

# Charged-current and neutral-current coherent pion productions — Theoretical status

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**Abstract.** The CC and NC coherent pion productions have been studied with PCAC-based models and microscopic models. Current status of the theoretical studies is reviewed.

## 1. Introduction

Recent neutrino oscillation experiments have driven quite a few nuclear theorists to study neutrino-nucleus interaction. Theoretical study of coherent pion productions (coh $\pi$ ) has been also on the trend. The charged-current (CC) coh $\pi^+$  is  $\nu_l + A_{g.s.} \rightarrow l^- + A_{g.s.} + \pi^+$ , while the neutral-current (NC) coh $\pi^0$  is  $\nu_l + A_{g.s.} \rightarrow \nu_l + A_{g.s.} + \pi^0$ , where  $\nu_l$  is the neutrino of  $l = e, \mu, \tau$ , and  $A_{g.s.}$  is a nucleus in its ground state. The process takes place under small momentum transfer from the lepton. The NC coh $\pi^0$  needs to be understood for the  $\nu_e$  appearance event, because when one of two photons from  $\pi^0 \rightarrow 2\gamma$  is not detected, the NC coh $\pi^0$  can fake the  $\nu_e$  appearance event. Theoretically, there are two main approaches to coh $\pi$ . One of them is based on the partially conserved axial current (PCAC) [1, 2, 3, 4, 5, 6], and the other is based on a microscopic dynamical model [7, 8, 9, 10, 11, 12, 13, 14]. In the following, I discuss both of the approaches, and then comparison among them and with recent experimental data.

## 2. PCAC-based models

The prominent PCAC-based model was proposed by Rein and Sehgal [1] (RS model). The model is based on the fact that a coh $\pi$  amplitude is proportional to the divergence of the axial-current in the limit of  $q^2 \rightarrow 0$  where  $q$  is the momentum transfer from the lepton. The PCAC dictates that the divergence of the axial-current is proportional to the pion field. Thus, the coh $\pi$  amplitude and the elastic  $\pi$ -nucleus scattering amplitude is related. Continuation of the cross section to  $q^2 \neq 0$  is parametrized solely by a dipole-type function. The elastic pion-nucleus cross section is simply given by the product of the pion-nucleon elastic cross section, the square of the nuclear form factor, and the pion absorption factor. The RS model works well for high energy neutrino ( $E_\nu \gtrsim 2$  GeV), from medium to heavy nuclei [15]. The RS model has been used in many analyses of neutrino experiments because of its success and simplicity.

However, recent neutrino experiments use neutrino beam of  $E_\nu \lesssim 2$  GeV and relatively light nuclei (*e.g.*,  $^{12}\text{C}$ ) as a target, for which the validity of the RS model is questionable. Indeed, recent experiments [16, 17] found that CC coh $\pi^+$  cross section of the RS model is significantly larger than their data. Rein and Sehgal corrected the RS model by taking account of the finite muon mass whose effect can be significant for low  $E_\nu$  [2]. They found 25% reduction of the CC

coh $\pi^+$  cross sections for  $E_\nu = 1.3$  GeV. Further improvements were attempted by Hernández *et al.* [3]. They pointed out that the elastic pion-nucleus scattering model in the RS model is rather off data. They improved the model by considering previously ignored  $t$ -dependence of the pion-nucleon cross sections, and by using a more realistic pion absorption factor. It was found that, although the pion-nucleus elastic scattering is significantly improved, the coh $\pi$  cross sections from the improved RS model are still rather different from those of microscopic calculations.

Meanwhile, another avenue for the PCAC-based model was explored [4, 5, 6]. There, the PCAC relation was used to relate the axial current-nucleus interaction to the pion-nucleus interaction (nuclear PCAC), rather than the axial current-nucleon interaction to the pion-nucleon interaction (nucleon PCAC); the latter was employed in the RS model. Thus, they can use the pion-nucleus cross section data directly to calculate the coh $\pi$  cross sections. They found that this change yields a drastic reduction ( $\sim 1/2$ ) of the coh $\pi$  cross sections, now the size is comparable to those of the microscopic calculations. Strictly speaking, however, the nuclear PCAC relates a coh $\pi$  amplitude to the half off-shell pion-nucleus scattering amplitude [18]. Therefore, the use of pion-nucleus cross section may call for some justification.

### 3. Dynamical microscopic models

A description of coh $\pi$  with a dynamical microscopic model consists of three basic ingredients: elementary amplitudes (neutrino-induced pion production off a single nucleon); nuclear medium effect on the  $\Delta$ -propagation; pion-nucleus final state interaction. In the following, I will discuss each of them used in the previous works.

