Soft & Next-to-Soft virtual Resummation for QCD Observables

Ajjath Abdul Hameed
aabdulhameed@lpthe.Jussie.fr

Diffraction & Low-x, 24-30 September, 2022

Phys.Rev.D 105(2022) 9, 094035, AAH, P.Mukherjee, V.Ravindran

Eur.Phys.J.C. 82 (2022) 3, 234, AAH, P.Mukherjee, V.Ravindran, A.Sankar, S.Tiwari

LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR

INTRODUCTION : WHAT IS SV & NSV

- In QCD improved Parton model, the hadronic cross section is the convolution of Parton distribution functions, $f(x)$, and partonic coefficient function, $\Delta_{ab}(z)$

$$\sigma(q^2,\tau) = \sigma_0(\mu_R^2) \sum_{ab} \int dx_1 \int dx_2 f_a(x_1,\mu_F^2)\, f_b(x_2,\mu_F^2) \Delta_{ab}(q^2,\mu_F^2,\mu_R^2,z)$$

$q^2 \longrightarrow$ Invariant mass of final state

$\hat{s} \longrightarrow$ partonic center of mass energy

$z = \dfrac{q^2}{\hat{s}} \rightarrow$ partonic scaling variable

$\mu_F \longrightarrow$ factorisation scale

$\mu_R \longrightarrow$ renormalisation scale

- Soft limit defines the kinematic limit where all the final real emissions has almost zero energy~ soft emissions : $z \rightarrow 1$

- In this limit, the functional form of partonic coefficient function could be organised in terms of their singular behaviour in z

$$\Delta_{ab} = \sum_{i=0}^{\infty} a_s^i \sum_{j=0}^{2i-1} C^{(0)}_{i,-1}\, \delta(1-z) + C^{(0)}_{i,j} \left(\frac{\ln^j(1-z)}{1-z}\right)_+ + \sum_{j=0}^{2i-1} C^{(1)}_{i,j}\, \ln^j(1-z) \quad + \mathcal{O}(1-z)$$

INTRODUCTION : WHAT IS SV & NSV

- In soft limit, the partonic coefficient function could be organised in terms of their singular behaviour in z

$$\Delta_{ab} = \sum_{i=0}^{\infty} a_s^i \sum_{j=0}^{2i-1} C_{i,-1}^{(0)}\,\delta(1-z) + C_{i,j}^{(0)}\left(\frac{\ln^j(1-z)}{1-z}\right)_+ + \sum_{j=0}^{2i-1} C_{i,j}^{(1)}\,\ln^j(1-z) + \mathcal{O}(1-z)$$

beyond NSV

- Leading singular distributions
- Contributions from soft real emissions+virtual corrections
- Resummation known to N3LL

Soft-Virtual (SV)

- Next-to-leading singular
- Collinear logarithms
- Resummation known to LL

Next to Soft-Virtual (NSV)

- $a_s \ln(1-z) \sim \mathcal{O}(1)$: resummation is required - threshold resummation

[Sterman] [Catani, Trentedue]

IS NSV RELEVANT?

- Though NSV are less singular to SV, their contributions are numerically sizeable [C Duhr, et al]

- For inclusive DY at a_s^2 at Q =200 GeV at. NNLO

SV		NSV	
$\mathcal{D}_3$	6.13%	$\ln^3(1-z)$	12.4%
$\mathcal{D}_2$	1.49%	$\ln^2(1-z)$	7.83%
$\mathcal{D}_1$	-3.24%	$\ln^1(1-z)$	-2.82%
$\mathcal{D}_0$	-4.74%	$\ln^0(1-z)$	-6.57%
$\delta(1-z)$	0.003%		
	-0.035%		10.8%

SV		NSV	
$\ln^4 N$	0.0144%	$\frac{\ln^4 N}{N}$	0%
$\ln^3 N$	0.125%	$\frac{\ln^3 N}{N}$	0.05%
$\ln^2 N$	2.70%	$\frac{\ln^2 N}{N}$	0.392%
$\ln N$	6.07%	$\frac{\ln N}{N}$	4.08%
$\ln^0 N$	17.7%	$\frac{1}{N}$	3.35%
	34.5%		7.87%

- For inclusive $gg \to H$ at a_s^2 at Q =125 GeV at Nnlo

$$D_i = \left(\frac{\ln^i(1-z)}{1-z}\right)_+$$

SV		NSV	
$\mathscr{D}_3$	45.3%	$\ln^3(1-z)$	52.64%
$\mathscr{D}_2$	4.87%	$\ln^2(1-z)$	37.34%
$\mathscr{D}_1$	−10.60%	$\ln^1(1-z)$	−7.45%
$\mathscr{D}_0$	−25.51%	$\ln^0(1-z)$	−23.62%
$\delta(1-z)$	1.75%		
15.81%		58.91%	

SV		NSV	
$\ln^4 N$	0.18%	$\frac{\ln^3 N}{N}$	0.50%
$\ln^3 N$	1.11%	$\frac{\ln^2 N}{N}$	2.71%
$\ln^2 N$	9.21%	$\frac{\ln N}{N}$	10.87%
$\ln N$	15.28%	$\frac{1}{N}$	6.32%
1	50.12%		
75.9%		20.4%	

THE THRESHOLD FORMALISM

- The approach is based on :
    - Collinear factorisation and renormalisation group invariance
    - Using the logarithmic structure of known higher order results

- The SV formalism is welll-studied for Color singlet processes. **[Ravindran '05,'06]**
    - Extend the same formalism to include the diagonal NSV logarithmic corrections.

- We start with Collinear factorisation :

$$\frac{1}{z}\hat{\sigma}_{a'b'}(q^2, z, \epsilon) = \sigma_0(\mu_R^2) \sum_{ab} \left(\Gamma^T\right)_{a'a}(z, \mu_F^2, \epsilon) \otimes \Delta_{ab}(z, q^2, \mu_F^2, \mu_R^2) \otimes \Gamma^{-1}_{bb'}(z, \mu_F^2, \epsilon)$$

$\hat{\sigma}_{ab} \longrightarrow$ bare partonic cross section with initial collinear divergences

$\Gamma \longrightarrow$ Altarelli Parisi splitting kernel

- No off-diagonal has been considered : then terms like $\Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{g\bar{q}}$ do not contribute.

  Hence, we can safely drop the sum in the above formula

THE THRESHOLD FORMALISM

- Perturbative structure of SV+NSV partonic coefficient function :

$$\Delta^{\rm SV+NSV}_{d,c\bar{c}}(q^2,z) = \left|F_c(q^2)\right|^2 \delta(1-z) \otimes S^{\rm SV+NSV}_c(q^2) \otimes (\Gamma^T_{cc})^{-1,{\rm SV+NSV}}(z,\mu_F^2) \otimes \Gamma^{-1,{\rm SV+NSV}}_{\bar{c}\bar{c}}(z,\mu_F^2)$$

Virtual Corrections

$$Q^2\frac{d}{dQ^2}\ln\hat{F}^I(\hat{a}_s,Q^2,\mu^2,\varepsilon) = \frac{1}{2}\left[K^I\left(\hat{a}_s,\frac{\mu_R^2}{\mu^2},\varepsilon\right)+G^I\left(\hat{a}_s,\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\varepsilon\right)\right]$$

[Sen,Sterman,Magnea]

$$\mu_R^2\frac{d\hat{F}_c}{d\mu_R^2} = 0$$

$$\hat{F} \longrightarrow \{A_I, B_I, f_I, \gamma_I, g_I\}$$

Universal anomalous dimensions
And process dependent constants

Altarelli-Parisi splitting kernels

$$\mu_F^2\frac{d}{d\mu_F^2}\Gamma_{ab}(z,\mu_F^2,\epsilon) = \frac{1}{2}\sum_{a'=q,\bar{q},g} P_{aa'}(z,a_s(\mu_F^2)) \otimes \Gamma_{a'b}(z,\mu_F^2,\epsilon)$$

$$P_{cc}(z,a_s(\mu_F^2)) = 2B^c(a_s(\mu_F^2))\delta(1-z) + 2\Big[A^c(a_s(\mu_F^2))\mathcal{D}_0(z) + C^c(a_s(\mu_F^2))\ln(1-z) + D^c(a_s(\mu_F^2))\Big]$$

[Moch,Vogt,Vermaseren]

