

CC and NC Coherent π Productions

Theoretical Status

Satoshi Nakamura (Jefferson Lab)

Introduction

coherent π production

$$\nu_l \ A_{g.s.} \rightarrow l^- \ \pi^+ \ A_{g.s.} \quad (\text{CC})$$

$$\nu_l \ A_{g.s.} \rightarrow \nu_l \ \pi^0 \ A_{g.s.} \quad (\text{NC})$$

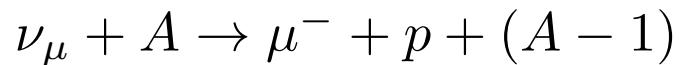
* Amplitude is approximately proportional to nuclear form factor

====> Forward scattering (small momentum transfer) is favored

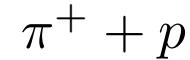
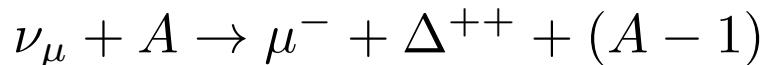
Recent interest

ν -oscillation experiments (K2K, T2K, MiniBooNE ... etc.) in sub-& few-GeV region

- QE



- 1π (Quasi-free Δ)



- Coherent π

- Improved description of π event in forward direction

- Excess of Electron-like Events in MiniBooNE ($\nu_\mu \rightarrow \nu_e$)

NC π^0 fake ? PRL **102**, 101802 (2009)

Theoretical interest

- * Elementary amplitude (spin-isospin non-flip part)
- * Medium effects on Δ -propagation

Challenge data

- * No evidence for CC K2K (2005), SciBooNE (2008)
- * Signature for NC MiniBooNE (2008), SciBooNE (2010)

Puzzling result because ...

- Naive expectation from isospin matrix element : $\sigma_{CC} \sim 2 \sigma_{NC}$
- Finite μ mass reduces phase space at low-energy

Theoretical approaches to coherent π production

- * **PCAC** (Partially Conserved Axial Current)-based model
 - Rein, Sehgal, NPB **223** (1983)
 - Kartavtsev et al., PRD **74** (2006)
 - Berger, Sehgal, PRD **79**, (2009)
 - Paschos, Schalla, PRD **80** (2009)
- * **Dynamical microscopic model**
 - Kelkar et al., PRC **55** (1997) ● Singh et al., PRL **96** (2006)
 - Alvarez-Ruso et al., PRC **75** **76** (2007) ● Amaro et al., PRD **79** (2009)
 - Hernandez et al., PRD **82** (2010) ● Leitner et al., PRC **79** (2009)
 - Martini et al., PRC **80** (2009) ● Nakamura et al, PRC **81** (2010)

PCAC-based model

Rein-Sehgal model, NPB **223**, 29 (1983)

- For $q^2 \rightarrow 0$ (q_μ : momentum transfer from lepton)

$$\mathcal{M}_{coh} \propto \langle q_\mu a^\mu \rangle \quad a^\mu : \text{axial - current}$$

- PCAC relation and LSZ reduction formula

$$q_\mu a^\mu = f_\pi m_\pi^2 \pi \quad (\text{PCAC})$$

$$\implies \mathcal{M}_{coh} |_{q^2=0} \propto -2 i f_\pi T (\pi A_{g.s.} \rightarrow \pi A_{g.s.}) |_{q^2=0}$$

Rein-Sehgal model (cont'd)

$$\frac{d\sigma^{NC}}{dq^2 dy dt} = \frac{G_F^2 f_\pi^2}{4\pi^2} \frac{1-y}{y} \left. \frac{d\sigma(\pi^0 A_{g.s.} \rightarrow \pi^0 A_{g.s.})}{dt} \right|_{q^2=0, E_\pi=q^0} G_A^2$$

$$t = (q - p_\pi)^2 , \quad y = q^0/E_\nu$$

- Continuation to $q^2 \neq 0 \Rightarrow G_A = (1 - q^2/1 \text{ GeV}^2)^{-1}$

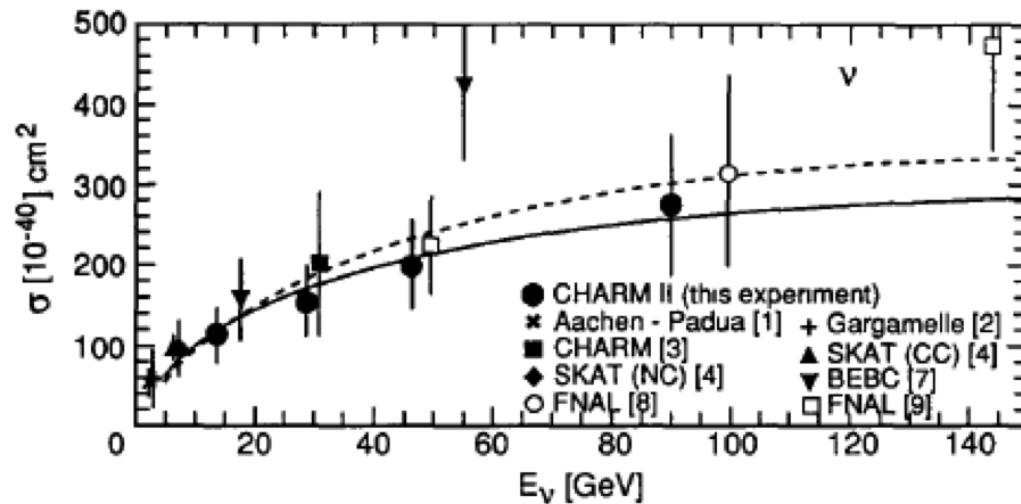
$$\frac{d\sigma(\pi^0 A_{g.s.} \rightarrow \pi^0 A_{g.s.})}{dt} = |F_A(t)|^2 F_{abs} \left. \frac{d\sigma(\pi^0 N \rightarrow \pi^0 N)}{dt} \right|_{q^2=0, E_\pi=q^0}$$

$$F_A(t) = \int d^3r e^{i(\vec{p}_\pi - \vec{q}) \cdot \vec{r}} \{\rho_p(\vec{r}) + \rho_n(\vec{r})\}$$

$$F_{abs} = \exp \left(-\frac{9A^{1/3}}{16\pi R_0^2} \sigma_{inel}^{\pi N} \right)$$

Rein-Sehgal model compared with data

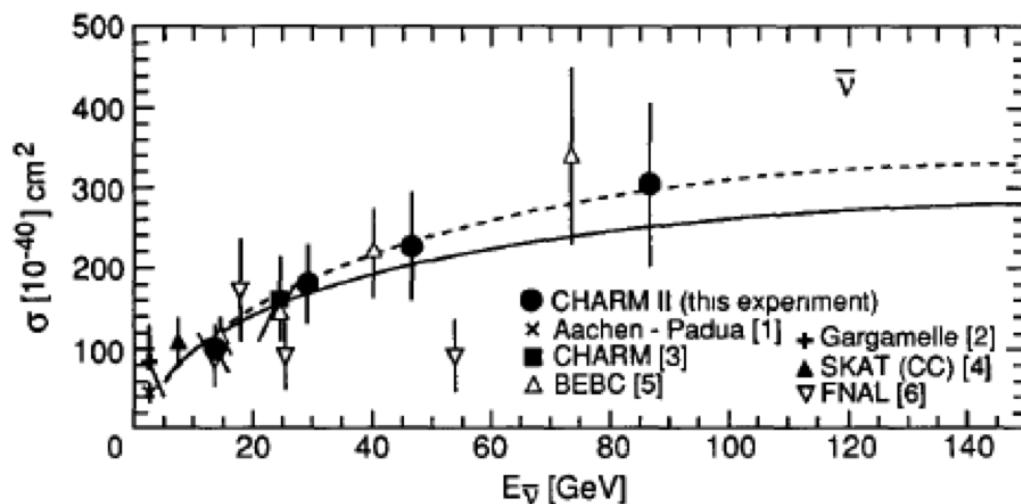
CHARM II Collaboration, PLB 313 (1993)



σ_{CC} and $2\sigma_{NC}$

Target : ^{27}Al , Freon (CCl_2F_2), ^{10}Ne

Glass ($\langle A \rangle \sim 20$), etc.



Work well for

* $E_\nu \gtrsim 2 \text{ GeV}$

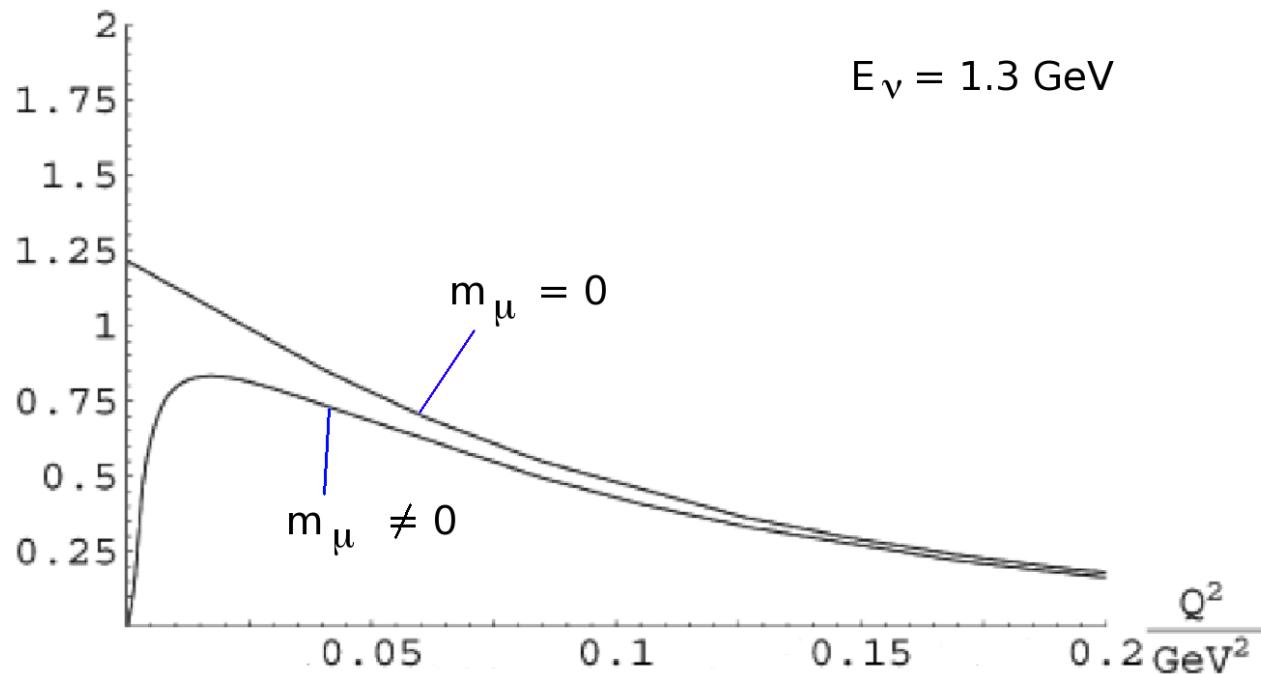
* Medium and heavy nuclei

Problems of Rein-Sehgal model and improvement

Too large CC cross sections for $E_\nu < 2 \text{ GeV}$ and Light nuclei (^{12}C)

