A parton branching algorithm with transverse momentum dependent splitting functions

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Diffraction and Low-x 2022

Universiteit Antwerpen

fwo Research Foundation Flanders Opening new horizons

Motivation

- Monte Carlo (MC) generators crucial for HEP predictions
- MC developments to reach high precision in HL LHC, LHeC, FCC, EIC...
- Baseline MCs based on collinear factorization
  but collinear physics has limitations
- Hot topic: 3D hadron structure
- recently new developments to include TMD physics in MCs
- MCs based on parton branching algorithms
  Today: parton branching with transverse momentum dependent (TMD) splitting functions (never explored so far!) [arXiv:2205.15873]

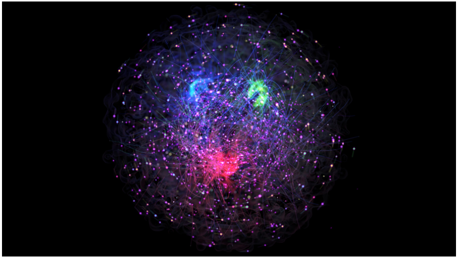


Image: James LaPlante/Sputnik Animation, MIT CAST & Jefferson Lab

Setting up the scene

Two elements which come together:

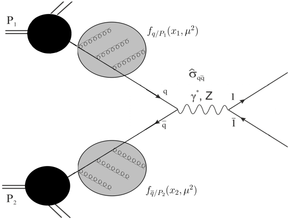
- **TMD splitting functions**, defined from the high-energy limit of partonic decay amplitudes
- Flexible parton branching algorithm, allowing to incorporate them: the **TMD Parton Branching (PB) method** [arXiv:1704.01757,arXiv:1708.03279]

first **step towards a full TMD MC generator covering the small-x** phase space.

A parton branching: basic concepts

- QCD: partons undergo an evolution
- DGLAP: Change of the PDF with the scale μ^2
- DGLAP splitting functions $P_{ab}(z, \mu^2)$: probability* that $b \to a$

  real branchings + virtual contributions (loops)
  P^R_{ab} P^V_a



Unitarity: $\mathbb{P}_{\mathrm{E}} + \mathbb{P}_{\mathrm{NE}} = 1$

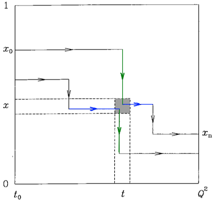
$\mathbb{P}_{\mathrm{NE}}$ can be expressed in terms of $\mathbb{P}_{\mathrm{E}}$ by exponentiating (**Sudakov form factor**)

$$\mathbb{P}_{\mathrm{NE}}\left(\mu^2, \mu_0^2\right) = \exp\left(-\int_{\mu_0^2}^{\mu^2} \mathrm{d}\mu'^2 \frac{\mathrm{d}\mathbb{P}_{\mathrm{E}}(\mu'^2)}{\mathrm{d}\mu'^2}\right)$$

$\mathbb{P}_{\mathrm{E}}$ ($\mathbb{P}_{\mathrm{NE}}$)- Probability of (no)emission

* probabilistic interpretation valid at LO

A parton branching for DGLAP evolution



Representation of branching by paths in (t,x) space. From *QCD and collider Physics*.

$$\mu^2 \frac{\mathrm{d}\widetilde{f}_a(x,\mu^2)}{\mathrm{d}\mu^2} = \underbrace{\sum_b \int_x^1 \mathrm{d}z\, P^R_{ab}(\mu^2, z)\widetilde{f}_b\left(\frac{x}{z}, \mu^2\right)}_{\text{real emissions}} - \underbrace{\widetilde{f}_a(x,\mu^2)\int_0^1 \mathrm{d}z P^V_a(\mu^2, z)}_{\text{virtual/non-resolvable}}$$

Momentum sum rule:

$$\sum_a \int_0^1 \mathrm{d}x\, x f_a(x,\mu^2) = 1 \;\rightarrow\; \sum_a \int_0^1 \mathrm{d}z\, z P_{ab}(z,\mu^2) = 0$$

Momentum sum rule allows to rewrite evolution in terms of P^R:

$$\mu^2 \frac{\mathrm{d}\widetilde{f}_a(x,\mu^2)}{\mathrm{d}\mu^2} = \sum_b \int_x^1 \mathrm{d}z\, P^R_{ab}(\mu^2, z)\widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) - \widetilde{f}_a(x,\mu^2)\sum_b \int_0^1 \mathrm{d}z\, z P^R_{ba}(\mu^2, z)$$

