Photon-photon transition form factors of axial vector quarkonia in a light front approach

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Diffraction and Low-x 2022, Sep 24-30, 2022 Corigliano Calabro, Italy

Outline

$\gamma^*\gamma^*$ transition form factors for the axial vector meson and spacelike photons

An exotic axial vector: $\chi_{c1}(3872)$, can one pin down its $c\bar{c}$ component?

Ultraperipheral collisions, $\gamma\gamma$ collisions at EIC

I. Babiarz, R. Pasechnik, W. Schäfer and A. Szczurek, "Light-front approach to axial-vector quarkonium $\gamma^*\gamma^*$ form factors," [arXiv:2208.05377 [hep-ph]], JHEP09(2022)170.

A. Cisek, W. Schäfer and A. Szczurek, "Structure and production mechanism of the enigmatic $X(3872)$ in high-energy hadronic reactions," [arXiv:2203.07827 [hep-ph]].

$\gamma^*\gamma^*$-transition form factors for $J^{PC} = 1^{++}$ axial mesons

$$\frac{1}{4\pi\alpha_{\rm em}}\mathcal{M}_{\mu\nu\rho} = i\left(q_1 - q_2 + \frac{Q_1^2 - Q_2^2}{(q_1+q_2)^2}(q_1+q_2)\right)_\rho \tilde{G}_{\mu\nu}\,\frac{M}{2X}F_{\rm TT}(Q_1^2, Q_2^2)$$
$$+ \; i e^L_\mu(q_1)\tilde{G}_{\nu\rho}\,\frac{1}{\sqrt{X}}F_{\rm LT}(Q_1^2, Q_2^2) + i e^L_\nu(q_2)\tilde{G}_{\mu\rho}\,\frac{1}{\sqrt{X}}F_{\rm TL}(Q_1^2, Q_2^2)\,.$$

- Above we introduced

$$\tilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta \,,\; X = (q_1 \cdot q_2)^2 - q_1^2 q_2^2$$

  and the polarization vectors of longitudinal photons

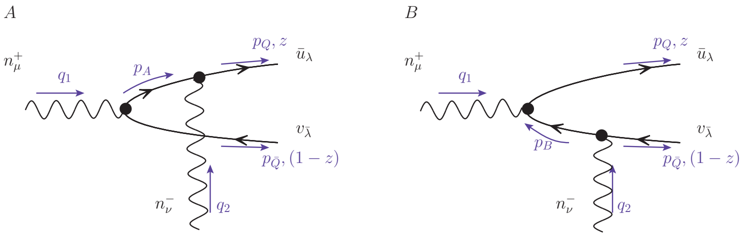
$$e^L_\mu(q_1) = \sqrt{\frac{-q_1^2}{X}}\left(q_{2\mu} - \frac{q_1\cdot q_2}{q_1^2}q_{1\mu}\right), \qquad e^L_\nu(q_2) = \sqrt{\frac{-q_2^2}{X}}\left(q_{1\nu} - \frac{q_1\cdot q_2}{q_2^2}q_{2\nu}\right).$$

- $F_{\rm TT}(0,0) = 0$, there is no decay to two photons (Landau-Yang).
- $F_{\rm LT}(Q^2, 0) \propto Q$ (absence of kinematical singularities). $f_{\rm LT} = \lim_{Q^2\to 0} F_{\rm LT}(Q^2,0)/Q$ gives rise to so-called "reduced width".
- in most practical applications, we are interested in the situation with one virtual and one real photon.

Helicity amplitude and form factors

The definition of the transition form factors $\gamma^*\gamma^* \to A$, where A is axial-vector meson

$$M_{\mu\nu\rho}(\gamma^*(q_1)\gamma^*(q_2) \to A)E^{\rho*}(\lambda_A) = 4\pi\alpha_{em} \sum_{i=\mathrm{TT,LT,TL}} t^i_{\mu\nu\rho} F_i(Q_1^2, Q_2^2)E^{\rho*}(\lambda_A)\,.$$



$$n^+_\mu n^-_\nu \mathcal{A}^{\lambda\bar{\lambda}}_{\mu\nu}\left(\gamma^*(q_1)\gamma^*(q_2) \to Q_\lambda(z, \vec{p}_{\perp Q})\, \bar{Q}_{\bar{\lambda}}(1-z, \vec{p}_{\perp \bar{Q}})\right)$$

$$= \bar{u}_\lambda(p_Q)\,\hat{n}^+ \frac{\hat{p}_A + m_Q}{p_A^2 - m_Q^2}\,\hat{n}^-\, v_{\bar{\lambda}}(p_{\bar{Q}}) \;+\; \bar{u}_\lambda(p_Q)\hat{n}^- \frac{\hat{p}_B + m_Q}{p_B^2 - m_Q^2}\,\hat{n}^+\, v_{\bar{\lambda}}\,(p_{\bar{Q}}),$$

