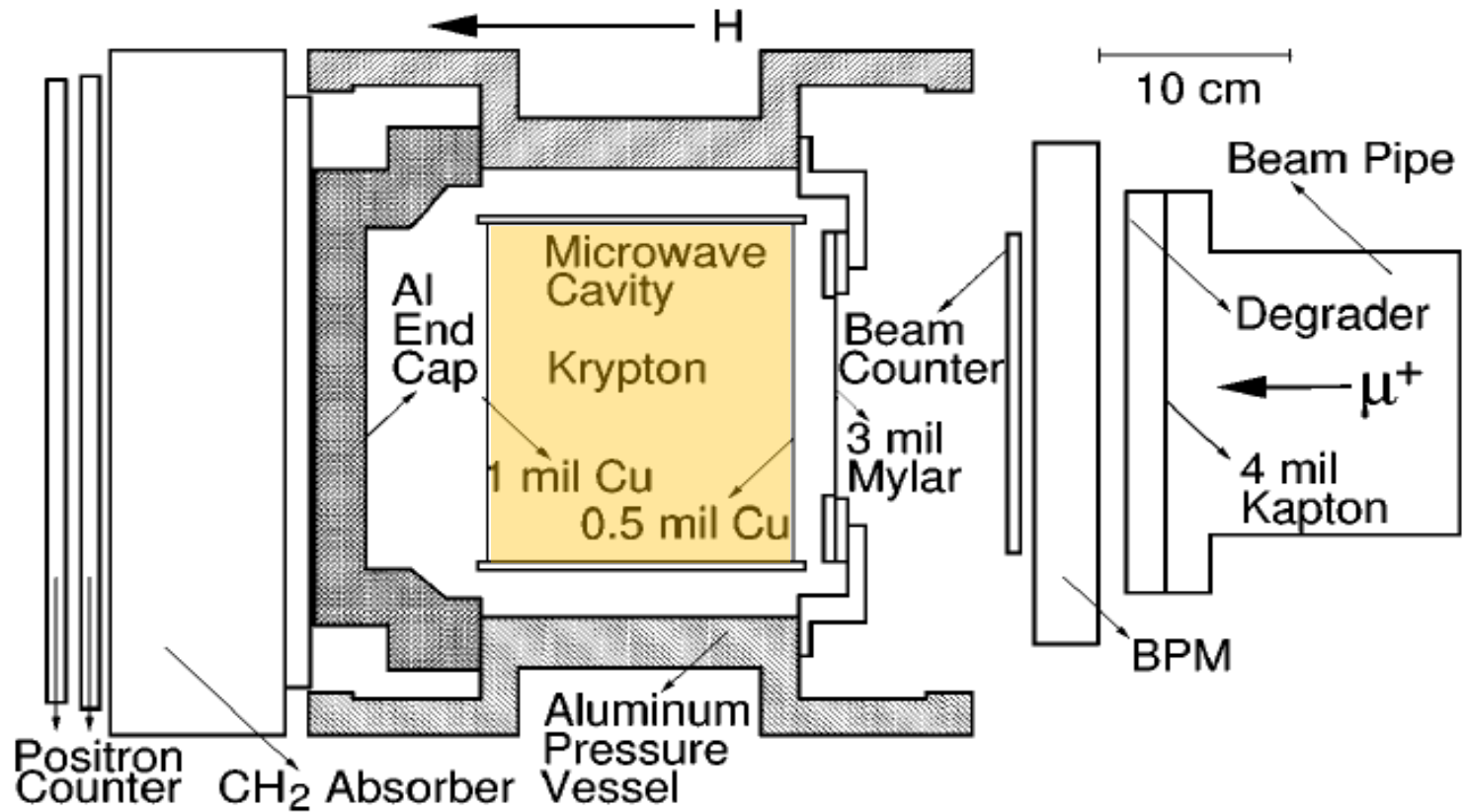
- ▶ Detailed talk by Mr. Sasaki (later)



LAMPF experiment set up



LAMPF experiment set up



Cavity

Two transitions to be measured in the same field strength

- ▶ ν_{12} in TM110 mode(1925.0MHz)
- ▶ ν_{34} in TM210 mode(2581.3MHz)

