Exclusive vector meson production at next-to-leading order in the Color Glass Condensate framework

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Diffraction and Low-x

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Exclusive vector meson production in deep inelastic scattering

- $\gamma^*_\lambda + A \to V_\lambda + A$
- Requires an exchange of at least two gluons

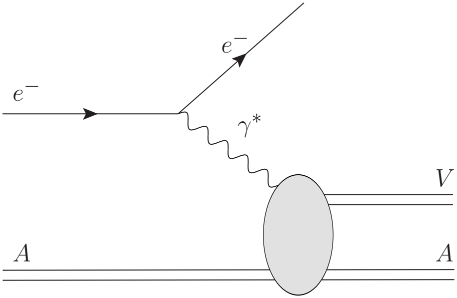
  Ryskin, Z.Phys.C 57 (1993) 89-92:

$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma(\gamma^* + A \to V + A) \sim [xg(x)]^2$$

$\Rightarrow$ Highly sensitive to the gluon structure of the nucleus

- Momentum transfer $\boldsymbol{\Delta}$ can be measured:
  - Conjugate to the impact parameter **b**

$\Rightarrow$ Access to the spatial distribution of gluons



This talk:

- Heavy mesons (quarkonia) in the nonrelativistic limit $\mathcal{O}(\alpha_s v^0, \alpha_s^0 v^2)$
- Light mesons $\mathcal{O}(\alpha_s)$ at high Q^2

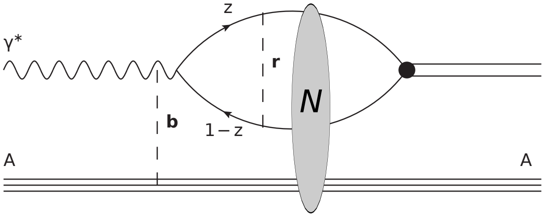
Vector meson production at the leading order in the dipole picture

- Factorization in the high-energy limit:

Invariant amplitude for exclusive vector meson production

$$-i\mathcal{A} = 2\int \mathrm{d}^2\mathbf{b}\,\mathrm{d}^2\mathbf{r}\,\frac{\mathrm{d}z}{4\pi}\,e^{-i\left(\mathbf{b}+\left(\frac{1}{2}-z\right)\mathbf{r}\right)\cdot\boldsymbol{\Delta}}\,\Psi_{\gamma^*}^{q\bar{q}}(\mathbf{r},z)\,N(\mathbf{r},\mathbf{b},Y)\,\Psi_V^{q\bar{q}*}(\mathbf{r},z)$$

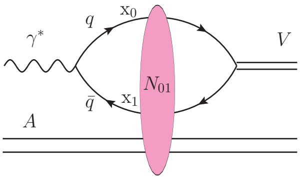
- $\Psi_{\gamma^*}^{q\bar{q}}$: Photon light-front wave function
- N: Dipole-target scattering amplitude
- $\Psi_V^{q\bar{q}}$: Vector meson light-front wave function



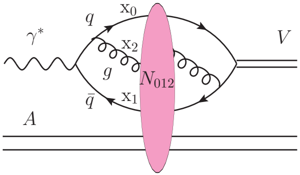
Exclusive vector meson production at NLO

Invariant amplitude for exclusive vector meson production

$$-i\mathcal{A}(t=0) = 2\int \mathrm{d}^2\mathbf{x}_0\,\mathrm{d}^2\mathbf{x}_1 \int \frac{\mathrm{d}z_0\,\mathrm{d}z_1}{(4\pi)}\delta(z_0+z_1-1)\Psi_{\gamma^*}^{q\bar{q}} N_{01} \Psi_V^{q\bar{q}*}$$
$$+2\int \mathrm{d}^2\mathbf{x}_0\,\mathrm{d}^2\mathbf{x}_1\,\mathrm{d}^2\mathbf{x}_2 \int \frac{\mathrm{d}z_0\,\mathrm{d}z_1\,\mathrm{d}z_2}{(4\pi)^2}\delta(z_0+z_1+z_2-1)\Psi_{\gamma^*}^{q\bar{q}g} N_{012} \Psi_V^{q\bar{q}g*}$$

