Effects of the Sudakov form factor in the Golec-Biernat Wüsthoff saturation model[1]

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Colour Dipole

In the dipole picture of DIS, the photon-proton interaction cross-section is written

$$\sigma_P^{\gamma^* p}(x, Q^2) = \int d^2\mathbf{r} \int_0^1 dz |\Psi_P(z, \mathbf{r}, Q^2)|^2 \sigma_{\mathrm{dp}}(x, r). \qquad (1)$$

- $\Psi(z, \mathbf{r}, Q^2)$ ... fluctuation of a photon into a quark-antiquark pair.
- $\sigma_{\mathrm{dp}}(x, r)$ ... interaction of the $q - \overline{q}$ pair with the target proton.

$\sigma_{\mathrm{dp}}(x, r)$ embodies the saturation effects.

Solving an evolution equation is computationally expensive...

$\rightarrow$ A simple model to describe the saturation effects!

GBW model

Golec-Biernat and Wüsthoff proposed a simple form[1],

$$\sigma_{\mathrm{GBW}}(x,r) = \sigma_0 \left(1 - e^{-\frac{r^2}{4} Q_0^2 \left(\frac{x_0}{x}\right)^{\lambda}}\right), \tag{2}$$

where $x = x_{bj}\left(1 + 4m_f^2/Q^2\right)$,
and whose important features are

$$\begin{array}{lll} \sigma(x,r) \sim r^2 & \mathit{for} & r \ll 2/Q_s(x), \\ \sigma(x,r) \sim \sigma_0 & \mathit{for} & r \gg 2/Q_s(x), \end{array}$$

where the saturation scale

$$Q_s(x)^2 = Q_0^2 \left(\frac{x_0}{x}\right)^{\lambda}. \tag{3}$$

BGK model

However, the small-r limit of the dipole cross-section is [2],

$$\sigma_{\rm dp}(x,r) \simeq \frac{\pi^2 r^2 \alpha_s(\mu^2) xg(x,\mu^2)}{3}, \tag{4}$$

where $g(x,\mu^2)$ is the integrated gluon distribution at the scale

$$\mu^2 = \frac{C}{r^2}.$$

In order to incorporate this behaviour, Bartels Golec-Biernat Kowalski proposed a new form:

$$\sigma_{\rm BGK}(x,r) = \sigma_0 \left[1 - \exp\left(-\frac{\pi^2 r^2 \alpha_s(\mu^2) xg(x,\mu^2)}{3\sigma_0}\right)\right], \tag{5}$$

where

$$\mu^2 = \frac{C}{r^2} + \mu_0^2, \tag{6}$$

and the initial condition

$$xg(x,Q_0^2) = A_g x^{-\lambda_g}(1-x)^{5.6}. \tag{7}$$

Sudakov form factor

We use the Sudakov factor in Ref. [3]:

$$S^{(1)}_{\rm pert} = \frac{C_A}{2\pi}\int_{\mu_b^2}^{Q^2} \alpha(\mu^2)\frac{d\mu^2}{\mu^2}\log\left(\frac{Q^2}{\mu^2}\right). \tag{8}$$

We employ a modified "b_*-prescription", proposed in Ref. [4], for both the lower limit μ_b, and μ in the gluon distribution of the BGK model:

$$\mu = \frac{\mu_0^2}{1 - e^{-r^2\frac{\mu_0^2}{C}}} \tag{9}$$

where, for the Sudakov factor, $C = 2e^{-\gamma_{\rm E}}$, and $\gamma_{\rm E} \approx 0.577$ is the Euler-Mascheroni constant.

New model

Using

$$\sigma_{\rm dp}(x,r) = \frac{2\pi}{3}\int \frac{d^2\mathbf{k}}{k^2}\alpha_s\mathcal{F}(x,k^2)|1-e^{-i\mathbf{k}\cdot\mathbf{r}}|^2, \qquad (10)$$

$$\alpha_s\mathcal{F}(x,k^2) = \frac{3}{16\pi^3}\int d^2\mathbf{r}\, e^{i\mathbf{k}\cdot\mathbf{r}}\nabla^2_{\mathbf{r}}\sigma_{\rm dp}(x,r), \qquad (11)$$

and replacing

$$\nabla^2_{\mathbf{r}}\sigma_{\rm dp}(x,r) \to e^{-S(r,Q^2)}\nabla^2_{\mathbf{r}}\sigma_{\rm dp}(x,r)$$

one obtains a formula

$$\sigma_{\rm dp}(x,r,Q^2) = \int_0^r dr'\,r'\log\left(\frac{r}{r'}\right) e^{-S(r',Q^2)}\nabla^2_{r'}\sigma_{\rm dp}(x,r'). \qquad (12)$$

Now, the dipole cross-section is Q dependent.

