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The ReBB model at 8 TeV: Odderon exchange is a certainty

based on

[Eur. Phys. J. C **82**, 827 \(19 September 2022\)](#) & [Eur. Phys. J. C **81**, 611 \(13 July 2021\)](#)

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Bialas-Bzdak p=(q,d) model

$$\tilde{\sigma}_{in}(\vec{b}) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} d^2\vec{s}_q d^2\vec{s}'_q d^2\vec{s}_d d^2\vec{s}'_d D(\vec{s}_q, \vec{s}_d) D(\vec{s}'_q, \vec{s}'_d) \sigma(\vec{s}_q, \vec{s}_d; \vec{s}'_q, \vec{s}'_d; \vec{b})$$

- quark-diquark distribution inside the proton:

$$D(\vec{s}_q, \vec{s}_d) = \frac{1 + \lambda^2}{R_{qd}^2 \pi} e^{-\frac{s_q^2 + s_d^2}{R_{qd}^2}} \delta^2(\vec{s}_q + \lambda \vec{s}_d)$$

$$\lambda = \frac{m_q}{m_d}$$

$$\vec{s}_d = -\lambda \vec{s}_q$$

$$\vec{s}'_d = -\lambda \vec{s}'_q$$

[A. Bialas, A. Bzdak, Acta Phys.Polon. B 38, 159-168 \(2007\), Phys.Lett.B 649: 263-268 \(2007\)](#)

- sum of inelastic interaction probabilities of the constituents:

$$\sigma(\vec{s}_q, \vec{s}_d; \vec{s}'_q, \vec{s}'_d; \vec{b}) = 1 - \prod_a \prod_b [1 - \sigma_{ab}(\vec{b} + \vec{s}'_a - \vec{s}_b)]$$

$$\sigma_{ab}(\vec{s}) = A_{ab} e^{-|\vec{s}|^2 / S_{ab}^2}$$

$$S_{ab}^2 = R_a^2 + R_b^2$$

$$a, b \in \{q, d\}$$

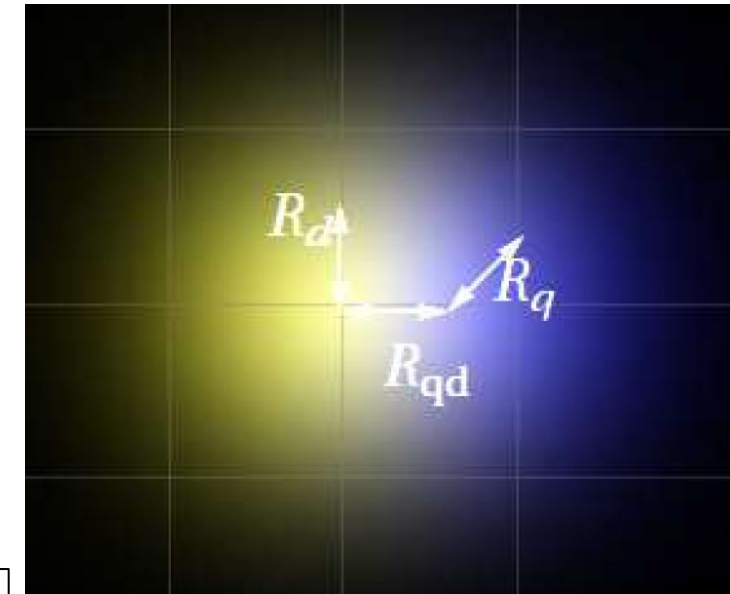
- inelastic cross-sections of quark, diquark scatterings :

$$\sigma_{ab,in} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sigma_{ab}(\vec{s}) d^2\vec{s}$$

$$\sigma_{qq,in} : \sigma_{qd,in} : \sigma_{dd,in} = 1 : 2 : 4$$

- free parameters:

$$A_{qq}, \lambda, R_q, R_d, R_{qd}, \quad (A_{qq} = 1 \text{ and } \lambda = 0.5 \text{ can be fixed})$$



Proton-(anti)proton scattering in the quark-diquark model (Glauber style calculation).

Unitarily Real Extended Bialas-Bzdak (REBB) model

- elastic scattering amplitude in the impact parameter space:

$$t_{el}(s, \vec{b}) = i \left[1 - e^{-\Omega(s, \vec{b})} \right]$$

arXiv:1505.01415

F. Nemes, T. Csörgő, M. Csanád, Int. J. Mod. Phys. A Vol. 30 (2015) 1550076

- the opacity function:

$$\Omega(s, \vec{b}) = \text{Re}\Omega(s, \vec{b}) + i \text{Im}\Omega(s, \vec{b})$$

$\text{Im}\Omega \neq 0$ as the real part of the amplitude is not negligibly small

$$\text{Re}\Omega(s, \vec{b}) = -\frac{1}{2} \ln[1 - \tilde{\sigma}_{in}(s, \vec{b})]$$

$$\text{Im}\Omega(s, \vec{b}) = -\alpha \tilde{\sigma}_{in}(s, \vec{b})$$

NEW FREE PARAMETER,
has different values for pp and $\bar{p}p$

- elastic scattering amplitude in momentum space:

$$T(s, t) = 2\pi \int_0^\infty t_{el}(s, |\vec{b}|) J_0(|\vec{\Delta}| |\vec{b}|) |\vec{b}| d|\vec{b}|$$

$$|\vec{\Delta}| \equiv \sqrt{-t} \text{ as } \sqrt{s} \rightarrow \infty$$

(t is the squared momentum transfer)

The pp and $p\bar{p}$ elastic scattering amplitude & the odderon

- according to the Regge formalism the strong scattering amplitude for pp and $p\bar{p}$ scattering is written in terms of $C = +1$ and $C = -1$ exchange components

$$T^{pp}(s, t) = T^+(s, t) - T^-(s, t)$$

$$T^{p\bar{p}}(s, t) = T^+(s, t) + T^-(s, t)$$

- for $\sqrt{s} \gtrsim 1$ TeV the mesonic reggeon exchanges are negligible and essentially only the gluonic Pomeron and Odderon exchanges are present implying that

$$T^+(s, t) \equiv T^P(s, t)$$

$$T^-(s, t) \equiv T^O(s, t)$$



$$T^P(s, t) = \frac{1}{2} (T^{pp}(s, t) + T^{p\bar{p}}(s, t))$$

$$T^O(s, t) = \frac{1}{2} (T^{p\bar{p}}(s, t) - T^{pp}(s, t))$$

- a simple and model independent consequence:

$$\text{if } \frac{d\sigma^{pp}}{dt}(s, t) \neq \frac{d\sigma^{p\bar{p}}}{dt}(s, t) \text{ for } \sqrt{s} \gtrsim 1 \text{ TeV then } T^O(s, t) \neq 0$$

Fit method for Odderon search

- least squares fitting with:

$$\chi^2 = \left(\sum_{j=1}^M \left(\sum_{i=1}^{n_j} \frac{(d_{ij} + \epsilon_{bj}\tilde{\sigma}_{bij} + \epsilon_{cj}d_{ij}\sigma_{cj} - th_{ij})^2}{\tilde{\sigma}_{ij}^2} \right) + \epsilon_{bj}^2 + \epsilon_{cj}^2 \right) + \left(\frac{d_{\sigma_{tot}} - th_{\sigma_{tot}}}{\delta\sigma_{tot}} \right)^2 + \left(\frac{d_{\rho_0} - th_{\rho_0}}{\delta\rho_0} \right)^2$$

$$\tilde{\sigma}_{ij}^2 = \tilde{\sigma}_{aij} \left(\frac{d_{ij} + \epsilon_{bj}\tilde{\sigma}_{bij} + \epsilon_{cj}d_{ij}\sigma_{cj}}{d_{ij}} \right)$$

$$\tilde{\sigma}_{kij} = \sqrt{\sigma_{kij}^2 + (d'_{ij}\delta_k t_{ij})^2}, \quad k \in \{a, b\}$$

[A. Adare et al. \(PHENIX Collab.\)](#)

[Phys. Rev. C 77, 064907](#)

- it takes into account (in M separately measured t ranges):

- the t -dependent statistical (a) and systematic (b) errors (both vertical σ_k and horizontal $\delta_k t$) $\rightarrow \epsilon_b$ parameters;
- the t -independent σ_c normalization uncertainties $\rightarrow \epsilon_c$ parameters;
- the measured total cross-section $d_{\sigma_{tot}}$ and ratio d_{ρ_0} and their total uncertainties $\delta\sigma_{tot}$ and $\delta\rho_0$.

- minimization with **CERN Root MINUIT**, parameter error estimation by **MINOS**.

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A. Adare et al. (PHENIX Collab.)

- The χ^2 parameters ϵ_i were considered as fit parameters in [T. Csörgő, I. Szanyi, Eur. Phys. J. C 81, 611 \(2021\)](#) decreasing NDF. [907](#) σ_k and horizontal $\delta_k t$ $\rightarrow \epsilon_b$ parameters;

 - the t -independent σ_c normalization uncertainties $\rightarrow \epsilon_c$ parameters;
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horizontal $(\delta_k, t) \rightarrow \epsilon_k$ parameters:

ϵ_i -s, in fact, must be considered as both measurements and fit parameters not effecting the NDF. This is done now.

- the measured total cross-section $\alpha_{\sigma_{tot}}$ and ratio α_{ρ_0} and their total uncertainties $\delta\sigma_{tot}$ and $\delta\rho_0$.

- minimization with **CERN Root MINUIT**, parameter error estimation by **MINOS**.

ReBB model analysis of pp and p \bar{p} data

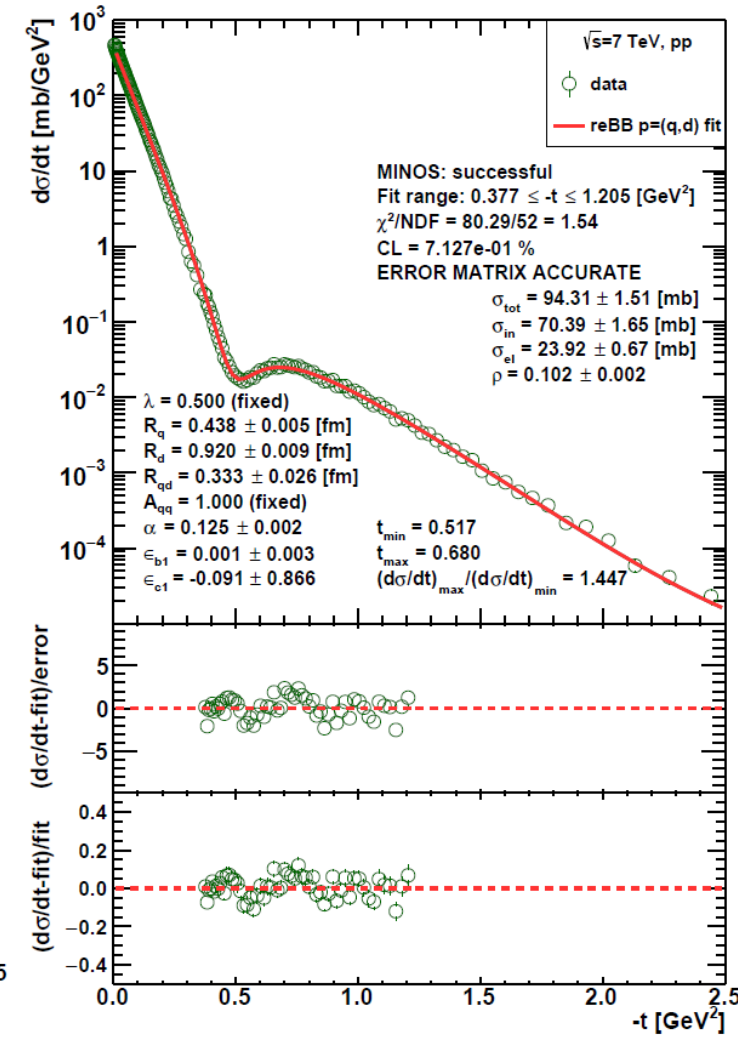
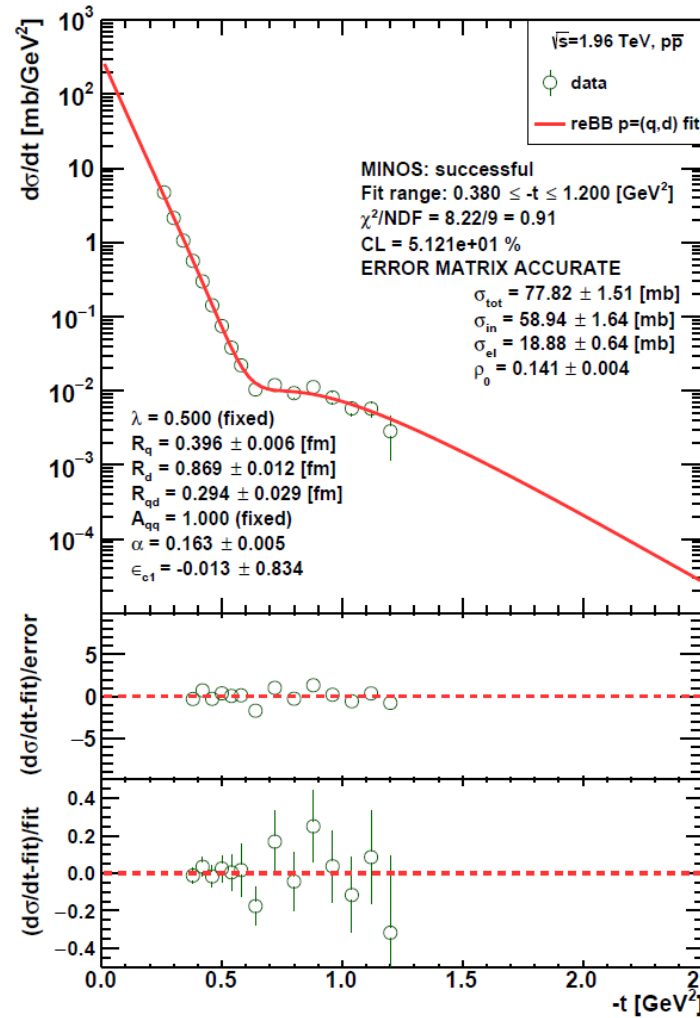
→ fits for pp d σ /dt data at 2.76 TeV and 7 TeV and for p \bar{p} d σ /dt data at 0.546 TeV and 1.96 TeV

