

Neutrino mass and grand unification

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Abstract. Simple grand unified theories (GUT) of matter and forces provide a natural home for the seesaw paradigm for understanding small neutrino masses within a framework that also connects quark flavor to that of leptons. This provides a promising possibility for a unified approach to the flavor puzzle. To see what recent indications of a "large" value for the neutrino mixing parameter θ_{13} mean for this approach, I report on investigations based on supersymmetric SO(10) GUTs with type II seesaw for neutrino masses and find that it seem to support this point of view, with other predictions testable in near future.

1. Introduction

Understanding neutrino masses and mixings is an integral part of attempts to unravel the flavor puzzle in particle physics. During the past decade, the large amount of information on neutrino masses and mixings gained from the study of accelerator, reactor, solar, and cosmic ray neutrino observations have provided additional information that may have moved the frontier of understanding in this field forward. Several crucial pieces of the puzzle must still be found before we can begin to have a complete picture of neutrino masses. They include Dirac vs Majorana nature of the neutrino masses, normal vs inverted nature of the mass hierarchy, as well as the values of the mixing angle θ_{13} , and CP phases. One major step in this direction is the announcement that the T2K experiment has possible indications for a non-zero value for θ_{13} [1]. The T2K lower limit, if correct, is not far below the current experimental upper bound obtained several years ago from the CHOOZ experiment [2] and the fact that it is so "large" has important theoretical implications. The MINOS experiment has also seen an excess of electron events which could be indicating a non-zero θ_{13} [3], and their allowed range for θ_{13} overlaps with the T2K one. There have also been analyses of existing oscillation data suggesting a non-zero θ_{13} [4].

Before we specifically discuss the implications of a "large" θ_{13} for our understanding of flavor, let us review in brief the already existing information to see what possible conclusions may be drawn about the nature of neutrinos and physics beyond the standard model from them. First, the fact the neutrino masses are all in the sub-eV range can be understood within a simple paradigm of the seesaw mechanism, which we will use throughout this talk. Depending on what kind of dimension five operator [5] generates neutrino masses i.e. $LHLH$ or $L^T C^{-1} \tau_2 \bar{\tau} L \cdot H^T \tau_2 \bar{\tau} H$, we call it type I [6] or type II seesaw mechanism [7]. These operators are scaled by one power of inverse mass and naively taking generic neutrino mass to be 0.1 eV implies that the scale $M \sim 10^{14}$ GeV. This is close to the familiar scale of supersymmetric grand unification (GUT) of 2×10^{16} GeV. It is therefore very suggestive that the seesaw scale is indeed

related to the grand unification scale just as the quark or charged lepton masses are related to the standard model weak scale. This naive argument becomes more compelling when one realizes that without any specific framework like that of grand unified theories (GUT), the type I or type II seesaw scales could be anywhere from a TeV to the GUT scale by simply adjusting the neutrino Yukawa coupling parameters of the underlying theory while in GUT theories, neutrino Yukawa couplings are related to quark Yukawas and are therefore determined thereby fixing the seesaw scale. The minimal GUT group is the $SO(10)$ group which unifies all fermions of each generation including the right handed neutrinos into a single **16**-dim. spinor representation. A lot of interesting results follow if the $SO(10)$ GUT is supplemented by the assumption of the type II seesaw mechanism for neutrino masses. Additional support for the GUT idea comes from the approximate equality of the bottom and tau masses when extrapolated to high scales via supersymmetric renormalization group equations. Also the approximate GUT scale relations like $m_\mu \simeq 3m_s$ are suggestive of specific details of GUT models and how appealing a GUT model is, is determined by how naturally they explain these relations.