*3.1 Elementary amplitude* : The most important mechanism for the pion production is the  $\Delta$ -excitation. Thus [7, 8, 9] used only the  $s$ -channel  $\Delta$  diagram for the elementary amplitude. Among the  $AN\Delta$  ( $A$ :axial-current;  $N$ :nucleon) form factors, the so-called  $C_5^A(q^2)$  gives a dominant contribution so that the coh $\pi$  cross section is roughly  $\sigma_{\text{coh}\pi} \propto |C_5^A|^2$ . In [7, 8, 9], the Goldberger-Treiman (GT) value for  $C_5^A(0)$  was taken:  $C_5^A(0) = g_{\pi N\Delta} f_\pi / \sqrt{6} m_N = 1.2$ . Martini *et al.* [12] used  $s$ - and  $u$ -channel nucleon and  $\Delta$  mechanisms as the elementary amplitudes; the value of  $C_5^A(0)$  is the GT value. Meanwhile, elementary amplitudes used in [10, 11, 14] consists of  $s$ - and  $u$ -channel nucleon and  $\Delta$  mechanisms,  $t$ -channel  $\pi$ -exchange mechanism, a contact and a pion pole terms [19]. Because of the background terms, the value of  $C_5^A(0)$  has to be smaller than the GT value by 20% in order to fit neutrino data. So far, all elementary amplitudes are based on tree-level calculations. On the other hand, Sato *et al.* proposed a model (SL model [20]) in which tree-level weak pion production mechanisms, which are similar to those of [19], are dressed by multiple  $\pi N$  rescattering to yield a unitary elementary amplitude. The SL amplitude contains significant pion cloud effect, which is unique in this model. Bare  $AN\Delta$  couplings were fixed using the constituent quark model. Our group used the elementary amplitudes from the SL model [13]. All elementary amplitudes used in various groups fairly reproduce available data. Unfortunately, available data are rather limited, and not enough to test each elementary amplitude model. More good quality data would help.

*3.2 Medium effect on  $\Delta$*  : In the vacuum, the  $\Delta$  propagates by repeating the decay into  $\pi N$  state and formation of  $\Delta$  (self energy). In a nuclear medium, this process becomes more complicated. Some  $\pi N$  states are forbidden by the Pauli blocking so that the  $\Delta$  decay is suppressed. A pion emitted from the  $\Delta$  can be absorbed by another nucleon, forming multiparticle-multipole configurations or RPA(random phase approximation)-type processes. The  $\Delta$  mass and width can shift due to these medium effects. Oset *et al.* calculated the medium effects on the  $\Delta$  propagation using a many-body approach [21], and parametrized the shift of the  $\Delta$  mass and width as a function of the density. Their result has been employed by many people for calculating the coh $\pi$  cross sections [7, 8, 9, 10, 11, 12, 14]. However, non-locality due to the recoil of the  $\Delta$  was not explicitly taken into account in these calculations of coh $\pi$ . Leitner *et al.* [22] pointed

out this, and showed that the non-local effect can reduce the total coh $\pi$  cross sections by half. Hernández *et al.* argued that the non-local effect is effectively (partially) taken into account in the course of fitting the medium shift of the  $\Delta$  to observables [14]. Our group [13] considered the non-locality of the  $\Delta$ -propagation as well as several medium effects explicitly. Multiparticle-multihole effects are simulated by a phenomenological spreading potential, parameters of which were fitted to pion-nucleus elastic and total (elastic+inelastic) cross sections. It is important to fit the total cross sections because the spreading potential describes pion absorptions.

*3.3 Final state interaction (FSI) :* In [7, 9, 10, 11, 14], pion-nucleus FSI is considered in distorted pion wave function obtained by solving the Klein-Gordon equation with an optical potential. The optical potential is based on the  $\Delta$ -hole model and the Lorentz-Lorentz correction, and reproduces spectra of pionic atoms and low-energy pion-nucleus elastic cross sections [23]. In Martini *et al.*'s approach [12], the elastic pion-nucleus cross section is obtained by solving a RPA equation with a bare polarization propagator as the driving term. The bare polarization propagator contains various nuclear excitation such as multiparticle-multihole configurations. The model well describes the elastic total cross sections in the  $\Delta$  region. Our group constructed a pion-nucleus optical potential from  $\pi N$  amplitudes of the SL model, combined with the medium effects on the  $\Delta$ . A pion-nucleus scattering amplitude is obtained by solving the Lippmann-Schwinger equation with the optical potential. The model well describes the elastic differential cross sections and the total cross sections in the  $\Delta$  region.

