UNIVERSITÀ DEGLI STUDI DI GENOVA

INFN

Next-to-Leading Order virtual correction to Higgs-induced DIS

Diffraction and Low-x 2022

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Paper in preparation with Simone Marzani and Giovanni Ridolfi (Università di Genova) and Marco Bonvini (INFN Roma) and Federico Silvetti (Università "La Sapienza" Roma)

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Small-x resummation:
How does it work?

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- Resummation ⟷ Factorization

We need some factorisation properties

- Mellin Transform

$$g(N, Q^2) = \int_0^1 dx\, x^N\, g\left(x, Q^2\right) \qquad \ln^k(x) \to \frac{1}{N^{k+1}}$$

Collinear factorization theorem

$$\sigma(N, Q^2) = \sum_{i=q,g} C_i\left(N, \alpha_s(Q^2)\right) f_i\left(N, Q^2\right)$$

Coefficient function

Parton distribution function (PDF)

$\ln(x)$

Our goal: resum NLL terms in the coefficient function

High energy factorization theorem

$$\sigma(N, Q^2) = \int_0^\infty dk_\perp^2\, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right)\, \mathscr{F}_g\left(N, k_\perp^2\right)$$

Off-shell coefficient function

Unintegrated PDF

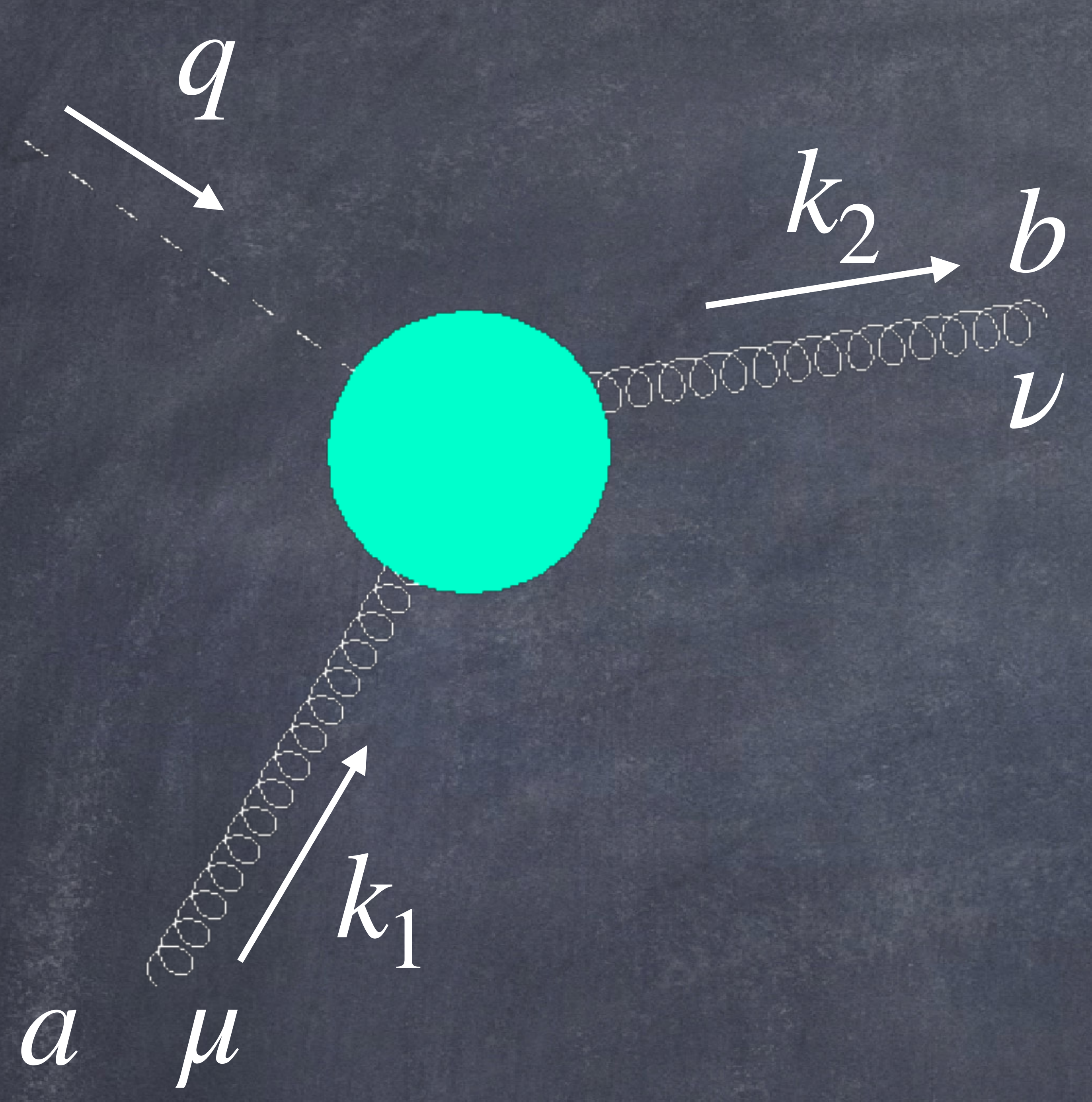
$$\mathscr{F}_g\left(N, k_\perp^2\right) = \mathscr{U}\left(N, k_\perp^2, Q^2\right) f_g(N, Q^2)$$

$$C_g(N, \alpha_s) = \int_0^\infty dk_\perp^2\, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right)\, \mathscr{U}\left(N, k_\perp^2, Q^2\right)$$

Bonvini, Marzani and Peraro (**2016**)

Higgs induced DIS

Higgs induced DIS



- $n_f = 0$

- Higgs gluon effective vertex:

$$M^{\mu\nu} = i\, c\, \delta_a^b \left[k_2^\mu k_1^\nu - g^{\mu\nu} k_1 \cdot k_2 \right]$$

- Off-shell coefficient function

$$k_1^2 = - \vec{k}_1^{\,2}$$

Higgs induced DIS

We want to resum NLL terms in the coefficient function

$$C_g(N, \alpha_s) = \int_0^\infty dk_\perp^2\, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right)\ \mathscr{U}\left(N, k_\perp^2, Q^2\right)$$

Higgs induced DIS

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We have to compute the one-loop off-shell coefficient function

