Evolution equation for elastic scattering of hadrons

Anderson Kohara

In collaboration with Hiren Kakkad and Piotr Kotko

Faculty of Physics, AGH-University of Science and Technology

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Diffraction and Low-x

Iceland Liechtenstein Norway grants

Hadronic Collider Experiments

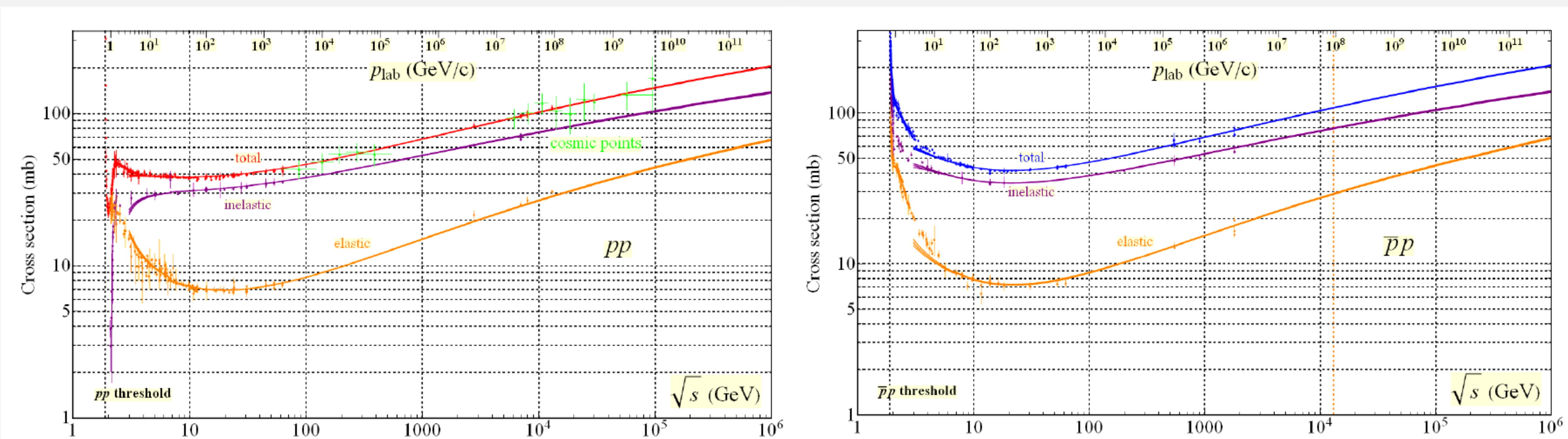
Intersecting Storage Rings-CERN, 1971–1984

Proton-Antiproton Collider(SPS)-CERN, 1981–1991

Tevatron-Fermilab, 1987–2011

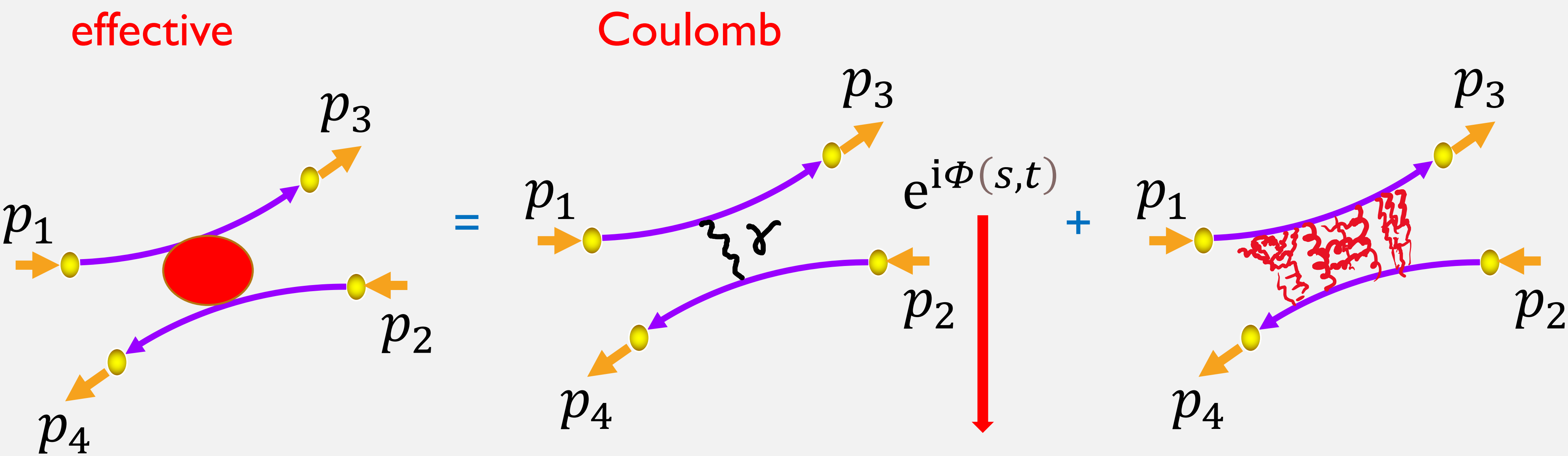
Relativistic Heavy Ion Collider-BNL, 2000–...

Large Hadron Collider-CERN, 2009–...



P.A. Zyla et al. (Particle Data Group), *Prog. Theor. Exp. Phys.* 2020, 083C01 (2020)

Relativistic Elastic Scattering



Coulomb phase

L.D. Solov'ev, *JETP* **22**, 205 (1966) 26;
H. Bethe, *Ann. Phys. (N.Y.)* **3**, 190 (1958) 27;
G.B. West, D.R. Yennie, *Phys. Rev.* **172**(5), 1413 (1968);
V. Kundrat and M. Lokajcek, *Phys. Lett. B* **611** (2005) 102 ;
R. Cahn, *Z. Phys. C* **15** (1982) 253.

Mandelstam variables

$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

Basic physical quantities we are interested in:

Forward quantities

Optical theorem $$\sigma_{tot} = 4\pi(\hbar c)^2 T_I^N(s,0)$$

Ratio of real and imaginary amplitudes $$\rho = \frac{T_R^N(S,0)}{T_I^N(s,0)}$$

And the slopes
$$B_I = \frac{2}{T_I^N(s,t)} \frac{\mathrm{d}}{\mathrm{d}t} T_I^N(s,t)\Big|_{t=0}$$

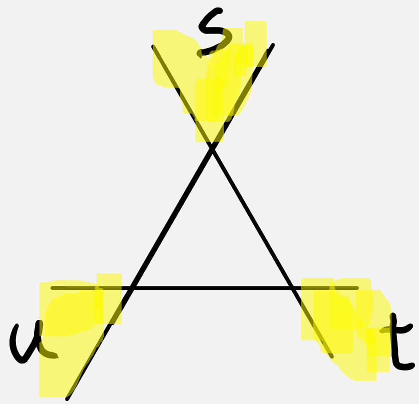
$$B_R = \frac{2}{T_R^N(s,t)} \frac{\mathrm{d}}{\mathrm{d}t} T_R^N(s,t)\Big|_{t=0}$$

$$B = \frac{\rho^2 B_R + B_I}{\rho^2 + 1}$$

Differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = |T(s,t)|^2$$

Assumptions



Analytic nuclear amplitude $A(s,t,u)$

Singularities have a physical meaning

Crossing symmetric amplitudes $A_{pp}(s,t,u) = A_{p\bar{p}}(u,t,s)$

Unitarity of S matrix $ss^{\dagger} = 1$

Theorems

Optical theorem $\sigma_T = \frac{1}{2|p|\sqrt{s}} \operatorname{Im} A(s,t)$

Froissart theorem/bound $\sigma_T(s) \leq C \log^2\left(\frac{s}{s_0}\right) \qquad s \to \infty$