Cavity length 19cm \rightarrow ~30cm

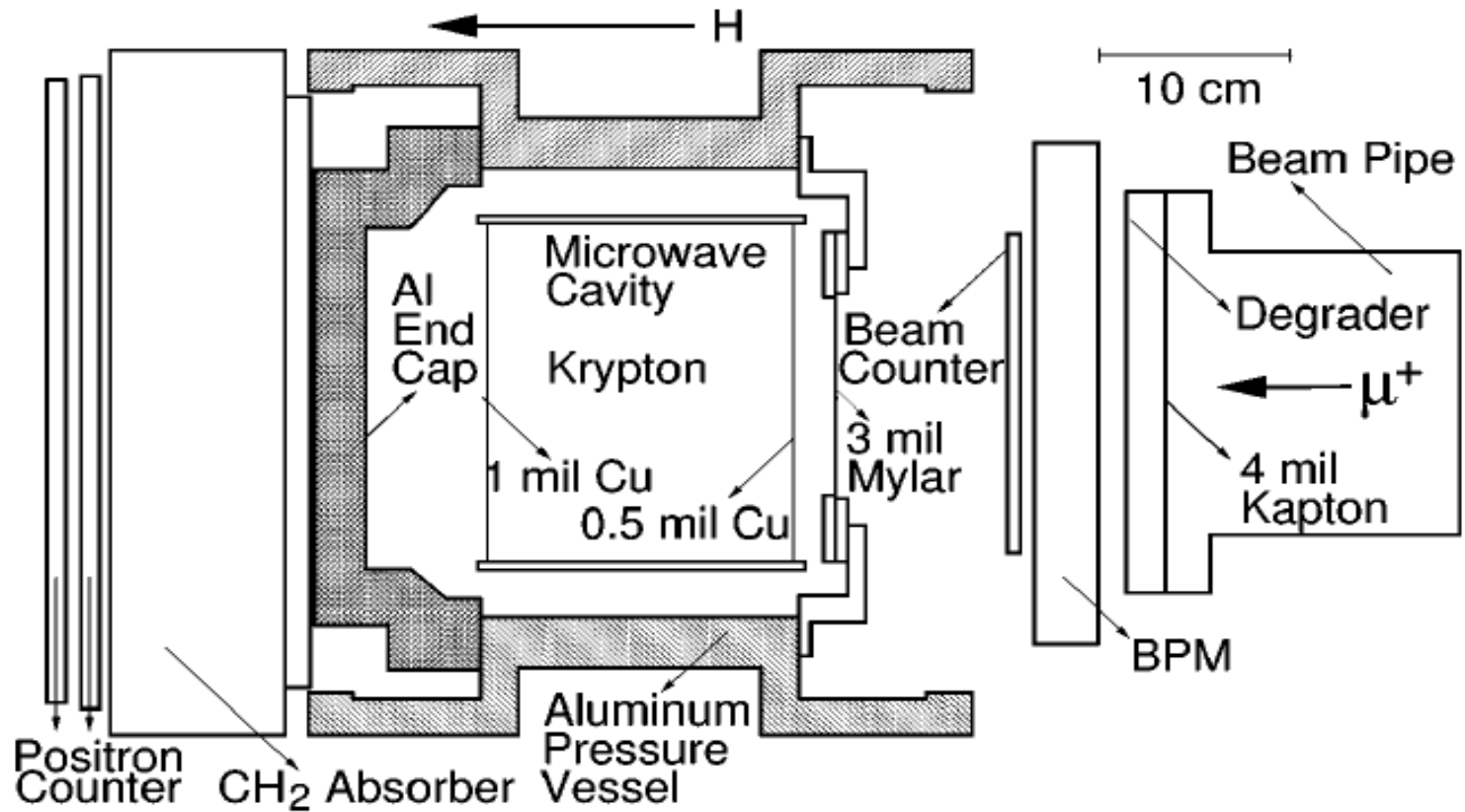
- ▶ Lower pressure gas for better determination of pressure shift
- ▶ gas pressure must be measured accurately