→ use of the χ^2 definition developed by PHENIX

→ determination of the energy dependences of the model parameters

→ satisfactory description in the kinematical range: $0.546 \leq \sqrt{s} \leq 7$ TeV & $0.37 \leq -t \leq 1.2$ GeV²

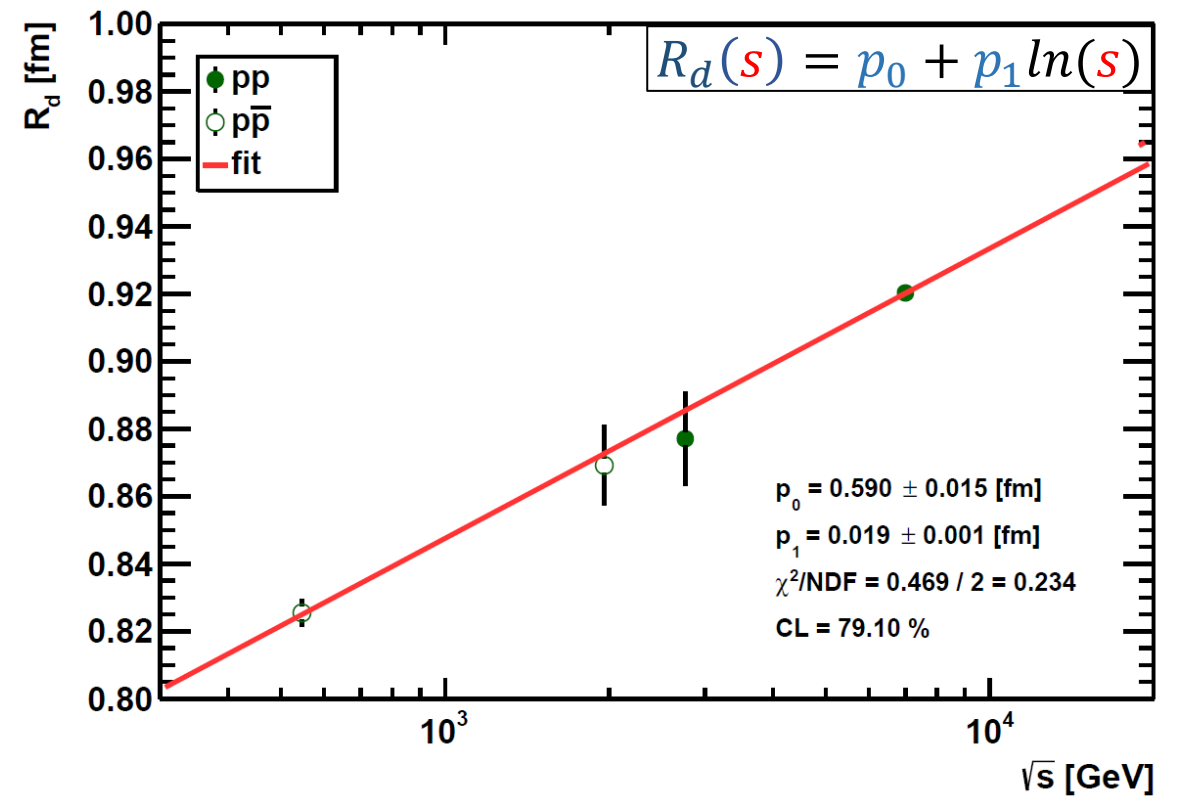
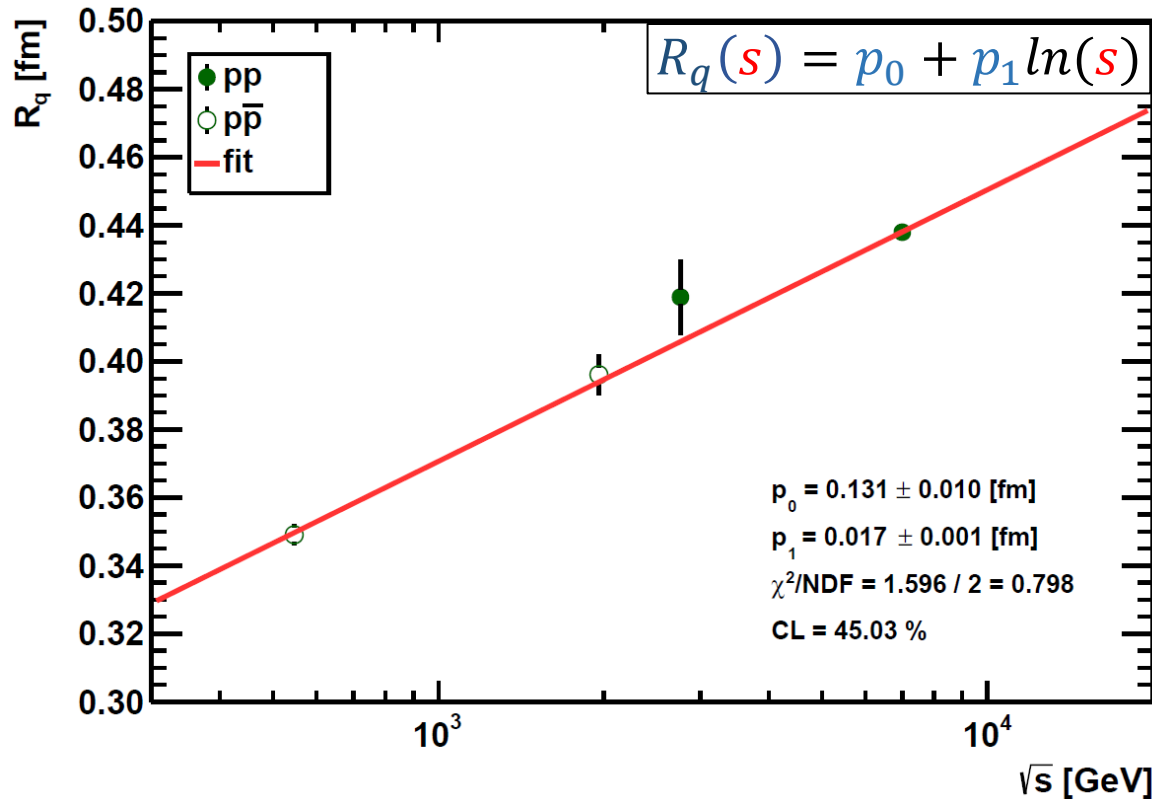
→ observation of an at least 7 σ Odderon signal (now corrected to 6.7 σ)



Examples of ReBB model fits for pp and p \bar{p} differential cross section data.

Energy dependences of the ReBB model parameters

T. Csörgő, I. Szanyi, *Eur. Phys. J. C* 81, 611 (2021)

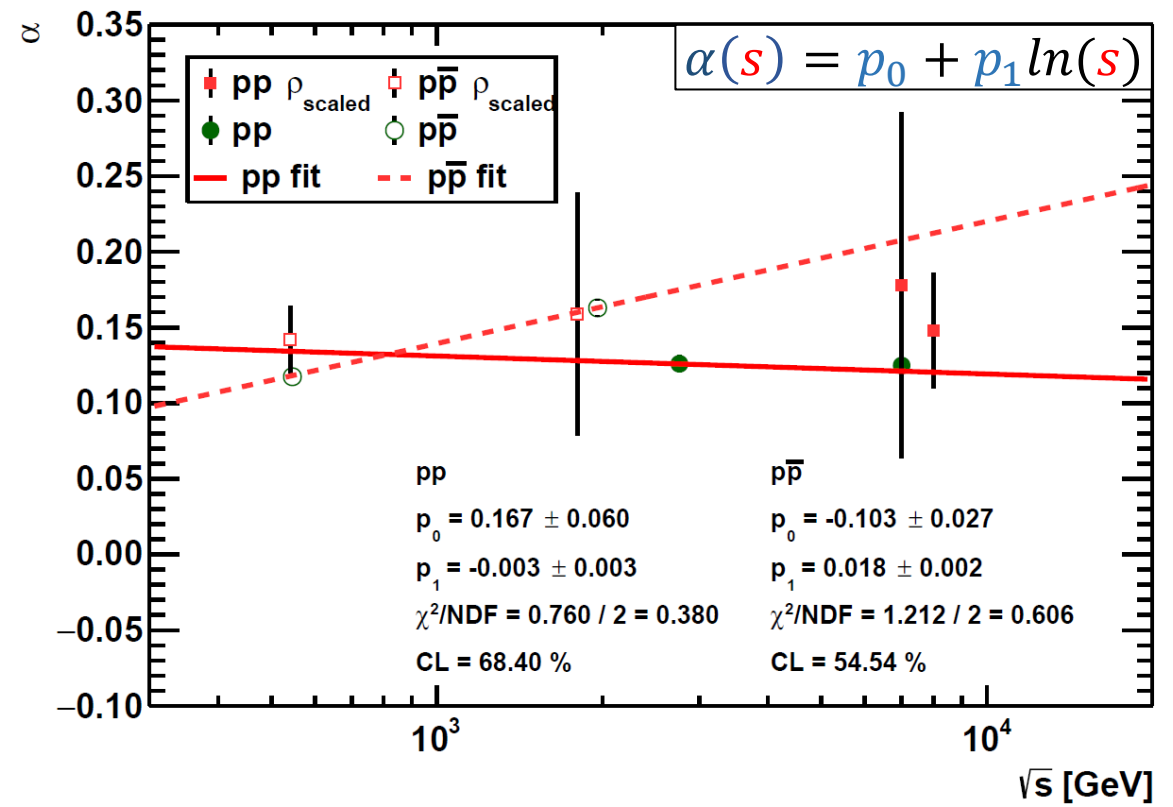
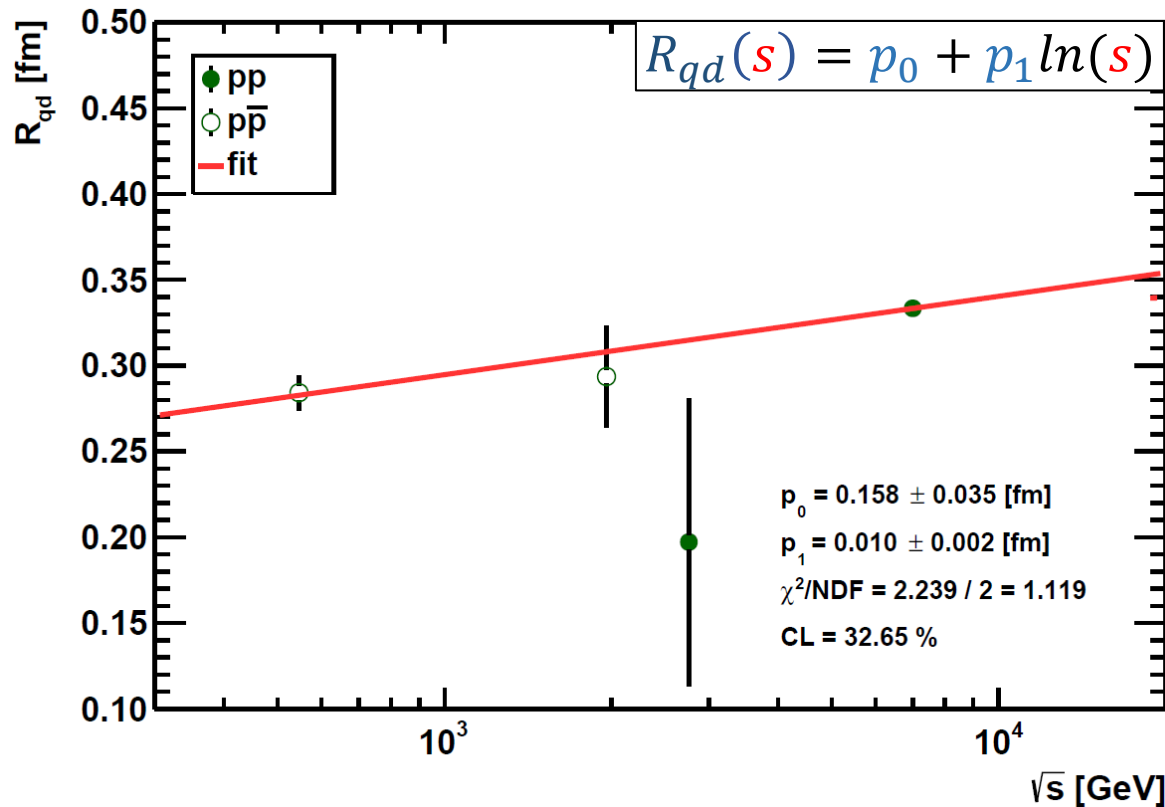


The energy dependences of the scale parameters, $R_q(s)$, $R_d(s)$, and $R_{qd}(s)$ are **linear logarithmic** and the **same** for pp and $p\bar{p}$ processes!