Pomeranchuck theorem $\frac{\sigma_T^{pp}(s)}{\sigma_T^{p\bar{p}}(s)} \to 1 \qquad s \to \infty$

b-space (geometric space)

Physical cross sections are written

$$\sigma_{el}(s) = \int \mathrm{d}^2\vec{b}\,\left|\tilde{T}(s,\vec{b})\right|^2 = \int \mathrm{d}^2\vec{b}\,\frac{\mathrm{d}\tilde{\sigma}_{el}}{\mathrm{d}^2\vec{b}}(s,\vec{b})$$

$$\sigma_{tot}(s) = 2\int \mathrm{d}^2\vec{b}\,\tilde{T}_I(s,\vec{b}) = \int \mathrm{d}^2\vec{b}\,\frac{\mathrm{d}\tilde{\sigma}_{tot}}{\mathrm{d}^2\vec{b}}(s,\vec{b})$$

$$\sigma_{inel}(s) = \int \mathrm{d}^2\vec{b}\,G_{inel}(s,\vec{b}) = \int \mathrm{d}^2\vec{b}\,\frac{\mathrm{d}\tilde{\sigma}_{inel}}{\mathrm{d}^2\vec{b}}(s,\vec{b})$$

Unitarity constraint in b space

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel}$$

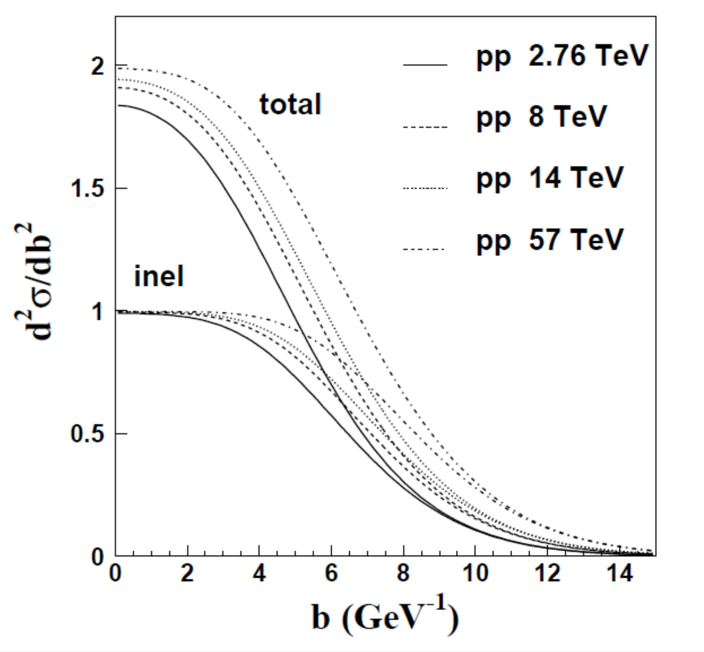
Assuming independent $\mathrm{d}^2\vec{b}$ increments

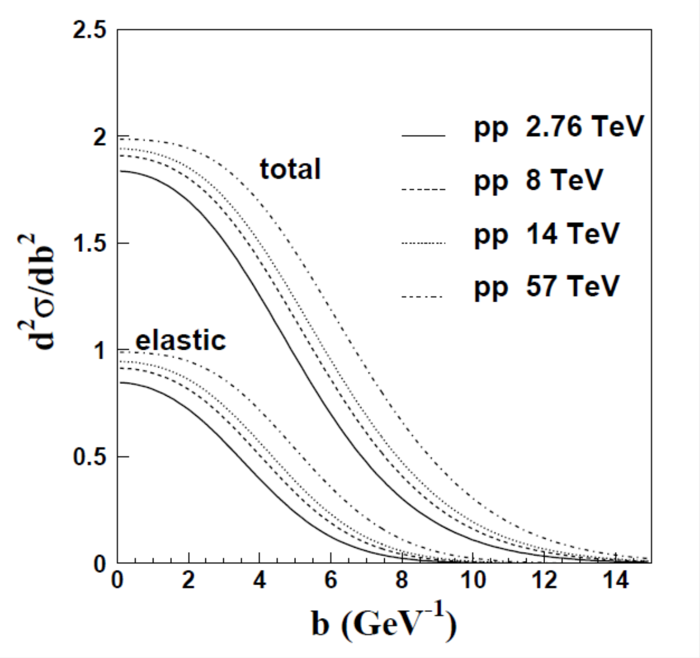
$$2\,\tilde{T}_I(s,b) = \left|\tilde{T}(s,b)\right|^2 + G_{inel}(s,b)$$

$$G_{inel}(s,b) \approx \tilde{T}_I(s,b)[2 - \tilde{T}_I(s,b)] - \cancelto{0}{\tilde{T}_R(s,b)^2}$$

$$G_{inel}(s,b) \leq 1$$

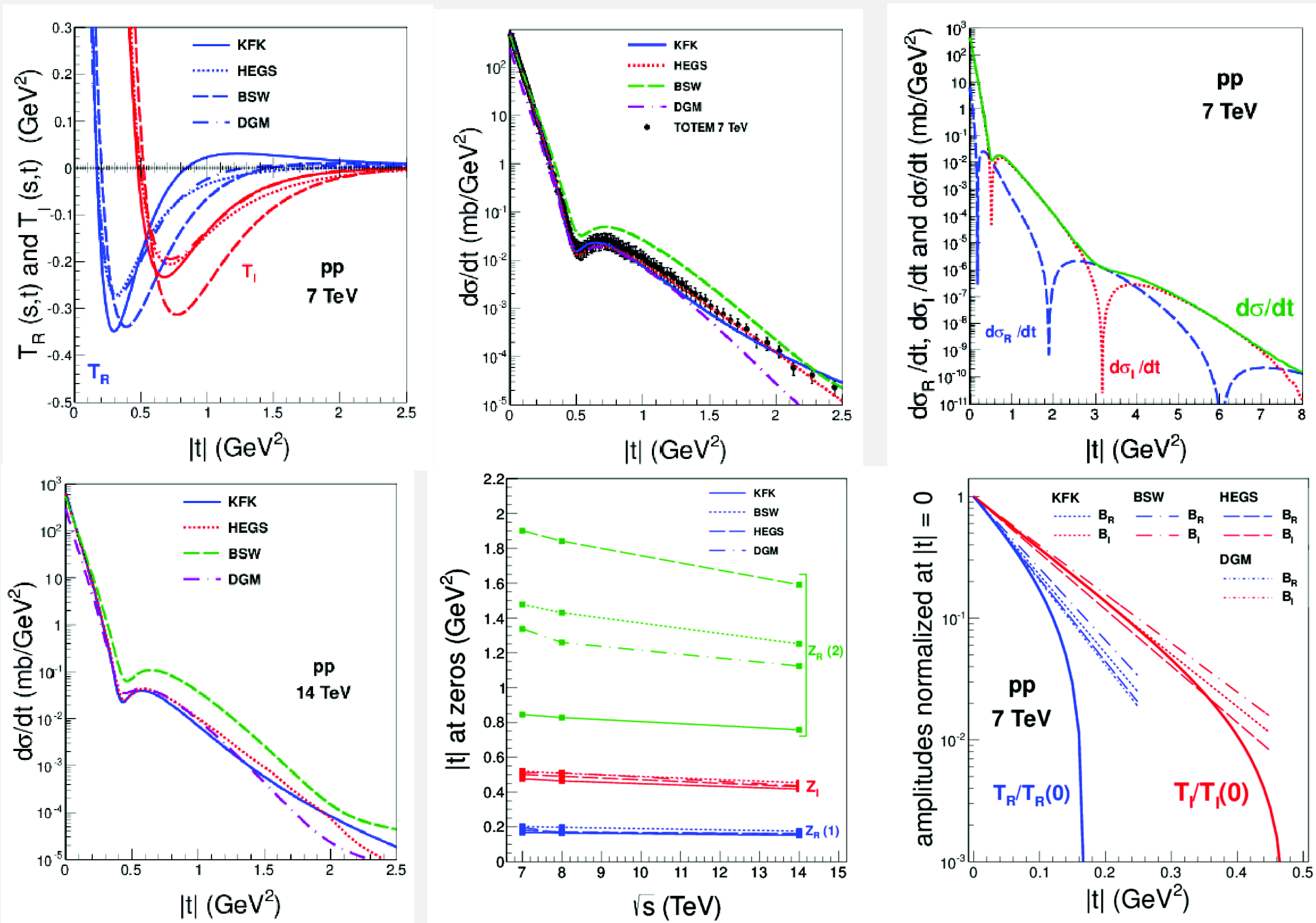
Monotonic results for elastic differential 'cross sections’ profile functions