K2K PRL **95** (2005); SciBooNE PRD **78** (2008)

Finite muon mass for CC Rein and Sehgal, PLB **657** (2007) \Rightarrow 25% reduction

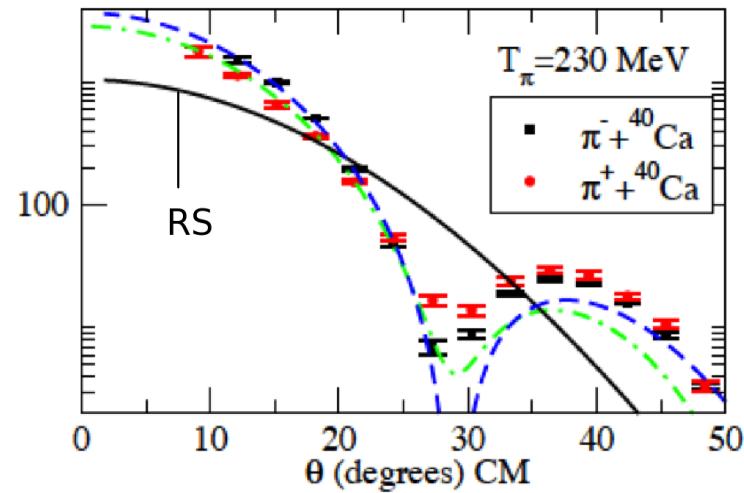
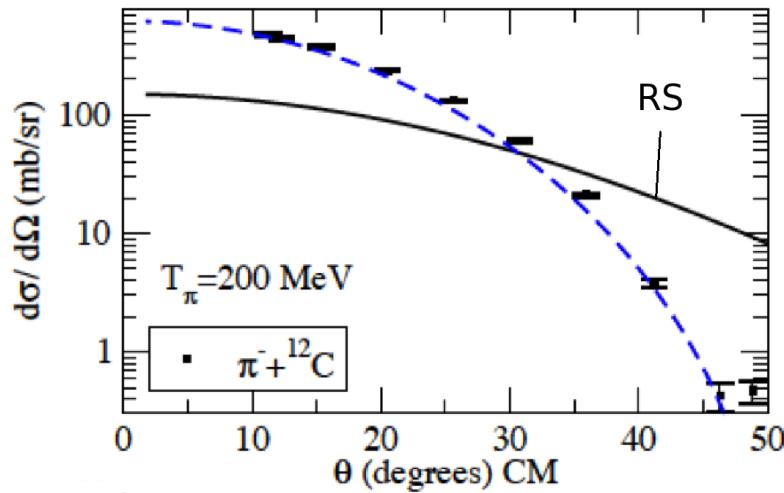


Problems of Rein-Sehgal

Hernandez et al., PRD **80** (2009)

$\pi - A_{g.s.}$ elastic scattering

$$\frac{d\sigma(\pi^0 A_{g.s.} \rightarrow \pi^0 A_{g.s.})}{dt} = |F_A(t)|^2 F_{abs} \left. \frac{d\sigma(\pi^0 N \rightarrow \pi^0 N)}{dt} \right|_{q^2=0, E_\pi=q^0}$$



Improvements

- t -dependence of $\pi - N$ scattering

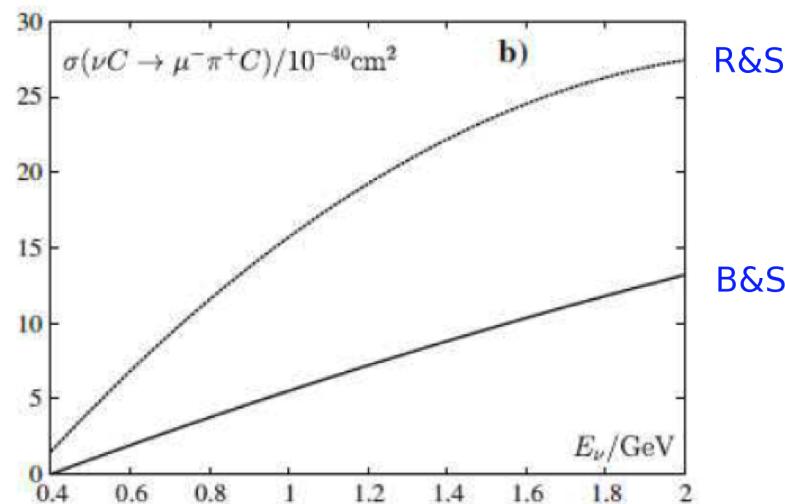
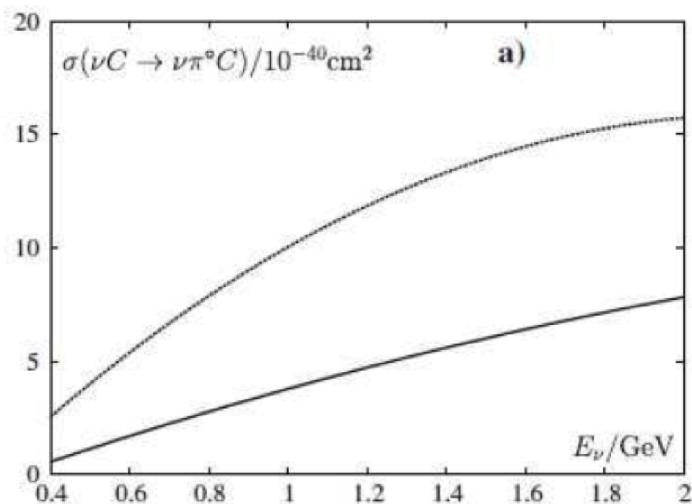
- More realistic F_{abs}

However, coherent π (p_π -distribution etc.) is rather different from microscopic model

Nucleon PCAC \implies Nuclear PCAC

Kartavtsev et al., PRD **74** (2006); Berger, Sehgal, PRD **79**, (2009); Paschos, Schalla, PRD **80** (2009)

Directly use $\pi - A_{g.s.}$ elastic data



Questions

Bell PRL **13** (1964)

- $\sigma_{coh\pi} \propto A$ but $\sigma_{\pi-A} \propto A^{2/3}$
- $T_{coh\pi} \sim T_{\pi-A}^{\text{off-shell}}$ by PCAC

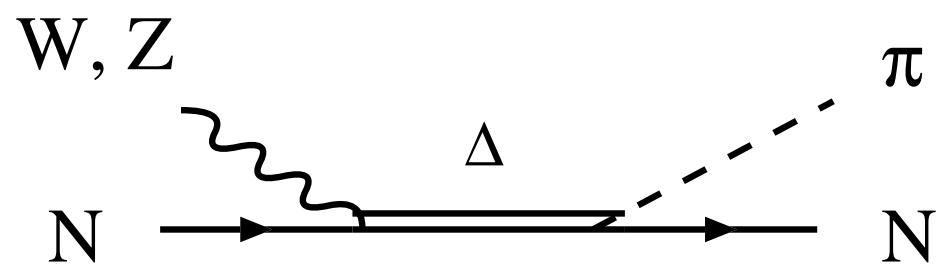
Dynamical model for coherent π production

- * Elementary amplitudes ($\nu N \rightarrow \mu^- \pi^+ N$, $\nu N \rightarrow \nu \pi^0 N$)
- * Medium effect on Δ (mass, width, non-locality)
- * Final state interaction (π -nucleus scattering)

Elementary amplitudes

Kelkar et al., PRC **55** (1997); Singh et al., PRL **96** (2006); Alvarez-Ruso et al., PRC **75** (2007)

only Δ



$$\sigma \sim |C_5^A(0)|^2$$

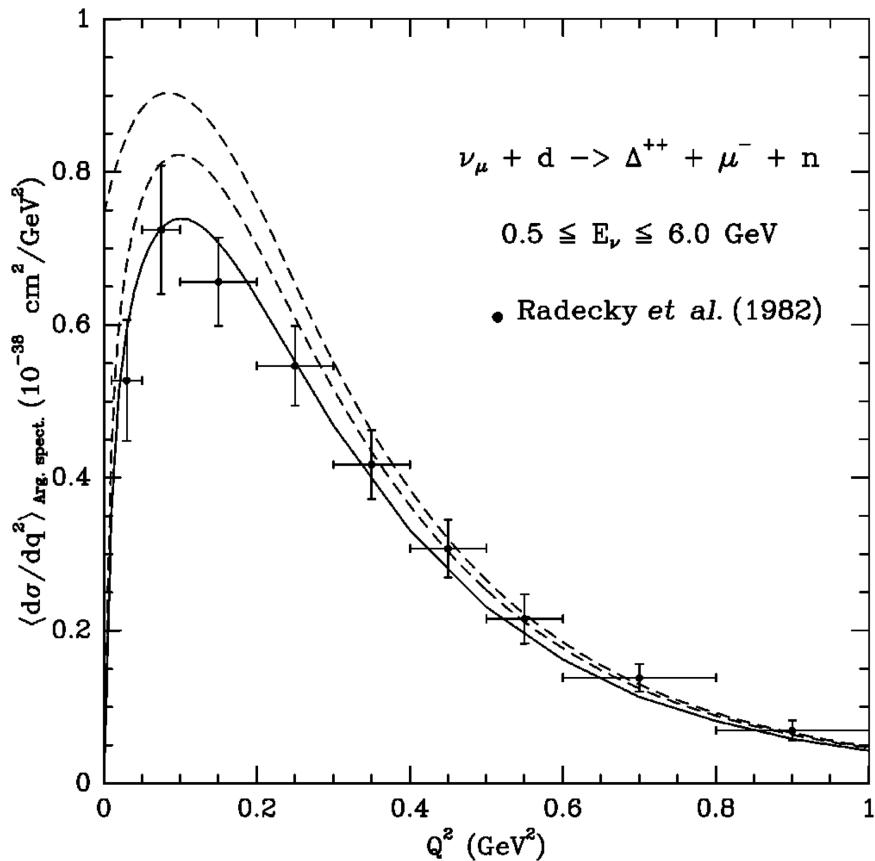
$$C_5^A(0) \sim 1.2 = g_{\pi N \Delta} f_\pi / \sqrt{6} m_N$$