The energy dependence of the α parameter, $\alpha(s)$ is **linear logarithmic** too, but **not** the same for pp and $p\bar{p}$ processes!

Energy dependences of the ReBB model parameters

T. Csörgő, I. Szanyi, *Eur. Phys. J. C* 81, 611 (2021)

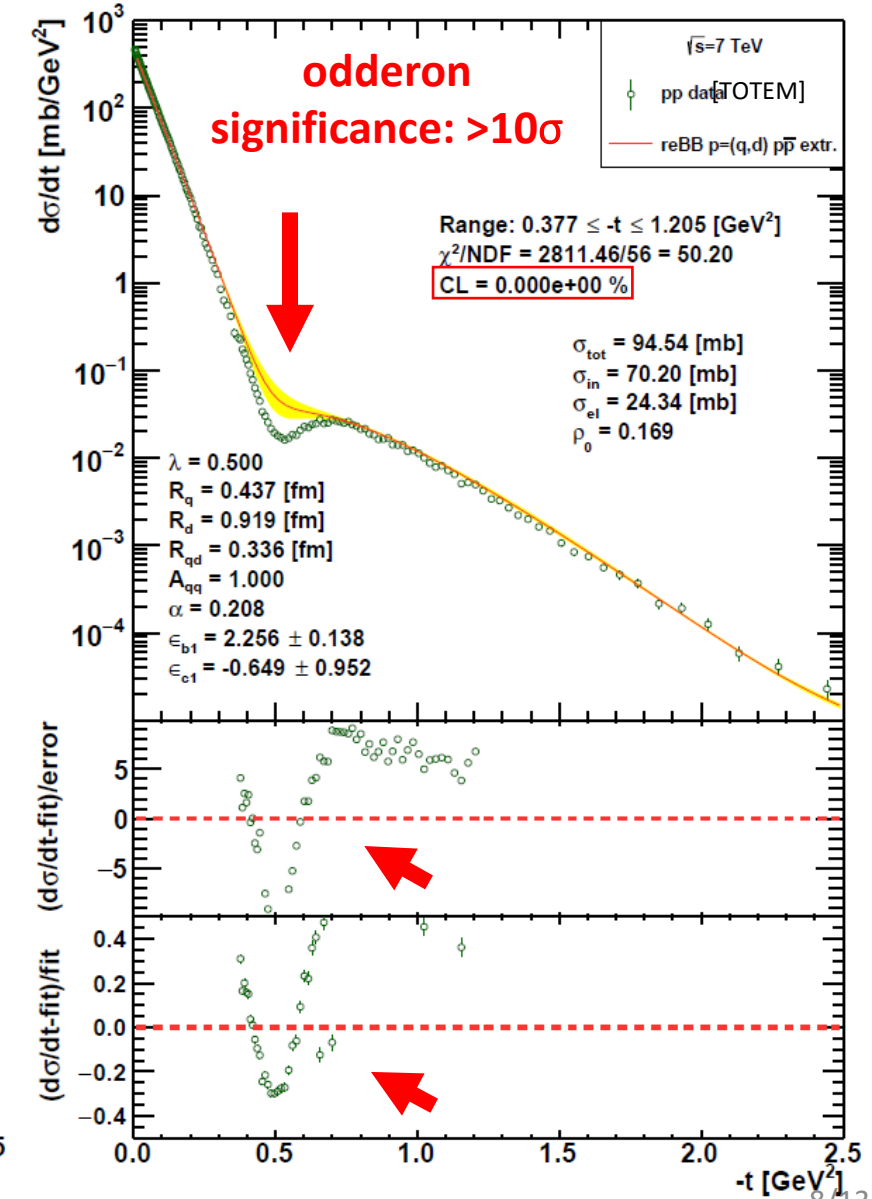
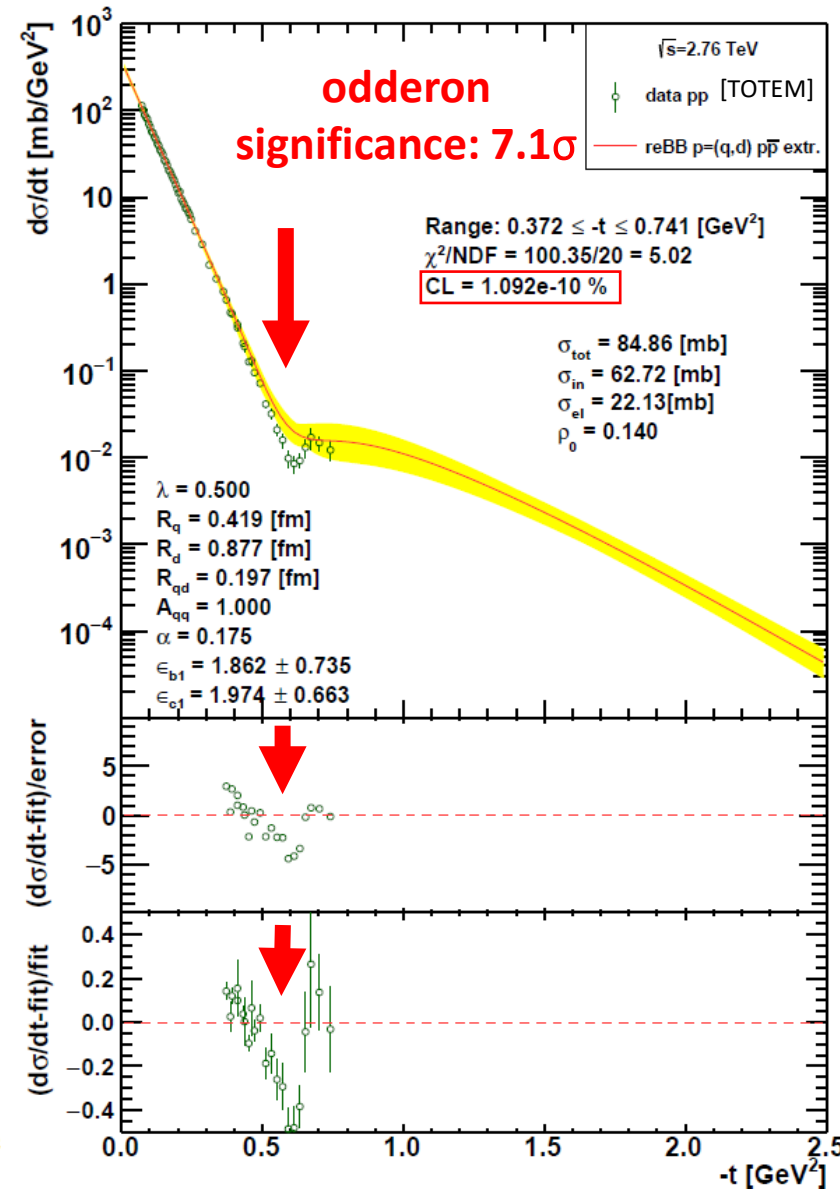
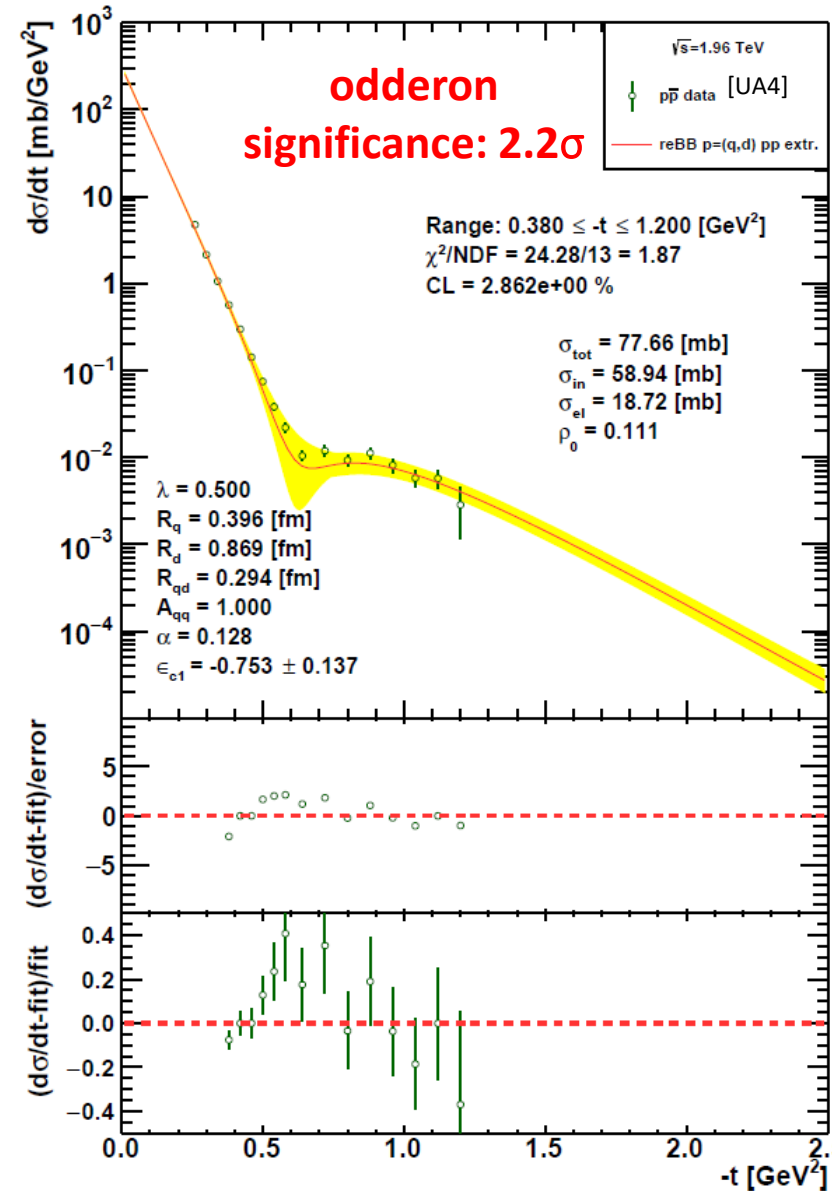


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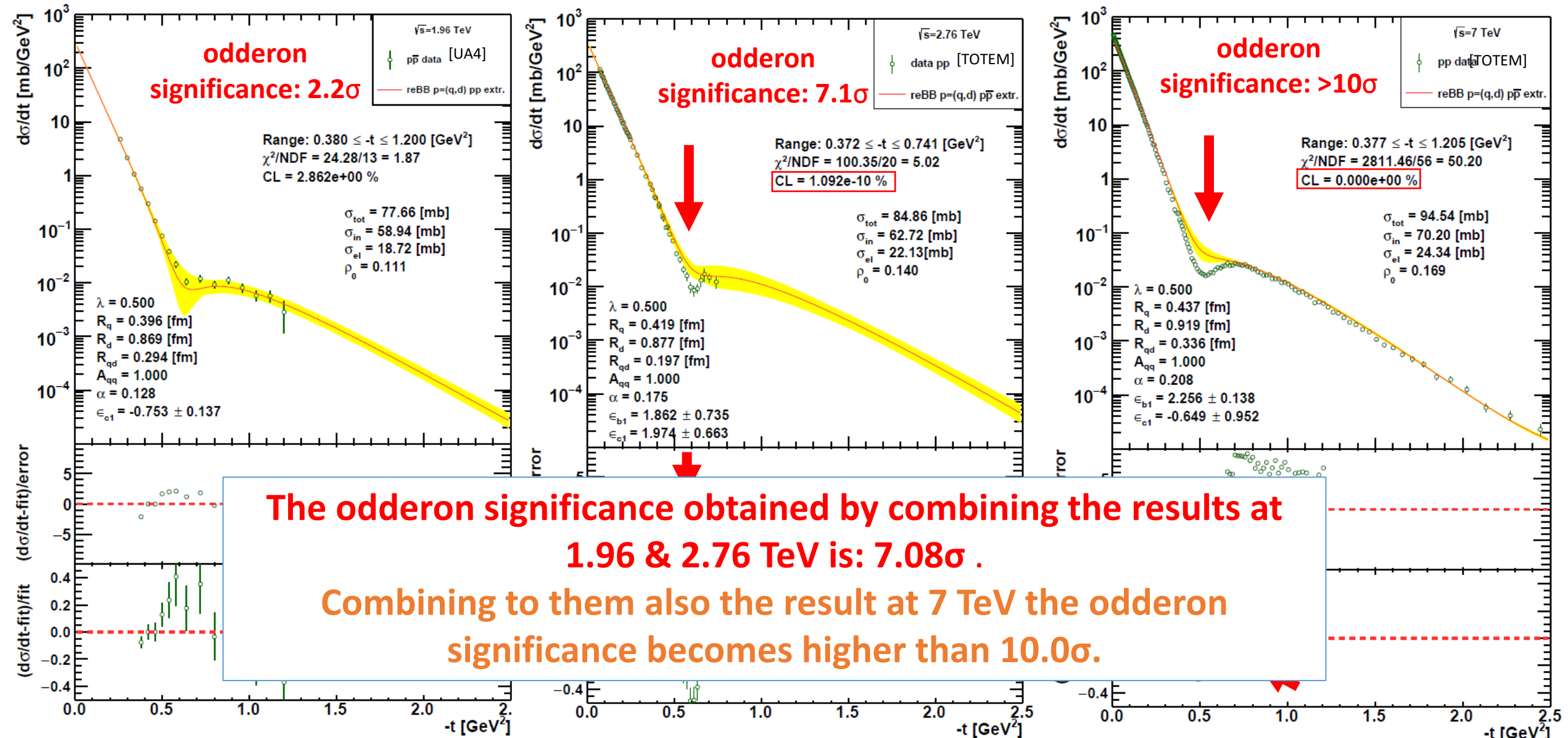
Extrapolations: ODDERON