THE THRESHOLD FORMALISM

- Owing to the first order diff. eq.s and RG invariance:

$$\Delta^{\mathrm{SV+NSV}}_{d,c\bar{c}}(q^2,z) = \left|F_c(q^2)\right|^2 \delta(1-z) \otimes S^{\mathrm{SV+NSV}}_c(q^2) \otimes (\Gamma^T_{cc})^{-1,\mathrm{SV+NSV}}(z,\mu_F^2) \otimes \Gamma^{-1,\mathrm{SV+NSV}}_{\bar{c}\bar{c}}(z,\mu_F^2)$$

Soft-collinear distributions - contributions from real emissions

$$q^2 \frac{d}{dq^2} \ln \mathcal{S}_c = \left[\overline{K}^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon, z\right) + \overline{G}^c\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon, z\right)\right]$$

- The solution could be :

$$\ln S^{\mathrm{SV+NSV}}(\hat{a}_s, q^2, z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \, S_\epsilon^i \left(\frac{q^2(1-z)^2}{\mu^2 z}\right)^{i\frac{\epsilon}{2}} \left(\frac{i\epsilon}{1-z}\right) \left[\hat{\phi}^{\mathrm{SV},(i)}(\epsilon) + (1-z)\, \hat{\phi}^{\mathrm{NSV},(i)}(z,\epsilon)\right]$$

Pure SV

Universal coefficients

For of z- dependency :

Process dependent coefficients

$$\sum_{k=0}^{i+j} \mathcal{G}^{(i,j,k)}_{NSV} \, \epsilon^j \, \ln^k(1-z) \}$$

- Logarithmic structure is obtained to N3LO from the fixed order corrections, we propose the same structure to all order.

INTEGRAL REPRESENTATION

- With the knowledge on the structure, we can formulate an Integral representation for Δ^c, which gives an understanding on the all order structure.

$$\Delta_c(q^2,\mu_R^2,\mu_F^2,z) = C_0^c(q^2,\mu_R^2,\mu_F^2)\ \mathscr{C}\exp\left(\int_{\mu_F^2}^{q^2(1-z)^2}\frac{d\lambda^2}{\lambda^2}P_{cc}(a_s(\lambda^2),z) + \mathscr{Q}^c(a_s(q^2(1-z)^2),z)\right)$$

- C_0 is proportional to $\delta(1-z)$. Finite part after cancelling of poles be F & S.

- The integrant is the finite part after cancellation poles between splitting kernels and soft collinear function

$$P'_{cc}\big(z,a_s(\mu_F^2)\big) = 2\Big[A^c(a_s(\mu_F^2))\mathcal{D}_0(z) + C^c(a_s(\mu_F^2))\ln(1-z) + D^c(a_s(\mu_F^2))\Big]$$

- The finite contribution comes completely from soft-collinear function

$\mathscr{C}$ denotes convoluted exponential.

$$\mathscr{Q}^c(a_s(q^2(1-z)^2),z) = \left(\frac{1}{1-z}2\mathscr{G}_{SV}(a_s(q^2(1-z)^2))\right)_+ + 2\mathscr{G}_{NSV}(a_s(q^2(1-z)^2),z)$$

RESUMMATION IN MELLIN SPACE

- Convolutions are easy to handle in Mellin space. Hence, generally we solve this integral representation in Mellin space - gives the resummation formula in Mellin N-space.
- Definition :

$$\Delta_N^c = \int_0^1 dz\; z^{N-1} \Delta_c(z)$$

- The soft limit then converts to : $z \to 1 \longrightarrow N \to \infty$

$a_s \ln N \sim \mathcal{O}(1)$ **when** a_s **is small : spoils the truncation of series**

- The large logarithms transforms to :

SV NSV

$$\left(\frac{\ln(1-z)}{1-z}\right)_+ \to \frac{\ln^2 N}{2} - \frac{\ln N}{2N} + \frac{1}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$\ln^k(1-z) \to \frac{\ln^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

REORGANISATION OF SERIES : RESUMMTION

- Solving the integral representation in Mellin N-space :

$$\Delta_N^{\mathrm{SV+NSV}}(q^2,\mu_R^2,\mu_F^2) = \left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\tilde{g}_{0,i}(q^2,\mu_R^2,\mu_F^2)\right) \exp\left(\Psi^c_{\mathrm{sv,N}} + \Psi^c_{\mathrm{nsv,N}}\right)$$

- The SV part is well-known to third logarithmic accuracy for Color singlet processes

$$\omega = 2\ a_s(\mu_R^2)\ \beta_0 \log N$$

[Sterman]

[Catani, Trentedue]

$$\Psi^c_{\mathrm{sv,N}} = g_1^c(\omega)\ln(N) + \sum_{i=0}^{\infty} a_s^i(\mu_R^2) g^c_{i+2}(\omega)$$

- The NSV part is the new result :

$$\Psi^c_{\mathrm{nsv,N}} = \frac{1}{N}\sum_{n=0}^{\infty} a_s^n \left[\bar{g}^c_{n+2}(\omega) + \sum_{k=0}^{n} h^c_{nk}(\omega)\ \log^k N\right]$$

ALL ORDER PREDICTIONS : SV&NSV

- Let us recollect the all order predations form SV resummation :

$$\mathcal{L}^i = \ln^i(N)$$

$$L_N^i = \frac{1}{N}\ln^i(N)$$

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta^{(2)}_{c,N}$	$\Delta^{(3)}_{c,N}$	$\Delta^{(i)}_{c,N}$
SV-LL	$\tilde{g}^c_{0,0}, g^c_1$	$\mathcal{L}^4$	$\mathcal{L}^6$	$\mathcal{L}^{2i}$
SV-NLL	$\tilde{g}^c_{0,1}, g^c_2$		$\mathcal{L}^5, \mathcal{L}^4$	$\mathcal{L}^{2i-1}, \mathcal{L}^{2i-2}$
SV-N^{2}LL	$\tilde{g}^c_{0,2}, g^c_3$			$\mathcal{L}^{2i-3}, \mathcal{L}^{2i-4}$
SV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$			$\mathcal{L}^{2i-2n-1}, \mathcal{L}^{2i-2n}$

- Similar way, the all order predictions from NSV resummation could found to be :

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta^{(2)}_{c,N}$	$\Delta^{(3)}_{c,N}$	$\Delta^{(i)}_{c,N}$
NSV-LL	$\tilde{g}^c_{0,0}, g^c_1, \overline{g}^c_1, h^c_0$	L^3_N	L^5_N	L^{2i-1}_N
NSV-NLL	$\tilde{g}^c_{0,1}, g^c_2, \overline{g}^c_2, h^c_1$		L^4_N	L^{2i-2}_N
NSV-N^{2}LL	$\tilde{g}^c_{0,2}, g^c_3, \overline{g}^c_3, h^c_2$			L^{2i-3}_N
NSV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$			$L^{2i-(n+1)}_N$

ALL ORDER PREDICTIONS : SV&NSV

- Let us recollect the all order predations form SV resummation :

$\mathcal{L}^i = \ln^i(N)$

$L_N^i = \frac{1}{N}\ln^i(N)$

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta_{c,N}^{(2)}$	$\Delta_{c,N}^{(3)}$	$\Delta_{c,N}^{(i)}$
SV-LL	$\tilde{g}_{0,0}^c, g_1^c$	$\mathcal{L}^4$	$\mathcal{L}^6$	$\mathcal{L}^{2i}$
SV-NLL	$\tilde{g}_{0,1}^c, g_2^c$		$\mathcal{L}^5, \mathcal{L}^4$	$\mathcal{L}^{2i-1}, \mathcal{L}^{2i-2}$
SV-N^{2}LL	$\tilde{g}_{0,2}^c, g_3^c$			$\mathcal{L}^{2i-3}, \mathcal{L}^{2i-4}$
SV-N^{n}LL	$\tilde{g}_{0,n}^c, g_{n+1}^c, \overline{g}_{n+1}^c, h_n^c$			$\mathcal{L}^{2i-2n-1}, \mathcal{L}^{2i-2n}$

- Similar way, the all order predictions from NSV resummation could found to be :