- We evaluate the $\gamma^*\gamma^* \to A$ amplitude in the frame where $q_{1\mu} = q_{1+} n^+_\mu + q^\perp_{1\mu}$ and $q_{2\mu} = q_{2-} n^-_\mu + q^\perp_{2\mu}$ with the effective polarization vectors n^+_μ and n^-_ν.

Helicity amplitude and the light front wave function

$$\int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3} \sum_{\lambda\bar{\lambda}} \Psi^{(\lambda_A)*}_{\lambda\bar{\lambda}} n^{+\mu} n^{-\nu} \mathcal{A}^{\lambda\bar{\lambda}}_{\mu\nu} = (-2) \int \frac{dz\, d^2\boldsymbol{k}}{\sqrt{z(1-z)}16\pi^3}$$

$$\times \Big\{ -m_Q \left[\frac{1}{\vec{l}_A^{\,2} + \varepsilon^2} - \frac{1}{\vec{l}_B^{\,2} + \varepsilon^2} \right]$$

$$\times \left(\sqrt{2}(\boldsymbol{e}(-)\boldsymbol{q}_1)\Psi^{(\lambda_A)*}_{++}(z,\boldsymbol{k}) + \sqrt{2}(\boldsymbol{e}(+)\boldsymbol{q}_1)\Psi^{(\lambda_A)*}_{--}(z,\boldsymbol{k}) \right)$$

$$+ \left(2z(1-z)\boldsymbol{q}_1^2 + (1-2z)(\boldsymbol{k}\cdot\boldsymbol{q}_1) \right) \left[\frac{1}{\vec{l}_A^{\,2} + \varepsilon^2} - \frac{1}{\vec{l}_B^{\,2} + \varepsilon^2} \right]$$

$$\times \left(\Psi^{(\lambda_A)*}_{+-}(z,\boldsymbol{k}) + \Psi^{(\lambda_A)*}_{-+}(z,\boldsymbol{k}) \right)$$

$$-(1-2z)(\boldsymbol{q}_1\cdot\boldsymbol{q}_2) \left[\frac{1-z}{\vec{l}_A^{\,2} + \varepsilon^2} + \frac{z}{\vec{l}_B^{\,2} + \varepsilon^2} \right] \left(\Psi^{(\lambda_A)*}_{+-}(z,\boldsymbol{k}) + \Psi^{(\lambda_A)*}_{-+}(z,\boldsymbol{k}) \right)$$

$$+ i[\boldsymbol{k},\boldsymbol{q}_1] \left[\frac{1}{\vec{l}_A^{\,2} + \varepsilon^2} - \frac{1}{\vec{l}_B^{\,2} + \varepsilon^2} \right] \left(\Psi^{(\lambda_A)*}_{+-}(z,\boldsymbol{k}) - \Psi^{(\lambda_A)*}_{-+}(z,\boldsymbol{k}) \right)$$

$$+ i[\boldsymbol{q}_1,\boldsymbol{q}_2] \left[\frac{1-z}{\vec{l}_A^{\,2} + \varepsilon^2} + \frac{z}{\vec{l}_B^{\,2} + \varepsilon^2} \right] \left(\Psi^{(\lambda_A)*}_{+-}(z,\boldsymbol{k}) - \Psi^{(\lambda_A)*}_{-+}(z,\boldsymbol{k}) \right) \Big\} .$$

- $\vec{l}_A = \boldsymbol{k} + z\boldsymbol{q}_2$, $\vec{l}_B = -\boldsymbol{k} + (1-z)\boldsymbol{q}_2$. Amplitude can be decomposed into the invariant FF's which are obtained as integrals over LFWF.

