Mueller Tang jets in next-to-leading BFKL

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Diffraction and low-x

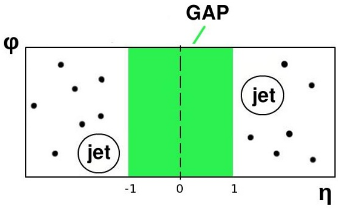
Corigliano Calabro 28.09.2022

Outline

- Introduction:
  - Review of PT picture of BFKL for Mueller-Tang jets
  - LL factorization formula
- Beyond LL approximation
  - Phenomenology with LL and NLL GGF
  - $\rightarrow$ need of a full NLL calculation?
- NLL impact factors
  - Structure of NLL impact factor
  - Implementation of NLL impact factors: numerical and conceptual issues
  - (Small) breaking of BFKL factorization at NLL level
- Numerical results
- Conclusions and outlook

Mueller-Tang jets

An important process for studying PT high-energy QCD and the Pomeron at hadron colliders *[Mueller, Tang '87]*



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- two jets with similar p_T
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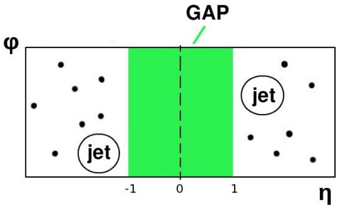

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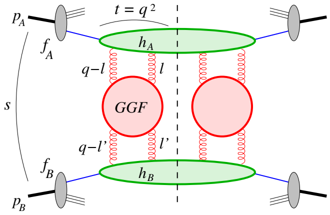
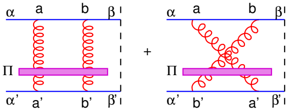


- Gap $\Longrightarrow$ mostly colour-singlet exchanges contribute to cross section
- $Y \gg 1 \Longrightarrow$ enhanced PT series $(\alpha_S Y)^n$ resummed into singlet BFKL GGF
- In LLA factorization formula holds

Mueller-Tang jets at LO and LL

- LO amplitude: box + crossed diagrams projected onto colour-singlet $\Pi^{ab,a'b'} = \delta^{ab}\delta^{a'b'}/(N_c^2-1)$



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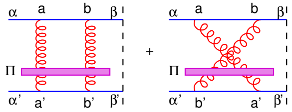
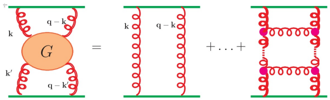


- Elastic amplitude at higher orders: affected by large $\log^n s$ due to gluon-ladder diagrams (UV and IR finite)
- All LL resummed in (colour-singlet) gluon Green function (GGF)



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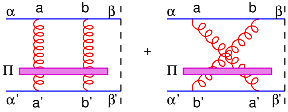

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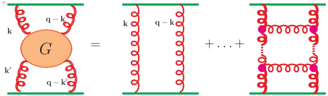
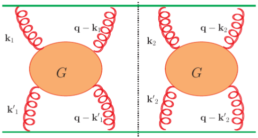
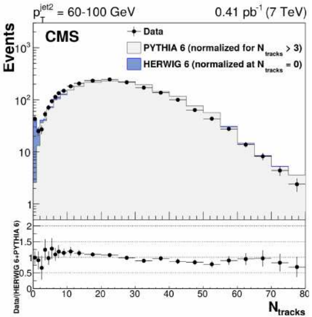


- LL partonic cross section: 2 GGF $*$ 2 (trivial) impact factors
- Two outgoing partons to be identified with the (back-to-back) jets

