Diffractive structure function in the dipole picture

T. Lappi

Academy of Finland Center of Excellence in Quark Matter,
University of Jyväskylä, Finland

Diffraction and low x 2022, Corigliano Calabro

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

ACADEMY OF FINLAND

HELSINKI INSTITUTE OF PHYSICS

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Outline

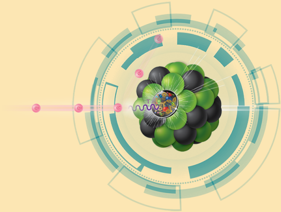
Outline of this talk

- Dipole + eikonal scattering picture of DIS
- Diffractive DIS
- Diffractive structure function at NLO: radiative part, $q\bar{q}g$ component
- Diffractive structure function at NLO: loops (future)
- Comparison to existing large-M_X and large-Q^2 results

G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161

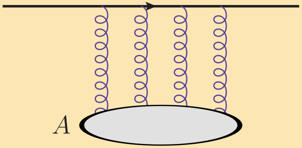
Process of interest

DIS in the high energy saturation regime



High energy collisions as eikonal scattering

Eikonal scattering off target of glue

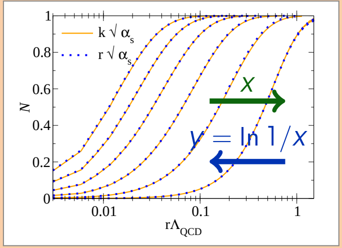


How to measure small-x glue?

- Dilute probe through target color field
- At high energy interaction is **eikonal**, $\perp$ coordinate conserved in scattering

- Amplitude for quark: **Wilson line**

$$\mathbb{P}\exp\left\{-ig\int^{x^+}\mathrm{d}y^+A^-(y^+,x^-,\mathbf{x})\right\}\underset{x^+\to\infty}{\approx}V(\mathbf{x})\in SU(N_c)$$



- Initial gluon field in AA: same $V(\mathbf{x})$
- Amplitude for color dipole

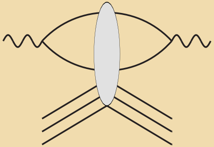
$$\mathcal{N}(r=|\mathbf{x}-\mathbf{y}|)=1-\left\langle\frac{1}{N_c}\mathrm{tr}\,V^\dagger(\mathbf{x})V(\mathbf{y})\right\rangle$$

- $r=0$: color transparency, $r\gg 1/Q_s$: saturation , nonperturbative!

Dipole picture of DIS

Limit of small x, i.e. high γ^*-target energy

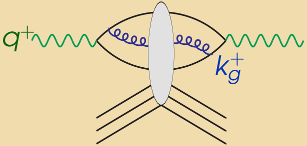
Leading order



- $\gamma^* \to q\bar{q}$ in vacuum
- $q\bar{q}$ interacts eikonally with target
- σ^{tot} is $2\times$Im-part of amplitude

"Dipole model": Nikolaev, Zakharov 1991 ; Mueller HERA fits starting with Golec-Biernat, Wüsthoff 1998

Leading Log: add **soft** gluon



$\Longrightarrow$ Large log $\displaystyle\int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$

Absorb into **BK-evolution** Balitsky 1995, Kovchegov 1999 of target

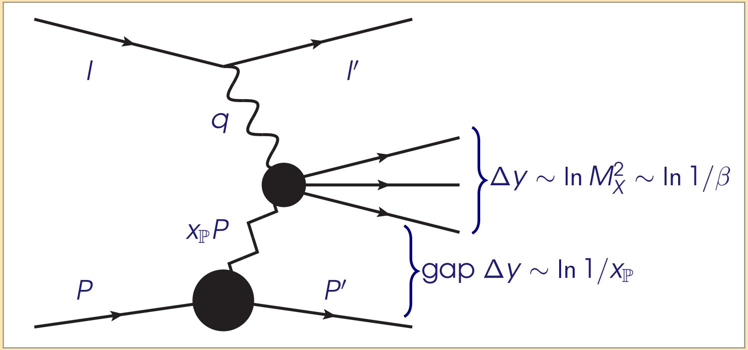
NLO

The same gluon with full kinematics

Diffractive structure function

Inclusive diffraction, kinematics

$\gamma^* + A \to X + A$, differential in M_X



- Momentum transfer $t = (P - P')^2$
- Gap size $x_{\mathbb{P}}$, target evolution rapidity $\sim \ln 1/x_{\mathbb{P}}$
- Diffractive system mass M_X^2, $\beta = Q^2/(Q^2 + M_X^2)$
- Virtuality Q^2