- Interesting diffusive behaviour with increasing energy
- Unitarity bound saturation

Different models: amplitudes, differential cross sections, zeros, slopes



*E. Ferreira, T. Kodama, A. K. K., D. Szilard, Acta Phys.Polon.Supp. **8** (2015) 1017*

Regge Field Theory

Reggeons propagate in two space dimensions $\vec{x}$ and imaginary time τ

The action for a free Pomeron field is written

$$A_0 = \int d^2\vec{x}\ \mathrm{d}\tau\, \mathcal{L}_0(\vec{x},\tau)$$

with the free Lagrangian

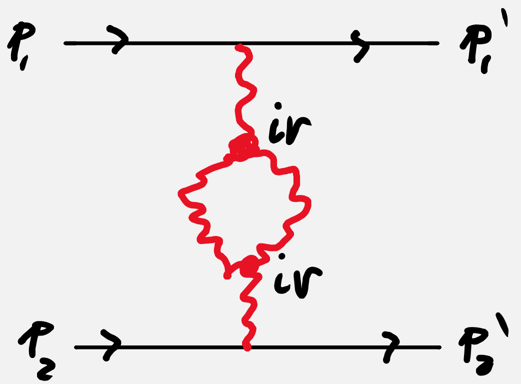
$$\mathcal{L}_0(\vec{x},\tau) = \frac{1}{2}\varphi^+(\vec{x},\tau)\frac{\overleftrightarrow{\partial}}{\partial\tau}\varphi(\vec{x},\tau) - \alpha_0'\nabla\varphi^+(\vec{x},\tau)\cdot\nabla\varphi(\vec{x},\tau) - \varepsilon_0\varphi^+(\vec{x},\tau)\varphi(\vec{x},\tau)$$

The interecting part is written in terms of triple Pomeron coupling

$$\mathcal{L}_I = -i\lambda[\varphi^+(\vec{x},\tau)\varphi^+(\vec{x},\tau)\varphi(\vec{x},\tau) + \varphi^+(\vec{x},\tau)\varphi(\vec{x},\tau)\varphi(\vec{x},\tau)]$$

H. D. Abarbanel, J. D. Bronzan, R. L. Sugar, and A. R. White, Physics Reports 21, 119 (1975).

In RFT the imaginary time τ is the rapidity $\tau = \ln(s)$



Typical graph contributing to the amplitude

To avoid the imaginary term one can transform the Gribov fields such as $q = i\varphi^{+}$ and $p = i\varphi$

$$\mathcal{L} = \frac{1}{2} q \overleftrightarrow{\partial_\tau} p + \alpha' \nabla_b q \cdot \nabla_b p - \varepsilon_0 q p + \lambda q (p+q) p$$

The Hamiltonian is given

$$H = \int d^2 b \left[-\alpha' \nabla_b q(b) \cdot \nabla_b p(b) + \varepsilon_0 q(b) p(b) - \lambda q(b)[p(b) + q(b)] p(b) \right]$$

Where *q* and *p* are creation and anihillation operators respectively satisfiying the comutation relation

$$[p(b,\tau), q(b',\tau)] = -\delta^{(2)}(b - b')$$

In discretized two-dimensional *b*-space lattice it was shown that for $\varepsilon_0 > 0$ the zero-energy ground state $|\phi_O >$ acquires a non-zero energy state $|\phi_1 >$ which approaches a coherent state

$$|\phi_1 >= e^{-\frac{\varepsilon_0}{\lambda} \int q(b,0)\, \mathrm{d}b} |\phi_0 > \qquad \text{such that} \qquad p(b,0)|\phi_1 >= \frac{\varepsilon_0}{\lambda} |\phi_1 >$$

A generalized state is written as

$$|\psi(\tau) >= e^{-\hat{A}(\tau)} |\phi_O >$$

With the operator

n+1 correlation functions

$$\hat{A}(\tau) = \sum_{n=1}^{\infty} \frac{1}{n!} \int \mathrm{d}^2 \overrightarrow{b_1} \ldots d^2 \overrightarrow{b_n}\; q(\overrightarrow{b_1}) \cdots q(\overrightarrow{b_n})\, G_n(\tau, \bar{b}_1, \ldots, \vec{b}_n)$$

V. Alessandrini, D. Amati, and M. Ciafaloni, Nucl. Phys. B 130, 429 (1977)

Expanding $|\psi> = e^{-\hat{A}(\tau)}|\phi_0> = (1 - \hat{A} + \frac{1}{2!}\hat{A}^2 + \cdots)\ |\phi_0>$

And solving the Schrodinger equation $\partial_\tau|\psi> = -H|\psi>$

By colecting powers of q such as $\int d^2b\, q(b), \int d^2b\, q(b)q(b)$ ...

One arrive to as set of coupled differential equations

$$\partial_\tau G_1(b,\tau) = (\varepsilon_0 + \alpha'\nabla_b^2)G_1(b,\tau) - \lambda G_1^2(b,\tau) - \lambda G_2(b,b,\tau)$$

$$\partial_\tau G_2(b,b',\tau) = (\varepsilon_0 + \alpha'\nabla_b^2)G_2(b,b',\tau) + (\varepsilon_0 + \alpha'\nabla_{b'}^2)G_2(b,b',\tau) - 2\lambda\,\delta^2(b-b')G_1(b,\tau)$$
$$-2\lambda[G_1(b,\tau) + G_1(b',\tau)]G_2(b,b',\tau) - 2\lambda G_3(b,b,b',\tau)$$

.

.