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta_{c,N}^{(2)}$	$\Delta_{c,N}^{(3)}$	$\Delta_{c,N}^{(i)}$
NSV-LL	$\tilde{g}_{0,0}^c, g_1^c, \overline{g}_1^c, h_0^c$	L_N^3	L_N^5	L_N^{2i-1}
NSV-NLL	$\tilde{g}_{0,1}^c, g_2^c, \overline{g}_2^c, h_1^c$		L_N^4	L_N^{2i-2}
NSV-N^{2}LL	$\tilde{g}_{0,2}^c, g_3^c, \overline{g}_3^c, h_2^c$			L_N^{2i-3}
NSV-N^{n}LL	$\tilde{g}_{0,n}^c, g_{n+1}^c, \overline{g}_{n+1}^c, h_n^c$			$L_N^{2i-(n+1)}$

1- loop info

Resumed terms

$a_s \frac{1}{N} \log N$

$a_s^2 \frac{1}{N} \log^3 N$

$a_s^3 \frac{1}{N} \log^5 N$

•
•
•

$a_s^i \frac{1}{N} \log^{2i-1} N$

LL

ALL ORDER PREDICTIONS : SV&NSV

- Let us recollect the all order predations form SV resummation :

$$\mathcal{L}^i = \ln^i(N)$$

$$L_N^i = \frac{1}{N}\ln^i(N)$$

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta_{c,N}^{(2)}$	$\Delta_{c,N}^{(3)}$	$\Delta_{c,N}^{(i)}$
SV-LL	$\tilde{g}_{0,0}^c, g_1^c$	$\mathcal{L}^4$	$\mathcal{L}^6$	$\mathcal{L}^{2i}$
SV-NLL	$\tilde{g}_{0,1}^c, g_2^c$		$\mathcal{L}^5, \mathcal{L}^4$	$\mathcal{L}^{2i-1}, \mathcal{L}^{2i-2}$
SV-N²LL	$\tilde{g}_{0,2}^c, g_3^c$			$\mathcal{L}^{2i-3}, \mathcal{L}^{2i-4}$
SV-NⁿLL	$\tilde{g}_{0,n}^c, g_{n+1}^c, \overline{g}_{n+1}^c, h_n^c$			$\mathcal{L}^{2i-2n-1}, \mathcal{L}^{2i-2n}$

Resumed terms

- Similar way, the all order predictions from NSV resummation could found to be :

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta_{c,N}^{(2)}$	$\Delta_{c,N}^{(3)}$	$\Delta_{c,N}^{(i)}$
NSV-LL	$\tilde{g}_{0,0}^c, g_1^c, \overline{g}_1^c, h_0^c$	L_N^3	L_N^5	L_N^{2i-1}
NSV-NLL	$\tilde{g}_{0,1}^c, g_2^c, \overline{g}_2^c, h_1^c$		L_N^4	L_N^{2i-2}
NSV-N²LL	$\tilde{g}_{0,2}^c, g_3^c, \overline{g}_3^c, h_2^c$			L_N^{2i-3}
NSV-NⁿLL	$\tilde{g}_{0,n}^c, g_{n+1}^c, \overline{g}_{n+1}^c, h_n^c$			$L_N^{2i-(n+1)}$

2- loop info

$$a_s^2 \frac{1}{N} \log^2 N$$

$$a_s^3 \frac{1}{N} \log^4 N$$

$$a_s^4 \frac{1}{N} \log^6 N$$

$$\vdots$$

$$a_s^i \frac{1}{N} \log^{2i-2} N$$

NLL

ALL ORDER PREDICTIONS : SV&NSV

➤ Let us recollect the all order predations form SV resummation :

$$\mathcal{L}^i = \ln^i(N)$$

$$L_N^i = \frac{1}{N}\ln^i(N)$$

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta^{(2)}_{c,N}$	$\Delta^{(3)}_{c,N}$	$\Delta^{(i)}_{c,N}$
SV-LL	$\tilde{g}^c_{0,0}, g^c_1$	$\mathcal{L}^4$	$\mathcal{L}^6$	$\mathcal{L}^{2i}$
SV-NLL	$\tilde{g}^c_{0,1}, g^c_2$		$\mathcal{L}^5, \mathcal{L}^4$	$\mathcal{L}^{2i-1}, \mathcal{L}^{2i-2}$
SV-N^{2}LL	$\tilde{g}^c_{0,2}, g^c_3$			$\mathcal{L}^{2i-3}, \mathcal{L}^{2i-4}$
SV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$			$\mathcal{L}^{2i-2n-1}, \mathcal{L}^{2i-2n}$

Resumed terms

➤ Similar way, the all order predictions from NSV resummation could found to be :

GIVEN		PREDICTIONS		
Logarithmic Accuracy	Resummed Exponents	$\Delta^{(2)}_{c,N}$	$\Delta^{(3)}_{c,N}$	$\Delta^{(i)}_{c,N}$
NSV-LL	$\tilde{g}^c_{0,0}, g^c_1, \overline{g}^c_1, h^c_0$	L^3_N	L^5_N	L^{2i-1}_N
NSV-NLL	$\tilde{g}^c_{0,1}, g^c_2, \overline{g}^c_2, h^c_1$		L^4_N	L^{2i-2}_N
NSV-N^{2}LL	$\tilde{g}^c_{0,2}, g^c_3, \overline{g}^c_3, h^c_2$			L^{2i-3}_N
NSV-N^{n}LL	$\tilde{g}^c_{0,n}, g^c_{n+1}, \overline{g}^c_{n+1}, h^c_n$			$L^{2i-(n+1)}_N$