Goldberger-Treiman value

Elementary amplitudes

Kelkar et al., PRC **55** (1997); Singh et al., PRL **96** (2006); Alvarez-Ruso et al., PRC **75** (2007)

only Δ



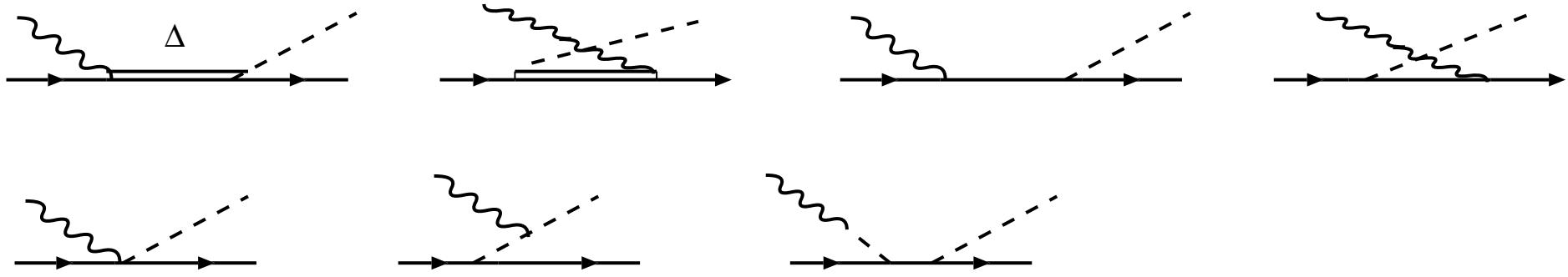
Alvarez-Ruso et al., PRC **59** (1999)

Elementary amplitudes

Alvarez-Ruso et al., PRC **76** (2007); Amaro et al., PRD **79** (2009) Hernandez et al., PRD **82** (2010)

Δ + non-resonant (tree-level)

Hernandez et al., PRD **76** (2006); **81** (2010)



$$C_5^A(0) \sim 1.0$$

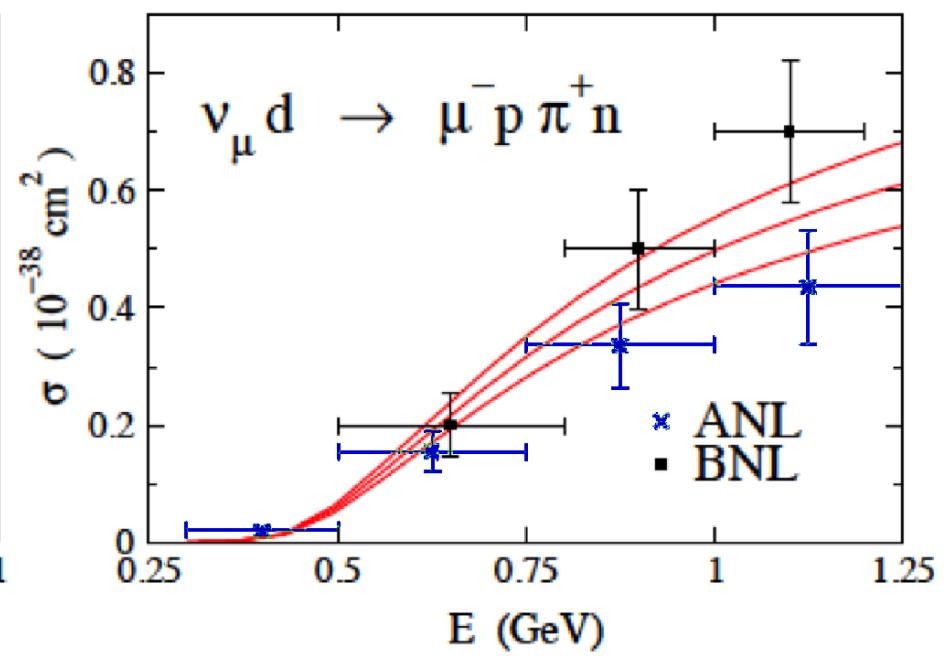
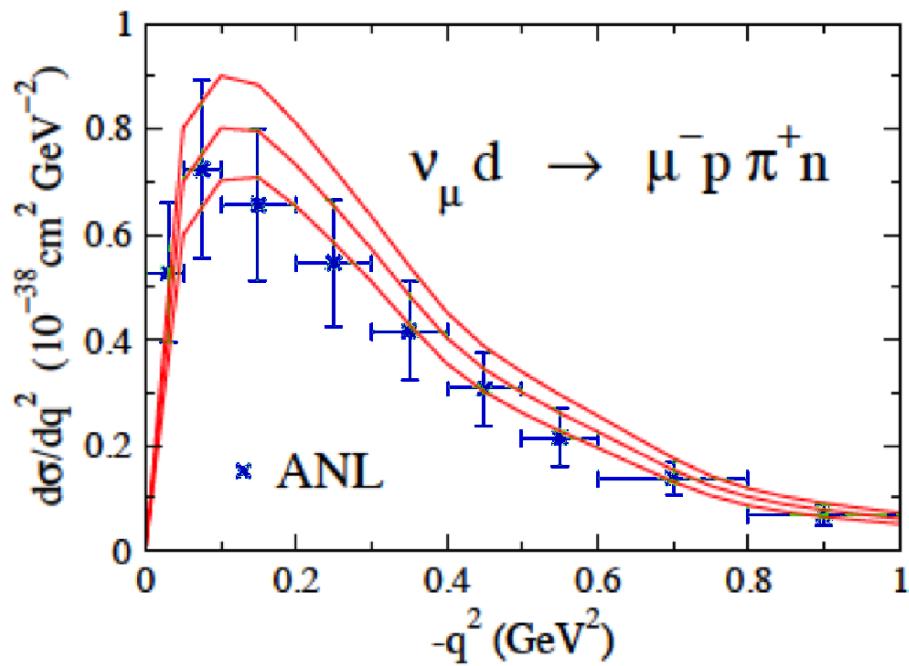
(20% smaller than GT ; fitted to ANL $\nu_\mu d \rightarrow \mu^- p \pi^+ n$ data, deuteron effects)

Elementary amplitudes

Alvarez-Ruso et al., PRC **76** (2007); Amaro et al., PRD **79** (2009) Hernandez et al., PRD **82** (2010)

$\Delta + \text{non-resonant}$ (tree-level)

Hernandez et al., PRD **76** (2006); **81** (2010)



Elementary amplitudes

Martini et al., PRC **80** (2009)

Δ + non-resonant (tree-level)



$$g_{\pi N \Delta} / g_{\pi NN} = g_{\gamma N \Delta} / g_{\gamma NN} = g_{AN \Delta} / g_{ANN} = 2.2$$

$$\implies C_5^A(0) \sim 1.12 \quad (\text{GT value})$$

Comparison with neutrino data ?

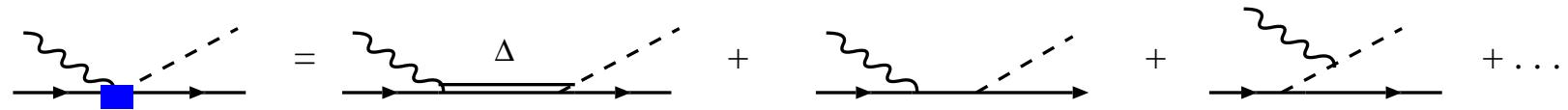
Elementary amplitudes

Nakamura et al, PRC **81** (2010)

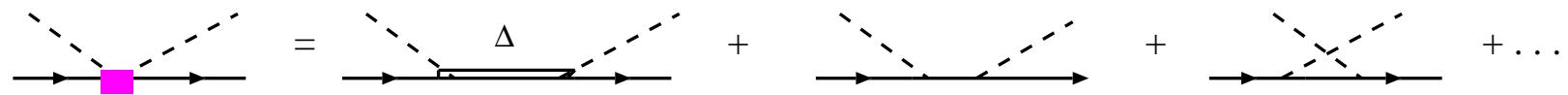
$\Delta + \text{non-resonant}$ (unitary)

Sato et al., PRC **67** (2003)

