Electron- vs Neutrino-Nucleus Scattering

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Electron scattering
  ▶ standard data representation
  ▶ theoretical description: the impulse approximation
Outline

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  - standard data representation
  - theoretical description: the impulse approximation

- Neutrino-nucleus scattering
  - impact of the flux average
  - flux-averaged electron scattering x-section: a numerical experiment
  - contributions of reaction mechanisms other than quasi elastic single nucleon knock out to the MiniBooNE CCQE data sample
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- **Electron scattering**
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- **Summary & Outlook**
The inclusive electron-nucleus x-section

- The x-section of the process

$$e + A \rightarrow e' + X$$

is generally analyzed at *fixed beam energy* and electron scattering angle, as a function of the electron energy loss $\omega = E_e - E_{e'}$.

- Different reaction mechanisms can be readily identified.
Kinematical range covered by the available data

- Carbon target

\[ Q^2 = 4E_e E_{e'} \sin^2 \frac{\theta_e}{2}, \quad x = \frac{Q^2}{2M\omega} \]
Double differential electron-nucleus cross section

\[
\frac{d\sigma_{eA}}{d\Omega_{e'}dE_{e'}} = \frac{\alpha^2}{Q^4} \frac{E_e' L_{\mu\nu} W_{\mu\nu}^A}{E_e}
\]

- \( L_{\mu\nu} \) is fully specified by the measured electron kinematical variables
- the determination of the target response tensor

\[
W_{\mu\nu}^A = \sum_n \langle 0| J_{\mu}^A | n \rangle \langle | J_{\nu}^A | 0 \rangle \delta^{(4)}(P_0 + k_e - P_n - k_{e'})
\]

requires a consistent description of the target internal dynamics and electromagnetic current, unavoidably implying approximations

Impulse approximation (IA): at large momentum transfer electron-nucleus scattering reduces to the incoherent sum of scattering processes involving individual nucleons
The impulse approximation

- The IA amounts to the replacement

\[ J_A^\mu = \sum_i j_i^\mu, \quad |X\rangle \rightarrow |x, p_x\rangle \otimes |\mathcal{R}, p_\mathcal{R}\rangle. \]

- Nuclear dynamics and electromagnetic interactions are decoupled

\[ d\sigma_A = \int d^3kdE d\sigma_N P(k, E) \]

- The electron-nucleon cross section \( d\sigma_N \) can be extracted from electron-proton and electron-deuteron scattering data

- The spectral function \( P(k, E) \), momentum and energy distribution of the knocked out nucleon, can be obtained from \textit{ab initio} many-body calculations
Theory vs data

- **Memento**: theoretical calculations involve no adjustable parameters
The quasi elastic (QE) sector

- Elementary interaction vertex described in terms of the vector form factors, $F_1^{(p,n)}$ and $F_2^{(p,n)}$, precisely measured over a broad range of $Q^2$.

- Position and width of the peak are determined by $P(k, E)$.

- The tail extending to the region of high energy loss is due to nucleon-nucleon correlations in the initial state, leading to the occurrence of two particle-two hole (2p2h) final states.
Consider the data sample of charged current QE data collected by the MiniBooNE Collaboration using a CH$_2$ target.

The measured double differential cross section is averaged over the energy of the incoming neutrino, distributed according to the flux $\Phi$:

$$
\frac{d\sigma_A}{dT_\mu d\cos\theta_\mu} = \frac{1}{N_\Phi} \int dE_\nu \Phi(E_\nu) \frac{d\sigma_A}{dE_\nu dT_\mu d\cos\theta_\mu}
$$

In addition to $F_1$ and $F_2$, the QE electron-nucleon cross section is determined by the axial form factor $F_A$, assumed to be of dipole form and parametrized in terms of the axial mass $M_A$.

According to the paradigm successfully employed to describe electron scattering data, in order to minimize the bias associated with nuclear effects, $M_A$ must be determined from measurement carried out using a deuterium target. The resulting value is $M_A = 1.03$ GeV.
Analysis of CCQE data

MiniBooNe flux

Theoretical calculations carried out setting $M_A = 1.03$
Analysis of CCQE data

- Theoretical calculations carried out setting $M_A = 1.03$

- The electron scattering paradigm fails to explain the *flux-averaged* neutrino scattering x-section

**MiniBooNE CCQE data**
In neutrino interactions the lepton kinematics is \textit{not} determined. The flux-averaged cross sections at fixed $T_\mu$ and $\cos \theta_\mu$ picks up contributions at different beam energies, corresponding to a variety of kinematical regimes in which different reaction mechanisms dominate.

- $x = 1 \rightarrow E_\nu 0.788$ GeV, $x = 0.5 \rightarrow E_\nu 0.975$ GeV
- $\Phi(0.975)/\Phi(0.788) = 0.83$
“Flux averaged” electron-nucleus x-section

- The electron scattering x-section off Carbon at $\theta_e = 37^\circ$ has been measured for a number of beam energies

- MIT-Bates data compared to theoretical calculations including QE scattering only
A numerical experiment

- Consider the cross section at $\theta = 37^\circ$ and $550 \leq T_{e'} \leq 650$ MeV (corresponding to the QE peak at 730 MeV)
- Data are available at $E_e = 730, 961, 1108, 1299$ and 1501 MeV
A numerical experiment

- Consider the cross section at $\theta = 37^\circ$ and $550 \leq T_e' \leq 650$ MeV (corresponding to the QE peak at 730 MeV)
- Data are available at $E_e = 730, 961, 1108, 1299$ and 1501 MeV
- Compute the flux averaged cross section using experimental data and assuming that the electron beam energies be distributed according to the MiniBooNE neutrino flux ($\Sigma_{exp}$)
- Compute the flux averaged cross section using the results of theoretical calculations including QE scattering only ($\Sigma_{th}$)

The above procedure yields $\Sigma_{exp} \Sigma_{th} \sim > 1.20$. The electron scattering paradigm fails to explain the flux averaged electron scattering cross section.
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\frac{\Sigma_{\text{exp}}}{\Sigma_{\text{th}}} \gtrsim 1.20
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- The electron scattering paradigm fails to explain the flux averaged electron scattering x-section
Where does the additional strength come from?

