

# Neutrino physics within and beyond the three flavour oscillation

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**Abstract.** I will discuss several possible choices for the mass scale  $M$  of the Majorana leptons, complementing the Standard Model to describe neutrino masses and oscillations. Depending on  $M$ , they can or cannot accommodate eV neutrino anomalies, lead to baryogenesis, provide the dark matter candidate, ensure the stability of the Higgs mass against radiative corrections, and be directly searched at some experiments.

## 1. Introduction

The discovery of neutrino oscillations provides an undisputed signal that the Standard Model (SM) of elementary particles is not complete. However, what kind of new physics it brings to us remains still unclear - we do not know yet the properties of new particles which are believed to be behind this phenomenon. In this talk I will overview a number of possibilities. The most conservative one is the scenario of three flavour oscillations, encoded in  $(3 \times 3)$  Majorana neutrino mass matrix for active neutrinos. The particle physics model in this case is nonrenormalisable, calling for an “integrating in” of some new degrees of freedom. The most attractive possibility is the extension of the SM by three right-handed neutrinos, making the leptonic sector similar to the quark one. The masses and couplings of new leptons remain largely unknown. The four different possibilities for the masses  $M$  of new leptons will be shortly discussed: (i) the “GUT see-saw” corresponding to the scale  $10^{10-16}$  GeV, (ii) “electroweak see-saw” with the scale  $10^{2-3}$  GeV, (iii) the  $\nu$ MSM (Neutrino Minimal SM) choice  $M \sim \text{keV to GeV}$ , and (iv) the eV scale, associated with the “canonical” sterile neutrinos with this mass.

## 2. Three flavour oscillations scenario

The most minimal way (no extra degrees of freedom are introduced) to incorporate non-zero neutrino masses into the Standard Model is to add to its Lagrangian the  $SU(3) \times SU(2) \times U(1)$  invariant five-dimensional operator [1, 2]

$$O_5 = A_{\alpha\beta} \left( \bar{L}_\alpha \tilde{\phi} \right) \left( \phi^\dagger L_\beta^c \right), \quad (1)$$

suppressed by the scale  $\Lambda$ . Here  $L_\alpha$  are the leptonic doublets ( $\alpha = e, \mu, \tau$ ),  $\phi$  is the Higgs field,  $\tilde{\phi}_i = \epsilon_{ij} \phi_j^*$ , and  $c$  is the sign of charge conjugation. Then the neutrino mass matrix is given by:

$$M_\nu \sim A_{\alpha\beta} \frac{v^2}{\Lambda}. \quad (2)$$

where  $v = 174$  GeV is the vacuum expectation value of the Higgs field. If  $|A_{\alpha\beta}| \sim 1$ , then the correct neutrino mass scale  $\mathcal{O}(0.1)$  eV is obtained at  $\Lambda \sim 3 \times 10^{14}$  GeV.

The matrix  $M_\nu$  depends on 9 physical parameters which potentially can be determined in low energy neutrino experiments. They are: 3 absolute values of  $\nu$  masses, (only mass square differences can be determined in neutrino oscillation experiments, and  $\delta m_{sol}^2$ ,  $\delta m_{atm}^2$  are known with good accuracy); 3 mixing angles  $\theta_{23}$ ,  $\theta_{12}$  and  $\theta_{13}$ , 1 Dirac CP-violating phase and 2 Majorana phases. Four parameters out of these 9 are not known (absolute value of neutrino masses and 3 CP-violating phases). The mixing angles and Dirac phase determine the PMNS mixing matrix [3, 4].

The ultimate experimental goal within 3 family scenario is to determine all 9 parameters with as high accuracy as possible. The self-consistency of this scheme would require the verification of the unitarity of the PMNS mixing matrix. The methods include long and short baseline neutrino oscillation experiments, search for neutrinoless double beta decay, determination of the end point of electron spectra in  $\beta$  decays. An incomplete list of experiments include T2K, MINOS, GERDA, CUORE, NO $\nu$ A, MiniBooNE, Majorana, Double Chooz, Opera, NEMO, RICE, KATRIN, SNO, RENO, IceCube, LAGUNA-LBNO, Daya Bay, LSND and Karmen. The theoretical challenges of the 3 family scenario were discussed in [5], and will not be addressed here. This scenario can be rejected if the existence of extra neutrino flavours, such as sterile neutrinos, will be firmly established.