In the semiclassical approximation $\partial_\tau G_1(b,\tau) = (\varepsilon_0 + \alpha'\nabla_b^2)G_1(b,\tau) - \lambda G_1^2(b,\tau)$

Assumption of our approach:

2-POINT CORRELATION FUNCTION $\propto$ ELASTIC SCATTERING AMPLITUDE

$$G_1(b,\tau) \propto i\tilde{T}(b,\tau)$$

We found similar ideias *R. Peschanski, Phys. Rev. D 79, 105014 (2009).*

The author explore the analytical properties of the imaginary amplitude with a noise term

We are focused in the complex equation $T(s,t) = T_R(s,t) + \mathrm{i}T_I(s,t)$

In b-space $\tilde{T}(s,\vec{b}) = \int \mathrm{d}^2\vec{q}\, \mathrm{e}^{-\mathrm{i}\vec{b}\cdot\vec{q}}\, T(s,-q^2)$

$$\partial_\tau(i\tilde{T}(b,\tau)) = \left(\varepsilon_0 + \alpha'\nabla_b^2\right)(i\tilde{T}(b,\tau)) - \lambda\left(i\tilde{T}_I(b,\tau)\right)^2$$

The imaginary and real parts respectively are *K. Kakkad, A. K. K. and P. Kotko, Eur. Phys. J. C* 82 (2022) 9, 830

$$\frac{\partial\tilde{T}_I}{\partial\tau} = \alpha'\frac{\partial^2\tilde{T}_I}{\partial b^2} + \varepsilon_0\left[\tilde{T}_I\left(1 - \frac{\lambda}{\varepsilon_0}\tilde{T}_I\right) + \frac{\lambda}{\varepsilon_0}\tilde{T}_R^{\,2}\right]$$

$$\frac{\partial\tilde{T}_R}{\partial\tau} = \alpha'\frac{\partial^2\tilde{T}_R}{\partial b^2} + \varepsilon_0\tilde{T}_R\left(1 - 2\frac{\lambda}{\varepsilon_0}\tilde{T}_I\right)$$

EVOLUTION EQUATION FOR COMPLEX AMPLITUDES

They look like BK type equation:

S. Munier and R. B. Peschanski, Phys. Rev. Lett. 91, 232001 (2003)

Y. V. Kovchegov, L. Szymanowski, and S. Wallon, Phys. Lett. B 586, 267 (2004),

To evolve the evolution equation for amplitudes we use as initial conditions two different models with analytical forms in b-space: KFK and BSW

A. K. K., E. Ferreira, and T. Kodama, Eur. Phys. J. C 74, 3175 (2014)

C. Bourrely, J. Soffer, and T. T. Wu, Phys. Rev. D 19, 3249 (1979)

Our equation is aimed at high energies, so we start at $\tau = \ln\frac{500GeV}{1GeV}$

Initial conditions starting at $\sqrt{s_{fix}} = 500\ GeV$

$$\tilde{T}_R(s_{fix}, b) = \tilde{T}_R^{KFK}(s_{fix}, b) \qquad \tilde{T}_R(s_{fix}, b) = \tilde{T}_R^{BSW}(s_{fix}, b)$$

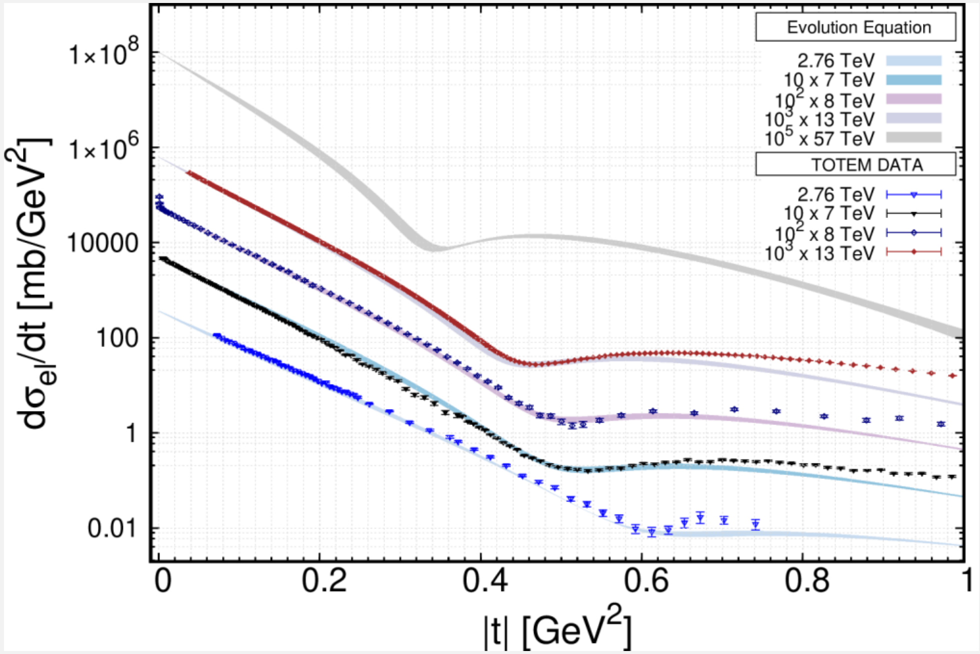
$$\tilde{T}_I(s_{fix}, b) = \tilde{T}_I^{KFK}(s_{fix}, b) \qquad \tilde{T}_I(s_{fix}, b) = \tilde{T}_I^{BSW}(s_{fix}, b)$$

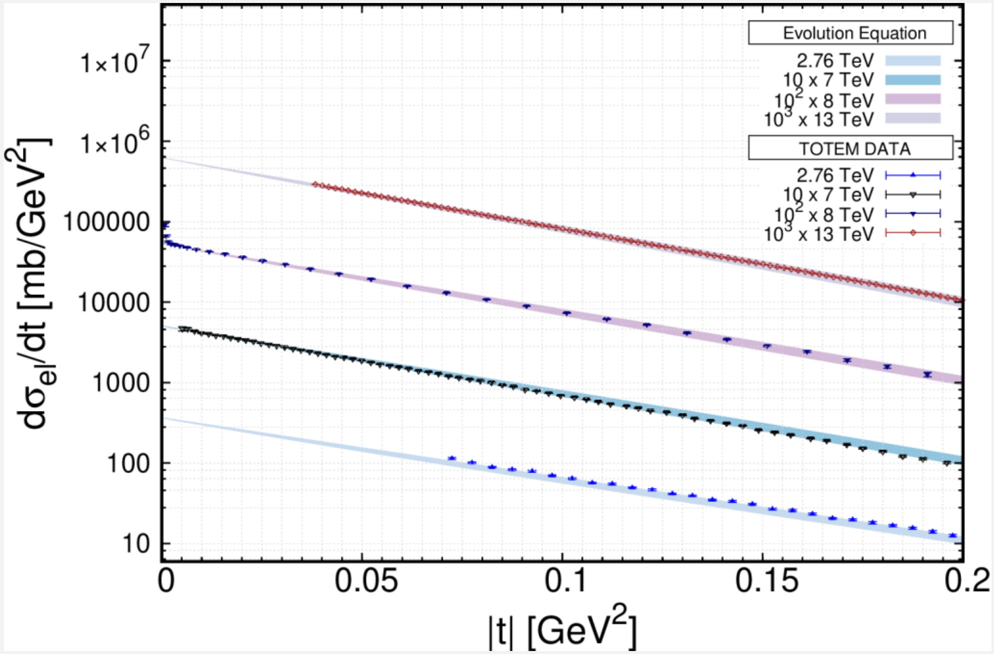
Boundary conditions set $\tilde{T}_I(s, b \to \infty) = \tilde{T}_R(s, b \to \infty) = 0$