n- loop info

$$a_s^n \frac{1}{N} \log^n N$$

⋮

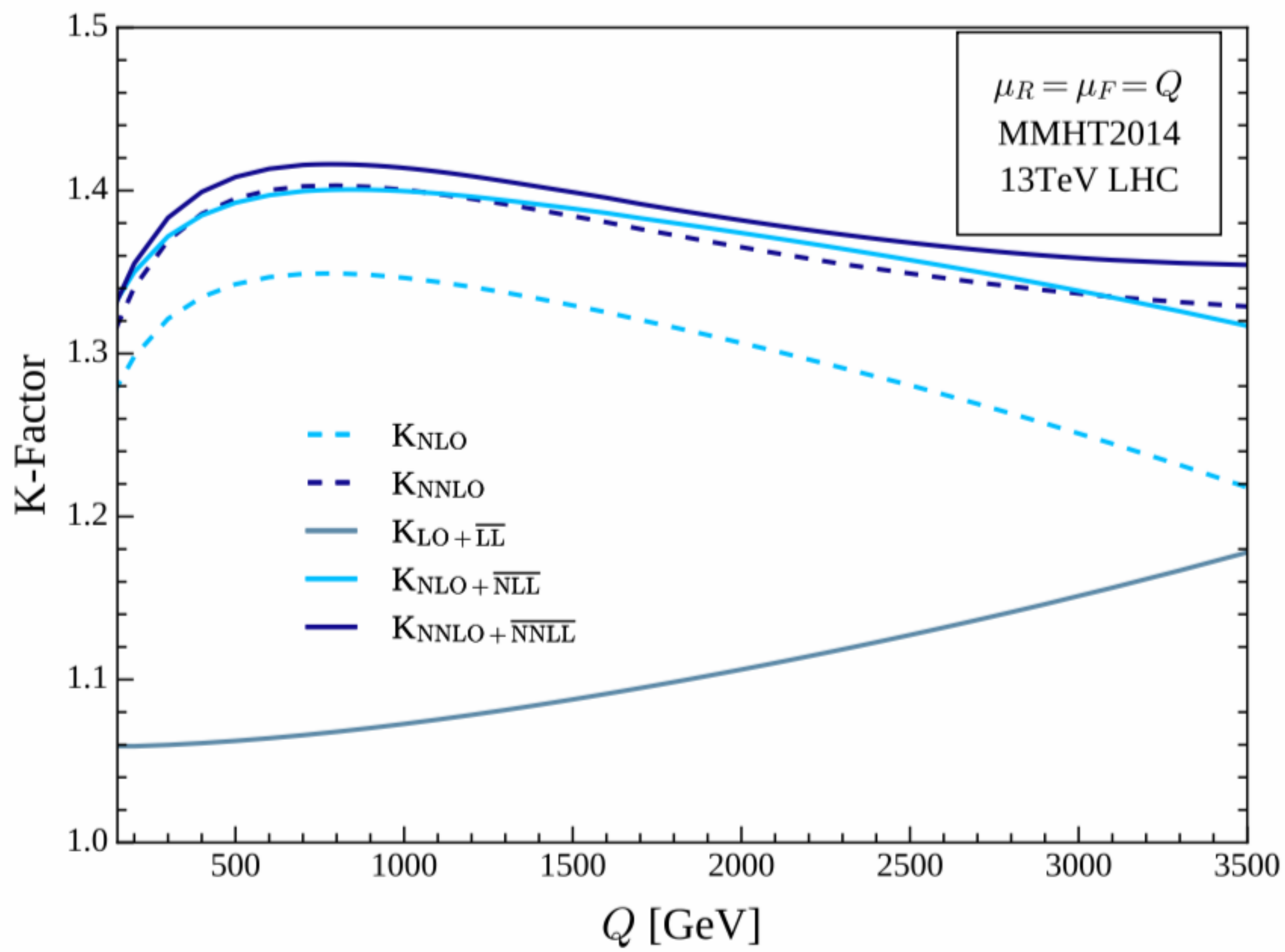
$$a_s^i \frac{1}{N} \log^{2i-n} N$$

N^{n}LL

RESUMMATION FOR DY : K– FACTOR

- **Now let us see how these NSV resummation is phenomenologically relevant.**

$\mu_R = \mu_F = Q$(GeV)	$\mathrm{LO} + \overline{\mathrm{LL}}$	NLO	$\mathrm{NLO} + \overline{\mathrm{NLL}}$	NNLO	$\mathrm{NNLO} + \overline{\mathrm{NNLL}}$
500	1.0624	1.3425	1.3925	1.3950	1.4082
1000	1.0728	1.3464	1.3995	1.4004	1.4138
2000	1.1062	1.3064	1.3739	1.3652	1.3818



Increment from FO → resum at Q=2000 GeV

- 10.6% for LO $\rightarrow \mathrm{LO} + \overline{\mathrm{LL}}$
- 5.2% for NLO $\rightarrow \mathrm{NLO} + \overline{\mathrm{NLL}}$
- 1.2% for NNLO $\rightarrow \mathrm{NNLO} + \overline{\mathrm{NNLL}}$

➤ $\mathrm{NLO} + \overline{\mathrm{NLL}}$ for quark part mimics the NNLO

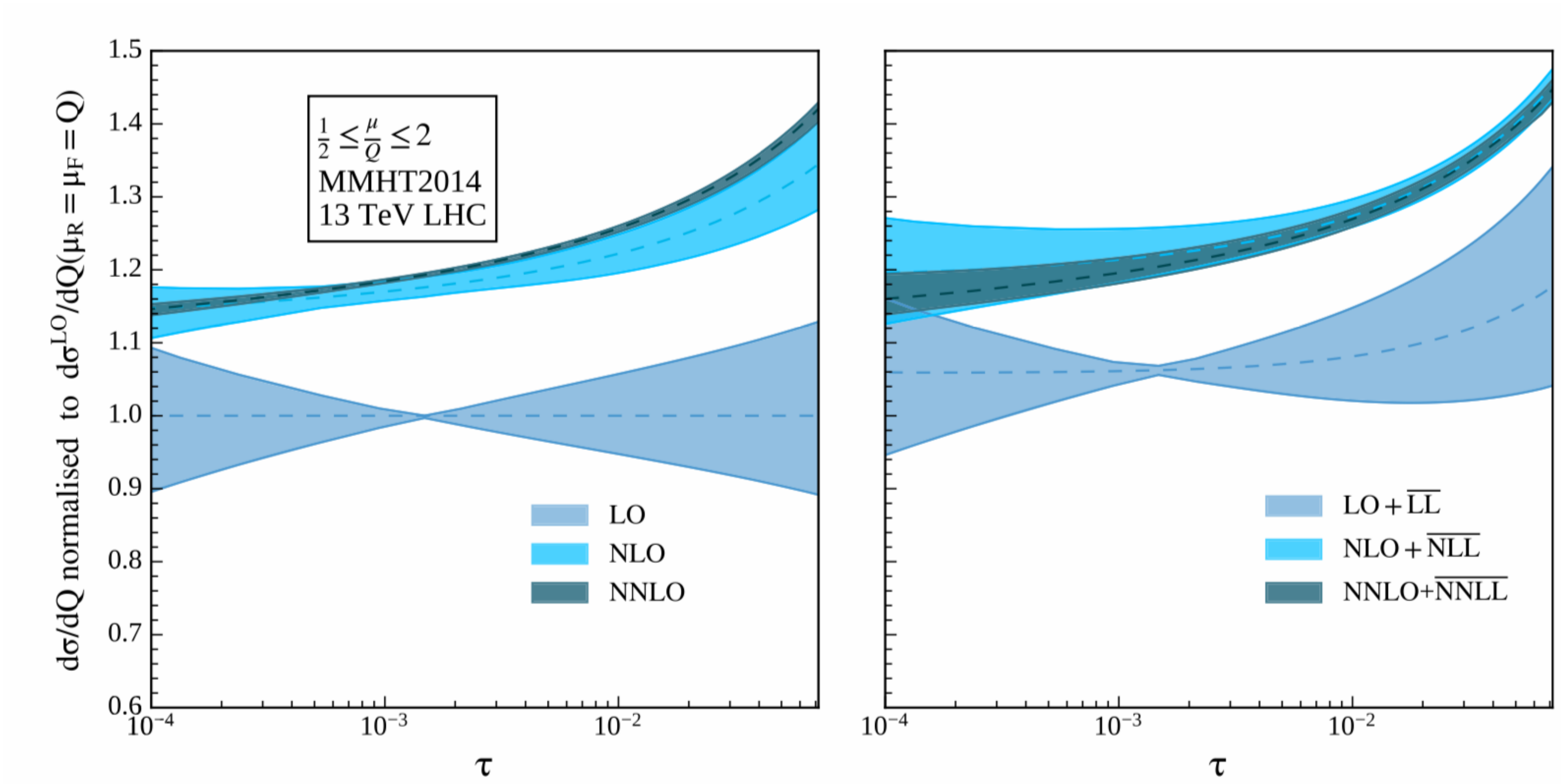
➤ Resummed predictions are closer compared to FO : improves the reliability of perturbative predictions

➤ Resummed corrections decreases as we go higher order.

RESUMMATION FOR DY : SCALE VARIATIONS

- Impact of μ_R & μ_F scales in the predictions using canonical 7-point variation $\frac{1}{2} \leq \frac{\mu_F}{Q}, \frac{\mu_R}{Q} \leq 2$

[AAH,P.Mukherjee,V.Ravindran,A.Sankar,S.Tiwari]



- FO from all channels
- resummed predictions from only diagonal channels $(q\bar{q})$

