Nuclear effects in coherent photoproduction of heavy quarkonia

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Outline

- The $\vec{b}$-$\vec{r}$ correlation between impact parameter of a collision $\vec{b}$ and dipole orientation $\vec{r}$
- The higher-twist nuclear shadowing
  $\Longleftrightarrow$ $Q\bar{Q}$ Fock state of the photon
- The leading-twist gluon shadowing
  $\Longleftrightarrow$ higher Fock $Q\bar{Q}nG$ components of the photon containing gluons
- Model predictions for $d\sigma/dt$ and $d\sigma/dy$ vs data
- Reduced effects of quantum coherence in the Balitsky-Kovchegov equation
  $\Rightarrow$ relative variance from calculations, which are frequently presented in the literature
- Summary & Outlook

$\vec{b}$-$\vec{r}$ correlation

- The dipole-nucleon electroproduction amplitude within the color dipole formalism has the following factorized form

$$\mathcal{A}^N(x,\vec{q}\,) = 2\int d^2b\, e^{i\vec{q}\cdot\vec{b}} \int d^2r \int_0^1 d\alpha\, \Psi_V^*(\vec{r},\alpha) \times \mathcal{A}^N_{\bar{Q}Q}(\vec{r},x,\alpha,\vec{b}\,)\, \Psi_{\gamma^*}(\vec{r},\alpha,Q^2)$$

- $\vec{q} \Rightarrow$ transverse component of momentum transfer
  $\alpha \Rightarrow$ fractional LF momentum carried by a heavy quark or antiquark of the $\bar{Q}Q$ Fock component of the photon, with the transverse separation $\vec{r}$. $\Longleftrightarrow$ **the lowest Fock state.**
- **Higher Fock components** contribute by default to the dipole-proton amplitude
- Considering **nuclear targets** $\Rightarrow$ these components are taken into account **separately**, due to different **coherence effects** in gluon radiation.

$\vec{b}$-$\vec{r}$ correlation

- The dipole-proton amplitude $\mathcal{A}^N_{\bar{Q}Q}(\vec{r}, x, \alpha, \vec{b}\,)$ depends on the transverse dipole size $\vec{r}$ and impact parameter of collision $\vec{b}$.
- The LF distribution functions $\Psi_{\gamma^*}(r, \alpha, Q^2)$ and $\Psi_V(r, \alpha)$ correspond to the transitions $\gamma^* \to \bar{Q}Q$ and $\bar{Q}Q \to V$, respectively.
- The essential feature of $\mathcal{A}^N_{\bar{Q}Q}(\vec{r}, x, \alpha, \vec{b}\,)$ is the $\vec{b}$-$\vec{r}$ corr.

$$\mathrm{Im}\mathcal{A}^N_{\bar{Q}Q}(\vec{r}, x, \alpha, \vec{b}\,) = \frac{\sigma_0}{8\pi\mathcal{B}(x)} \left\{ \exp\left[-\frac{[\vec{b} + \vec{r}(1-\alpha)]^2}{2\mathcal{B}(x)}\right] + \exp\left[-\frac{(\vec{b} - \vec{r}\alpha)^2}{2\mathcal{B}(x)}\right] - 2\,\exp\left[-\frac{r^2}{R_0^2(x)} - \frac{[\,\vec{b} + (1/2-\alpha)\vec{r}\,]^2}{2\mathcal{B}(x)}\right]\right\},$$

- with $\mathcal{B}(x) = B^{\bar{q}q}_{el}(x, r \to 0) - \frac{1}{8}R_0^2(x)$

[B.Z. Kopeliovich, et al. Phys.Rev. D**103**, 094027 (2021)]

- Interaction vanishes if $\vec{r} \perp \vec{b}$, reaches max.strength if $\vec{r} \parallel \vec{b}$

$\vec{b}$-$\vec{r}$ correlation

- After integration over $\vec{b}$ at $q = 0 \Rightarrow$ correct reproduction of the dipole cross section of the standard saturated form

$$\sigma_{Q\bar{Q}}(r, x) = \mathrm{Im}\mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q} = 0) =$$

$$2 \int d^2b\, \mathrm{Im}\mathcal{A}^N_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{b}) = \sigma_0\,(1 - \exp[-r^2/R_0^2(x)])$$

- In our calculations $\Rightarrow$ Golec-Biernat-Wusthoff (GBW) and Bartels-Golec-Biernat-Kowalski (BGBK) param.
- From known production amplitude $\Rightarrow$ diff. cross section

$$\frac{d\sigma^{\gamma N \to V N}(x, t = -q^2)}{dt} = \frac{1}{16\,\pi} \left| \mathcal{A}^{\gamma N \to V N}(x, \vec{q}) \right|^2$$

$$x = \frac{M_V^2 + Q^2}{s} = \frac{M_V^2 + Q^2}{W^2 + Q^2 - m_N^2}$$

$\vec{b}$-$\vec{r}$ correlation

- We have incorporated the **real part** of the $\gamma N \to V N$ amplitude via a substitution $\Rightarrow$

$$\mathrm{Im}\mathcal{A}^{N}_{\bar{Q}Q}(\vec{r},x,\alpha,\vec{b}\,) \Rightarrow \mathrm{Im}\mathcal{A}^{N}_{\bar{Q}Q}(\vec{r},x,\alpha,\vec{b}\,)\cdot\left(1-i\,\frac{\pi\,\Lambda}{2}\right)$$

with $\Lambda = \partial\ln(\,\mathrm{Im}\mathcal{A}^{N}_{\bar{Q}Q}(\vec{r},x,\alpha,\vec{b}\,)\,)/\partial\ln(1/x)$

[J.B. Bronzan, et al. Phys.Lett. B**49**, 272 (1974); J. Nemchik, et al. Z.Phys. C**75**, 71 (1997); J.R. Forshaw, et al. Phys.Rev. D**69**, 094013 (2004)]

- The **skewness correction** has been included via the following modification $\Rightarrow$

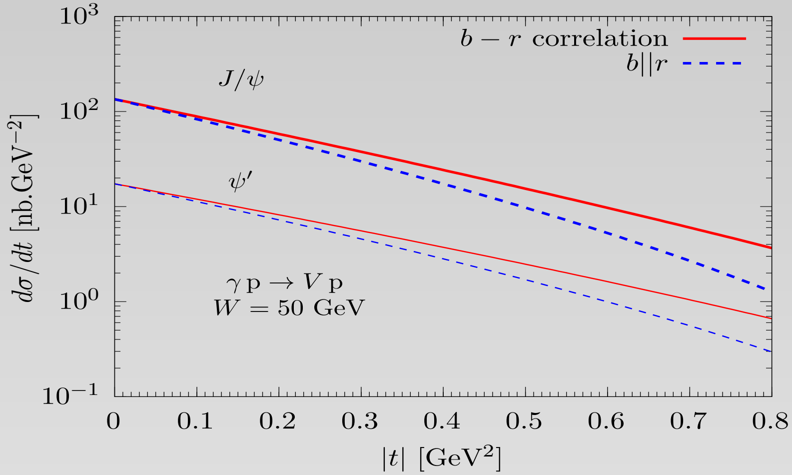
$$\mathrm{Im}\mathcal{A}^{N}_{\bar{Q}Q}(\vec{r},x,\alpha,\vec{b}\,) \Rightarrow \mathrm{Im}\mathcal{A}^{N}_{\bar{Q}Q}(\vec{r},x,\alpha,\vec{b}\,)\cdot R_S(\Lambda)$$