The obtained parameters are

	α' (GeV^{-2})	ε_0	λ/ε_0
KFK	0.105	0.129	0.712
BSW	0.090	0.140	0.820

Our predictions for differential cross section

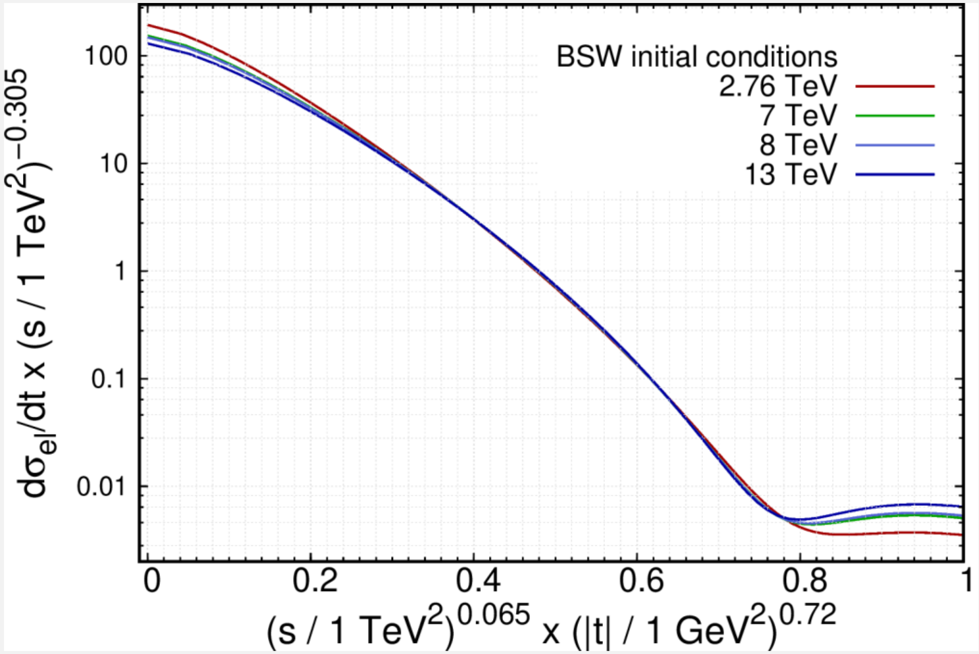


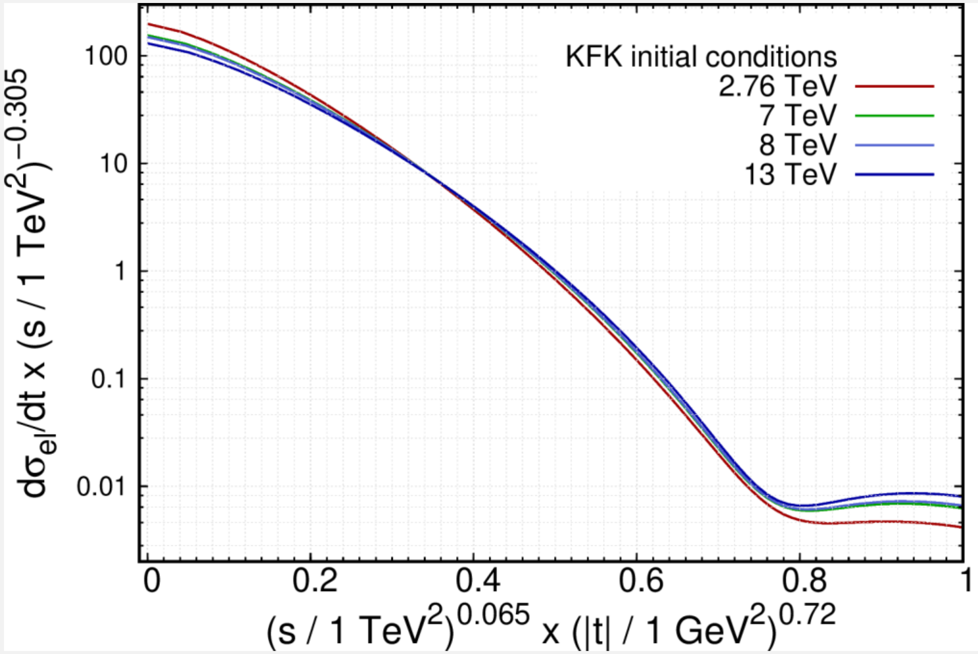


Differential cross section ‘acidental’ scaling behaviour among LHC energies

C. Baldenegro, C. Royon and A. M. Stasto, Phys. Lett. B 830, 137141 *(2022)*

Our results with different initial conditions





