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# The nucleon axial mass and the MiniBooNE CCQE neutrino-nucleus data

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- [arXiv:1106.5374 \[hep-ph\]](#)
  - [arXiv:1102.2777 \[hep-ph\]](#): PRC 83 (2011) 045501
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- [nucl-th/0408005](#): PRC 70 (2004) 055503
  - [hep-ph/0604042](#): PLB 638 (2006) 325

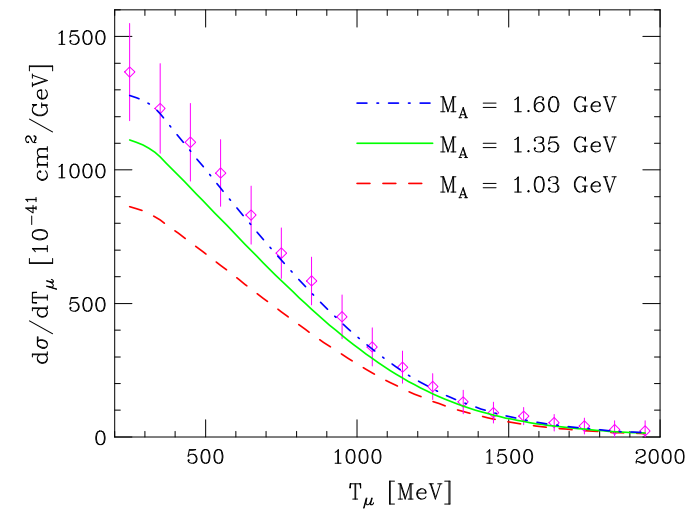
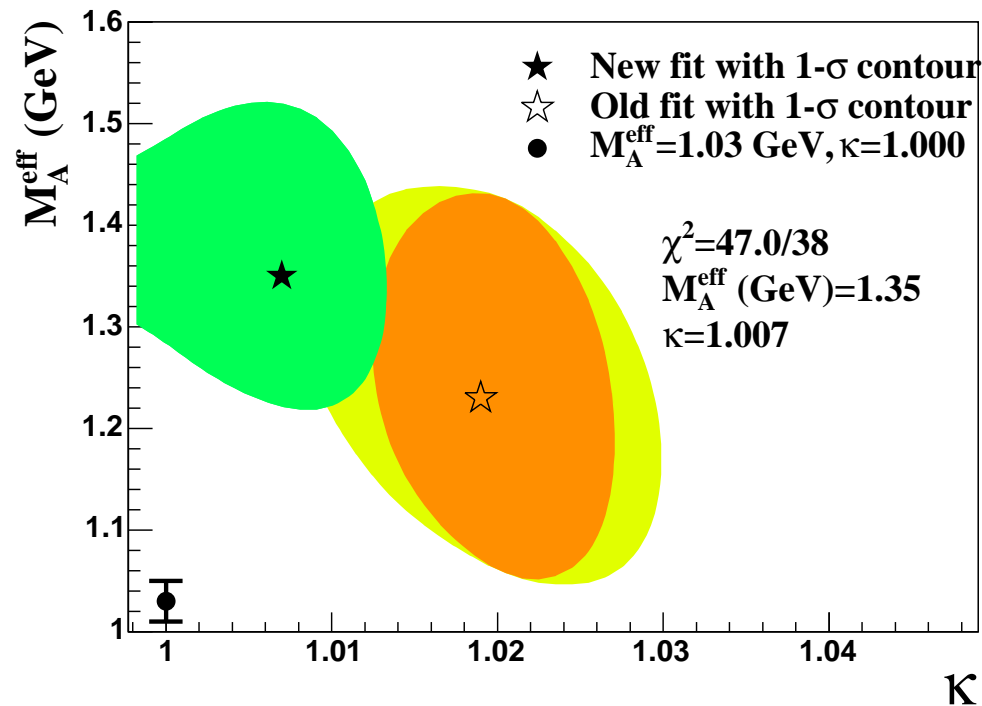
**Motivation: MiniBooNE CCQE**  
(PRD 81, 092005)

$$M_A^{\text{eff}} = 1.35 \text{ GeV}$$

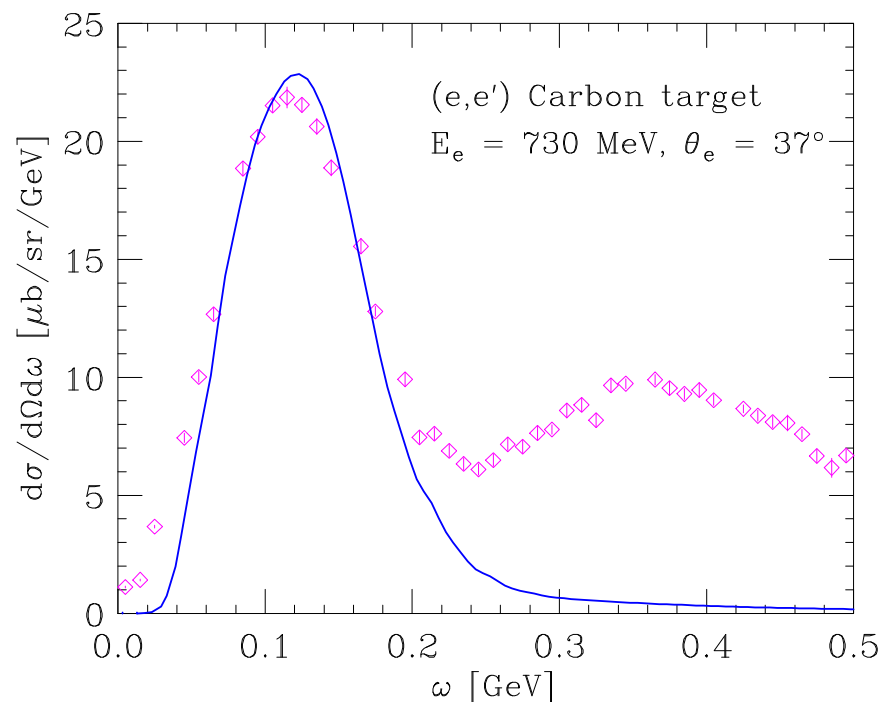
vs

$$1.03 \text{ GeV (world avg)}$$

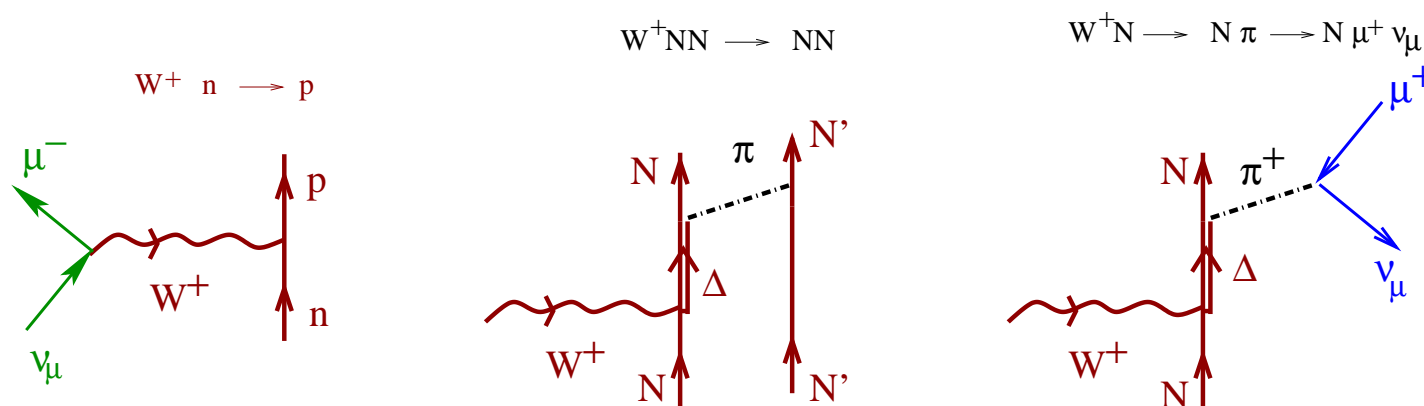
confirmed by many other groups,  
for instance by Benhar et al. (PRL  
105, 132301)



The problem turned out to even more worrying since the height, position, and width of the **QE peak in the case of electron scattering are well reproduced in most of used models**, for instance see results of Benhar et al. at similar energies and in carbon



...but key observation (Martini et al., PRC 81, 045502): in most **theoretical** works QE is used for processes where the gauge boson  $W^\pm$  or  $Z^0$  is absorbed by just one nucleon, which together with a lepton is emitted, **however in the recent MiniBooNE measurements, QE is related to processes in which only a muon is detected** (ejected nucleons are not detected !)  $\equiv$  **CCQE-like**



It **includes multinucleon processes and others like  $\pi$  production followed by absorption** (MBooNE analysis Monte Carlo corrects for those events). **It discards pions coming off the nucleus, since they will give rise to additional leptons after their decay.**

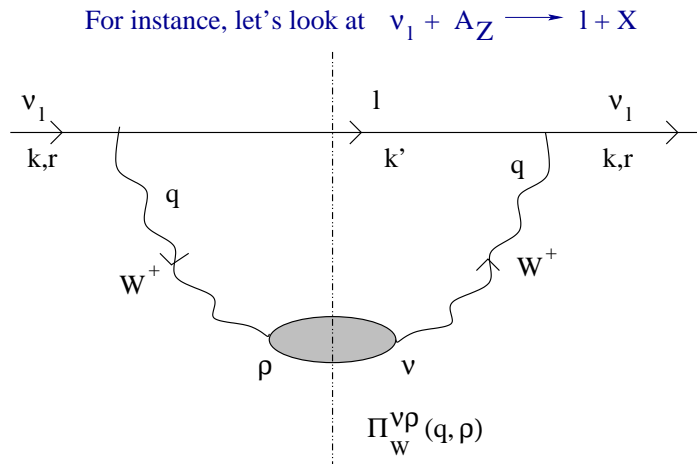
- MiniBooNE experimental results cannot be directly compared to most theoretical previous calculations!
- We present a microscopic calculation of the CCQE-like double differential cross section  $\frac{d\sigma}{dT_\mu d\cos\theta_\mu}$  measured by MiniBooNE and we will use these data to extract  $M_A$

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To describe the propagation of particles inside of the nuclear medium  $\Rightarrow$  microscopic framework:

- Pauli Blocking
- **RPA** and Short Range Correlations (**SRC**)
- **$\Delta(1232)$** –Degrees of Freedom
- Spectral Function (**SF**) + Final State Interaction (**FSI**)
- Meson Exchange Currents (**MEC**)

compute the imaginary part of the lepton-selfenergy inside of the nucleus:



$$\frac{d^2\sigma}{d\Omega(\hat{k}')dE'} = \frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{4\pi^2} L_{\mu\sigma} W^{\mu\sigma}$$