$\Longleftrightarrow$ **sum rule allows to fix the exact form of virtual piece**

one can introduce **Sudakov form factor** *

$$\Delta_a\left(\mu^2, \mu_0^2\right) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \int_0^{z_M} \mathrm{d}z\, z P^R_{ba}\left(z, \mu^2\right)\right)$$

to rewrite:

$$\widetilde{f}_a\left(x, \mu^2\right) = \Delta_a\left(\mu^2, \mu_0^2\right)\widetilde{f}_a\left(x, \mu_0^2\right) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \Delta_a\left(\mu^2, \mu'^2\right) \int_x^{z_M} \mathrm{d}z\, z P^R_{ab}\left(z, \mu'^2\right) \widetilde{f}_b\left(\frac{x}{z}, \mu'^2\right)$$

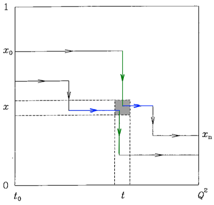
Simple intuitive **probabilistic interpretation** of an evolution structure easy to **solve by Monte Carlo** (MC) techniques

---

$\widetilde{f} = xf$, $(x/x_1 = z)$

* soft gluon resolution scale z_M introduced to treat non-resolvable branchings, partons with $z > z_M$ non-resolvable

A parton branching for DGLAP evolution



Representation of branching by paths in (t,x) space. From *QCD and collider Physics*.

$$\mu^2 \frac{d\widetilde{f}_a(x,\mu^2)}{d\mu^2} = \underbrace{\sum_b \int_x^1 dz\, P^R_{ab}(\mu^2, z)\widetilde{f}_b\left(\frac{x}{z}, \mu^2\right)}_{\text{real emissions}} - \underbrace{\widetilde{f}_a(x,\mu^2)\int_0^1 dz P^V_a(\mu^2, z)}_{\text{virtual/non-resolvable}}$$

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$\Longleftrightarrow$ **sum rule allows to fix the exact form of virtual piece**

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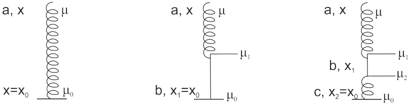
The TMD Parton Branching (PB) method



DGLAP evolution is collinear
TMD PB method extends it to the **transverse momentum dependent (TMD) PDFs (TMDs)** case

$$\widetilde{A}_a\left(x,k_\perp^2,\mu^2\right) = \Delta_a\left(\mu^2,\mu_0^2\right)\widetilde{A}_a\left(x,k_\perp^2,\mu_0^2\right) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}^2\mu_{\perp 1}}{\pi\mu_{\perp 1}^2}\Theta\left(\mu_{\perp 1}^2-\mu_0^2\right)\Theta\left(\mu^2-\mu_{\perp 1}^2\right)$$
$$\times\Delta_a\left(\mu^2,\mu_{\perp 1}^2\right)\int_x^{z_M} \mathrm{d}z P^R_{ab}\left(z,\mu_{\perp 1}^2\right)\widetilde{A}_b\left(\frac{x}{z}, |k_\perp+(1-z)\mu_{\perp 1}|^2,\mu_{\perp 0}^2\right)\Delta_b\left(\mu_{\perp 1}^2,\mu_{\perp 0}^2\right)+\dots$$

$\mathbf{k}$ of the propagating parton is a sum of intrinsic transverse momentum and **all emitted transverse momenta** $\mathbf{k}=\mathbf{k}_0-\sum_i \mathbf{q}_i$ → **TMDs from branchings!**

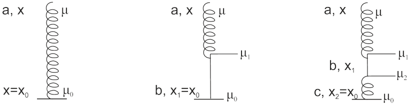
collinear PDF from TMD: $\int \mathrm{d}k_\perp^2 \widetilde{A}_a\left(x,k_\perp^2,\mu^2\right)=\widetilde{f}_a\left(x,\mu^2\right)$

in the limit $z_M \to 1$ & $\alpha_s(\mu_i^2)$ DGLAP reproduced

Notice: $P^R(z,\mu^2)$ still collinear → **natural extension to include also the TMD splitting functions**!