### 3. Neutrino physics beyond three flavour oscillations

An effective field theory approach, described in Section 2 allows to “solve” phenomenological aspects of the problem of neutrino masses and oscillations. However, it leaves unanswered the following fundamental questions:

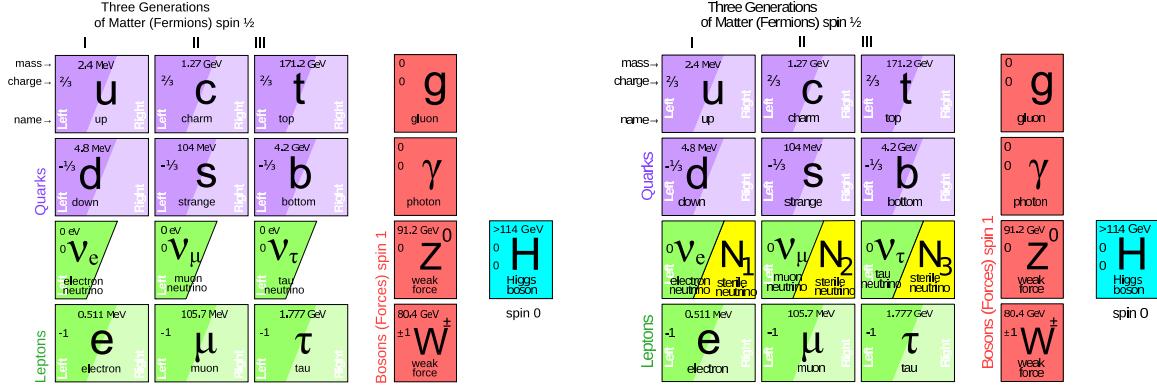
- What is the physics behind the non-renormalizable terms?
- What is the energy scale of physics beyond the SM, leading to neutrino masses?

The success of relativistic quantum field theory, associated with the fact that the SM agrees with most experiments, strongly indicates that the origin of neutrino masses is the existence of new unseen particles and that the complete theory should be a renormalisable extension of the Standard Model. From the SM quantum numbers of active neutrinos one can identify several possible sources for neutrino masses. If no new fermionic degrees of freedom are introduced, one needs to have a Higgs triplet with weak hypercharge 2. Another option is an introduction of singlet (with respect to the SM gauge group) Majorana fermions  $N_I$  (other names for them are sterile neutrinos or heavy neutral leptons). In this talk I will consider only the latter possibility. The dimension five operator in (1) appears then due to an exchange of the singlet fermions.

This extension of the SM is associated with the Lagrangian

$$L = L_{\text{SM}} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + h.c., \quad (3)$$

where  $L_{\text{SM}}$  is the Lagrangian of the SM. Since  $N_I$  are  $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$  singlets, Majorana mass terms for them are consistent with the symmetries of the SM. The number of singlet fermions cannot be deduced from symmetry principles; the minimal number is 2, to get 2 different mass square differences in active neutrino sector. We take it to be 3 in analogy with the number of generations of quarks and leptons. These particles complement nicely the fermionic content of the SM, making it left-right symmetric in neutrino sector as well, see Fig. 1. This Lagrangian is usually used for the explanation of the small values of neutrino masses via the see-saw mechanism [6, 7, 8, 9], which *assumes* that the Yukawa coupling constants  $F_{\alpha I}$  of the singlet fermions are of the order of the similar couplings of the charged leptons or quarks. We are not going to make such an assumption.



**Figure 1.** Particle content of the SM and its extension in neutrino sector. In the SM (left) right-handed partners of neutrinos are missing. In the  $\nu$ MSM (right) all fermions have both left and right-handed components.