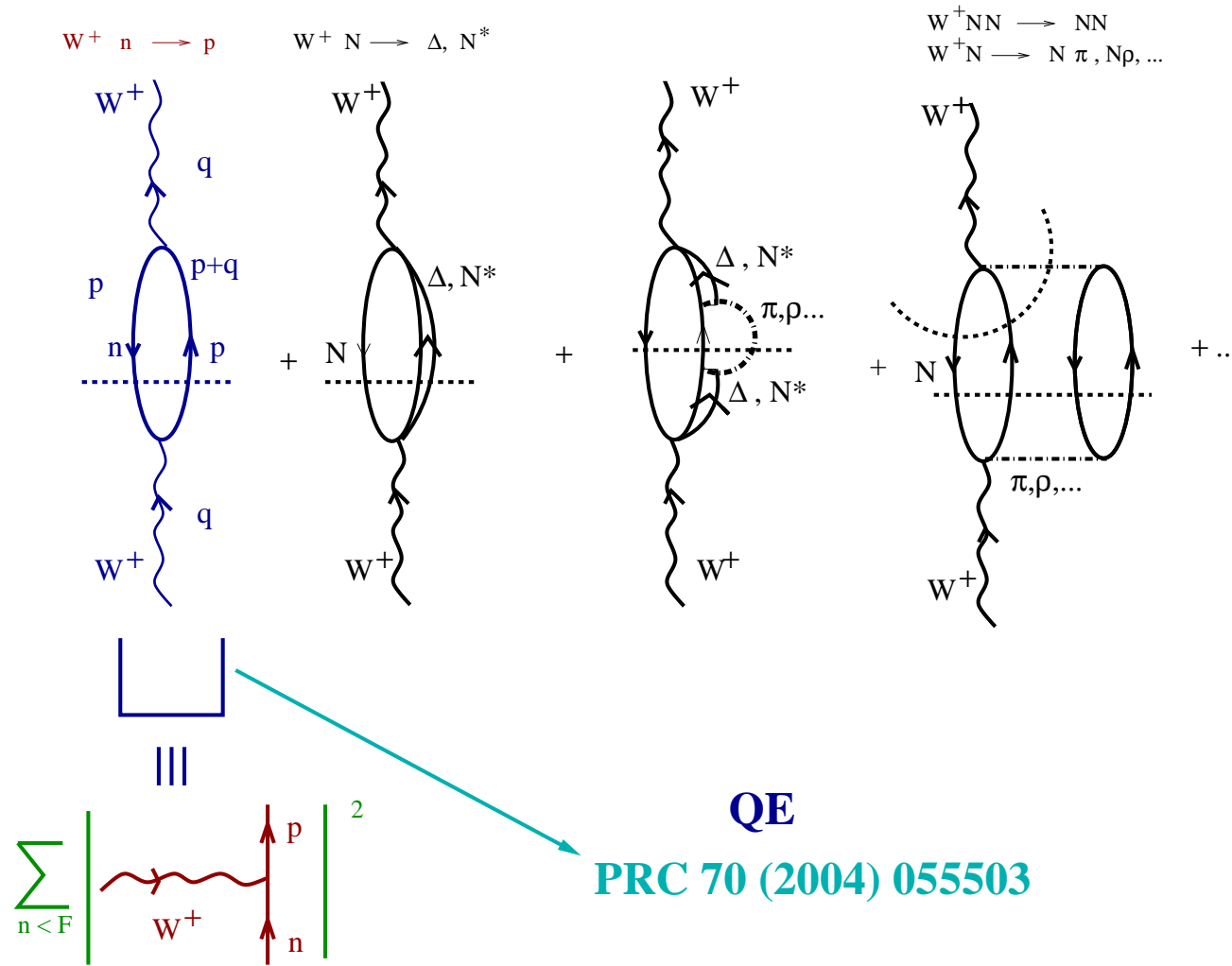
$$L_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k \cdot k' + i\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k^\beta$$

$$W^{\mu\sigma} = W_s^{\mu\sigma} + iW_a^{\mu\sigma}$$

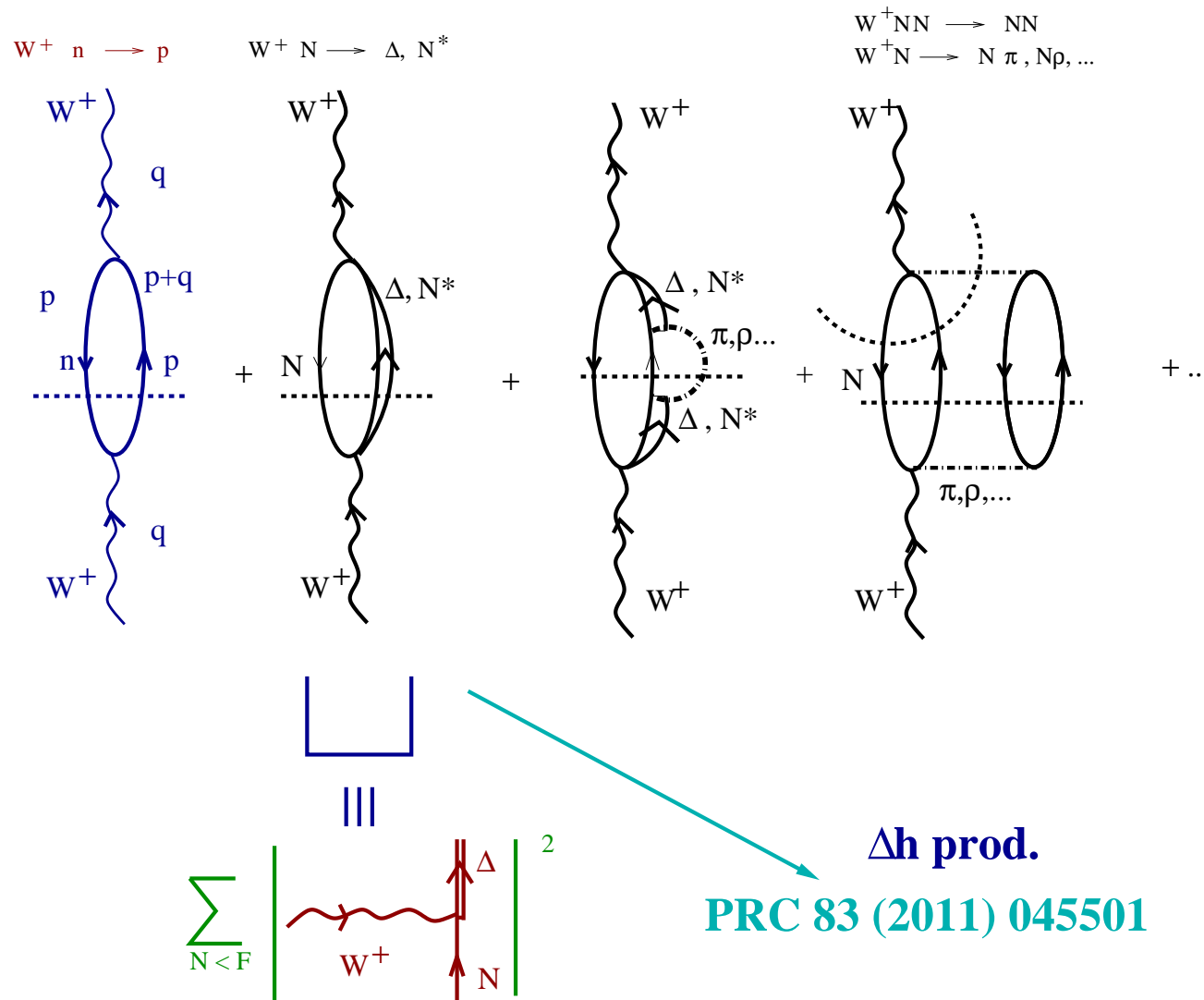
$$W_s^{\mu\sigma} \propto \int \frac{d^3r}{2\pi} \text{Im} \left\{ \Pi_W^{\mu\sigma}(q, \rho) + \Pi_W^{\sigma\mu}(q, \rho) \right\} \Theta(q^0)$$

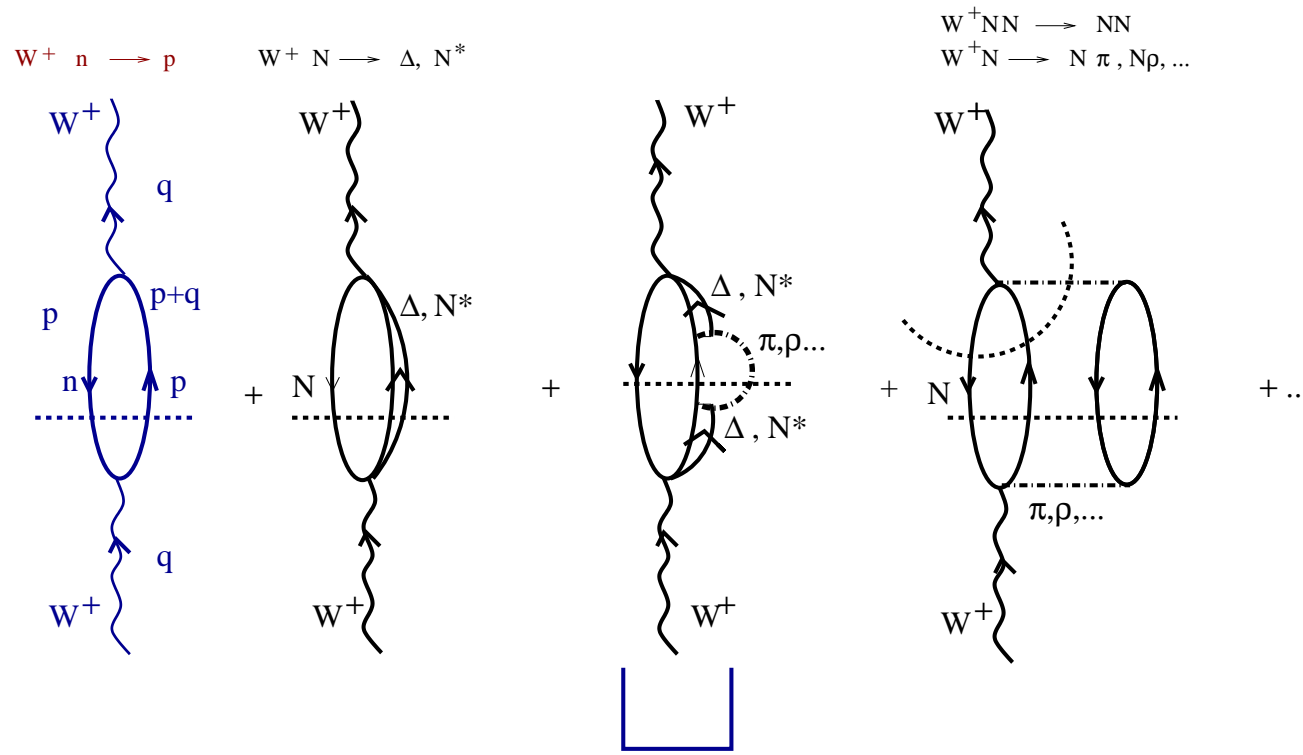
$$W_a^{\mu\sigma} \propto \int \frac{d^3r}{2\pi} \text{Re} \left\{ \Pi_W^{\mu\sigma}(q, \rho) - \Pi_W^{\sigma\mu}(q, \rho) \right\} \Theta(q^0)$$

**Basic object**  $\Pi_{W,Z^0,\gamma}^{\nu\rho}(q, \rho)$   $\equiv$  Selfenergy of the Gauge Boson ( $W^\pm, Z^0, \gamma$ ) inside of the nuclear medium. Perform a Many Body expansion, where the relevant gauge boson absorption modes should be systematically incorporated: absorption by one N, or NN or even 3N, real and virtual (MEC) meson ( $\pi, \rho, \dots$ ) production,  $\Delta$  excitation, etc...