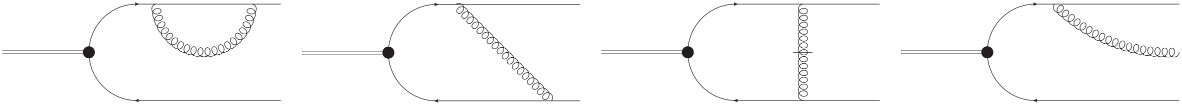


- Also contribution from the $q\bar{q}g$ state
- Need the wave functions for $q\bar{q}$ state at NLO
- Dipole amplitude N:
  - Perturbative evolution in rapidity (JIMWLK/BK)
  - Needs a nonperturbative initial condition (fit HERA Beuf et al, 2007.01645)

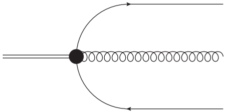


Perturbative corrections to the wave functions

Meson wave function: nonperturbative part (leading-order wave function) + perturbative part



- In principle: also contribution from the nonperturbative part
- In the both cases considered in this talk this can be neglected
  - Heavy mesons: suppressed by the velocity v of the quark $\Rightarrow \mathcal{O}(\alpha_s v^2)$
  - Light mesons: suppressed by the photon virtuality $Q^2 \Rightarrow \mathcal{O}(\frac{1}{Q^3})$



Photon wave function: Completely perturbative

Beuf, 1606.00777, 1708.06557; Hänninen, Lappi, Paatelainen, 1711.08207

Beuf, Lappi, Paatelainen, 2103.14549, 2112.03158, 2204.02486

Regularization and divergences

- IR cutoff α for the gluon longitudinal momentum fraction: $k_2^+ = z_2 q^+ > \alpha q^+ \Rightarrow z_2 > \alpha$
- Dimensional regularization in the transverse coordinates

The results are free of any divergences after the following are taken into account:

- Some divergences cancel between $q\bar{q}$ and $q\bar{q}g$ parts of the calculation
  - In practice: subtract UV divergences from $q\bar{q}g$ and add them to the $q\bar{q}$ part
- Balitsky-Kovchegov equation of the dipole amplitude N_{01}

$$\frac{\partial}{\partial Y} N_{01} = \frac{N_c \alpha_s}{2\pi^2} \int \mathrm{d}^2 \mathbf{x}_2 \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2 \mathbf{x}_{21}^2} \left[N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right], \; Y = \ln z_2 + \ln\left(q^+ / P^+\right)$$

- Renormalization the $V \to q\bar{q}$ vertex (leading-order wave function)
  - Different for heavy and light mesons

Heavy vector meson production

Based on Mäntysaari, JP, 2104.02349 and Mäntysaari, JP, 2204.14031

Nonrelativistic expansion

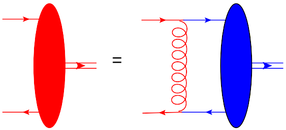
- Non-Relativistic QCD (NRQCD): Parametrically $v \sim \alpha_s(vM_V) > \alpha_s(M_V)$

  $\Rightarrow$ Expansion in v and α_s: $1 > \alpha_s \gtrsim v^2 > \alpha_s^2 \dots$

Nonrelativistic expansion Escobedo, Lappi, 1911.01136

$$\Psi_V^n = \sum_{m,k} \underbrace{C^k_{n\leftarrow m}}_{\text{perturbative corrections}} \underbrace{\int_0^1 \frac{\mathrm{d}z'}{4\pi} \left(\frac{1}{m_q}\nabla\right)^k \phi^m(\mathbf{r}=0, z')}_{\text{nonperturbative constant}}$$

- ϕ^m = leading-order wave function (LOWF) for Fock state m
- α_s corrections included in $C^k_{n\leftarrow m}$
- Relativistic corrections go as v^k in the index k



Escobedo, Lappi, 1911.01136

NLO calculation in the nonrelativistic limit

- Nonrelativistic limit: Leading-order wave function $\phi^{q\bar{q}}(\vec{k}) \sim (2\pi)^3\delta^3(\vec{k})$
- Only include the leading terms v^0 in heavy quark velocity:

$$\Psi_V^{q\bar{q}} = C^0_{q\bar{q}\leftarrow q\bar{q}} \int_0^1 \frac{\mathrm{d}z'}{4\pi}\phi^{q\bar{q}}(\mathbf{r}=0,z') \qquad \Psi_V^{q\bar{q}g} = C^0_{q\bar{q}g\leftarrow q\bar{q}} \int_0^1 \frac{\mathrm{d}z'}{4\pi}\phi^{q\bar{q}}(\mathbf{r}=0,z')$$