To reduce the uncertainties in the further discussion, we will assume the validity of this theory up to the Planck scale. This may be a pretty safe bet: no supersymmetry, or extra dimensions, or technicolor are seen at the LHC at present.

In comparison with the SM, this theory contains 18 new parameters: 3 Majorana masses of new neutral fermions  $N_I$ , and 15 new Yukawa couplings in the leptonic sector, corresponding to 3 Dirac neutrino masses, 6 mixing angles and 6 CP-violating phases. The number of parameters is almost doubled in comparison with the SM; none of them can be determined theoretically within this model, in complete analogy with the SM parameters (which are all taken from experiment).

The new parameters can be divided in two different groups. The first one is the new mass scale - a generic value of the Majorana neutrino mass (denoted by  $M$ ), and the second one is the typical amplitude of the Yukawa coupling constants  $Y$ , which may be defined as  $Y^2 = \text{Trace}[F^\dagger F]$ . We know very little about the actual values of  $Y$  and  $M$ . Basically,  $M$  can have any value between zero (corresponding to Dirac neutrinos) to  $10^{16}$  GeV, whereas  $Y$  can vary from  $10^{-13}$  (Dirac neutrino case) to 1 (the onset of the strong coupling). The admitted region is shown in Fig. 2 (left).

There is no preference to any value of  $M$  or  $Y$  inside the triangle shown in Fig. 2 from the point of view of particle theory described by the Lagrangian (3). The four “preferred” choices, relying on different physics and marked by “GUT see-saw”, “LHC”, “ $\nu$ MSM” and “LSND”, will be discussed below.

### 3.1. GUT see-saw

The key *assumption* of the see-saw mechanism [6, 7, 8, 9] is that the Yukawa couplings of  $N$  to the Higgs and left-handed lepton doublets are similar to those in quark or charged lepton sector (say,  $Y \sim 1$ , as for the top quark). This leads to an estimate the mass scale of the singlet leptons:

$$M_I \simeq \frac{v^2}{m_{\text{atm}}} \simeq 6 \times 10^{14} \text{ GeV} , \quad (4)$$

where  $m_{\text{atm}} = \sqrt{\Delta m_{\text{atm}}^2} \simeq 0.05 \text{ eV}$  is the atmospheric mass difference. This scale is rather close to that of Grand Unification, providing an argument that the gauge coupling unification and neutrino masses might be related [10]. In addition, the decays of superheavy  $N_I$  in the early Universe may lead to baryogenesis through leptogenesis [11] and rapid anomalous fermion number non-conservation at high temperatures [12] (see also [13]). There are several challenges

of the GUT see-saw. One of them is the stabilisation of the Higgs mass hierarchy  $m_H \ll M$  against radiative corrections to  $m_H$  involving the loops with superheavy Majorana leptons [14]. It requires low energy supersymmetry. If SUSY is not discovered at the LHC then so small value of  $m_H$  in comparison with  $M$  is “unnatural” and requires an enormous fine-tuning. From more phenomenological side, this choice of  $M$  does not provide a Dark Matter candidate and thus calls for a further extension of the model.

The experimental neutrino physics within GUT see-saw is identical with that within the three flavour oscillations - new particles are too heavy to be searched directly at any experiment. However, if the see-saw mechanism is embedded in some grand unified theory, one should expect to have a proton decay.

### 3.2. EW see-saw

The key assumption of the “EW see-saw” is that the masses of new Majorana leptons are related to (unknown) physics of electroweak symmetry breaking and are thus of the order of 100 GeV - 1 TeV. If all the Yukawa coupling are of the same order of magnitude, then they are too small to make these particles directly observable at any type of experiment (this corresponds to the lower part of the triangle in Fig. 2). However, extra symmetries can allow for cancellations [15, 16], and some of Yukawa couplings can be of the order of 1, corresponding to the upper horizontal line in Fig. 2. If true, LHC can find them [16, 17]. This question was discussed in [18]; the low energy signatures of the EW see-saw were analysed in [20, 19]. As in the GUT see-saw case, the baryogenesis through resonant leptogenesis is possible [21]. However, no dark matter candidate exists with this choice of parameters.