$\widetilde{A}=xA$

The TMD Parton Branching (PB) method



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$$\times\Delta_a\left(\mu^2,\mu_{\perp 1}^2\right)\int_x^{z_M}\mathrm{d}zP_{ab}^R\left(z,\mu_{\perp 1}^2\right)\widetilde{A}_b\left(\frac{x}{z},|k_\perp+(1-z)\mu_{\perp 1}|^2,\mu_{\perp 0}^2\right)\Delta_b\left(\mu_{\perp 1}^2,\mu_{\perp 0}^2\right)+\ldots$$

$\mathbf{k}$ of the propagating parton is a sum of intrinsic transverse momentum and **all emitted transverse momenta** $\mathbf{k}=\mathbf{k}_0-\sum_i\mathbf{q}_i$ $\rightarrow$ **TMDs from branchings!**

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$\widetilde{A}=xA$

The TMD PB method: more than just parton branching

- **evolution equation**
  flexibility! different scenarios studied
  (e.g. ordering conditions, resolution scales [1704.01757, 2103.09741, 1908.08524])
- **fit** procedure to obtain parameters of the initial distribution (xFitter [1410.4412])
- PB TMDs and PDFs [1804.11152, 2106.09791, 2102.01494] available in **TMDlib** [2103.09741] and LHAPDF format
  to be used in (TMD) MC generators,
  e.g. **TMD MC generator Cascade** [2101.10221]
  TMD initial state **parton shower**, with the backward shower guided by the PB TMDs
- **matching** method to match PB TMDs with NLO ME [1906.00919]
- TMD **merging** [2107.01224]
- example of applications to **measurements**:
  DY process at LHC [1906.00919]
  DY process at low masses and energies [2001.06488]
  DY + jets [2204.01528]
  jets [2112.10465]
  lepton-jet correlations in DIS [2108.12376]
  Notice applicability both at low and high $p_\perp$ (TMD effects at high $p_\perp$!)
- theory: ongoing comparison with standard low $q_\perp$ resummation methods (CSS) [2108.04099, 2206.01105]
- ...

The TMD PB method: flexible, widely applicable MC approach to obtain QCD high energy predictions based on TMDs

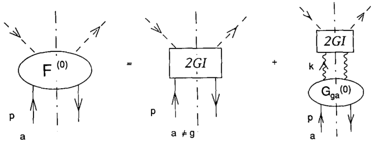
TMD splitting functions

- Concept from **high-energy factorization** [hep-ph/9405388]

  $k_\perp$ – factorization for DIS:

  $$F^0(x, Q^2) = \int [\mathrm{d}\mathbf{k}] \int \frac{\mathrm{d}z}{z} \hat{\sigma}(z, \mathbf{k}, Q^2, \mu) \mathcal{G}^0 \left(\frac{x}{z}, \mathbf{k}, \mu\right)$$

  $\mathcal{G}^0$ – solution of BFKL equation

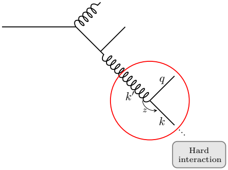


- originally TMD Pqg calculated

$$P_{qg}\left(\alpha_s, z, k'_\perp, \tilde{q}_\perp\right) =$$

$$\frac{\alpha_s T_F}{2\pi} \frac{\tilde{q}_\perp^2 \, z(1-z)}{(\tilde{q}_\perp^2 + z(1-z)k'^2_\perp)^2} \left[\frac{\tilde{q}_\perp^2}{z(1-z)} + 4(1-2z)\tilde{q}_\perp \cdot k'_\perp - 4\frac{(\tilde{q}_\perp \cdot k'_\perp)^2}{k'^2_\perp} + 4z(1-z)k'^2_\perp\right]$$

where $\tilde{q}_\perp = k_\perp - zk'_\perp$



Properties:

- **well defined collinear and high energy limits**:

  – for $k'^2_\perp \ll k^2_\perp$, after angular average: TMD P_{qg} $\to$ LO DGLAP P_{qg}

  – for finite $k'^2_\perp$, $k'^2_\perp \sim \mathcal{O}(k^2_\perp)$: expansion in $(k'^2_\perp / \tilde{q}^2_\perp)^n$, with z-dependent coefficients

  resummation of $\ln \frac{1}{z}$ at all orders in α_s via convolution with TMD gluon Green's functions

- positive definite

Other channels calculated in [1511.08439, 1607.01507, 1711.04587]
virtual pieces still missing

TMD PB method and TMD P

Idea: **replace DGLAP P by TMD P** [2205.15873]
What to do with the Sudakov form factor?

$$\widetilde{A}_a\left(x,k_\perp^2,\mu^2\right) = \Delta_a\left(\mu^2,\mu_0^2\right)\widetilde{A}_a\left(x,k_\perp^2,\mu_0^2\right) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}^2\mu_{\perp 1}}{\pi\mu_{\perp 1}^2}\Theta\left(\mu_{\perp 1}^2-\mu_0^2\right)\Theta\left(\mu^2-\mu_{\perp 1}^2\right)\Delta_a\left(\mu^2,\mu_{\perp 1}^2\right) \times \int_x^{z_M} \mathrm{d}z P_{ab}^R\left(z,k_\perp+(1-z)\mu_{\perp 1},\mu_{\perp 1}\right)\widetilde{A}_b\left(\frac{x}{z},|k_\perp+(1-z)\mu_{\perp 1}|^2,\mu_{\perp 1}^2\right)$$