$$x_{Bj} = x_{\mathbb{P}} \beta$$

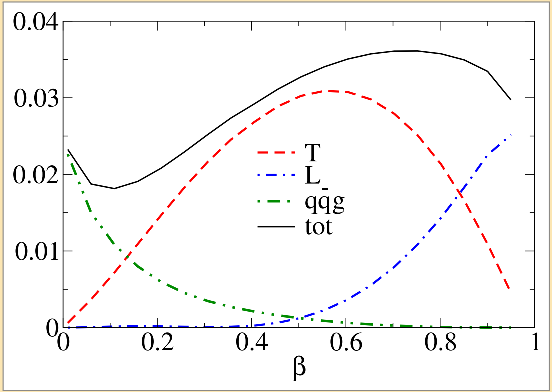
This talk: $x_{\mathbb{P}}$ small, β not.

Dependence on β, i.e. M_X

M_X^2 = photon remnants.

Essential regimes:

- Large $\beta \to 1$ — small M_X: longitudinal $q\bar{q}$
- Medium $\beta \sim 0.5$ — ${M_X}^2 \sim Q^2$: transverse $q\bar{q}$
- Small $\beta \ll 1$ — large ${M_X}^2$: higher Fock states ($q\bar{q}g$ etc.)



Proton F_{2p}^{D}, $Q^2 = 5\text{GeV}^2$, $x_{\mathbb{P}} = 10^{-3}$

H. Kowalski, T.L., C. Marquet and R. Venugopalan, [arXiv:0805.4071 [hep-ph]].

LO and radiative corrections to F_2^D NLO

Calculational setup: Light Cone Perturbation Theory

- Know free particle Fock states: $|\gamma^*\rangle_0, \quad |q\bar{q}\rangle_0, \quad |q\bar{q}g\rangle_0$ etc.
- **Interacting** states are superpositions of these:

$$|\gamma^*\rangle = (1 + \dots)|\gamma^*\rangle_0 + \psi^{\gamma^* \to q\bar{q}} \otimes |q\bar{q}\rangle_0 + \psi^{\gamma^* \to q\bar{q}g} \otimes |q\bar{q}g\rangle_0 + \dots$$

- Calculate in QM perturbation theory, e.g. ground state $|0\rangle$ wavefunction:

$$\psi^{0 \to n} = \sum_n \frac{\langle n| \hat{V} |0\rangle}{E_n - E_0} + \dots$$

  - $1/\Delta E \sim$ lifetime of the quantum fluctuation from 0 to n
  - "Energy" E is conjugate to "time", LC time is $x^+ \implies$ LC energy k^-
  - Note: energy "not conserved"!
  - Matrix elements $\langle n| \hat{V} |m\rangle \leftarrow$ vertices in Feynman rules
  - LC energy denominators $\leftarrow$ propagators, integrating over k^- pole

$\gamma^* \implies$ partons: this part of dipole model is LCPT calculation (no CGC, saturation here)

DIS at NLO: Fock state expansion

Incoming photon in DIS:

$$\left|\gamma_\lambda^*(q^+, \mathbf{q}; Q^2)\right\rangle_D = \sqrt{Z_{\gamma_\lambda^*}} \Bigg\{ \text{Non-QCD Fock states} + \widetilde{\sum_{q_0\bar{q}_1 \text{ F. s.}}} \widetilde{\Psi}_{\gamma_\lambda^* \to q_0\bar{q}_1} \left|q_0\bar{q}_1\right\rangle + \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. s.}}} \widetilde{\Psi}_{\gamma_\lambda^* \to q_0\bar{q}_1 g_2} \left|q_0\bar{q}_1 g_2\right\rangle + \cdots \Bigg\},$$

DIS at NLO: Fock state expansion

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$$\left|\gamma_\lambda^*(q^+,\mathbf{q};Q^2)\right\rangle_D = \sqrt{Z_{\gamma_\lambda^*}}\Bigg\{\text{Non-QCD Fock states} + \widetilde{\sum_{q_0\bar{q}_1 \text{ F. s.}}} \widetilde{\Psi}_{\gamma_\lambda^*\to q_0\bar{q}_1}\left|q_0\bar{q}_1\right\rangle$$

$$+ \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. s.}}} \widetilde{\Psi}_{\gamma_\lambda^*\to q_0\bar{q}_1 g_2}\left|q_0\bar{q}_1 g_2\right\rangle + \cdots\Bigg\},$$

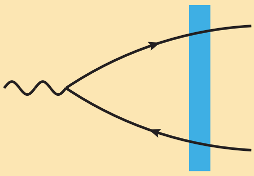
Coefficients: **Light cone wave functions** for $\gamma \to$ partons