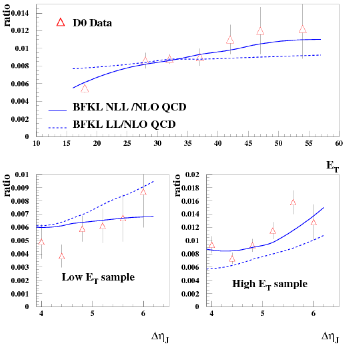


CMS analysis at 7 TeV



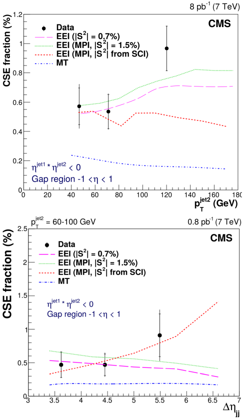
- Charged-particle multiplicity in the gap region between the tagged jets compared to PYTHIA and HERWIG predictions.
- HERWIG 6: include contributions from color singlet exchange (CSE), based on BFKL at LL.
- PYTHIA 6: inclusive dijets (tune Z2*), no-CSE.

D0 and CMS analysis at 7 TeV



Left: LL & NLL BFKL at Tevatron [hep-ph/1012.3849].

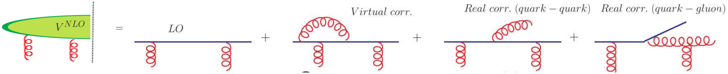
- Ratio $R = \frac{NLL^* BFKL}{NLOQCD}$ of jet-gap-jet events to inclusive dijet events as a function of p_t and the rapidity gap Y.



NLL* BFKL calculations different implementations of the soft rescattering processes (EEI models), describe many features of the data, but none of the implementations is able to simultaneously describe all the features of the measurement. Ekstedt, Enberg, Ingelman, [1703.10919]

NL impact factors

- Compelling to include all NLL corrections into the game



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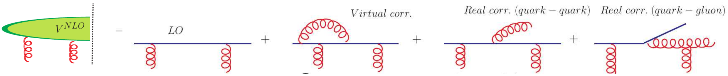
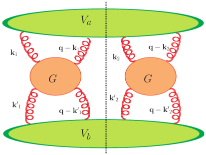


- Idea: generalize MT factorization formula at NLL
- BFKL GGF at NLL known since long *[Fadin, Fiore et al.]*



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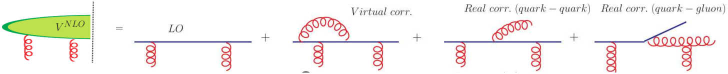

- Idea: generalize MT factorization formula at NLL
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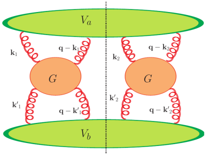
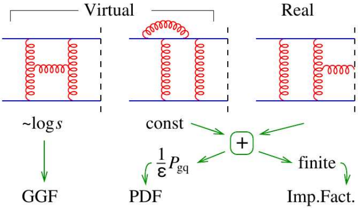
- NL impact factors determined by NLO calculation, with IR (soft and collinear) divergencies





Not a trivial statement:

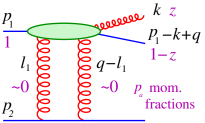
- all $\log(s)$ terms must reproduce LL kernel (GGF at 1st order)
- all IR singularities (taken away collinear ones proportional to splitting functions) must cancel

NL impact factors

$$\Phi(\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{q}) = \frac{\alpha_S^3}{2\pi(N_c^2-1)} \int_0^1 \mathrm{d}z \int \mathrm{d}^2\boldsymbol{k}$$

$$\times\, S_J(\boldsymbol{k}, \boldsymbol{q}, z)\, C_F \frac{1+(1-z)^2}{z}$$

$$\times \left\{ C_F^2 \frac{z^2 \boldsymbol{q}^2}{\boldsymbol{k}^2(\boldsymbol{k}-z\boldsymbol{q})^2} + C_F C_A f_1(\boldsymbol{l}_{1,2}, \boldsymbol{k}, \boldsymbol{q}, z) + C_A^2 f_2(\boldsymbol{l}_{1,2}, \boldsymbol{k}, \boldsymbol{q}) \right\}$$