$\Rightarrow$ only one unknown constant $\int_0^1 \frac{\mathrm{d}z'}{4\pi}\phi^{q\bar{q}}(\mathbf{r}=0,z')$

- $C^0_{q\bar{q}\leftarrow q\bar{q}}$, $C^0_{q\bar{q}g\leftarrow q\bar{q}}$ at $\mathcal{O}(\alpha_\mathrm{s})$: Escobedo, Lappi, 1911.01136
- Renormalization of the LOWF: $\int \frac{\mathrm{d}z'}{4\pi}\phi^{q\bar{q}} = \int \frac{\mathrm{d}z'}{4\pi}\phi^{q\bar{q}}_{\mathrm{DR}} \times \left[1 - \frac{\alpha_s C_F}{2\pi}\frac{1}{\alpha}\right]$
- $\int \frac{\mathrm{d}z'}{4\pi}\phi^{q\bar{q}}_{\mathrm{DR}}$ in terms of NRQCD matrix elements: Lappi, Mäntysaari, JP, 2006.02830

Including relativistic corrections at LO

Use the distributional identity Escobedo, Lappi, 1911.01136:

$$\phi^{q\bar{q}}(\mathbf{r},z) = \sum_{k_1 k_2 k_3} \frac{1}{k_1! k_2! k_3!}(m_q r_1)^{k_1}(m_q r_2)^{k_2} 4\pi \left(-\frac{1}{2i}\partial_z\right)^{k_3} \delta(z-1/2) \times \overbrace{\int_0^1 \frac{\mathrm{d}z'}{4\pi} \frac{1}{m_q^{k_1+k_2}} \partial_{r_1}^{k_1}\partial_{r_2}^{k_2} \phi^{q\bar{q}}(\mathbf{r}=0,z')[2i(z'-1/2)]^{k_3}}^{\text{nonperturbative constants}}$$

- Keep terms up to v^2 in heavy quark velocity
- Nonperturbative constants related to NRQCD matrix elements Lappi, Mäntysaari, JP, 2006.02830
- Relativistic corrections $v^2\alpha_s^0$ to the amplitude:

$$-i\mathcal{A}^T_{\text{rel}} = \sqrt{\frac{N_c}{2}} \frac{e e_f m_q}{\pi} 2 \int \mathrm{d}^2\mathbf{x}_{01}\, N_{01}(Y_{\text{dip}}) \frac{1}{\sqrt{m_q}} \frac{\nabla^2\phi_{\text{RF}}(0)}{6m_q^2} \left[\frac{1}{2}m_q^2\mathbf{x}_{01}^2 K_0(\zeta) - \frac{\mathbf{x}_{01}^2 Q^2}{8\zeta}K_1(\zeta) - \frac{1}{2}\zeta K_1(\zeta)\right]$$

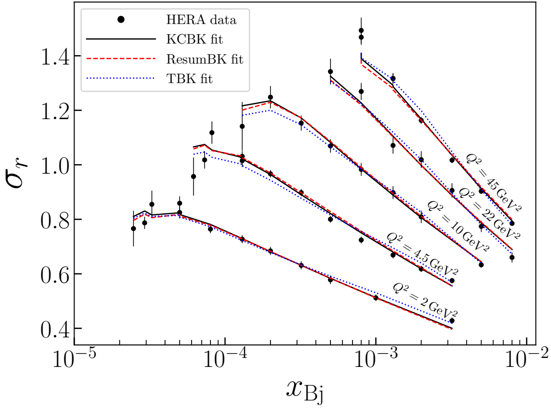
where the derivative of the rest-frame wave function ϕ_{RF} can be written as

$$\nabla^2\phi_{\text{RF}}(0) = -\langle \vec{q}^{\,2}\rangle_V \frac{1}{\sqrt{2N_c}} \sqrt{\langle\mathcal{O}_1\rangle_V}$$

$\Rightarrow$ first NRQCD corrections $v^0\alpha_s$ and $v^2\alpha_s^0$ included in the production amplitude