2. From data to physics via the neutrino mass matrix

Before discussing the GUT models, we note some features of the neutrino observations and their broad implications for theory. First, the maximal atmospheric neutrino mixing ($\tan \theta_{23} = 1$) suggests a form for the 2 – 3 sector of the neutrino mass matrix that exhibits an underlying discrete $\mu - \tau$ symmetry (denoted by $Z_{2,\mu-\tau}$). When this symmetry is exact for the whole mass matrix, it leads to vanishing θ_{13} [8]. Depending on how this symmetry is broken (e.g. in the μ -sector or e -sector), the resulting value of θ_{13} can either be very small or not so small (or “large”). The neutrino mass matrix has also been suspected to have a larger symmetry beyond this from the observation that the value of the solar mixing angle seems to have a geometric value ($\tan \theta_{12} = 1/\sqrt{2}$). The resulting lepton mixing (PMNS) matrix known as the tri-bi-maximal mixing matrix [9] (TBM for short) arises from a mass matrix that has a larger symmetry given by the larger symmetry Z_2 symmetries $Z_{2,S} \times Z_{2,\mu-\tau}$ [10]. This full symmetry of course also leads to zero θ_{13} . The form of the TBM neutrino mass matrix is given by

$$\mathcal{M}_\nu = \begin{pmatrix} b & c & c \\ c & a+b & c-a \\ c & c-a & a+b \end{pmatrix}, \quad (1)$$

which has only three parameters given by the three neutrino masses. In fact, in the above matrix, one could set $b = 0$, without changing the TBM mixing matrix. It only affects the masses of the neutrinos. This matrix is very different from the known mass matrices in the quark sector and could be a possible clue to a unified understanding of the quark-lepton flavor puzzle. A non-zero θ_{13} suggests that the TBM PMNS mixing is not the right form, and that “large” corrections to both the $\mu - \tau$ symmetry and TBM matrix must be present. Thus the value of θ_{13} could eventually guide us not only towards a complete determination of the neutrino mass matrix but also give indications for what kind of physics lies behind neutrino masses. For instance if observations require that the corrections to TBM mass matrix are “large”, it would not be too implausible to think that the symmetry approach may only be illusory and some other mechanism is at work. We will take this point of view and explore whether the framework of $SO(10)$ grand unification provides such an alternative to symmetry approach and may provide answers to such questions as: do the large atmospheric and solar angles as well as the curious feature of masses that $\frac{m_{solar}}{m_{atmos}} \sim \theta_{Cabibbo}$ get an easy explanation in GUT models without invoking any symmetries. The post- $SO(10)$ theory may of course contain symmetries as a way to understand the parameters of the theory in the same way as the Yukawa couplings of SM may be reflection of some underlying symmetries.

3. Neutrino mixings in renormalizable SO(10) GUT

As noted $SO(10)$ is the minimal GUT that embeds the seesaw paradigm for neutrino masses and among many $SO(10)$ models, the most predictive class [11] without extra symmetries appears to be the ones with renormalizable Yukawa couplings which use the Higgs fields $\mathbf{10}$, $\mathbf{126}$ and possibly $\mathbf{120}$ or another $\mathbf{10}$. This is a generalization of a minimal model with only three Higgs fields $\mathbf{10}+\mathbf{126}+\mathbf{210}$ discussed early on in connection with coupling unification [12]. Many of the attractive features of the model relating to fermion masses can be illustrated with two $\mathbf{10}$'s plus $\mathbf{126}$. Denoting the $\mathbf{10}$ Higgs fields (by H, H') and the $\mathbf{126} + \overline{\mathbf{126}}$ fields by $\Delta + \bar{\Delta}$, the $SO(10)$ invariant Yukawa couplings of the model can be written as:

$$\mathcal{L}_Y = h_0 \psi \psi H + f_0 \psi \psi \bar{\Delta} \quad (2)$$

where ψ 's denote the $\mathbf{16}$ dimensional spinors of $SO(10)$. The Yukawa couplings, h and f are 3×3 matrices in generation space. After symmetry breaking, one obtains the following formulae for charged fermion as well as Dirac mass for neutrinos:

$$\begin{aligned} M_u &= h + r_2 f, \\ M_d &= \frac{r_1}{\tan \beta} (h + f), \end{aligned} \quad \text{label eq : 2} \quad (3)$$

$$\begin{aligned} M_\ell &= \frac{r_1}{\tan \beta} (h - 3f), \\ M_{\nu D} &= h - 3r_2 f. \end{aligned} \quad (4)$$

where we have absorbed the vev of Higgs doublets into h, f . The neutrino mass formula is a combination of a type I and type II term:

$$M_\nu = f v_L - M_{\nu D} \frac{1}{f v_{BL}} M_{\nu D} \quad (5)$$

If in the above neutrino mass formula, the type II term dominates, there is a relation between neutrino and charged fermion masses of the form [13]

$$M_\nu = c(M_d - M_\ell) \quad (6)$$

It was pointed out in [14], this formula combined with $b - \tau$ mass equality at GUT scale from RGER extrapolation, provides a natural explanation of large atmospheric neutrino mixing angle. Soon afterwards, it was noted in [15] that once this idea is extended to three generations, it not only explains the large solar angle but it also explains the relation $\frac{m_{\text{solar}}}{m_{\text{atmos}}} \sim \theta_{\text{Cabibbo}}$. Furthermore, it leads to a value for $\theta_{13} \sim \theta_{\text{Cabibbo}} \sim 9 - 10^\circ$ (for the case when $h' = 0$). These results are extremely interesting since without any symmetries, the simple fact of $b - \tau$ unification seems to qualitatively reproduce all observations. To see how this comes about, observe that M_d and M_ℓ are both hierarchical mass matrices- the former due to small CKM angles and latter due to the constraint of $SO(10)$ grand unification in Eq. 3. Thus one can write

$$M_\nu = c m_b(M_U) \begin{pmatrix} \sim \lambda^4 & \sim \lambda^3 & \sim \lambda^3 \\ \sim \lambda^3 & \sim \lambda^2 & \sim \lambda^2 \\ \sim \lambda^3 & \sim \lambda^2 & 1 \end{pmatrix} - c m_\tau(M_U) \begin{pmatrix} \sim \lambda^4 & \sim \lambda^3 & \sim \lambda^3 \\ \sim \lambda^3 & \sim \lambda^2 & \sim \lambda^2 \\ \sim \lambda^3 & \sim \lambda^2 & 1 \end{pmatrix} \quad (7)$$

where $\lambda \sim 0.22$, the Cabibbo angle. Note that at GUT scale if $m_b(M_U) \sim m_\tau(M_U)(1 + \lambda^2)$, the 3×3 entries cancel out in the above difference and one naturally gets for the neutrino mass matrix:

$$M_\nu = c m_\tau(M_U) \lambda^2 \begin{pmatrix} \sim \lambda^2 & \sim \lambda & \sim \lambda \\ \sim \lambda & 1 + \lambda & 1 \\ \sim \lambda & 1 & 1 \end{pmatrix} \quad (8)$$

where we have omitted order one coefficients in the matrix elements. This mass matrix for neutrinos clearly implies that both the solar and the atmospheric mixing angles are large and $\frac{m_{\text{solar}}}{m_{\text{atmos}}} \sim \lambda$ and also $\theta_{13} \sim \lambda$ [15]. Furthermore note that without CP violation, the model has only eleven parameters and yet it fits quark and lepton masses and mixings as well as neutrino mixings and masses so well.

This model was subsequently reanalyzed after including CP violation to see if the right CKM phase can be accommodated while at the same time explaining neutrino mixings [16]. It was found that this was possible only in a small parameter space of the model where it predicted $\theta_{13} \sim 6^\circ$ and the Dirac phase $\delta \sim 4^\circ$. Extensions of this model by the addition of a **120** Higgs multiplet [17] was then considered and shown to considerably broaden the parameter space while still preserving the “large” θ_{13} prediction. Recall that **10**, **126** and **120** are the only three irreducible representations that appear in the product of two spinors of $SO(10)$; so in some sense it was a natural thing to consider. A particularly interesting version of the 10+126+120 model is one that assumes that CP is a good symmetry prior to spontaneous breaking by the vev of the **120** field [18], which is assumed to be odd under CP. Since **120** gives anti-symmetric Yukawa couplings, its CP odd property after symmetry breaking hermitean fermion mass matrices, responsible for CP violation both in the quark and lepton sectors. As a result, CKM phase and the Dirac phase of the neutrinos get connected. The model has a total of 16 parameters in the fermion mass sector (12 from Yukawa couplings and four from the vevs) keeping the model still predictive. Three inputs from the neutrino sector e.g. solar and atmospheric mass differences and the atmospheric mixing angle in addition to thirteen from the charged fermion sector allow one to predict θ_{12} , θ_{13} and the neutrino CP phases.