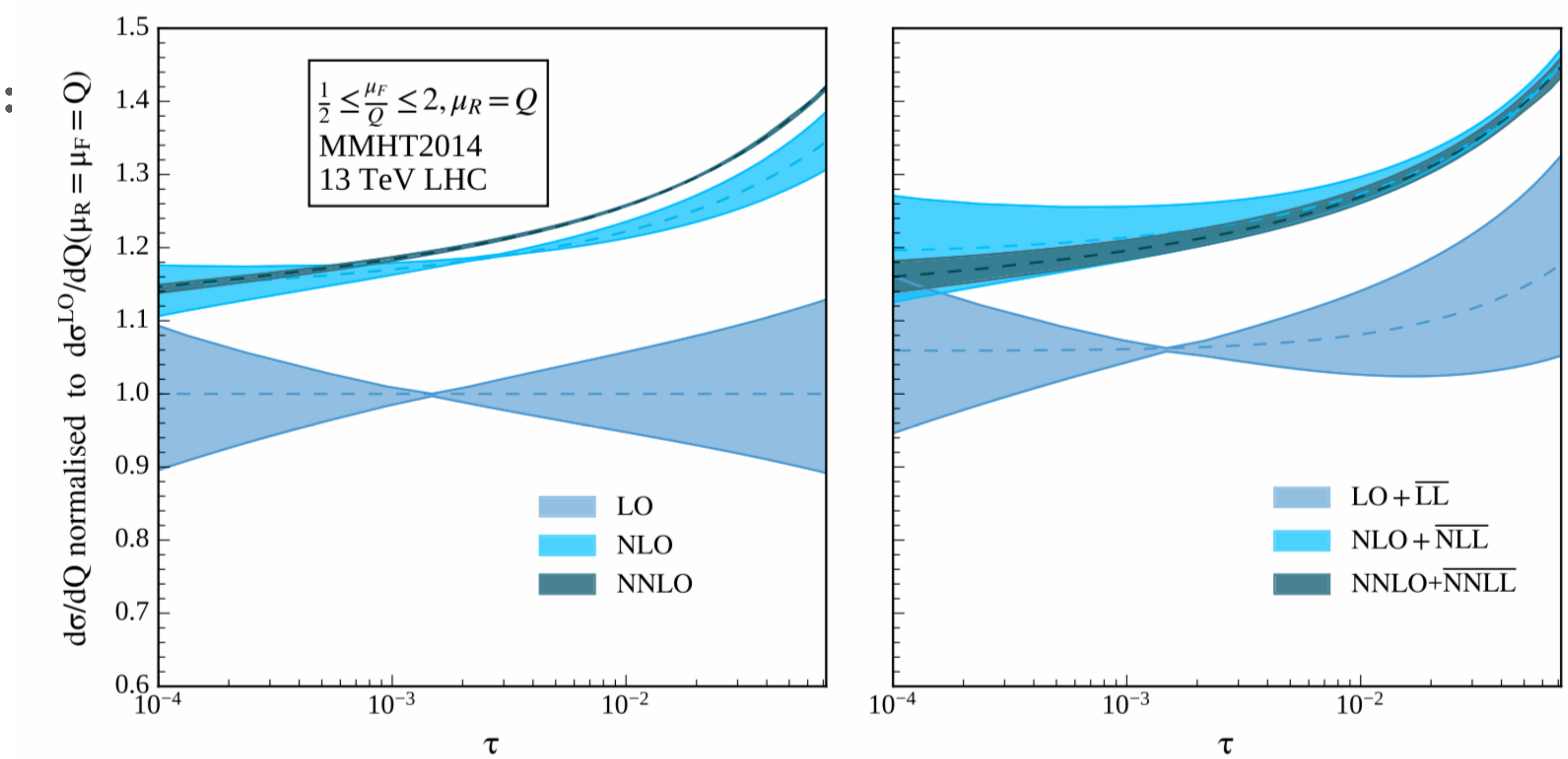
$\tau = \frac{Q^2}{S}$

➤ NLO to $\mathrm{NLO}+\overline{\mathrm{NLL}}$: band width decreases. Large scale uncertainty at $\mathrm{NNLO}+\overline{\mathrm{NNLL}}$

➤ $\mathrm{NNLO}+\overline{\mathrm{NNLL}}$ is within $\mathrm{NLO}+\overline{\mathrm{NLL}}$ unlike fixed order case at high energies : notable NSV corrections from diagonal channels

RESUMMATION FOR DY : μ_F - SCALE

- **In order to understand the cause of large uncertainty at $\mathrm{NNLO} + \overline{\mathrm{NNLL}}$, we study the scales separately.**
- **The μ_F - variation :**

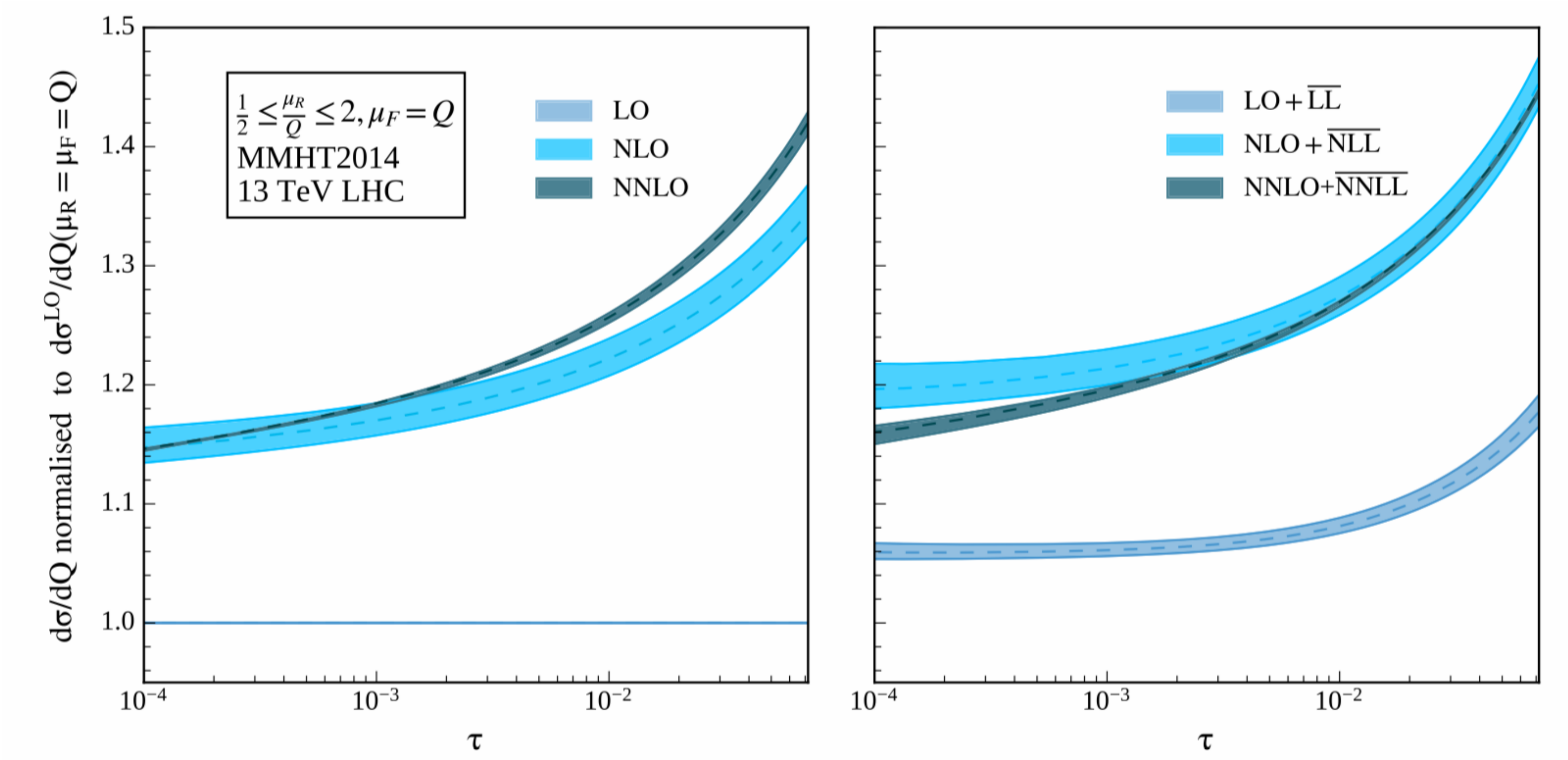


- Looks similar to 7-point scale variation : large band in 7-point scale could be due to μ_F-uncertainties
- Reason could be due to lack of off-diagonal contribution! Collinear logarithms arises from $q\bar{q}$ & qg channels
  - At NLO : $q\bar{q} \rightarrow 22\%$ & $qg \rightarrow -5\,\%$
  - At NNLO : $q\bar{q} \rightarrow 4.9\%$ & $qg \rightarrow -2.5\,\%$

Bigger cancellation at NNLO. Lack of qg- resummed predictions cause the larger uncertainty at $\mathrm{NNLO} + \overline{\mathrm{NNLL}}$

RESUMMATION FOR DY : μ_R - SCALE

- **To emphasise the role of collinear resummation, we see the μ_R - variation :**



- NLO : $^{+1.46\%}_{-1.28\%}$
- NLO + $\overline{\text{NLL}}$: $^{+1.35\%}_{-1.23\%}$
- NNLO : $^{+0.37\%}_{-0.46\%}$
- NNLO + $\overline{\text{NNLL}}$: $^{+0.02\%}_{-0.23\%}$

