Small-x resummation in the coefficient function for differential heavy-quarks production

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Small-x resummation

Perturbative coefficient functions and splitting functions contain logarithms of x that are single-logarithmically enhanced:

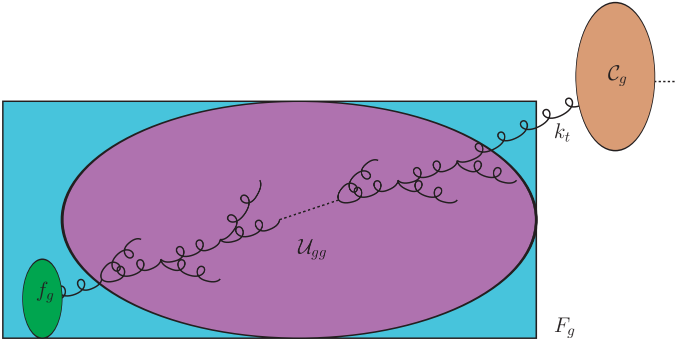
$$
\begin{aligned}
P(x,\alpha_s) \text{ or } C(x,\alpha_s) = \quad & a_0 \\
& + \alpha_s\left[a_1 \ln\frac{1}{x} + b_1\right] \\
& + \alpha_s^2\left[a_2 \ln^2\frac{1}{x} + b_2 \ln\frac{1}{x} + c_2\right] \\
& + \alpha_s^3\left[a_3 \ln^3\frac{1}{x} + b_3 \ln^2\frac{1}{x} + c_3 \ln\frac{1}{x} + d_3\right] \\
& + \alpha_s^4\left[a_4 \ln^4\frac{1}{x} + b_4 \ln^3\frac{1}{x} + c_4 \ln^2\frac{1}{x} + d_4 \ln\frac{1}{x} + e_4\right] \\
& + \dots
\end{aligned}
$$

When $\alpha_s \ln\frac{1}{x} \sim 1$ the fixed-order expansion is no longer predictive!

$\downarrow$

Resummation

k_t-factorisation and small-x resummation



k_t-factorisation: [Catani, Hautmann hep-ph/9405388]

$$\sigma(x,Q^2) = \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}{k_t}^2\, \mathcal{C}_g\left(\frac{x}{z}, \alpha_s, Q^2, {k_t}^2\right) \mathcal{F}_g\left(z, Q^2, {k_t}^2\right)$$

The off-shell coefficient function

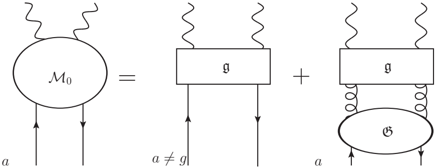


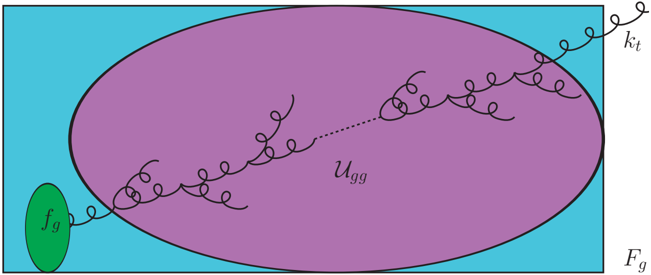
Figure: The only relevant part for x leading logs is the second one on the RHS

$\mathcal{C}(\cdots)$ is obtained as an off-shell continuation of the Born counterpart.
In practice one requires:

- Light-cone gauge $\rightarrow$ no mass singularities in 2PI amplitudes
- Off-shellness of the incoming parton (gluon)

$$k_{\text{collinear}} \rightarrow k_{\text{in}} = z p_1 + k_t, \quad \frac{1}{2}\sum_{\lambda} \varepsilon^{\mu}_{\lambda}(k_{\text{in}})\varepsilon^{*\nu}_{\lambda}(k_{\text{in}}) \rightarrow -\frac{{k_t}^{\mu}{k_t}^{\nu}}{{k_t}^2} \tag{1}$$

- This is the only process dependent part of the computation.
- 2 Gluon Irreducible (2GI) by construction $\Longrightarrow$ gauge-invariance at Born level.



$$\mathcal{F}_g\left(x, Q^2, {k_t}^2\right) = \int_x^1 \frac{\mathrm{d}z}{z} U_{gg}\left(\frac{x}{z}, {k_t}^2, Q^2\right) f_g\left(z, Q^2\right) \tag{2}$$