### 3.3. The $\nu$ MSM

The  $\nu$ MSM approach [22, 23] (for reviews see [24, 25]) is driven by neutrino experiments and cosmology. It requires that three new particles - Majorana leptons - must explain simultaneously neutrino masses and oscillations, Dark Matter, and baryon asymmetry of the Universe. Once the range of the parameters of the model, leading to these facts, is determined, one can figure out how and where to search for the new particles.

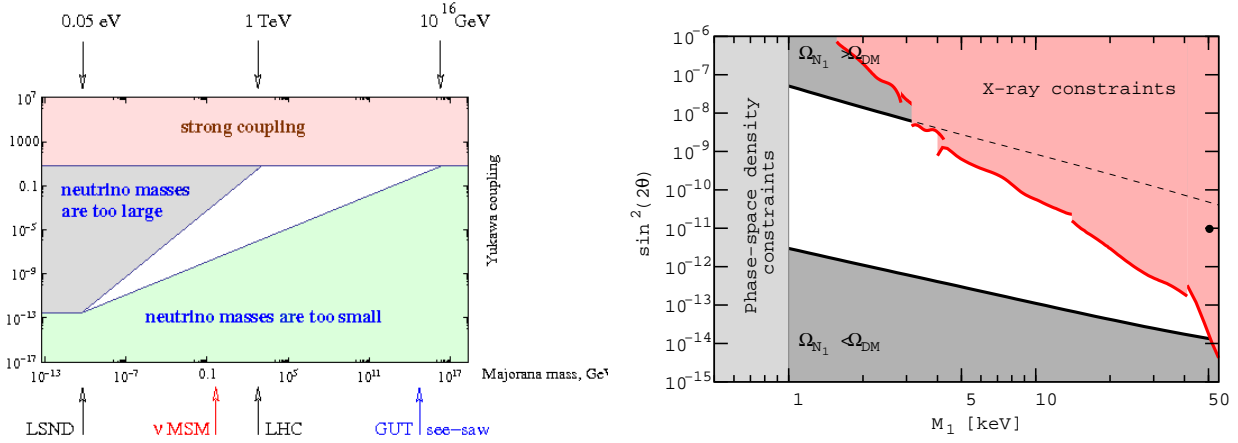
*3.3.1. Dark matter* Though the  $\nu$ MSM does not have any extra stable particle in comparison with the SM, the lightest singlet fermion,  $N_1$ , may have a life-time  $\tau_{N_1}$  greatly exceeding the age of the Universe and thus play a role of a dark matter particle [26, 27, 28, 29]. The following considerations determine the range of masses and couplings of the DM sterile neutrino:

- (i) Cosmological production.  $N_1$  are created in the early Universe in reactions  $l\bar{l} \rightarrow \nu N_1$ ,  $q\bar{q} \rightarrow \nu N_1$ , etc. We should get the correct DM abundance.
- (ii) Structure formation. If  $N_1$  is too light it may have considerable free streaming length and erase fluctuations on small scales. This can be checked by the study of Lyman- $\alpha$  forest spectra of distant quasars and structure of dwarf galaxies.
- (iii) X-rays.  $N_1$  decays radiatively,  $N_1 \rightarrow \gamma\nu$ , producing a narrow line which can be detected by X-ray telescopes (such as Chandra or XMM-Newton). This line has not been seen yet.

The summary of these constraints is presented in Fig. 2 where the mixing angle  $\theta$  is the ratio of the Dirac and Majorana masses,

$$\theta = \frac{m_D}{M_1} . \quad (5)$$

The interactions of  $N_1$  with particles of the SM is weaker than the weak interactions by a factor  $\theta$  (in the amplitude). So, they fall into the SuperWIMP category of the DM particle physics candidates. It is important that the DM sterile neutrino production requires the presence of large,  $\Delta L/L > 2 \times 10^{-3}$  lepton asymmetry at temperature  $T \sim 100$  MeV. It can only be produced in the  $\nu$ MSM [33] (the GUT, EW, or eV see-saw cannot make it).