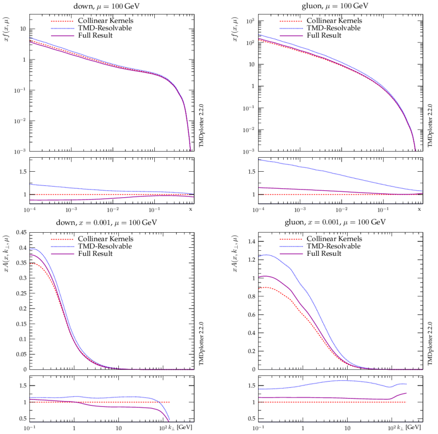
Two models investigated:

- with collinear Sudakov $\Delta_a\left(\mu^2,\mu_0^2\right)$, i.e. TMD P in real splittings only
- with **newly constructed TMD Sudakov**

$$\Delta_a\left(\mu^2,\mu_0^2\right) \to \Delta_a\left(\mu^2,\mu_{\perp 1}^2,k_\perp^2,\right) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2}\frac{\mathrm{d}\mu'^2}{\mu'^2}\int_0^{z_M}\mathrm{d}z\, z\overline{P}_{ba}^R\left(z,k_\perp^2,\mu'^2\right)\right)$$

$\overline{P}$ - angular averaged P
momentum sum rule & unitarity allowed us to construct the missing virtual term
i.e. we demonstrated analytically that with TMD Sudakov form factor momentum sum rule is satisfied and with collinear Sudakov it's broken

TMDs and PDFs



[2205.15873]

- for the study, the same starting distribution was used for all scenarios:
  collinear P,
  TMD real emissions, collinear Sudakov,
  TMD P both in real emissions and Sudakov
- the effect of the TMD splitting functions visible both in TMD and iTMD distributions
- iTMDs: significant differences, especially in the low x
- TMDs: effects in the whole $k_\perp$ range
- Red and purple curve obey momentum sum rule, differences due to dynamical effects in the splitting functions
- Blue curve violates momentum sum rule

Momentum sum rule check

[2205.15873]

μ^2 (GeV^2)	Full Result		
μ^2 (GeV^2)	$\alpha_s(\mu^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.000	1.000	1.000
10	0.999	0.999	0.999
10^2	0.997	0.996	0.997
10^3	0.994	0.992	0.994
10^4	0.991	0.987	0.991
10^5	0.984	0.978	0.983
	TMD-Resolvable		
μ^2 (GeV^2)	$\alpha_s(\mu^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.029	1.038	1.000
10	1.087	1.139	1.007
10^2	1.156	1.304	1.045
10^3	1.195	1.413	1.091
10^4	1.219	1.478	1.129
10^5	1.229	1.507	1.148
	Collinear Kernels		
μ^2 (GeV^2)	$\alpha_s(\mu^2)$ fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.000	1.000	1.000
10	0.999	0.999	0.999
10^2	0.997	0.997	0.997
10^3	0.995	0.993	0.995
10^4	0.992	0.989	0.992
10^5	0.986	0.981	0.984

In the table: $\sum_a \int_{x_0}^1 \mathrm{d}x \int \mathrm{d}k_\perp^2 \widetilde{A}(x, k^2\perp, \mu^2)$, with $x_0 = 10^{-5}$

The three columns: three different boundary conditions on α_s and z_M.

Scenarios with the same splitting functions in real emissions and non-resolvable part **fulfil momentum sum rule**

Scenarios with TMD splitting functions in real emissions and collinear Sudakov form factor **violate momentum sum rule**

Conclusions & Prospects

a parton branching algorithm to TMDs and PDFs which for the first time includes TMD splitting functions and fulfils momentum sum rule

- the TMD splitting functions, defined from the high-energy limit of partonic decay processes, with well defined collinear and high energy limits
- TMD splitting functions $\longleftrightarrow$ resummation $\ln \frac{1}{z}$
- new TMD Sudakov form factor thanks to momentum sum rule and unitarity

Studies presented today for forward evolution but it's the **first step towards a full TMD MC generator covering the small-x** phase space.
next steps:

- fits
- implement in PS:
  Cascade - perfect candidate

Other directions: further develop evolution equation

- extend formalism to CCFM-inspired scenarios (phase space, non-sudakov form factor)

Thank you!