- Tree level $\widetilde{\Psi}_{\gamma_\lambda^*\to q_0\bar{q}_1}$: LO
- $\widetilde{\Psi}_{\gamma_\lambda^*\to q_0\bar{q}_1}$: known to 1-loop, but not yet implemented in F_2^D
- $\widetilde{\Psi}_{\gamma_\lambda^*\to q_0\bar{q}_1 g_2}$: contribution now calculated
- Non-QCD Fock states: EW corrections, not needed

LO cross section: general result

Earlier literature: approximate treatment of impact parameter

$$\frac{\mathrm{d}\sigma^{D}_{\lambda,\,q\bar{q}}}{\mathrm{d}{M_X}^2\,\mathrm{d}|t|} = \frac{N_c}{4\pi}\int_0^1 \mathrm{d}z \int_{\mathbf{x}_0\mathbf{x}_1\bar{\mathbf{x}}_0\bar{\mathbf{x}}_1} \mathcal{I}^{(2)}_{\boldsymbol{\Delta}}\mathcal{I}^{(2)}_{M_X}$$
$$\times \sum_{f,h_0,h_1} \left(\widetilde{\psi}_{\gamma^*_\lambda \to q_{\bar{0}}\bar{q}_{\bar{1}}}\right)^\dagger \left(\widetilde{\psi}_{\gamma^*_\lambda \to q_0\bar{q}_1}\right)\left[S^\dagger_{\bar{0}\bar{1}} - 1\right]\left[S_{01} - 1\right],$$



"Transfer functions:" relate coordinates at shockwave to:

- Momentum transfer $t = -\boldsymbol{\Delta}^2$

$$\mathcal{I}^{(2)}_{\boldsymbol{\Delta}} = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2}\delta(\boldsymbol{\Delta}^2 - |t|)e^{i\boldsymbol{\Delta}\cdot(z\mathbf{x}_{\bar{0}0}-(1-z)\mathbf{x}_{\bar{1}1})} = \frac{1}{4\pi}J_0\left(\sqrt{|t|}\,\|z\mathbf{x}_{\bar{0}0} - (1-z)\mathbf{x}_{\bar{1}1}\|\right)$$

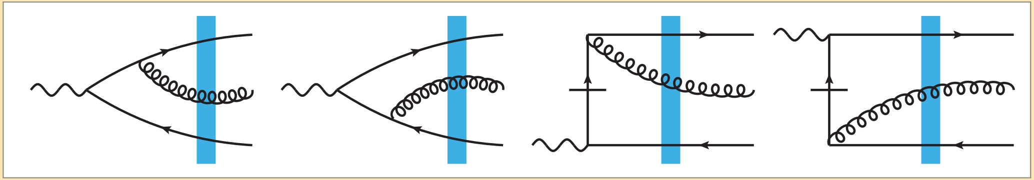
(Clearly $\mathbf{b} = \mathbf{x}_0 - \mathbf{x}_1$ is not good variable, but center of mass $z\mathbf{x}_0 - (1-z)\mathbf{x}_1$)

- Invariant mass

$$\mathcal{I}^{(2)}_{M_X} = \int \frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\delta(\mathbf{l}^2 - z(1-z){M_X}^2)e^{i\mathbf{l}\cdot(\mathbf{x}_{\bar{0}\bar{1}}-\mathbf{x}_{01})} = \frac{1}{4\pi}J_0\left(\sqrt{z(1-z)}M_X\|\bar{\mathbf{r}} - \mathbf{r}\|\right)$$

NLO radiative

New in G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari `arXiv:2206.13161` , we evaluate and square:



- Finite subset of full NLO structure function
- Generalizes earlier approximate results to "exact eikonal kinematics"

Explicit expressions (next slides) involve:

- 3-particle phase space, transfer functions $\mathcal{I}^{(3)}_{M_X}\mathcal{I}^{(3)}_{\boldsymbol{\Delta}}$ from $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_{\bar{0}}, \mathbf{x}_{\bar{1}}, \mathbf{x}_{\bar{2}}$ to M_X and t
- Explicit squared gluon emission wavefunction $\left(\widetilde{\psi}_{\gamma^*_\lambda \to q_{\bar{0}}\bar{q}_{\bar{1}}g_{\bar{2}}}\right)^\dagger \left(\widetilde{\psi}_{\gamma^*_\lambda \to q_0\bar{q}_1 g_2}\right)$

3-particle phase space, coordinate scales

- Argument of Bessel K function: $q\bar{q}g$ formation time / photon lifetime

$$X_{012}^2 = z_0 z_1 \mathbf{x}_{01}^2 + z_0 z_2 \mathbf{x}_{02}^2 + z_1 z_2 \mathbf{x}_{12}^2$$