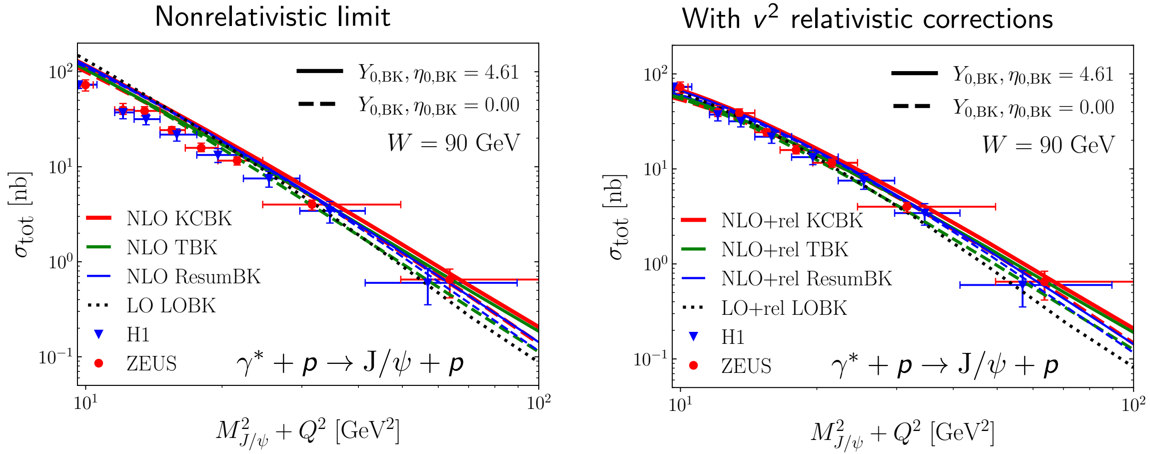
Initial condition fit for the dipole amplitude at NLO

- Fit the initial condition of the dipole amplitude to the HERA structure function data
- NLO calculation: needs an NLO fit
- NLO BK: numerically heavy
  - Use different approximations: KCBK, ResumBK, TBK
  - Two starting points for the BK evolution: $Y_{0,\mathrm{BK}} = 0.00$ and $Y_{0,\mathrm{BK}} = 4.61$
- $3 \times 2 = 6$ different NLO dipole amplitude fits
- Note: only massless quarks in this fit



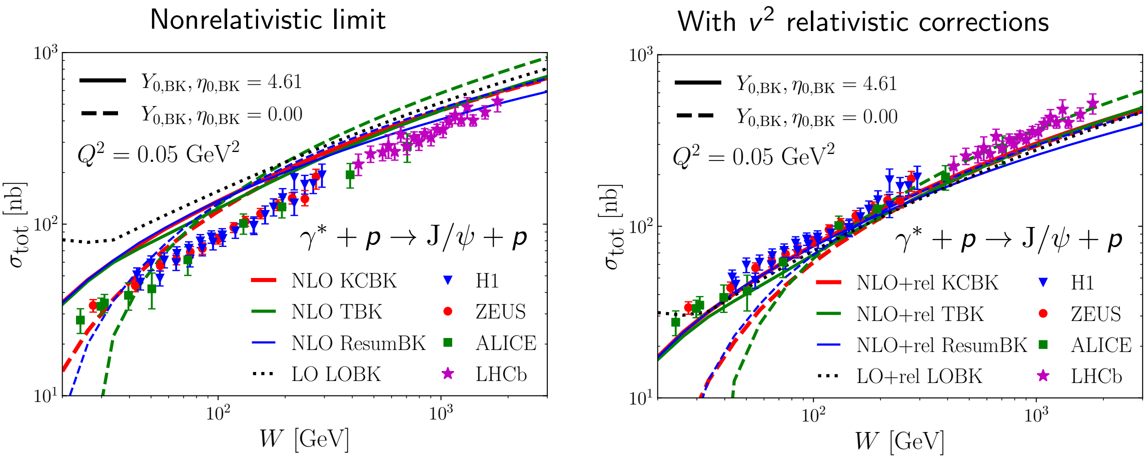
Beuf et al, 2007.01645

Total J/ψ production – dependence on the photon virtuality Q^2



- NLO corrections moderate
- Good agreement with the data, v^2 corrections important at low Q^2