T. Csörgő, I. Szanyi, *Eur. Phys. J. C* 81, 611 (2021)



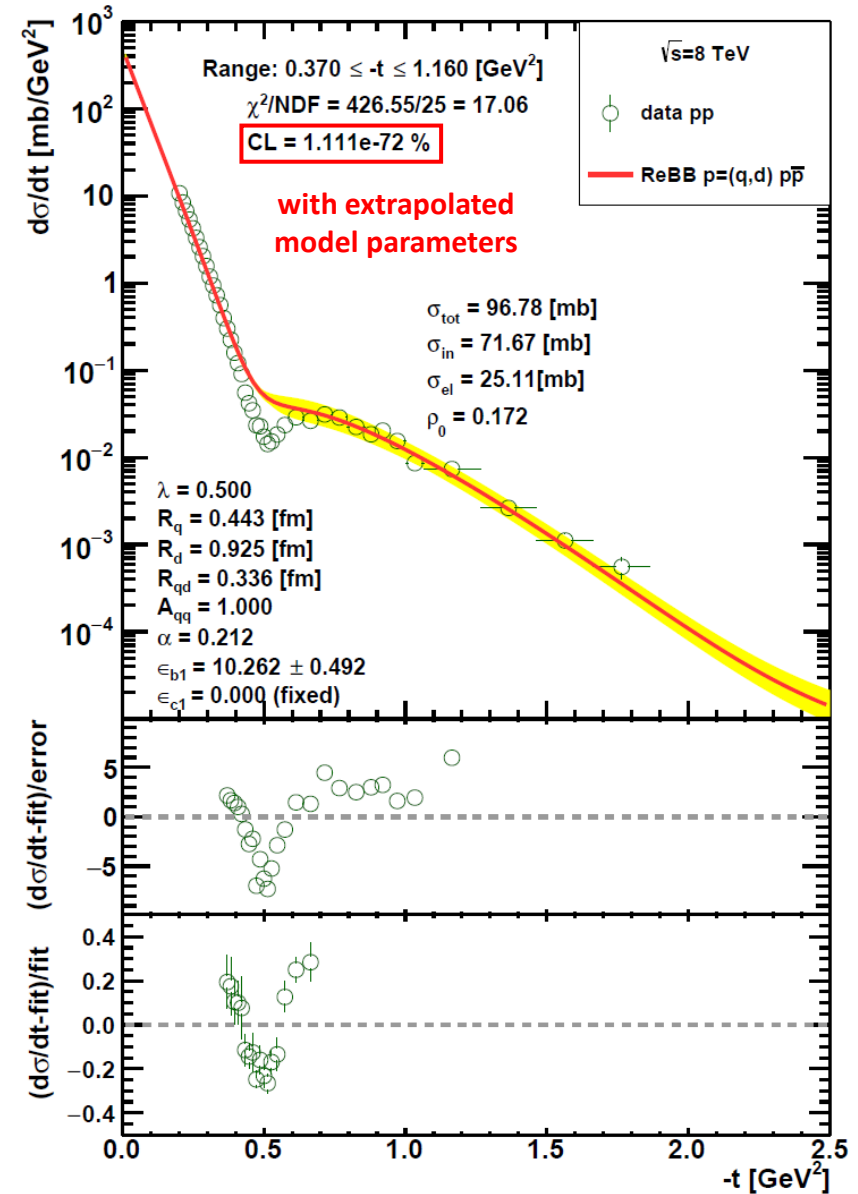
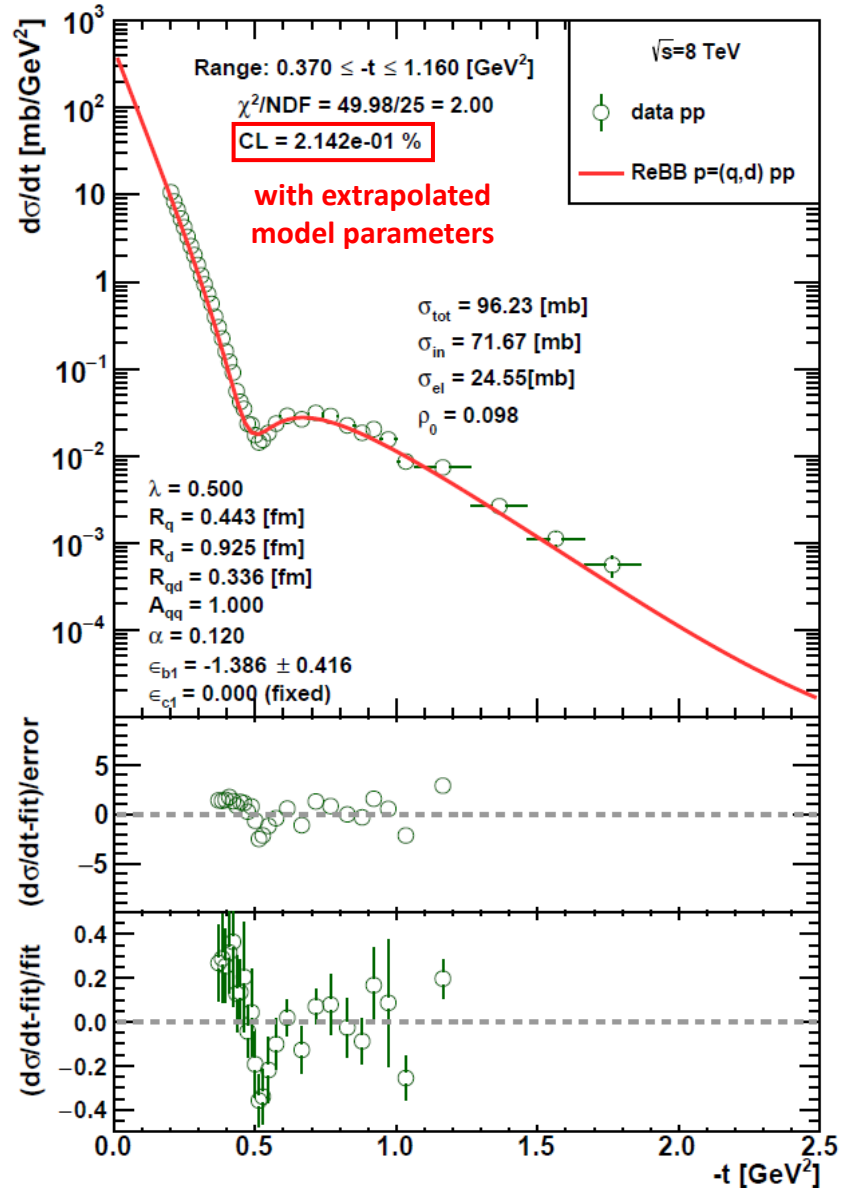
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ReBB model & Odderon @ 8 TeV

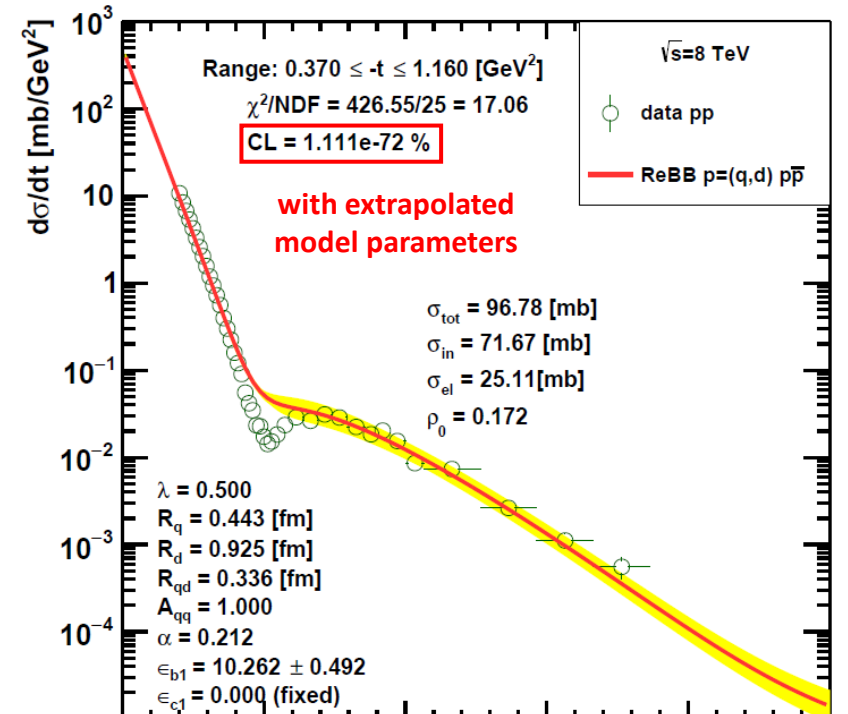
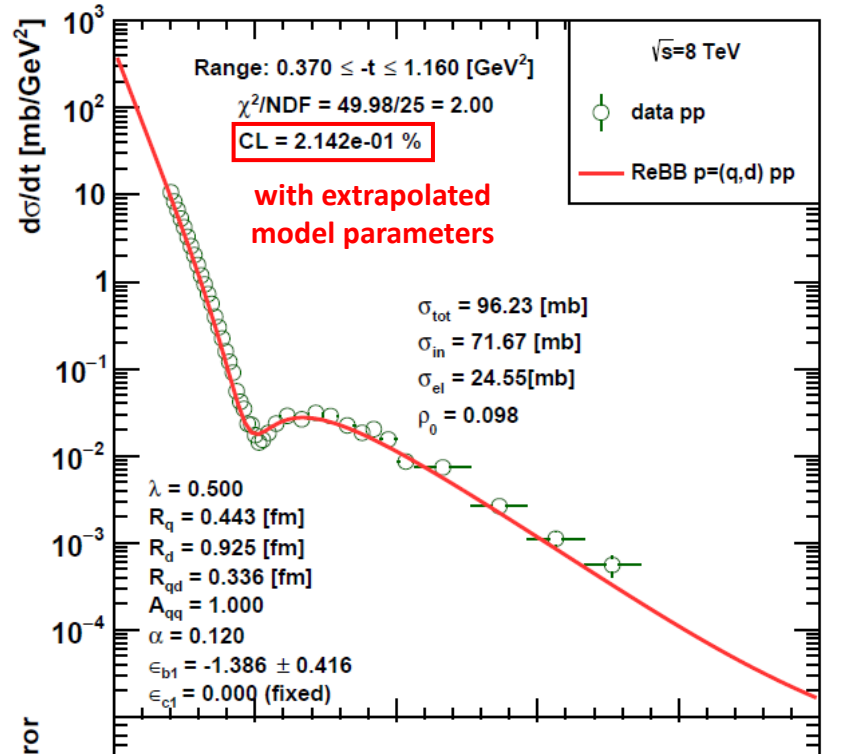
I. Szanyi, T. Csörgő, *Eur. Phys. J. C* 82, 827 (2022)