Light-cone wave functions from potential models

- For the weakly bound systems a procedure to obtain the LFWF from Schrödinger WFs has been proposed by Terentev. In this case the helicity dependent WF $\Psi^{(\lambda_A)}_{\lambda\bar{\lambda}}(z,\boldsymbol{k})$ factorizes into a "radial" part, and a spin-orbit part obtained by a Melosh-rotation $R(z,\boldsymbol{k})$.
- Rest frame:

$$\begin{aligned}\Psi^{(\lambda_A)}_{\tau\bar{\tau}}(\vec{k}) &= \sum_{L_z+S_z=\lambda_A} Y_{1L_z}(\hat{k})\left\langle \frac{1}{2}\frac{1}{2}\tau\bar{\tau}\middle| 1S_z\right\rangle \langle 11L_zS_z|1\lambda_A\rangle \frac{u(k)}{k} \\ &= \frac{1}{2}\sqrt{\frac{3}{4\pi}}\,\xi_Q^{\tau\dagger}\,\left(\vec{\sigma}\cdot\frac{\vec{k}\times\vec{E}(\lambda_A)}{k}\right) i\sigma_2\,\xi_{\bar{Q}}^{\bar{\tau}*}\,\frac{u(k)}{k}\,.\end{aligned}$$

- Light front:

$$\Psi^{(\lambda_A)}_{\lambda\bar{\lambda}}(z,\boldsymbol{k}) = \chi_Q^{\lambda\dagger}\,\mathcal{O}'_{\lambda_A}\,i\sigma_2\,\chi_{\bar{Q}}^{\bar{\lambda}*}\,\psi(z,\boldsymbol{k})\,\sqrt{2(M^2_{Q\bar{Q}}-4m_Q^2)}\,,$$

with:

$$\mathcal{O}'_{\lambda_A} = \sqrt{\frac{3}{2}}\,R^\dagger(z,\boldsymbol{k})\left(\vec{\sigma}\cdot\frac{\vec{k}\times\vec{E}(\lambda_A)}{\sqrt{2}k}\right)R(1-z,-\boldsymbol{k})\,.$$

and

$$\psi(z,\boldsymbol{k}) = \frac{\pi\sqrt{M_{Q\bar{Q}}}}{2\sqrt{2}}\,\frac{u(k)}{k^2}\,,$$

where k is properly expressed through LF variables.

Form factors and LFWF

- The form factors of the LF-amplitude

$$\Phi_1(Q_1^2, Q_2^2) = -4\sqrt{\frac{3}{2}} \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \psi(z,\boldsymbol{k})(1-2z)\Big\{(\boldsymbol{k}^2 + m_Q^2)\Big(\frac{1}{\boldsymbol{l}_A^2+\varepsilon^2} - \frac{1}{\boldsymbol{l}_B^2+\varepsilon^2}\Big) - (\boldsymbol{q}_2\cdot\boldsymbol{k})\Big(\frac{1-z}{\boldsymbol{l}_A^2+\varepsilon^2} + \frac{z}{\boldsymbol{l}_B^2+\varepsilon^2}\Big)\Big\},$$

$$\Phi_2(Q_1^2, Q_2^2) = -8\sqrt{\frac{3}{2}}\frac{Q_1^2}{Q_2^2} \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \psi(z,\boldsymbol{k}) z(1-z)(\boldsymbol{q}_2\cdot\boldsymbol{k})\Big(\frac{1}{\boldsymbol{l}_A^2+\varepsilon^2} - \frac{1}{\boldsymbol{l}_B^2+\varepsilon^2}\Big).$$

- ...can be used to evaluate helicity FF's:

$$Q_1 F_{\rm LT} = \frac{e_f^2\sqrt{N_c}}{2}\Big\{(\nu - Q_1^2)\Phi_A - (\nu + Q_1^2)\Phi_S\Big\},$$

$$Q_2 F_{\rm TL} = \frac{e_f^2\sqrt{N_c}}{2}\Big\{(\nu - Q_2^2)\Phi_A + (\nu + Q_2^2)\Phi_S\Big\},$$