Temperature must be stabilized

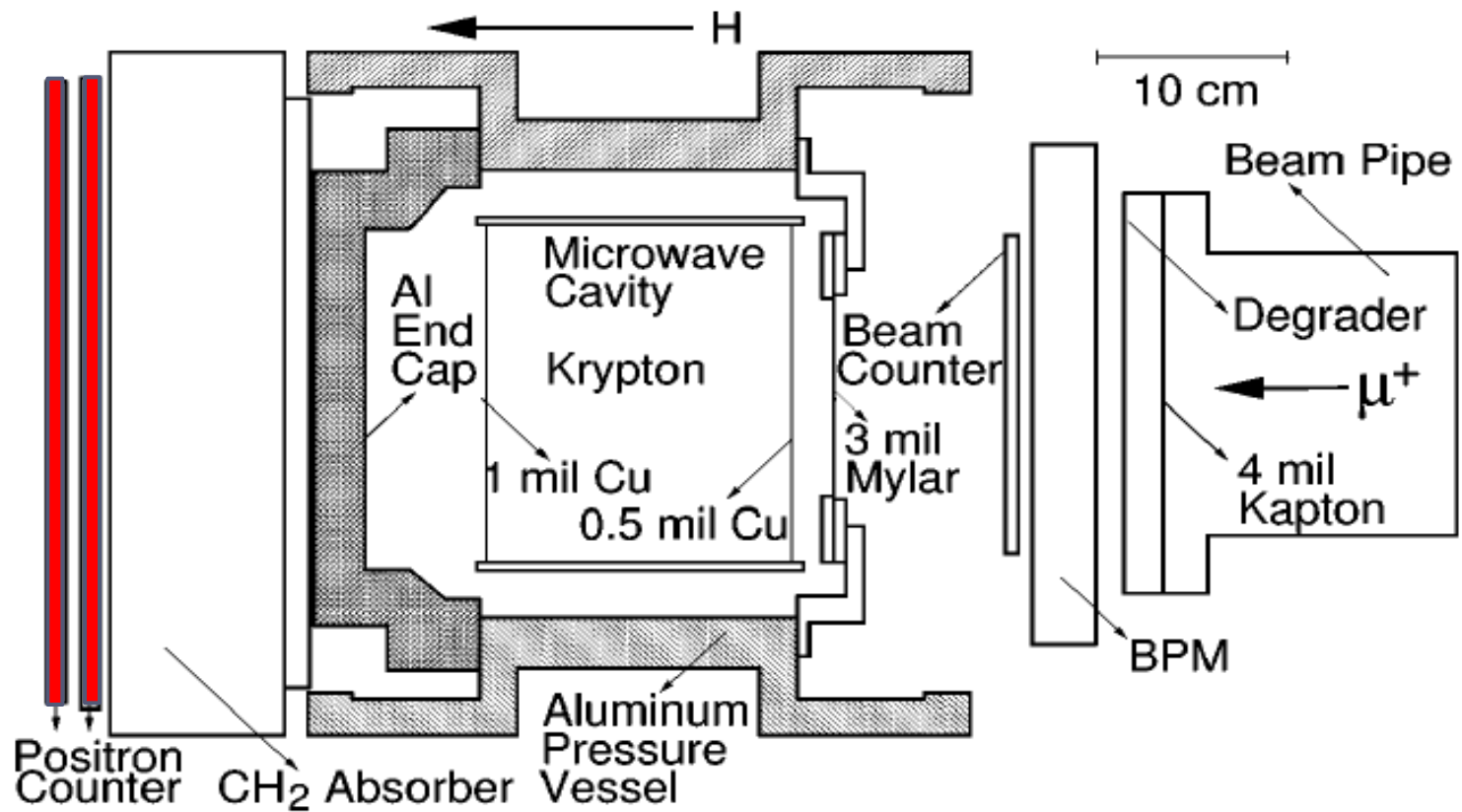
- ▶ Detailed talk by Mr. Tanaka (later)