$\times$ h.c. $(\boldsymbol{l}_1 \leftrightarrow \boldsymbol{l}_2)$

- The calculation of NL impact factors for Mueller-Tang jets was performed by *[Hentchinski, Madrigal Martinez, Murdaca, Sabio Vera, '14]* using Lipatov's effective action (and confirmed by F.Deganutti and myself)

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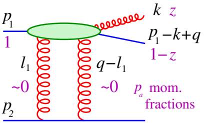

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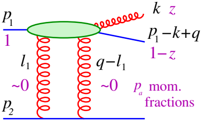
- Phase-space integration restricted by IR-safe jet algorithm (e.g., kt $\simeq$ cone)
- The two partons in the same hemisphere form (at least) one jet:
  - $\Delta\Omega \equiv \sqrt{\Delta y^2 + \Delta\phi^2} < R \quad \Longrightarrow J = \{qg\}$ composite jet
  - $\Delta\Omega > R \Longrightarrow J = \{g\}$ and q outside jet cone or $J = \{q\}$ and g outside

Problem with NL impact factor

$$\Phi(\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{q}) = \frac{\alpha_S^3}{2\pi(N_c^2-1)} \int_0^1 \mathrm{d}z \int \mathrm{d}^2\boldsymbol{k} \times \boldsymbol{S_J}(\boldsymbol{k}, \boldsymbol{q}, z)\, C_F \frac{1+(1-z)^2}{z} \times \left\{ C_F^2 \frac{z^2\boldsymbol{q}^2}{\boldsymbol{k}^2(\boldsymbol{k}-z\boldsymbol{q})^2} + C_F C_A f_1(\boldsymbol{l}_{1,2}, \boldsymbol{k}, \boldsymbol{q}, z) + C_A^2 f_2(\boldsymbol{l}_{1,2}, \boldsymbol{k}, \boldsymbol{q}) \right\}$$



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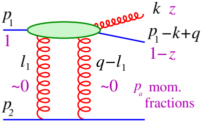

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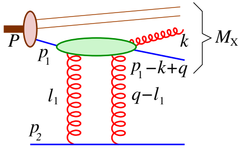
- There is a problem in the C_A^2 term, due to $\int_0^1 \mathrm{d}z/z$ integration
- If integration is not constrained, we have a divergence
- Such region $z \to 0$ corresponds to gluon in central (and backward) region, where the emission probability of the gluon turns out to be flat in rapidity: $\int_0^1 \mathrm{d}z/z = \int_{-\infty}^{\log\sqrt{s}/\boldsymbol{k}} \mathrm{d}y$
- If we believe the IF calculation to be reliable at least in the forward hemisphere $(y>0) \implies \int_0^{\log\sqrt{s}/\boldsymbol{k}} \mathrm{d}y = \int_{\boldsymbol{k}/\sqrt{s}}^1 \mathrm{d}z/z = \frac{1}{2}\log(s/\boldsymbol{k}^2)$
- But a $\log(s)$ in IFs is not acceptable within the spirit of BFKL factorization

Constraint on diffractive invariant mass

In order to solve this problem, *[HMMS]* constrain mass of diffractive system $M_X^2 \equiv (P+q)^2 < M^2_{\max}$



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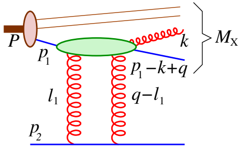

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- Diffractive mass requires measuring outgoing proton or its remnants
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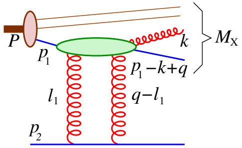

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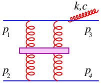
...and it is definitely different from the JGJ experimental definition

Violation of BFKL factorization



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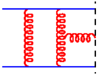

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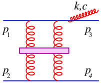

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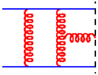


$\Longrightarrow$ $\log s$ unavoidable without constraints

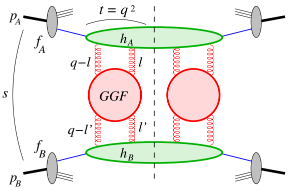
- MT event selection constrains particles not to be emitted within the gap provided they are above some energy threshold $E_{\rm th}$ (cal resolution)
- Only particles below threshold can be emitted at any rapidity
- This prescription is IR safe because inclusive for $E_g < E_{\rm th}$

  But gluons below threshold can have any rapidity $\Longrightarrow \sigma \ni C_A^2 \frac{E_{\rm th}^2}{E_J^2} \log \frac{s}{E_J^2}$

With such "minimal" experimental prescription, BFKL factorization is violated (impact factors depend on s). However violation is expected to be small.

