On NLO BFKL impact factors from NLO collinear amplitudes

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Goal and outline

1. The goal: construct a general approach to determine NLO impact factors in k_T-factorization from NLO collinear amplitudes

2. Outline:
   - A bottom-up approach: decomposition of NLO collinear amplitudes with auxillary partons into objects that can be interpreted within the High Energy Factorization (HEF) framework
   - Insight into calculational technique and results
   - Remarks on evolution scheme

3. Based on results of A. van Hameren, L. Motyka and G. Ziarko, arXiv:2205.09585

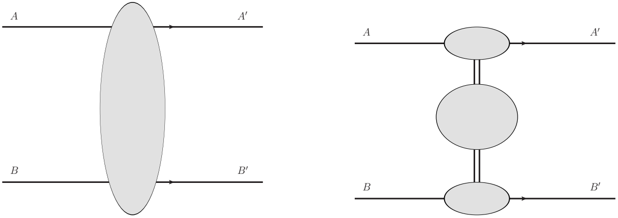
The HE / Regge factorization

At NLO in QCD Regge Factorization is proven:

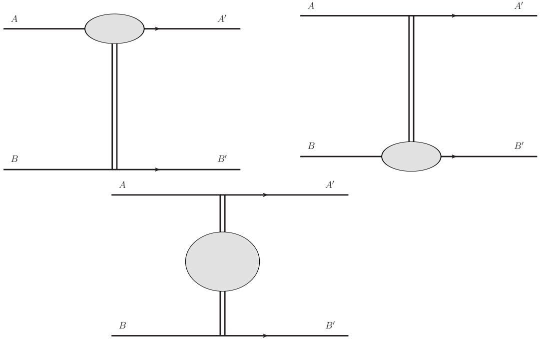
1. The QCD amplitudes at high energies may be factorized into projectile and target impact factors and the Green's function that incorporates the evolution (implicitly the BFKL evolution)
2. Convolution of the target impact factor and the Green's function gives the unintegrated (or transverse momentum dependent) gluon density function
3. There is a residual ambiguity in defining the impact factors and the Green's function related to the choice of the energy scale

The HE / Regge factorization

Regge:



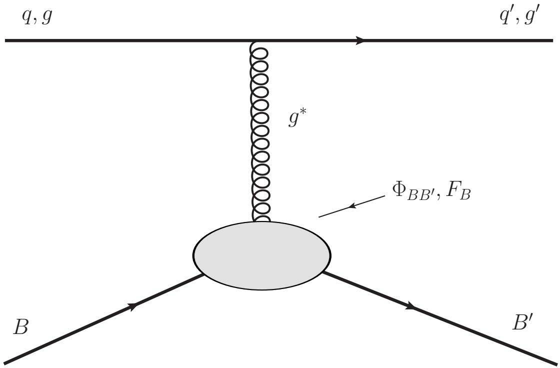
NLO:



The auxillary parton method at LO

- At LO in QCD the definition of the impact factors and the unintegrated gluon density may be formulated using an auxillary fast parton A — quark or gluon, related to a Wilson line
- The impact factor of such a fast (eikonal) particle introduces no momentum dependence — it is a constant function of g^* transverse momentum
- The prescription to obtain LO scattering amplitudes $B + g^* \to B'$ with an off-shell gluon g^*:
  Start from a collinear amplitude $q + B \to q' + B'$ or $g + B \to g' + B'$ and factor out the quark or gluon constant impact factor
- In result, a universal "partonic" amplitude is found $B + g^* \to B'$ that does not depend on the kind of auxillary parton used

The auxillary parton method at LO



The auxillary partons at NLO

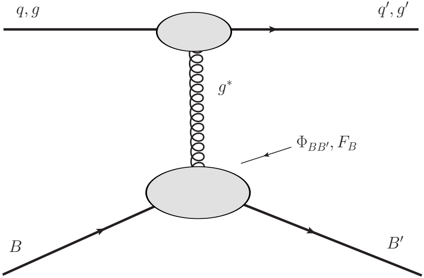
- A naive approach at NLO: in analogy to the auxillary parton method at LO — take NLO collinear amplitudes with auxillary parton and factor out constant couplings of auxillary partons
- Unfortunately at NLO it does not work: the impact factors $B + g^* \to B'$ depend on the choice of the auxillary parton (q or g)
- A rather expected solution: the auxillary parton coupling has to include the NLO effects. It is necessary to reinterpret the auxillary parton as a physical parton and dress it up in NLO QCD effects