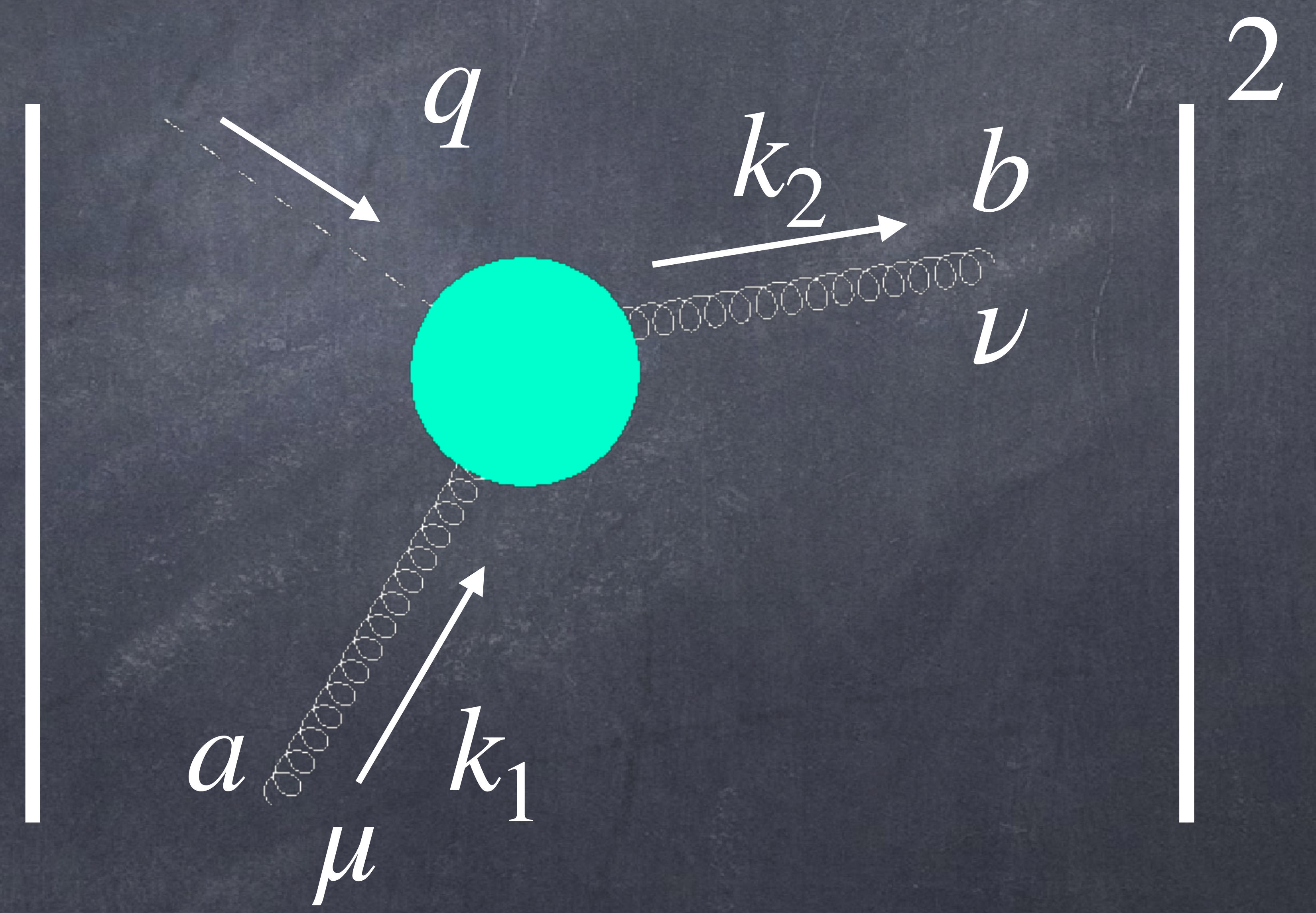
Key points

1. We have to work in axial gauge: $A \cdot n = 0$

The off-shell coefficient function is free from logs if we work in axial gauge

Catani and Hautmann (**1994**)

2. We have to understand the "sum over polarisation" of an off-shell gluon at NLL



Axial gauge

- Growing number of terms due to gauge choice

$$\Pi^{\mu\nu}_{a,b}(k,n) = i\,\delta_{a,b}\left[\frac{-g^{\mu\nu}}{k^2} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k^2\, k\cdot n}\right]$$

- Non covariant loop integrals

$$I = \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2(k-k_1)^2(k-k_2)^2(k\cdot n)}$$

- Counterterms in axial gauge

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Non covariant loop integrals

$$I_n = \int \frac{d^d k}{(2\pi)^d} \frac{f(k \cdot n)}{D_1 D_2 \ldots D_n}$$

Covariant denominators

Non-covariant part:

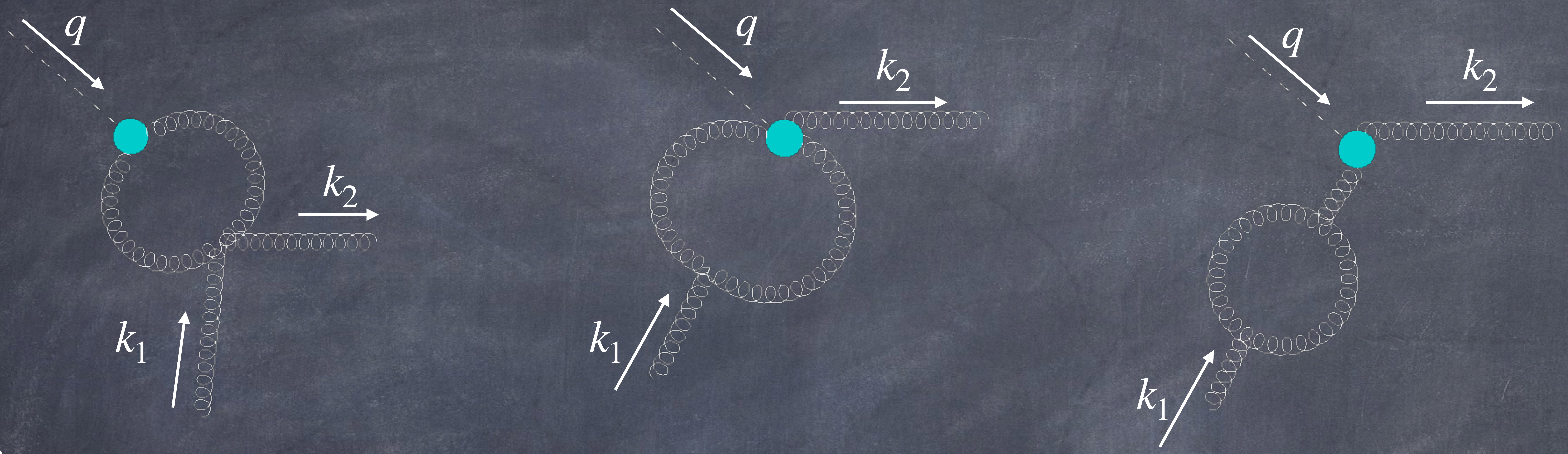
$$\frac{1}{(k \cdot n)}$$

Principal value prescription

$$\frac{1}{(k \cdot n)} \to \frac{k \cdot n}{(k \cdot n)^2 + \delta^2}$$

Curci, Furmanski and Petronzio (**1980**)

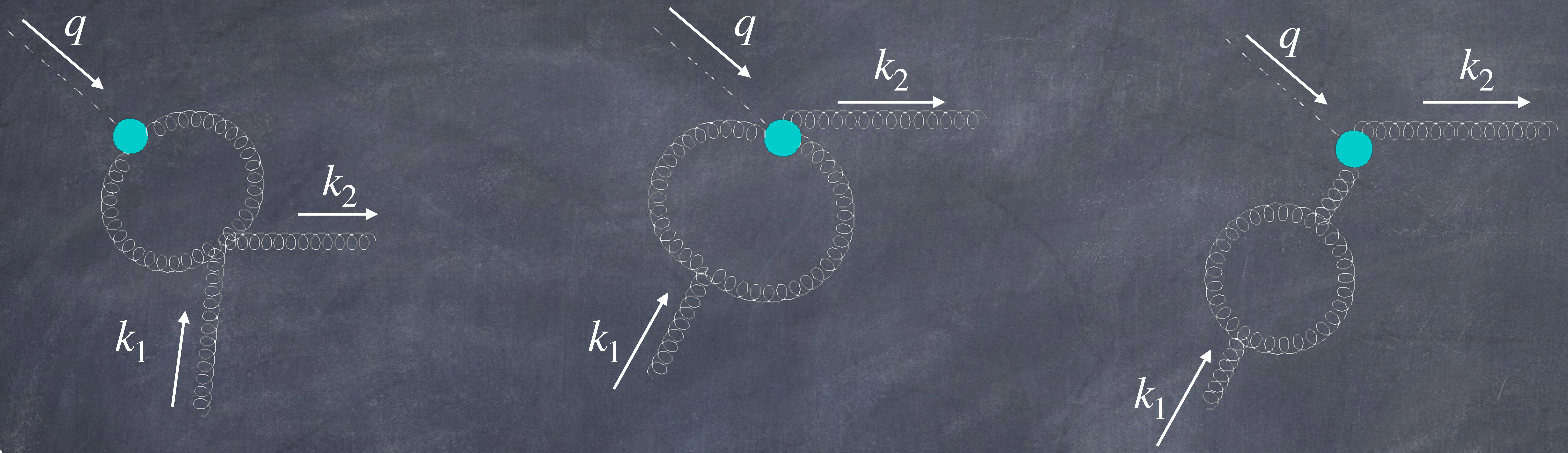
Non covariant loop integrals



$$k_1^2 \neq 0$$

$$I_2 = \int \frac{d^d k}{(2\pi)^d} \frac{f\left(k \cdot n\right)}{k^2 (k-l)^2}$$