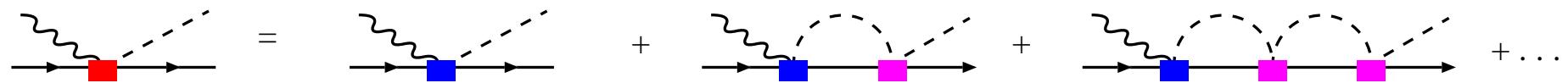
$W(Z) N \rightarrow \pi N$ potential



πN potential



$W(Z) N \rightarrow \pi N$ amplitude



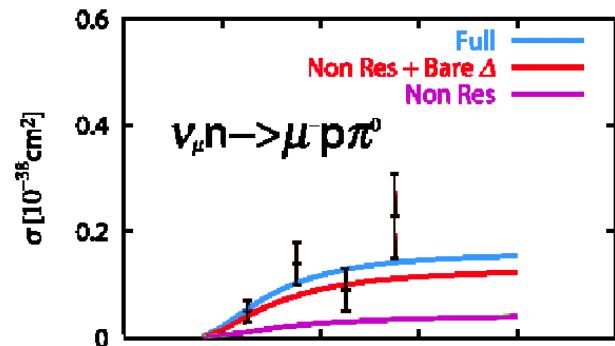
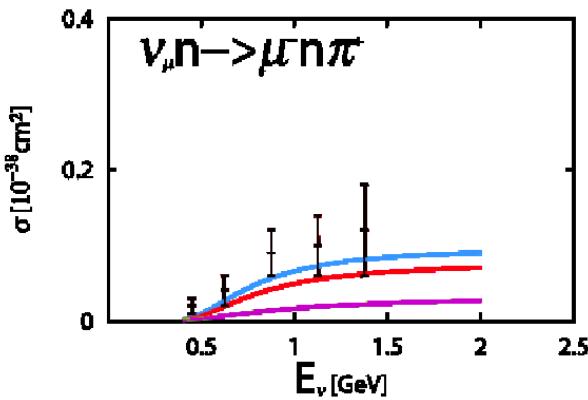
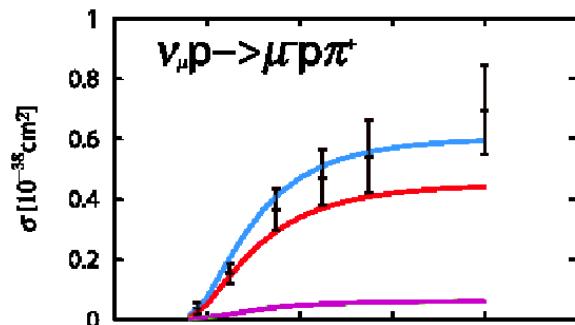
$\pi N \rightarrow \pi N, \gamma N \rightarrow \pi N, W(Z)N \rightarrow \pi N$ are related by **unitarity**

Elementary amplitudes

Nakamura et al, PRC **81** (2010)

Δ + non-resonant (unitary)

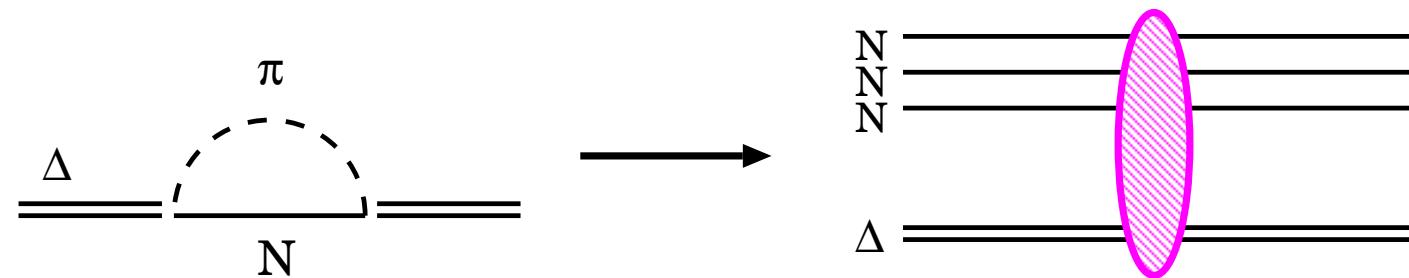
Sato et al., PRC **67** (2003)



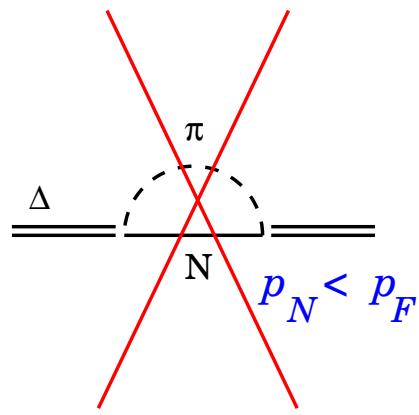
Data : Barish et al. (ANL), PRD **19**(1979)

- Significant pion cloud effect
- Important non-resonant mechanism (interference)

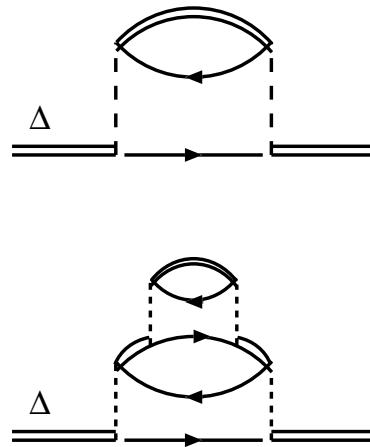
Medium effect on Δ



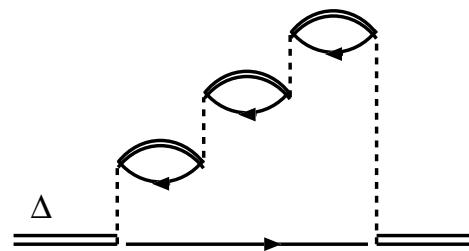
Medium effect on Δ



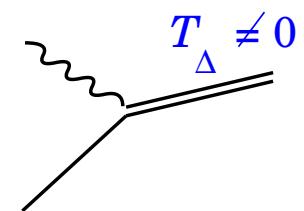
Pauli Blocking



2p-2h, 3p-3h



RPA



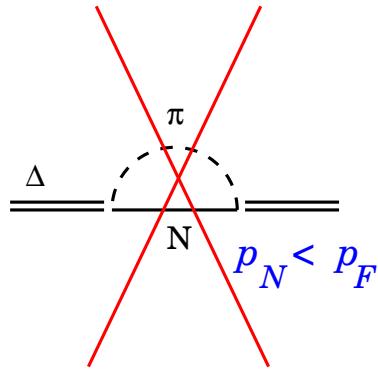
Non-locality

Medium effect on Δ

Kelkar et al., PRC **55** (1997); Singh et al., PRL **96** (2006); Alvarez-Ruso et al., PRC **75** (2007)

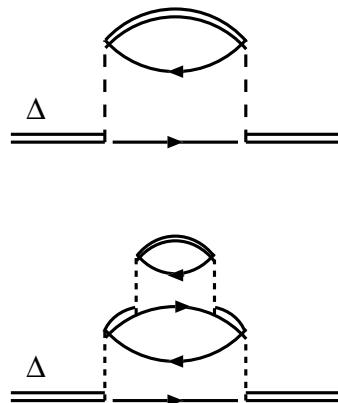
Amaro et al., PRD **79** (2009); Hernandez et al., PRD **82** (2010); Martini et al., PRC **80** (2009)

Many-body approach

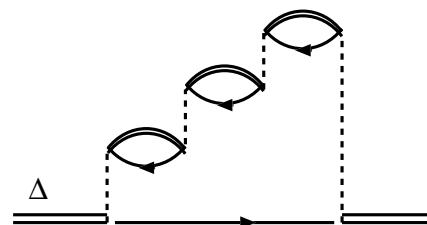


Pauli Blocking

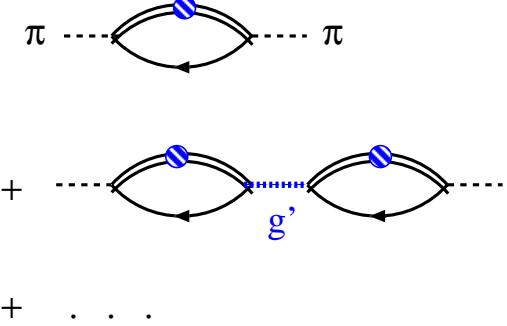
Oset and Salcedo, NPA **468** (1987); Carrasco and Oset, NPA **536** (1992)



2p-2h, 3p-3h



RPA



Short-range repulsion

- Δ mass and width are shifted
- Non-locality (recoil of Δ) is missing

Medium effect on Δ

Nakamura et al, PRC **81** (2010)

$$\frac{1}{E - m_\Delta^0 - \Sigma_\Delta} \implies \frac{1}{E - m_\Delta^0 - \Sigma_\Delta - H_\Delta - \Sigma_{Pauli} - \Sigma_{spr}}$$

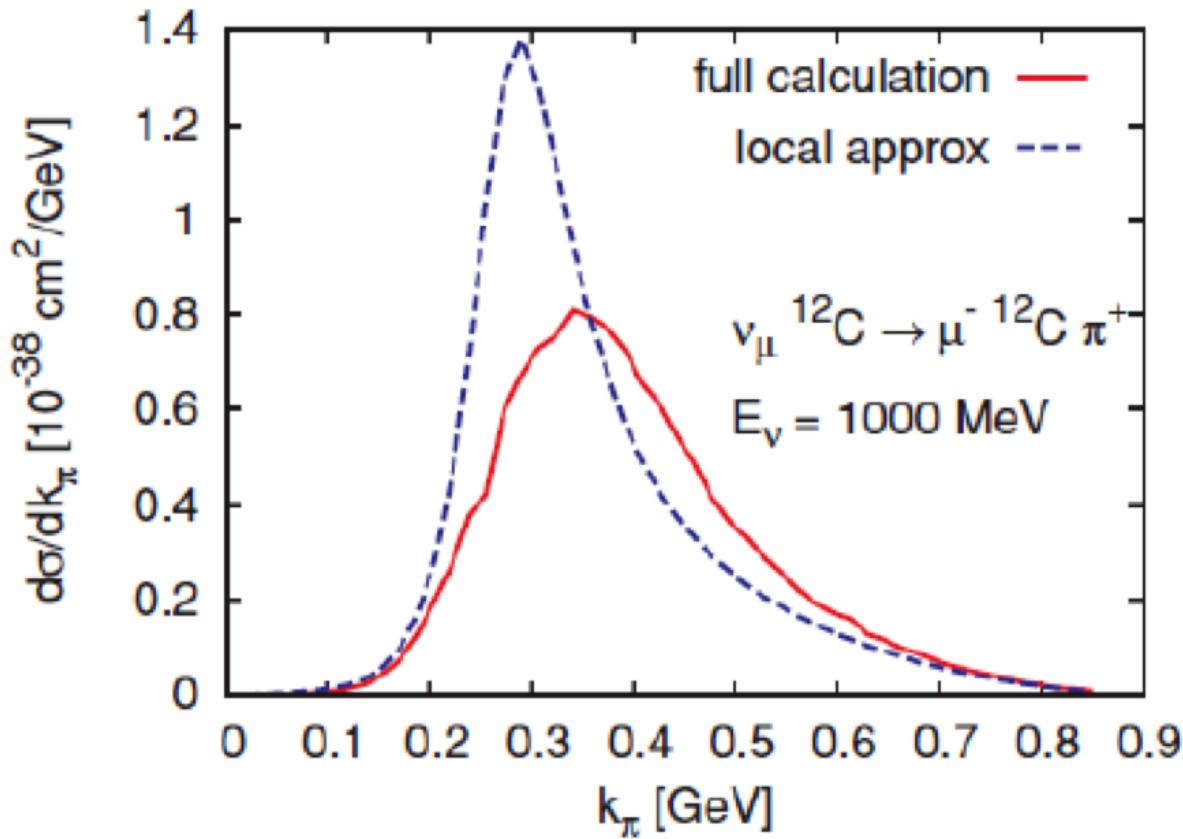
$$H_\Delta = T_\Delta + V_\Delta + H_{A-1}, \quad T_\Delta \Rightarrow \text{non-local effect}$$