- Final state, transfer from $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_{\bar{0}}, \mathbf{x}_{\bar{1}}, \mathbf{x}_{\bar{2}}$ to M_X (not obvious result)

$$\mathcal{I}_{M_X}^{(3)} = 2\frac{z_0 z_1 z_2}{(4\pi)^2} \frac{M_X}{Y_{012}} \mathrm{J}_1 \left(M_X Y_{012}\right),$$

with

$$\mathbf{Y}_{012}^2 = z_0 z_1 \left(\mathbf{x}_{\bar{0}0} - \mathbf{x}_{\bar{1}1}\right)^2 + z_1 z_2 \left(\mathbf{x}_{\bar{2}2} - \mathbf{x}_{\bar{1}1}\right)^2 + z_0 z_2 \left(\mathbf{x}_{\bar{2}2} - \mathbf{x}_{\bar{0}0}\right)^2.$$

- Momentum transfer is conjugate to 3-particle center-of-mass

$$\mathcal{I}_{\mathbf{\Delta}}^{(3)} = \frac{1}{4\pi} \mathrm{J}_0 \left(\sqrt{-t}\left\|z_0 \mathbf{x}_{\bar{0}0} + z_1 \mathbf{x}_{\bar{1}1} + z_2 \mathbf{x}_{\bar{2}2}\right\|\right).$$

Radiative corrections: result, longitudinal

$$
\begin{aligned}
x_{\mathbb{P}} F^{\mathrm{D(4)\,NLO}}_{L,\,q\bar{q}g}(x_{Bj}, Q^2, \beta, t) = {}& 4\frac{\alpha_s N_c C_F Q^4}{\beta} \sum_f e_f^2 \int_0^1 \frac{\mathrm{d}z_0}{z_0} \int_0^1 \frac{\mathrm{d}z_1}{z_1} \int_0^1 \frac{\mathrm{d}z_2}{z_2} \delta(z_0+z_1+z_2-1) \\
& \times \int_{\mathbf{x}_0} \int_{\mathbf{x}_1} \int_{\mathbf{x}_2} \int_{\bar{\mathbf{x}}_0} \int_{\bar{\mathbf{x}}_1} \int_{\bar{\mathbf{x}}_2} \mathcal{I}^{(3)}_{M_X} \mathcal{I}^{(3)}_{\boldsymbol{\Delta}} 4 z_0 z_1 Q^2 K_0\left(QX_{012}\right) K_0\left(QX_{\bar{0}\bar{1}\bar{2}}\right) \\
& \times \Bigg\{ z_1^2 \Bigg[\left(2z_0(z_0+z_2)+z_2^2\right) \left(\frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2} \cdot \left(\frac{\mathbf{x}_{\bar{2}\bar{0}}}{\mathbf{x}_{\bar{2}\bar{0}}^2} - \frac{1}{2}\frac{\mathbf{x}_{\bar{2}\bar{1}}}{\mathbf{x}_{\bar{2}\bar{1}}^2} \right) - \frac{1}{2}\frac{\mathbf{x}_{\bar{2}\bar{0}} \cdot \mathbf{x}_{21}}{\mathbf{x}_{\bar{2}\bar{0}}^2 \mathbf{x}_{21}^2} \right) + \frac{z_2^2}{2} \left(\frac{\mathbf{x}_{\bar{2}\bar{0}} \cdot \mathbf{x}_{21}}{\mathbf{x}_{\bar{2}\bar{0}}^2 \mathbf{x}_{21}^2} + \frac{\mathbf{x}_{20} \cdot \mathbf{x}_{\bar{2}\bar{1}}}{\mathbf{x}_{20}^2 \mathbf{x}_{\bar{2}\bar{1}}^2} \right) \Bigg] \\
& \quad + z_0^2 \Bigg[\left(2z_1(z_1+z_2)+z_2^2\right) \left(\frac{\mathbf{x}_{21}}{\mathbf{x}_{21}^2} \cdot \left(\frac{\mathbf{x}_{\bar{2}\bar{1}}}{\mathbf{x}_{\bar{2}\bar{1}}^2} - \frac{1}{2}\frac{\mathbf{x}_{\bar{2}\bar{0}}}{\mathbf{x}_{\bar{2}\bar{0}}^2} \right) - \frac{1}{2}\frac{\mathbf{x}_{20} \cdot \mathbf{x}_{\bar{2}\bar{1}}}{\mathbf{x}_{20}^2 \mathbf{x}_{\bar{2}\bar{1}}^2} \right) + \frac{z_2^2}{2} \left(\frac{\mathbf{x}_{\bar{2}\bar{0}} \cdot \mathbf{x}_{21}}{\mathbf{x}_{\bar{2}\bar{0}}^2 \mathbf{x}_{21}^2} + \frac{\mathbf{x}_{20} \cdot \mathbf{x}_{\bar{2}\bar{1}}}{\mathbf{x}_{20}^2 \mathbf{x}_{\bar{2}\bar{1}}^2} \right) \Bigg] \Bigg\} \\
& \times \left[1 - S^{(3)\dagger}_{\bar{0}\bar{1}\bar{2}}\right] \left[1 - S^{(3)}_{012}\right],
\end{aligned}
$$