Non covariant loop integrals

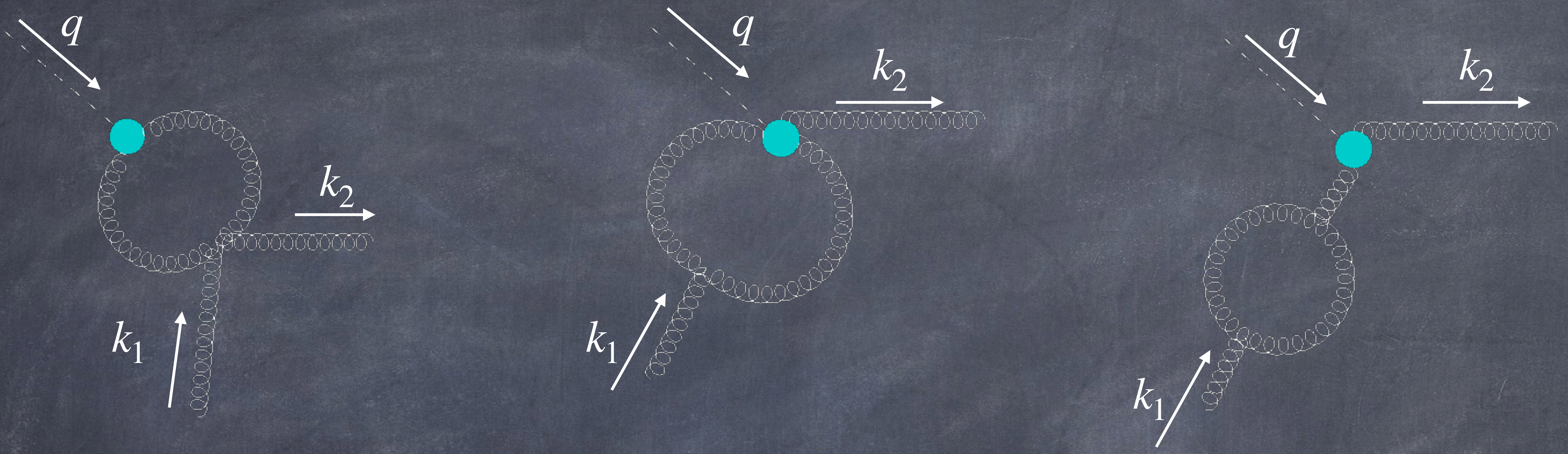


$k_1^2 \neq 0$

$$I_2 = \int \frac{d^dk}{(2\pi)^d} \frac{f\left(k \cdot n\right)}{k^2(k-l)^2} = \frac{i}{16\pi^2}\left(\frac{4\pi}{-l^2}\right)^{\epsilon} \frac{\Gamma(1+\epsilon)}{\epsilon} \int_0^1 dz\, f\left(l_+ z\right) z^{-\epsilon}(1-z)^{-\epsilon}$$

$$l_+ = l \cdot n$$

Non covariant loop integrals



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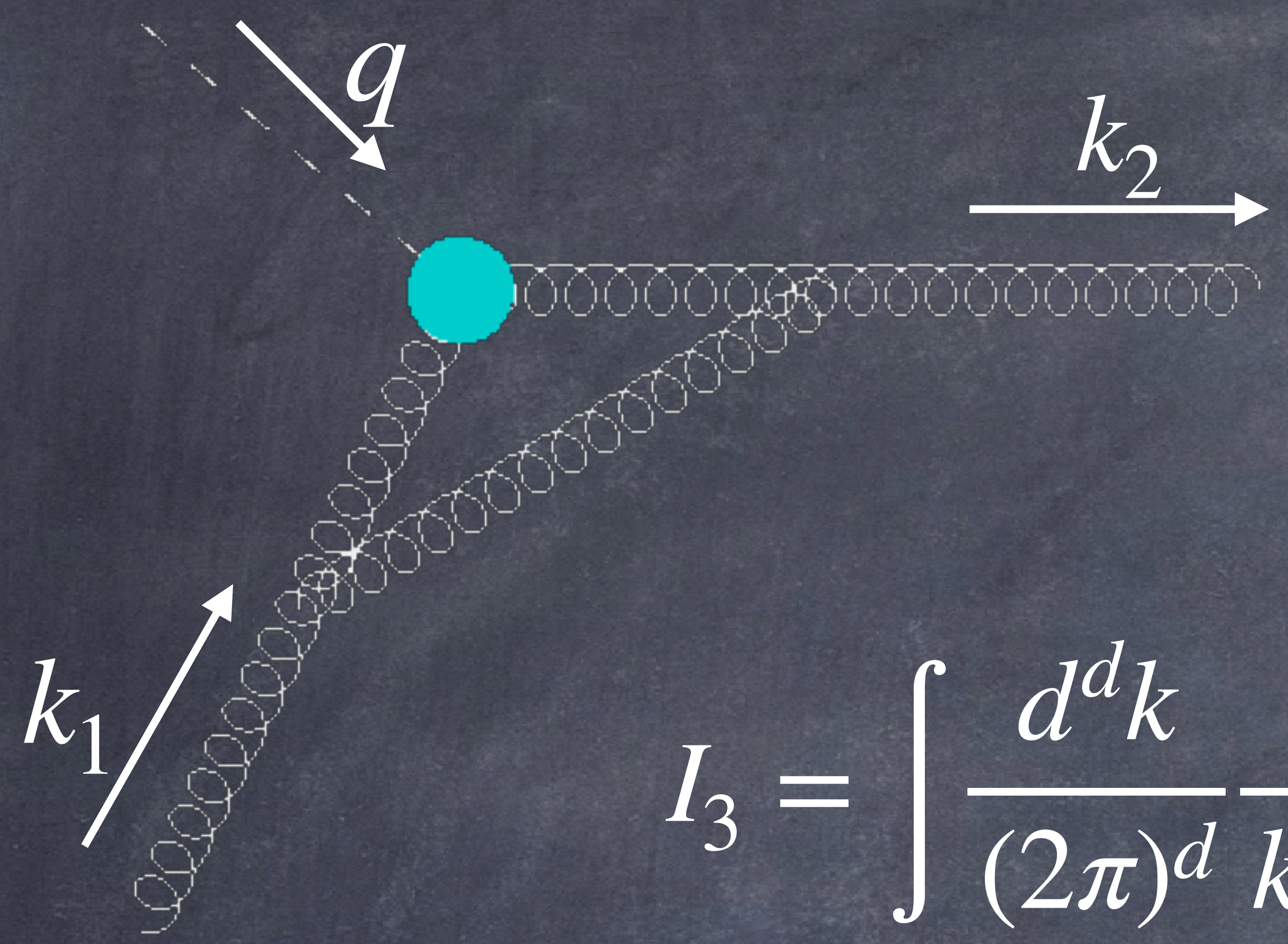
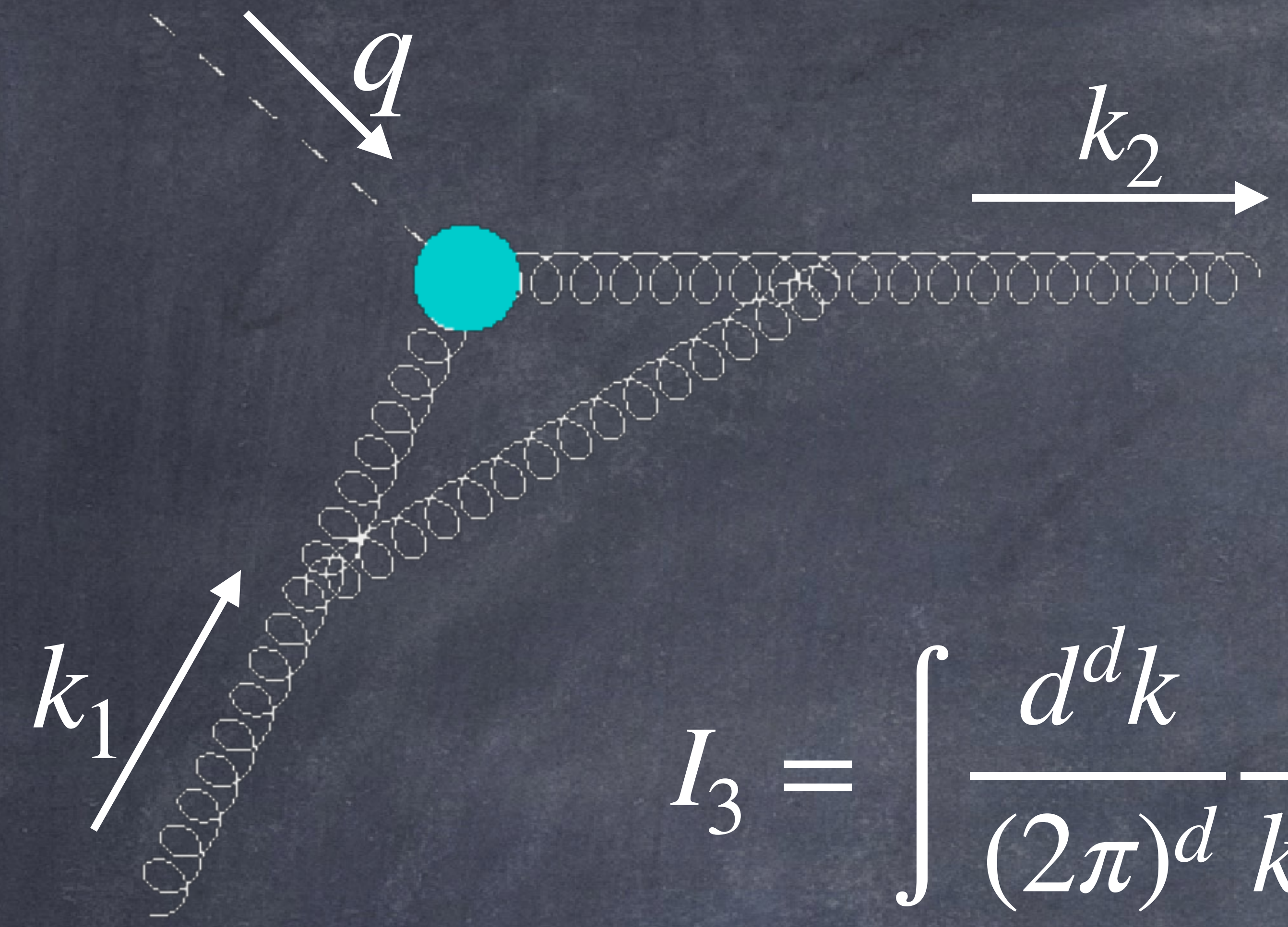