and

$$F_{\rm TT} = -\frac{1}{M}\Big\{Q_1 F_{\rm LT} + Q_2 F_{\rm TL}\Big\}.$$

where

$$\Phi_A = \Phi_1 + \Phi_2, \, , \Phi_S = \Phi_1 - \Phi_2 \, .$$

Transition FFs for two virtual photons for $\chi_{c1}(1P)$

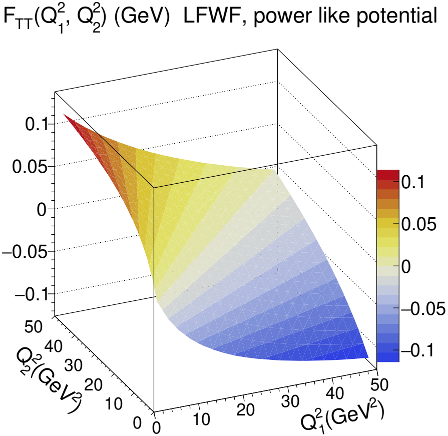


Figure: Dependence of form factors $F_{\rm TT}(Q_1^2, Q_2^2)$ and $F_{\rm LT}(Q_1^2, Q_2^2)$ on the two photon virtualities. Here we used the LFWF obtained from the power-like potential model.

$\gamma^*\gamma^*$ cross sections

$$\sigma_{ij} = \frac{32\pi(2J+1)}{N_i N_j} \frac{\hat{s}}{2M\sqrt{X}} \frac{M\Gamma}{(\hat{s}-M^2)^2 + M^2\Gamma^2} \Gamma^{ij}_{\gamma^*\gamma^*}(Q_1^2, Q_2^2, \hat{s}) \,,$$

where $\{i,j\} \in \{\mathrm{T},\mathrm{L}\}$, and $N_\mathrm{T} = 2, N_\mathrm{L} = 1$ are the numbers of polarization states of photons. In terms of our helicity form factor, we obtain for the LT configuration, putting at the resonance pole $\hat{s} \to M^2$, and $J = 1$ for the axial-vector meson:

$$\Gamma^{\mathrm{LT}}_{\gamma^*\gamma^*}(Q_1^2, Q_2^2, M^2) = \frac{\pi\alpha_{\mathrm{em}}^2}{3M} F^2_{\mathrm{LT}}(Q_1^2, Q_2^2) \,.$$

$$\tilde{\Gamma}(A) = \lim_{Q^2\to 0} \frac{M^2}{Q^2} \Gamma^{\mathrm{LT}}_{\gamma^*\gamma^*}(Q^2, 0, M^2) = \frac{\pi\alpha_{\mathrm{em}}^2 M}{3} f^2_{\mathrm{LT}} \,, \quad \text{with } f_{\mathrm{LT}} = \lim_{Q^2\to 0} \frac{F_{\mathrm{LT}}(Q^2,0)}{Q} \,,$$

which provides a useful measure of size of the relevant e^+e^- cross section in the $\gamma\gamma$ mode. For a $c\bar{c}$ state:

$$f_{\mathrm{LT}} = -e_f^2 M^2 \frac{\sqrt{3N_c}}{8\pi} \int_0^\infty \frac{dk\, k^2 u(k)}{(k^2+m_Q^2)^2} \frac{1}{\sqrt{M_{Q\bar{Q}}}} \left\{ \frac{2}{\beta^2} - \frac{1-\beta^2}{\beta^3} \log\left(\frac{1+\beta}{1-\beta}\right) \right\},$$

with

$$\beta = \frac{k}{\sqrt{k^2+m_Q^2}} \,, \qquad M_{Q\bar{Q}} = 2\sqrt{k^2+m_Q^2} \,.$$

Nonrelativistic limit

- Simple explicit expressions in terms of derivative of WF at the origin can be found in the nonrelativistic limit, which fully agree with the earlier results of Schuler, Berends, van Gulik '98:

$$
\begin{aligned}
F_{\mathrm{TT}}(Q_1^2, Q_2^2) &= 2e_f^2 \sqrt{\frac{6N_c}{\pi M^3}} R'(0)\, \frac{Q_1^2 - Q_2^2}{\nu}\,, \\
F_{\mathrm{LT}}(Q_1^2, Q_2^2) &= -2e_f^2 \sqrt{\frac{6N_c}{\pi M}} R'(0) \frac{(\nu + Q_2^2) Q_1}{\nu^2}\,, \\
F_{\mathrm{TL}}(Q_1^2, Q_2^2) &= 2e_f^2 \sqrt{\frac{6N_c}{\pi M}} R'(0) \frac{(\nu + Q_1^2) Q_2}{\nu^2}\,,
\end{aligned}
$$

  with $\nu = (M^2 + Q_1^2 + Q_2^2)/2$.