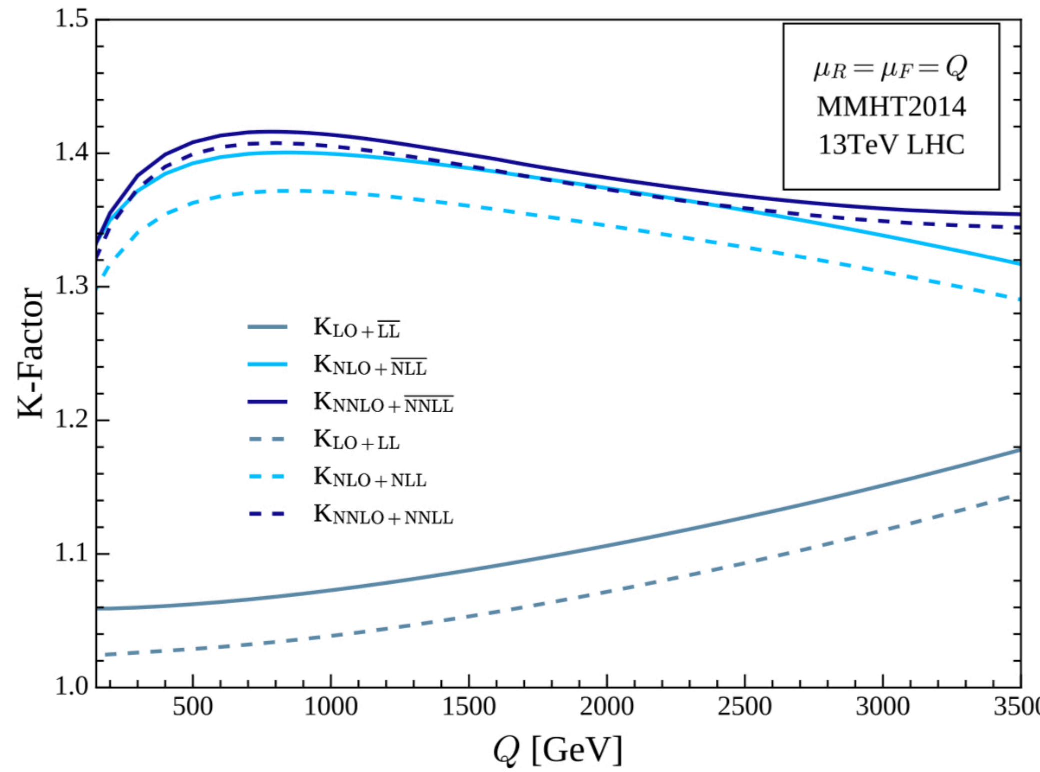
➤ Substantial scale reduction at NNLO + $\overline{\text{NNLL}}$

➤ The μ_R cancellation happens within each partonic channels.

➤ Inclusion of resummed predictions improves the μ_R uncertainty remarkably

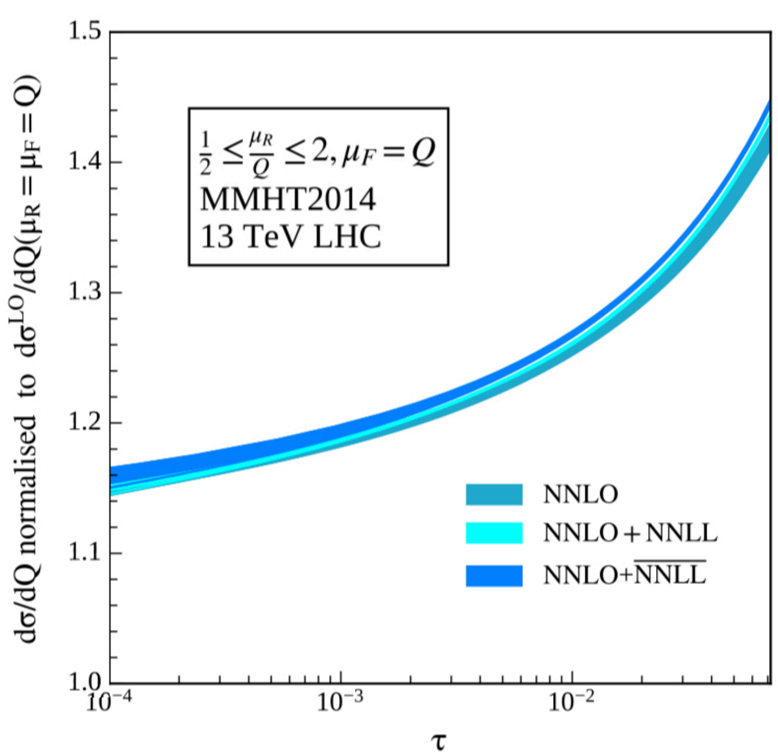
RESUMMATION FOR DY : SV & NSV

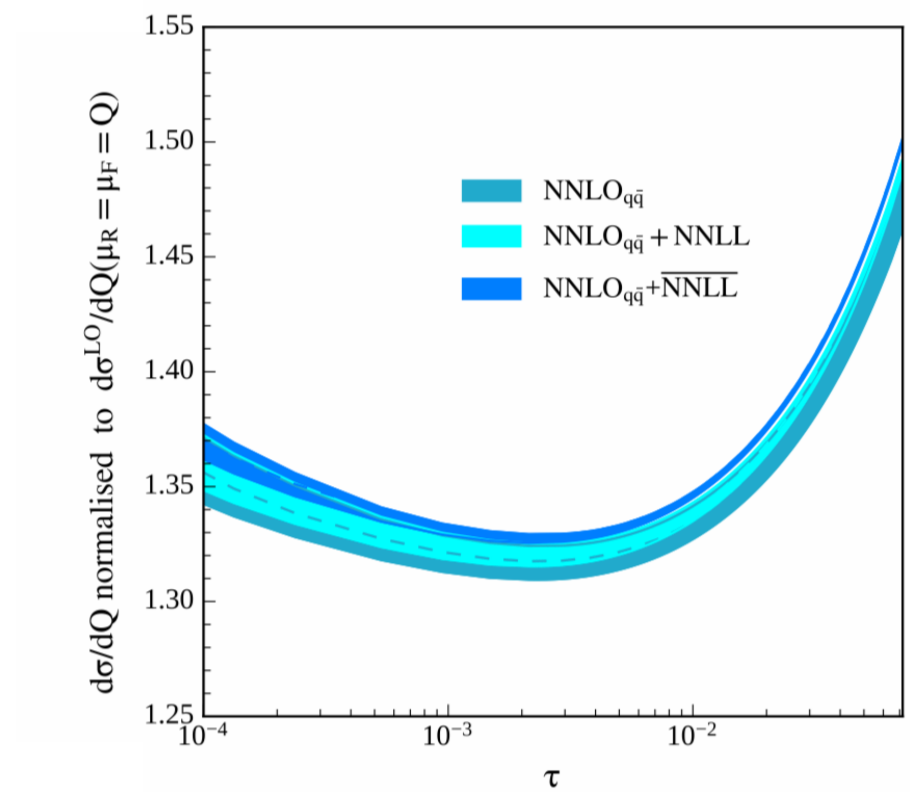
- **Comparing SV resummation, SV+NSV resummation with NNLO results**



Increment from SV resum → NSV resum at Q=2000 GeV

- 2.1% for NLO → $\mathrm{NLO} + \overline{\mathrm{NLL}}$
- 0.64% for NNLO → $\mathrm{NNLO} + \overline{\mathrm{NNLL}}$





SUMMARY

- Require threshold resummation in the kinematic limit $z \to 1$
- We extend the threshold framework with inclusion of NSV large logarithms
- This enhance the resummed predictions and improves the better perturbative convergence
- However, large scale uncertainties shows the need of including off-diagonal resummation for large collinear logarithms, for which works are on the way!

SUMMARY

- Require threshold resummation in the kinematic limit $z \to 1$
- We extend the threshold framework with inclusion of NSV large logarithms
- This enhance the resummed predictions and improves the better perturbative convergence
- However, large scale uncertainties shows the need of including off-diagonal resummation for large collinear logarithms, for which works are on the way!

THANKS FOR THE ATTENTION !