Radiative corrections: result, transverse

$$x_{\mathbb{P}} F_{T,\, q\bar{q}g}^{\mathrm{D}(4)\,\mathrm{NLO}}(x_{Bj}, Q^2, \beta, t) = 2\frac{\alpha_s N_c C_F Q^4}{\beta} \sum_f e_f^2 \int_0^1 \frac{dz_0}{z_0} \int_0^1 \frac{dz_1}{z_1} \int_0^1 \frac{dz_2}{z_2} \delta(z_0+z_1+z_2-1) \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,\bar{\mathbf{x}}_0,\bar{\mathbf{x}}_1,\bar{\mathbf{x}}_2} \mathcal{I}_{M_X}^{(3)} \mathcal{I}_{\mathbf{\Delta}}^{(3)}$$

$$\times \frac{z_0 z_1 Q^2}{X_{012} X_{\bar{0}\bar{1}\bar{2}}} K_1(QX_{012}) K_1(QX_{\bar{0}\bar{1}\bar{2}}) \left\{ \Upsilon_{\mathrm{reg.}}^{(|b|^2)} + \Upsilon_{\mathrm{reg.}}^{(|c|^2)} + \Upsilon_{\mathrm{inst.}}^{d} + \Upsilon_{\mathrm{inst.}}^{e} + \Upsilon_{\mathrm{interf.}}^{b\times c} \right\} \left[1 - S_{\bar{0}\bar{1}\bar{2}}^{(3)\dagger}\right] \left[1 - S_{012}^{(3)}\right]$$

with (b $\leftrightarrow$ c and d $\leftrightarrow$ e related by $q \leftrightarrow \bar{q}$)

$$\Upsilon_{\mathrm{reg.}}^{(|b|^2)} = z_1^2 \Bigg[(2z_0(z_0+z_2)+z_2^2)(1-2z_1(1-z_1)) \left(\mathbf{x}_{\bar{0}+\bar{2};\bar{1}} \cdot \mathbf{x}_{0+2;1}\right) \frac{(\mathbf{x}_{\bar{2}\bar{0}} \cdot \mathbf{x}_{20})}{\mathbf{x}_{\bar{2}\bar{0}}^2 \mathbf{x}_{20}^2}$$

$$- z_2(2z_0+z_2)(2z_1-1) \frac{\left(\mathbf{x}_{\bar{0}+\bar{2};\bar{1}} \cdot \mathbf{x}_{\bar{2}\bar{0}}\right)\left(\mathbf{x}_{0+2;1} \cdot \mathbf{x}_{20}\right) - \left(\mathbf{x}_{\bar{0}+\bar{2};\bar{1}} \cdot \mathbf{x}_{20}\right)\left(\mathbf{x}_{0+2;1} \cdot \mathbf{x}_{\bar{2}\bar{0}}\right)}{\mathbf{x}_{\bar{2}\bar{0}}^2 \mathbf{x}_{20}^2} \Bigg],$$

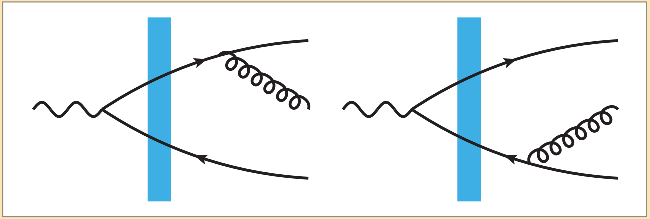
$$\Upsilon_{\mathrm{inst.}}^{d} = \frac{z_0^2 z_1^2 z_2^2}{(z_0+z_2)^2} - \frac{z_0^2 z_1^3 z_2}{z_0+z_2} \left(\frac{\mathbf{x}_{0+2;1} \cdot \mathbf{x}_{20}}{\mathbf{x}_{20}^2} + \frac{\mathbf{x}_{\bar{0}+\bar{2};\bar{1}} \cdot \mathbf{x}_{\bar{2}\bar{0}}}{\mathbf{x}_{\bar{2}\bar{0}}^2} \right) + \frac{z_0^2 z_1 (z_1+z_2)^2 z_2}{z_0+z_2} \left(\frac{\mathbf{x}_{0;1+2} \cdot \mathbf{x}_{21}}{\mathbf{x}_{21}^2} + \frac{\mathbf{x}_{\bar{0};\bar{1}+\bar{2}} \cdot \mathbf{x}_{\bar{2}\bar{1}}}{\mathbf{x}_{\bar{2}\bar{1}}^2} \right),$$