Parameters and the fit quality

The models were fitted[2] to HERA data[6] in the range

$$x < 10^{-2} \qquad 0.045\ \mathrm{GeV}^2 \leq Q^2 \leq 650\ \mathrm{GeV}^2$$

type	light quark mass [GeV^2]	σ_0 [mb]	$x_0(10^{-4})$	λ	χ^2/dof
GBW	$m_l^2 = 0.0196$	23.8	1.12	0.308	5.27
$\mathrm{GBWS_{pert}}$	$m_l^2 = 0.0196$	23.0	1.32	0.287	3.37
GBW	$m_l^2 = 0.0$	19.1	2.58	0.322	4.44
$\mathrm{GBWS_{pert}}$	$m_l^2 = 0.0$	18.6	3.11	0.299	2.66

type	light quark mass [GeV^2]	σ_0 [mb]	A_g	λ_g	C	μ_0^2 [GeV^2]	χ^2/dof
BGK	$m_l^2 = 0.0196$	33.1	1.33	0.0553	0.420	1.76	1.62
$\mathrm{BGKS_{pert}}$	$m_l^2 = 0.0196$	30.7	6.28	-0.380	0.677	3.09	1.27
BGK	$m_l^2 = 0.0$	23.3	1.18	0.0832	0.329	1.87	1.56
$\mathrm{BGKS_{pert}}$	$m_l^2 = 0.0$	22.2	8.67	-0.500	0.670	3.83	1.21

- 4.44$\rightarrow$2.66 for GBW
- 1.56$\rightarrow$1.21 for BGK

[2] MINUIT package[5] was used.

Dependence on Q^2_{up}

We can see how well the models do for different ranges of Q^2 of the data.

Q^2_{up} [GeV^2]	GBW	GBWS
5	1.55	1.55
25	1.46	1.41
50	1.97	1.83
100	2.36	2.15
650	4.44	2.66

Q^2_{up} [GeV^2]	BGK	BGKS
5	1.63	1.59
25	1.42	1.30
50	1.52	1.23
100	1.55	1.25
650	1.56	1.21

Better tolerance for wider ranges of Q!

Significance of the non-perturbative Sudakov factor

We have investigated possible effects of the non-pertirbative Sudakov factor of the form in Ref. [7]

$$S_{\rm np}(r, Q^2) = g_1 r^2 + g_2 \log\left(\frac{r}{r_*}\right) \log\left(\frac{Q}{Q_0}\right), \qquad (13)$$

where $r_* = C_S/\mu_{0S}$.

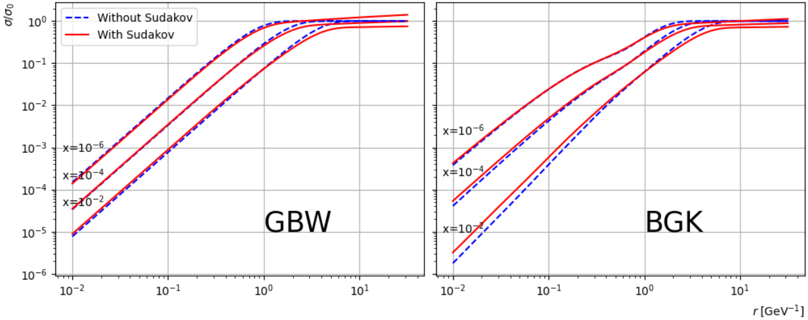
$\mu_{0S}^2[\mathrm{GeV}^2]$	$\mathrm{GBWS}_{\mathrm{pert}}$	$\mathrm{GBWS}_{\mathrm{np}}$
1	2.71	2.72
2	2.66	2.67
3	2.64	2.65
4	2.64	2.64
5	2.64	2.65

$\mu_{0S}^2[\mathrm{GeV}^2]$	$\mathrm{BGKS}_{\mathrm{pert}}$	$\mathrm{BGKS}_{\mathrm{np}}$
1	1.18	1.17
2	1.21	1.17
3	1.25	1.21
4	1.29	1.21
5	1.32	1.22