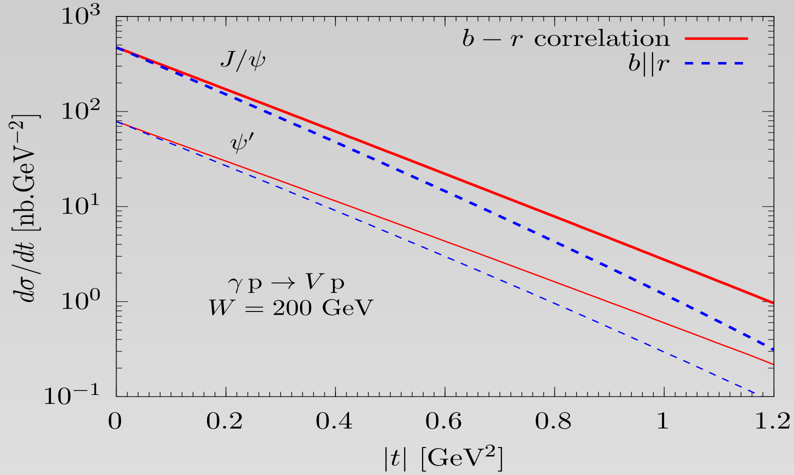
where the skewness factor

$$R_S(\Lambda) = (2^{2\Lambda+3}/\sqrt{\pi}\,)\;\cdot\Gamma(\Lambda+5/2)/\Gamma(\Lambda+4)$$

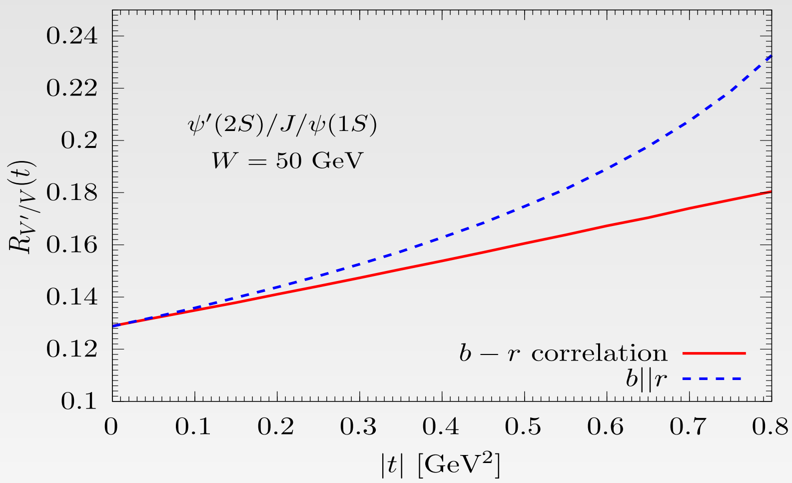
[A.G. Shuvaev et al. Phys.Rev. D**60**, 014015 (1999)]

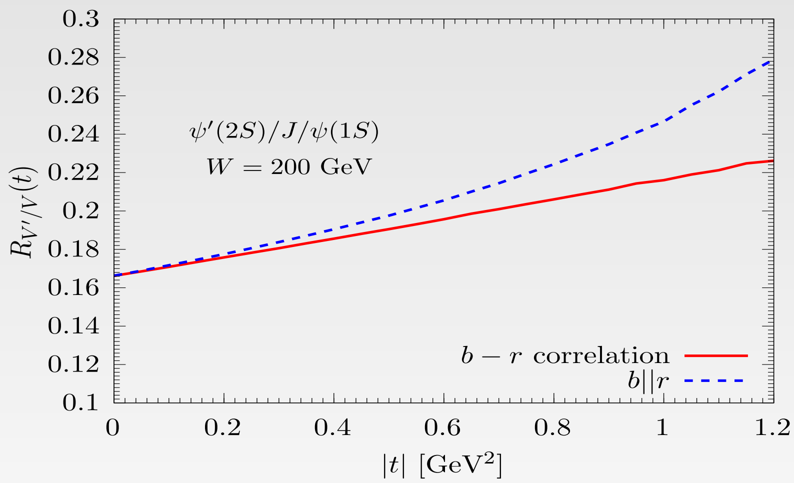
$\vec{b}$-$\vec{r}$ correlation





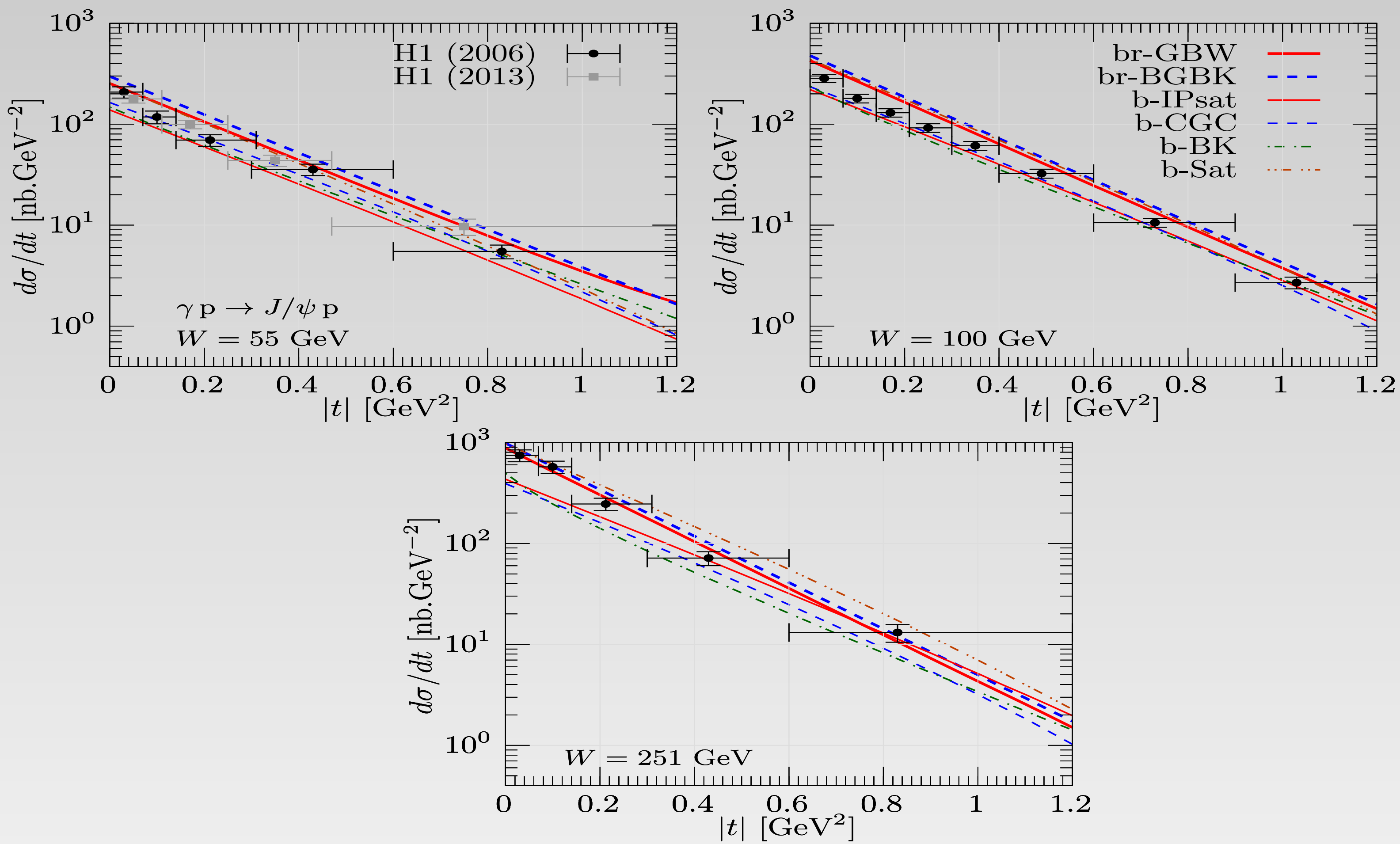
Demonstration of an importance of $\vec{b}$-$\vec{r}$ correlation in photoproduction of charmonia on proton target at $W = 50$ GeV and $W = 200$ GeV