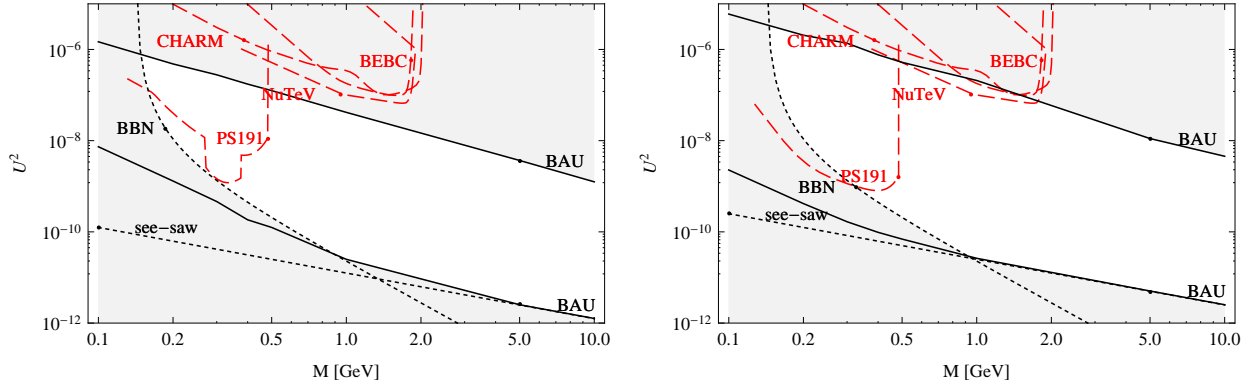
**Figure 2.** *Left:* The admitted values of the Yukawa couplings as a function of the Majorana fermion mass. *Right:* The allowed region of parameters for dark matter sterile neutrinos produced via mixing with active neutrinos (unshaded region). The two thick black lines bounding this region represent production curves for zero lepton asymmetry (upper line) and for the maximal lepton asymmetry attainable in the  $\nu$ MSM. The red shaded region in the upper right corner represents X-ray constraints. The region below 1 keV is ruled out according to the phase-space density arguments [30, 31]. The Lyman- $\alpha$  constraints are in general stronger but depend essentially on lepton asymmetry, see [32] for detailed discussion. For zero lepton asymmetry the lower bound on  $M_1$  is around 8 keV, while for large asymmetries it is as small as 2 keV. From Ref. [25].

The constraints shown in Fig. 2 (right) allow to make a number of predictions for neutrino physics [22, 34]. The minimal number of sterile neutrinos, which can explain the dark matter in the Universe and neutrino oscillations, is  $\mathcal{N} = 3$ . Only one sterile neutrino can be the dark matter. Moreover, it practically decouples and does not contribute to active neutrino masses. Also, the absolute neutrino mass scale is fixed: the mass of the lightest active neutrino is bounded from above by  $m_1 \leq 2 \cdot 10^{-3}$  eV. This leads to the following values of the masses of other active neutrinos:  $m_2 = [9.05_{-0.1}^{+0.2}] \cdot 10^{-3}$  eV  $\simeq \sqrt{\Delta m_{solar}^2}$ ,  $m_3 = [4.8_{-0.5}^{+0.6}] \cdot 10^{-2}$  eV  $\simeq \sqrt{\Delta m_{atm}^2}$  (normal hierarchy), or  $m_{2,3} = [4.7_{-0.5}^{+0.6}] \cdot 10^{-2}$  eV (inverted hierarchy). Yet another prediction is the effective Majorana mass  $m_{\beta\beta}$  for neutrinoless double  $\beta$  decay [35]:  $1.3$  meV  $< m_{\beta\beta} < 3.4$  meV (normal hierarchy) and  $13$  meV  $< m_{\beta\beta} < 50$  meV (inverted hierarchy). Moreover, knowing  $m_{\beta\beta}$  experimentally will allow to fix Majorana CP-violating phases in neutrino mass matrix, provided  $\theta_{13}$  and Dirac phase  $\delta$  are known.