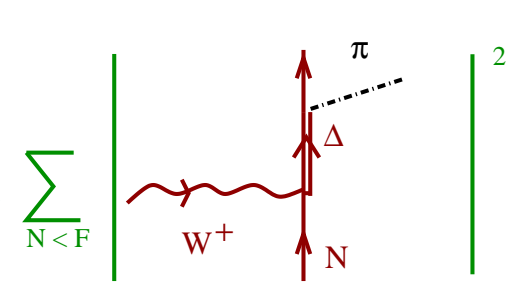


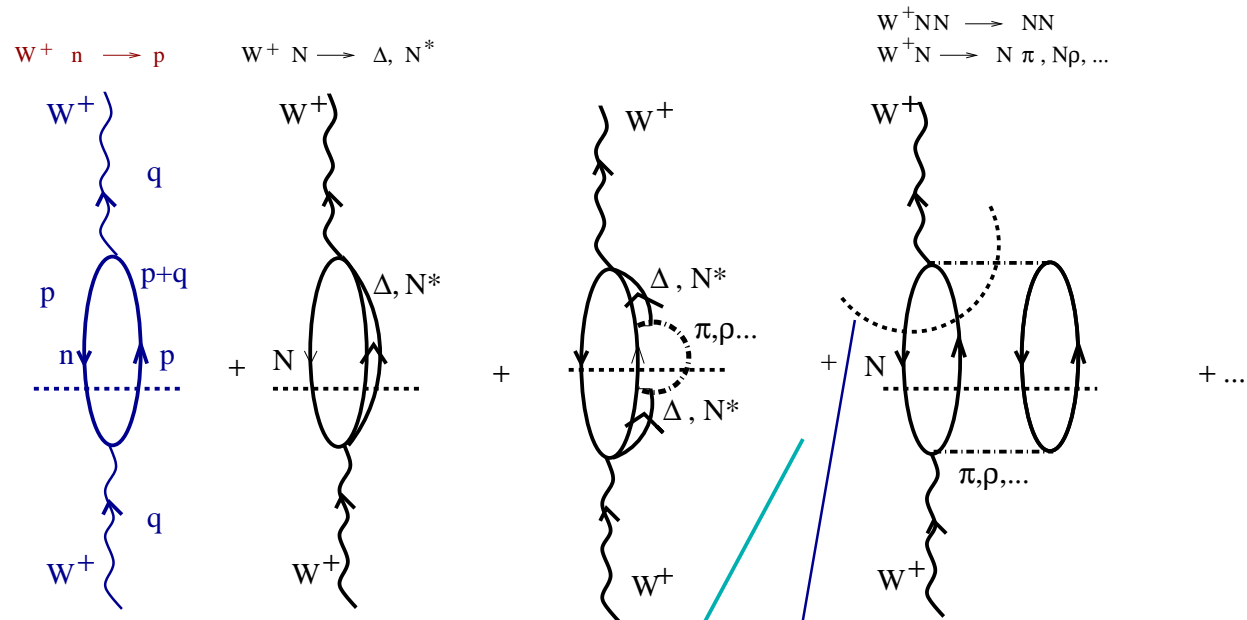




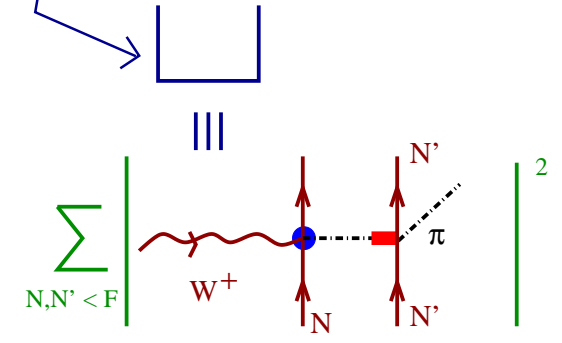
PRC 83 (2011) 045501

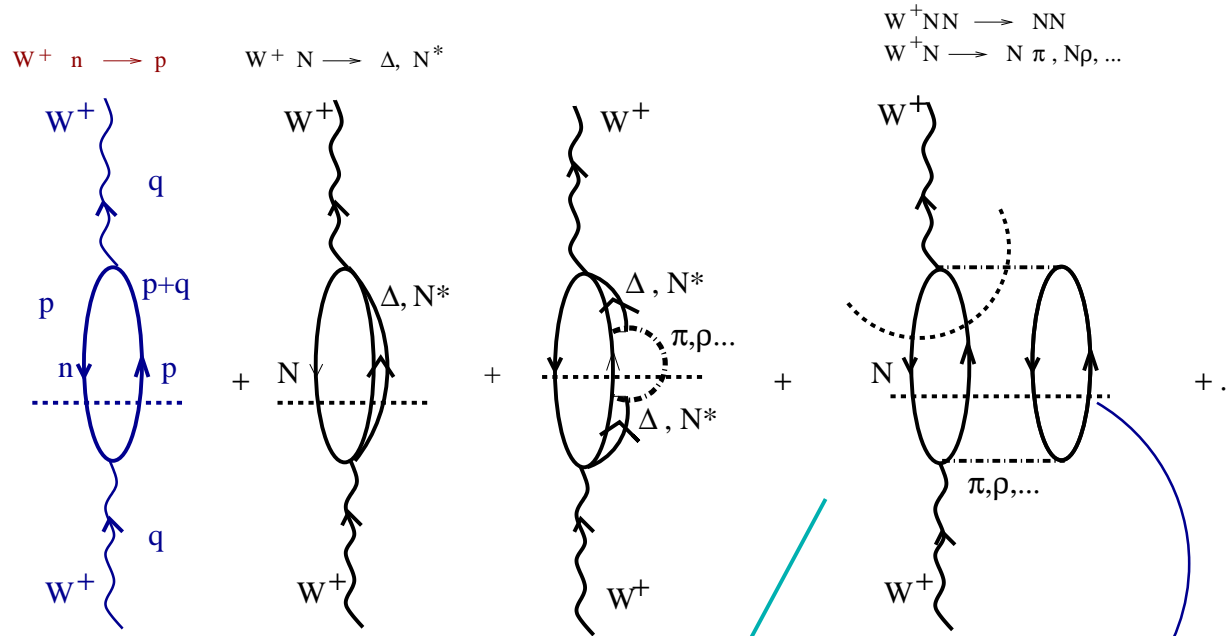
$\pi$  prod.





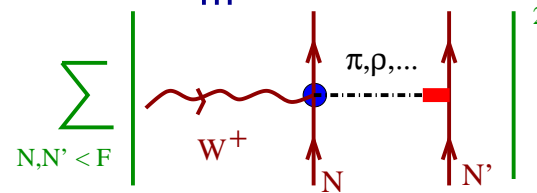
PRC 83 (2011) 045501  
 $\pi$  prod. + rescatt.





**QE-like !**

[PRC 83 \(2011\) 045501](#)  
**2N absorption (MEC)**



## Inclusive QE processes [f.i. $(\nu_l, l)$ ]

$(W^\pm, Z^0$  absorption by one nucleon)

First ingredient: M.E. of the CC/NC current between nucleons.

$$\langle p; \vec{p}' = \vec{p} + \vec{q} | j_{\text{CC}}^\alpha(0) | n; \vec{p} \rangle = \bar{u}(\vec{p}') [V^\alpha - A^\alpha] u(p)$$

$$V^\alpha = 2 \cos \theta_c \times \left( F_1^V(q^2) \gamma^\alpha + i \mu_V \frac{F_2^V(q^2)}{2M} \sigma^{\alpha\nu} q_\nu \right)$$

$$A^\alpha = \cos \theta_c G_A(q^2) \times \left( \gamma^\alpha \gamma_5 + \frac{2M}{m_\pi^2 - q^2} q^\alpha \gamma_5 \right) \quad (\text{PCAC})$$

with vector form factors related to the electromagnetic ones and

$$G_A(q^2) = \frac{g_A}{(1 - q^2 / M_A^2)^2}, \quad g_A = 1.257$$

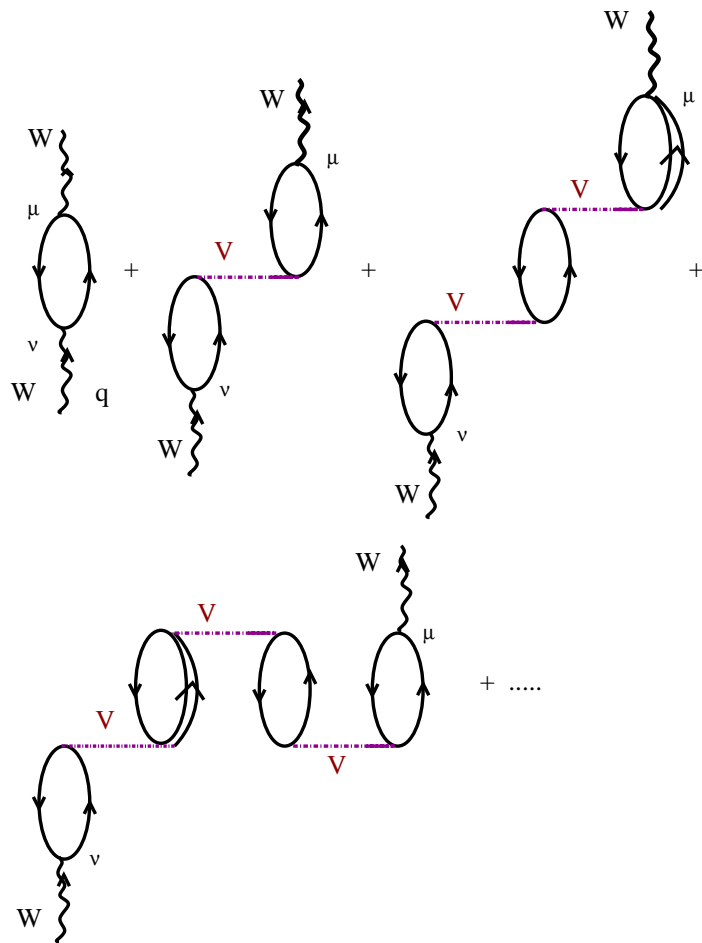
One finds (quasielastic peak)

$$\begin{aligned}
 W_{s,a}^{\mu\nu}(q) &= -\frac{1}{2M^2} \int_0^\infty \mathbf{drr}^2 \left\{ 2 \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\vec{p})} \frac{M}{E(\vec{p} + \vec{q})} \Theta(q^0) \right. \\
 &\times \Theta(\mathbf{k}_F^{\mathbf{n}}(\mathbf{r}) - |\vec{p}|) \Theta(|\vec{p} + \vec{q}| - \mathbf{k}_F^{\mathbf{p}}(\mathbf{r})) \\
 &\times \left. (-\pi) \delta(q^0 + E(\vec{p}) - E(\vec{p} + \vec{q})) A_{s,a}^{\mu\nu}(p, q) \right\}
 \end{aligned}$$

**Relativistic Local Fermi Gas that includes Pauli Blocking !**

**in addition we include some nuclear corrections...**

- Polarization (RPA) effects. Substitute the  $ph$  excitation by an RPA response: series of  $ph$  and  $\Delta h$  excitations.