The auxillary partons at NLO

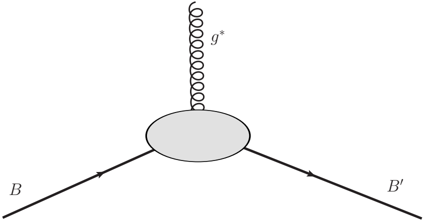
- The Regge factorization at NLO guarantees universality of the remaining part of the amplitude: the $B + g^* \longrightarrow B'$ impact factor and /or the unintegrated gluon density in the target B
- Explicit checks performed for several different processes — with the NLO auxillary parton impact factors the $B + g^* \longrightarrow B'$ amplitudes do not depend on the kind of the auxillary parton
- Jumping ahead: the NLO auxillary parton impact factors agree the NLO quark and gluon inclusive impact factors obtained by M. Ciafaloni and D. Colferai

The auxillary partons at NLO

Collinear amplitude with auxillary parton:



Target impact factor / unintegrated gluon in B:



Towards defining the off-shell partons at NLO: familiar and unfamiliar contributions

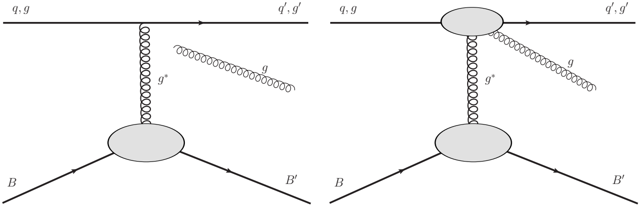
- We focus on obtaining the NLO amplitudes $B + g^* \to B'$. An interesting question emerges how they differ from their collinear counterparts: $B + g \to B'$
- In order to address it, at one loop we decompose the collinear amplitude with auxillary parton into a sum of a contribution $B + g \to B'$ with a collinear gluon — we call it a **familiar contribution**, — and the reminder, which we call an **unfamiliar contribution**
- The familiar contribution assumes implicitly the LO auxillary parton impact factors. The unfamiliar contribution includes the NLO effects in the auxillary parton (the NLO impact factor) and one iteration of the gluon Regge trajectory
- The unfamiliar contribution in the real radiation includes effects of the resolution of the projectile auxillary parton into two fast objects: e.g. $q \to qg$ splitting with gluon

Towards defining off-shell partons at NLO: familiar and unfamiliar contributions

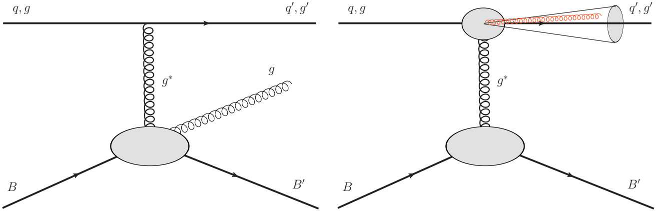
- The gluon Regge trajectory is thus interpreted as a change of the off-shell gluon parton definition due to NLO effects
- The separation into familiar and unfamiliar contributions is quite useful: the familiar contribution allows to use directly the usually known results obtained in collinear QCD and ensures cancellations of IR divergencies that occur in collinear amplitudes, and also with the NLO IR divergencies related to the incoming collinear partons
- The unfamiliar contribution turns out to be simple and boils down to the NLO auxillary quark or gluon impact factor and the one gluon Regge trajectory at the NLO

Familiar and unfamiliar contributions

Real gluon radiation and unfamiliar contribution



Familiar contributions: target region and collinear to auxillary parton



Technical didascalia

1. Analysis is carried out in the high energy limit. The collision energy squared $S = \Lambda\nu^2 \gg \mu_i^2$. ν^2 is fixed $\sim$ hadronic mass squared and Λ is a large dimensionless parameter. The leading power of Λ is retained
2. We use the dimensional regularization of UV and IR singularities and perform the renormalization of the UV divergencies
3. Scattering amplitudes are obtained using the formalism of helicity amplitudes at NLO, including some general identities
4. Not discussed here: a general analysis of NLO virtual corrections performed showing the Regge factorization in virtual contributions