LAMPF experiment set up



LAMPF experiment set up



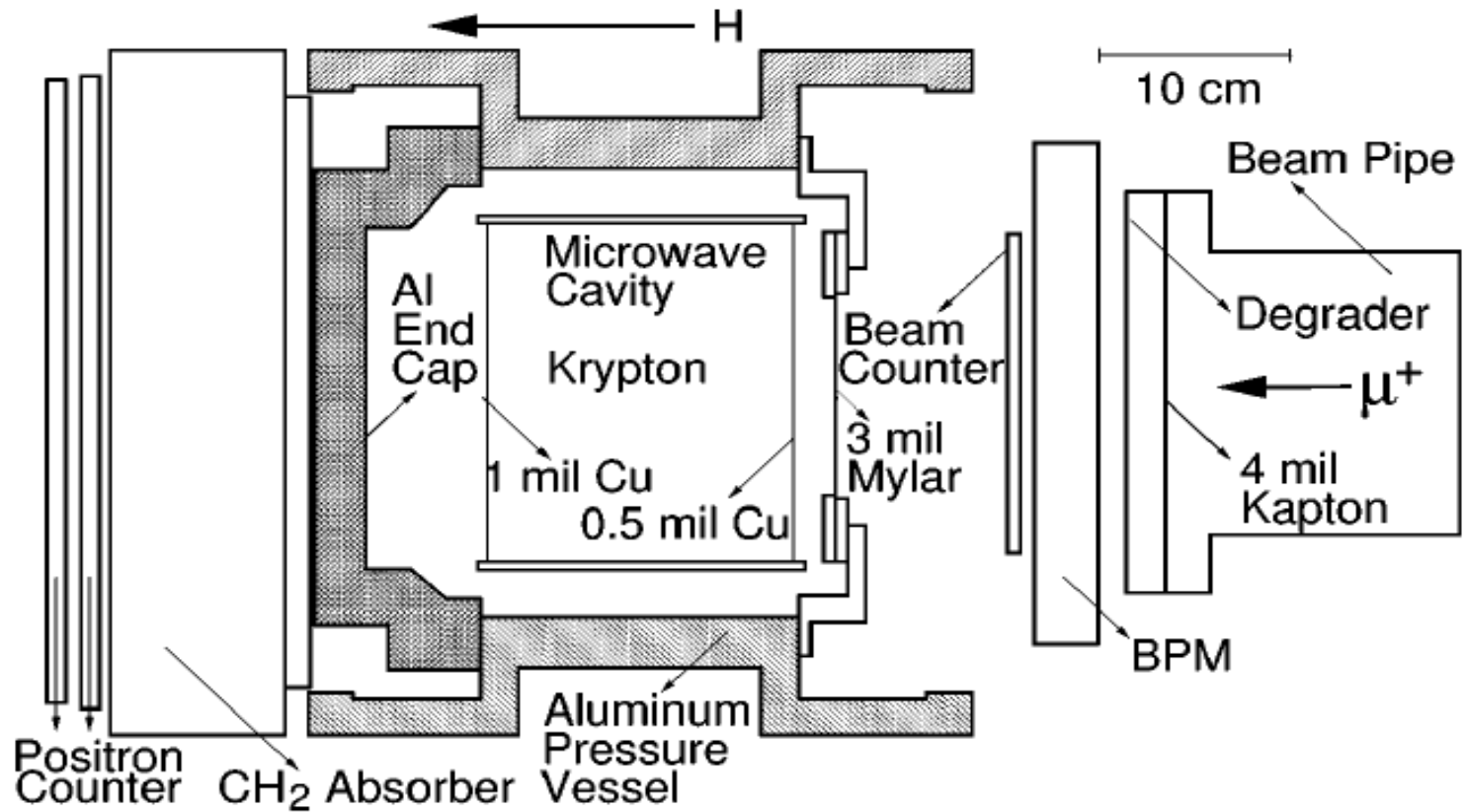
Positron detector

Very high instantaneous rate of muon decay

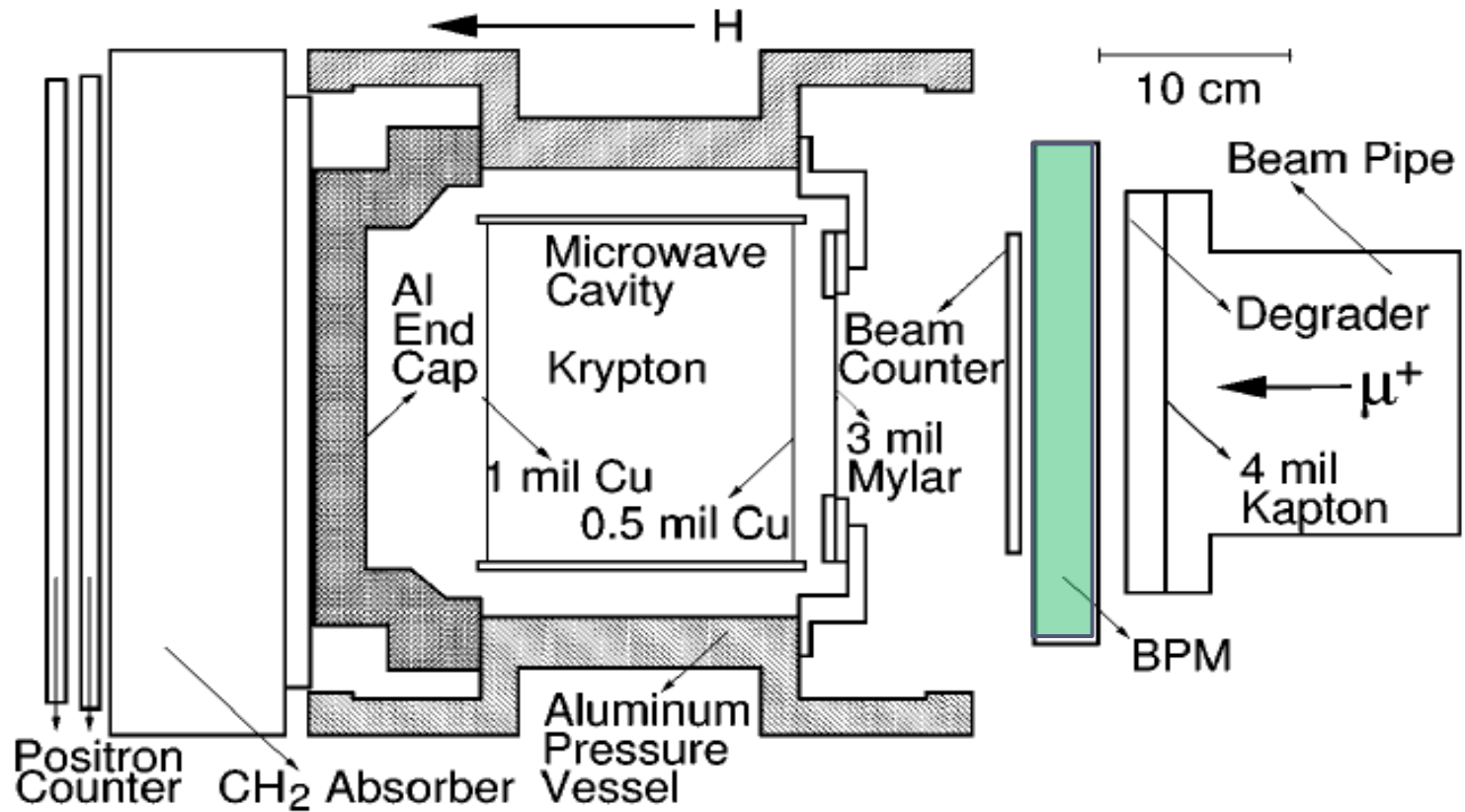
- ▶ $3 \times 10^8 \mu^+/\text{s}$ at H-line $\rightarrow 10^7 \mu^+/\text{pulse}$
 - ▶ $4.5 \times 10^{12} \text{ Hz}$ at $t=0$
 - ▶ $2.3 \times 10^{11} \text{ Hz}$ at $t=3\tau$
 - ▶ $3.1 \times 10^{10} \text{ Hz}$ at $t=5\tau$
- ▶ With “old muonium” ($t > 3\tau$) method, and highly segmented detectors ($N=500$), each detector should be able to handle $5 \times 10^8 \text{ Hz}$ ($\sim 2 \text{ ns}$ interval) count rate.
- ▶ Sharing R&D with μSR group at J-PARC MLF