The strategy for search of DM sterile neutrino was discussed in a number of papers, for a review see [25]. In short, one should use the X-ray telescopes (such as Chandra and XMM Newton) to look for a narrow  $\gamma$  line against astrophysical background. The astrophysical objects leading to the best signal to background ratio are the dwarf satellite galaxies and the Milky Way. So, the answer to John Womersley question: “Do neutrinos play a role in dark matter, especially if there is no light neutralino?” is: “Yes, sterile neutrino with mass from 1 keV to 50 keV is an excellent warm or cold DM candidate. Search for it with X-ray telescopes in space!”

**3.3.2. Baryon asymmetry** In addition to DM sterile neutrino the  $\nu$ MSM contains a pair of more heavier singlet fermions,  $N_2$  and  $N_3$ . The parameters of these particles can be constrained from the following conditions:

(i) BAU generation via singlet fermion oscillations [36, 23] requires out of equilibrium: mixing



**Figure 3.** Constraints on  $U^2$  coming from the baryon asymmetry of the Universe (solid lines), from the see-saw formula (dotted line) and from the big bang nucleosynthesis (dotted line) [37]. Experimental searched regions are in red - dashed lines. *Left:* normal hierarchy. *Right:* inverted hierarchy.

angle of  $N_{2,3}$  to active neutrinos cannot be too large. In addition, due to the smallness of the Yukawa couplings, the asymmetry generation must have a resonant character, leading to the requirement that  $N_{2,3}$  must be almost degenerate.

- (ii) Neutrino masses: mixing angle of  $N_{2,3}$  to active neutrinos cannot be too small.
- (iii) BBN: decays of  $N_{2,3}$  must not spoil Big Bang Nucleosynthesis.
- (iv) Experiment:  $N_{2,3}$  have not been seen yet.

The summary of these constraints is presented in Fig. 3 where the mixing angle  $U^2$  is defined in full analogy with (5).

The answer to Sachio Komamiya question: “ $\theta_{13}$  and  $\delta$  are similar to  $V_{ub}$  and CP phase in the quark sector. In heavy flavours, angles are not really the interesting thing - CP violation there is found to be insufficient to generate baryogenesis. How about neutrinos? We know about lepto-genesis, is there further fundamental physics in neutrinos beyond the numerical values of the angle and phase?” is: “The knowledge of  $\theta_{13}$  and  $\delta$  is necessary, but not sufficient to find theoretically baryon asymmetry of the Universe. It is not zero even if  $\theta_{13} = 0$  or  $\delta = 0$ . To compute it, one *must* find  $N$ 's and determine their properties experimentally”. The ways how this can be done is discussed below.

**3.3.3. Experimental searches of  $N_{2,3}$**  Several distinct strategies can be used for the experimental search of these particles [38]. The first one is related to their production ( $U^2$  effect). The singlet fermions participate in all the reactions the ordinary neutrinos do with a probability suppressed roughly by a factor  $U^2$ . Since they are massive, the kinematics of, say, two body decays  $K^\pm \rightarrow \mu^\pm N$ ,  $K^\pm \rightarrow e^\pm N$  or three-body decays  $K_{L,S} \rightarrow \pi^\pm + e^\mp + N_{2,3}$  changes when  $N_{2,3}$  is replaced by an ordinary neutrino. Therefore, the study of *kinematics* of rare  $K$ ,  $D$ , and  $B$  meson decays can constrain the strength of the coupling of heavy leptons. This strategy has been used in a number of experiments for the search of neutral leptons in the past [39, 40], where the spectrum of electrons or muons originating in decays  $\pi$  and  $K$  mesons has been studied. The precise study of kinematics of rare meson decays is possible in  $\Phi$  (like KLOE), charm, and B factories, or in experiments with kaons where their initial 4-momentum is well known (like NA48 or E787 experiments).