Main result: factorization formula at NLO

As a result we obtain a hybrid factorization formula at NLO,

$$d\sigma^{(1)} = \int dx_{\mathrm{in}}\, d^2k_\perp\, d\bar{x}_{\overline{\mathrm{in}}} \Big\{$$

$$F(x_{\mathrm{in}}, |k_\perp|)\, f(\bar{x}_{\overline{\mathrm{in}}}) \Big[d\mathrm{V}^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}}) + d\mathrm{R}^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}}) \Big]_{\mathrm{cancelling}}$$

$$+ \Big[F^{(1)}(x_{\mathrm{in}}, |k_\perp|) + F(x_{\mathrm{in}}, |k_\perp|)\Delta_{\mathrm{unf}} + \Delta^\star_{\mathrm{coll}} \Big] f(\bar{x}_{\overline{\mathrm{in}}})\, d\mathrm{B}^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}})$$

$$+ F(x_{\mathrm{in}}, |k_\perp|) \Big[f^{(1)}(\bar{x}_{\overline{\mathrm{in}}}) + \Delta_{\overline{\mathrm{coll}}} \Big] d\mathrm{B}^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}}) \Big\}$$

with f and F — the collinear and k_T-dependent PDFs resp. and $f^{(1)}$ and $F^{(1)}$ are the NLO corrections to the PDFs.

B is for Born, and V – virtual, R – real corrections matrix elements and the related part of the phase space. This formula still holds for the cross section with auxillary parton, which is a part of Δ_{unf}

Unfamiliar contribution and NLO parton impact factors

$$\Delta_{\rm unf} = \frac{a_\epsilon N_c}{\epsilon}\left(\frac{\mu^2}{|k_\perp|^2}\right)^\epsilon \left[2\mathcal{J}_{\rm univ} + \mathcal{J}_{\rm aux} - 2\ln\frac{s_{\rm in}}{|k_\perp|^2}\right]$$

with

$$\mathcal{J}_{\rm univ} = \frac{11}{6} - \frac{n_f}{3N_c} - \frac{\mathcal{K}}{N_c}(-\epsilon) \quad \text{writing} \quad \mathcal{K} = N_c\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9}$$

and

$$\mathcal{J}_{\text{aux-q}} = \frac{3}{2} - \frac{1}{2}(-\epsilon) \quad , \quad \mathcal{J}_{\text{aux-g}} = \frac{11}{6} + \frac{n_f}{3N_c^3} + \frac{n_f}{6N_c^3}(-\epsilon)$$

$\mathcal{J}_{\rm univ} + \mathcal{J}_{\rm aux}$ equals to the NLO quark and gluon impact factors found by M. Ciafaloni and D. Colferai. $\mathcal{J}_{\rm univ}$ is related to running coupling effects.

Non-cancelling terms

In the cross sections at NLO we find contributions that have non-cancelling IR poles

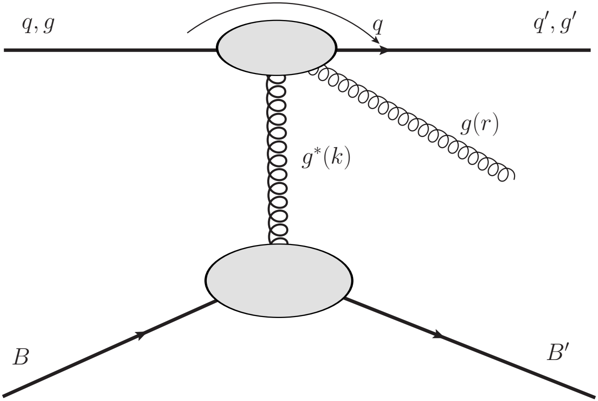
$$\Delta_{\overline{\mathrm{coll}}} = -\frac{a_\epsilon}{\epsilon}\int_{\bar{x}_{\overline{\imath\mathrm{n}}}}^{1} dz\,\left[\mathcal{P}^{\mathrm{reg}}_{\overline{\imath\mathrm{n}}}(z) + \gamma_{\overline{\imath\mathrm{n}}}\delta(1-z)\right]\frac{1}{z}f\left(\frac{\bar{x}_{\overline{\imath\mathrm{n}}}}{z}\right)$$

$$\Delta^{\star}_{\mathrm{coll}} = -\frac{a_\epsilon}{\epsilon}\int_{x_{\mathrm{in}}}^{1} dz\left[\frac{2N_{\mathrm{c}}}{[1-z]_+} + \gamma_{\mathrm{g}}\delta(1-z)\right]\frac{1}{z}F\left(\frac{x_{\mathrm{in}}}{z}, |k_\perp|\right)$$

- They appear because the amplitudes describe the collinear partons at NLO, and they should be used with NLO collinear parton densities. Then the IR poles of the amplitudes and the PDF-s cancel
- In $\Delta^{\star}_{\mathrm{coll}}$ the original IR singularity of the auxillary parton propagates ino the emission of a gluon g^*

Treatment of the real radiation: kinematics

Kinematics of the real gluon radiation:



Treatment of the real radiation

The emission from the auxillary parton is obtained in a well-known form

$$\mathcal{Q}_{\rm aux}(x_q, q_\perp, x_r, r_\perp) = x_q x_r \mathcal{P}_{\rm aux}(x_q, x_r)\, |k_\perp|^2 \Bigg(\frac{c_{\bar{q}}}{|r_\perp|^2 |r_\perp + k_\perp|^2} + \frac{c_q\,(1-x_r)^2}{|r_\perp + k_\perp|^2 |r_\perp + x_r k_\perp|^2} + \frac{c_r\, x_r^2}{|r_\perp|^2 |r_\perp + x_r k_\perp|^2} \Bigg)$$

$$\mathcal{R}_{\rm aux} = \frac{a_\epsilon}{2\pi_\epsilon \mu^{\bar{\epsilon}}} \int_0^u dx\, \mathcal{P}_{\rm aux}(1-x, x) \int d^{2+\bar{\epsilon}} r_\perp\, \theta\big(0 < |r_\perp|^2 < \nu^2 x \bar{x}_{\overline{\rm in}} \Lambda'\big) \times |k_\perp|^2 \Bigg(\frac{c_{\bar{q}}}{|r_\perp|^2 |r_\perp + k_\perp|^2} + \frac{c_q\,(1-x)^2}{|r_\perp + k_\perp|^2 |r_\perp + x k_\perp|^2} + \frac{c_r\, x^2}{|r_\perp|^2 |r_\perp + x k_\perp|^2} \Bigg)$$

$$\mathcal{P}_{\rm aux}(1-x, x) = \frac{\mathcal{P}^{\rm pole}_{\rm aux}}{x} + \mathcal{P}^{\rm rest}_{\rm aux}(x)$$

Treatment of the real radiation

1. Separation between the impact factor and the real part of the evolution kernel is necessary: the term in the gluon splitting function $1/z$ generates logs of energy after the z-integration — so it is part of the evolution kernel and is not included in the impact factor
2. When the z-integration extends to 1, the splitting function develops soft singularities, besides collinear singularities appear for $r_\perp \to 0$
3. When the radiation becomes collinear to the initial state auxillary parton, it belongs to the part of familiar contribution, that is associated with this parton and it is not part of the kernel
4. The **angular ordering** condition on gluon emission ensures that the soft-collinear divergencies are cancelled in the unfamiliar term — in agreement with classical results of M. Ciafaloni and M. Ciafaloni, D. Colferai

Hint on evolution — BFKL and soft logs

1. We analyze the cross sections at fixed order — NLO. Therefore we have no means to derive the evolution equation that necessarily requires an all order nested (ladder) structure.
2. The obtained NLO amplitude, however, should contain a one step of the evolution equation. In particular, the real gluon radiation leads to a following change of the unintegrated gluon density:

$$\tilde{F}(x_{\rm in}, k_\perp) = \frac{2a_\epsilon N_c}{\pi_\epsilon \mu^{\bar\epsilon}} \int_{x_{\rm in}}^1 \frac{dz}{z(1-z)} \int \frac{d^{2+\bar\epsilon} r_\perp}{|r_\perp|^2} \frac{|k_\perp|^2}{|k_\perp + r_\perp|^2}\, \theta\Big(|r_\perp| < |k_\perp|(1-z)\Big) F\left(\frac{x_{\rm in}}{z}, k_\perp + r_\perp\right)$$

3. This expression generetares the BFKL log (the $1/z$ factor) and the soft log (the $1/(1-z)$ factor). It closely resembles the real emission part of the firstly proposed CCFM equation. Also consistent with recent proposal of M. Nefedov

Towards a general approach to NLO impact factor

The obtained results aim to provide a general framework to compute NLO k_T dependent impact factors $B + g^* \longrightarrow B'$ from NLO collinear amplitudes with an auxillary parton:

1. Factor out the NLO auxillary parton impact factor from the collinear amplitude
2. Compute the contribution of the resolved real radiation at the $B \longrightarrow B'$ side
3. Subtract one step of the LO evolution kernel (issue of the soft logs!)
4. Integrate over the radiated gluon phase space

Conclusions and outlook

1. We develop an approach to k_T-factorization at NLO based on embedding the off-shell amplitudes into collinear ones
2. The NLO formula in hybrid factorization proposed
3. A scheme is proposed to extract the NLO k_T dependent impact factors from NLO collinear amplitudes
4. In turn, a possible deffinition of an unitegrated gluon using an enveloping collinear amplitude and the NLO parton impact factors

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THANKS!