4. $SO(10)$ with type II seesaw and connection to TBM mass matrix

Recently, two interesting connections to the standard TBM model discussion has been noted in $SO(10)$ models with type II seesaw. In [19], it was observed that the type II formula for neutrino masses allows a TBM form for the neutrino mass matrix by simply a choice of fermion basis, with no additional symmetries; corrections to the TBM form then arise from the form of the charged lepton matrix, which, in our case, is determined by the $SO(10)$ constraints from quark masses and mixings. Strictly speaking, no bottom-tau unification is invoked explicitly in this approach but clearly, consistency requires that bottom-tau unification must hold for this basis choice to eventually lead to fermion mass fits. Detailed numerical analyses of these models have been carried out and lead to excellent fits for models with **10**, **126** and **120** Higgs fields [19, 20], and, yet again, a large θ_{13} is predicted. The near-tri-bi-maximal PMNS form in this class of GUT theories is related to the dynamics of the model rather than to any symmetry. Of course, to understand the particular Yukawa textures, one may need to invoke some symmetries broken above the GUT scale, but still those symmetries are not directly related to the θ_{13} value.

It was shown in a separate work [21] that the GUT scale mass relations $m_b \simeq m_\tau$ and $m_\mu \simeq -3m_s$ emerge out of a post-GUT S_4 flavor symmetry that determines the Yukawa texture of the $SO(10)$ GUT with **10** + **126** + **10'** Higgses while successfully giving TBM form of the neutrino mass matrix (or **126** Yukawa coupling matrix). As a consequence, in this model, no basis choice is required to get TBM form for the neutrino matrix. Since the same **126**-Yukawa coupling which gives the TBM neutrino mass matrix also contributes to charged fermion mass matrix, the model is tightly constrained and charged fermion mass fits lead to a prediction for θ_{13} . A detailed quantitative analysis of the model was carried out in [22] by supplementing the minimal model [21] by additional effective GUT scale Yukawa couplings that could follow from S_4 with a total of 12 parameters. Below we summarize the basic elements and results of this.

5. Details of the model

The effective Yukawa couplings f_0, h_0, h'_0 corresponding to $H, \bar{\Delta}$ and H' Higgs, are assumed to have descended from a higher scale theory which has an S_4 symmetry broken by flavon fields with particularly aligned vacuum expectation values (vevs) (see e.g. [21]). We do not need to know the detailed form of these flavon interactions for our analysis, and we will simply write down the effective form of the h_0, h'_0, f_0 that follow from it. The effective GUT scale Yukawa superpotential in this case is

$$\mathcal{L}_Y = h_0 \psi \psi H + f_0 \psi \psi \bar{\Delta} + h'_0 \psi \psi \Sigma \quad (9)$$

which after symmetry breaking leads to the fermion mass formulae

$$\begin{aligned} M_u &= h + r_2 f + r_3 h', \\ M_d &= \frac{r_1}{\tan \beta} (h + f + h'), \\ M_\ell &= \frac{r_1}{\tan \beta} (h - 3f + h'), \end{aligned} \quad (10)$$

$$(11)$$

where we have absorbed the Higgs vevs into h, f, h' . The neutrino mass matrix is unaffected by this extension. Turning to the flavor structure of the various couplings, we note that S_4 symmetry [21] puts the neutrino mass matrix in a form that, upon diagonalization, leads to tri-bi-maximal mixing prior to charged lepton corrections i.e. an f_0 coupling in the form:

$$f_0 \propto \mathcal{M}_\nu = \kappa \begin{pmatrix} 0 & m_1 & m_1 \\ m_1 & m_0 & m_1 - m_0 \\ m_1 & m_1 - m_0 & m_0 \end{pmatrix}, \quad (12)$$