$$x = \frac{k_2 \cdot n}{k_1 \cdot n} \qquad b = -\frac{k_1^2}{2k_1 \cdot k_2}$$

$$I_3 = \int \frac{d^d k}{(2\pi)^d} \frac{f\left(k \cdot n\right)}{k^2 (k - k_1)^2 (k - k_2)^2}$$

Non covariant loop integrals



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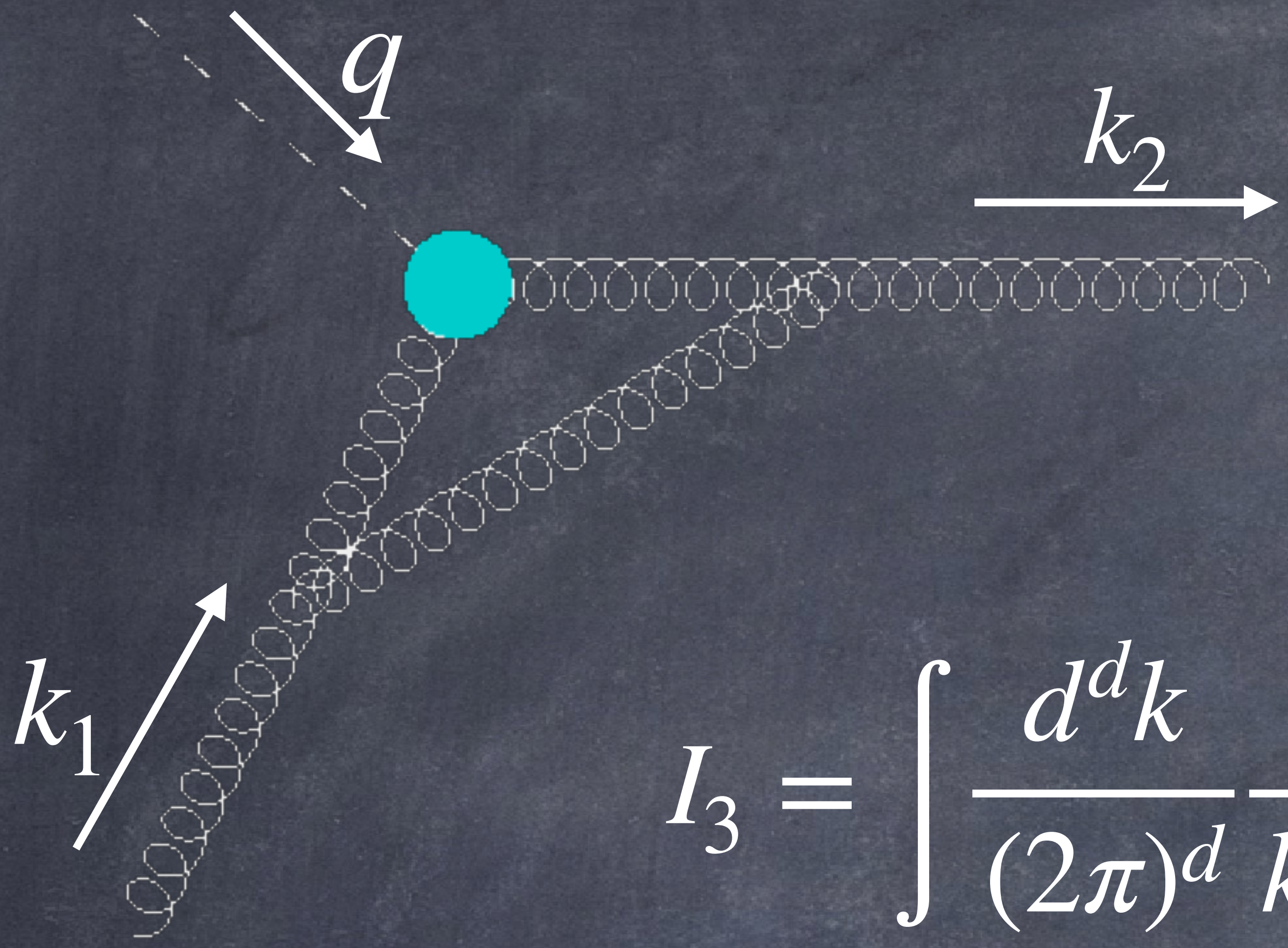
$$\left[\frac{b^{-1-\epsilon}}{x} \int_0^x dz\, f(l_+ z)\, z^{-\epsilon}(1-z)^{-1-\epsilon} {}_2F_1\left(1-\epsilon, 1; 1+\epsilon; \frac{(1-b\,x)}{b\,x} \frac{z}{1-z}\right)\right.$$

$$-b^{-\epsilon}(b\,x-1)^{\epsilon} \int_x^1 dy\, f(l_+ y)(1-y)^{-\epsilon}(b\,x-y)^{-1-\epsilon} {}_2F_1\left(1+\epsilon, -\epsilon; 1-\epsilon; \frac{(1-y)b\,x}{b\,x-y}\right)$$

$$\left. + \frac{(b\,x-1)^{\epsilon}(1-x)^{\epsilon}}{(b-1)^{\epsilon}} \int_x^1 dy\, f(l_+ y)(1-y)^{-\epsilon}(b\,x-y)^{-1-\epsilon} {}_2F_1\left(1+\epsilon, -\epsilon; 1-\epsilon; \frac{(b-1)\,x}{(1-x)} \frac{(1-y)}{(b\,x-y)}\right)\right]$$