### 1. Effective Landau-Migdal interaction

$$V(\vec{r}_1, \vec{r}_2) = c_0 \delta(\vec{r}_1 - \vec{r}_2) \left\{ \boxed{f_0(\rho)} + f'_0(\rho) \vec{\tau}_1 \vec{\tau}_2 \right. \\ \left. + \boxed{g_0(\rho) \vec{\sigma}_1 \vec{\sigma}_2} + g'_0(\rho) \vec{\sigma}_1 \vec{\sigma}_2 \vec{\tau}_1 \vec{\tau}_2 \right\}$$

Isoscalar terms  $\boxed{\phantom{x}}$  do not contribute to CC

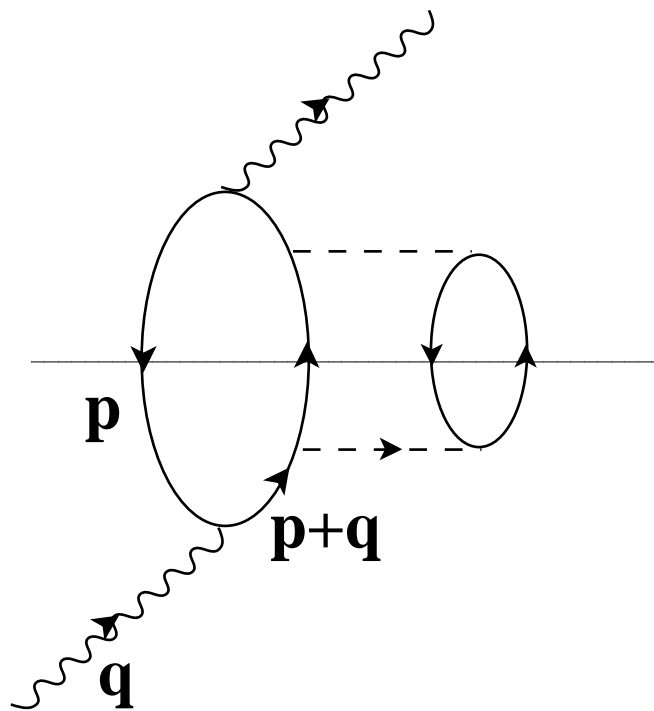
2.  $S = T = 1$  channel of the  $ph-ph$  interaction  $\rightarrow$  s longitudinal ( $\pi$ ) and transverse ( $\rho$ ) + SRC

$$g'_0 \vec{\sigma}_1 \vec{\sigma}_2 \vec{\tau}_1 \vec{\tau}_2 \rightarrow [V_l(q) \hat{q}_i \hat{q}_j + V_t(q) (\delta_{ij} - \hat{q}_i \hat{q}_j)] \sigma_1^i \sigma_2^j \vec{\tau}_1 \vec{\tau}_2$$

$$V_{l,t}(q) = \frac{f_{\pi NN, \rho NN}}{m_{\pi, \rho}^2} \left( F_{\pi, \rho}(q^2) \frac{\vec{q}^2}{q^2 - m_{\pi, \rho}^2} + g'_{l,t}(q) \right)$$

3. Contribution of  $\Delta h$  excitations important

- **Spectral Function (SF) + Final State Interaction (FSI):** dressing up the nucleon propagator of the hole (SF) and particle (FSI) states in the  $ph$  excitation



– Change of nucleon dispersion relation:

- \* hole  $\Rightarrow$  Interacting Fermi sea (SF)
- \* particle  $\Rightarrow$  Interaction of the ejected nucleon with the final nuclear state (FSI)

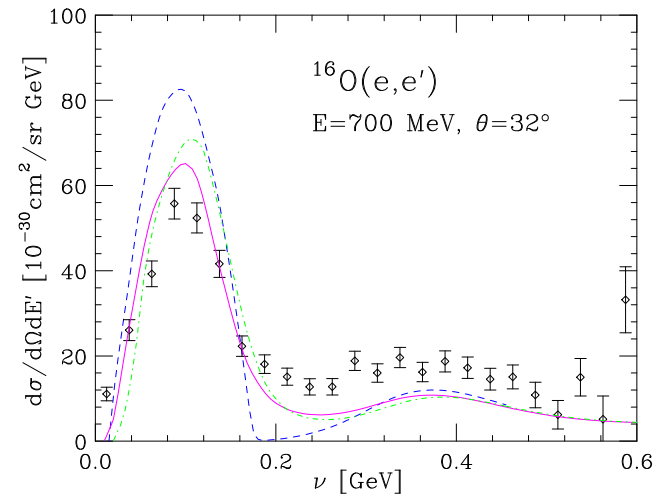
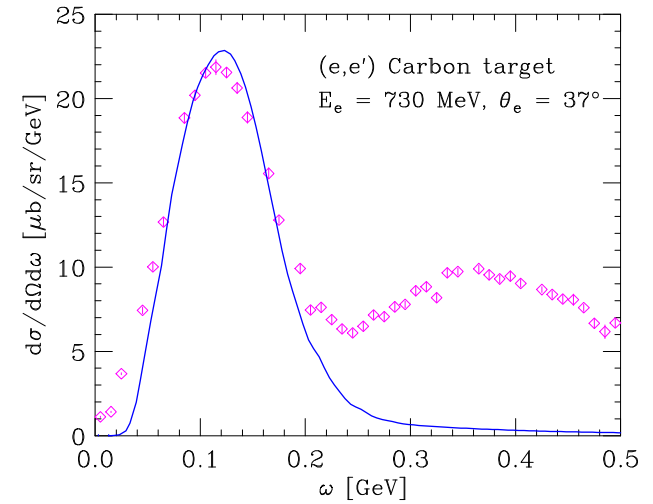
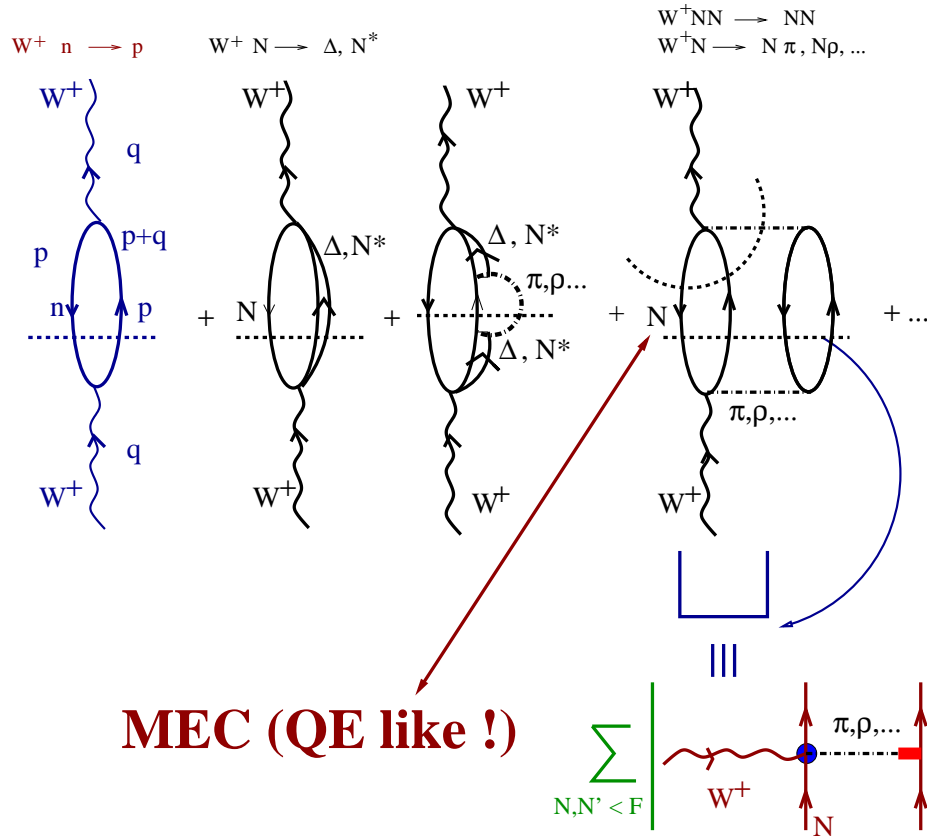
$$G(p) \rightarrow \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, \vec{p})}{p^0 - \omega - i\epsilon} + \int_{\mu}^{+\infty} d\omega \frac{S_p(\omega, \vec{p})}{p^0 - \omega + i\epsilon}$$

The hole and particle spectral functions are related to nucleon self-energy  $\Sigma$  in the medium,

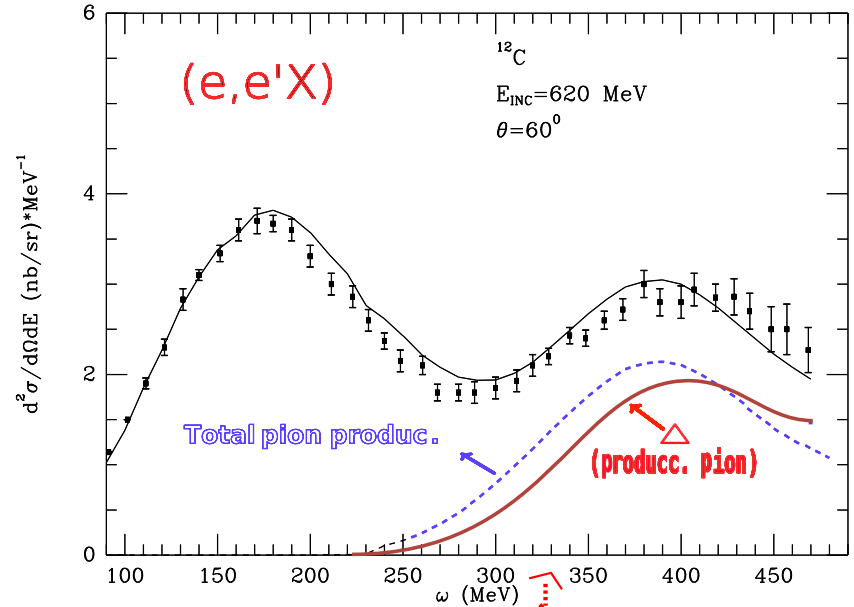
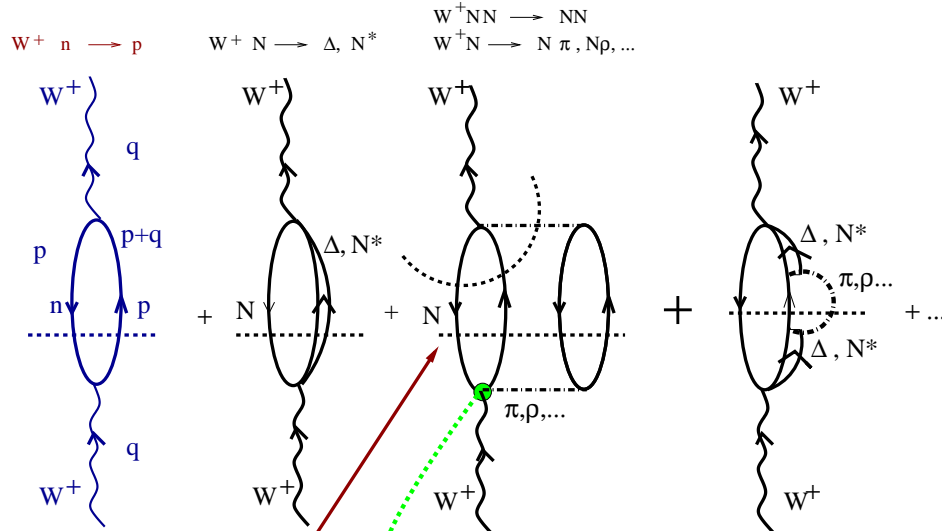


Above QE Region:  $\pi$  Production

(e,e') PRL 105, 132301 & PRD 72 053005

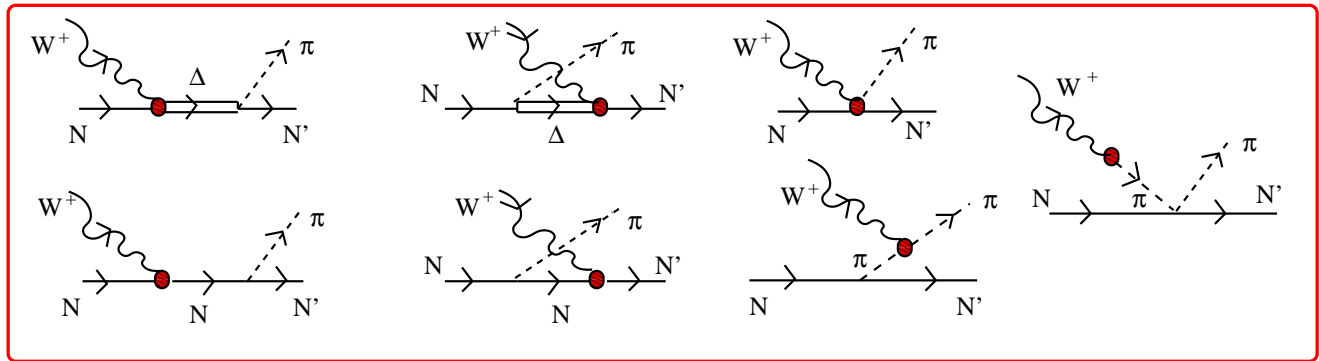


Above QE Region:  $\pi$  Production



MEC  $\rightarrow$  QE like !