Analogous demonstration of $\vec{b}$-$\vec{r}$ correlation in photoproduction of the $\psi'(2S)$-to-$J/\psi(1S)$ ratio [B.Z. Kopeliovich, et al. Phys. Rev. D**103**, 094027 (2021)]

$\vec{b}$-$\vec{r}$ correlation



- Comparison among different models at various photon energies $W = 55, 100$ and 251 GeV [B.Z. Kopeliovich, et al. Phys.Rev. D**103**, 094027 (2021)]
- However, treating nuclear targets, the effect of $\vec{b}$-$\vec{r}$ correlation is diluted, except the nuclear periphery, where the nuclear density steeply varies with $\vec{b}$

The higher-twist nuclear shadowing

CFRJS

- The lowest Fock component of the projectile photon is $|Q\bar{Q}\rangle \Rightarrow$ small transverse dipole size $\propto 1/m_Q \Rightarrow$ small shadowing corrections $\propto 1/m_Q^2 \Rightarrow$ should be treated as a higher twist effect
- For the amplitude of coherent quarkonium electroproduction on a nuclear target, $\gamma^* A \to V A$, one can employ the above expression for $\mathcal{A}^N(x,\vec{q}\,)$, but replacing the dipole-nucleon by dipole-nucleus amplitude,

$$\mathcal{A}^{\gamma^* A \to V A}(x, Q^2, \vec{q}\,) = 2 \int d^2 b_A \, e^{i\vec{q}\cdot\vec{b}_A} \int d^2 r \int_0^1 d\alpha \; \Psi_V^*(\vec{r},\alpha)\, \mathcal{A}^A_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{b}_A)\, \Psi_{\gamma^*}(\vec{r},\alpha,Q^2)$$

The higher-twist nuclear shadowing

- In UPC at the LHC, the photon virtuality $Q^2 \sim 0$ and the photon energy in the nuclear rest frame is sufficiently high $\Rightarrow$ the coherence length

$$l_c^{Q\bar{Q}} = 1/q_L = \frac{W^2 + Q^2 - m_N^2}{m_N\,(M_V^2 + Q^2)}\Big|_{Q^2\sim 0} \approx \frac{W^2}{m_N M_V^2} \gg R_A$$

- Lorentz time delation freezes the fluctuations of the dipole size, and one can rely on the eikonal form for the dipole-nucleus partial amplitude at impact parameter $\vec{b}_A$

$$\mathrm{Im}\mathcal{A}^A_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{b}_A)\Big|_{l_c^{Q\bar{Q}}\gg R_A} =$$

$$1 - \left[1 - \frac{1}{A}\int d^2b\;\mathrm{Im}\mathcal{A}^N_{Q\bar{Q}}(\vec{r},x,\alpha,\vec{b}\,)\,T_A(\vec{b}_A+\vec{b}\,)\right]^A$$

- $T_A(\vec{b}_A) = \int_{-\infty}^{\infty} dz\,\rho_A(\vec{b}_A,z)$ is the nuclear thickness function normalized as $\int d^2b_A\,T_A(\vec{b}_A) = A$

The higher-twist nuclear shadowing

- Expression for the differential cross sections in the limit $l_c^{Q\bar{Q}} \gg R_A$ is analogous to that for proton,

$$\frac{d\sigma^{\gamma^* A \to V A}(x, Q^2, t = -q^2)}{dt}\bigg|_{l_c^{Q\bar{Q}} \gg R_A} = \frac{1}{16\,\pi}\left|\mathcal{A}^{\gamma^* A \to V A}(x, Q^2, \vec{q})\right|^2$$

- In a heavy-ion UPC $\Rightarrow$ the photon field of one nucleus can produce a photo-nuclear reaction in the other. Within the

one-photon-exchange approximation $\Rightarrow$ diff. cross sec.,

$$k\frac{d\sigma}{dk} = \int d^2\tau \int d^2b_A\, n(k, \vec{b}_A - \vec{\tau}, y)\, \frac{d^2\sigma_A(s, b_A)}{d^2b_A} + \left\{y \to -y\right\}$$

- the rapidity variable $y = \ln[s/(M_V\sqrt{s_N})] \approx \ln[(2km_N + m_N^2)/(M_V\sqrt{s_N})]$
  $\vec{\tau} \Rightarrow$ the relative impact parameter of a nuclear collision
  $\vec{b}_A \Rightarrow$ the impact parameter of the photon-nucleon collision relative to the center of one of the nuclei.

The higher-twist nuclear shadowing

CFRJS

- The photon flux induced by the projectile nucleus with Lorentz factor $\gamma \Rightarrow$

$$n(k,\vec{b}_A) = \frac{\alpha_{em} Z^2 k^2}{\pi^2 \gamma^2}\left[K_1^2\left(\frac{b_A k}{\gamma}\right) + \frac{1}{\gamma^2} K_0^2\left(\frac{b_A k}{\gamma}\right)\right]$$

where $\gamma = 2\gamma_{col}^2 - 1$ with $\gamma_{col} = \sqrt{s_N}/2m_N$

- Within the "frozen approximation" $\Rightarrow l_c^{Q\bar{Q}} \gg R_A \Rightarrow$

$$\left.\frac{d^2\sigma_A^{coh}(s,b_A)}{d^2 b_A}\right|_{l_c^{Q\bar{Q}} \gg R_A} = \left|\int d^2 r \int_0^1 d\alpha\, \Sigma_A^{coh}(r,\alpha,s,b_A)\right|^2$$

$$\Sigma_A^{coh} = \Psi_V^*(\vec{r},\alpha)\left(1 - \left[1 - \frac{1}{2\,A}\sigma_{Q\bar{Q}}(\vec{r},s) T_A(\vec{b}_A)\right]^A\right)\Psi_\gamma(\vec{r},\alpha)$$

- The universal $\sigma_{Q\bar{Q}}(\vec{r},s)$ depends on c.m. energy squared $s = M_V\,\sqrt{s_N}\,\exp[y]$ or, alternatively, on a variable $x = M_V^2/s = M_V\,\exp[-y]/\sqrt{s_N}$

The leading-twist gluon shadowing

- GS $\Longleftrightarrow$ to higher Fock components of the photon containing besides the $Q\bar{Q}$ pair additional gluons, $|Q\bar{Q}\,g\rangle$, $|Q\bar{Q}\,2g\rangle$, etc.
- In a $\gamma^* p$ collision such components $\Rightarrow$
  - ● correspond to gluon radiation processes, which should be treated as higher-order corrections to the gluonic exchange, which take part in the building of the Pomeron exchange in the diffractive $\gamma^* p$ interaction
  - ● are included in the $Q\bar{Q}$-dipole interaction with the proton
- In an electro-production on a nucleus such components contribute to the amplitude $\mathcal{A}^N_{\bar{Q}Q}(\vec{r}, x, \alpha, \vec{b})$ in the eikonal formula for the dipole-nucleus cross section (partial amplitude) $\Rightarrow$ we expect the Bethe-Heitler regime of radiation, when each of multiple interactions produce independent gluon radiation without interferences

The leading-twist gluon shadowing

CFRJS

- However, the pattern of multiple interactions changes in the regime of long $l_c^{Q\bar{Q}g} \gg d$, where $d \approx 2\,\mathrm{fm}$ is the mean separation between bound nucleons
- The gluon radiation length reads,

$$l_c^{Q\bar{Q}g} = \frac{2k\alpha_g(1-\alpha_g)}{k_T^2 + (1-\alpha_g)m_g^2 + \alpha_g M_{Q\bar{Q}}^2},$$