- reduced width:

$$
\tilde{\Gamma}(A) = \frac{2\alpha_{\mathrm{em}}^2 e_f^4 N_c}{m_Q^4} \left| R'(0) \right|^2 ,
$$

- for one virtual & one real photon one obtains

$$
F_{\mathrm{TT}}(Q^2, 0) = -\frac{Q}{M} F_{\mathrm{LT}}(Q^2, 0)\,.
$$

  This relation remains true after relativistic corrections!

$\gamma^*\gamma^*$-transition form factors for $\chi_{c1}(1P)$ axial mesons

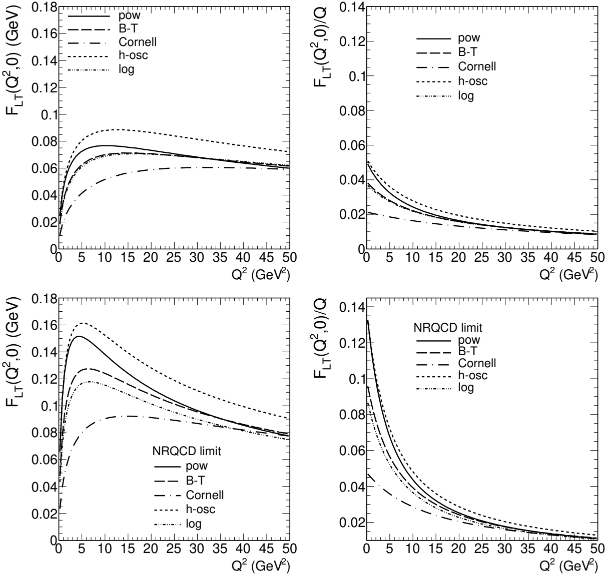


Figure: Form factors $F_{\rm LT}(Q^2,0)$ for one virtual photon (left and middle panels) and $F_{\rm LT}(Q^2,0)/Q$ (right panel). The top panels: our results in the LFWF approach and the bottom panels: nonrelativistic limit.

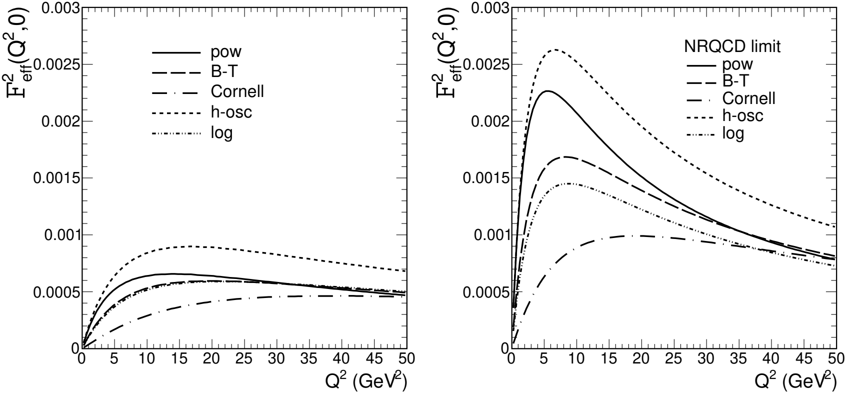


Figure: The square of the effective form factor as a function of photon virtuality within LFWF approach (on the l.h.s.) and in the nonrelativistic limit (on the r.h.s.).

$$\sigma_{\rm tot}^{\gamma^*\gamma}(Q^2,0) = 16\pi^3\alpha_{\rm em}^2\delta(\hat{s}-M^2)\frac{Q^2}{Q^2+M^2}\left(1+\frac{Q^2}{2M^2}\right)\left(\frac{F_{\rm LT}(Q^2,0)}{Q}\right)^2$$

$$\equiv 16\pi^3\alpha_{\rm em}^2\delta(\hat{s}-M^2)\,{\rm F}^2_{\rm eff}(Q^2)\,.$$