- It has been suggested that 2p2h final states provide a large contribution to the measured neutrino cross section.
- Two particle-two hole final states may be produced through different mechanisms:
  - **Initial state correlations**: lead to the tail extending to large energy loss, clearly visible in the calculated QE cross section. The corresponding strength is consistent with the measurements of the coincidence \((e, e'p)\) x-section carried out by the JLAB E97-006 Collaboration.
  - **Final state interactions**: lead to a redistribution of the inclusive strength, mainly affecting the region of \(x > 1\), corresponding to low energy loss.
  - **Coupling to the two-body current**: leads to the appearance of strength at \(x < 1\), in the dip region between the QE and \(\Delta\)-excitation peaks.
Measured correlation strength

- The correlation strength in the 2p2h sector has been measured by the JLAB E97-006 Collaboration using a carbon target.

- Strong energy-momentum correlation: \( E \sim E_{\text{thr}} + \frac{A-2}{A-1} \frac{k^2}{2m} \)

![Graph showing measured correlation strength](image)

- Measured correlation strength \(0.61 \pm 0.06\), to be compared with the theoretical predictions 0.64 (CBF) and 0.56 (G-Matrix).

- Correlated nucleons (most of the times a proton and a neutron) have momenta > 250 MeV, pointing in opposite directions.
Two-body current

- The nuclear electromagnetic current involves two-nucleon contributions, arising from nucleon-nucleon interactions and nucleon excitations

\[ J_A^{\mu} = \sum_{i=1}^{A} j_1^{\mu}(i) + \sum_{j>1=1}^{A} j_2^{\mu}(i,j) \]

- Meson exchange current (MEC) arising from processes involving pions

- In electron scattering MEC produce a significant enhancement of the $^3$He and $^4$He inclusive x-section in the transverse channel
MEC effect on the transverse electromagnetic response

- L/T separation of the the inclusive electron-nucleus x-section

\[
\frac{d^2\sigma}{d\Omega_{e'}dE_{e'}} = \left( \frac{d\sigma}{d\Omega_e} \right)_M \left[ \frac{Q^4}{|q|^4} R_L(|q|, \omega) + \left( \frac{1}{2} \frac{Q^2}{|q|^2} + \tan^2 \frac{\theta}{2} \right) R_T(|q|, \omega) \right]
\]

- Calculations by M.J. Dekker et al (Fermi gas model, AD 1991)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{graphs.png}
\end{figure}
The nuclear electromagnetic current must satisfy the continuity equation

\[ \partial_\mu J^\mu = \nabla \cdot J + i[H, J^0] = 0, \]

where \( H \) is the nuclear hamiltonian.

The two-body current can be separated into a model-independent term, fully determined from the interaction, and a model-dependent one associated with the \( \rho \pi \gamma \) and \( \omega \pi \gamma \) electromagnetic couplings.

A fully consistent treatment of the contribution of 2p2h final states requires the inclusion of initial state nucleon-nucleon correlations, final state interactions and MEC.

\textit{ab initio} calculations based on realistic nuclear hamiltonians and wave functions provide a very good description of the electron scattering x-sections of light nuclei.
May inclusion of MEC contributions explain the MiniBooNE data?

- A circumstantial evidence: the disagreement between data and theoretical calculations not including MEC turns out to be less pronounced at small $\theta_\mu$.
Summary and outlook

- In electron-nucleus scattering, the energy loss spectra at fixed beam energy allow for a clearcut identification of different reaction mechanisms.

- This feature has been exploited to carry out theoretical calculations of the *inclusive* cross section, providing a quantitative description of the large body of data over a broad kinematical range.

- Due to flux average, the measured “quasi elastic” neutrino-nucleus cross section picks up contributions from different kinematical regions, where reaction mechanisms other than single nucleon knock out are dominant.

- Systematic studies of the flux-averaged electron scattering cross section may prove very useful to pin down the relevant reaction mechanisms.

- The development of a theoretical description of the neutrino x-section *consistently* taking into account all the relevant mechanisms will shed light on the controversial issue of medium modifications of the axial form factor.
Background slides
Local Density Approximation (LDA) \( P(k, E) \) for oxygen

\[
P(k, E) = P_{MF}(k, E) + P_{corr}(k, E)
\]

- \( P_{MF}(k, E) \rightarrow \) from \((e, e'p)\) data
- \( P_{corr}(p, E) \rightarrow \) from uniform nuclear matter calculations at different densities:

\[
P_{MF}(k, E) = \sum_n Z_n |\phi_n(k)|^2 F_n(E - E_n)
\]

\[
P_{corr}(k, E) = \int d^3r \rho_A(r) P_{corr}^{NM}(k, E; \rho = \rho_A(r))
\]
Spectral function and momentum distribution of Oxygen

- FG model: $P_h(k, E) \propto \theta(k_F - |k|) \delta(E - \sqrt{|k|^2 + m^2 + \epsilon})$
- shell model states account for $\sim 80\%$ of the strength
- the remaining $\sim 20\%$, arising from NN correlations, is located at high momentum and large removal energy ($k \gg k_F, E \gg \epsilon$)
Nuclear structure described according to the Relativistic Fermi Gas model