The second strategy is to use the proton beam dump ( $U^4$  effect). As a first step, the proton beam heating the fixed target creates  $K$ ,  $D$  or  $B$  mesons, which decay and produce  $N_{2,3}$ . The

	N mass	$\nu$ masses	eV $\nu$ anomalies	BAU	DM	$M_H$ stability	direct search	experiment
GUT see-saw	$10^{-16}$ – 10 GeV	YES	NO	YES	NO	NO	NO	–
EWSB	$10^{2-3}$ GeV	YES	NO	YES	NO	YES	YES	LHC
$\nu$ MSM	keV – GeV	YES	NO	YES	YES	YES	YES	a'la CHARM
$\nu$ scale	eV	YES	YES	NO	NO	YES	YES	a'la LSND

**Figure 4.** This table shows whether the corresponding choice of the mass for Majorana fermions may explain neutrino masses and oscillations, accommodate eV neutrino anomalies, lead to baryogenesis, provide the dark matter candidate, ensure the stability of the Higgs mass against radiative corrections, and be directly searched at some experiments.

second step is a search for decays of  $N$  in a near detector, looking for the processes “nothing”  $\rightarrow$  leptons and hadrons [41, 42, 43, 44]. To this end, quite a number of already existing or planned neutrino facilities (related, e.g., to CERN SPS, MiniBooNE, MINOS or J-PARC), complemented by a near *dedicated* detector, can be used. Finally, these two strategies can be unified, so that the production and the decay occurs inside the same detector [45].

For the mass interval  $M_N < M_K$ , both strategies can be used. According to the estimates, an upgrade of NA62 experiment at CERN would allow the finding or exclusion of singlet fermions with the mass below that of the kaon. If  $m_K < M_{2,3} < m_D$ , the search for the missing energy signal, potentially possible at beauty, charm, and  $\tau$  factories, is unlikely to gain the necessary statistics. Thus, the search for decays of neutral fermions is the most effective opportunity. The dedicated experiments on the basis of the SPS proton beam at CERN can touch a very interesting parameter range for  $M_N < 1.8$  GeV. The sensitivity is proportional to total delivered protons on target (PoT); for  $2.5 \times 10^{20}$  PoT the constraints shown in Fig. 3 can be improved by one order of magnitude (without accounting for improvement of experimental technique). An upgrade of the LHCb experiment, allowing to use the combination of two strategies, could potentially enter in a cosmologically interesting region for masses and mixing angles of singlet fermions. Going above  $D$ -meson but still below  $B$ -meson thresholds is very hard if not impossible with the present or planned proton machines or B-factories. To enter into a cosmologically interesting parameter space would require the increase in the present intensity of, say, CERN SPS beam by two orders of magnitude or to produce and study the kinematics of more than  $10^{10}$  B-mesons.

### 3.4. eV scale: “canonical” sterile neutrinos

The key *assumption* of the “eV see-saw” is that the new scale coincides with that found in neutrino oscillations. This leads to existence of eV sterile neutrinos which can be used to explain a number of anomalies (LSND, MiniBOONE) indicating that the three neutrino oscillation paradigm may not be complete. This possibility has been discussed in detail at this conference (for theory and phenomenology see [46, 47, 5, 48, 49], while experimental situation was discussed in [50, 51]). I would like just to add that the theory (3) with the eV mass for sterile neutrinos cannot explain baryon asymmetry of the Universe and does not provide a dark matter candidate, so that an extra extension of the theory (3) is required.

## 4. Conclusions

Even the simplest extension of the Standard Model by three Majorana fermions leads to rich and interesting physics. Independently on the masses of new leptons, they can explain oscillations of

active neutrinos. For some ranges of parameters, they can provide the Dark Matter candidate, lead to baryogenesis, and allow for direct experimental search. The summary of different possibilities is given in Fig. 4. Hopefully, future experiments in neutrino and flavour physics will allow to clarify what is the physics beyond the three flavour oscillation.

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