Non covariant loop integrals



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Axial gauge

- Growing number of terms due to gauge choice ✔

$$\Pi^{\mu\nu}_{a,b}(k,n) = i\,\delta_{a,b}\left[\frac{-g^{\mu\nu}}{k^2} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k^2\, k\cdot n}\right]$$

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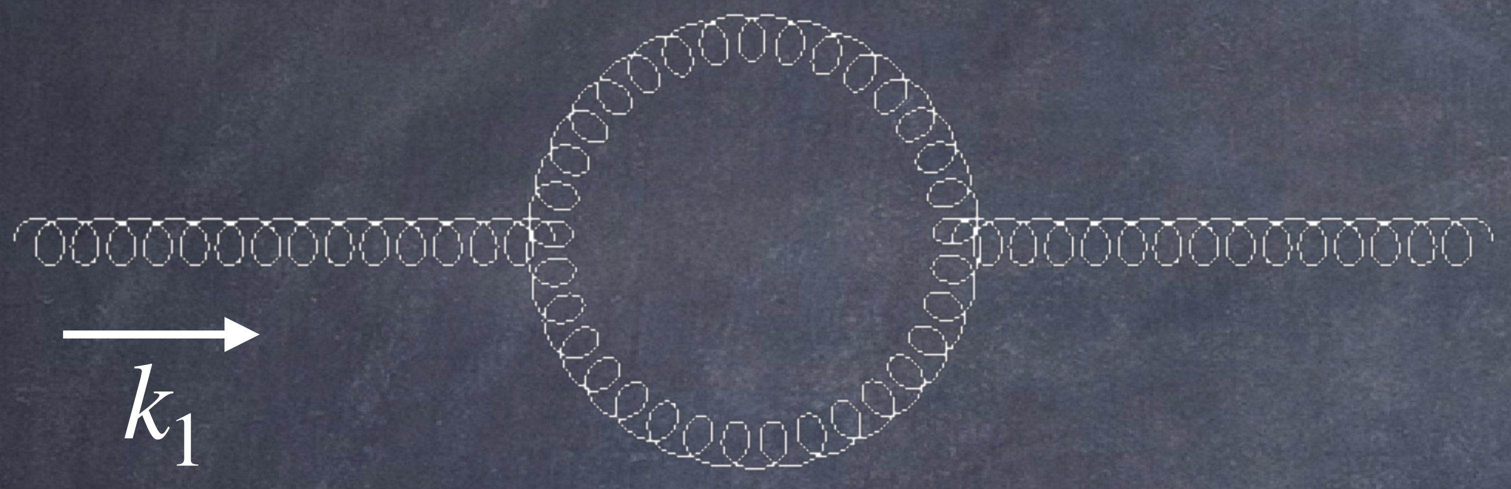
$$I = \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k-k_1)^2(k-k_2)^2(k\cdot n)}$$

- Counterterms in axial gauge

Counterterms in axial gauge: gluon propagator



$n_f = 0$

Feynman gauge:

$$\Pi^{\mu\nu}(k_1, n) = \mathrm{i}\frac{\alpha_s}{4\pi}\frac{C_A\delta_{a,b}}{\epsilon}Z_A\left(k_1^\mu k_1^\nu - k_1^2 g^{\mu\nu}\right)$$

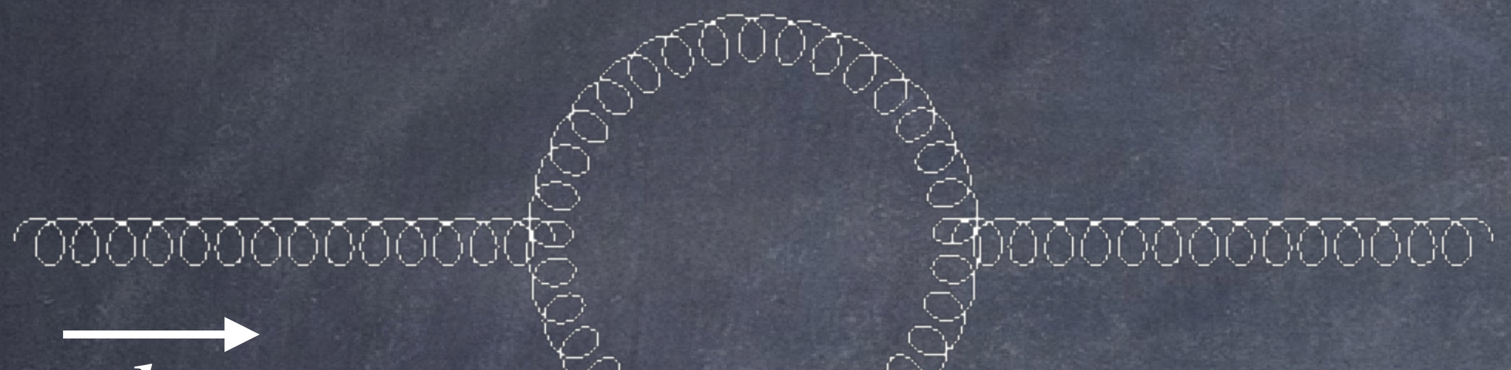
$$I_0 = \int_0^1 \frac{du\, u}{u^2+\delta^2} = -\ln(\delta)$$

Axial gauge:

$$\Pi^{\mu\nu}(k_1, n) = -\mathrm{i}\frac{\alpha_s}{4\pi}\frac{C_A\delta_{a,b}}{\epsilon}\left[\left(\frac{11}{3} - 4I_0\right)\left(k_1^\mu k_1^\nu - k_1^2 g^{\mu\nu}\right)\right.$$

$$\left.-4\left(1 - I_0\right)\left(k_1^\mu k_1^\nu - \frac{k_1^2}{k_1\cdot n}(k_1^\mu n^\nu + k_1^\nu n^\mu) + \frac{k_1^4}{(k_1\cdot n)^2}n^\mu n^\nu\right)\right]$$

Counterterms in axial gauge: gluon propagator



k_1

$n_f = 0$

Feynman gauge:

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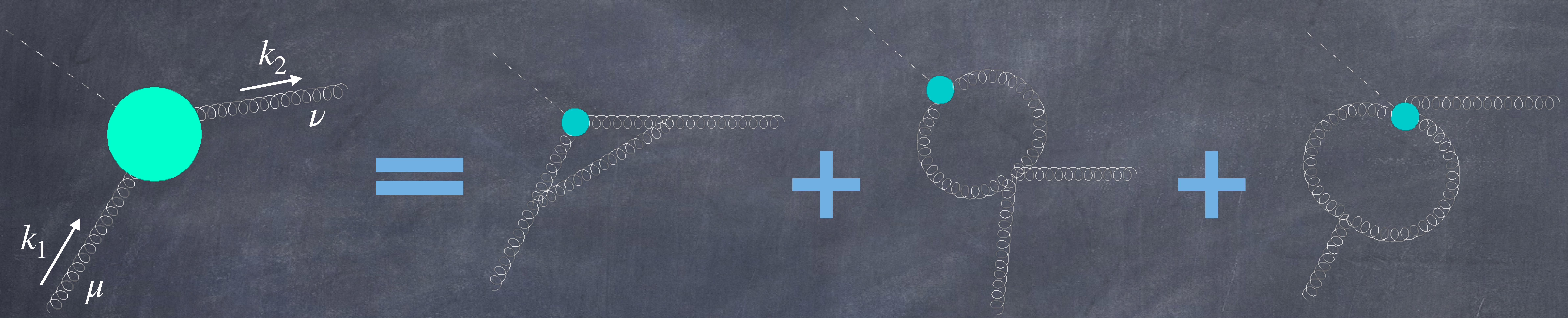
$$k_{1\mu}\Pi^{\mu\nu}(k_1, n) = 0$$