PRD D76 (2007) 033005  $\rightarrow$



## $(e, e')$ Results

Same formalism applied to the study of **inclusive processes**  $(e, e')$ ,  $(e, e'N)$ ,  $(e, e'NN)$ ,  $(e, e'\pi)$ , ... in nuclei at intermediate energies [**Gil+Nieves+Oset, NPA 627 (1997) 543-619**] leads to excellent results both in the quasielastic and  $\Delta$  excitation regions. To describe the  **$\Delta$  peak and the “dip” regions**, we include  **$\Delta h$  and MEC contributions + ...**

## Real Photon Results

Same formalism applied to the study of the interaction of Real Photons with Nuclei at Intermediate Energies: **Total Photo-absorption cross section**  $\gamma A_Z \rightarrow X$  [Carrasco + Oset, NPA 536 (1992) 445] and **Inclusive**  $(\gamma, \pi)$ ,  $(\gamma, N)$ ,  $(\gamma, NN)$  and  $(\gamma, N\pi)$  reactions [ Carrasco + Oset + Salcedo NPA 541 (1992) 585 and Carrasco+Vicente-Vacas+ Oset NPA 570 (1994) 701]

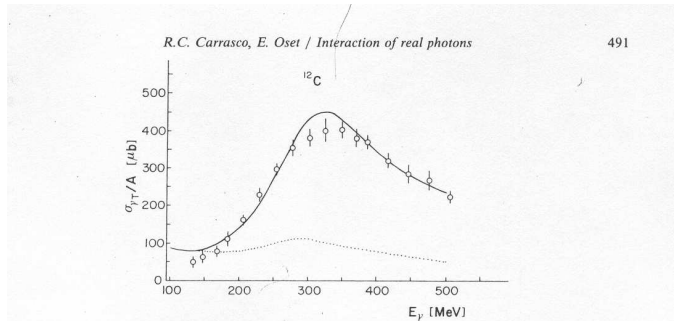


Fig. 45. Results for  $\sigma_A/A$  as a function of the photon energy for  $^{12}\text{C}$ . Experiment from ref. <sup>6)</sup>. The lower curve is the result for direct photon absorption.

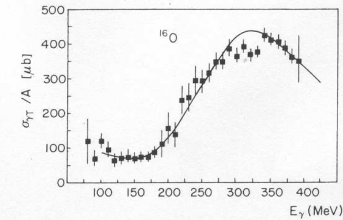


Fig. 46. Results for  $\sigma_A/A$  as a function of the photon energy for  $^{16}\text{O}$ . Experiment from ref. <sup>5)</sup>.

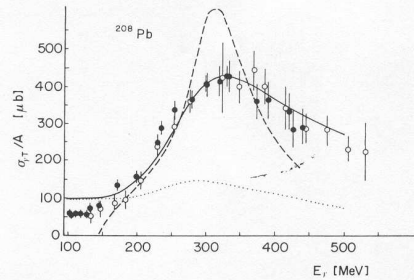


Fig. 47. Continuous line: results for  $\sigma_A/A$  as a function of the photon energy for  $^{208}\text{Pb}$ . The dashed line shows the impulse approximation result  $(Z\sigma_{\gamma p} + N\sigma_{\gamma n})/A$  for comparison. The dotted line is the result for direct photon absorption. Experimental data: dark dots from ref. <sup>3)</sup>, white dots from ref. <sup>5)</sup>.

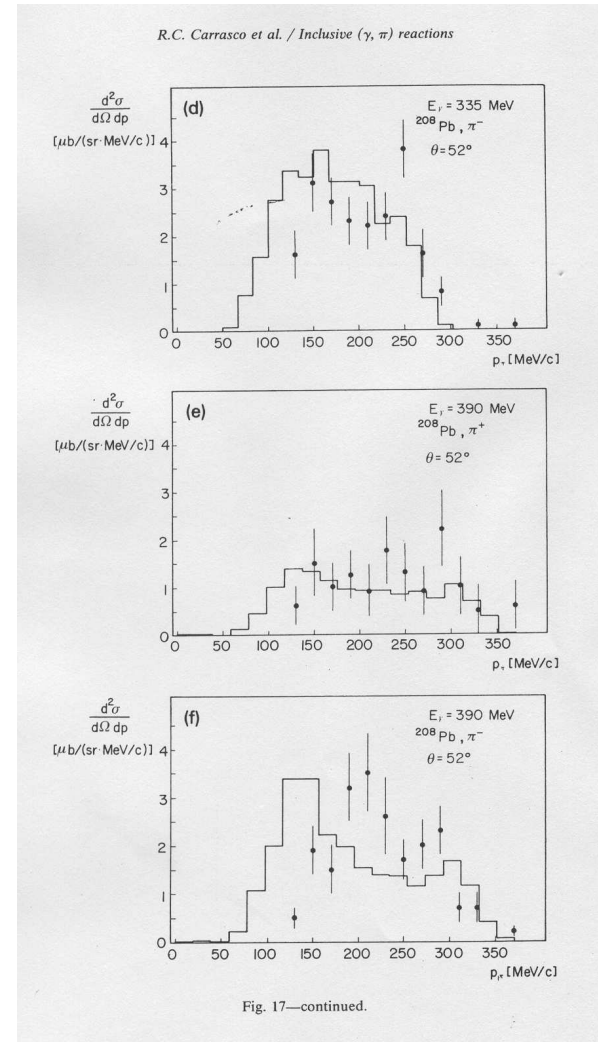
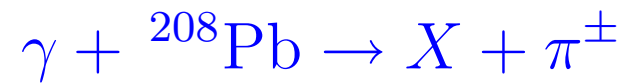


Fig. 17—continued.



## Pion Physics

Same Many Body framework applied to the study of different nuclear processes involving pions at intermediate energies. For instance, pionic atoms, elastic and inelastic pion-nucleus scattering,  $\Lambda$  hypernuclei, etc.. Oset+Toki+Weise, Phys. Rep. 83 (1982) 281

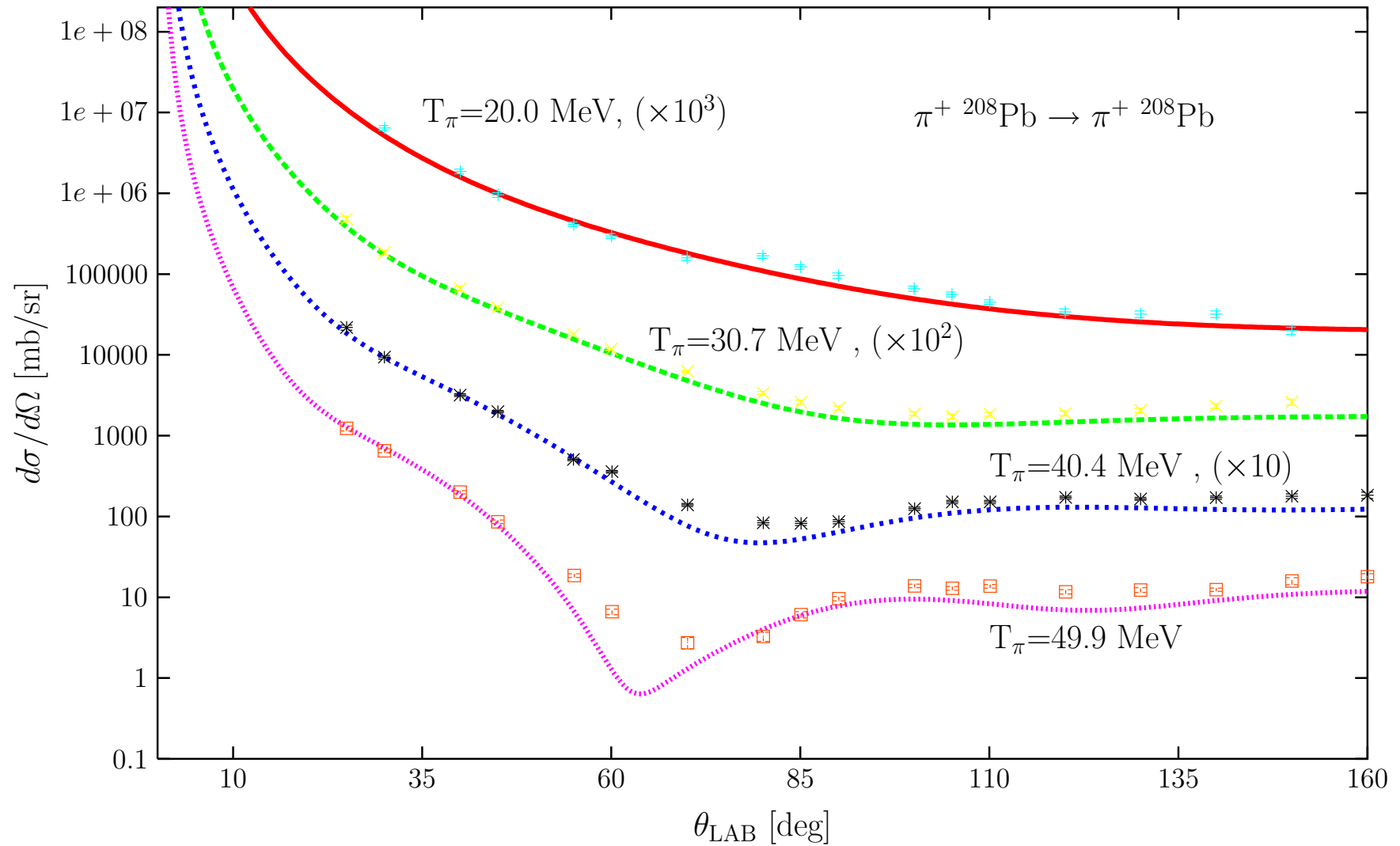
García-Recio+Oset+Salcedo+Strottman, NPA 526, 685