Back up slides

DIAGONAL AND OFF DIAGONAL

$$
\begin{aligned}
\frac{\hat{\sigma}_{q\bar{q}}}{z\sigma_0} &= \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{g\bar{q}} + \Gamma^T_{qq} \otimes \Delta_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \\
&+ \Gamma^T_{qg} \otimes \Delta_{gq} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{qg} \otimes \Delta_{gg} \otimes \Gamma_{g\bar{q}} + \Gamma^T_{qg} \otimes \Delta_{g\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \\
&+ \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}q} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}g} \otimes \Gamma_{g\bar{q}} + \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \,.
\end{aligned}
$$

$$
\frac{\hat{\sigma}^{\rm sv+nsv}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\rm sv+nsv}_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \,.
$$

$$
\begin{aligned}
\frac{\hat{\sigma}_{qg}}{z\sigma_0} &= \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{qg} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{gg} + \Gamma^T_{qq} \otimes \Delta_{q\bar{q}} \otimes \Gamma_{\bar{q}g} \\
&+ \Gamma^T_{qg} \otimes \Delta_{gq} \otimes \Gamma_{qg} + \Gamma^T_{qg} \otimes \Delta_{gg} \otimes \Gamma_{gg} + \Gamma^T_{qg} \otimes \Delta_{g\bar{q}} \otimes \Gamma_{\bar{q}g} \\
&+ \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}q} \otimes \Gamma_{qg} + \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}g} \otimes \Gamma_{gg} + \Gamma^T_{q\bar{q}} \otimes \Delta_{\bar{q}\bar{q}} \otimes \Gamma_{\bar{q}g} \,.
\end{aligned}
$$

$$
\frac{\hat{\sigma}^{\rm sv+nsv}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\rm sv+nsv}_{q\bar{q}} \otimes \Gamma_{\bar{q}g} + \Gamma^T_{qq} \otimes \Delta^{\rm sv+nsv}_{qg} \otimes \Gamma_{gg} \,.
$$

ϵ & z DEPENDENCY OF SV AND NSV

- SV

$$\hat{\phi}^{c(1)}_{SV}(\epsilon) = \frac{1}{\epsilon^2}\left(2A^c_1\right) + \frac{1}{\epsilon}\left(\overline{\mathcal{G}}^c_1(\epsilon)\right)$$

$$\hat{\phi}^{c(2)}_{SV}(\epsilon) = \frac{1}{\epsilon^3}\left(-\beta_0 A^c_1\right) + \frac{1}{\epsilon^2}\left(\frac{1}{2}A^c_2 - \beta_0\overline{\mathcal{G}}^c_1(\epsilon)\right) + \frac{1}{2\epsilon}\overline{\mathcal{G}}^c_2(\epsilon)$$

$$\hat{\phi}^{c(3)}_{SV}(\epsilon) = \frac{1}{\epsilon^4}\left(\frac{8}{9}\beta_0^2 A^c_1\right) + \frac{1}{\epsilon^3}\left(-\frac{2}{9}\beta_1 A^c_1 - \frac{8}{9}\beta_0 A^c_2 - \frac{4}{3}\beta_0^2\overline{\mathcal{G}}^c_1(\epsilon)\right) + \frac{1}{\epsilon^2}\left(\frac{2}{9}A^c_3 - \frac{1}{3}\beta_1\overline{\mathcal{G}}^c_1(\epsilon) - \frac{4}{3}\beta_0\overline{\mathcal{G}}^c_2(\epsilon)\right) + \frac{1}{\epsilon}\left(\frac{1}{3}\overline{\mathcal{G}}^c_3(\epsilon)\right)$$

- NSV

$$\hat{\varphi}^{(1)}_c(z,\epsilon) = \frac{1}{\epsilon}\mathcal{G}^c_{L,1}(z,\epsilon)\,,$$
$$\hat{\varphi}^{(2)}_c(z,\epsilon) = \frac{1}{\epsilon^2}(-\beta_0\mathcal{G}^c_{L,1}(z,\epsilon)) + \frac{1}{2\epsilon}\mathcal{G}^c_{L,2}(z,\epsilon)\,,$$
$$\hat{\varphi}^{(3)}_c(z,\epsilon) = \frac{1}{\epsilon^3}\left(\frac{4}{3}\beta_0^2\mathcal{G}^c_{L,1}(z,\epsilon)\right) + \frac{1}{\epsilon^2}\left(-\frac{1}{3}\beta_1\mathcal{G}^c_{L,1}(z,\epsilon) - \frac{4}{3}\beta_0\mathcal{G}^c_{L,2}(z,\epsilon)\right) + \frac{1}{3\epsilon}\mathcal{G}^c_{L,3}(z,\epsilon)\,,$$

$$\mathcal{G}^c_{L,i}(z,\epsilon) = L^c_i(z) + \overline{\chi}^c_{L,i}(z) + \sum_{j=1}^{\infty}\epsilon^j\mathcal{G}^{c,(j)}_{L,i}(z)\,, \qquad \mathcal{G}^{c,(j)}_{L,i}(z) = \sum_{k=0}^{i+j-1}\mathcal{G}^{c,(j,k)}_{L,i}\ln^k(1-z)\,.$$

MORE PLOTS : N & $\bar{N}$ SCHEMES

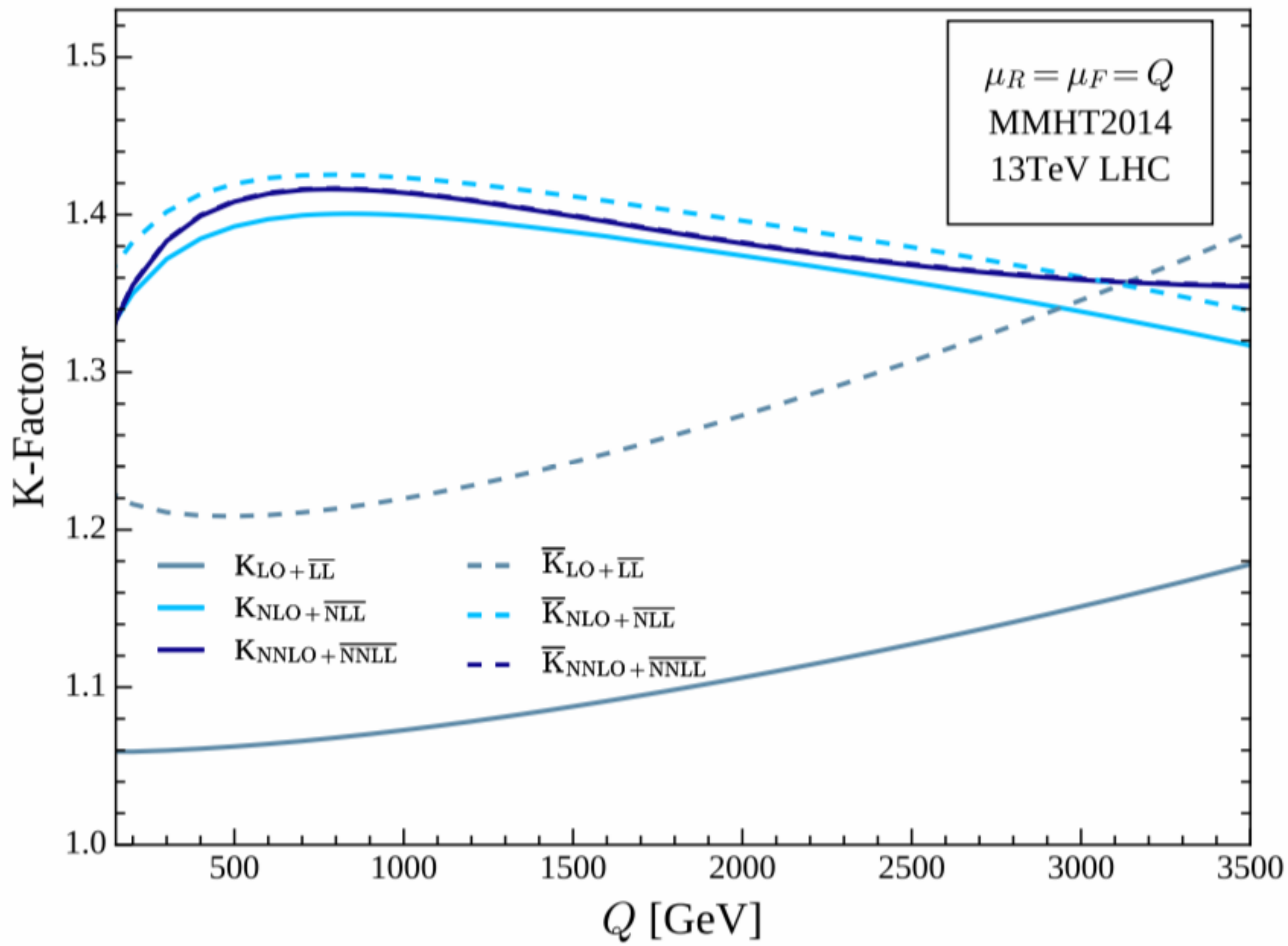


Fig. 14 Comparison between the K-factors for the N and $\overline{N}$ exponentiation at the central scale $Q = \mu_R = \mu_F$