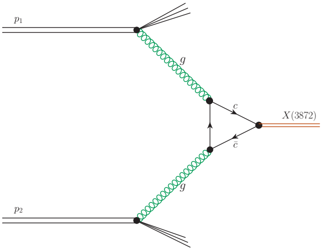
Reduced width of $\chi_{c1}(1P)$

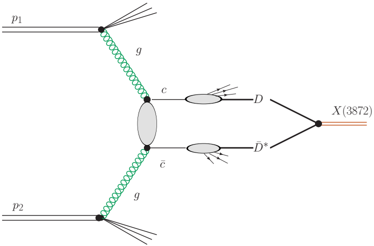
Table: Reduced width

potential model	m_c (GeV)	$\lvert R'(0)\rvert$ (GeV$^{5/2}$)	$\tilde{\Gamma}(\chi_{c1})_{\mathrm{NRQCD}}$ (keV)	$\tilde{\Gamma}(\chi_{c1})$ (keV)
power-law	1.33	0.22	0.97	0.50
Buchmüller-Tye	1.48	0.25	0.82	0.30
Cornell	1.84	0.32	0.56	0.09
harmonic oscillator	1.4	0.27	1.20	0.53
logarithmic	1.5	0.24	0.72	0.27

- Considerably larger values of $\tilde{\Gamma}(\chi_{c1})$ are quoted in the literature. For example Danilkin & Vanderhaeghen (2017) report a value of $\tilde{\Gamma}(\chi_{c1}) \approx 1.6\,\mathrm{keV}$ from a sum rule analysis. Li et al. (2022) obtain $\tilde{\Gamma}(\chi_{c1}) \approx 3\,\mathrm{keV}$ from a LFWF approach.
- A measurement of the reduced width would therefore be very valuable.

Hadroproduction of $X(3872)$ (or $\chi_{c1}(3872)$)





- Structure of $\chi_{c1}(3872)$ ($J^{PC} = 1^{++}$) still enigmatic. Its situation near the threshold of $D\bar{D}^*$ suggests its interpretation as the weakly bound "molecule".
- What about a $\chi_{c1}(2P)$ component?
- Production at large p_T (hard process) is often suggested to serve as a probe of structure.
- P_T distributions have been measured by ATLAS, CMS and LHCb in the $J/\psi\pi\pi$ channel.
- Do the sizeable production cross sections rule out the large size molecule?
- We use the k_T-factorization approach in which gluons carry transverse momentum and are off-shell. It efficiently includes some NLO corrections at small-x. Note that for on-shell gluons $gg \to 1^{++}$ vanishes!

$c\bar{c}$-state, molecule, mixture of both...

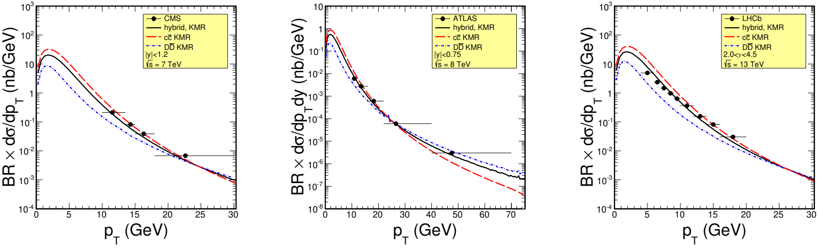
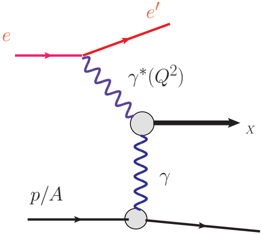


Figure: Transverse momentum distribution of $X(3872)$ for the CMS, ATLAS and LHCb experiments. Shown are results for the KMR UGDF. Here BR $=$ 0.038 for CMS and LHCb, and $\mathrm{BR} =$ 0.038 $\cdot$ 0.0596 for ATLAS. We show results for different combinations of α and β as specified in the figure legend. Here the feedown contributions are included.