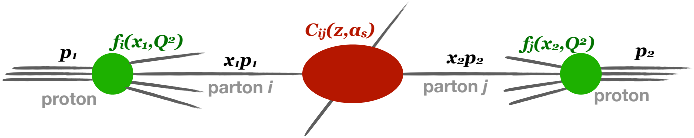
Mellin-space formalism, fixed coupling:

$$U(N, \xi) = \partial_\xi \xi^{\gamma(\alpha_s, N)}\,,$$

Anomalous dimension resummation [Altarelli, Ball, Forte 0802.0032]
Direct space formalism [Bonvini, Marzani, Peraro 1607.02153]

LHC phenomenology

Resummation of LHC observables



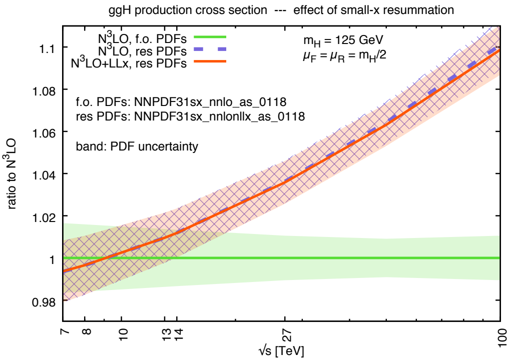
Challenges:

- two protons in the initial state ✓
- want to describe differential distributions ✓

Processes considered so far in `HELL`:

- $gg \to H$ (inclusive cross section) [Bonvini 1805.08785]
- $c\bar{c}$, $b\bar{b}$ pair production (fully differential) *(almost done)*
- Drell-Yan (fully differential) *(in progress)*
- $gg \to H$ (fully differential) *(should be straightforward given the first two)*

$gg \to H$ inclusive cross section [Bonvini,Marzani 1802.07758] [Bonvini 1805.08785]



ggH cross section at FCC-hh can be $\sim 10\%$ larger than expected with only fixed order theory!

At LHC $+1\%$ inclusive effect, larger in some kinematic region? $\to$ differential study.

LHC observables in collinear and k_t factorization

Differential cross section in collinear factorization

$$\frac{d\sigma}{dQ^2 dY...} = \int_\tau^1 \frac{dz}{z} \int d\hat{y}\, \mathscr{L}_{ij}\left(\frac{\tau}{z}, \hat{y}, Q^2\right) \frac{dC_{ij}}{dy...}(z, Y-\hat{y}, ..., \alpha_s)$$

$$\mathscr{L}_{ij}(x, \hat{y}, Q^2) = f_i(\sqrt{x}e^{\hat{y}}, Q^2)\, f_j(xe^{-\hat{y}}, Q^2)\, \theta(e^{-2|\hat{y}|} - x)$$

and in k_t factorization [Caola,Forte,Marzani 1010.2743] [Muselli 1710.09376]

$$\frac{d\sigma}{dQ^2 dY...} =$$

$$\int_\tau^1 \frac{dz}{z} \int d\hat{y} \int_0^\infty dk_1^2 \int_0^\infty dk_2^2\, \mathcal{L}_{ij}\left(\frac{\tau}{z}, \hat{y}, k_1^2, k_2^2\right) \frac{d\mathcal{C}_{ij}}{dy...}(z, Y-\hat{y}, k_1^2, k_2^2, \ldots, \alpha_s)$$

$$\mathcal{L}_{\mathrm{ij}}(x, \hat{y}, k_1^2, k_2^2) = \mathcal{F}_{\mathrm{i}}(\sqrt{x}e^{\hat{y}}, k_1^2)\, \mathcal{F}_{\mathrm{j}}(\sqrt{x}e^{-\hat{y}}, k_2^2)\, \theta(e^{-2|\hat{y}|} - x)$$

$\tau = Q^2/s, \quad y = Y - \frac{1}{2}\log\frac{x_1}{x_2}$

k_t factorisation allows to gather the small-x logarithms away from the coefficient function.