Counterterms in axial gauge: effective vertex



$$\mathrm{CT}^{\mu\nu}(k_1, k_2, n) = -\left[D_1^{\mu\nu}(k_1, k_2, n)\,|_{UV} + D_2^{\mu\nu}(k_1, k_2, n)\,|_{UV} + D_3^{\mu\nu}(k_1, k_2, n)\,|_{UV} \right]$$

$k_1^2 \neq 0$

$k_2^2 \neq 0$

$$D_1^{\mu\nu}\,|_{UV} = C_{k_1k_1}(k_1, k_2, n)k_1^\mu k_1^\nu + C_{k_2k_2}(k_1, k_2, n)k_2^\mu k_2^\nu$$
$$+ C_{k_1k_2}(k_1, k_2, n)k_1^\mu k_2^\nu + C_{k_2k_1}(k_1, k_2, n)k_2^\mu k_1^\nu$$
$$+ C_g(k_1, k_2, n)g^{\mu\nu} + \ldots$$

Axial gauge

- Growing number of terms due to gauge choice ✔

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- Non covariant loop integrals ✔

$$I = \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k-k_1)^2(k-k_2)^2(k\cdot n)}$$

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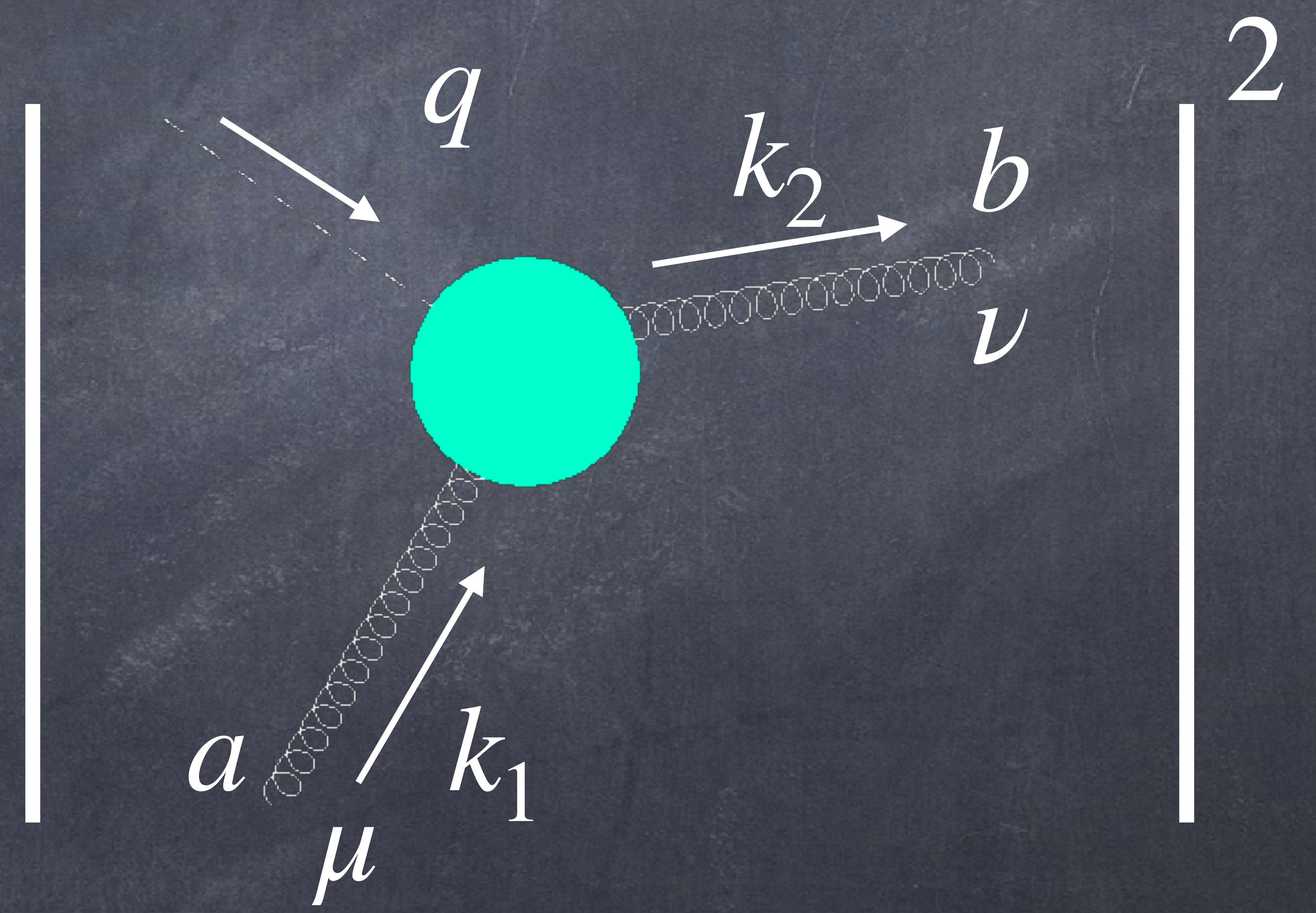
Key points

1. We have to work in axial gauge: $A \cdot n = 0$

The off-shell coefficient function is free from logs if we work in axial gauge

Catani and Hautmann (**1994**)