- Uncertainty is smaller for N-Exponentation

MELLIN N-SPACE STRUCTURE

- Partonic cross section in N-space in terms of large logarithms $\ln N$ & $\dfrac{\ln N}{N}$

$$
\begin{aligned}
\hat{\sigma}_N = 1 &+ a_s\Big[c_1^2 \ln^2 N + \cdots + c_1^0 + d_1^1 \frac{\ln N}{N} + d_1^0 \frac{1}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big)\Big] \\
&a_s^2\Big[c_2^4 \ln^4 N + \cdots + c_2^0 + d_2^3 \frac{\ln^3 N}{N} + \cdots + d_2^0 \frac{1}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big)\Big] \\
&a_s^3\Big[c_3^6 \ln^6 N + \cdots + c_3^0 + d_3^5 \frac{\ln^5 N}{N} + \cdots + d_3^0 \frac{1}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big)\Big] \\
&+\cdots \\
&a_s^n\Big[c_n^{2n} \ln^{2n} N + \cdots + c_n^0 + d_n^{2n-1} \frac{\ln^{2n-1} N}{N} + \cdots + d_3^0 \frac{1}{N} + \mathcal{O}\Big(\frac{1}{N^2}\Big)\Big]
\end{aligned}
$$

$a_s \ln N \sim \mathcal{O}(1)$ when a_s is small : spoils the truncation of series

SV

Resummation is well understood.

[sterman et al]

[Catani et al]

NSV Ongoing research!

SV+NSV : PREDICTIONS

	PREDICTIONS		
	$\Delta_c^{(2)}$	$\Delta_c^{(3)}$	$\Delta_c^{(i)}$
Given one loop coefficients	$\mathcal{D}_3, \mathcal{D}_2$ L_z^3	$\mathcal{D}_5, \mathcal{D}_4$ L_z^5	$\mathcal{D}_{(2i-1)}, \mathcal{D}_{(2i-2)}$ $L_z^{(2i-1)}$
Given two loop coefficients		$\mathcal{D}_3, \mathcal{D}_2$ L_z^4	$\mathcal{D}_{(2i-3)}, \mathcal{D}_{(2i-4)}$ $L_z^{(2i-2)}$
Given three loop coefficients			$\mathcal{D}_{(2i-5)}, \mathcal{D}_{(2i-6)}$ $L_z^{(2i-3)}$
			$\mathcal{D}_{(2i-(2n-1))}, \mathcal{D}_{(2i-2n)}$ $L_z^{(2i-n)}$

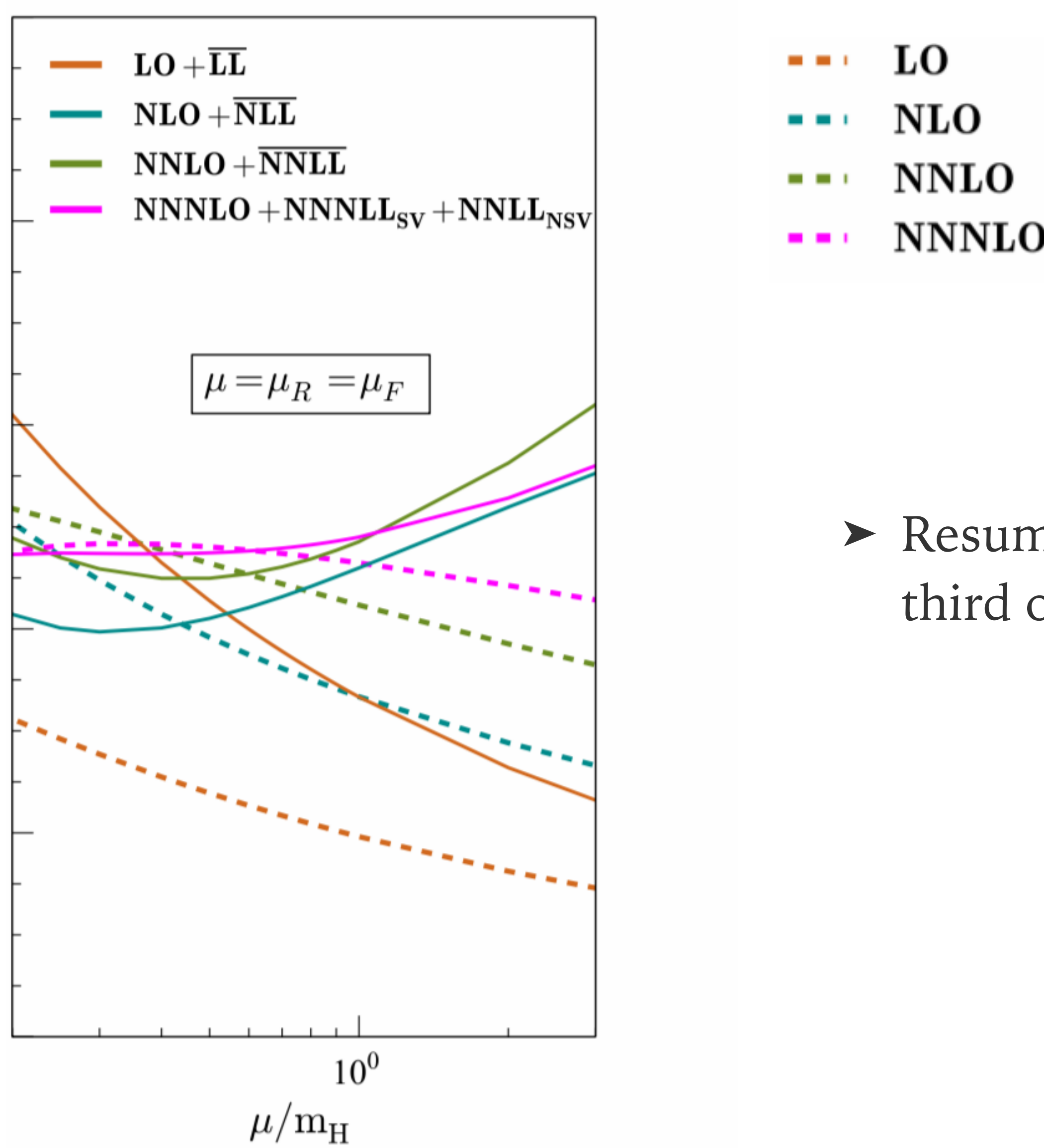
$$D_i = \left(\frac{\ln^i(1-z)}{1-z}\right)_+$$

$$L_z^i = \ln^i(1-z)$$

RESUMMATION FOR $gg \rightarrow H$

- **The μ variation with $\mu_R = \mu_F = \mu$ in pb at 13 TeV with central scale $\mu_R = \mu_F = \frac{m_H}{2}$**

LO	LO+$\overline{\text{LL}}$	NLO	NLO+$\overline{\text{NLL}}$	NNLO	NNLO+$\overline{\text{NNLL}}$
$23.8940^{+5.33}_{-4.27}$	$42.7612^{+13.03}_{-9.44}$	$39.1681^{+7.99}_{-5.82}$	$41.0325^{+4.95}_{-1.32}$	$46.4304^{+4.13}_{-4.09}$	$44.9685^{+3.58}_{-0.004}$



➤ Resummed cross section decreases at second and third order indicating better perturbative convergence

RESUMMATION FOR $gg \to H$: SV & NSV

Table 10 Values of resummed SV + NSV cross section (in pb) at various orders in comparison to the fixed order results and resummed SV predictions at central scale value $\mu_R = \mu_F = \frac{m_H}{2}$

NLO	NLO + NLL	NLO + $\overline{\text{NLL}}$	NNLO	NNLO + NNLL	NNLO + $\overline{\text{NNLL}}$
$39.1681^{+9.09}_{-6.73}$	$38.0142^{+7.06}_{-5.70}$	$41.0325^{+7.06}_{-5.97}$	$46.4304^{+4.11}_{-4.70}$	$45.0904^{+4.32}_{-4.52}$	$44.9685^{+2.94}_{-3.74}$

- No considerable improvement at first order due to them NSV resummed predictions compared to SV ones
- But better behaviour at NNLO + $\overline{\text{NNLL}}$
- Reason could be due to the considerable NSV contributions at NNLO and not at NLO
  - at NLO : 73.16% SV and 45.81% NSV
  - at NNLO : 15.8% SV and 58.9% NSV