- $$X(3782)\rangle = \alpha c\bar{c}\rangle + \frac{\beta}{\sqrt{2}}\left(D\bar{D}^*\rangle + \bar{D}D^*\rangle\right) \ .$$

Ultraperipheral collisions

Photon-photon process in the electron-proton or electron-ion collision:



- proton ($Z = 1$) or ion ($Z = 82$ for Pb) is a source of quasireal Weizsäcker-Williams photons)

$$n(x) = \frac{Z^2\alpha_{\rm em}}{\pi} \int \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \left(\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + x^2 m_p^2} \right)^2 F^2_{\rm ch}(\boldsymbol{q}^2 + x^2 m_p^2)$$

- "anti-tagged" electrons, say $Q^2 < 0.1\,\mathrm{GeV}^2$, are also the source of quasireal photons $\longrightarrow$ $\gamma\gamma$-fusion.
- at finite Q^2 we have access to a whole polarization density matrix of virtual photons,
- the intact nuclei in the final state are not measured. Photon exchange is associated with a large rapidity gap.

(Rough) Estimates for EIC energies

- Can we pin down the $c\bar{c}$ component of $\chi_{c1}(3872)$? Work in progress with I. Babiarz, R. Pasechnik and A. Szczurek.
- for $Q^2 \ll 2M^2$ longitudinal photons will dominate. Note, that the total cross section does not have the dQ^2/Q^2 logarithmic integral.

$$Q^2 \frac{d\sigma(eA \to e' X(3872) A)}{dQ^2} = \frac{\alpha_{\rm em}}{\pi} \int_{y_{\rm min}}^1 \frac{dy}{y} \frac{dx}{x} f_L(y) n_{\gamma/A}(x) \delta(xys - M^2) 16\pi^3 \alpha_{\rm em}^2 \mathrm{F}_{\rm eff}^2(Q^2)$$

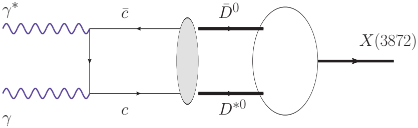
- We assume the Q^2-dependence of a $c\bar{c}$-state, and a reduced width of $\tilde{\Gamma} = 0.5\,\mathrm{keV}$.
- limit on reduced width from Belle (updated, Achasov, 2022): $24\,\mathrm{eV} < \tilde{\Gamma}(\chi_{c1}(3872)) < 615\,\mathrm{eV}$.

Table: Cross sections on proton and ^{208}Pb

$\sqrt{s_{eN}}$ [GeV]	$\sigma(ep \to epX)$ [pb]	$\sigma(eA \to eAX)$ [pb]
50	0.06	60
140	0.16	340

- Hadronic contribution?

Possible molecule contribution to $\tilde{\Gamma}$?



- apparently nothing (?) is known about the molecular contribution to the reduced width.
- What about the analogous contribution to the one we adopted in the hadronic case? Say $\gamma^*\gamma \to c\bar{c} \to \bar{D}D^*$, and FSI of $D\bar{D}^*$ generates the $X(3872)$.
- Spins of heavy quarks in $X(3872)$ are entangled to be in the spin-triplet state (M. Voloshin, 2004). But near threshold the $c\bar{c}$ state produced via $\gamma\gamma$-fusion is in the $^1S_0$ state. (It's different for gluons, where color octet populates $^3S_1$!)
- $\to$ "handbag mechanism" suppressed in heavy quark limit.
- Purely hadronic models?

Summary

- We have derived the LFWF representation of axial meson $\gamma^*\gamma^*$ transition form factors.
- These FFs contain valuable information on the structure of the meson. They also appear as building blocks of charmonium production in a k_T-factorization approach within the color-singlet approach.
- The reduced width of the ground state $\chi_{c1}(1P)$, for one londitudinal and one real photon $\tilde{\Gamma}$ is obtained in the ballpark of ~ 0.5 keV.
- We considered prompt hadroproduction of $\chi_{c1}(3872)$ at LHC energies for a $c\bar{c}$ state, a molecule, or a mixture of both.
- The molecule production has the hardest behaviour as a function of p_T. This is expected from to the color-octet contribution. Shape of molecule alone does not agree well with data.
- Production of $c\bar{c}$ state gives reasonable behaviour, as does a mixture of $c\bar{c}$ and molecule.
- Electroproduction of $\chi_{c1}(1P), \chi_{c1}(3872)$ in the Coulomb field of a heavy nucleus may give access to form factor $F_{\rm LT}(Q^2, 0)$. This is additional information on the structure. We know how to calculate it for $c\bar{c}$, or possibly tetraquark states.
- What about the molecule? Can one calculate its reduced width to $\gamma_L^*\gamma$?