Spreading potential $\Sigma_{spr} = V_C \rho(r) + V_{LS}(r) \vec{L}_\Delta \cdot \vec{S}_\Delta$

Parameters (complex) : $V_C, V_{LS} \rightarrow \pi$ -nucleus (total & elastic) scattering data

Non-locality

Leitner et al., PRC **79** (2009)



- $\sigma/\sigma_{\text{local}} \sim 0.5 (0.6)$ for $E_\nu = 0.5 (1)$ GeV
- Similar analysis including medium effect on Δ and FSI [Nakamura et al, PRC **81** (2010)]
- Effectively (partially) contained in $\delta\Sigma_\Delta$ [Hernandez et al., PRD **82** (2010)]

Final State Interaction

Kelkar et al., PRC **55** (1997); Alvarez-Ruso et al., PRC **75****76** (2007)

Amaro et al., PRD **79** (2009); Hernandez et al., PRD **82** (2010)

Klein-Gordon equation

$$(-\Delta - p_\pi^2 + 2\omega_\pi V_{\text{opt}}) \phi_\pi^*(r) = 0$$

$V_{\text{opt}}(r)$: (Δ -hole model) + (Lorentz-Lorentz correction)

⇒ Pionic atoms, low-energy $\pi - A$ elastic scattering Nieves et al., NPA **554** (1993)

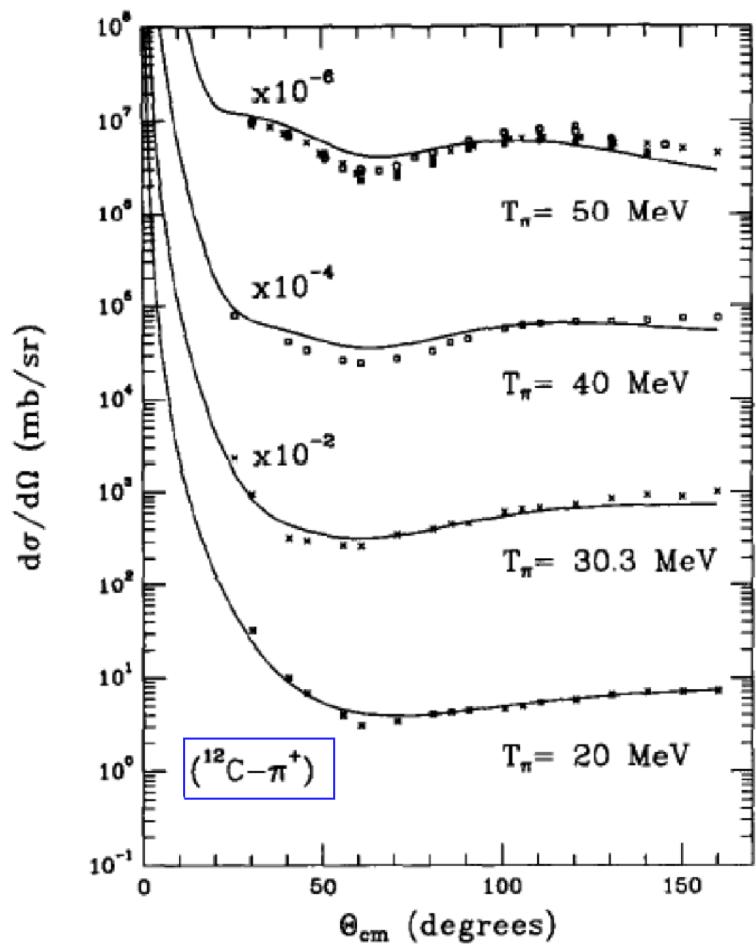
Singh et al., PRL **96** (2006)

Eikonal approximation

Final State Interaction

Kelkar et al., PRC **55** (1997); Alvarez-Ruso et al., PRC **7576** (2007)

Amaro et al., PRD **79** (2009); Hernandez et al., PRD **82** (2010)



Nieves et al., NPA **554** (1993)

Final State Interaction

Martini et al., PRC **80** (2009)

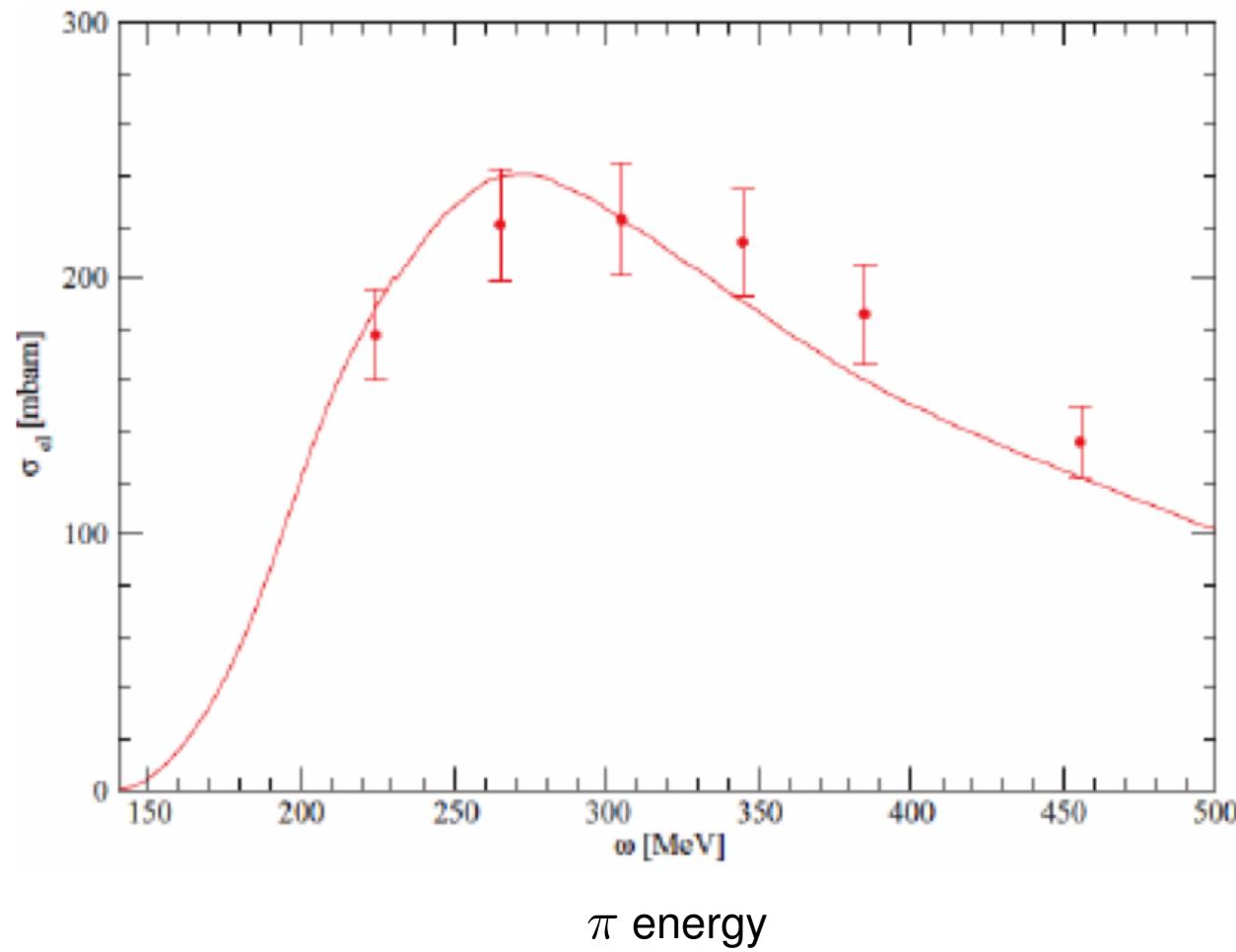
RPA equation

$$\text{---} \circlearrowleft \text{---} = \text{---} \circlearrowleft \Pi_0 \text{---} + \text{---} \circlearrowleft \Pi_0 \text{---} \circlearrowleft \text{---}$$

$$\begin{aligned} \text{---} \circlearrowleft \Pi_0 \text{---} &= \text{---} \circlearrowleft \text{---} + \text{---} \circlearrowleft \text{---} + \text{---} \circlearrowleft \text{---} + \text{---} \circlearrowleft \Delta \text{---} + \text{---} \circlearrowleft \Delta \text{---} \\ &+ \text{---} \circlearrowleft \Delta \text{---} + \text{---} \circlearrowleft \Delta \text{---} + \dots \end{aligned}$$

