Axial and Vector SFs for Lepton-Nucleon Scattering

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Neutrino Cross Section

- Quasi-Elastic / elastic (W=M): $\nu_\mu + n \rightarrow \mu^- + p$
  - by form factors
- Resonance (low $Q^2$, W< 2): $\nu_\mu + p \rightarrow \mu^- + p + \pi$
  - by Rein and Seghal model (overlap with DIS)
- Deep Inelastic Scattering: $\nu_\mu + p \rightarrow \mu^- + X$
  - by quark-parton model (non-pQCD effect, high x PDFs)

- Describe DIS, resonance within quark-parton model: with PDFS, it is easy to convert $\sigma(e)$ into $\sigma(\nu)$
- Challenges
  - High x PDFs at very low $Q^2$
  - Resonance scattering within quark-parton model
  - What happens at $Q^2=0$?
  - Axial vector contribution
Modeling neutrino cross sections

- **NNLO pQCD +TM approach:** describes the DIS and resonance data very well:

- **Bodek-Yang LO approach:** (pseudo NNLO)
  Use effective LO PDFs with a new scaling variable, $\xi_w$ to absorb target mass, higher twist, missing QCD higher orders

\[ \xi_w = \frac{Q^2 + B}{\{ M N [1 + \sqrt{(1+Q^2)/v^2}] + A \} } \]

\[ F_2(x,Q^2) \rightarrow \frac{Q^2}{Q^2 + C} F_2(\xi_w, Q^2) \]
1. Start with GRV98 LO ($Q^2_{\text{min}}=0.80$)

2. Replace $x_{tij}$ with a new scaling, $\xi_w$

3. Multiply all PDFs by $K$ factors for photo production limit and higher twist

   \[
   [\sigma(\gamma) = 4\pi \alpha / Q^2 * F_2(x, Q^2)]
   \]

   \[
   K_{\text{sea}} = Q^2 / [Q^2 + C_{\text{sea}}]
   \]

   \[
   K_{\text{val}} = [1 - G_D^2 (Q^2)] * [Q^2 + C_{2v}] / [Q^2 + C_{1v}]
   \]

   motivated by Adler Sum rule

   where $G_D^2 (Q^2) = 1 / [1 + Q^2 / 0.71]^4$

4. Freeze the evolution at $Q^2 = Q^2_{\text{min}}$

   - $F_2(x, Q^2 < 0.8) = K(Q^2) * F_2(\xi_w, Q^2=0.8)$

5. Fit all DIS $F_2(p/D)$ with low $x$ HERA data, photo-production data

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<tr>
<th></th>
<th>A</th>
<th>B</th>
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<tbody>
<tr>
<td></td>
<td>=0.621</td>
<td>=0.380</td>
</tr>
<tr>
<td>$C_{2v}(u)$</td>
<td>=0.264</td>
<td>$C_{2v}(d)$</td>
</tr>
<tr>
<td>$C_{1v}(u)$</td>
<td>=0.417</td>
<td>$C_{1v}(d)$</td>
</tr>
<tr>
<td>$C_{\text{sea}}(u)$</td>
<td>=0.369</td>
<td>$C_{\text{sea}}(d,s)$</td>
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Excellent Fitting:
• red solid line: effective LO using $\xi_w$
• black dashed line: $x_{bj}$
Low x HERA and NMC data

- Fit works at low x
Additional $K^{LW}$ factor for valence quarks:

$$K_{\text{val}} = K^{LW} \times [1 - G_D^2 (Q^2)] \times \frac{Q^2 + C_{2V}}{Q^2 + C_{1V}}$$

$$K^{LW} = \frac{\nu^2 + C^y}{\nu^2}$$

This makes a duality work all the way down to $Q^2 = 0$ (for charged leptons)

Photo-production data with $\nu > 1$ GeV are included in the fitting
Predictions are in good agreement with resonance data (not included in the fitting); duality works for electrons and muons for our effective LO PDFs
Effective LO model with $\xi w$ describe all DIS and resonance $F_2$ data as well as photo-production data ($Q^2=0$ limit): vector contribution works well

Neutrino Scattering:
- Effective LO model works for $xF_3$?
- Nuclear correction using $e/\mu$ scattering data
- Axial vector contribution at low $Q^2$?
- Use $R=R1998$ to get $2xF_1$
- Implement charm mass effect through $\xi w$ slow rescaling algorithm for $F_2$, $2xF_1$, and $xF_3$
Effective LO model for $xF_3$?