In Mellin-Fourier space, the **evolutor** U enstablishes a relation between collinear and off-shell PDFs [Bonvini, Marzani, Peraro 1607.02153].

$$\mathcal{F}_g\left(N \pm \frac{ib}{2}, k_t^2\right) = U\left(N \pm \frac{ib}{2}, k_t^2, Q^2\right) f_g\left(N \pm \frac{ib}{2}, Q^2\right)$$

Then the resummed coefficient function is given as

$$\frac{d\tilde{C}_{\rm gg}}{dy...}(N, b, ...) = \int_0^\infty dk_1^2 \int_0^\infty dk_2^2 \, \frac{d}{dk_1^2} U\left(N + \frac{ib}{2}, k_1^2, Q^2\right) \frac{d}{dk_1^2} U\left(N - \frac{ib}{2}, k_2^2, Q^2\right) \times \frac{d\tilde{\mathcal{C}}_{\rm gg}}{dy...}(N, b, k_1^2, k_2^2, ..., \alpha_s)$$

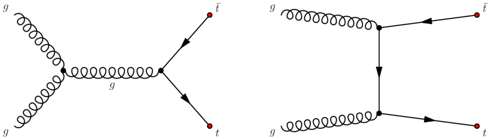
Numerical integration → HELL public code

$$\frac{dC_{\rm gg}}{dy...}(z, y, ...) = \int_0^\infty dk_1^2 \int_0^\infty dk_2^2 \int_z^1 \frac{dx}{x} \int d\hat{y} \, \frac{d\mathcal{C}_{\rm gg}}{dy...}(x, y - \hat{y}, k_1^2, k_2^2, ..., \alpha_s) \times \frac{d}{dk_1^2} U\left(\sqrt{\frac{z}{x}} e^{\hat{y}}, k_1^2, Q^2\right) \frac{d}{dk_2^2} U\left(\sqrt{\frac{z}{x}} e^{-\hat{y}}, k_2^2, Q^2\right) \theta\left(e^{-2|\hat{y}|} - \frac{z}{x}\right)$$

Transverse final-state variables do not play a role (except for kinematic constraints)

Charm (and bottom) pair production at LHC

Fully differential heavy-quark pair production [Bonvini,FS (in preparation)]



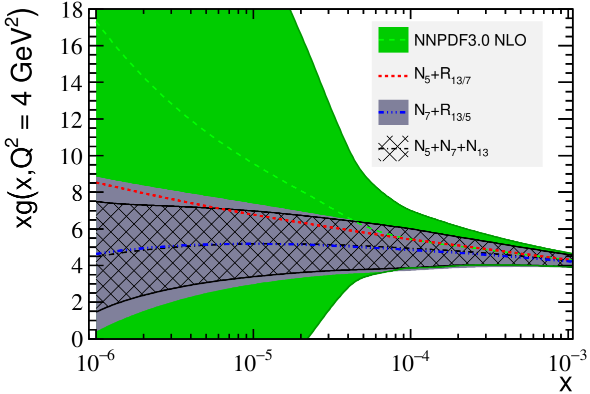
We have resummed the cross section for different kinematics:

- heavy-quark pair: $\dfrac{d\sigma}{dQ^2\, dY\, dq_t}$ $\rightarrow$ quarkonium production
- single heavy quark: $\dfrac{d\sigma}{dy\, dp_t}$ $\rightarrow$ D/B mesons production

Small-x resummation crucial for **charm and bottom production**

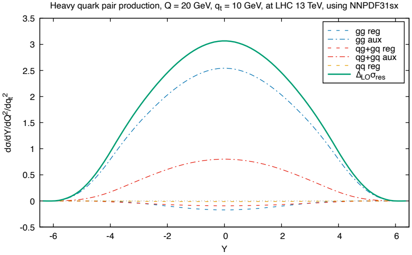
- key process at forward physics experiment e.g. FPF [Feng et al 2203.05090]
- sensitive to very small x $\rightarrow$ constrain the PDFs [Gauld, Rojo 1610.09373]

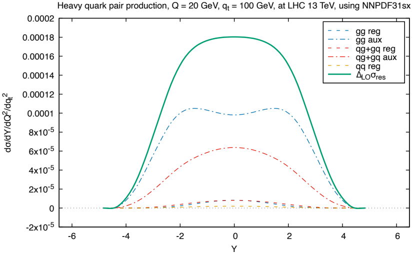
Impact of heavy meson data on gluon PDFGluon PDF uncertainty reduction from heavy quarks in LHC Run 2



[Gauld, Rojo 1610.09373]