LAMPF experiment set up



LAMPF experiment set up



Beam profile monitor / stopping distribution measurement

- ▶ Magnetic field is not completely homogeneous
- ▶ Muon stopping distribution needs to be taken account to calculate average B seen by muonium.
- ▶ Ideally, muon stopping distribution should be measured with “real” beam, but the rate is too high.
- ▶ Beam profile should be monitored during the data taking. Again the high flux gives difficulty.
- ▶ Discussion underway with muon group at J-PARC MLF



Budget

- ▶ Total budget ~Y240M
 - ▶ Superconducting magnet ~ Y130M
 - ▶ RF cavity and gas handling ~ Y50M
 - ▶ Positron detector system (inc. DAQ) ~ Y50M

Good

- ▶ Secured “Kakenhi-KibanA” (2011-2013) ~ Y37M
- ▶ Great support from KEK (IMSS, IPNS, CSC...)

Bad

- ▶ Still short of resources. Another budget proposal is under review.

Ugly

- ▶ Uncertainty due to the earthquake and tsunami.
-



The final words...

“天時不如地利 地利不如人和”

孟子 (Meng Tzu: 372 – 289 BCE)

Three conditions for a successful project

- ▶ 天の時: opportunity given by heaven = good scientific case
- ▶ 地の利: advantage of the territory (= facility)
- ▶ 人の和: people united for the purpose



The final words...

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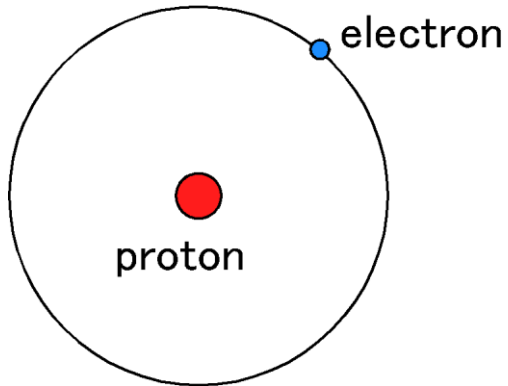
We sincerely welcome new ideas, suggestions, collaborators !!



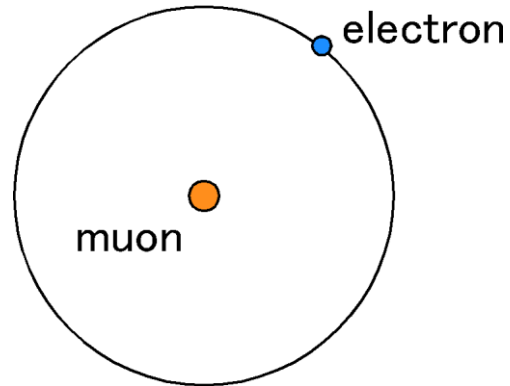


Muonium is μ^+e^- or $\mu^+\mu^-$?