The χ^2 values gets closer at smaller μ_{0S}

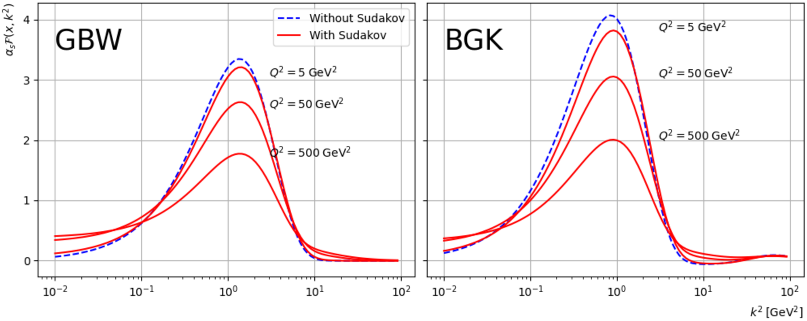
$\rightarrow$ It is neglegible for small μ_{0S}!

The dipole cross-section



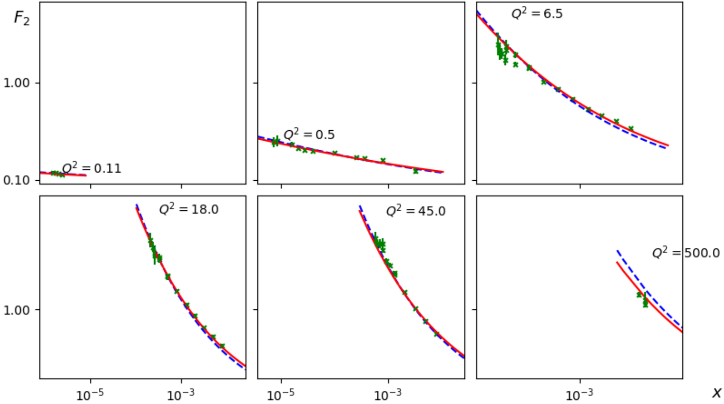
- The Sudakov factor affects the large-r regon.
- No longer a constant in the $r \to \infty$ limit, but rise logarithmically.
- Also, the differences in paramters affect small-r region.

The unintegrated gluon distribution



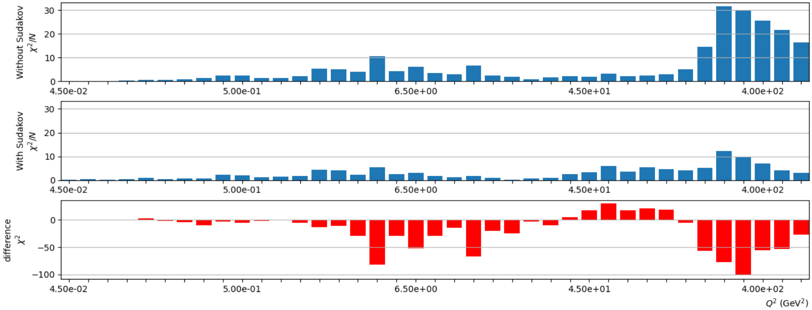
- The gluon distribution becomes wider, and particularly, more gluons in small-k_t regon.

Comparison of F_2 with data at selected Q^2: GBW



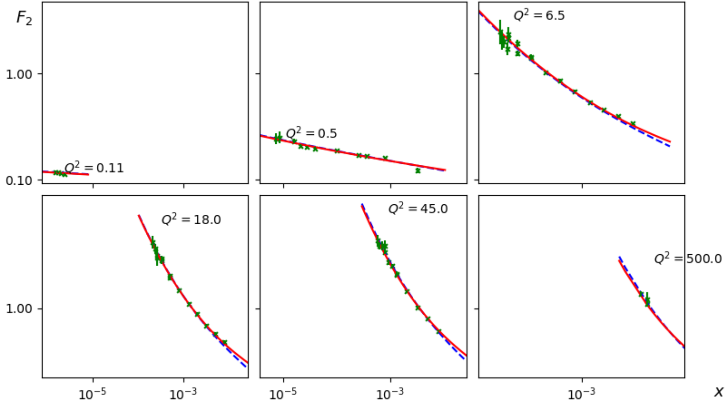
- Significant improvement at large Q^2.
- Differences in gradient are well noticeable for all range of Q^2.