Total J/ψ production – dependence on the center-of-mass energy W



- $Y_{0,\mathrm{BK}} = 0.00$: unphysical results at low W

  $\Rightarrow$ Additional constraints for the dipole amplitude fit from vector meson production

Light vector meson production

Based on Mäntysaari, JP, 2203.16911

Light vector meson production at high Q^2

- High photon virtuality: $Q^2 \gg M_V^2$

  $\Rightarrow$ Contribution only from small dipoles $\mathbf{r}^2 \sim \frac{1}{Q^2} \Rightarrow \Psi_V^{q\bar{q}}(\mathbf{r}, z) = \Psi_V^{q\bar{q}}(\mathbf{0}, z) + \mathcal{O}\left(\frac{1}{Q^2}\right)$
- Leading-order wave function is then:

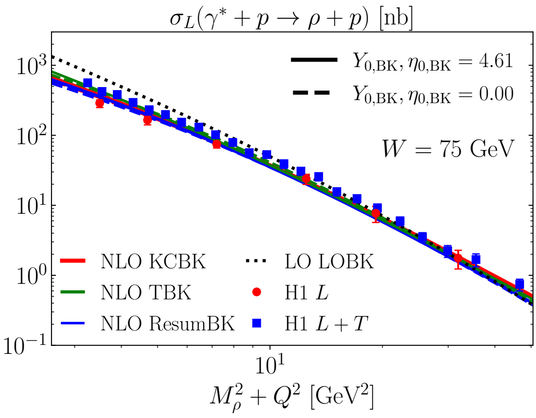
  $\Psi_V^{q\bar{q}}(\mathbf{k}, z) \sim \phi(z) \times (2\pi)^2 \delta^2(\mathbf{k})$ where $\phi(z)$ is the *distribution amplitude* of the meson
- Distribution amplitude renormalized in the $\overline{\text{MS}}$ scheme
- Renormalized distribution amplitude satisfies the ERBL evolution equation:

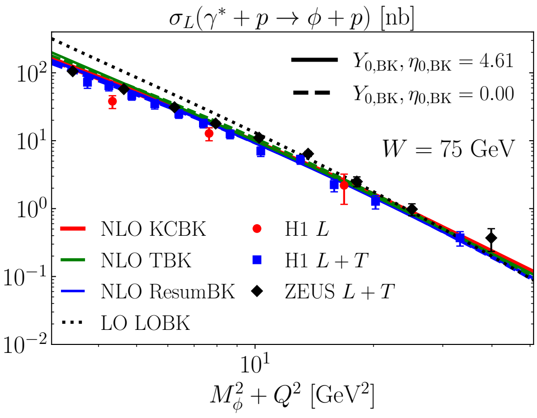
$$\frac{\partial}{\partial \ln \mu_F^2} \phi(z, \mu_F) = \frac{\alpha_s C_F}{2\pi} \int_0^1 \mathrm{d}z' \, K(z, z') \phi(z', \mu_F), \text{ where } \mu_F = \text{factorization scale}$$

- ϕ, ρ mesons: distribution amplitude close to the asymptotic form $\phi(z) = 6z(1-z)$
- Longitudinal production dominates: $\sigma_T / \sigma_L \sim M_V^2 / Q^2 \Rightarrow$ neglect transverse production

See also Boussarie et al, 1612.08026 for a different approach to NLO light vector meson production

ρ and ϕ production – dependence on the photon virtuality Q^2





- $\sigma_L + \sigma_T \approx \sigma_L$ for $Q^2 \gg M_V^2 \Rightarrow$ Can compare to total production data
- Good description of HERA data, except ϕ at low Q^2

Summary

- We have considered exclusive vector meson production at NLO in two different limits
  - Heavy vector meson: nonrelativistic limit in NRQCD 2104.02349, 2204.14031
  - Light vector meson: limit of high photon virtuality Q^2 2203.16911
- NLO corrections moderate
  - NLO effects can be mostly captured by the LO dipole amplitude fit
- Future considerations
  - NLO dipole amplitude fit with massive quarks
- Important developments: precise measurements expected at ultra-peripheral collisions at the LHC and the future Electron-Ion Collider