Nieves+Oset+García-Recio, NPA 554 (1993), 509-579

Nieves+Oset, PRC 47 (1993) 1478

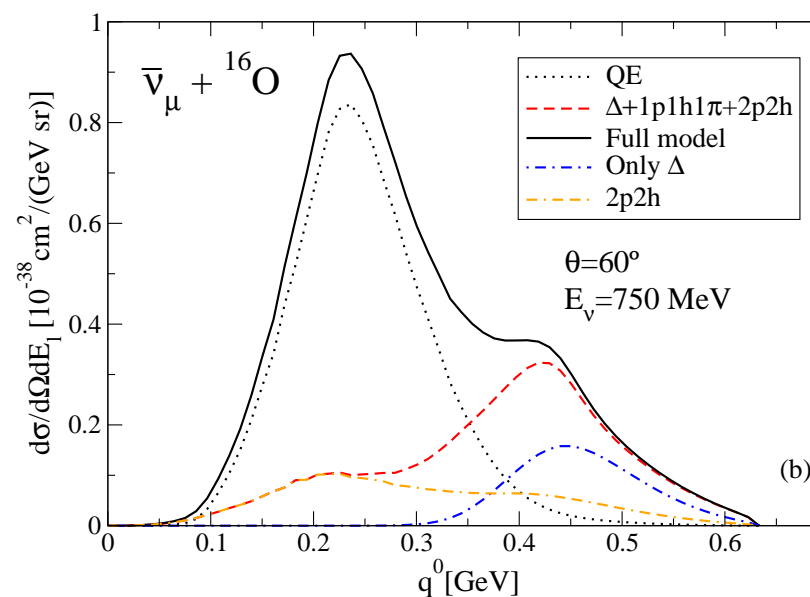
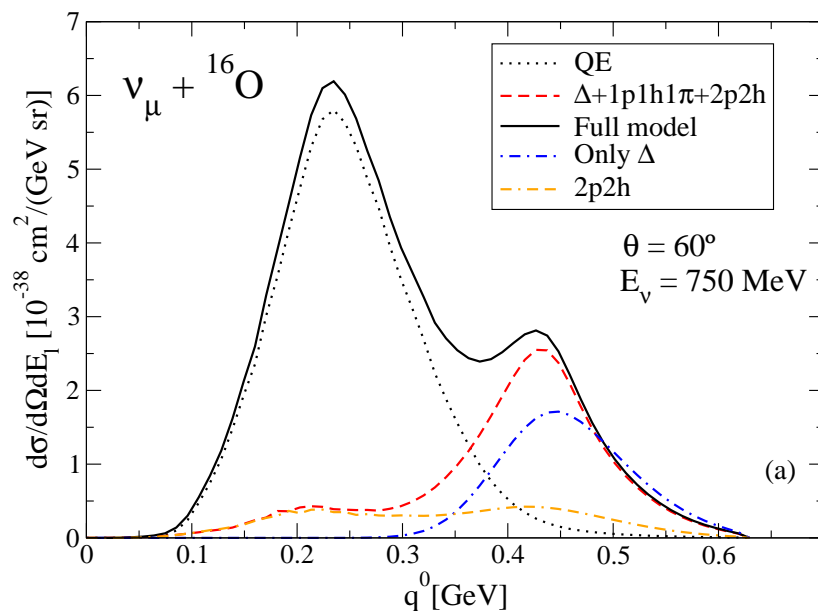
Amaro+Nieves, PRL 89 (2002) 032501

Albertus+Amaro+Nieves, PRC 67 (2003) 034604

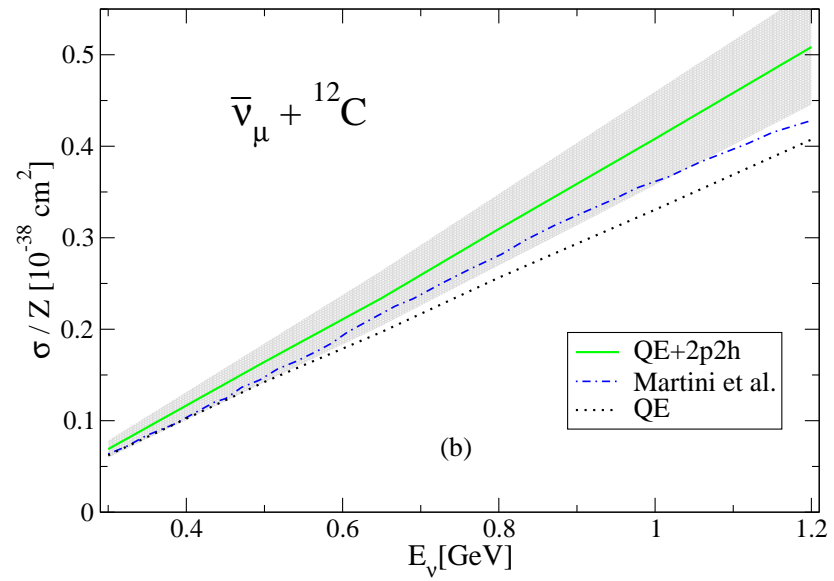
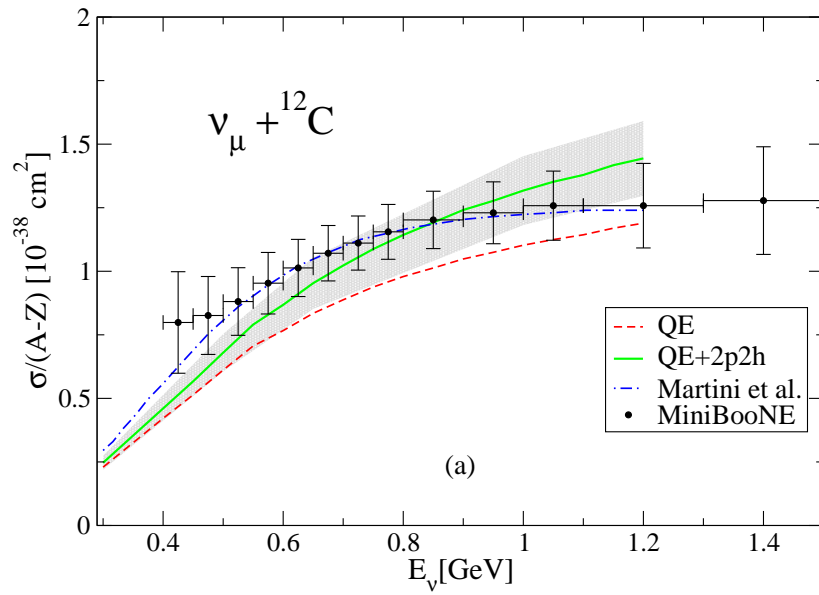
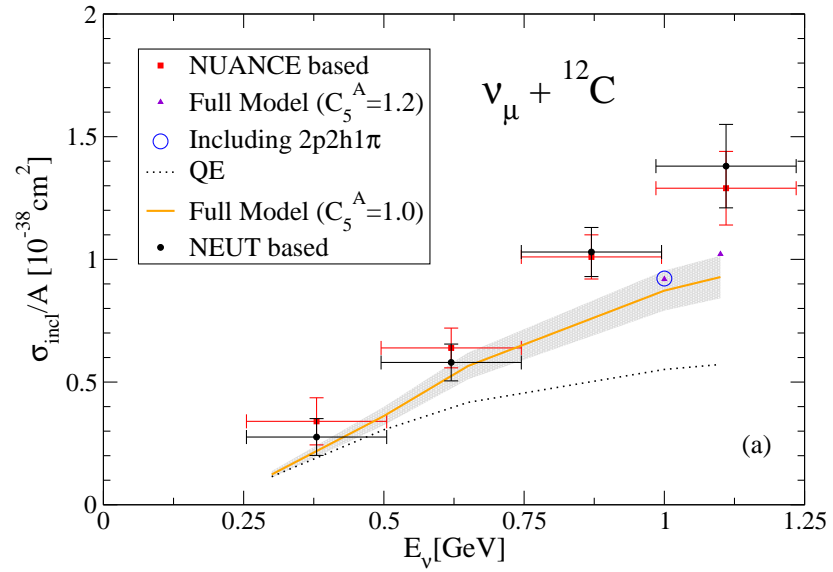


# $(\nu_\mu, \mu^-)$ Results

PRC 83 (2011) 045501 [ $M_A = 1.049$  GeV]







## MiniBooNE CCQE-like double differential cross section $\frac{d\sigma}{dT_\mu d\cos\theta_\mu}$

We define a **merit function** and consider our **QE+2p2h results**

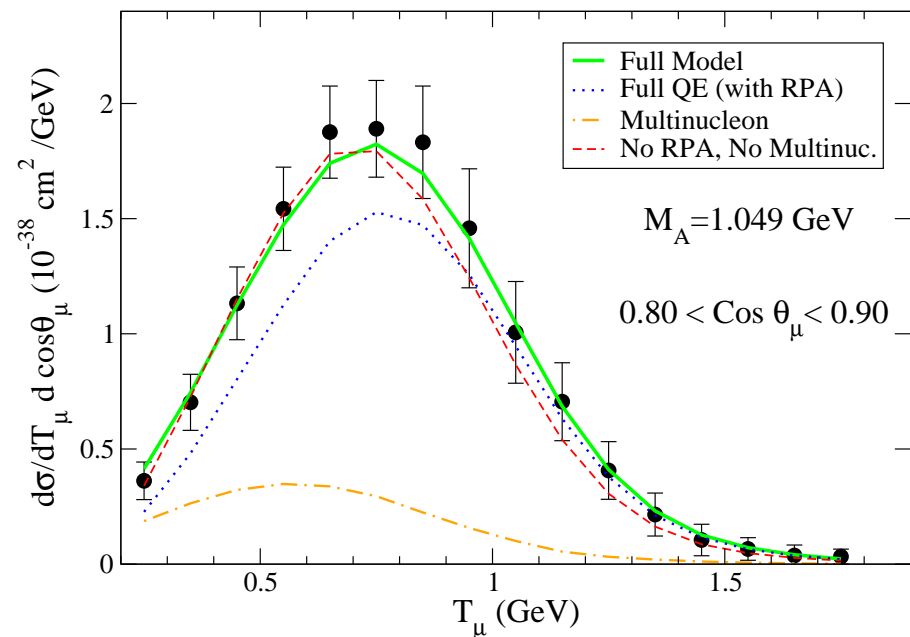
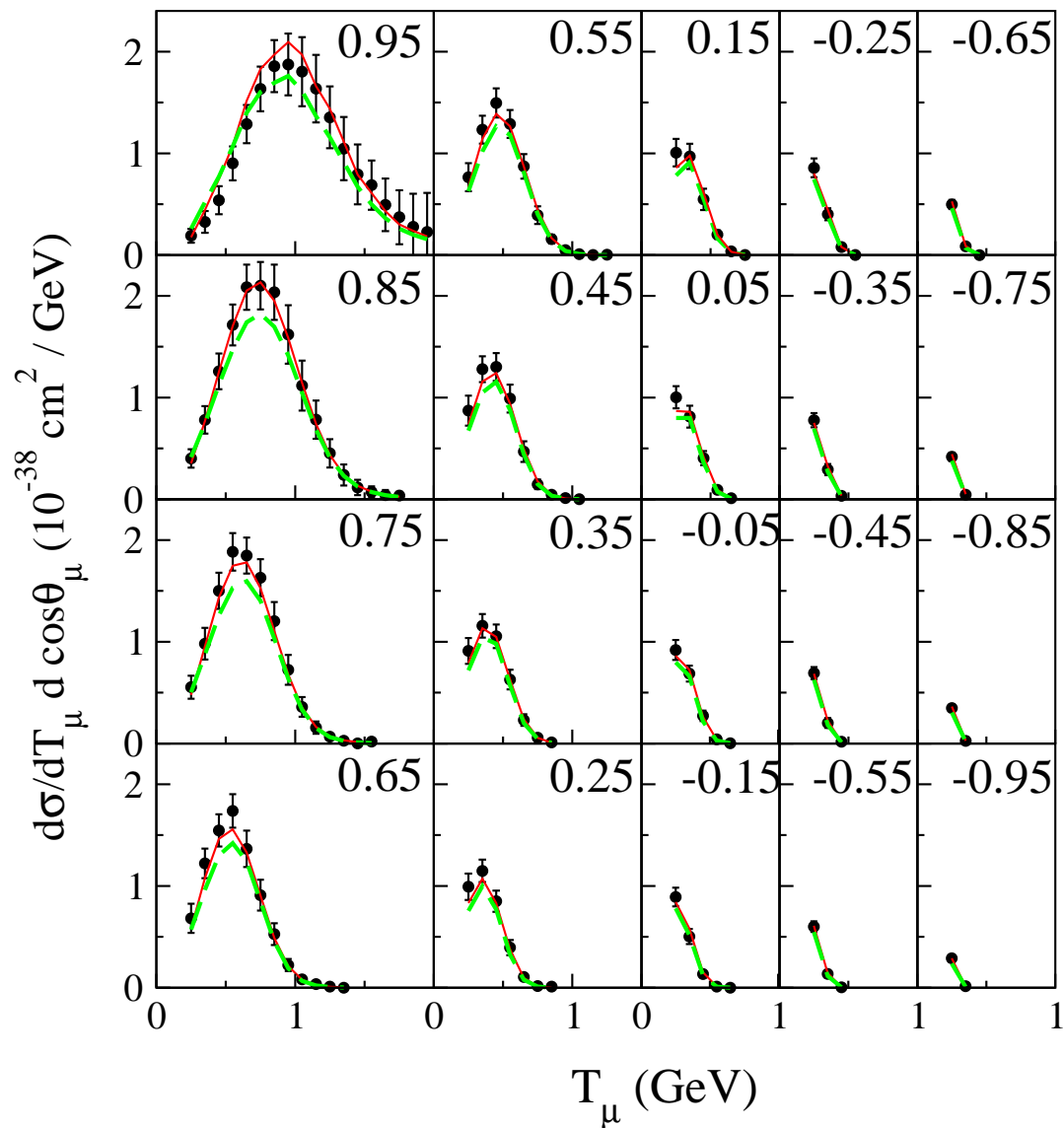
$$\chi^2 = \sum_{i=1}^{137} \left[ \frac{\lambda \left( \frac{d^2\sigma^{exp}}{dT_\mu d\cos\theta} \right)_i - \left( \frac{d^2\sigma^{th}}{dT_\mu d\cos\theta} \right)_i}{\lambda \Delta \left( \frac{d^2\sigma}{dT_\mu d\cos\theta} \right)_i} \right]^2 + \left( \frac{\lambda - 1}{\Delta\lambda} \right)^2,$$