$\alpha_g \Rightarrow$ the LF fraction of the photon momentum carried by the g,
$M_{Q\bar{Q}} \Rightarrow$ the effective mass of the $Q\bar{Q}$ pair,
$m_g \approx 0.7\,\mathrm{GeV} \Rightarrow$ the effective gluon mass fixed by data on gluon radiation [B.Z. Kopeliovich, et al. PR D**62**, 054022 (2000); PR D**76**, 094020 (2007)]

- Such a rather large $m_g \Rightarrow l_c^{Q\bar{Q}g} = l_c^{Q\bar{Q}}/f_g$, where the factor $f_g \approx 10$ [B.Z. Kopeliovich, et al. Phys.Rev. C**62**, 035204 (2000)]

The leading-twist gluon shadowing

- At long $l_c^{Q\bar{Q}g} \gg d$
  $\Rightarrow$ the Landau-Pomeranchuk effect is at work
  $\Rightarrow$ radiation does not resolve multiple interactions acting as one accumulated kick
  $\Rightarrow$ intensity of gluon radiation is reduced compared to the Bethe-Heitler regime $\equiv$ gluon shadowing
- Gluon shadowing $\Rightarrow$

- is a part of Gribov corrections
- corresponds to higher Fock components $|\bar{Q}Qg\rangle$, $|\bar{Q}Q2g\rangle$, ...
- requires eikonalization of these components
- differently from $\bar{Q}Q$ fluct., a $Q\bar{Q}g$ component does not reach the "frozen" size regime, because of divergent $d\alpha_g/\alpha_g$ behavior
- variation of the $Q\bar{Q} - g$ dipole size must be taken into account adopting the Green function technique

[Yu. Ivanov, at al. Phys.Rev. C**66**, 024903 (2002); B. Kopeliovich, et al. Phys.Rev. D**105**, 054023 (2022)]

The leading-twist gluon shadowing

CFRJS

- The Gribov correction, related to the $Q\bar{Q}g$ component of the photon, to the partial nuclear cross section at impact parameter b_A, reads

$$\Delta\sigma_{tot}^{\gamma^* A}(b_A) = \frac{1}{2}\,\mathrm{Re}\int\limits_{-\infty}^{\infty} dz_2\,\rho_A(b_A,z_2)\int\limits_{-\infty}^{z_2} dz_1\,\rho_A(b_A,z_1)\int d^2\rho_1$$

$$\int d^2\rho_2\int\frac{d\alpha_g}{\alpha_g}A^{\dagger}_{\gamma^*\to\bar{Q}Qg}(\alpha_g,\vec{\rho}_2)G_{gg}(z_2,\vec{\rho}_2;z_1,\vec{\rho}_1)A_{\gamma^*\to\bar{Q}Qg}(\alpha_g,\vec{\rho}_1)$$

- It contains the product of conjugated amplitudes of diffractive transitions, $\gamma^* + N \to \bar{Q}Qg + N$, on bound nucleons with longitudinal coordinates $z_{1,2}$ and the Green function $G_{gg}(z_2,\vec{\rho}_2;z_1,\vec{\rho}_1)$ describes the propagation of the $\bar{Q}Qg$ system in the nuclear medium

The leading-twist vs. higher-twist shadowing

CFRJS

- The $Q\bar{Q}g$ Fock state is characterized by **two scales** $\Rightarrow$
  - One scale determines the small $Q\bar{Q}$ separation $\approx 1/m_Q \Rightarrow$ **a higher twist effect** $\Rightarrow$ at large m_Q it can be treated as point-like color-octet system.
  - The second scale determines much larger $Q\bar{Q}$-g transverse size, which is independent of m_Q (up to Log corrections) and depends on $m_g \approx 0.7$ GeV $\Rightarrow$ the $Q\bar{Q} - g$ system

    — is **strongly asymmetric** and controls GS $\equiv$ the **leading twist effect**, since it is hardly dependent (only Log) on the m_Q

    — can be treated with high precision as glue-glue dipole [B.Z. Kopeliovich, et al. Phys.Rev. D**62**, 054022 (2000);] with the tranverse size $\approx 1/m_g$

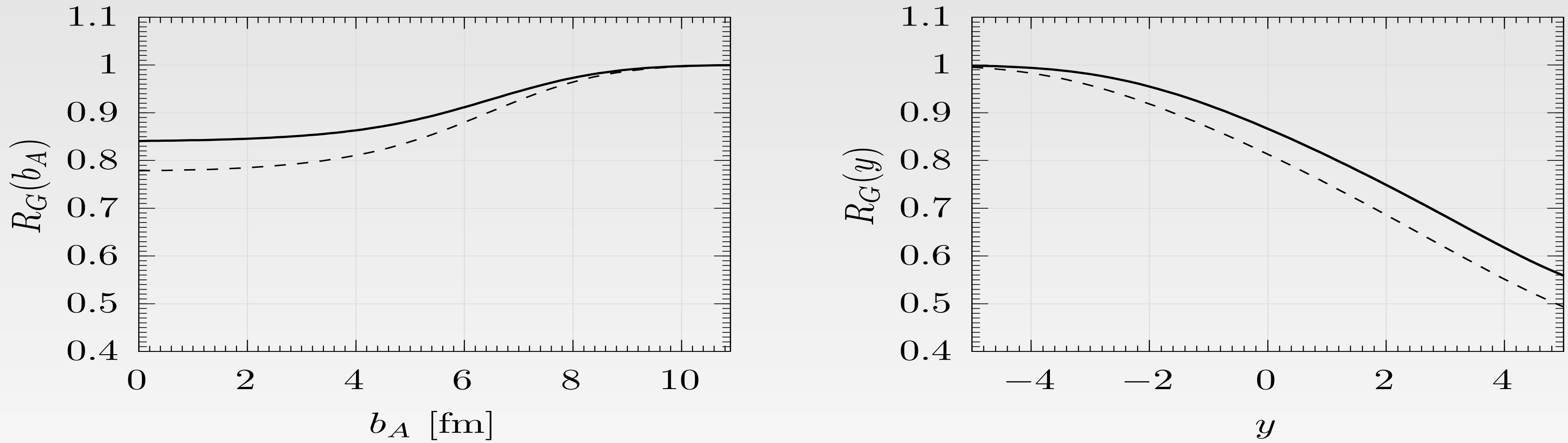
The leading-twist gluon shadowing

- The fractional Gribov correction to the partial nuclear cross section

$$R_G(b_A) = 1 - \frac{\Delta\sigma_{tot}^{\gamma^* A}(b_A)}{T_A(b_A)\sigma_{tot}^{\gamma^* N}},$$