In [21], the S_4 symmetry constrains h_0 to be a rank one matrix of the form:

$$h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M \end{pmatrix}, \quad (13)$$

and we choose h' of the form [22]

$$h'_0 = \begin{pmatrix} \delta'' & \delta & -\delta + \delta' \\ \delta & 0 & d \\ -\delta + \delta' & d & 0 \end{pmatrix}, \quad (14)$$

which can be also generated by choice of flavon fields and the alignment of their vevs. The model has eleven parameters if we choose all except δ' real (twelve parameters when we allow δ complex to study the allowed range of Dirac CP phase). Recall that the model with **10**, **126** and **120** has a total of seventeen parameters [18, 19]. The best fit values for the input parameters that leads to the minimum χ^2 is given in Table 1, and the resulting mass and mixing parameter values are given in Table 1. Note that while we get a higher value for m_b and slightly lower values for m_s and m_c , all are within reasonable statistical deviation from extrapolated values in [23]. Similarly, our predictions for m_d and m_u are somewhat higher than those obtained in [23], but we believe there could easily be instanton corrections to the light quark masses, which could change these extrapolated values. It is nevertheless remarkable that we are able to reproduce all other parameters in the charged fermion sector so well. In the figures 1 and 2 the

Table 1. Best fit values for the model parameters. Note that adding a small imaginary part to δ will give us a non-negligible Dirac CP phase [22].

	$\tan \beta = 10$	$\tan \beta = 55$
M (GeV)	88.2	106.2
m_0 (GeV)	1.435	1.382
m_1 (GeV)	0.275	0.275
δ (GeV)	0.285	0.2605
δ' (GeV)	0.4632 - 0.279 <i>i</i>	0.529 - 0.335 <i>i</i>
δ'' (GeV)	-0.0652	-0.0767
d (GeV)	3.78	4.31
$r_1/\tan \beta$	0.0153	0.01586
r_2	0.13	0.129
r_3	-0.06	-0.07

Table 2. The best fit values of the quark and charged lepton masses and the most relevant quark mixing parameters. The 1σ experimental values extrapolated by MSSM renormalization group (RG) equations to the GUT scale [23] are also shown for comparison.

	best fit	RG extrapolated	best fit	RG extrapolated value
m_e (MeV)	0.3587	$0.3585^{+0.0003}_{-0.0003}$	0.3563	$0.3565^{+0.0002}_{-0.0010}$
m_μ (MeV)	75.6865	$75.6715^{+0.0578}_{-0.0501}$	75.3359	$75.2938^{+0.0515}_{-0.1912}$
m_τ (GeV)	1.2927	$1.2922^{+0.0013}_{-0.0012}$	1.6272	$1.6292^{+0.0443}_{-0.0294}$
m_d (MeV)	3.8587	$1.5036^{+0.4235}_{-0.2304}$	4.0202	$1.4967^{+0.4157}_{-0.2278}$
m_s (MeV)	23.6026	$29.9454^{+4.3001}_{-4.5444}$	23.1619	$29.8135^{+4.1795}_{-4.4967}$
m_b (GeV)	1.3726	$1.0636^{+0.1414}_{-0.0865}$	1.7078	$1.4167^{+0.4803}_{-0.1944}$
m_u (MeV)	1.9772	$0.7238^{+0.1365}_{-0.1467}$	3.6311	$0.7244^{+0.1219}_{-0.1466}$
m_c (MeV)	177.3862	$210.3273^{+19.0036}_{-21.2264}$	177.6719	$210.5049^{+15.1077}_{-21.1538}$
m_t (GeV)	88.3886	$82.4333^{+50.2676}_{-14.7686}$	106.3806	$95.1486^{+69.2836}_{-20.6590}$
V_{us}	0.2230	0.2243 ± 0.0016	0.2233	0.2243 ± 0.0016
V_{ub}	0.0032	0.0032 ± 0.0005	0.0032	0.0032 ± 0.0005
V_{cb}	0.3489	0.0351 ± 0.0013	0.0352	0.0351 ± 0.0013
δ_{CKM}	-64.35°	$(-60 \pm 14)^\circ$	-61.84°	$(-60 \pm 14)^\circ$

correlation between the various θ_{13} predictions and other mixing angles such as θ_{12} or θ_3 are displayed. This is a way to test the model.