$$\Upsilon_{\mathrm{interf.}}^{b\times c} = -z_0 z_1 \left[z_1(z_0+z_2)+z_0(z_1+z_2)\right]\left[z_0(z_0+z_2)+z_1(z_1+z_2)\right] \left[\left(\mathbf{x}_{\bar{0}+\bar{2};\bar{1}} \cdot \mathbf{x}_{0;1+2}\right) \frac{(\mathbf{x}_{\bar{2}\bar{0}} \cdot \mathbf{x}_{21})}{\mathbf{x}_{\bar{2}\bar{0}}^2 \mathbf{x}_{21}^2} + \left(\mathbf{x}_{\bar{0};\bar{1}+\bar{2}} \cdot \mathbf{x}_{0+2;1}\right) \frac{(\mathbf{x}_{\bar{2}\bar{1}} \cdot \mathbf{x}_{20})}{\mathbf{x}_{\bar{2}\bar{1}}^2 \mathbf{x}_{20}^2} \right] + z_0 z_1 z_2 (z_0 - z_1)^2$$

$$\times \left[\frac{\left(\mathbf{x}_{\bar{0}+\bar{2};\bar{1}} \cdot \mathbf{x}_{\bar{2}\bar{0}}\right)\left(\mathbf{x}_{0;1+2} \cdot \mathbf{x}_{21}\right) - \left(\mathbf{x}_{\bar{0}+\bar{2};\bar{1}} \cdot \mathbf{x}_{21}\right)\left(\mathbf{x}_{0;1+2} \cdot \mathbf{x}_{\bar{2}\bar{0}}\right)}{\mathbf{x}_{\bar{2}\bar{0}}^2 \mathbf{x}_{21}^2} + \frac{\left(\mathbf{x}_{\bar{0};\bar{1}+\bar{2}} \cdot \mathbf{x}_{\bar{2}\bar{1}}\right)\left(\mathbf{x}_{0+2;1} \cdot \mathbf{x}_{20}\right) - \left(\mathbf{x}_{\bar{0};\bar{1}+\bar{2}} \cdot \mathbf{x}_{20}\right)\left(\mathbf{x}_{0+2;1} \cdot \mathbf{x}_{\bar{2}\bar{1}}\right)}{\mathbf{x}_{\bar{2}\bar{1}}^2 \mathbf{x}_{20}^2} \right].$$

Loops for F_2^D NLO: to be done

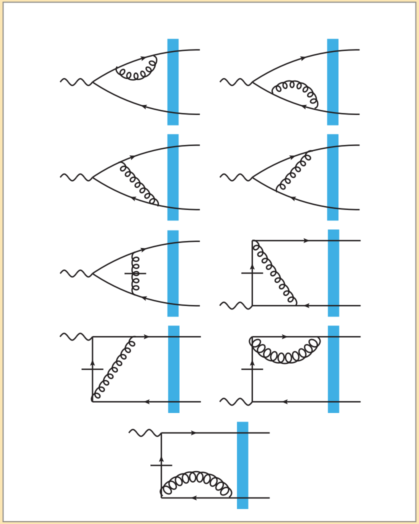
NLO radiative: emission after



- We are not including these "emission after shockwave" contributions yet
- Would have collinear divergence — to be cancelled by wavefunction renormalization — which again also has UV divergence . . .
- Thus these go together with virtual corrections, to get finite result

NLO virtual

Several **virtual** corrections **not included yet**



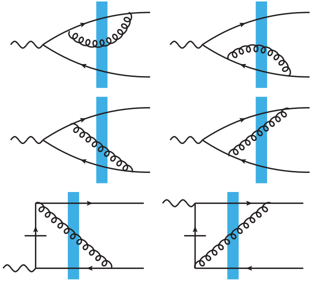
- Vertex correction diagrams: known 1-loop $\gamma \to q\bar{q}$ wavefunction

See also Boussarie et al 2014: diffractive jets,

also Caucal et al 2021 inclusive

NLO virtual

Several **virtual** corrections **not included yet**



- Vertex correction diagrams: known 1-loop $\gamma \to q\bar{q}$ wavefunction
- Gluon crosses shockwave, but not cut:
  - Loop corrections to amplitude, tree level wavefunctions
  - Similar to $\sigma_{\text{tot}} \sim \mathcal{M}_{\gamma^* \to \gamma^*}$
  - Subtraction of UV divergence
  - BK/JIMWLK evolution of amplitude