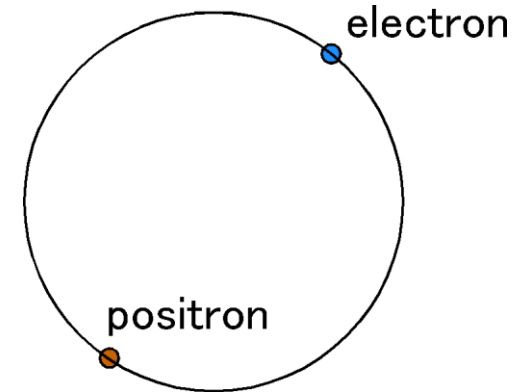
${}^1\text{H}$ = protium



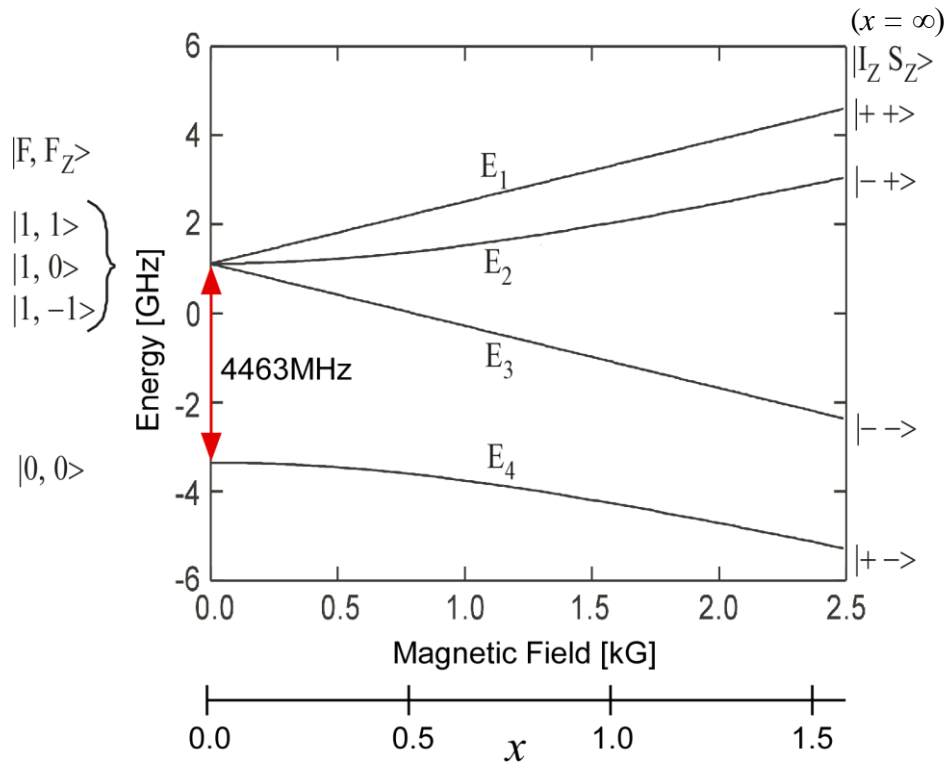
Muonium



Positronium



Breit-Rabi diagram



$$\phi_1 = |++\rangle$$

$$E_1 = \hbar \left(\frac{\omega_0}{4} + \omega_- \right)$$

$$\phi_2 = \alpha |+-\rangle + \beta |-+\rangle$$

$$E_2 = \hbar \left(-\frac{\omega_0}{4} + \sqrt{\omega_+^2 + \frac{\omega_0^2}{4}} \right)$$

$$\phi_3 = |--\rangle$$

$$E_3 = \hbar \left(\frac{\omega_0}{4} - \omega_- \right)$$

$$\phi_4 = \beta |+-\rangle - \alpha |-+\rangle$$

$$E_4 = \hbar \left(-\frac{\omega_0}{4} - \sqrt{\omega_+^2 + \frac{\omega_0^2}{4}} \right)$$

$$\alpha = \frac{1}{\sqrt{2}} \left(1 - \frac{x}{\sqrt{1+x^2}} \right)^{1/2}$$

$$\beta = \frac{1}{\sqrt{2}} \left(1 + \frac{x}{\sqrt{1+x^2}} \right)^{1/2}$$

$$x = B(\gamma_e + \gamma_\mu) / A$$

$x = 0 \rightarrow \alpha = 1/\sqrt{2}, \beta = 1/\sqrt{2}$: three $F=1$ states degenerate.