that takes into account the **global normalization uncertainty** ( $\Delta\lambda = 0.107$ ) claimed by the MiniBooNE collaboration.

**We fit  $\lambda$  to data with a fixed value of  $M_A (=1.049 \text{ GeV})$ .**

**We obtain  $\chi^2/\# \text{ bins} = 52/137$  with  $\lambda = 0.89 \pm 0.01$ .**

**The microscopical model, with no free parameters, agrees remarkably well with data! The shape is very good and  $\chi^2$  strongly depends on  $\lambda$ , which is strongly correlated with  $M_A$ .**



Model	Scale	$M_A$ (GeV)	$\frac{\chi^2}{\#bins}$
LFG	$0.96 \pm 0.03$	$1.32 \pm 0.03$	35/137
<b>Full</b>	<b><math>0.92 \pm 0.03</math></b>	<b><math>1.08 \pm 0.03</math></b>	<b>50/137</b>
Full $ q  > 0.4^\dagger$ GeV	$0.83 \pm 0.04$	$1.01 \pm 0.03$	30/123

$^\dagger$  : As suggested by Sobczyk et al. PRC 82, 045502

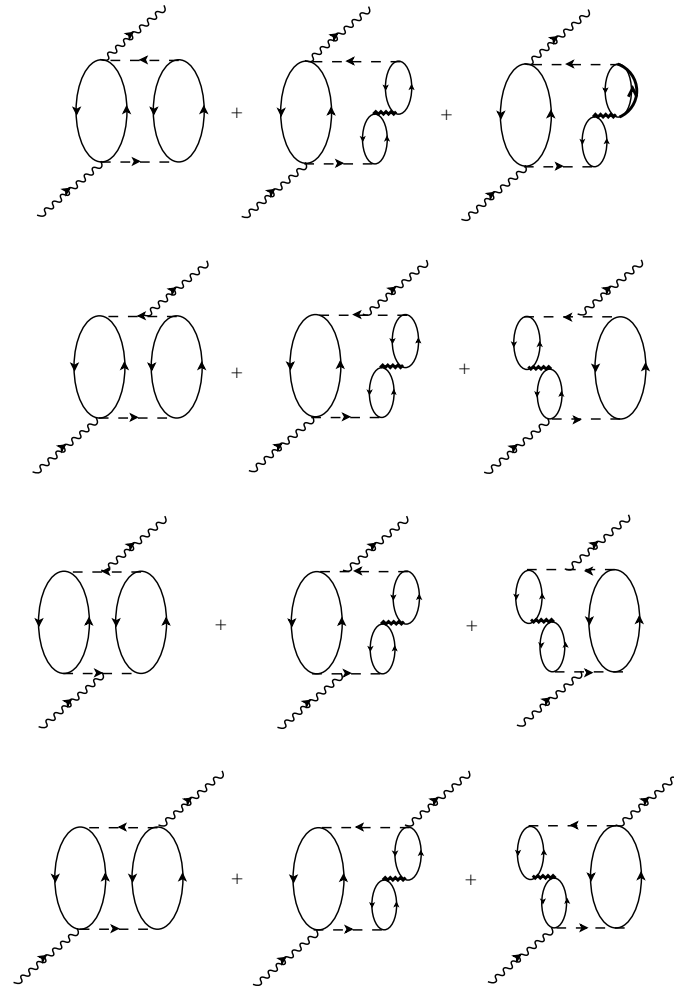
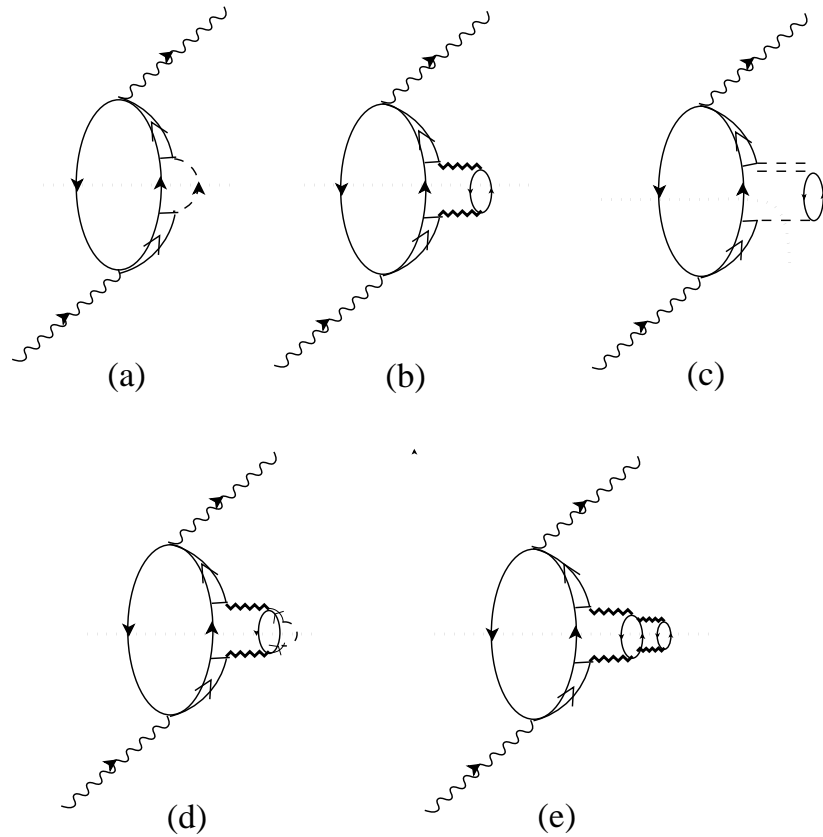
## Conclusions

- We have analyzed the MiniBooNE CCQE  $\frac{d\sigma}{dT_\mu d\cos\theta_\mu}$  data using a theoretical model that has proved to be quite successful in the analysis of nuclear reactions with electron, photon and pion probes and contains no additional free parameters.
- RPA and multinucleon knockout have been found to be essential for the description of the data.
- MiniBooNE CCQE-like data are fully compatible with former determinations of  $M_A$  in contrast with several previous analyses. We find,  $M_A = 1.08 \pm 0.03$ .
- The  $\nu_\mu$  flux could have been underestimated ( $\sim 10\%$ )

- The procedure commonly used to reconstruct the neutrino energy for quasielastic events from the muon angle and energy could be unreliable for a wide region of the phase space, due to the large importance of multinucleon events.

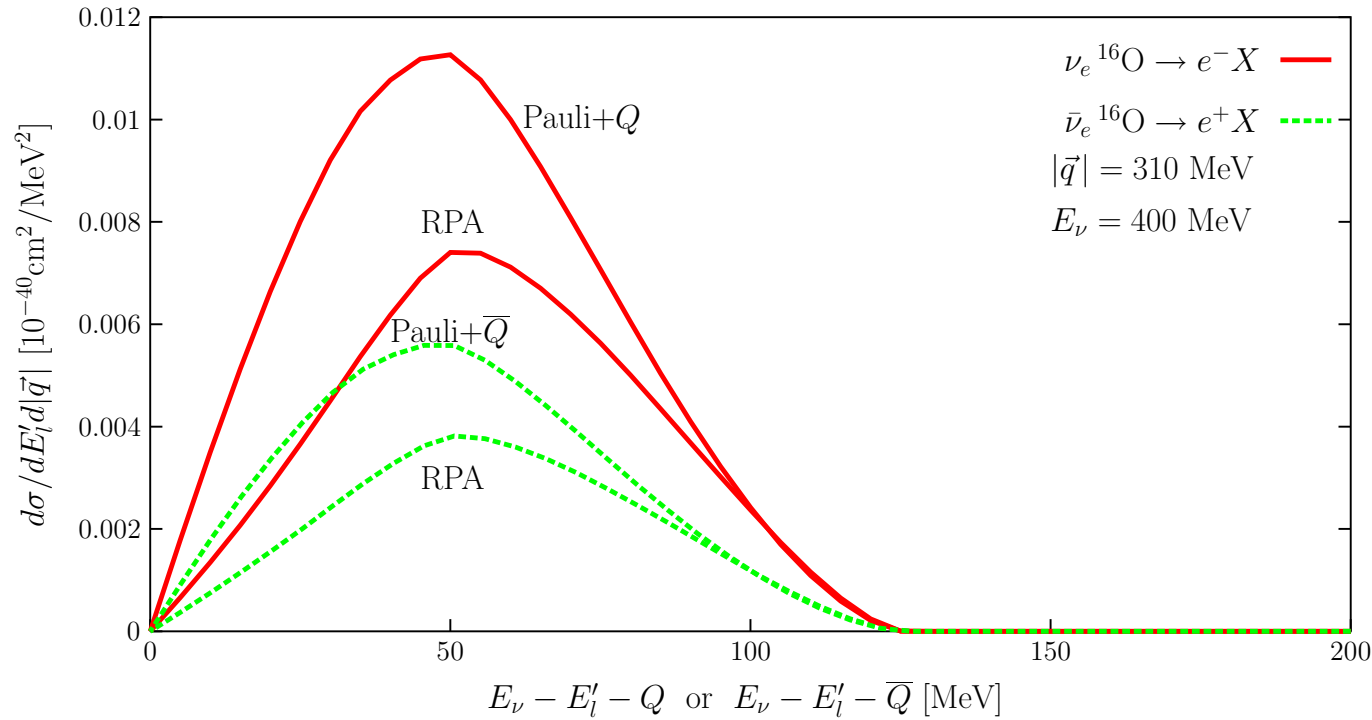
# Back up material

- Differences with the work of Martini et al. (PRC80,065501)
  1. Similar for the 2p2h contributions driven by  $\Delta$ h excitation (both groups use the same model for the  $\Delta$ –selfenergy in the medium).
  2. Martini et al. do not consider 2p2h contributions driven by contact, pion pole and pion in flight terms.
  3. Martini et al. give approximate estimates (no microscopical calculation) for the rest of 2p2h contributions.