2. We have to understand the "sum over polarisation" of an off-shell gluon at NLL



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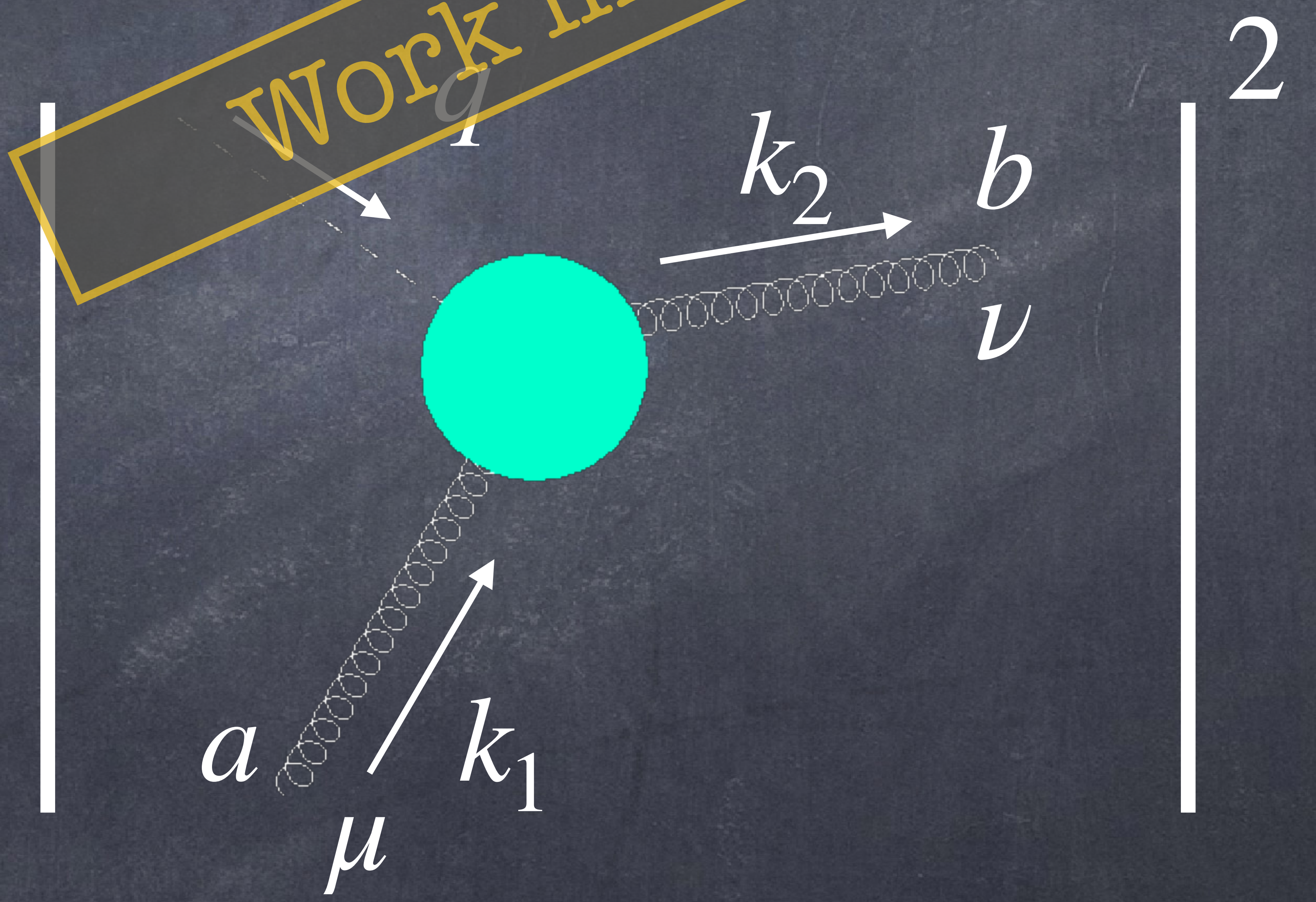
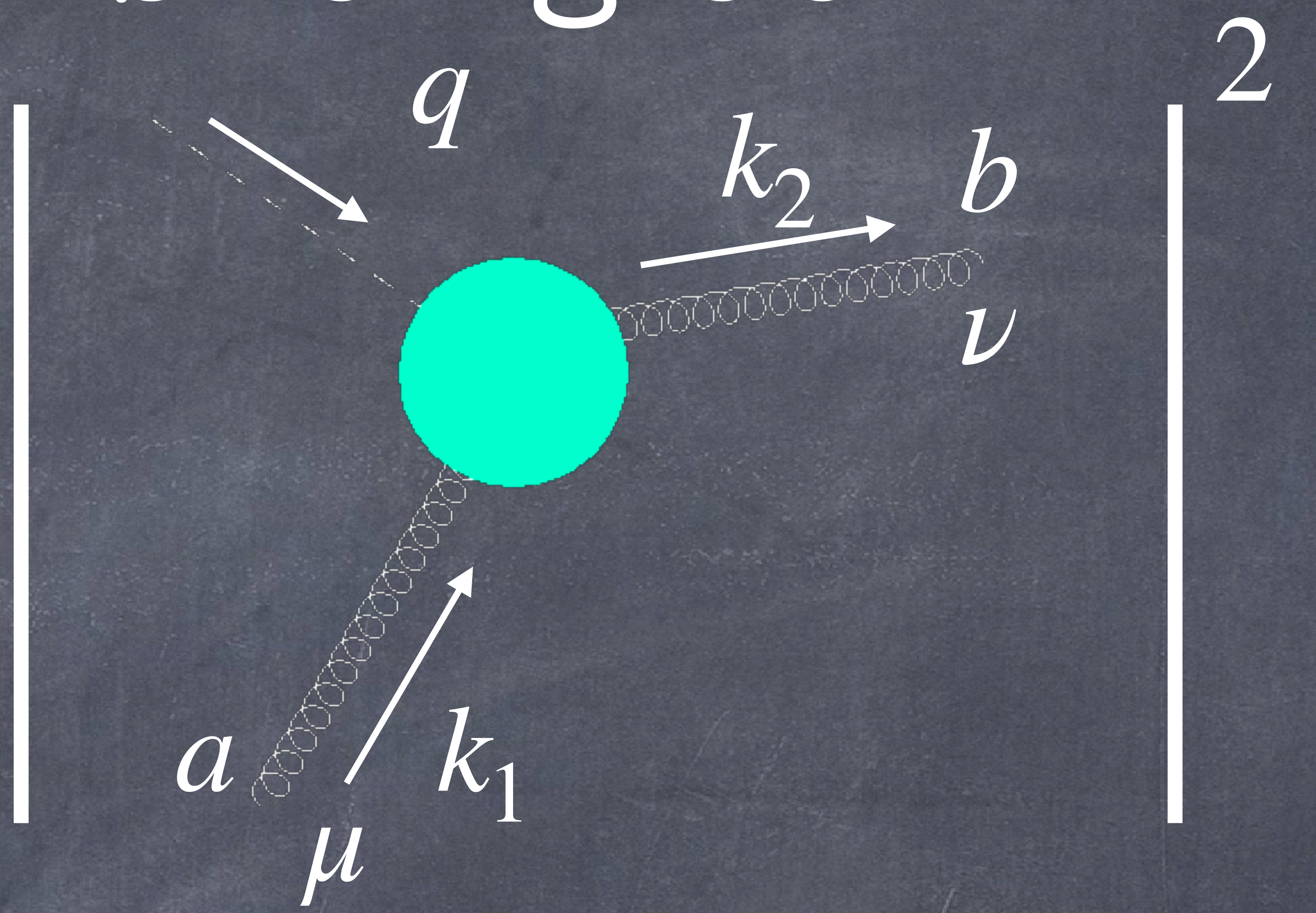


Sum over polarisation of an off-shell gluon

$$k_1^\mu = k^\mu + k_\perp^\mu$$
$$k_1^2 = k_\perp^2$$

$$d_{CH}^{\mu\nu} = (d-2)\frac{k_\perp^\mu k_\perp^\nu}{\vec{k}_\perp^{\,2}}$$



1. $d_{\rm CH}^{\mu\nu}$ selects the dominant part of the amplitude in the leading logarithm approximation

2. $\lim_{\vec{k}_\perp^{\,2}\to 0} \langle d_{CH}^{\mu\nu}\rangle = g_\perp^{\mu\nu}$

Catani and Hautmann (**1994**)