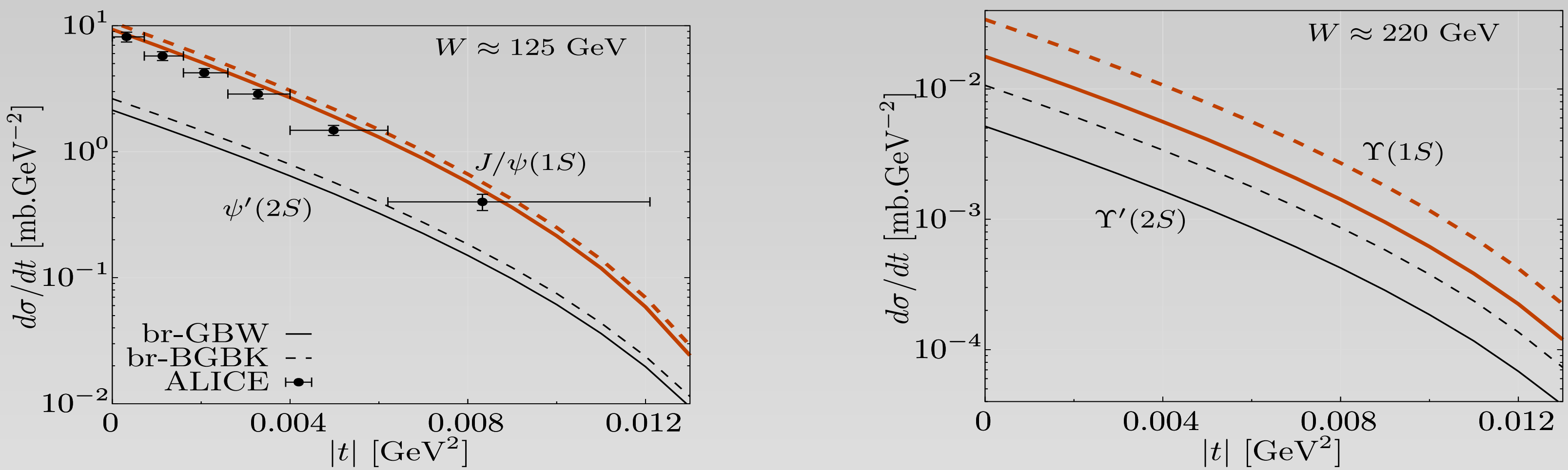
- In the parton model it is interpreted as the ratio of gluon densities $\Rightarrow$ reduction of the gluon density by factor R_G $\Rightarrow$ renormalization of the nucleon amplitude

$$\mathrm{Im}\mathcal{A}^N_{\bar{Q}Q}(\vec{r}, x, \alpha, \vec{b}) \Rightarrow \mathrm{Im}\mathcal{A}^N_{\bar{Q}Q}(\vec{r}, x, \alpha, \vec{b}) \cdot R_G(x, |\vec{b}_A + \vec{b}|)$$

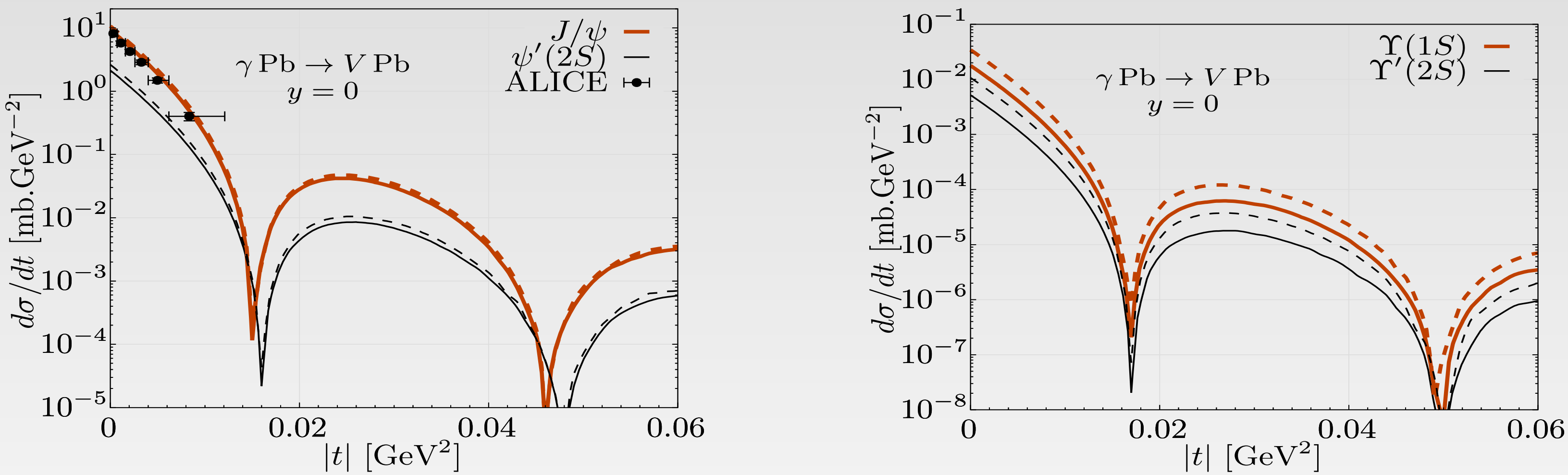
- 

Gluon shadowing factor $R_G(b_A)$ for photoproduction of J/ψ on lead as function of impact parameter b_A (left), and rapidity y (right). Solid and dashed curves correspond to c.m. collision energies $\sqrt{s_N} = 5.02$ TeV and 13 TeV, respectively.

Model predictions for $d\sigma/dt$ vs data



Predictions for $d\sigma^{\gamma Pb\to V Pb}/dt$ in comparison with ALICE data.

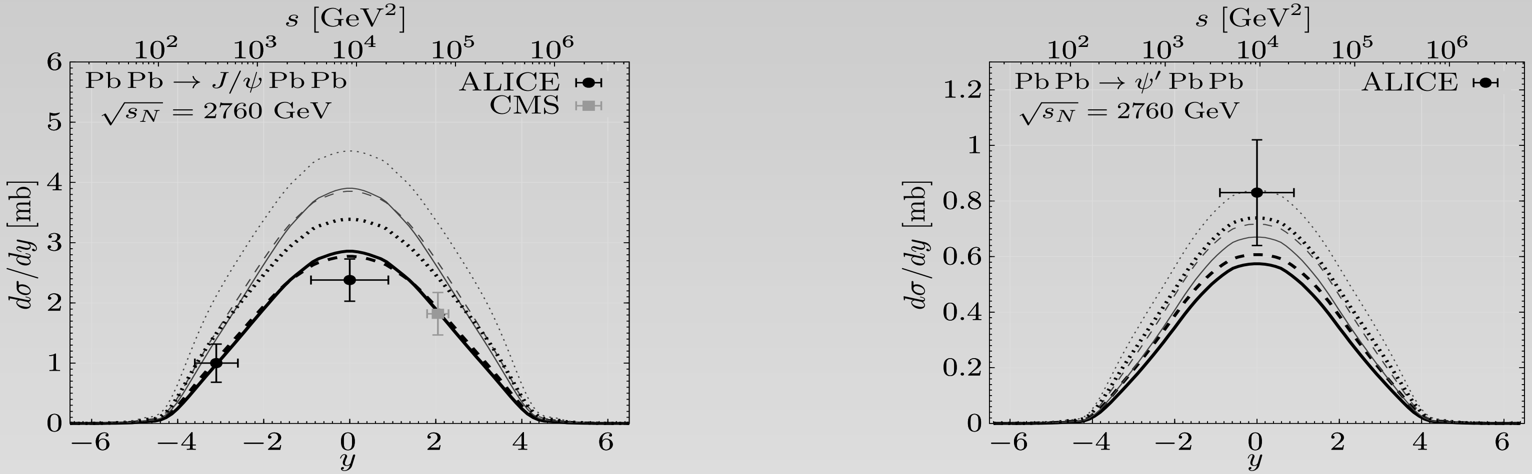


Predictions for $d\sigma^{\gamma Pb\to V Pb}/dt$ for a wider $|t|$-range, at c.m. energy