$$\begin{aligned}
A_s^{\mu\nu}(p, q) &= 16(F_1^V)^2 \left\{ (p+q)^\mu p^\nu + (p+q)^\nu p^\mu + \frac{q^2}{2} g^{\mu\nu} \right\} \\
&+ 2q^2(\mu_V F_2^V)^2 \left\{ 4g^{\mu\nu} - 4\frac{p^\mu p^\nu}{M^2} - 2\frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} \right. \\
&- \left. q^\mu q^\nu \left( \frac{4}{q^2} + \frac{1}{M^2} \right) \right\} - 16F_1^V \mu_V F_2^V (q^\mu q^\nu - q^2 g^{\mu\nu}) \\
&+ 4G_A^2 \left\{ 2p^\mu p^\nu + q^\mu p^\nu + p^\mu q^\nu + g^{\mu\nu} \left( \frac{q^2}{2} - 2M^2 \right) \right. \\
&- \left. \frac{2M^2(2m_\pi^2 - q^2)}{(m_\pi^2 - q^2)^2} q^\mu q^\nu \right\} \\
A_a^{\mu\nu}(p, q) &= 16G_A (\mu_V F_2^V + F_1^V) \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta
\end{aligned}$$



**Examples** of the RPA effect

$$G_A^2 \delta^{ij} \rightarrow G_A^2 \left( \frac{\hat{q}^i \hat{q}^j}{|1 - U(q)V_l(q)|^2} + \frac{\delta^{ij} - \hat{q}^i \hat{q}^j}{|1 - U(q)V_t(q)|^2} \right)$$

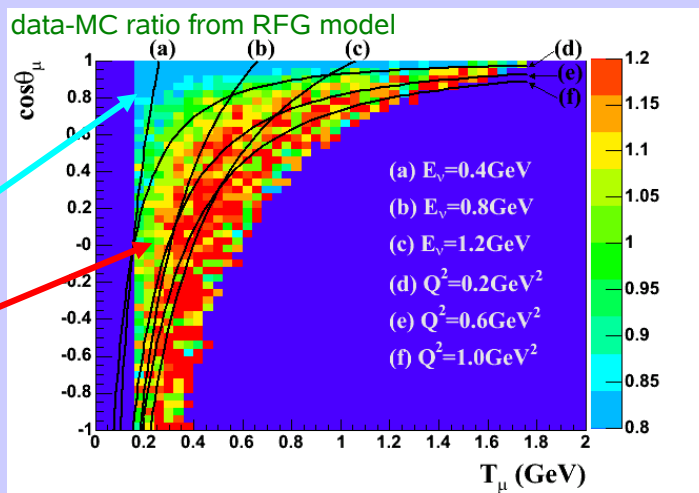
$$(F_1^V)^2 \rightarrow \frac{(F_1^V)^2}{|1 - c_0 f'_0(\rho)U_N(q)|^2}, \quad \text{etc...}$$

The Lindhard function  $U(q) = U_N + U_\Delta$  [ph + Δh]

### 3. CCQE data-MC comparison

CCQE kinematics phase space  
The data-MC agreement is not great

The data-MC disagreement is characterized by 2 features;  
(1) data deficit at low  $Q^2$  region  
(2) data excess at high  $Q^2$  region



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Teppei Katori, Indiana University, N

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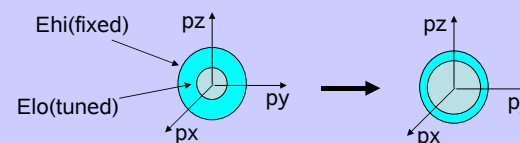
RPA effects might explain the data deficit at low  $Q^2$  in the MiniBooNE CCQE events reported in PRL 100, 032301.

### 3. CCQE data-MC comparison

Pauli blocking parameter "kappa" :  $\kappa$

To enhance the Pauli blocking at low  $Q^2$ , we introduced a new parameter  $\kappa$ , which is the scale factor of lower bound of nucleon sea and controls the size of nucleon phase space

$$E_{lo} = \kappa \sqrt{(p_F^2 + M^2)} - w + E_b$$



This modification gives significant effect only at low  $Q^2$  region

We tune the nuclear parameters in RFG model using  $Q^2$  distribution;

$M_A$  = tuned

$P_F$  = fixed

$E_B$  = fixed

$\kappa$  = tuned

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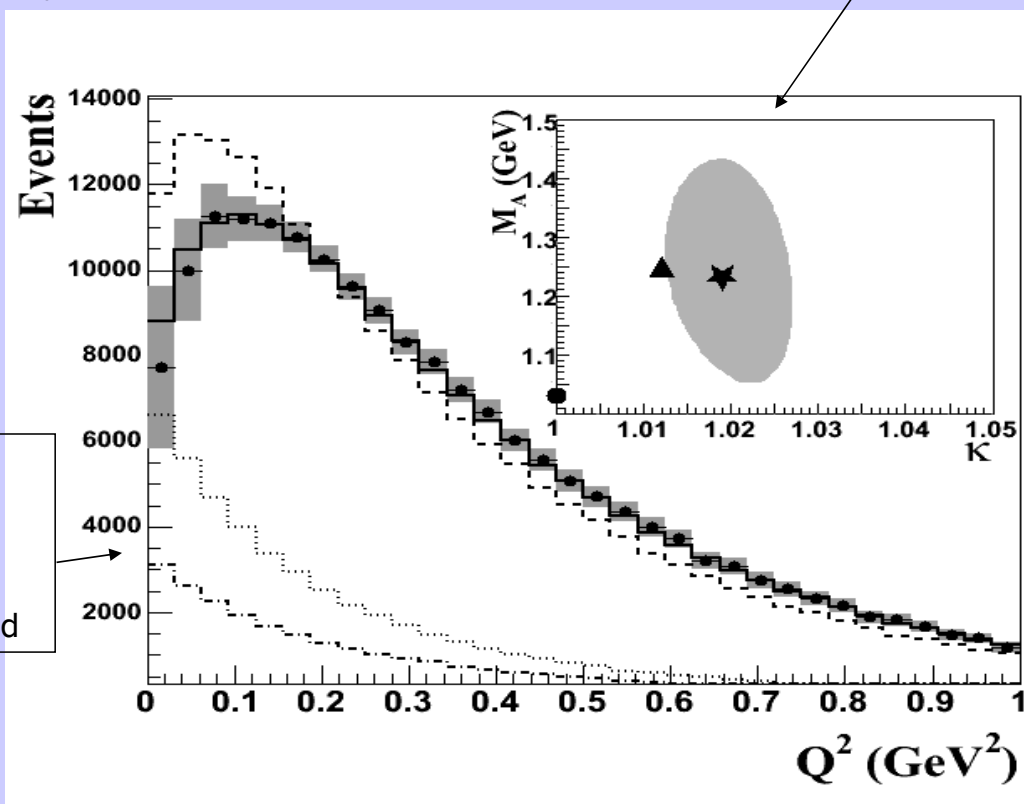
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$M_A - \kappa$  fit result

$$M_A = 1.23 \pm 0.20(\text{stat+sys})$$

$$\kappa = 1.019 \pm 0.011(\text{stat+sys})$$

circle: before fit  
 star: after fit with 1-sigma contour  
 triangle: bkgd shape uncertainty



dots : data with error bar  
 dashed line : before fit  
 solid line : after fit  
 dotted line : background  
 dash-dotted : non-CCQElike bkgd

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$$S_{p,h}(\omega, \vec{p}) = \mp \frac{1}{\pi} \frac{\text{Im}\Sigma(\omega, \vec{p})}{\left[\omega - \frac{\vec{p}^2}{2M} - \text{Re}\Sigma(\omega, \vec{p})\right]^2 + [\text{Im}\Sigma(\omega, \vec{p})]^2}$$

with  $\omega \geq \mu$  or  $\omega \leq \mu$  for  $S_p$  and  $S_h$ , respectively.

$$\text{chemical potential : } \mu = \frac{k_F^2}{2M} + \text{Re}\Sigma(\mu, k_F)$$

For non interacting fermions  $\boxed{\Sigma = 0}$ ,

$$S_p(\omega, \vec{p}) = \theta(|\vec{p}| - k_F) \delta\left(\omega - \frac{\vec{p}^2}{2M}\right)$$

$$S_h(\omega, \vec{p}) = \theta(k_F - |\vec{p}|) \delta\left(\omega - \frac{\vec{p}^2}{2M}\right)$$

and only Pauli blocking is incorporated!!

To take into account SF+FSI we should replace  $\text{Im}\bar{U}_R^N(q)$  by a new response function:

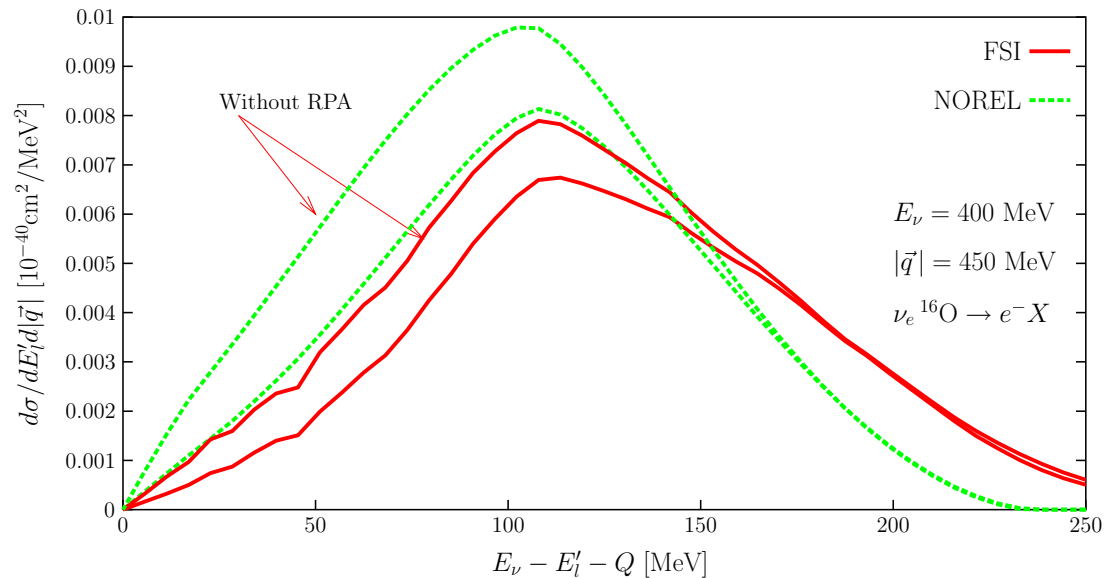
$$-\frac{1}{2\pi} \int_0^{+\infty} dp p^2 \int_{-1}^{+1} dx \int_{\mu-q^0}^{\mu} d\omega \mathbf{S}_h(\omega, \vec{p}) \mathbf{S}_p(\mathbf{q}^0 + \omega, \mathbf{t})$$

with  $t^2 = \vec{p}^2 + \vec{q}^2 + 2|\vec{p}||\vec{q}|x$ .

This nuclear effect is additional to those due to RPA (long range) correlations !!

$$G_A^2 \delta^{ij} \rightarrow G_A^2 \left( \frac{\hat{q}^i \hat{q}^j}{|1 - U(q)V_l(q)|^2} + \frac{\delta^{ij} - \hat{q}^i \hat{q}^j}{|1 - U(q)V_t(q)|^2} \right)$$

$$(F_1^V)^2 \rightarrow \frac{(F_1^V)^2}{|1 - c_0 f'_0(\rho) U_N(q)|^2}, \quad \text{etc...}$$



- Sizeable reduction of the strength at the QE peak, which is slightly shifted. Neutrino energy re-construction uses  $q^0 = -q^2/2M$ , problems??
- Enhancement of the high energy transfer tail, which partially compensates the above reduction and thus the effect on the total (integrated) cross section is smaller.