See also Boussarie et al 2014: diffractive jets,

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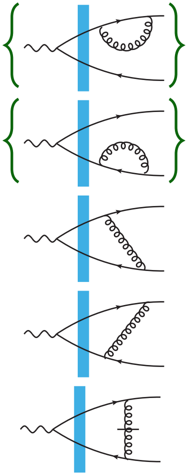
NLO virtual

Several **virtual** corrections **not included yet**



- Vertex correction diagrams: known 1-loop $\gamma \to q\bar{q}$ wavefunction
- Gluon crosses shockwave, but not cut:
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  - Similar to $\sigma_{\text{tot}} \sim \mathcal{M}_{\gamma^* \to \gamma^*}$
  - Subtraction of UV divergence
  - BK/JIMWLK evolution of amplitude
- Final state interactions

  (Propagator corrections $\{\}$ $\to$ state renormalization)

See also Boussarie et al 2014: diffractive jets,

also Caucal et al 2021 inclusive

Known limits

Large M_X

$$x_{\mathbb{P}} F^{\mathrm{D}\,(\mathrm{MS})}_{T,q\bar{q}g}(x_{\mathbb{P}}, \beta = 0, Q^2) = \frac{\alpha_s N_c C_F Q^2}{16\pi^5 \alpha_{\mathrm{em}}} \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int \mathrm{d}^2\mathbf{x}_2 \int_0^1 \frac{\mathrm{d}z}{z(1-z)} \left| \widetilde{\psi}^{\mathrm{LO}}_{\gamma^*_\lambda \to q_0 \bar{q}_1} \right|^2$$
$$\times \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{02}^2 \mathbf{x}_{12}^2} \left[N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right]^2 .$$

- LO $\gamma \to q\bar{q}$ wavefunction
- BK kernel for $q\bar{q} \to q\bar{q}g \approx q\bar{q}q\bar{q}$
- Squared BK Wilson line operator $(N_{ij} = 1 - S_{ij})$

Obtained by:

- Approximate gluon soft $z_2 \to 0$, $M_X{}^2 \sim 1/z_2$
- Remove M_X by $\int \mathrm{d}z_2 \delta(M_X{}^2 - \mathbf{p}_2^2/z_2)$
- Unconstrained final state integration introduced divergence: cure by including final state emissions

Large Q^2

We recover (somewhat mysterious) "Wüsthoff result" in large Q^2 limit

$$x_{\mathbb{P}} F^{\mathrm{D\,(GBW)}}_{T,q\bar{q}g} = \frac{\alpha_s \beta}{8\pi^4} \sum_f e_f^2 \int \mathrm{d}^2\mathbf{b} \int_\beta^1 \mathrm{d}z \left[\left(1 - \frac{\beta}{z}\right)^2 + \left(\frac{\beta}{z}\right)^2 \right] \int_0^{Q^2} \mathrm{d}k^2 k^4 \ln \frac{Q^2}{k^2}$$

$$\times \left[\int_0^\infty \mathrm{d}r r K_2(\sqrt{z} k r) J_2(\sqrt{1-z} k r) \frac{\mathrm{d}\tilde{\sigma}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, x_{\mathbb{P}}) \right]^2 .$$

Features:

- Explicit $\ln Q^2$
- $g \to q\bar{q}$ DGLAP splitting function: target evolution
- Color-octet small-size $q\bar{q}$ is "effective gluon" $\tilde{g}$ $\Longrightarrow$ adjoint dipole
- J_2, K_2 from curious "effective gluon wavefunction"

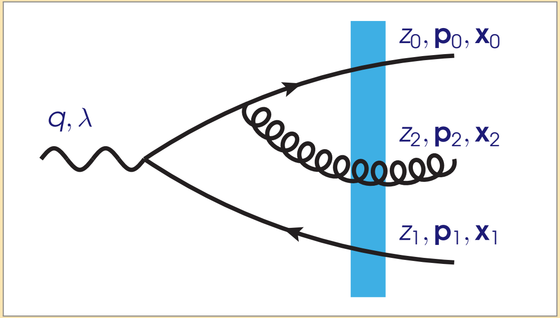
$$\psi^{\gamma \to g\tilde{g}} \sim k^i k^j - \frac{1}{2}\mathbf{k}^2 \delta^{ij}$$

Deriving large Q^2 limit: aligned jet limit

Leading large Q^2 from:

- $z_0 \gg z_1 \gg z_2$
- $\mathbf{p}_0^2 \sim \mathbf{p}_1^2 \gg \mathbf{p}_2^2$
- $p_0^- \sim p_1^- \sim p_2^-$
  $\Longrightarrow$ Wüsthoff momentum fractions