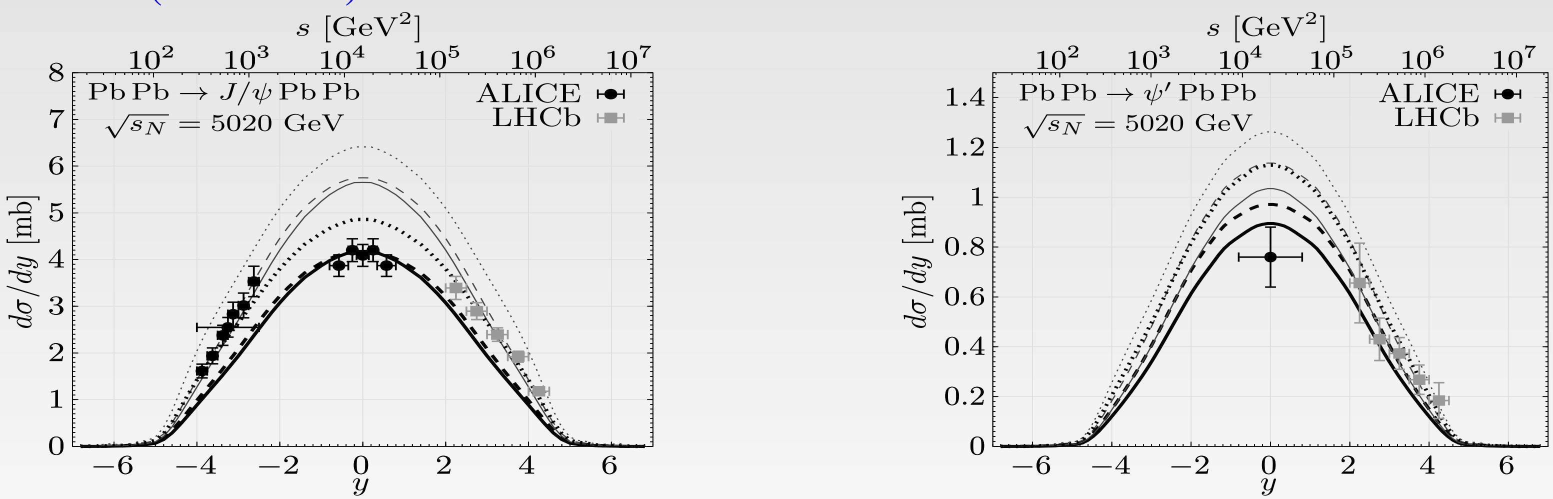
$W \approx \sqrt{M_V}(s_N)^{1/4}$ corresponding to kinematic regions of UPC at the LHC at $y = 0$

[B.Z. Kopeliovich, et al. Phys.Rev. D**105**, 054023 (2022)]

Model predictions for $d\sigma/dy$ vs data



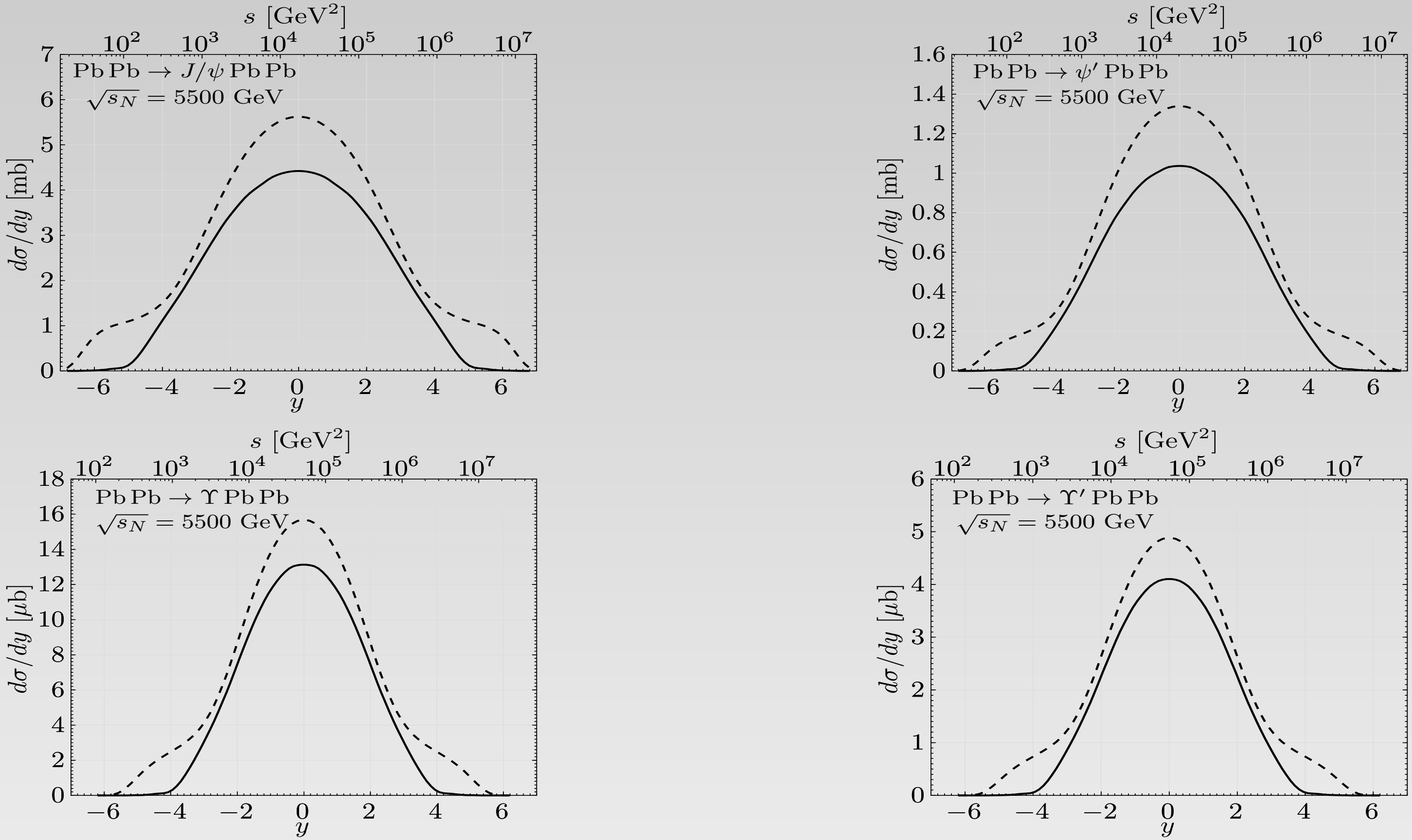
Predictions for $d\sigma^{PbPb\to V\,PbPb}/dy$ in comparison with ALICE and CMS data at $\sqrt{s_N} = 2.76$ TeV adopting GBW (solid lines), KST (dashed lines) and BGBK (dotted lines) models for $\sigma_{Q\bar{Q}}(r,x)$. Charmonium WFs are generated by the POW (thin lines) and BT (thick lines).



Analogous predictions for $d\sigma^{PbPb\to V\,PbPb}/dy$ in comparison with ALICE and LHCb data at $\sqrt{s_N} = 5.02$ TeV

[B.Z. Kopeliovich, et al. Phys.Rev. D**105**, 054023 (2022); e-Print: 2008.05116 [hep-ph]]

Model predictions for $d\sigma/dy$



Predictions for $d\sigma^{PbPb \to V\,PbPb}/dy$ at $\sqrt{s_N} = 5.5$ TeV. Solid lines ⇒ corrections for linite $l_c^{Q\bar{Q}}$ and GS, dashed lines ⇒ without such corrections.