Results

- We go on and use NL IF and GGF for $\mathrm{d}\sigma/\mathrm{d}Y$
- Numerical implementation requires 10-dimensional integrals



Leading VS Next-to-leading cross section

We follow CMS setup:

- $\sqrt{s} = 14$ TeV
- $E_J \geq 40$ GeV
- $1.5 \leq |y_J| \leq 5$
- $3 \leq Y \equiv y_{J1} - y_{J2} \leq 9$
- $y_{\rm gap} \in [-1, 1] \;\rightarrow\; \Delta Y_{\rm gap} = 2$
- $E_{\rm thresh} = 1.0$ GeV

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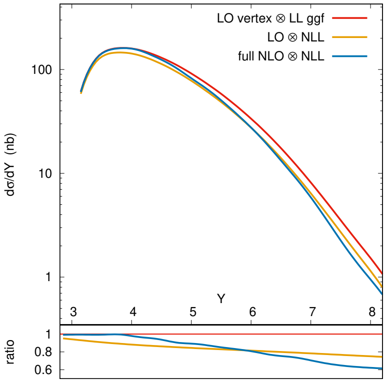
- NLL corrections of the impact factors are negative

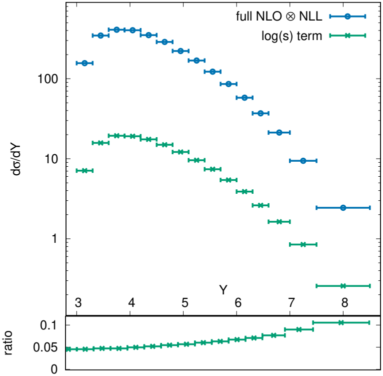
- $\sigma_{\mathrm{full}} \lesssim \sigma_{LL}$ with a slightly steeper decrease



Factorization violation

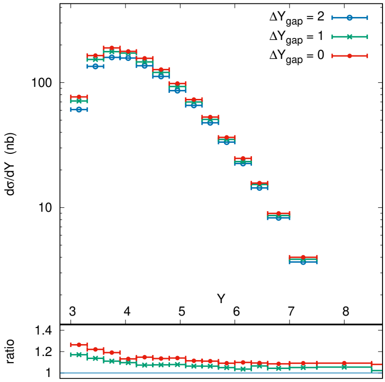
Contribution of the term $C_A^2 \log \frac{s}{E_J^2}$ that violates factorization:

- Violation of factorization is small, $\sim 6\%$
- Resummation of such logarithms not necessary for phenomenology



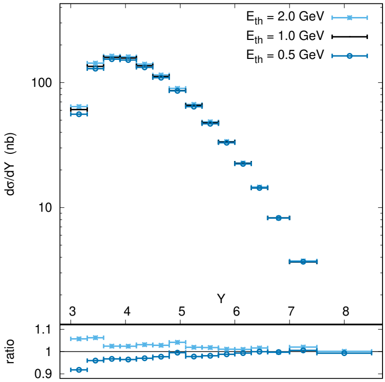
Dependence on gap width

- Cross section slightly increases while decreasing $\Delta Y_{\rm gap}$ and saturates with no gap
- Emission from singlet exchange in central region is dynamically suppressed