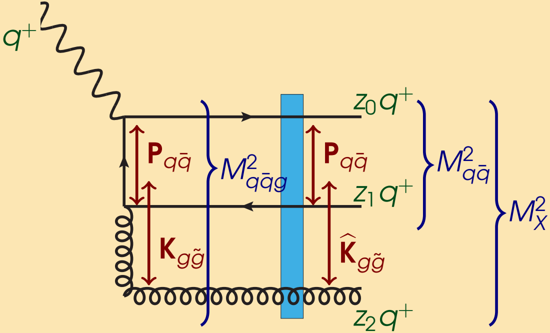
Consistently taking this limit derivation is straightforward:

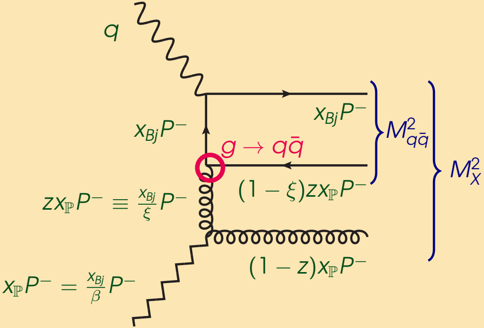


- q and $\bar{q}$ close: not resolved by target $\Longrightarrow$ point-like "effective gluon" $\tilde{g}$
- Consequently relative momentum of $q\bar{q}$ pair does not change in shockwave
  only relative momentum of $g\tilde{g}$
- Explicit log from $q\bar{q}$ internal dynamics, $g \to q\bar{q}$ (!) splitting function
- Rank-2 tensor for $\gamma^* \to g\tilde{g}$ See talk Triantafyllopoulos for a more elegant way

Deriving large Q^2 limit: matching

Identification with collinear variables by looking at invariant masses





$\mathbf{K}_{g\tilde{g}}$: before shock, Fourier-transform — $\widehat{\mathbf{K}}_{g\tilde{g}}$: final state, fixed

$$M^2_{q\bar{q}} \approx \mathbf{P}^2_{q\bar{q}}/z_1 \approx (1/\xi - 1)Q^2$$

$$M^2_{q\bar{q}g} \approx M^2_{q\bar{q}} + \mathbf{K}^2_{g\tilde{g}}/z_2 \qquad M^2_X = (1/\beta - 1)Q^2 \approx M^2_{q\bar{q}} + \widehat{\mathbf{K}}^2_{g\tilde{g}}/z_2$$

Conclusions

- General push to NLO in dilute-dense processes
- Inclusive diffraction: accessible at EIC
- Calculated radiative part:
  - Generalizes earlier results in approximate kinematics
  - In particular new derivation of the widely-used large-Q^2 result
  - Explicit, finite, ready for numerical implementation
- Outlined ingredients for full NLO calculation

LO, recovering known result

We can recover known results Golec-Biernat, Wüsthoff, Marquet et al :

$$x_{\mathbb{P}} F^{\mathrm{D}}_{L,q\bar{q}}\left(\beta, x_{\mathbb{P}}, Q^2\right) = \frac{N_c Q^4}{2\pi^3 \beta} \sum e_f^2 \int \mathrm{d}^2\mathbf{b} \int_0^1 \mathrm{d}z\, z^3 (1-z)^3 Q^2 \Phi_0(z, \beta, Q, \mathbf{b}),$$

$$x_{\mathbb{P}} F^{\mathrm{D}}_{T,q\bar{q}}\left(\beta, x_{\mathbb{P}}, Q^2\right) = \frac{N_c Q^4}{8\pi^3 \beta} \sum e_f^2 \int \mathrm{d}^2\mathbf{b} \int_0^1 \mathrm{d}z\, z^2 (1-z)^2 \left(z^2 - (1-z)^2\right) Q^2 \Phi_1(z, \beta, Q, \mathbf{b}),$$

$$\Phi_n(z, \beta, Q, \mathbf{b}) = \left[\int \mathrm{d}r\, r J_n\left(\sqrt{z(1-z)} M_X r\right) K_n\left(\sqrt{z(1-z)} Q\, r\right) (1 - S(r, b))\right]^2 .$$

Requires approximations

- Dependence on center-of-mass impact parameter $z_0 \mathbf{x}_0 + z_1 \mathbf{x}_1$ factorizes
- Dipole amplitude does not depend on $\mathbf{b}$, $\mathbf{r}$-angle:
  $\Longrightarrow$ Bessel function index from angular structure of $\gamma^*_\lambda \to q\bar{q}$ vertex