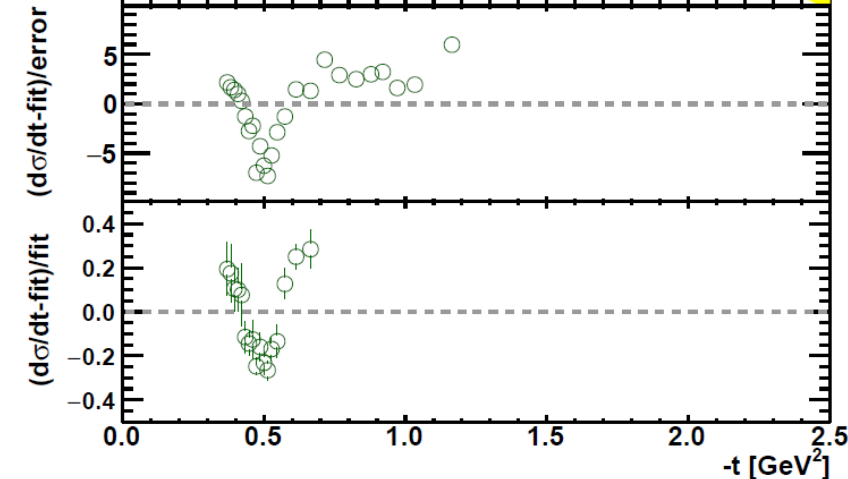
data: TOTEM Collab., *Eur. Phys. J. C* 82, 263 (2022)

ReBB model & Odderon @ 8 TeV

I. Szanyi, T. Csörgő, *Eur. Phys. J. C* 82, 827 (2022)



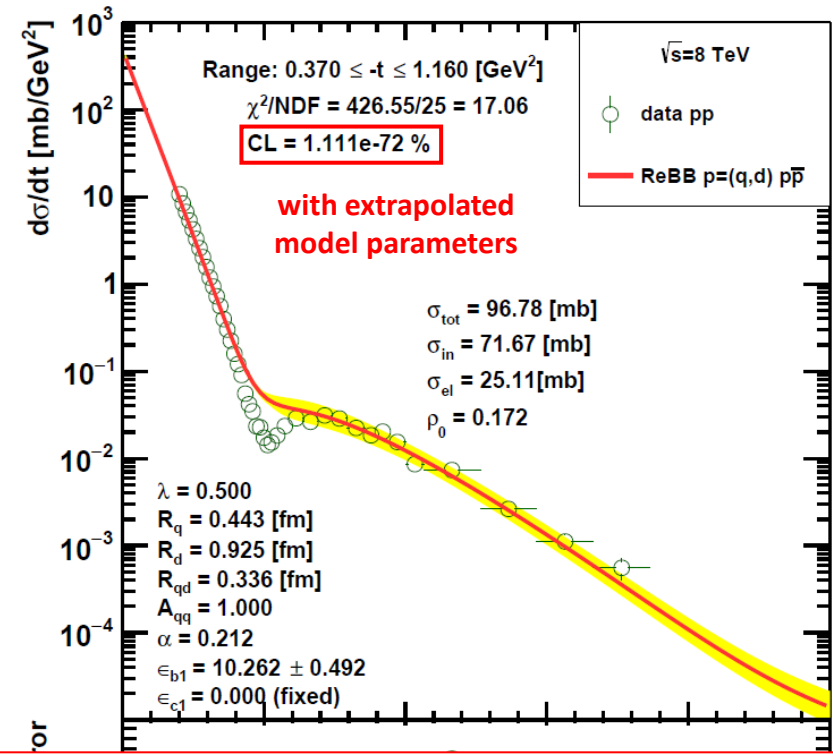
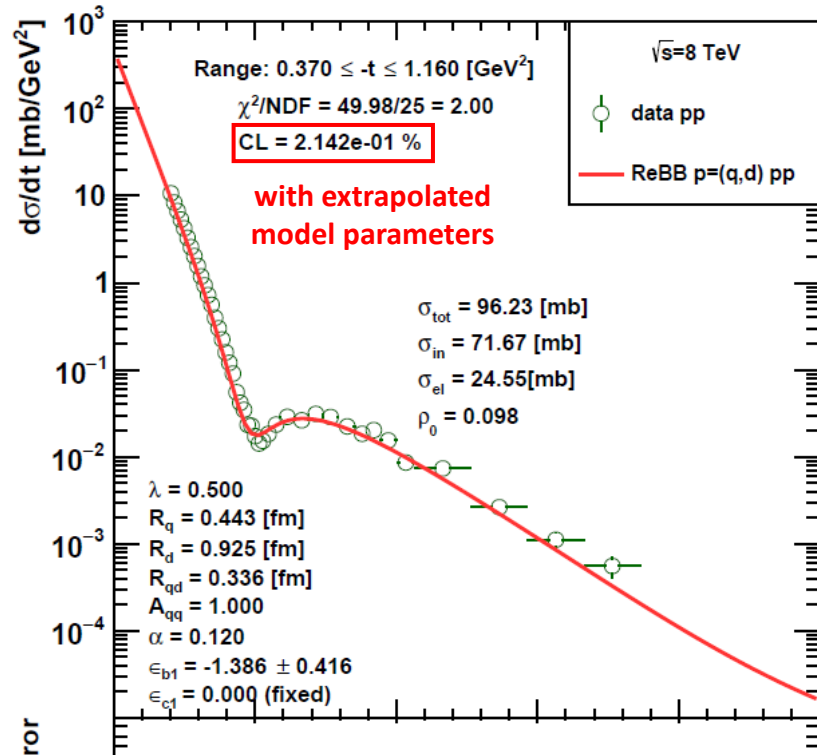
CL > 0.1 % from the comparison of pp data and pp ReBB model curve
 → ReBB model gives a statistically acceptable description @ 8 TeV



data: TOTEM Collab., *Eur. Phys. J. C* 82, 263 (2022)

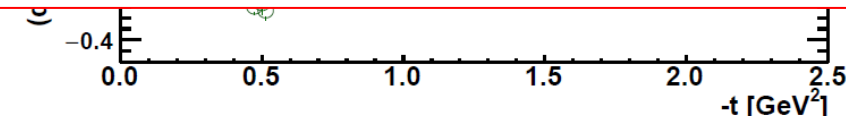
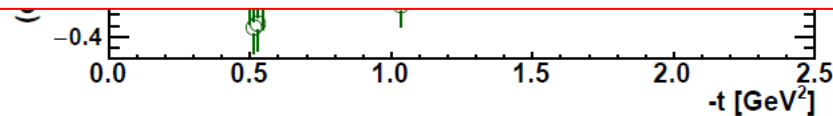
ReBB model & Odderon @ 8 TeV

I. Szanyi, T. Csörgő, *Eur. Phys. J. C* 82, 827 (2022)



CL > 0.1 % from the comparison of pp data and pp ReBB model curve
 → ReBB model gives a statistically acceptable description @ 8 TeV

CL ≤ 1.111 × 10⁻⁷² % from the comparison of pp data and p-pbar ReBB model curve
 → a model-dependent Odderon significance ≥ 18 σ @ 8 TeV



data: TOTEM Collab., *Eur. Phys. J. C* 82, 263 (2022)

Analytical approximation for significance calculation

I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (2022)

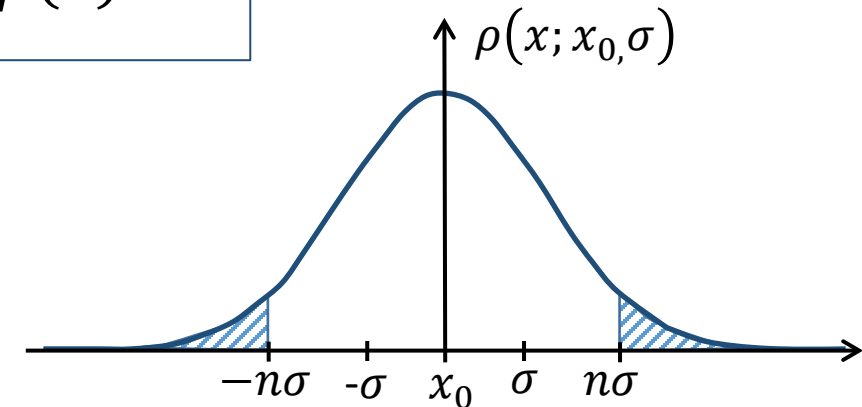
- the Gaussian probability density function with mean x_0 and variance σ^2 :

$$\rho(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} dx \rho(x) = 1$$

- the confidence level corresponding to $n\sigma$ significance:

$$CL = 2 \int_{x_0+n\sigma}^{\infty} dx \rho(x)$$



- applying a variable change, $x \rightarrow x' = x - (x_0 + n\sigma)$:

$$CL = \sqrt{\frac{2}{\pi\sigma^2}} \int_0^{\infty} dx' e^{-\frac{(x'+n\sigma)^2}{2\sigma^2}} = \sqrt{\frac{2}{\pi\sigma^2}} e^{-\frac{n^2}{2}} \int_0^{\infty} dx' e^{-\frac{x'^2+2x'n\sigma}{2\sigma^2}} \leq \sqrt{\frac{2}{\pi\sigma^2}} e^{-\frac{n^2}{2}} \int_0^{\infty} dx' e^{-\frac{x'n}{\sigma}}$$

$$CL \leq \sqrt{\frac{2}{\pi n^2}} e^{-\frac{n^2}{2}}$$

this formula gives the lower limit for the significance n in σ -s corresponding to a CL value

Odderon observation within the ReBB model analysis

I. Szanyi, T. Csörgő, *Eur. Phys. J. C* 82, 827 (2022)

\sqrt{s} (TeV)	χ^2	NDF	CL	significance (σ)
1.96	24.283	14	0.0423	2.0
2.76	100.347	22	5.6093×10^{-12}	6.89
7	2811.46	58	$< 7.2853 \times 10^{-312}$	> 37.7
8	426.553	25	1.1111×10^{-74}	≥ 18.2

\sqrt{s} of combined data (TeV)	χ^2	NDF	CL	combined significance (σ)	
				χ^2 /NDF method	Stouffner method
1.96 & 2.76	124.63	36	1.0688×10^{-11}	6.79	6.3
1.96 & 2.76 & 7	2936.09	94	$< 9.1328 \times 10^{-312}$	> 37.7	> 26.9
1.96 & 2.76 & 8	551.183	61	4.6307×10^{-80}	> 18.9	> 15.7
1.96 & 2.76 & 7 & 8	3362.64	119	$< 8.0654 \times 10^{-312}$	> 37.7	> 32.4

- combination of significances by summing the individual χ^2 and *NDF* values:

$$\chi^2 = \sum_i \chi_i^2$$

$$NDF = \sum_i NDF_i$$

- combination of significances s_i by Stouffner method:

$$S = \frac{\sum_{i=1}^N s_i}{\sqrt{N}}$$

Odderon observation within the ReBB model analysis

I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (2022)

> 6 σ combined Odderon signal from the data-model comparison at the two lowest energies, 1.96 & 2.76 TeV

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I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (2022)

> **6 σ combined Odderon signal from the data-model comparison at the two lowest energies, 1.96 & 2.76 TeV**

> **30 σ combined Odderon signal from the data-model comparison at all the four energies, 1.96, 2.76, 7 & 8 TeV**

\sqrt{s} of combined data (TeV)	χ^2	NDF	CL	combined significance (σ) χ^2 /NDF method	combined significance (σ) Stouffner method
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Summary and conclusions

- the ReBB model represents all available pp and $p\bar{p}$ $d\sigma/dt$ data in the kinematical ranges $0.546 \leq \sqrt{s} \leq 8$ TeV and $0.37 \leq -t \leq 1.2$ GeV² in a statistically acceptable manner
- a comparative study of pp and $p\bar{p}$ differential cross sections is done with ReBB model by interpolations & extrapolations to the same kinematical regions
- **model-dependent evidence for Odderon exchange in t-channel is observed**
 - the combined significance is $> 6 \sigma$ for the two lowest energies at TeV scale, i.e. 1.96 and 2.76 TeV
 - the combined significance is $> 30 \sigma$ for four energies at the TeV scale i.e. 1.96, 2.76, 7 and 8 TeV
- **Conclusion: the Odderon exchange is a certainty in the ReBB model study**

I. Szanyi, T. Csörgő, Eur. Phys. J. C 82, 827 (19 September 2022)

Thank you for your attention!