$x = \infty \rightarrow \alpha = 0, \beta = 1$: states are best described 完全に切り離され、 I_Z, S_Z (ミュオンと電子のスピン) の向き) で状態を表すことができるということです。

$$\omega_{\pm} = \frac{\omega_e \pm \omega_\mu}{2}$$

In reality...

- Precise calculation of the hydrogen(-like) atom is daunting task.

diagrams shown in M.I.Eides et al. Phys. Rep. 342, 63 (2001)

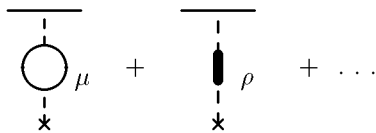


Fig. 16. Muon-loop and hadron contributions to the polarization operator. ³

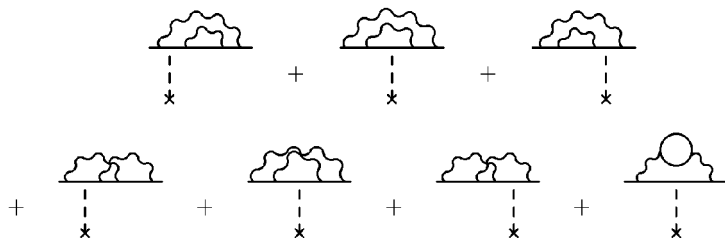


Fig. 12. Two-loop electron formfactor.

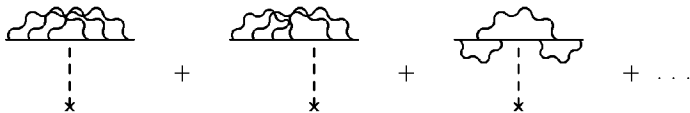


Fig. 14. Examples of the three-loop contributions for the electron form factor.

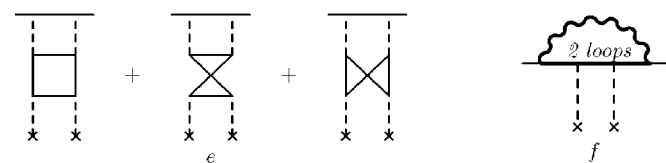
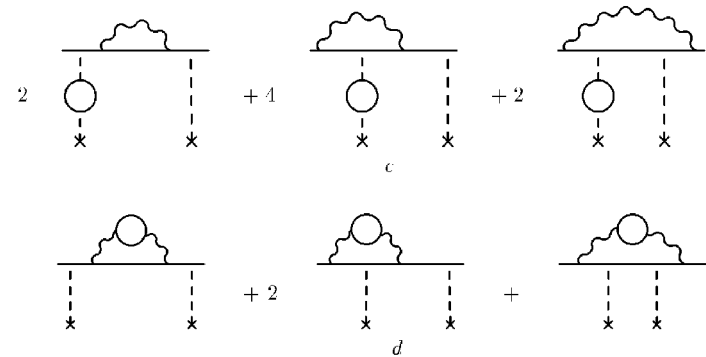
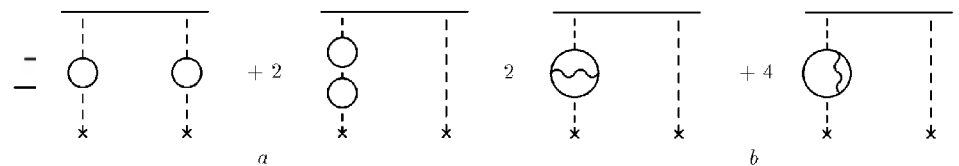
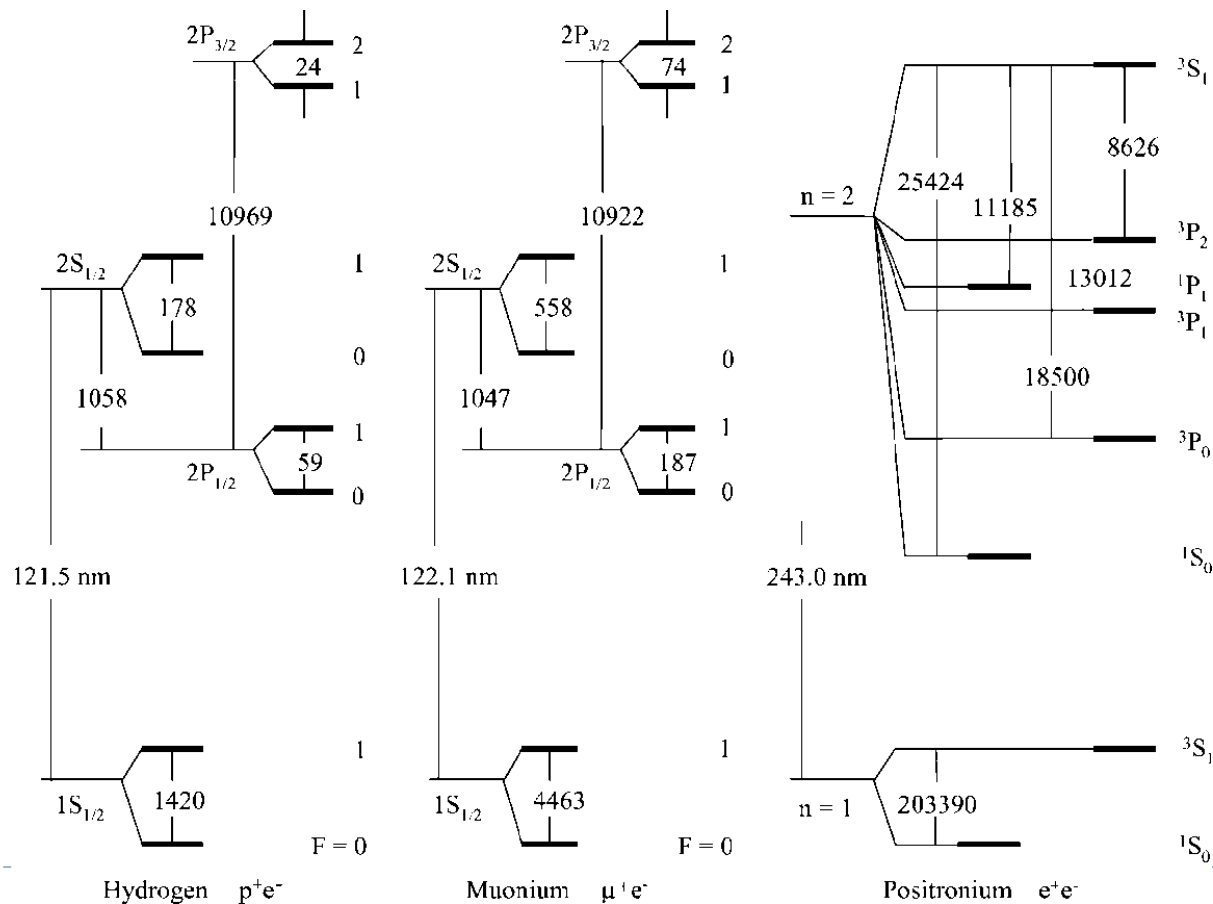