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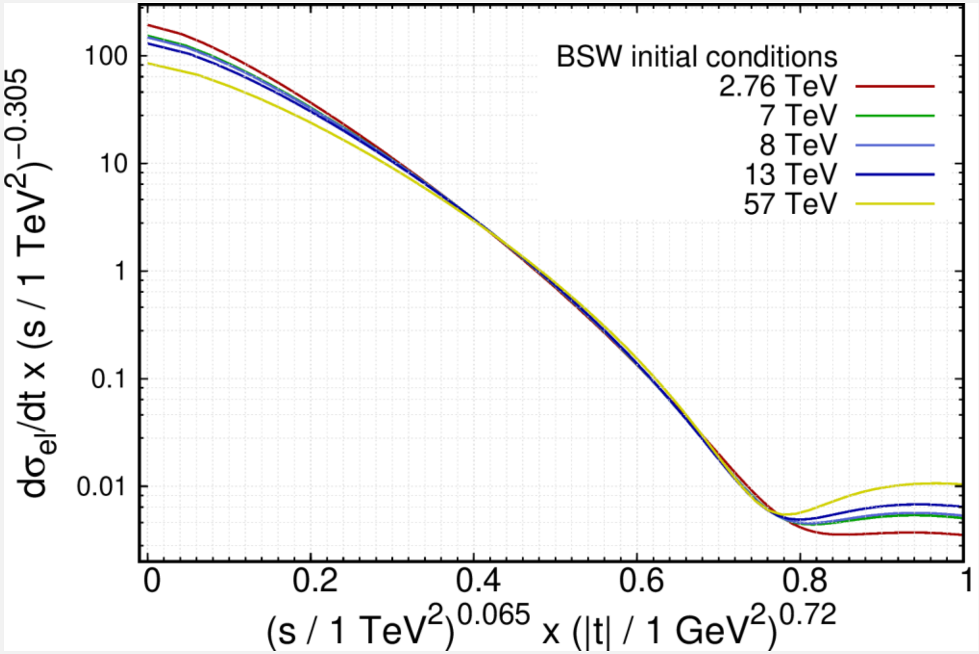
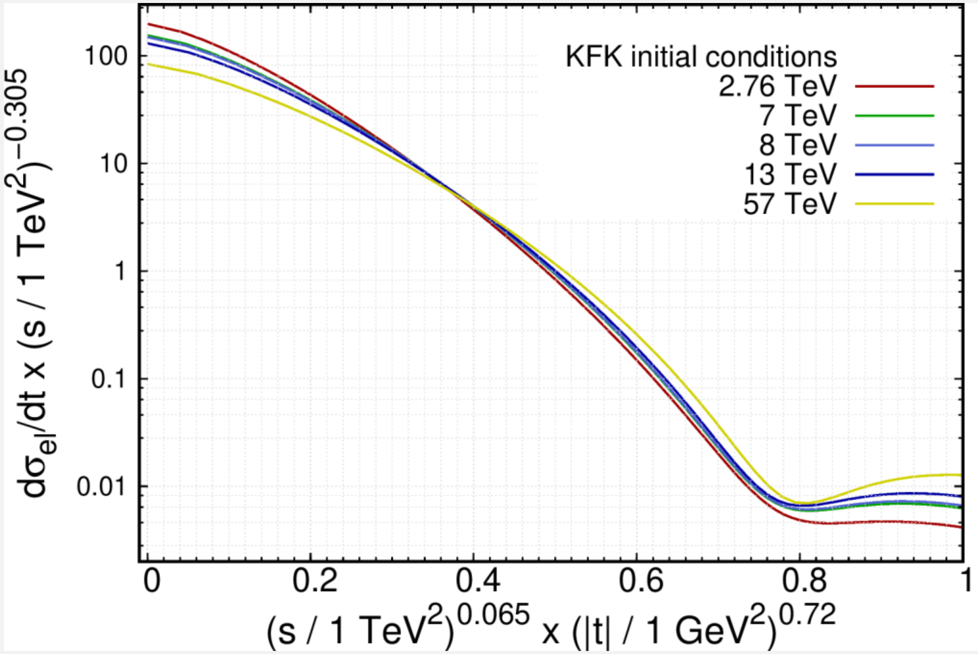
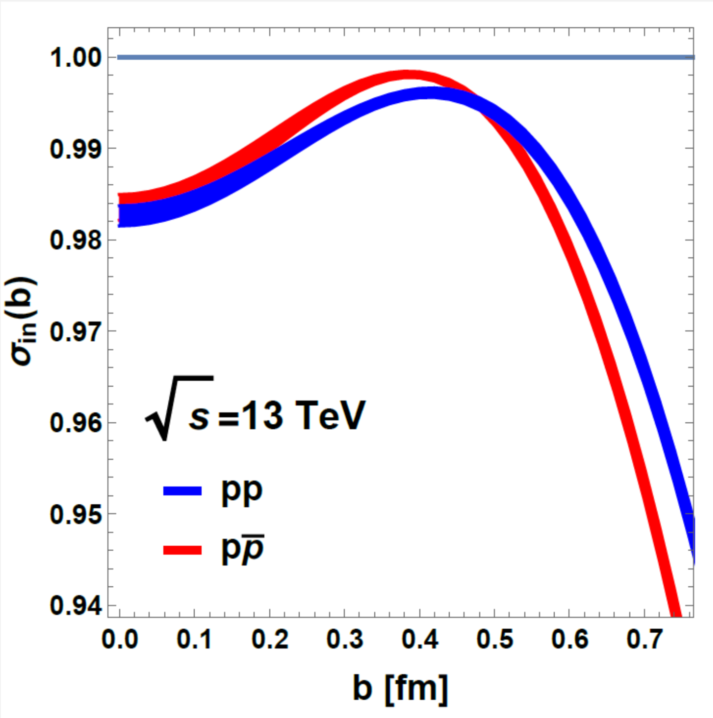
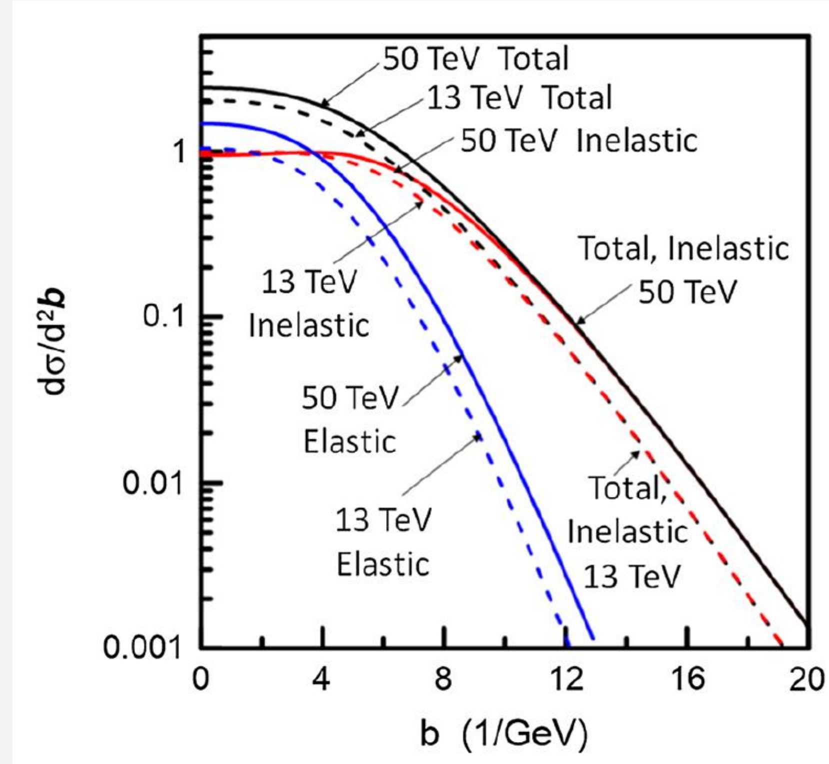




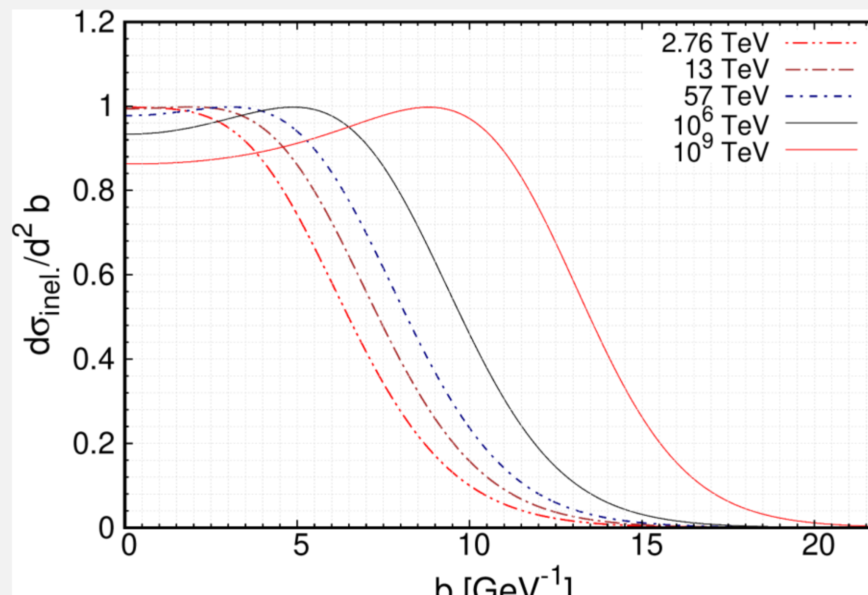
Something is going on at 13 TeV



W. Broniowski, L. Jenkovszky, E. R. Arriola, I. Szanyi, *Phys. Rev. D* **98**, 074012 (2018); E. R. Arriola, W. Broniowski, *Few Body Syst.* **57** (2016) 7, 485-490; *Phys. Rev. D* **95** (2017) 7, 074030