Sum over polarisation of an off-shell gluon

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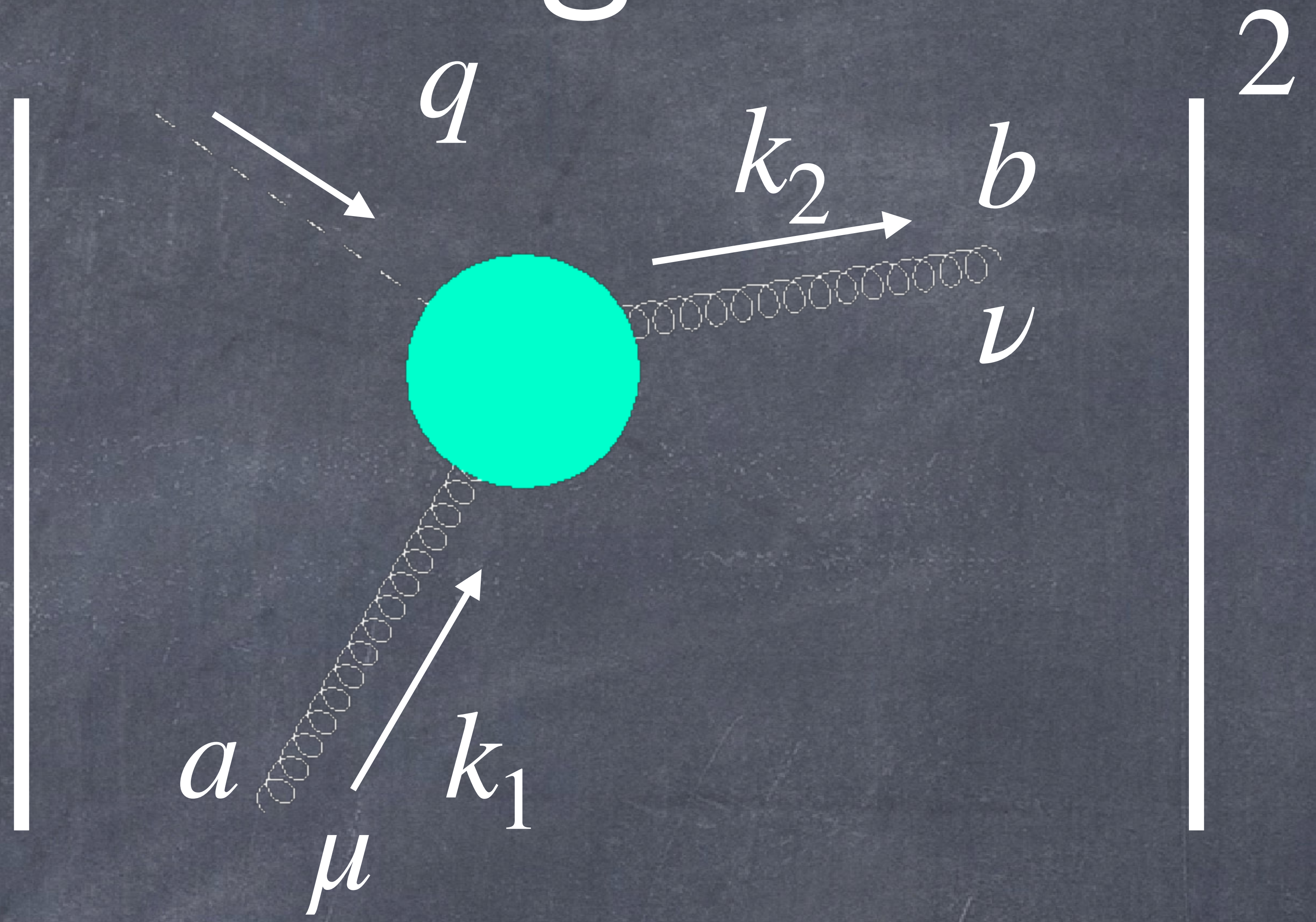


$$d_{CH}^{\mu\nu} = (d-2)\frac{k_\perp^\mu k_\perp^\nu}{\vec{k}_\perp^{\,2}}$$

1. $d^{\mu\nu}_{\mathrm{CH}}$ selects the dominant part of the amplitude in the leading logarithm approximation **Still true at NLL?**

2. $\lim_{\vec{k}_\perp^{\,2}\to 0} \langle d_{CH}^{\mu\nu}\rangle = g_\perp^{\mu\nu}$

Catani and Hautmann (**1994**)

Conclusions

Where are we?

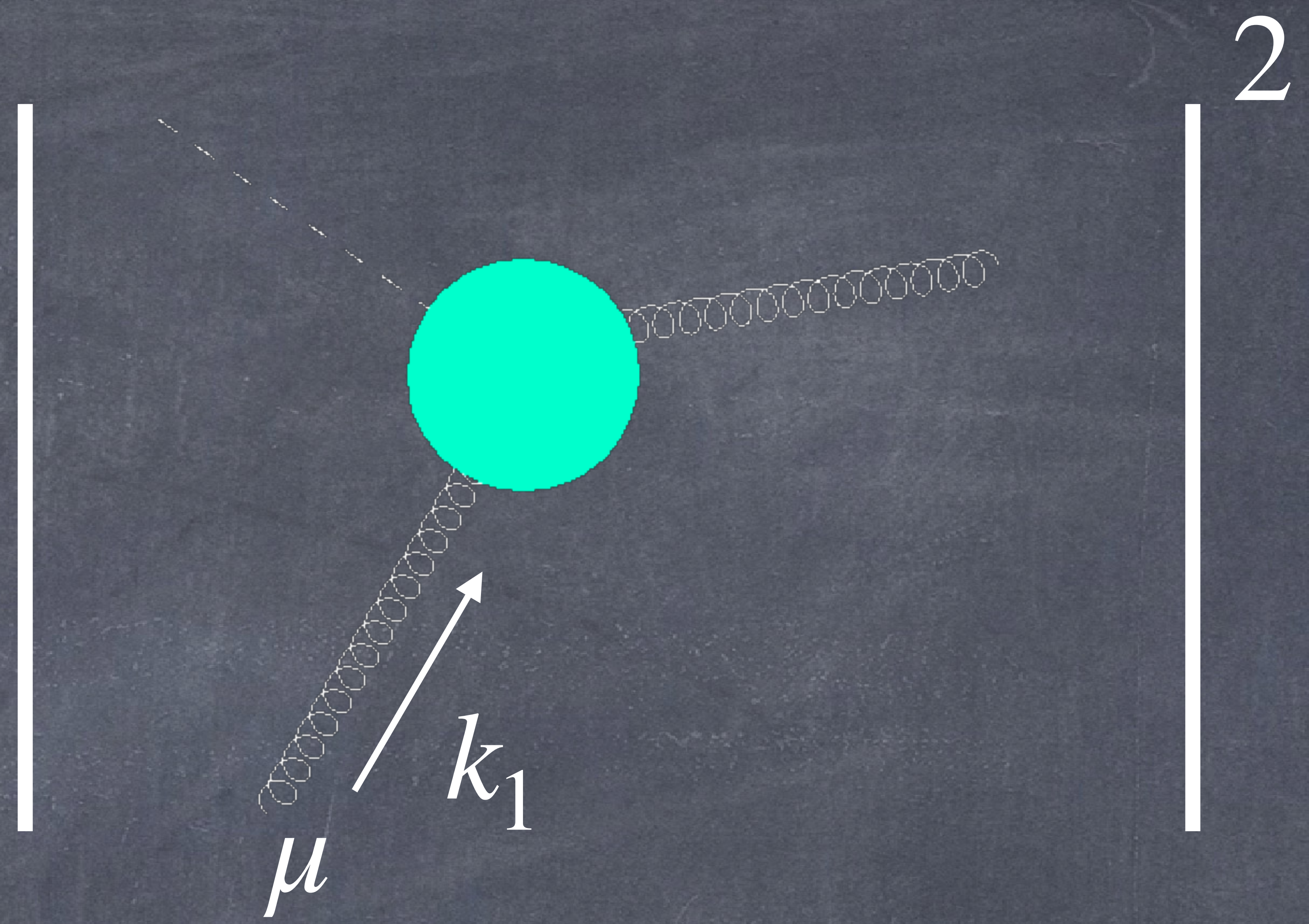
$$C_g(N, \alpha_s) = \int_0^\infty dk_\perp^2\, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right)\, \mathscr{U}\left(N, k_\perp^2, Q^2\right)$$

- Solved main issues due to the choice of axial gauge
- Virtual contribution ✔
- Real contribution Work in progress
- Cross checks

Thank you!

Back up

$$|M|^2 = \mathscr{M}^{\mu\nu}(k_1, k_2, n)\, d_{\mu\nu}(k_1, n)$$



$$\mathscr{M}^{\mu\nu}(k_1, k_2, n) = C_{k_1k_1} k_1^\mu k_1^\nu + C_{k_2k_2} k_2^\mu k_2^\nu + \frac{1}{2} C_{k_1k_2}\left(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu\right) + C_g g^{\mu\nu} + \ldots$$

$$n^\mu d_{\mu\nu}(k_1, n) = 0$$

$$C_{k_ik_j} = \frac{a}{\epsilon} + c + d\ln(\delta)$$

IR singularity: must be cancelled by real contribution

Principal value prescription for spurious singularities: must be cancelled by real contribution