Backup

Backup - Rapidity in heavy vector meson production

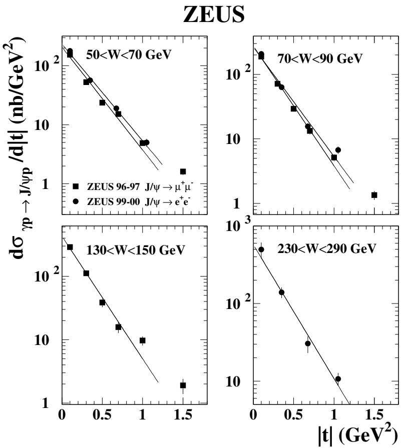
- The rapidity in the dipole amplitude is defined as $Y = \ln z_2 + \ln(q^+/P^+)$
  - $q^+, P^+ =$ longitudinal momenta of the virtual photon and the target
- Eikonal approximation: the invariant mass of the $q\bar{q}g$ system has to satisfy $M^2_{q\bar{q}g} \ll W^2$
  $\Rightarrow z_2 > z_{\text{min}} = \frac{P^+}{q^+} = e^{Y_0} \frac{Q_0^2}{W^2+Q^2-m_N^2}$, $Q_0^2 =$ transverse momentum scale of the target
- In total, we have three different rapidites in the expression:
  - $Y_0 =$ the initial rapidity
  - $Y_{\text{q}\bar{\text{q}}\text{g}} = Y_0 + \ln \frac{z_2}{z_{\text{min}}}$, evolution rapidity in the real contribution
  - $Y_{\text{dip}} = Y_0 + \ln \frac{z_0}{2z_{\text{min}}}$, evolution rapidity in the virtual contribution
- The amount of evolution in rapidity: $\ln 1/2z_{\text{min}} \approx \ln W^2/2Q_0^2$
- Following Beuf et al. 2007.01645, we choose $Q_0^2 = 1\,\text{GeV}^2$ and $Y_0 = 0$

Backup – Performing the t integral

- These results valid at $t = 0$
- Need the t-integrated cross section for comparisons with experimental data
- Use the experimental parametrization for t dependence:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = e^{-b|t|} \times \frac{\mathrm{d}\sigma}{\mathrm{d}t}(t = 0)$$

- $b \approx$ transverse size of the target-meson system
- b taken from a fit to experimental data



ZEUS collaboration, hep-ex/0201043

Backup – Renormalization of the LOWF for heavy vector meson

Has to be taken into account to cancel the $1/\alpha$ divergence in the dipole part

Decay width scheme

- Solve the LOWF from the NLO leptonic width in the nonrelativistic limit

$$\Gamma(V \to e^- e^+) \propto \left| \int \frac{\mathrm{d}z'}{4\pi} \phi^{q\bar{q}} \right|^2 \left[1 + \frac{2\alpha_s C_F}{\pi} \left(\frac{1}{2\alpha} - 2 \right) \right]$$

- Was used for the longitudinal production
- Problem: Hard to implement relativistic corrections for transverse production

Wave function scheme

- Connect the LOWF in our reg. scheme to the LOWF in dim.reg. 1911.01136

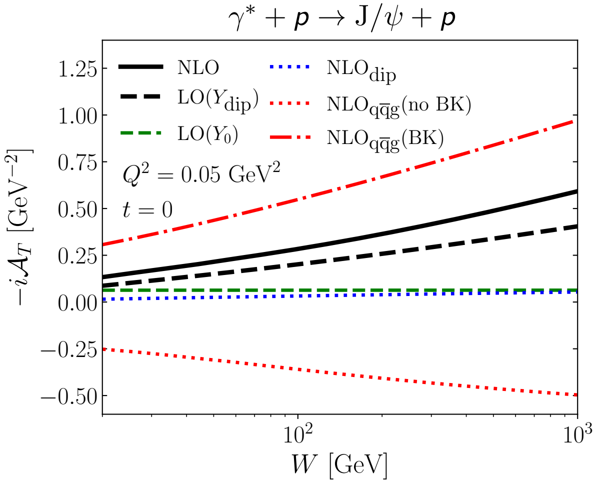
$$\int \frac{\mathrm{d}z'}{4\pi} \phi^{q\bar{q}} = \int \frac{\mathrm{d}z'}{4\pi} \phi^{q\bar{q}}_{\mathrm{DR}} \times \left[1 - \frac{\alpha_s C_F}{2\pi} \frac{1}{\alpha} \right]$$