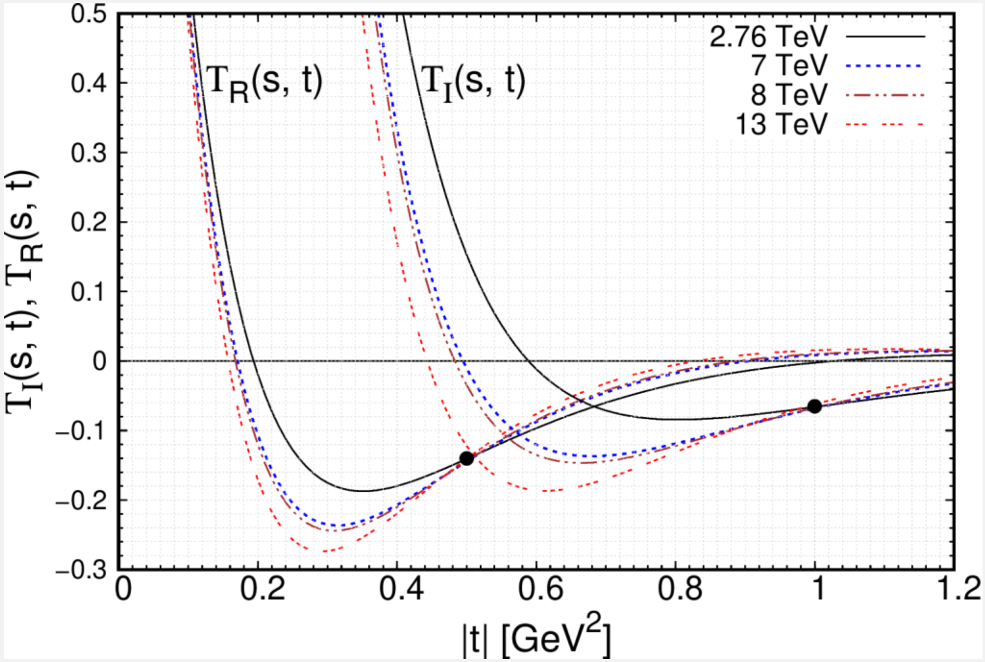


E. Ferreira, A. K. K. and T. Kodama; *Eur. Phys. J. C* **81** (2021) 4, 290



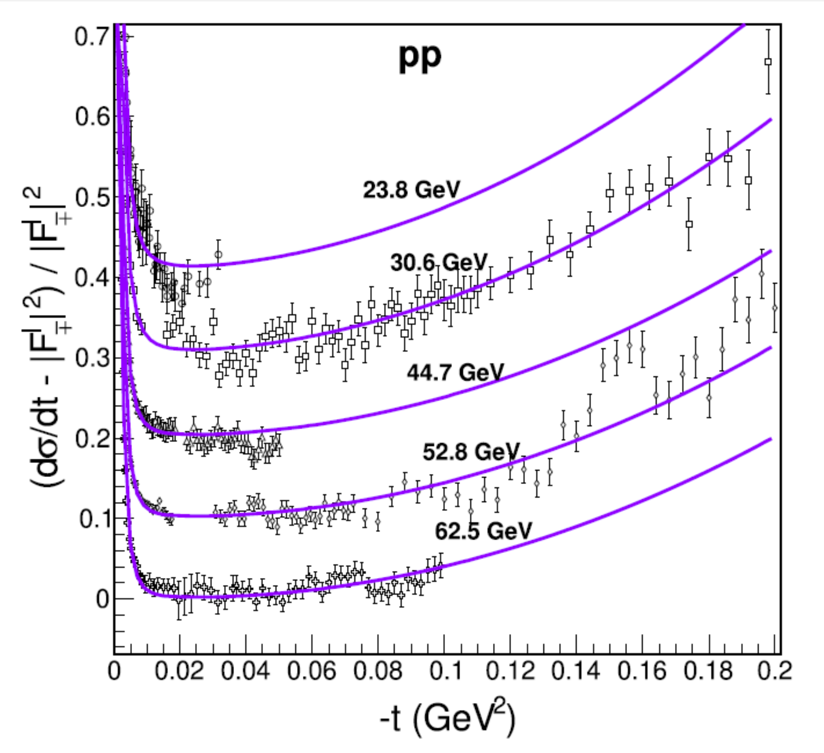
For LHC energies we also observe the ‘halo’ effect