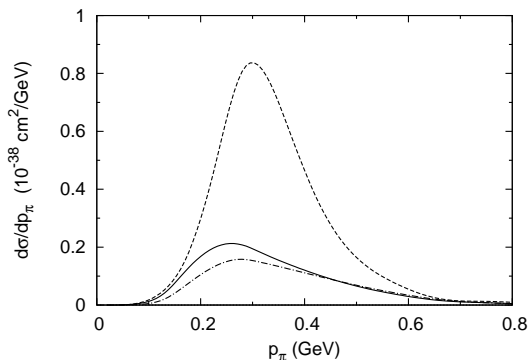
*3.4 Coherent pion production :* Combining the above-discussed three ingredients, one can calculate coh $\pi$  cross sections. First let us discuss a photo-coh $\pi^0$ . This is a great testing ground for microscopic models of coh $\pi$  because data of good quality are available. Also, some dynamical models like ours [13] can predict coh $\pi$  cross sections without any adjustable parameters; all parameters have been fitted to pion-nucleus scattering data. Thus we have tested our prediction for photo-coh $\pi$  with data, and found a good agreement [13]. It is noteworthy that the medium effects on  $\Delta$  and FSI reduce the cross sections by about half to reach the good agreement. This success serves as a good basis to apply our model to the neutrino-coh $\pi$ . The pion momentum distribution in CC coh $\pi^+$  at  $E_\nu = 1$  GeV is shown in Fig. 1. Significant medium effects are observed here. The dashed curve contain neither the  $\Delta$  spreading potential nor the FSI. Including these nuclear effects lowers the dashed curve into the solid curve. The dash-dotted curve contains only resonant amplitude, showing significant contribution from non-resonant unitary amplitude. This is in contrast with calculations with tree-level elementary amplitude [19], because the non-resonant amplitude hardly contributes.

#### 4. Comparison among models and data

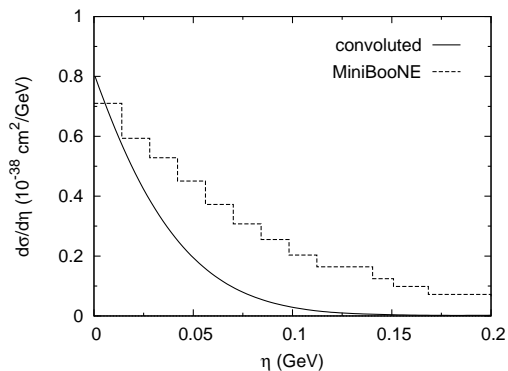
At NUINT09, CC and NC coh $\pi$  cross sections from many theoretical models and Monte Carlo (MC) codes were compared [24]. Although details are different, recent theoretical models [4, 5, 6, 9, 10, 11, 12, 13, 14] give fairly similar CC and NC coh $\pi$  cross sections. However, the MC code outputs, which are based on the RS model with different modifications, are very different from those of the recent theoretical models.

There have been large discrepancies between experimental data and theoretical calculations on the coh $\pi$  cross sections. The latest data from SciBooNE [25] gave the ratio between CC coh $\pi^+$  and NC coh $\pi^0$ :  $\sigma_{CC}/\sigma_{NC} = 0.14^{+0.30}_{-0.28}$ . On the other hand, all recent theoretical calculations gave  $\sigma_{CC}/\sigma_{NC} = 1.5 \sim 2$ , which is expected from the isospin factor.

One may suspect that this unsatisfactory situation is due to using the RS model in the analyses of the NC data. In fact, several authors [11, 13, 14] have shown that  $\eta$ -distribution ( $\eta \equiv E_\pi(1 - \cos \theta_\pi)$ ) of the RS model rather deviates from those of their microscopic models (Fig. 2). The  $\eta$ -distribution has been used in the analyses of the NC data in order to decompose  $\pi^0$  events into each production mechanism. The use of the RS model in the analyses could have



**Figure 1.** Pion momentum distribution in CC coh $\pi$  at  $E_\nu = 1$  GeV from [13]. See the text for more.



**Figure 2.**  $\eta$ -distribution for NC coh $\pi^0$  on  $^{12}\text{C}$ . The model calculation [13] convoluted with MiniBooNE flux (—); MC output (- - -).

overestimated the NC coh $\pi^0$  cross sections. It is very interesting to re-analyze the data with a more realistic coh $\pi$  model.

### Acknowledgments

I thank the Nufact11 organizers for the invitation and the financial support for my participation in the workshop. This work is supported by the U.S. Department of Energy, Office of Nuclear Physics Division, under Contract No. DE-AC05-06OR23177 under which Jefferson Science Associates operates Jefferson Lab.

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