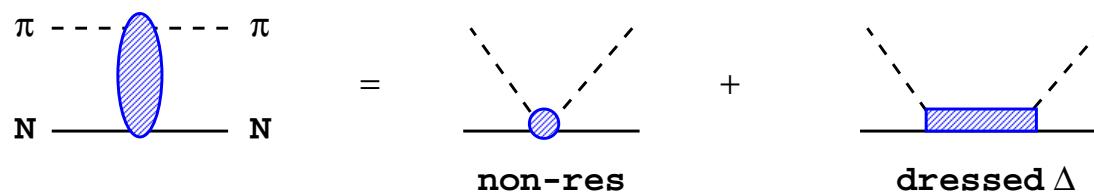
Final State Interaction

Martini et al., PRC **80** (2009) $\pi\text{-}^{12}\text{C}$ elastic scattering

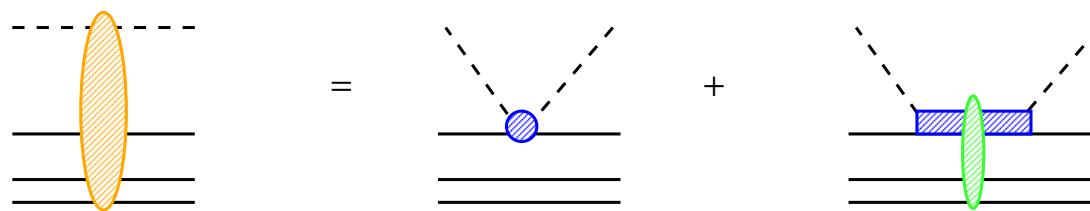


Final State Interaction Nakamura et al, PRC **81** (2010)

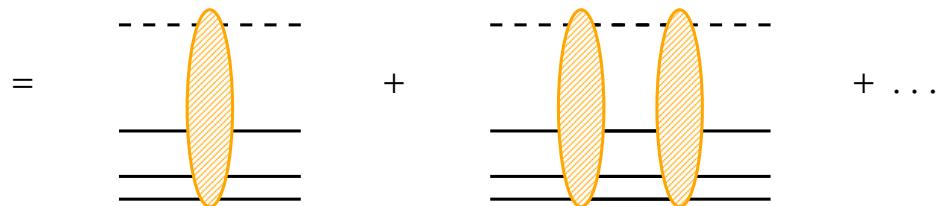
πN t-matrix



πA potential



πA t-matrix (Lippmann-Schwinger equation)



Final State Interaction Nakamura et al, PRC **81** (2010)

$\pi^\pm - {}^{12}\text{C}$ scattering [Data : NPB **17**, 168 (1970), PRC **29**, 561 (1984)]

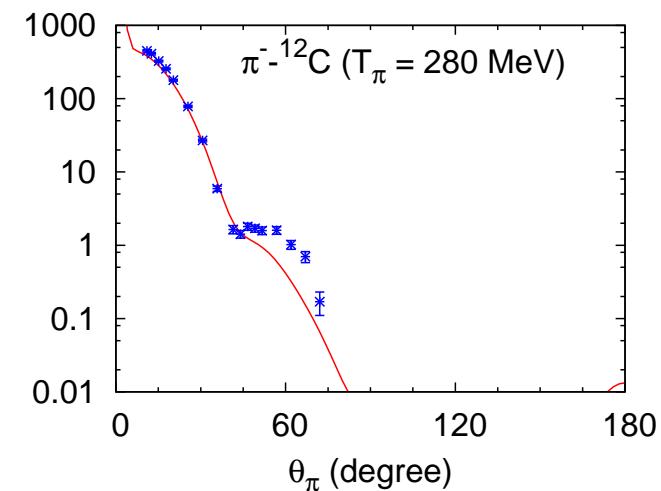
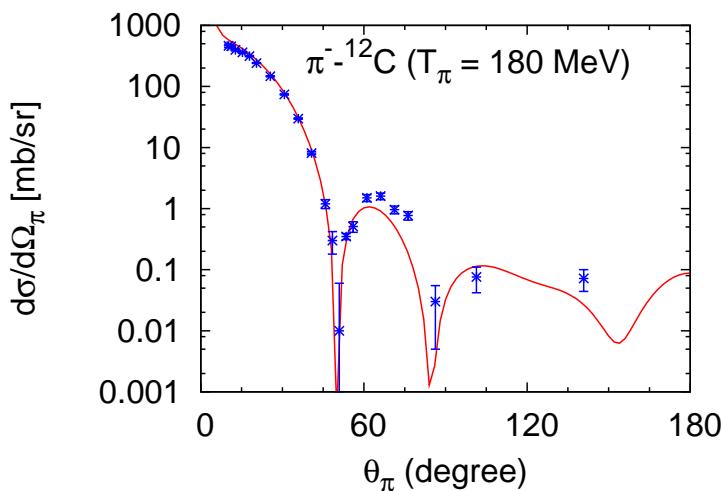
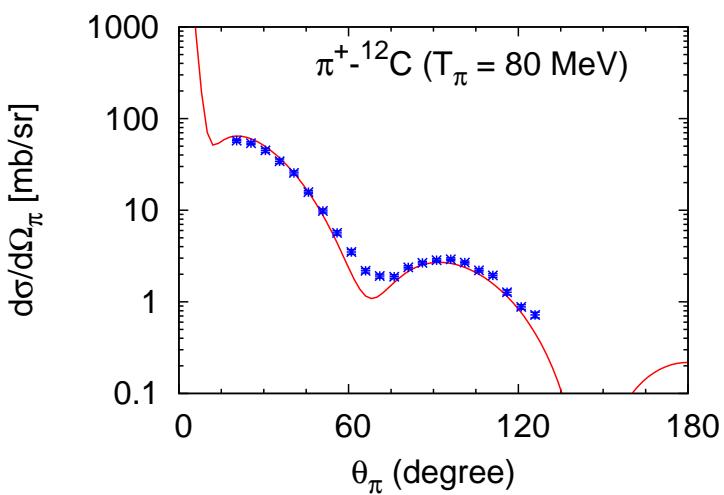
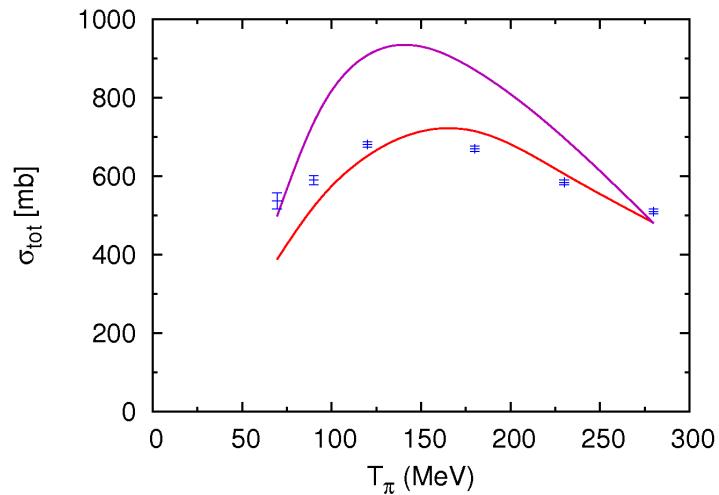
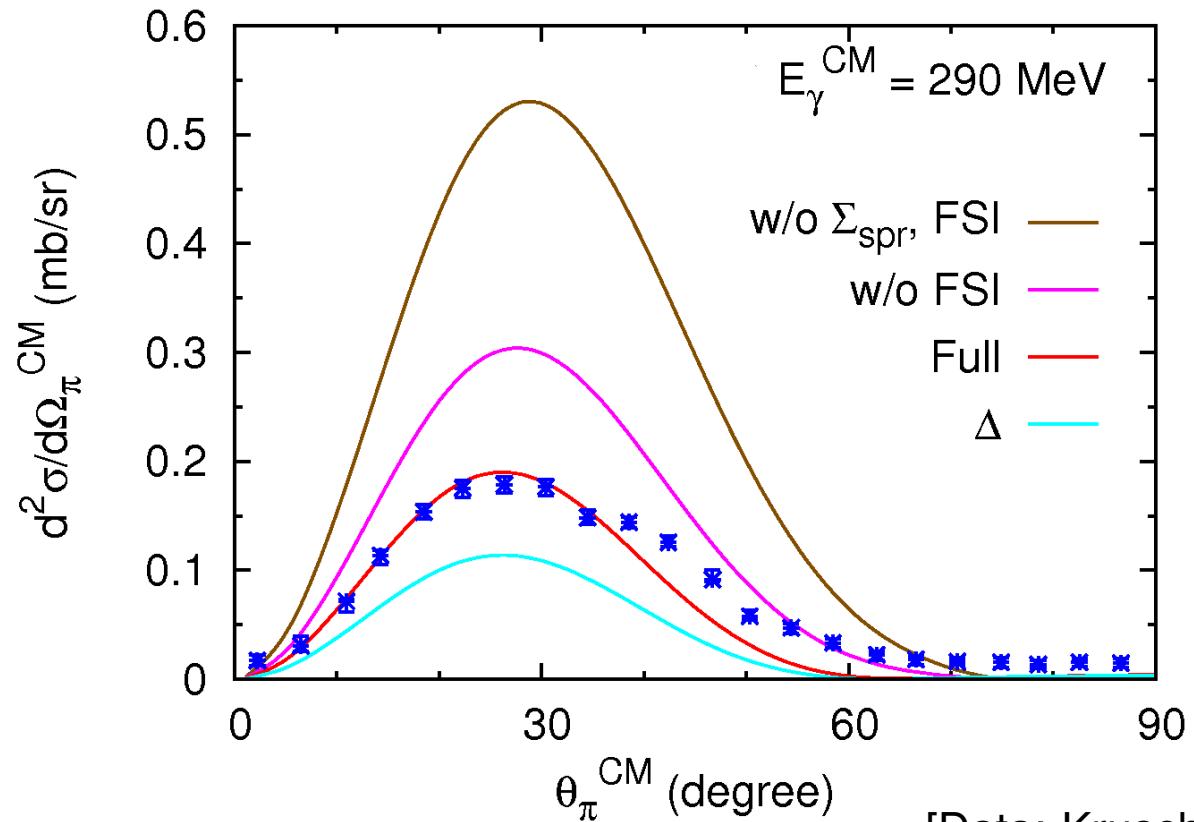


Photo coherent π production on ^{12}C

Nakamura et al, PRC **81** (2010)