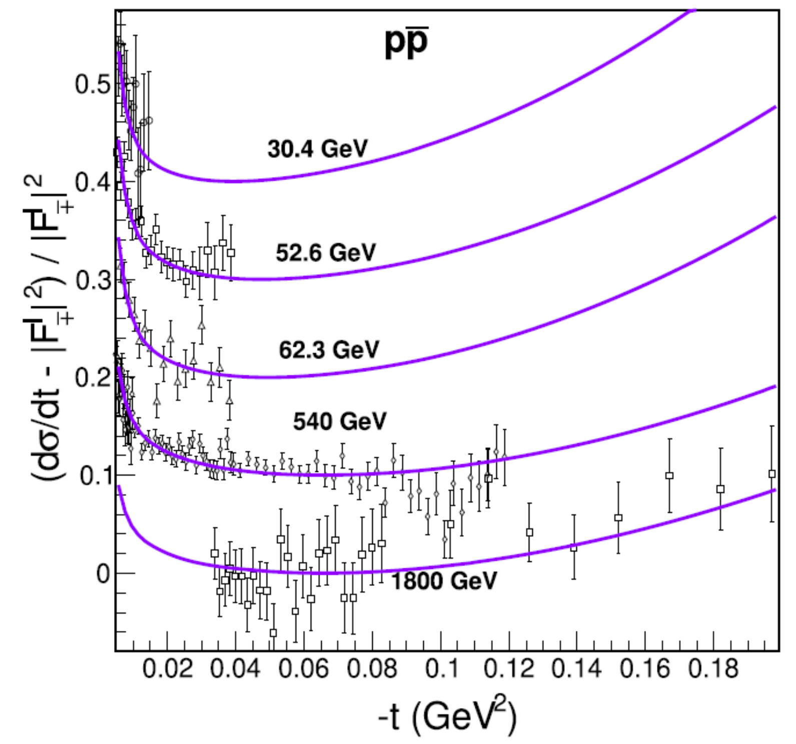
The complex amplitudes in t-space are

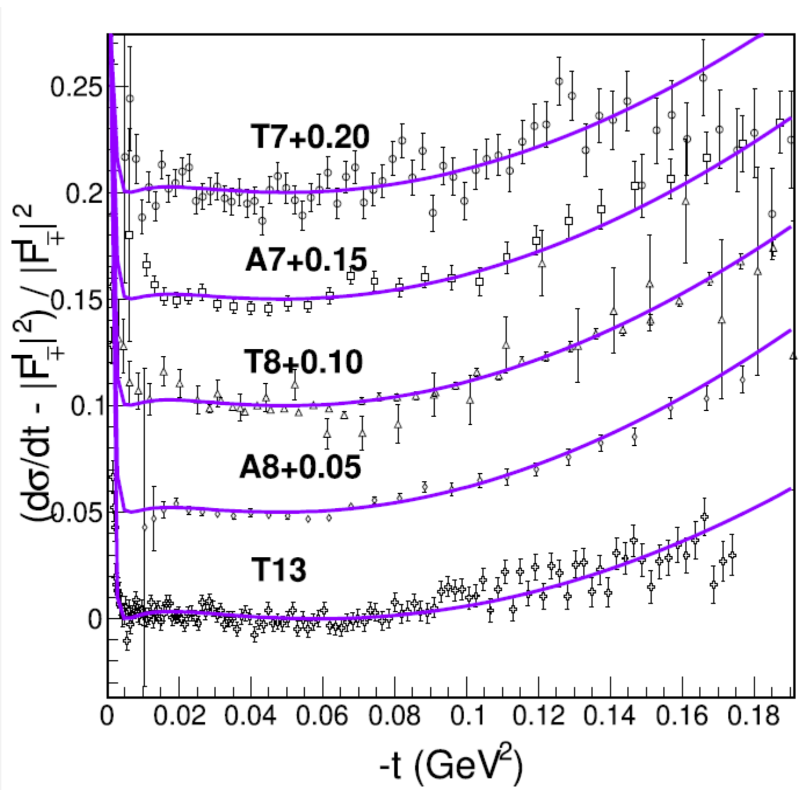


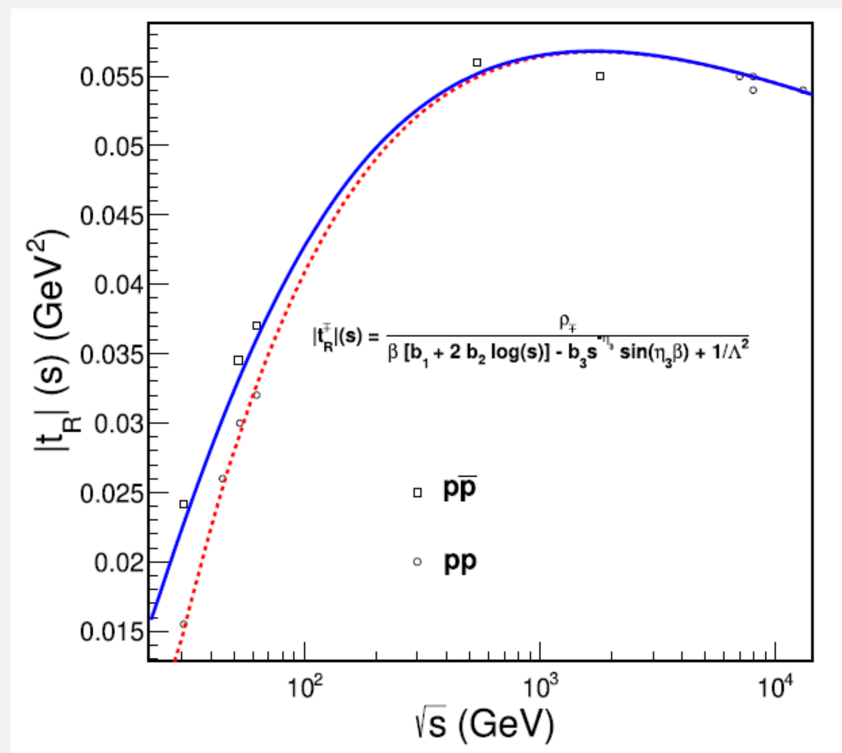
We also observe estationary points in t-space. This was observed by Csorgo et. al

T. Csorgo and I. Szanyi, in International Scientific Days - Femtoscopy Session (2022) https://indico.cern.ch/event/1152630/.


pp
23.8 GeV
30.6 GeV
44.7 GeV
52.8 GeV
62.5 GeV
(dσ/dt - |F^I_∓|^2) / |F^I_∓|^2
-t (GeV^2)

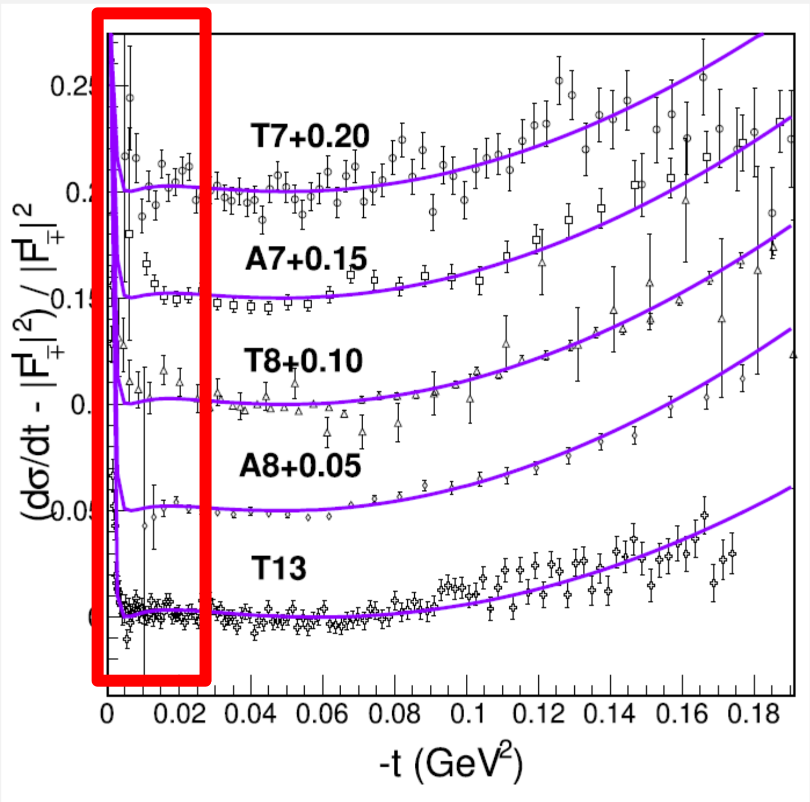

pp̄
30.4 GeV
52.6 GeV
62.3 GeV
540 GeV
1800 GeV
(dσ/dt - |F^I_∓|^2) / |F^I_∓|^2
-t (GeV^2)


T7+0.20
A7+0.15
T8+0.10
A8+0.05
T13
(dσ/dt - |F^I_∓|^2) / |F^I_∓|^2
-t (GeV^2)


|t_R^∓|(s) = ρ_∓ / (β [b_1 + 2 b_2 log(s)] - b_3 s^{-η_3} sin(η_3β) + 1/Λ^2)
pp̄
pp
|t_R| (s) (GeV^2)
√s (GeV)

Is it possible to observe any dip due to the interplay between real and Coulomb amplitude?

$$T_R(s,t) + T_C(t) = 0$$



The integrated quantities are

	$\sqrt{s}$ [TeV]	σ_{tot} [mb]	ρ	B [GeV^{-2}]
KFK initial condition	2.76	84.31	0.123	17.28
	7	99.07	0.117	18.47
	8	101.32	0.116	18.65
	13	109.78	0.113	19.32
	57	138.32	0.105	21.55
BSW initial condition	2.76	83.14	0.143	19.69
	7	98.40	0.134	21.12
	8	100.73	0.132	21.34
	13	109.52	0.127	22.15
	57	142.30	0.115	24.87
TOTEM	2.76	84.7 ± 3.3	–	17.1 ± 0.30
	7	98.0 ± 2.5	0.145 ± 0.091	19.73 ± 0.40
	8	101.7 ± 2.9	0.12 ± 0.03	19.74 ± 0.28
	13	110.6 ± 3.4	0.10 ± 0.01	20.40 ± 0.01
ATLAS	7	95.35 ± 0.38	0.14 (fix)	19.73 ± 0.14
	8	96.07 ± 0.18	0.136 (fix)	19.74 ± 0.05
AUGER	57	133 ± 29	–	–

For the future

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- Use other models as initial conditions

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- Implement the white noise term in the evolution equation

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- Use other models as initial conditions
- Implement the white noise term in the evolution equation
- Include odderon fields and higher order Pomeron couplings

Thank you!