- Scaling variable, $\xi w$ absorbs higher order effect for $F_2$, but the higher order effects for $F_2$ and $xF_3$ are not the same.
- Use NLO QCD to get double ratio

$$H(x) = \frac{xF_3^{(NLO)}}{xF_3^{(LO)}} / \frac{F_2^{(NLO)}}{F_2^{(LO)}}$$

not 1 but indep. of $Q^2$
- Enhance anti-neutrino cross section by 3%
Nuclear Effects: use e/μ data

Fe/D

D/(n+P)

Updated Fe/D

Lead/Fe
Axial Vector Structure Functions

- **Type I:** Axial Vector = Vector (A=V)

- **Type II:**
  \[
  K_{\text{sea}}^{\text{vector}} = \frac{Q^2}{Q^2 + C} \quad \Rightarrow \quad K_{\text{sea}}^{\text{axial}} = \frac{Q^2 + 0.6C^{\text{axial}}}{Q^2 + C^{\text{axial}}}
  \]
  \[C^{\text{axial}} = 0.3\]

  - 0.6 was chosen to satisfy the prediction from PCAC by Kulagin, agrees with CCFR/CHROUS data for \(F_2\) extrapolation to \((Q^2=0)\)

  \[
  K_{\text{val}}^{\text{axial}} = \frac{Q^2 + 0.1 \times 0.18}{Q^2 + 0.18}
  \]

- But, the non-zero PCAC component of \(F_2^{\text{axial}}\) at low \(Q^2\): purely longitudinal

  \[2xF_1^{\text{axial}} = 2xF_1^{\text{vector}}\]
Comparison with CCFR (Fe), CHORUS (Pb) data

- Blue point: CHORUS/theory (type II)
- Blue line: theory (type I)/(type II)
- Red point: CCFR/theory (type II)
Comparison with CCFR(Fe), CHORUS (Pb) data
Test of the Adler Sum Rule

- This sum rule should be valid at all values of $Q^2$

\[
|F_V(Q^2)|^2 + \int_{v_0}^{\infty} W_{2n-sc}^{\nu-vector}(v, Q^2)\,dv
- \int_{v_0}^{\infty} W_{2p-sc}^{\nu-vector}(v, Q^2)\,dv = 1
\]

\[
|F_A(Q^2)|^2 + \int_{v_0}^{\infty} W_{2n-sc}^{\nu-axial}(v, Q^2)\,dv
- \int_{v_0}^{\infty} W_{2p-sc}^{\nu-axial}(v, Q^2)\,dv = 1
\]

Vector

Axial
Total cross sections

- BY(DIS, W > 1.4)
- + Q.E. + Resonance
Summary of changes from 2004

- 2004: BY Model (currently implemented in Neutrino Monte Carlo) has $V=A$ and $H(x)=1$ (same scaling violation in $F_2$ and $xF_3$). It has been used for $W>1.8$ GeV.

- 2011: Addition of $H(x)$ correction to $xF_3$ plus axial $K$ factor by PCAC ($A=PCAC$) to $F_2$.
  
  - Change anti-neutrino cross sections by ~6%; better agreement with experimental data.
  - Better agreement in $d\sigma/dx dy$ at low $Q^2$.
  - Better agreement with the total cross sections.
  - Addition of low $W$ $K$ factor ($K^{LW}$) extend the validity of model down to $W=1.4$ GeV, thus providing overlap with resonance models.
Summary & Discussions

- BY Effective LO model with $\xi_w$ describe all e/µ DIS and resonance data as well as photo-production data (down to $Q^2=0$): provide a good reference for vector SF for neutrino cross section

- $d\sigma/dx dy$ data favor updated BY(DIS) type II model

- $K$ factors for axial vectors in BY(DIS) type II model are based on PCAC and could be further tuned with new neutrino data ($Q^2<0.3$, e.g. MINERνA)

- BY(DIS) type II model (axial=PCAC) provide a good reference for both neutrino and anti-neutrino cross sections ($W>1.8$). Low energy neutrino experiments can normalize their data to our model to extract their flux

- Model also works well down to $W=1.4$ GeV, thus providing overlap with resonance models