Fig. 20. Six gauge invariant sets of diagrams for corrections of order $\alpha^2(Z\alpha)^5 m$.

Muonium as an analog of hydrogen

- ▶ The spectrum of Muonium is very similar to that of hydrogen.



Precise spectroscopy of Hydrogen

- ▶ 1s-2s transition : 2 466 061 413 187 074(34) Hz (0.013ppt)
 - ▶ Fischer et al. PRL92, 230802 (2004)
 - ▶ Experimental precision far exceeds theoretical calculation, whose uncertainty is **dominated by the uncertainty of the proton's structure**.
- ▶ Using 2S HFS and 1S HFS, one can remove most of uncertainty from proton's structure

$$D_{21} = 8f_{HFS}(2S) - f_{HFS}(1S)$$

- ▶ Experiment : 48923(54)Hz
Kolachevsky et al. PRL102, 213002(2009)
agrees to a theoretical calculation.

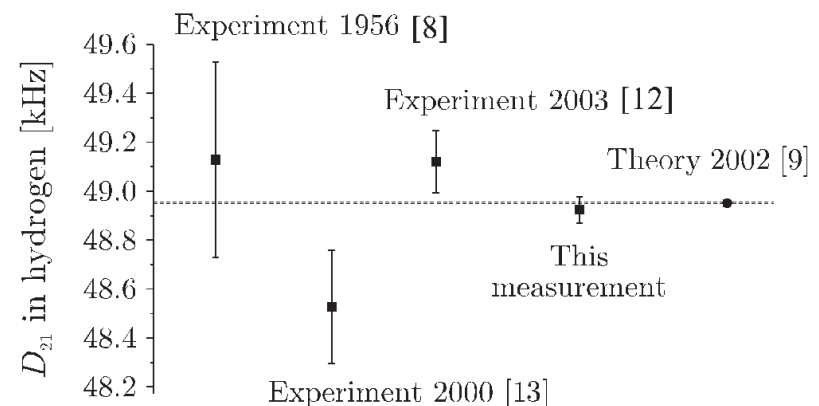


FIG. 4. Experimental and theoretical values for the $D_{21} = 8f_{HFS}(2S) - f_{HFS}(1S)$ difference in atomic hydrogen.