Backup slides

Measurable quantities

- differential cross section:

$$\frac{d\sigma}{dt}(s, t) = \frac{1}{4\pi} |T(s, t)|^2$$

- total, elastic and inelastic cross sections:

$$\sigma_{tot}(s) = 2\text{Im}T(s, t = 0)$$

$$\sigma_{el}(s) = \int_{-\infty}^0 \frac{d\sigma(s, t)}{dt} dt$$

$$\sigma_{in}(s) = \sigma_{tot}(s) - \sigma_{el}(s)$$

- ratio ρ_0 :

$$\rho_0(s) = \lim_{t \rightarrow 0} \rho(s, t) \equiv \frac{\text{Re}T(s, t \rightarrow 0)}{\text{Im}T(s, t \rightarrow 0)}$$

- slope of $d\sigma/dt$:

$$B(s, t) = \frac{d}{dt} \left(\ln \frac{d\sigma}{dt}(s, t) \right)$$

$$B_0(s) = \lim_{t \rightarrow 0} B(s, t)$$

Proportionality between $\rho_0(s)$ and $\alpha(s)$

$$t_{el}(s, b) = i \left(1 - e^{i \alpha \tilde{\sigma}_{in}(s, b)} \sqrt{1 - \tilde{\sigma}_{in}(s, b)} \right)$$

$$\alpha \tilde{\sigma}_{in} \ll 1$$

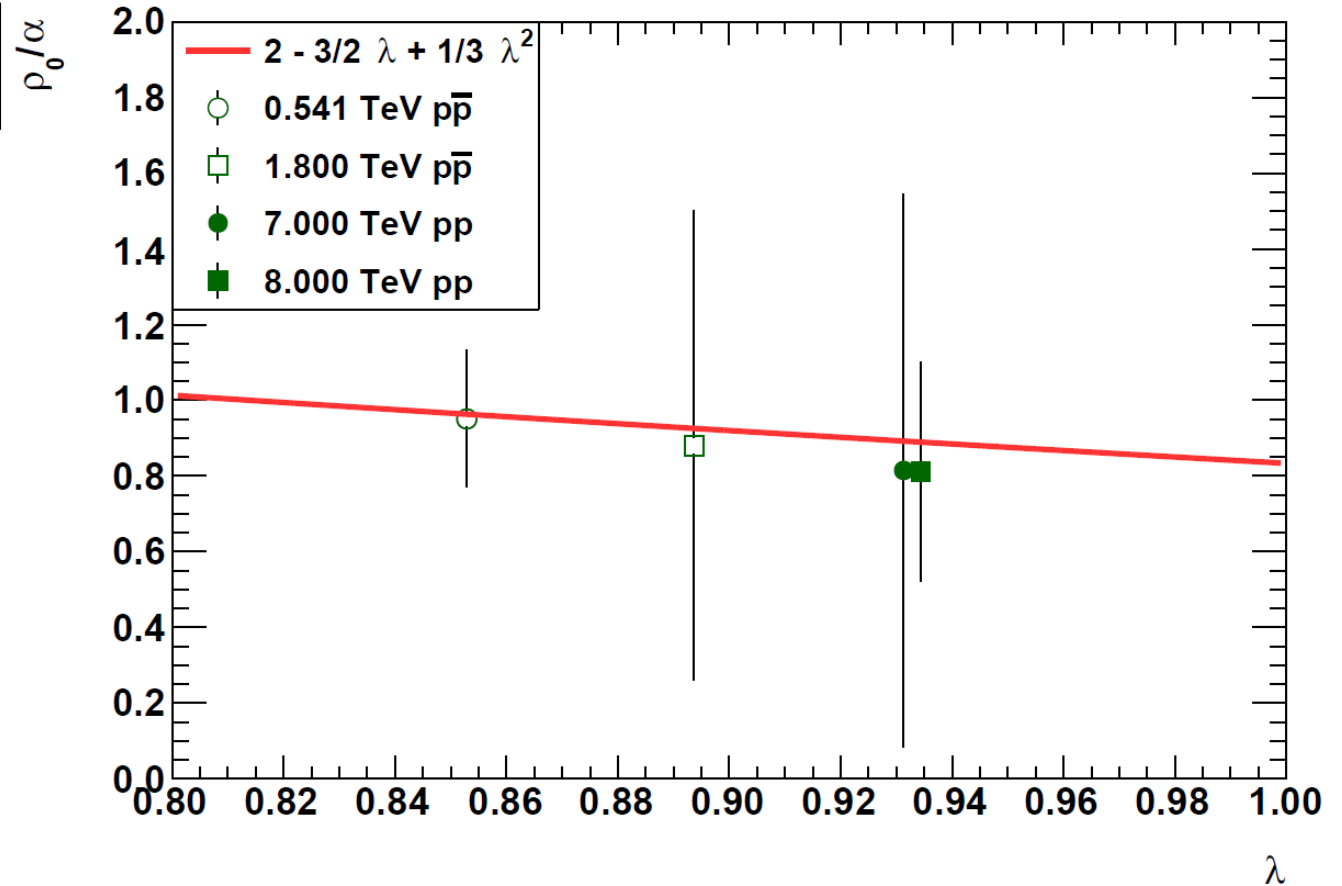
$$\text{Im } t_{el}(s, b) \simeq \lambda(s) \exp\left(-\frac{b^2}{2R^2(s)}\right)$$



$$\rho_0(s) = \alpha(s) \left(2 - \frac{3}{2} \lambda(s) + \frac{1}{3} \lambda^2(s) \right)$$

$$\lambda(s) = \text{Im } t_{el}(s, b = 0)$$

→ by rescaling one can get additional α parameter values at energies where ρ_0 is measured (and vice versa)



The dependence of ρ_0/α on $\lambda = \text{Im } t_{el}(s, b = 0)$ in the TeV energy range. The data points are generated numerically by using the trends of the ReBB model scale parameters and the experimentally measured ρ -parameter values.

H(x) scaling of the ReBB model

- the H(x) scaling is present in the energy range $\sqrt{s_1} \leq \sqrt{s} \leq \sqrt{s_2}$ if the

$$H(x, s) = \frac{1}{B_0(s)\sigma_{el}(s)} \left. \frac{d\sigma_{el}}{dt} \right|_{x=-tB_0(s)}$$

scaling function is energy independent in that range i.e. $H(x, s_1) = H(x, s_2)$.

- conditions for ReBB model H(x) scaling to be present:

T. Csörgő, T. Novák, R. Pasechnik, A. Ster,
and I. Szanyi, Eur. Phys. J. C 81, 180 (2021)

- energy independence for the α parameter (or ρ_0)

$$\alpha(s) = \alpha(s_0) \quad (\text{or } \rho_0(s) = \rho_0(s_0) \text{ since } \alpha \sim \rho_0)$$

- the energy dependence of the scale parameters is determined by the same factorizable $b(s)$ scaling function

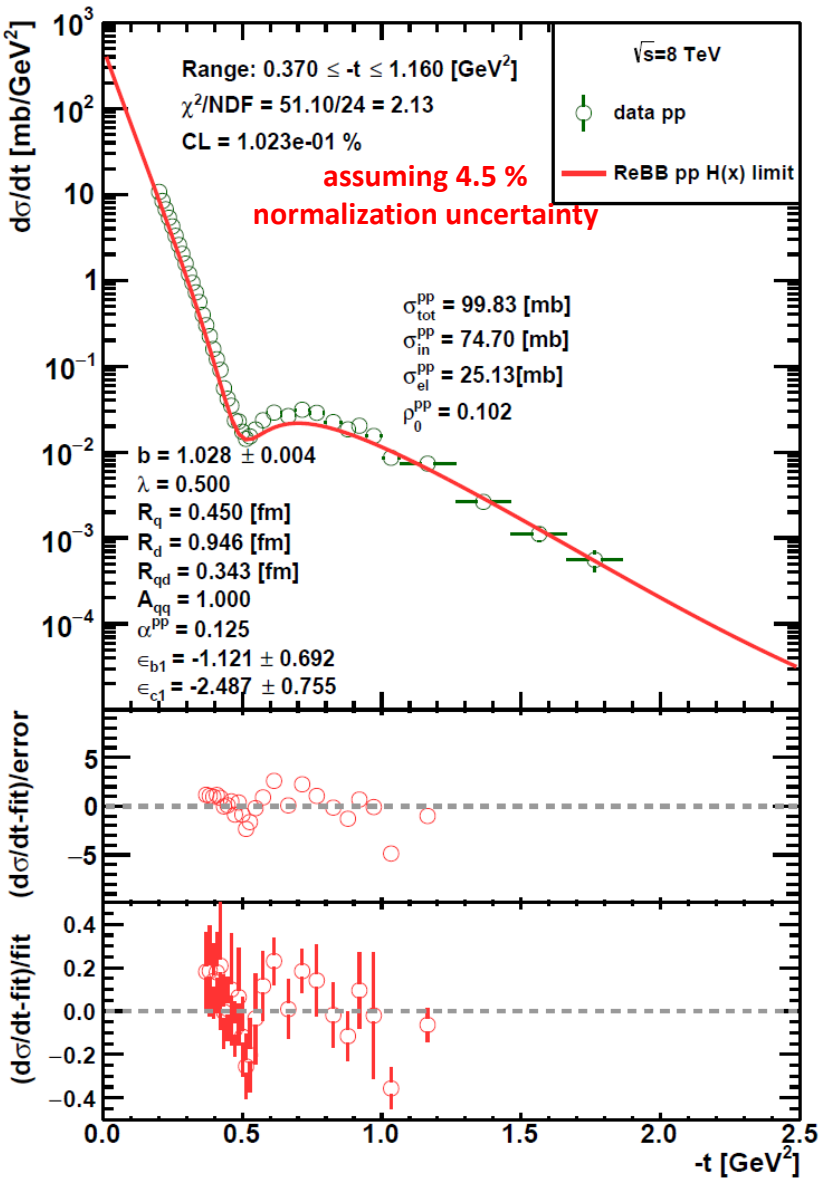
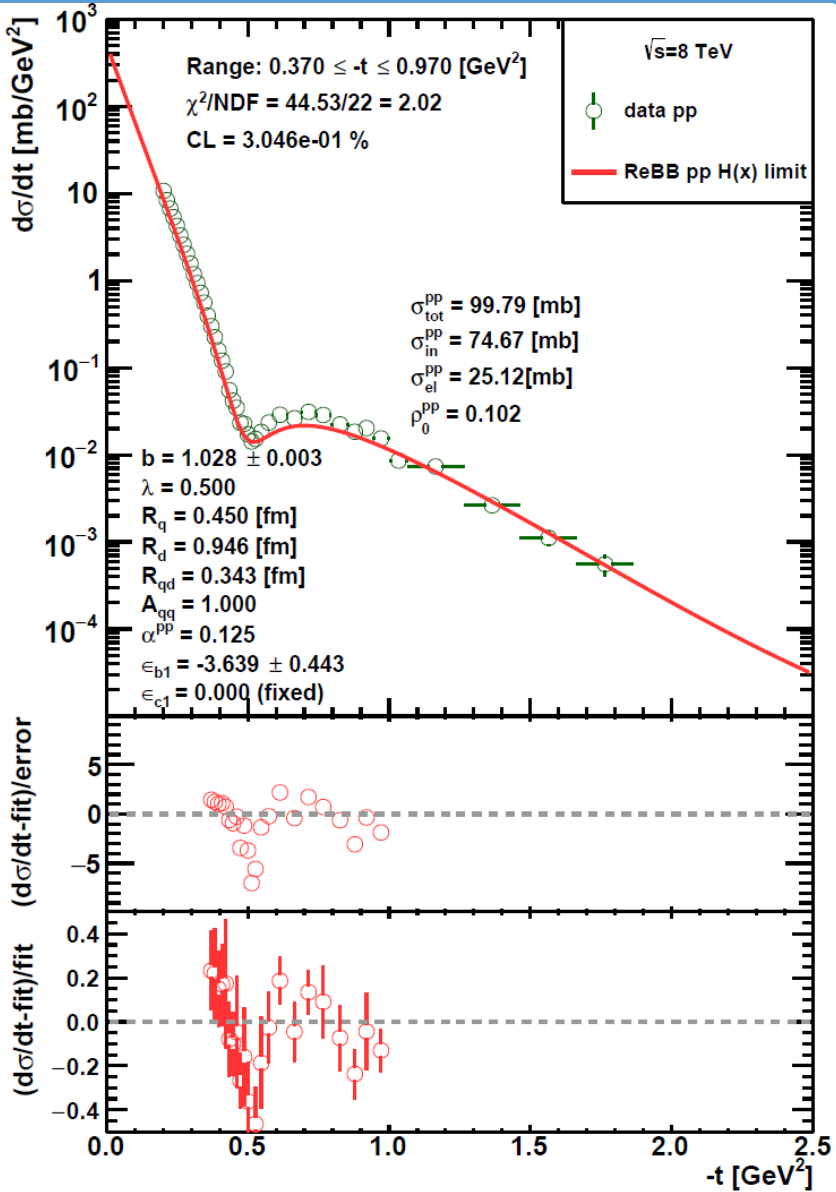
$$R_q(s) = b(s)R_q(s_0)$$