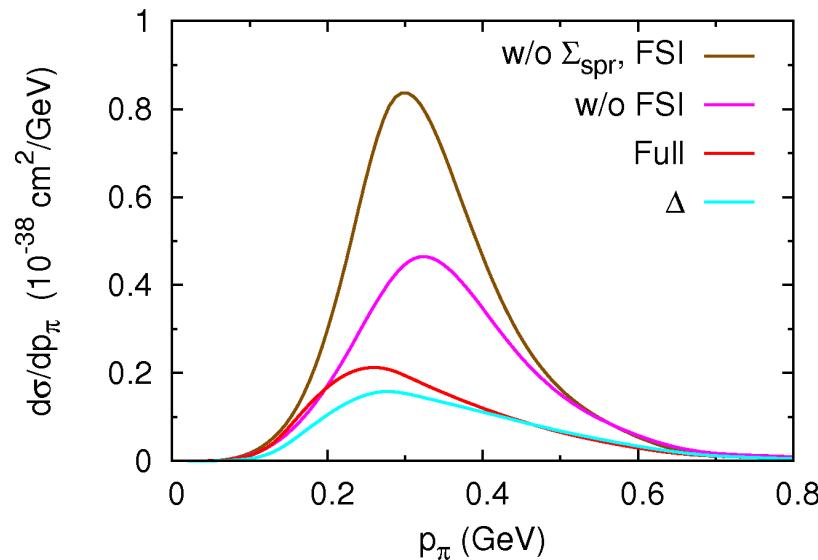
[Data: Krusche et al., PLB **526** (2002)]

- Parameter-free prediction
- Good testing ground for microscopic models
- Important medium effects

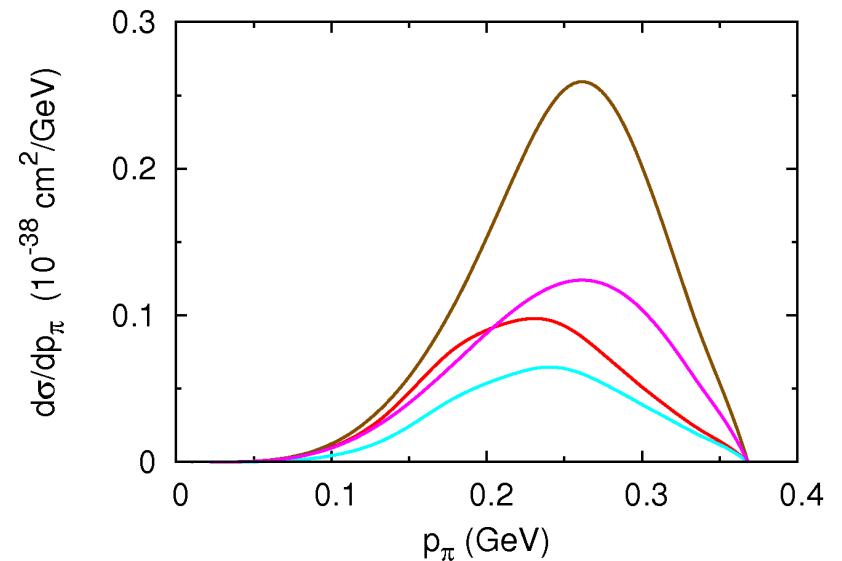
CC Coherent π production on ^{12}C

Nakamura et al, PRC **81** (2010)

CC $E_\nu = 1 \text{ GeV}$



CC $E_\nu = 0.5 \text{ GeV}$

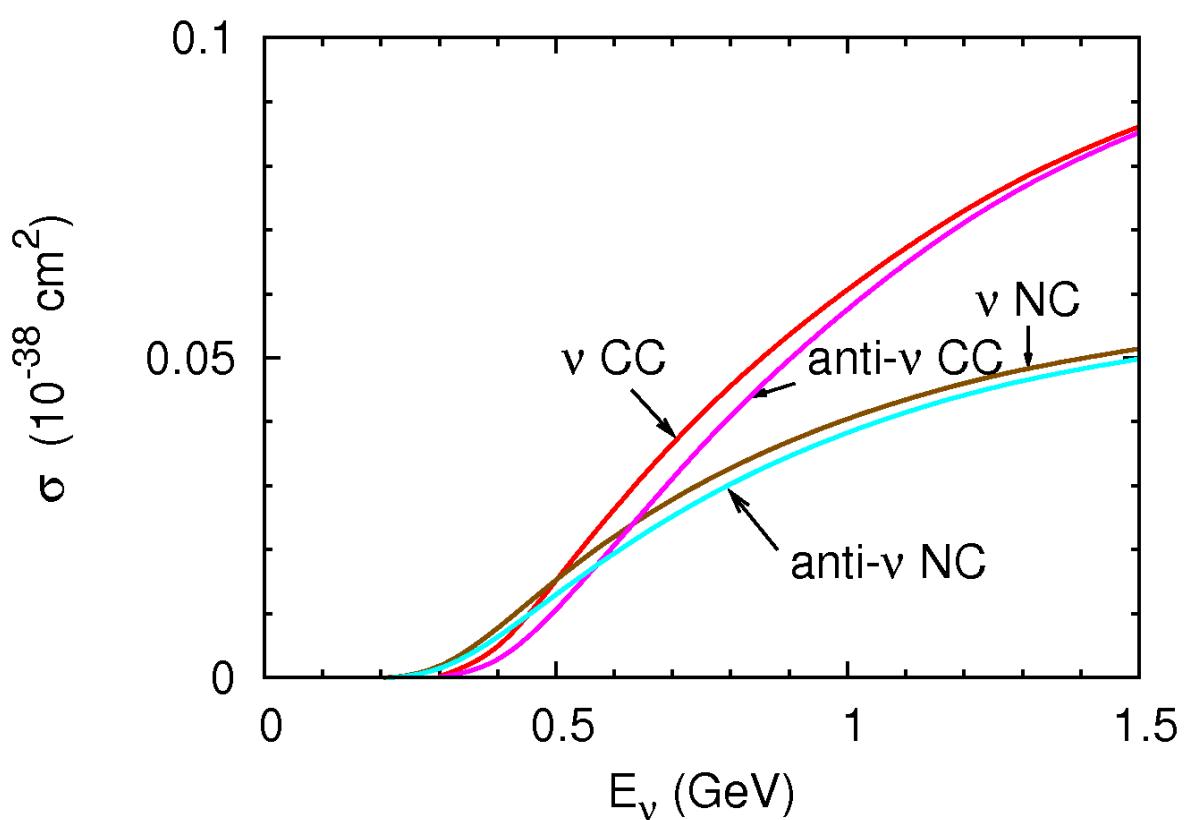


- Large medium effects in Δ region
- Enhancement due to non-resonance (interference with Δ)
32 (10) % at $E_\nu = 0.5$ (1) GeV
- No contribution from (tree-level) non-resonant mechanism

Amaro et al., PRD **79** (2009); Hernandez et al., PRD **82** (2010)

ν -induced coherent π production on ^{12}C

Nakamura et al, PRC **81** (2010)



CC

$$\sigma_{\text{ave}} = 6.3 \times 10^{-40} \text{ cm}^2$$

$$\sigma_{\text{K2K}} < 7.7 \times 10^{-40} \text{ cm}^2$$

[PRL **95**, 252301 (2005)]

NC

$$\sigma_{\text{ave}} = 3.9 \times 10^{-40} \text{ cm}^2$$

$$\sigma_{\text{MiniBooNE}} = 7.7 \pm 1.6 \pm 3.0 \times 10^{-40} \text{ cm}^2$$

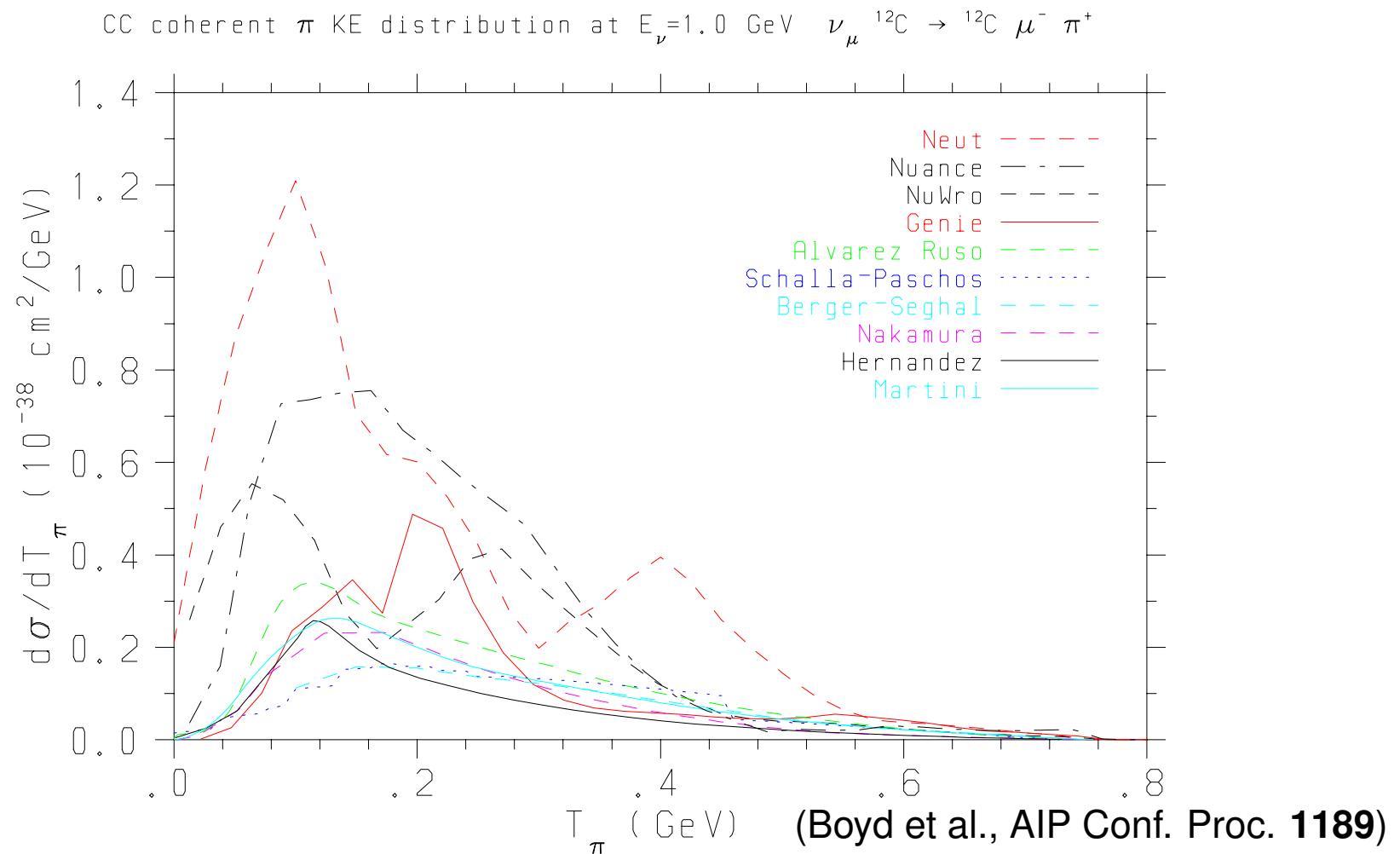
[Raaf, PhD thesis]