- Determine the dim.reg. LOWF from the leptonic width
- Can be consistently used for relativistic corrections

Scheme dependence higher order in α_s

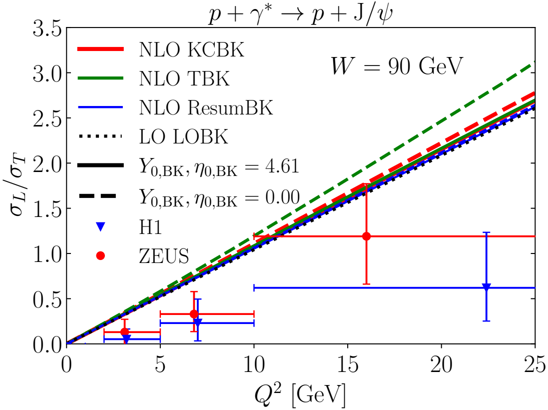
Backup – Decomposition of the production amplitude

- The leading order $\mathrm{LO}(Y_{\mathrm{dip}})$ result includes the resummation of the large logs $\sim \alpha_s \log 1/x$
- $\mathrm{NLO} = \mathrm{LO}(Y_0) + \mathrm{NLO}_{\mathrm{dip}} + \mathrm{NLO}_{\mathrm{q\bar{q}g}}$
- Here the same dipole amplitude used for both LO and NLO
  $\Rightarrow \mathrm{NLO} - \mathrm{LO}(Y_{\mathrm{dip}})$ tells about the largeness of the NLO correction terms

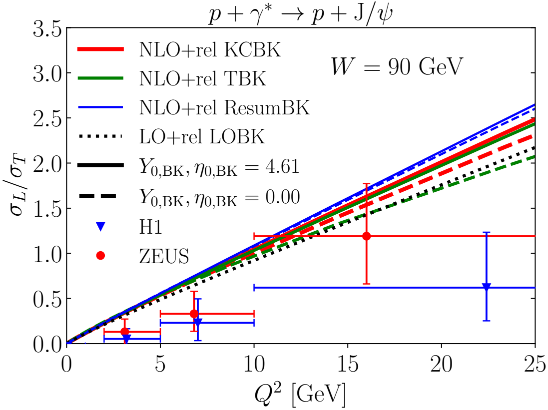


Backup – Longitudinal-to-transverse ratio for J/ψ production

Nonrelativistic limit



With v^2 relativistic corrections



- NLO corrections moderate

Backup – Renormalization of the distribution amplitude

- We renormalize the distribution amplitude in $\overline{\text{MS}}$ scheme by

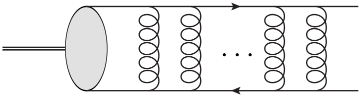
$$\phi(z,\mu_F) = \phi_0(z) + \frac{\alpha_s C_F}{2\pi}\int_0^1 \mathrm{d}z'\, K(z,z')\phi_0(z')\left(\frac{2}{D-4} + \gamma_E - \ln(4\pi) + \ln\left(\frac{\mu_F^2}{\mu^2}\right)\right)$$

where $K(z,z')$ is the ERBL kernel

$$K(z,z') = \frac{z}{z'}\left(1+\frac{1}{z'-z}\right)\theta(z'-z-\alpha) + \frac{1-z}{1-z'}\left(1+\frac{1}{z-z'}\right)\theta(z-z'-\alpha) + \left(\frac{3}{2}+\ln\left(\frac{\alpha^2}{z(1-z)}\right)\right)\delta(z'-z)$$