Energy threshold dependence

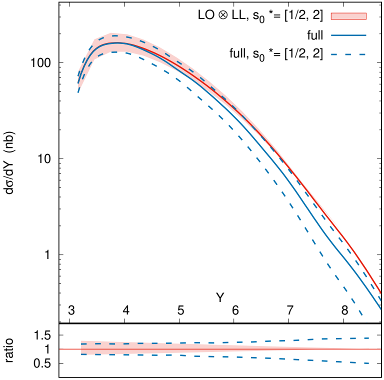
- Mild dependence on energy threshold of the gap
- Most of the dependence is due to soft dynamics (IR singularities) not to high-energy dynamics ($\log s$ terms)



Energy scale dependence

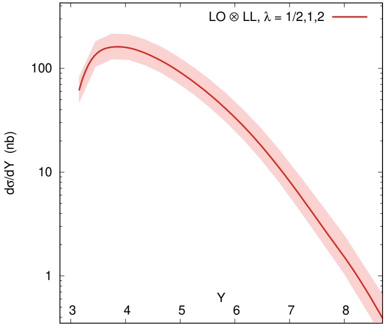
Energy scale s_0 is a parameter needed in order to define the $\log(s/s_0)$

- Reduced dependence on energy scale at moderate Y
- which however increases at large Y



Renormalization scale dependence

Running coupling $\alpha_S(Q^2)$ at phyisical scale $Q = \lambda(E_{J1} + E_{J2})$



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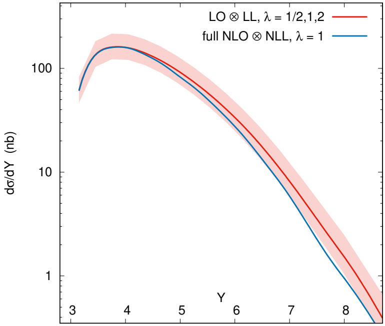

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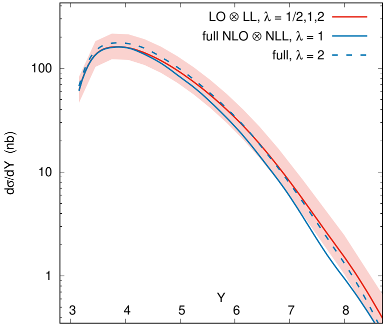

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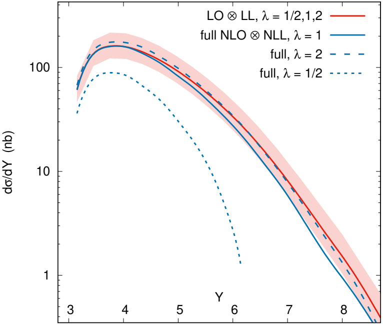
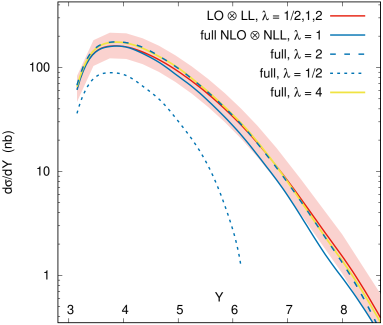
- Renorm scale dependence still large
- Need for scale fixing procedure (BLM, PMS, ...)



Renormalization scale dependence

Running coupling $\alpha_S(Q^2)$ at physical scale $Q = \lambda(E_{J1} + E_{J2})$

- Renorm scale dependence still large
- Need for scale fixing procedure (BLM, PMS, ...)
- Minimum sensitivity reached at $\lambda = 4$



Conclusions and outlook

- Complete numerical implementation of MT jets at LHC in NLLA with collinear resummation of BFKL kernel; cross section slightly lower and steeper than in LLA
- Gap survival probability still to be taken into account
- Strictly speaking jet-gap-jet observable violates BFKL factorization in NLLA
- Nevertheless the violation is small and factorization formula is expected to work well for LHC (non-asymptotic) kinematics.
- Good stability w.r.t. gap/threshold parameters
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- Improvements could include hadronization, resummation of $\log s$ term in IFs, and inclusion of gap survival probability