- **Corrections for a finite $l_c^{Q\bar{Q}}$** ⇒ dominate at forward and backward rapidities

$$\frac{d^2\sigma_A^{coh}(s,b)}{d^2b} = \left.\frac{d^2\sigma_A^{coh}(s,b)}{d^2b}\right|_{l_c \gg R_A} \cdot F^{coh}\Big(s, l_c(s)\Big)$$

Correction factors F^{coh} were calculated adopting the Green function formalism

- **Gluon shadowing corrections** ⇒ dominate at midrapidity

Coherence length for multi-gluon components

- Virtual photon with energy k develops a hadronic fluctuation $Q\bar{Q}$ for a lifetime

$$l_c^{Q\bar{Q}} = \frac{2k}{Q^2 + M_{Q\bar{Q}}^2} = \frac{1}{x m_N} P_q = l_c^{max} P_q$$

- $Q\bar{Q}$ effective mass $\Rightarrow M_{Q\bar{Q}}^2 = (m_Q^2 + k_T^2)/\alpha(1-\alpha)$
- the factor $$P_q = \frac{1}{1 + M_{Q\bar{Q}}^2/Q^2}$$
- naive estimation $\Rightarrow P_q = 1/2 \Longleftrightarrow$ usual approximation to assume $M_{Q\bar{Q}}^2 \approx Q^2$
- exact calculations in [B.Z. Kopeliovich, et al. Phys.Rev. C **62**, 035204 (2000)] $\Rightarrow \langle P_q \rangle_{T,L} \approx 0.36(0.75)\ at\ x \sim 10^{-4} \div 10^{-3}$
- $\langle P_q \rangle_L > \langle P_q \rangle_T \Rightarrow$ L photon develops lighter fluctuations than a T one $\Rightarrow$ asymmetric fluctuations $[\alpha$ or $1-\alpha \to 0]$ are suppressed in L photons by the WF

Coherence length for multi-gluon components

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- Higher Fock component $Q\bar{Q}g \Rightarrow$ coherence length

$$l_c^{Q\bar{Q}g} = \frac{2k}{Q^2 + M^2_{Q\bar{Q}g}} = \frac{1}{x m_N} P_g = l_c^{max} P_g$$

- $Q\bar{Q}g$ effective mass $\Rightarrow M^2_{Q\bar{Q}g} = \frac{M^2_{Q\bar{Q}} + k_T^2}{1-\alpha_g} + \frac{m_g^2 + k_T^2}{\alpha_g}$
  $M^2_{Q\bar{Q}g} \approx M^2_{Q\bar{Q}}(1 + \frac{\gamma}{\alpha_g})$ where $\gamma = 2m_g/M^2_{Q\bar{Q}}$
- the above factor $\quad P_g = \dfrac{\alpha_g}{\alpha_g + \gamma}$
- averaging P_g over gluon radiation spectrum $d\alpha_g/\alpha_g$ and fixing the $Q\bar{Q} - g$ transverse separation at the mean value $1/m_g \Rightarrow \langle P_g \rangle \equiv \langle P_g \rangle / \langle P_q \rangle = 0.12$
  [B.Z. Kopeliovich, et al. Phys.Rev. D **105**, 054023 (2022)]
- Exact calculations based on the Green function formalism $\Rightarrow \langle P_g \rangle / \langle P_q \rangle \approx 0.1$ [B.Z. Kopeliovich, et al. PR C **62**, 035204 (2000)]

Coherence length for multi-gluon components

CFRJS

- 2-gluon Fock component $Q\bar{Q}2g \Rightarrow$ effective mass $M^2_{Q\bar{Q}2g} \approx M^2_{Q\bar{Q}}(1 + \frac{\gamma}{\alpha_{g1}} + \frac{\gamma}{\alpha_{g2}})$
- coherence length

$$l_c^{Q\bar{Q}2g} = \frac{2k}{Q^2 + M^2_{Q\bar{Q}2g}} = \frac{1}{x m_N} P_{2g} = l_c^{max} P_{2g}$$

- analogously we get $P_{2g} = \frac{\alpha_{g1}\alpha_{g2}}{\alpha_{g1}\alpha_{g2} + \gamma\alpha_{g1} + \gamma\alpha_{g2}} \Rightarrow$ after averaging process over radiation spectra $d\alpha_{g1}/\alpha_{g1}$ and $d\alpha_{g2}/\alpha_{g2} \Rightarrow \langle P_{2g} \rangle \equiv \langle P_{2g} \rangle / \langle P_q \rangle = 0.035$
- straightforwardly for higher multi-gluon photon components $\Rightarrow$ strong inequalities $\Rightarrow$ $M^2_{Q\bar{Q}} \ll M^2_{Q\bar{Q}g} \ll M^2_{Q\bar{Q}2g} \ll \cdots \ll M^2_{Q\bar{Q}ng}$ $l_c^{Q\bar{Q}} \gg l_c^{Q\bar{Q}g} \gg l_c^{Q\bar{Q}2g} \gg \cdots \gg l_c^{Q\bar{Q}ng}$

Coherence length for multi-gluon components

	$\langle P_{ng}\rangle/\langle P_q\rangle$	$\langle P_{ng}\rangle/\langle P_g\rangle$	$\langle l_c\rangle$, [fm] UPC $y=0$
$Q\bar{Q}$	——-	——-	120.0
$Q\bar{Q}g$	0.11940	1.0000	14.2
$Q\bar{Q}2g$	0.03560	0.2980	4.2
$Q\bar{Q}3g$	0.01630	0.1370	1.9
$Q\bar{Q}4g$	0.00952	0.0798	1.1
$Q\bar{Q}5g$	0.00639	0.0535	0.7
$Q\bar{Q}6g$	0.00462	0.0388	0.5
$Q\bar{Q}7g$	0.00342	0.0287	0.4
$Q\bar{Q}8g$	0.00256	0.0217	0.3

Fractions of the coherence length for $Q\bar{Q}$ Fock state related to higher photon components containing different number of gluons

Reduced coherence effects in BK eq.

- $\exists$ strong inequalities
  $l_c^{Q\bar{Q}} \gg l_c^{Q\bar{Q}g} \gg l_c^{Q\bar{Q}2g} \gg \cdots \gg l_c^{Q\bar{Q}ng}$
  However, in the LHC energy range
  $l_c^{Q\bar{Q}}, l_c^{Q\bar{Q}g} \gg R_A$ but $l_c^{Q\bar{Q}2g}, ..., l_c^{Q\bar{Q}ng} \lesssim R_A$
- Two dominant sources of shadowing effects at the LHC
  — higher twist quark shadowing $\Longleftrightarrow$ $Q\bar{Q}$ component of the photon
  — leading twist gluon shadowing $\Longleftrightarrow$ $Q\bar{Q}g$ component of the photon
  Higher Fock states $|Q\bar{Q}2g\rangle, ..., |Q\bar{Q}ng\rangle$ have rather small or negligible contribution to shadowing
- BK equation in combination with the eikonal expression for dipole-nucleus cross section $\Rightarrow$ sumation of all photon components with $l_c^{Q\bar{Q}}, l_c^{Q\bar{Q}g}, ..., l_c^{Q\bar{Q}ng} \gg R_A$
- **This leads to exaggeration of shadowing effects**

Reduced coherence effects in BK eq.

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- The effects of reduced coherence length can be included using the Green function formalism ⇒ t-integrated cross section of the process $\gamma^* A \to V A$

$$\sigma^A_{coh}(x, Q^2) = \int d^2 b_A \left| \int dz\, \rho_A(\vec{b}_A, z)\, F(b_A, z, x) \right|^2$$

$$F(b_A, z, x) = \int d^2\rho_1 \int d^2\rho_2 \int_0^1 d\alpha\, \Psi^*_V(\vec{\rho}_2, \alpha) \mathcal{B}_{Q\bar{Q}}(z', \vec{\rho}_2; z, \vec{\rho}_1 | b_A, x) \Psi_{\gamma^*}(\vec{\rho}_1, \alpha, Q^2) \Big|_{z' \to \infty}$$