- Introduces a factorization scale μ_F
- μ_F-dependence given by the ERBL equation:

$$\frac{\partial}{\partial \ln \mu_F^2}\phi(z,\mu_F) = \frac{\alpha_s C_F}{2\pi}\int_0^1 \mathrm{d}z'\, K(z,z')\phi(z',\mu_F)$$



$$\mathbf{k}_1^2 \ll \mathbf{k}_2^2 \ll \ldots < \mu_F^2$$

Brodsky, Lepage, Phys. Rev. D 22 2157; Efremov, Radyushkin, Phys. Lett. B 94 245

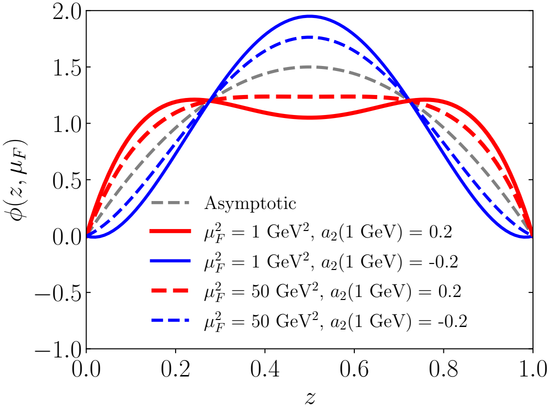
Backup – ERBL evolution of the distribution amplitude

- Expand the distribution amplitude in terms of the eigenfunctions of the ERBL kernel:

$$\phi(z, \mu_F) = \sum_{n=0}^{\infty} a_n(\mu_F) f_n(z)$$

where $f_n(z) = 6z(1-z)C_n^{(\frac{3}{2})}(2z-1)$

- ERBL equation evolves the coefficients $a_n(\mu_F)$
- Asymptotic limit: $\phi(z, \mu_F = \infty) \to 6z(1-z)$
- Higher a_n generally small: $a_2(1 \ \mathrm{GeV}) \approx 0.1$ for ϕ, ρ mesons



Polyakov, Son, 2008.06270

Backup - Factorization scale μ_F for light vector meson production

- Dependence on μ_F formally $\mathcal{O}(\alpha_s^2)$
- In practice: numerical results depend on the choice

$$i\mathcal{A} \sim \int_0^1 \mathrm{d}z' \, \phi(z', \mu_F) \times \left\{ \delta(z - z') + \frac{\alpha_s C_F}{2\pi} \left[-K(z, z') \ln\left(\frac{\mu_F^2 \mathbf{r}^2 e^{2\gamma_E}}{4} \right) + \dots \right] \right\}$$

r scheme

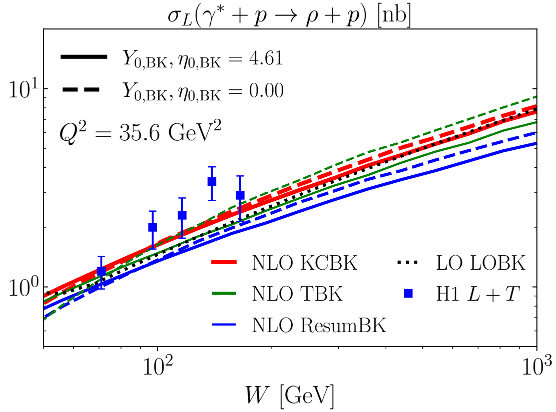
- Dependence on the dipole size $\mathbf{r}$: $\mu_F^2 = 4e^{-2\gamma_E} / \mathbf{r}^2$
- Motivated by the perturbation theory
- Cons: The factorization scale isn't a constant

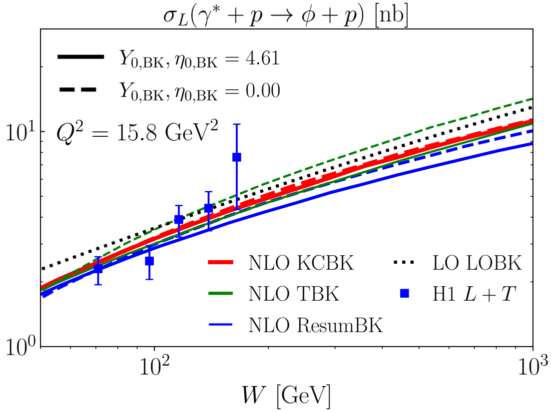
Q scheme

- Momentum scale of the process is $\sim Q$ $\Rightarrow$ Choose $\mu_F^2 = Q^2$
- Cons: Can have a significant numerical contribution from the logarithm $\ln\left(\mu_F^2 \mathbf{r}^2\right)$

Has only a small effect $\sim 5\%$ to the numerical results

Backup – ρ and ϕ production: W dependence





- W dependence also described well
- NLO corrections moderate