New data

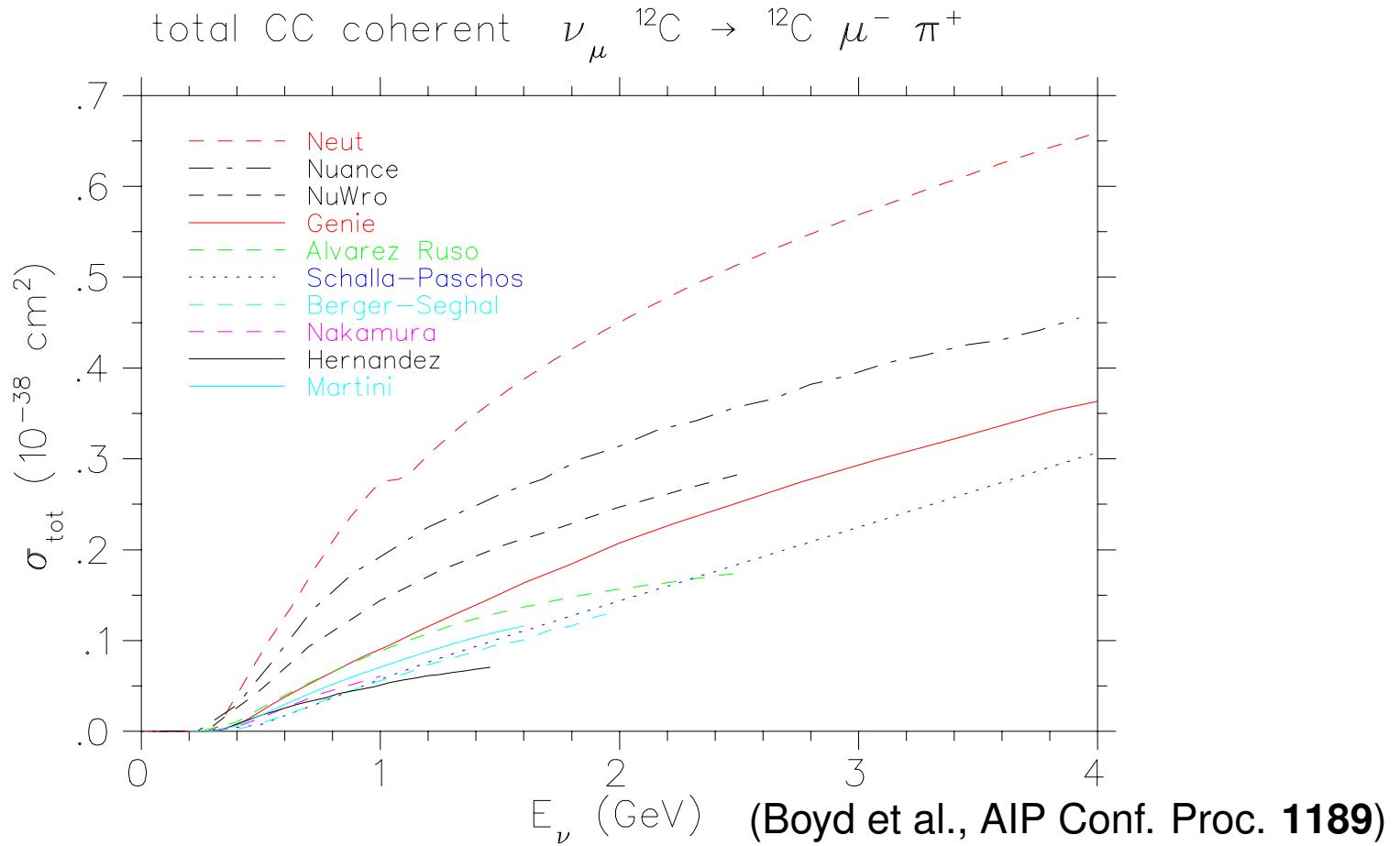
$$\sigma(CC\pi^+)/\sigma(NC\pi^0) = 0.14$$

[Kurimoto et al. PRD **81** 111102 (2010)]

Comparison of models



Comparison of models



- Large discrepancy between MC code (RS model) and recent calculations
- Fair agreement among recent calculations (but **details are different**)

Comparison with Data

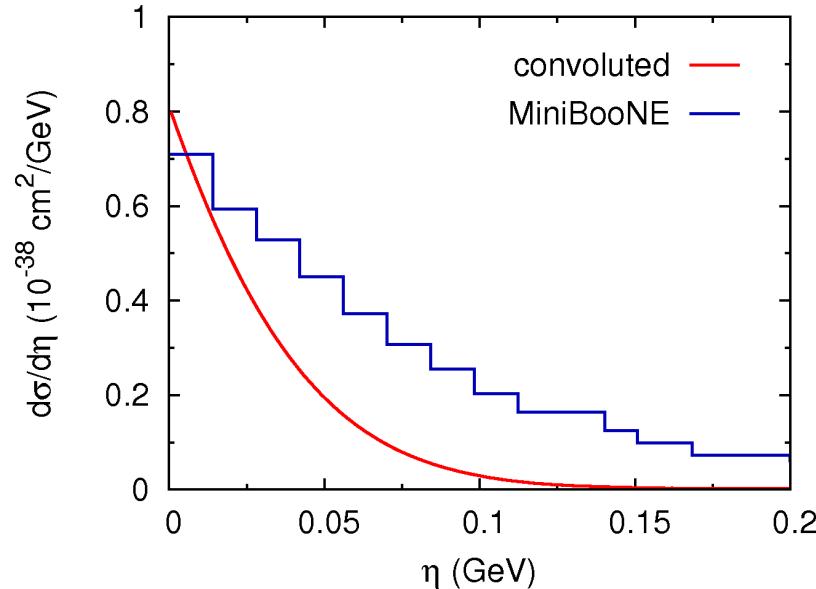
Large discrepancy between data and theory in CC π^+ / NC π^0

$$\sigma(\text{CC}\pi^+) / \sigma(\text{NC}\pi^0) = 0.14^{+0.30}_{-0.28} \quad \text{Kurimoto et al., PRD } \mathbf{81} \text{ (2010)}$$

$$\sigma(\text{CC}\pi^+) / \sigma(\text{NC}\pi^0) = 1 \sim 2 \quad \text{all theoretical calculations}$$

Data analysis of coherent NC π^0 with Rein-Sehgal model

$\eta \equiv E_\pi(1 - \cos \theta_\pi)$ distribution

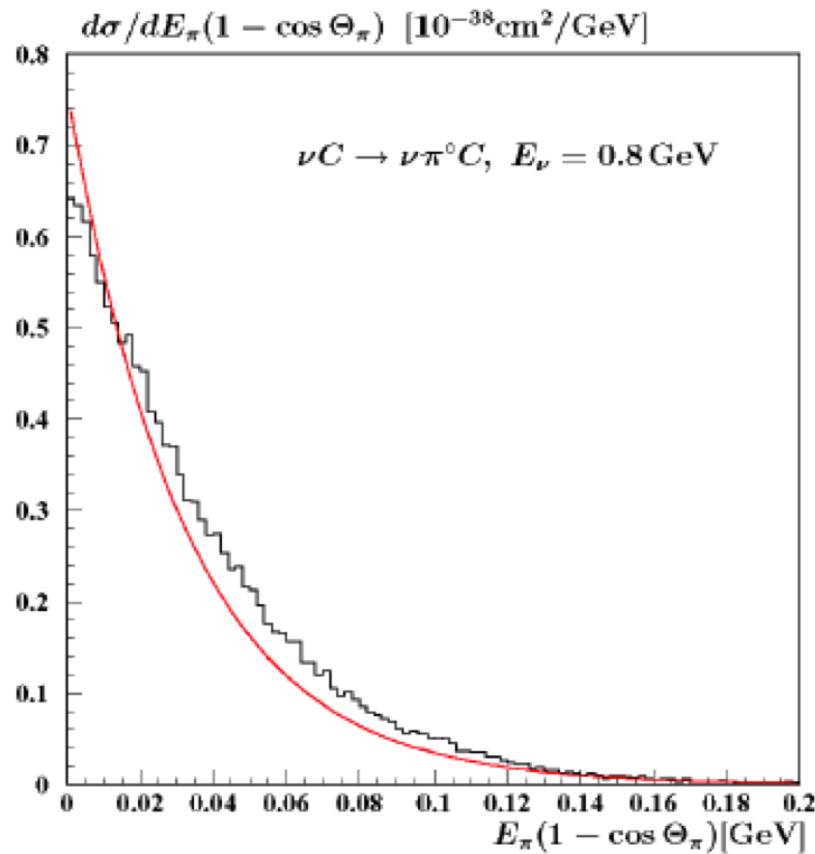


[MC from MiniBooNE PLB **664**, 41 (2008)]

- η is useful to break degeneracy of several pion productions in data
- Discrepancy between Monte Carlo (RS model) and microscopic models
Amaro et al., PRD **79** (2009); Hernandez et al., PRD **80** (2009); **82** (2010)
Nakamura et al, PRC **81** (2010)
⇒ possible overestimation of NC cross section

$\eta \equiv E_\pi(1 - \cos \theta_\pi)$ distribution from PCAC model

Berger and Sehgal, PRD **79**, (2009); Berger, AIP Conf. Proc. **1189** (2009)



red : Hernandez et al., PRD **80** (2009)

black : Berger and Sehgal