- The function $\mathcal{B}_{Q\bar{Q}}(z', \vec{\rho}_2; z, \vec{\rho}_1 \,|\, b_A, x)$ satisfies the BK evolution equation

$$\frac{\partial \mathcal{B}_{Q\bar{Q}}(z_2, \vec{r}\,'; z_1, \vec{r} \,|\, b_A, y)}{\partial y} = \int d^2 r_1\, K(\vec{r}, \vec{r}_1, \vec{r}_2) \Big[\mathcal{B}_{Q\bar{Q}}(z_2, \vec{r}\,'; z_1, \vec{r}_1 \,|\, b_A, y) + \mathcal{B}_{Q\bar{Q}}(z_2, \vec{r}\,'; z_1, \vec{r}_2 \,|\, b_A, y) - \mathcal{B}_{Q\bar{Q}}(z_2, \vec{r}\,'; z_1, \vec{r} \,|\, b_A, y) - \mathcal{B}_{Q\bar{Q}}(z_2, \vec{r}\,'; z_1, \vec{r}_1 \,|\, b_A, y) \cdot \mathcal{B}_{Q\bar{Q}}(z_2, \vec{r}\,'; z_1, \vec{r}_2 \,|\, b_A, y) \Big]$$

- with the initial condition ⇒

$$\mathcal{B}_{Q\bar{Q}}(z_2, \vec{r}\,'; z_1, \vec{r} \,|\, b_A, y = 0) = \sigma_0 N(\vec{r}, y = 0) \cdot g_{Q\bar{Q}}(z_2, \vec{r}\,'; z_1, \vec{r} \,|\, b_A, y = 0)$$

the rapidity variable ⇒ $y = \ln(x_0/x) \approx \ln\Big[(2\, k\, x_0\, m_N)/(Q^2 + m_V^2)\Big]$

Reduced coherence effects in BK eq.

- The Green function $g_{Q\bar{Q}}$ satisfies the following Schrödinger eq.,

$$i\,\frac{d}{dz_2}\,g_{Q\bar{Q}}(z_2,\vec{r}_2;z1,\vec{r}_1\,|\,b_A,y) =$$

$$\left[\frac{\epsilon^2-\Delta_{r_2}}{2\,k\,\alpha\,(1-\alpha)}+V_{Q\bar{Q}}(z_2,\vec{r}_2,\alpha,b_A)\right]g_{Q\bar{Q}}(z_2,\vec{r}_2;z_1,\vec{r}_1\,|\,b_A,y)$$

  with the initial condition

$$g_{Q\bar{Q}}(z_2,\vec{r}_2;z_1,\vec{r}_1\,|\,b_A,y)|_{z_2=z_1} = \delta^{(2)}(\vec{r}_1-\vec{r}_2)$$

- Reduced coherence effects $\Rightarrow$ in terms of relative variation from results based on eikonalization of the dipole-nucleon cross section
- Approximations $\Rightarrow$
  - an uniform nuclear density $\rho_A(b_A,z)=\rho_0\,\Theta(R_A^2-b_A^2-z^2)$
  - the quadratic form for $\sigma_{Q\bar{Q}}^N(\vec{r},y)=C(y)\cdot r^2$
  - the quadratic shape for the LF $Q-\bar{Q}$ interaction potential
- $\Longrightarrow$ Gaussian shape of the LF quarkonium WF and analytical harmonic oscillatory form for the Green function

Reduced coherence effects in BK eq.

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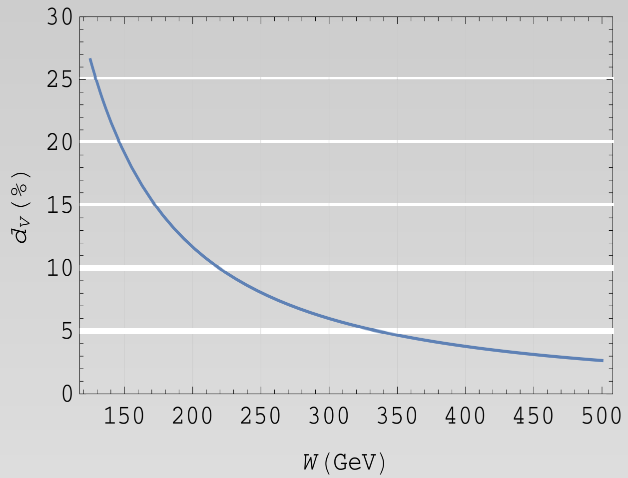

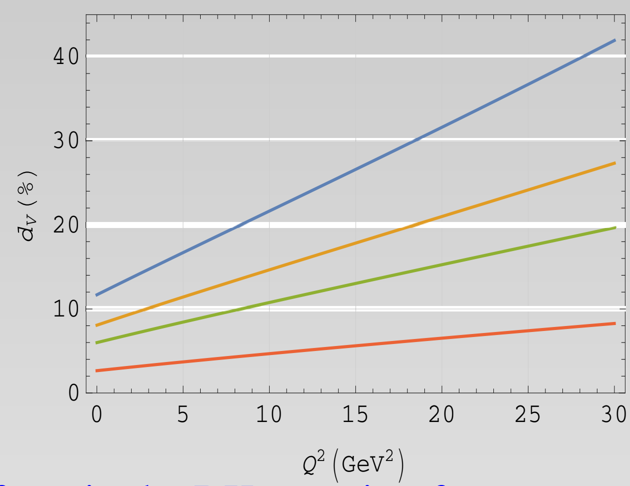

- **(left)-** Relative impact of reduced coherence effects in the BK equation for $\gamma Pb \to J/\psi Pb$ in terms of the c.m. energy W dependence of the relative difference $d_V = |\sigma_{coh}^{GF} - \sigma_{coh}^{eik}|/\sigma_{coh}^{eik}$ of t-integrated cross sections obtained from the BK eq. **(right)-** the Q^2 dependence of the factor d_V at several fixed values of $W = 200, 250, 300$ and 500 GeV depicted by lines from top to bottom.
- Even at the LHC collision energy $\sqrt{s_N} = 5.02$ TeV, corresponding to $W = 125$ GeV, the frequently used traditional calculations based on the eikonal formula cause an overestimation of shadowing effects by about 27%.
- The factor d_V rather slowly decreases with $W \Rightarrow$ one needs quite large $W \gtrsim 500$ GeV in order to use the "frozen" eikonal approximation with a reasonable accuracy.

Summary

- We studied $d\sigma/dt$ and $d\sigma/dy$ for coherent photoproduction of heavy quarkonia on nuclei, in the framework of the QCD color dipole formalism $\Rightarrow$ a good accord with ALICE data
- The LF WF of a photon was expanded over Fock states (FS) $|Q\bar{Q}\rangle$, $|Q\bar{Q}g\rangle$, ... Each of them contributes independently to heavy quarkonium production. The cross section $\gamma A \to V A$ was calculated for a particular FS separately in accordance with the corresponding coh. lengths.
- At the energies of UPC the CL $l_c^{Q\bar{Q}} \gg R_A \Rightarrow$ one can eikonalize the dipole $Q\bar{Q} - N$ amplitude. Here we have included the $\vec{b}$-$\vec{r}$ correlation The corresponding **quark shadowing** is a **higher twist effect**, so it is small at the scale imposed by the heavy quarkonium mass.
- The QCD dipole formalism also includes **gluon shadowing**, which is related to higher $|Q\bar{Q}ng\rangle$ components. The corresponding shadowing effect is a **leading twist** due to large, nearly scale independent size, $\propto 1/m_g$, of the $Q\bar{Q} - g$ dipoles. The effect of GS is much stronger than the higher twist quark shadowing, controlled by the small-size of $Q\bar{Q}$ dipoles.
- The CL $l_c^{Q\bar{Q}ng} \ll l_c^{Q\bar{Q}}$ and radiation of every additional gluon significantly reduces the CL $\Rightarrow l_c^{Q\bar{Q}ng} \ll l_c^{Q\bar{Q}(n-1)g} \Rightarrow$ at LHC energies the $|Q\bar{Q}g\rangle$ FS represents a dominant contribution to shadowing. Higher components have too short CL to produce a sizeable shadowing effect $\Rightarrow$ **BK equation cannot be applied to nuclear targets**.