$$R_d(s) = b(s)R_d(s_0)$$

$$R_{qd}(s) = b(s)R_{qd}(s_0)$$

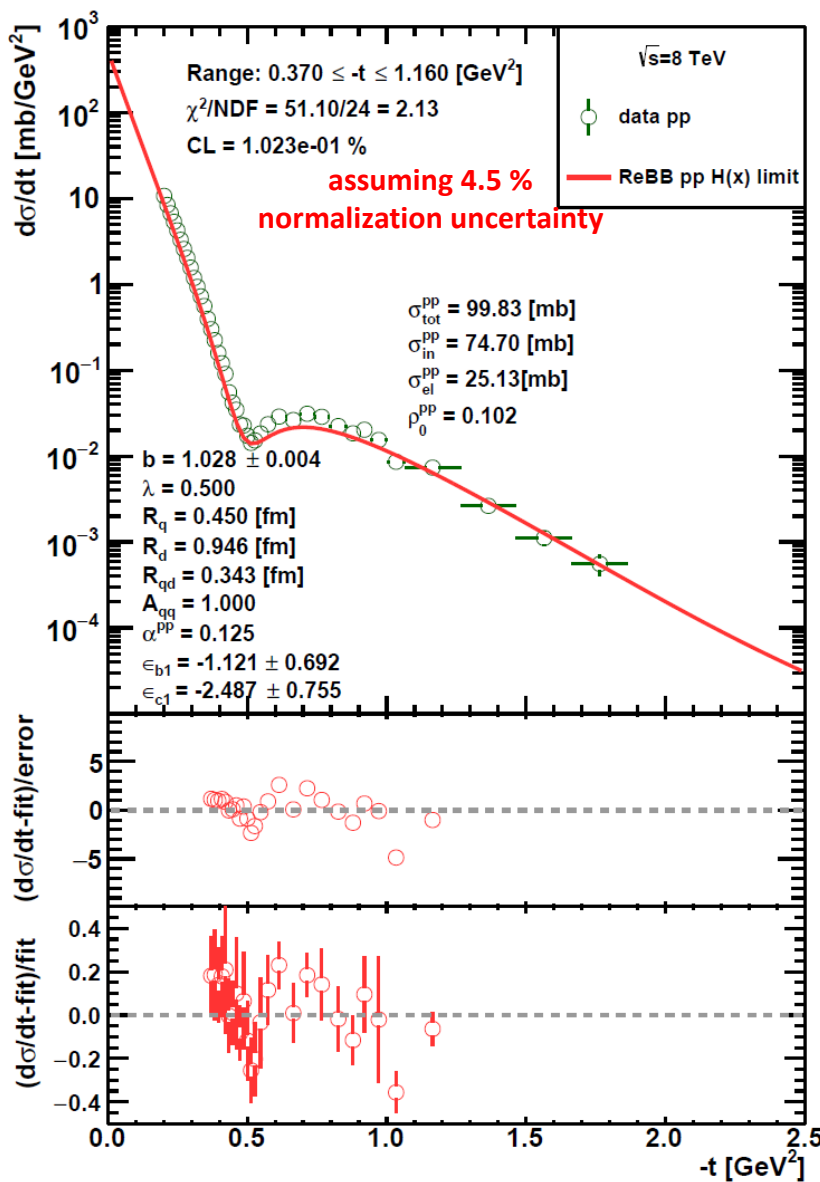
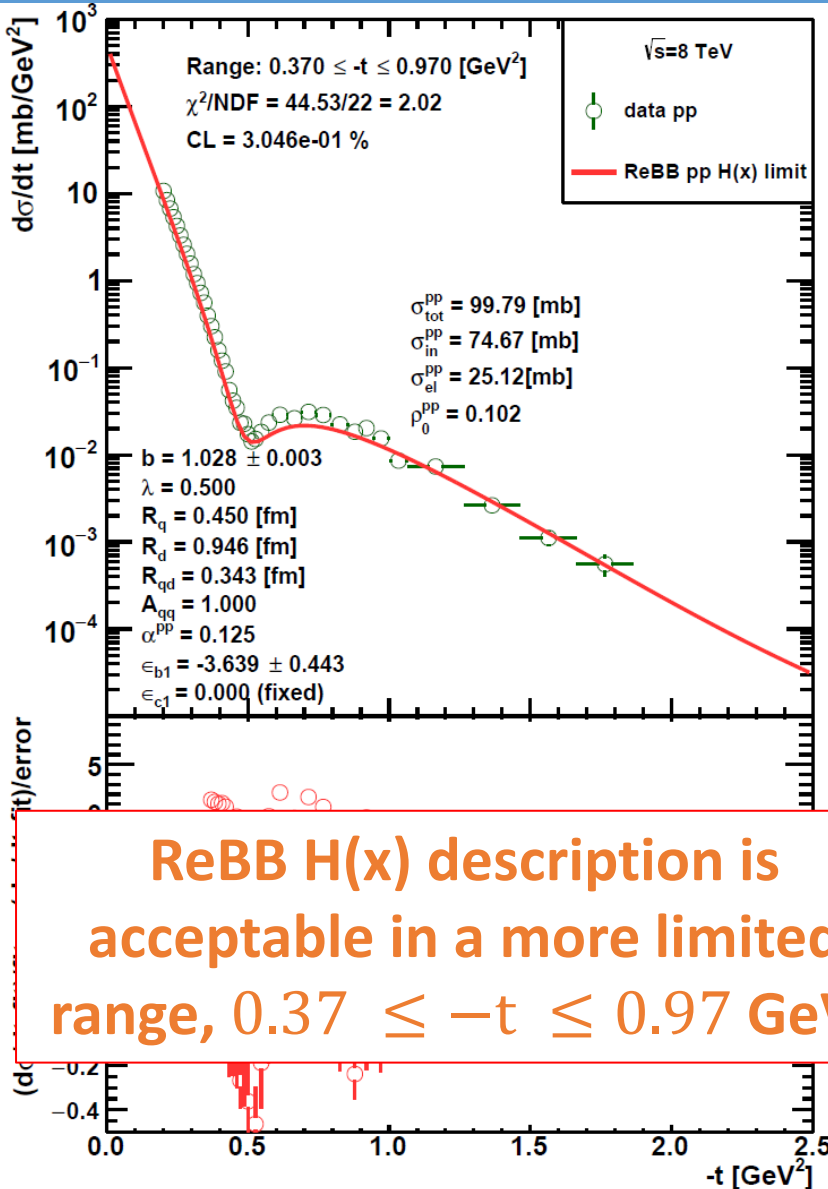
($\sqrt{s_0}$ is a reference energy to be chosen)

ReBB H(x) model @ 8 TeV



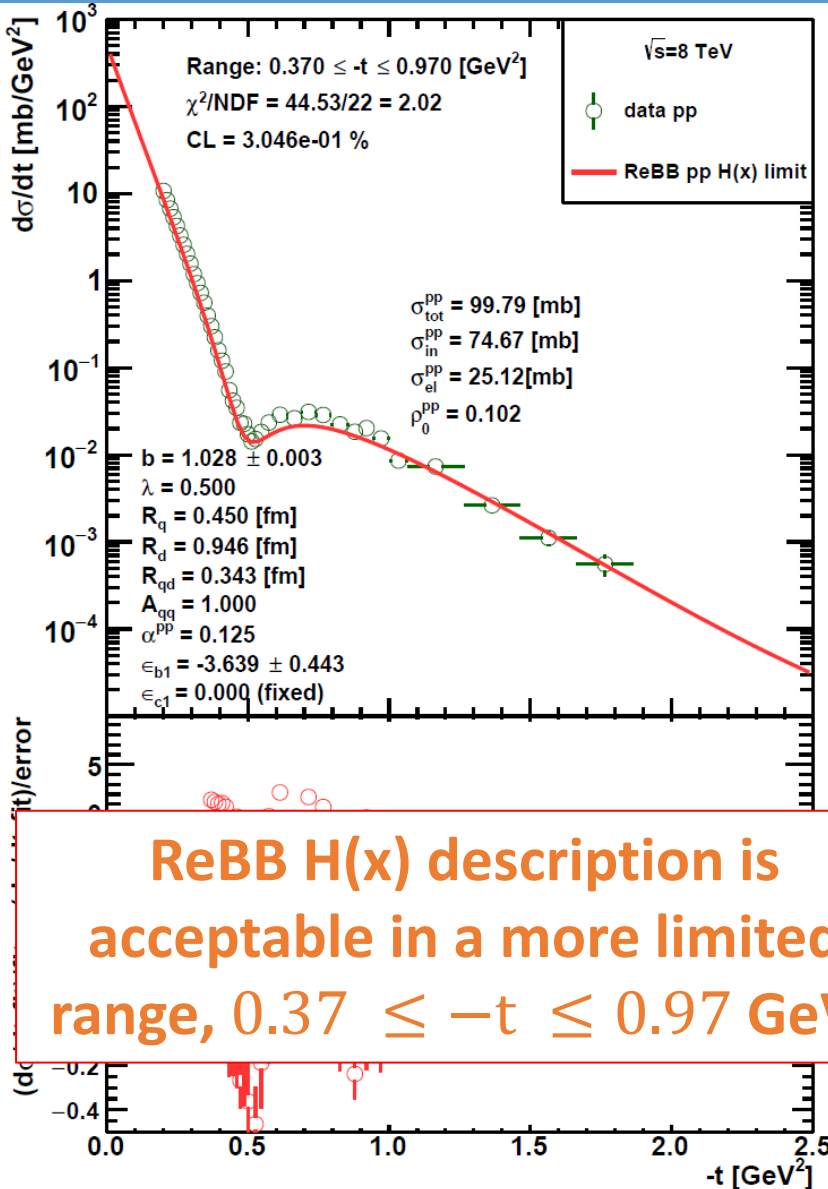
data: TOTEM Collab., *Eur. Phys. J. C* 82, 263 (2022)

ReBB H(x) model @ 8 TeV

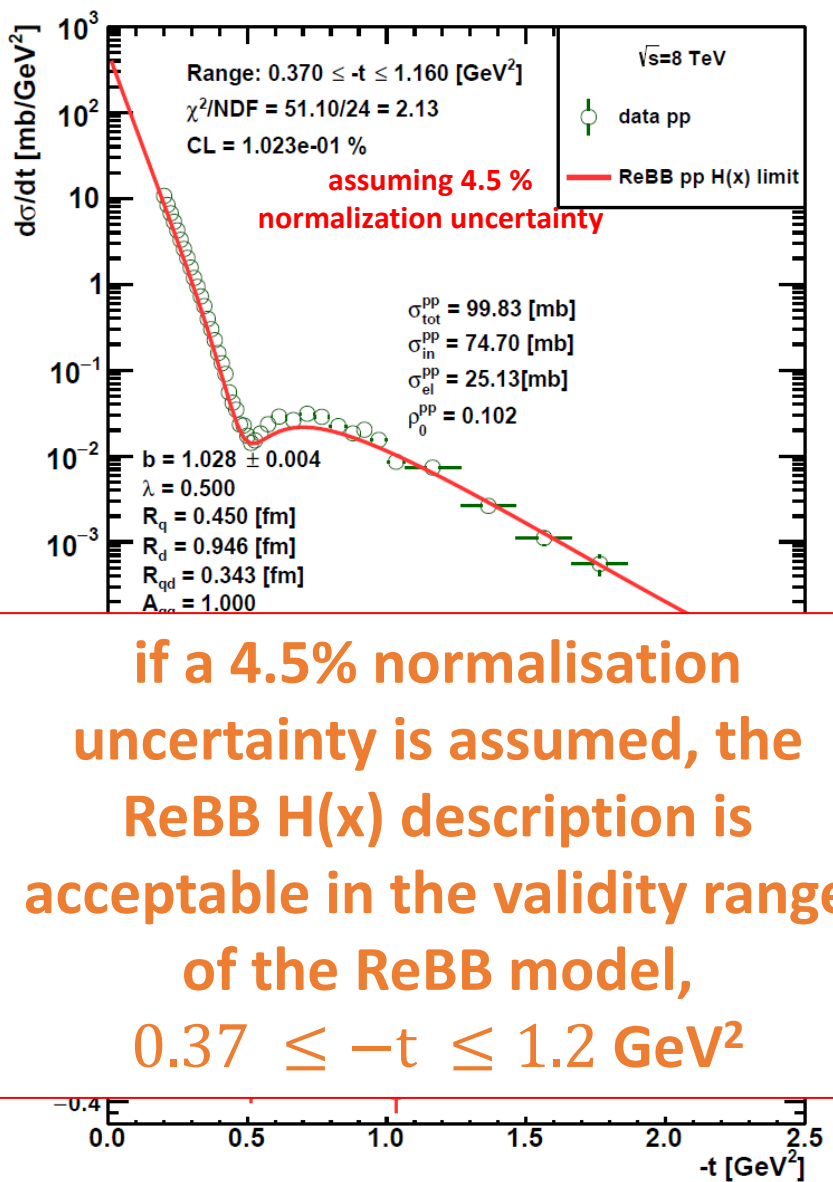


data: TOTEM Collab., *Eur. Phys. J. C* 82, 263 (2022)