Results for $\frac{d\sigma}{dQ^2\, dY\, dq_t}$ in **pair kinematics**



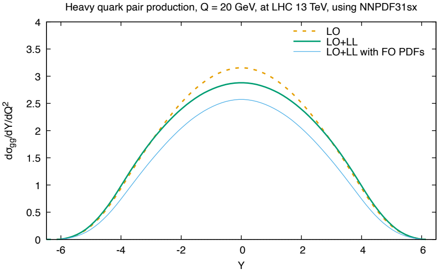


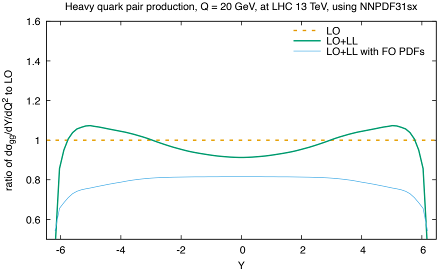
- Pure resummed → match to fix order, and assess resummation impact
- Open heavy quark final state → add fragmentation to compare with exp. data
- Auxilliary channels provide dominant contribution → **hybrid factorisation**

$$\mathcal{F}_g(N,\xi) = \left[U_{\rm reg}\left(N, Q^2\xi, \mu_{\rm F}^2\right) + \delta(\xi)\right] f_g(N, \mu_{\rm F}^2) + \frac{C_F}{C_A} U_{\rm reg}\left(N, Q^2\xi, \mu_{\rm F}^2\right) f_q(N, \mu_{\rm F}^2).$$

New results for $\frac{d\sigma}{dQ^2\,dY}$ in **pair kinematics**

- Open heavy quark final state
- PDFs convolution included
- Important impact at central rapidity, both from PDF and coefficient function
- sizable contribution at large rapidity, but lesser than PDFs





Heavy-quark pair production at LHC: preliminary results

New results for $\frac{d\sigma}{dQ^2\,dY}$ in **pair kinematics**

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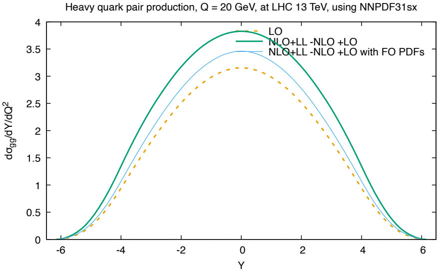
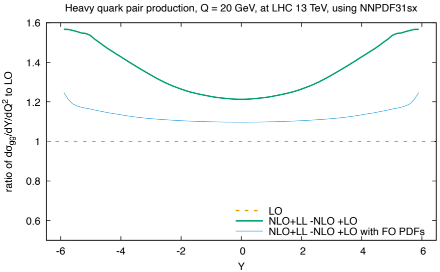
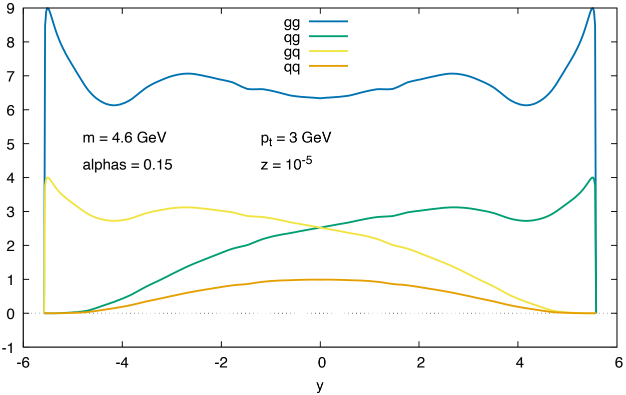




Heavy-quark pair production at LHC: preliminary results

Results for $\frac{d\sigma}{dy\,dp_t}$ in **single-quark kinematics**



- Pure resummed → match to fix order, and assess resummation impact
- Parton level → convolve with PDFs
- Open heavy quark final state → add fragmentation to compare with exp. data
- 5D numerical integral → code tuning in progress

Concluding remarks

Key messages:

- resummation is needed at small-x
- significant impact expected at LHC at low invariant mass and large rapidity in several processes

completion $\rightarrow$ outlook:

- for $c\bar{c}, b\bar{b}$ we considered only quark-level final states
  $\rightarrow$ add hadronization and refit PDFs with LHCb data [Aaij et al. 1510.01707]
- at forward rapidities one parton is at large x
  $\rightarrow$ combine with threshold resummation
- low accuracy (two log orders known for $P_{\rm gg}$ only, for everything else just one)
  $\rightarrow$ extension to the next logarithmic order **[Anna's talk]**

Many thanks for your attention