Change in χ^2 at each Q^2



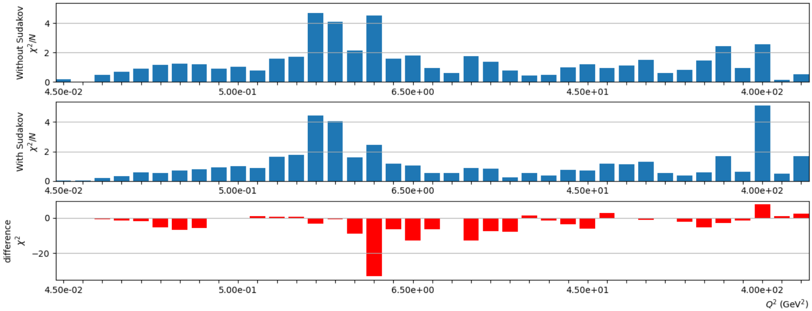
Improvements in the large-Q^2 region.

Comparison of F_2 with data at selected Q^2: BGK



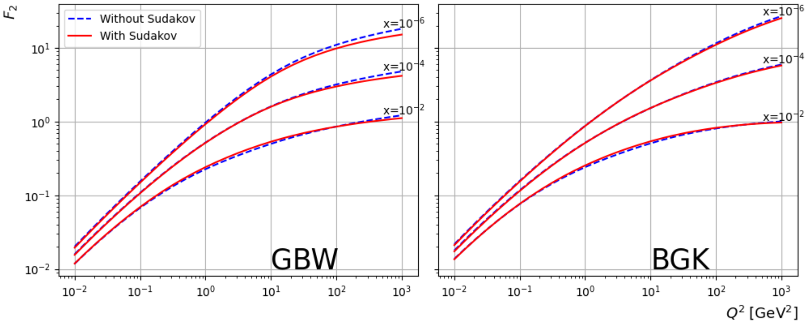
- The main improvement is at large x.
- The effect is most noticeable in the mid Q^2 region.

Change in χ^2 at each Q^2



More even improvement over Q^2 than the GBW model.

F_2



- Separations between the lines are smaller.

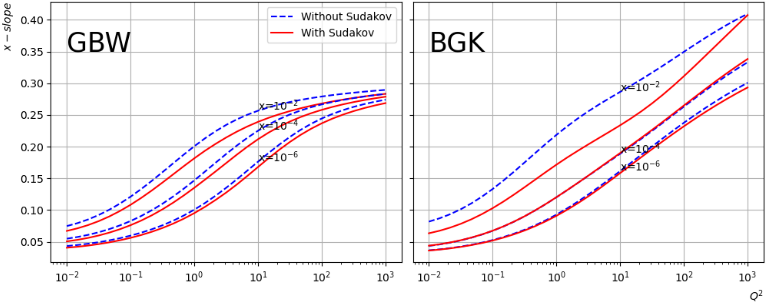
  $\rightarrow$ less gradient in x.

Effective slope of F_2

The differences are easier to see in the effective slope λ_{eff}:

$$F_2 \sim x^{-\lambda_{eff}}, \qquad (14)$$

for small x.

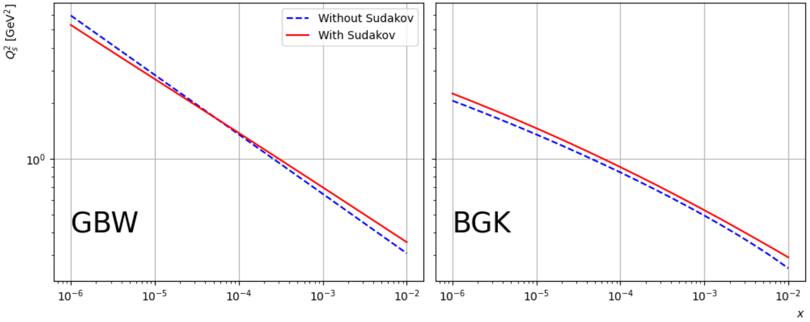


- The slopes are lower for the GBW model.
- A big change in the slope at large x for the BGK model.

The saturation scale

We define the saturation scale as the peak of the unintegrated gluon density:

$$\left.\frac{\partial \alpha_s \mathcal{F}(x,k)}{\partial k}\right|_{k=Q_s} = 0 \qquad (15)$$



- The saturation scale does not change dramatically, yet the shape of the dipole cross-section suggests that the transiton is milder.

Summary

To summarize,

- We included the leading order Sudakov form factor in the GBW and BGK models.
- χ^2/dof of both the models have improved noticeably.
  - 4.44→2.66 for GBW
  - 1.56→1.21 for BGK
- The Sudakov-improved BGK model no longer shows deterioration for wide range of Q^2!
- Transition to the saturation region become milder over a wider range.

Thank you!!

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