ReBB H(x) model @ 8 TeV

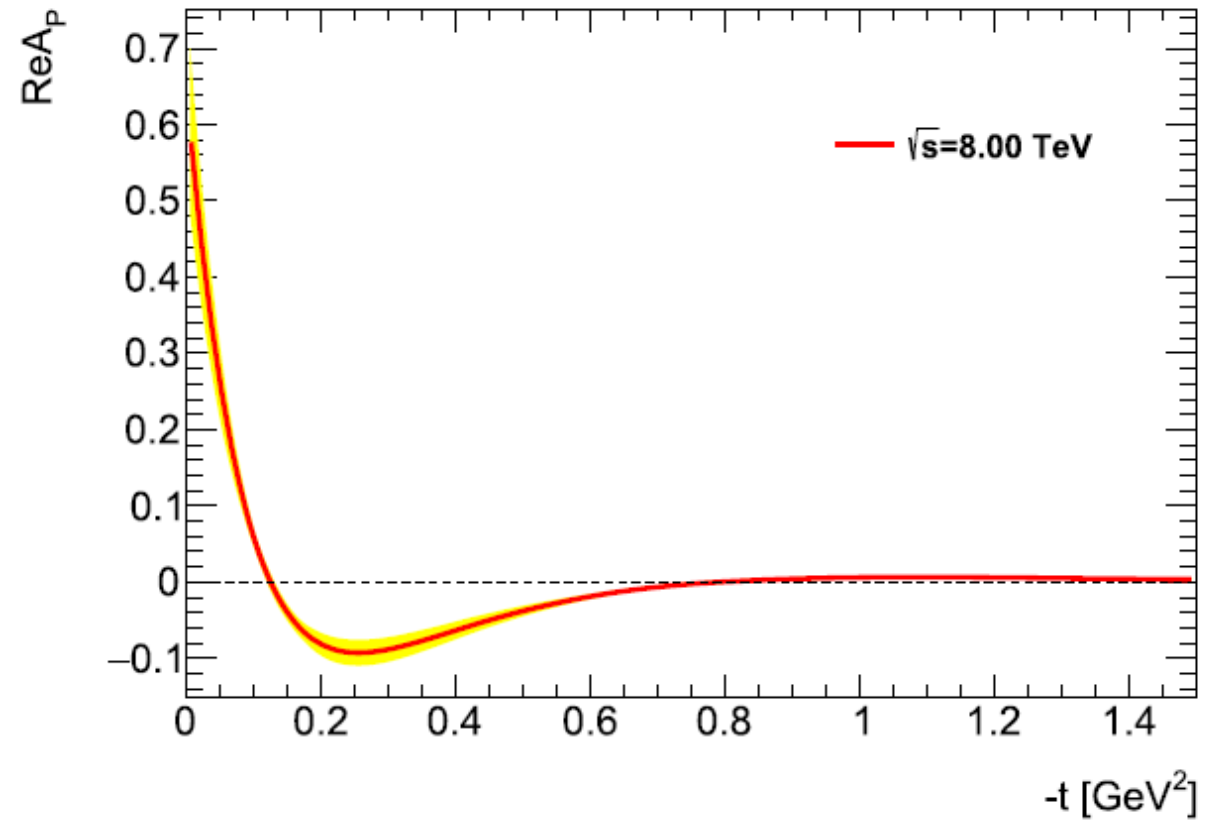
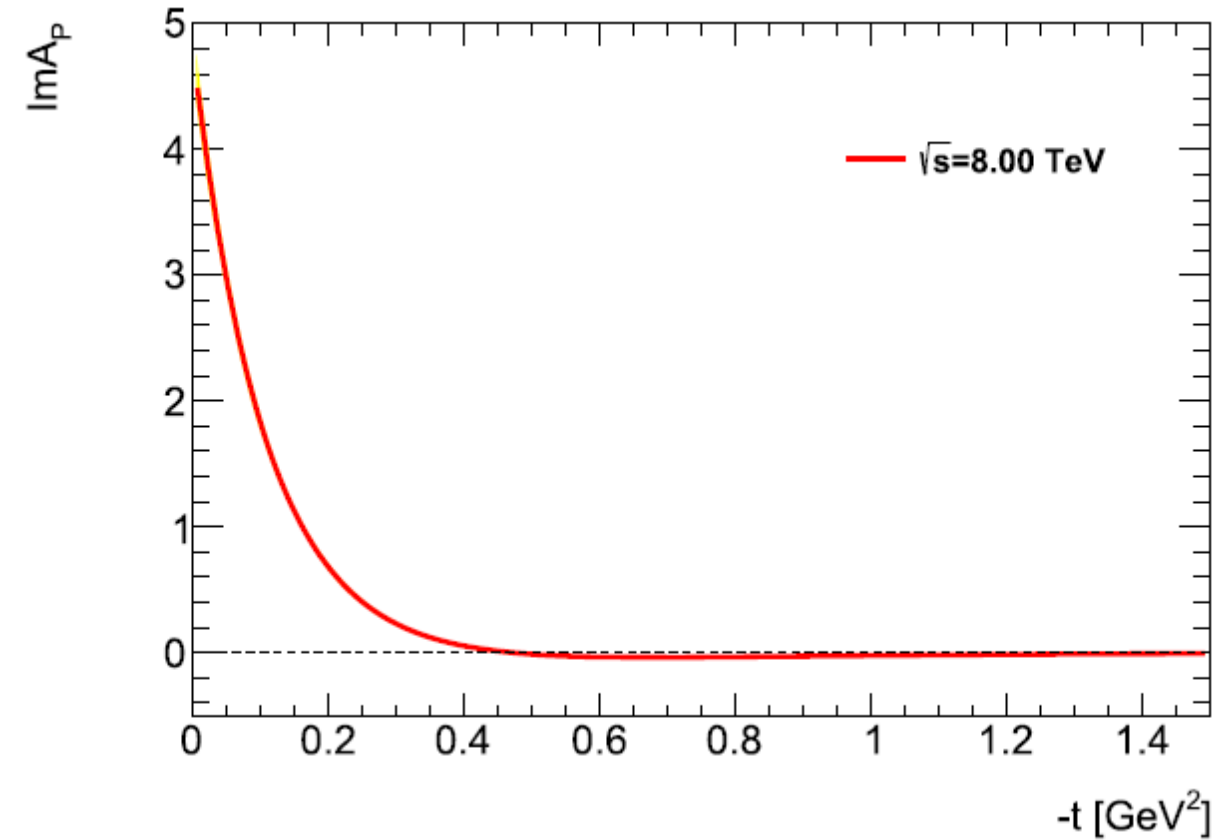


ReBB H(x) description is acceptable in a more limited range, $0.37 \leq -t \leq 0.97 \text{ GeV}^2$

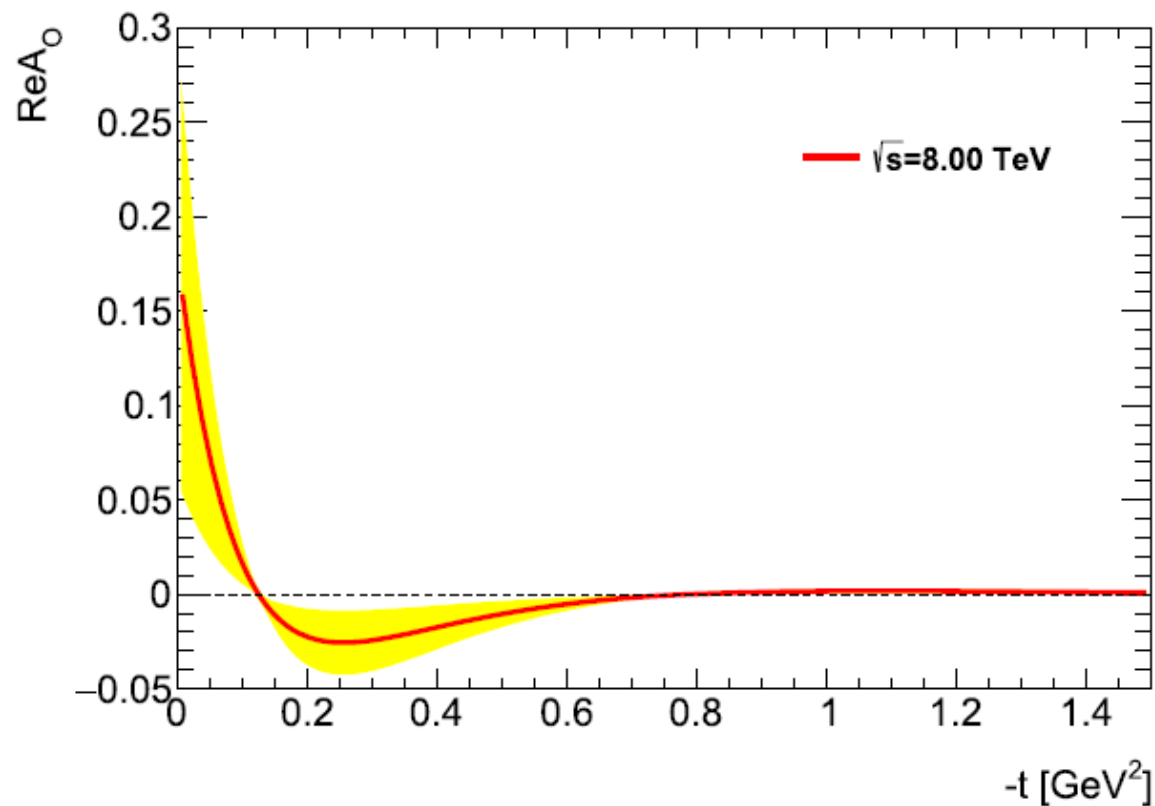
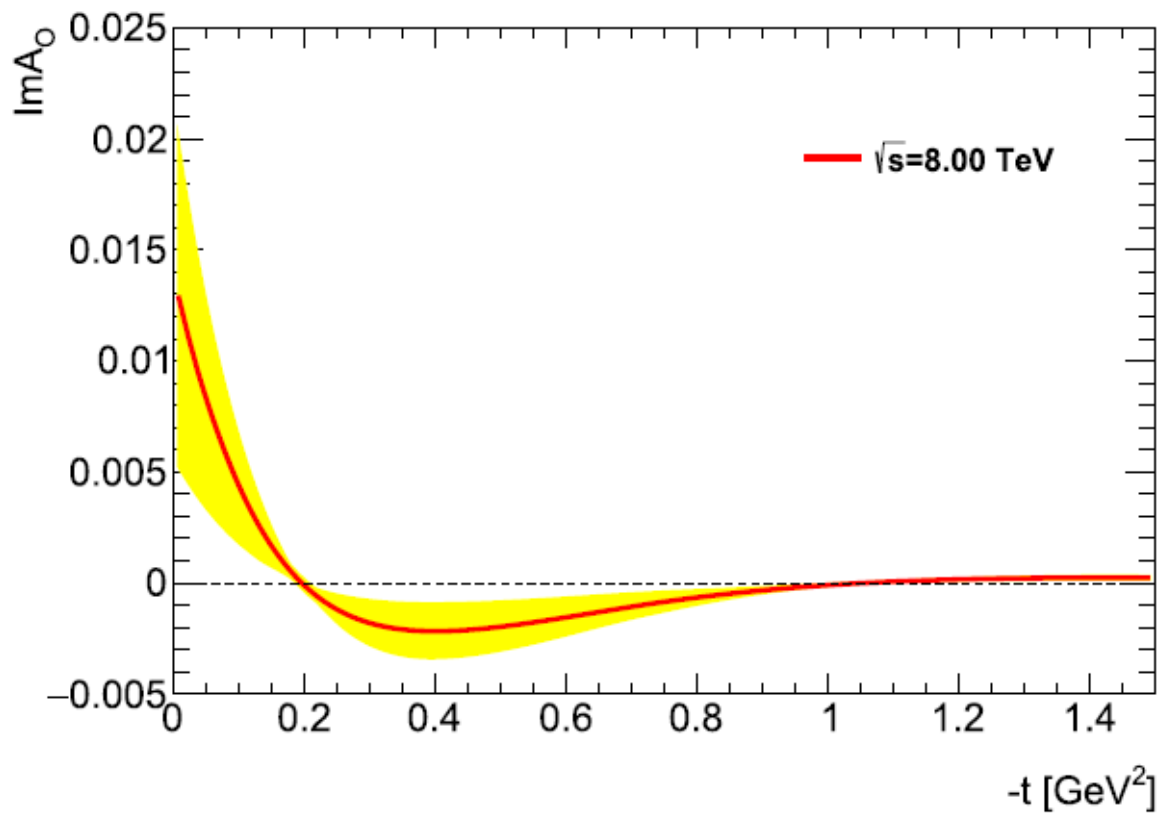


if a 4.5% normalisation uncertainty is assumed, the ReBB H(x) description is acceptable in the validity range of the ReBB model, $0.37 \leq -t \leq 1.2 \text{ GeV}^2$

ReBB pomeron amplitudes @ 8 TeV



ReBB odderon amplitudes @ 8 TeV



Unitarity and the elastic amplitude

- unitarity of the scattering matrix S :

$$SS^\dagger = I$$

$$S = I + iT$$

$$T - T^\dagger = iTT^\dagger$$

- unitarity equation in impact parameter b representation:

$$2 \operatorname{Im} t_{el}(s, \vec{b}) = |t_{el}(s, \vec{b})|^2 + \tilde{\sigma}_{in}(s, \vec{b})$$

(s is the squared CM energy)

- the elastic amplitude can be written as a solution of the unitarity equation in terms of $\tilde{\sigma}_{in}$
- $0 \leq \tilde{\sigma}_{in}(s, \vec{b}) \leq 1$ and $\tilde{\sigma}_{in}$ can be calculated using the rules of the probability calculus based on Glauber's multiple scattering theory