The exact numerical results for neutrino mixing corresponding to the quark sector fit from above are given in Figures 1-2 which show the relationships between $(\theta_{23}, \theta_{13})$ and $(\theta_{12}, \theta_{13})$, respectively. Note that the value of θ_{13} is large, though not so large as the 6° - 8° central value of T2K result gives. Also note that the atmospheric mixing angle θ_{23} is always larger than the maximal value of 45° . These correlations between different mixing parameters could be used to test the model once the current uncertainties in both θ_{12} and θ_{23} are reduced and a more precise value for θ_{13} has been determined. Our model also predicts small Majorana phases ($\sim 1^\circ$). Other predictions are the normal mass hierarchy for neutrinos which is also a generic features of most GUT models and an effective mass in neutrinoless double beta decay in the few meV

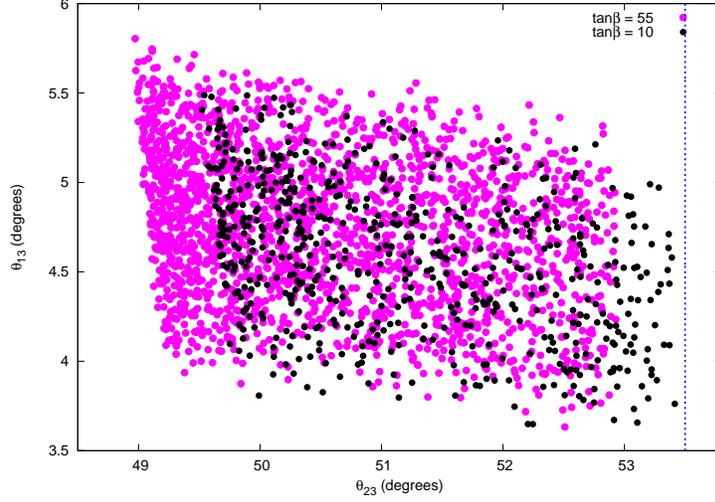


Figure 1. Correlation between θ_{13} and θ_{23} predicted in our model satisfying all the charged fermion sector constraints.

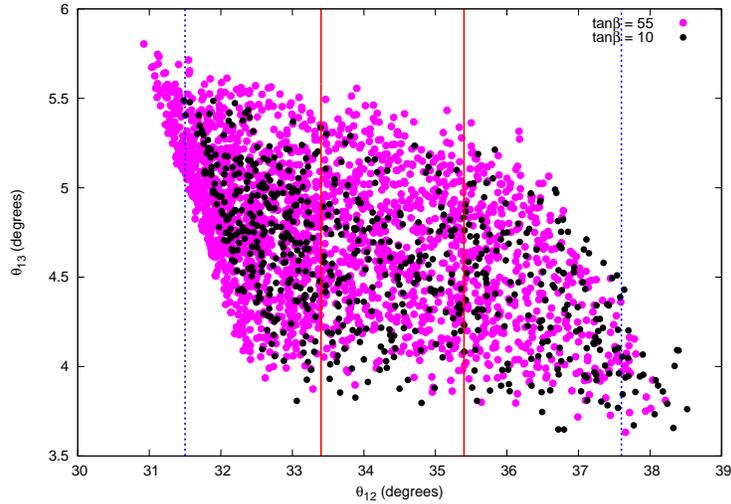


Figure 2. Correlation between θ_{13} and θ_{12} predicted in our model. The solid and dotted vertical lines are the current 1σ and 3σ limits respectively for the solar mixing angle.

range.

6. Summary

In this brief overview, an alternative to the symmetry approach to understanding the current neutrino observations is discussed based on the hypothesis of grand unification of forces and matter. A particular class of GUT models based on $SO(10)$ with type II seesaw seems to explain several observations in a simple way with testable predictions. Furthermore, the type II dominance advocated here can also be demonstrated in $SO(10)$ models, both supersymmetric [24] and non-supersymmetric [25]. Finally, all this discussion ignores the sterile neutrino possibility and will require drastic modification if they are confirmed. Mirror duplication may be one possibility which can accommodate the sterile neutrinos with least “damage” to the above

predictions.

Acknowledgments

I thank Bhupal Dev, B. Dutta, Y. Mimura and M. Severson for collaboration and discussions. The present work is supported by the National Science Foundation grant number PHY-0968854.

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