Quark and Gluon helicity evolution at small x: Revised and updated

F. Cougoulic (IGFAE), in collaboration with Y. V. Kovchegov (OSU), A. Tarasov (OSU, BNL-SBU, NC State), and Y. Tawabutr (OSU, JYU).

Based on [2204.11898]

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Proton spin puzzle:

In 1988, the European Muon Collaboration (EMC) observed for the proton

$$\int_{\mathbf{0.01}}^{\mathbf{0.7}} g_1(x)\,\mathrm{d}x = 0.114 \pm 0.012(\text{stat.}) \pm 0.026(\text{syst.}) \tag{1}$$

Reminder

$$g_1^{\gamma} = \frac{1}{2}\sum_q e_q^2\,\left(\Delta q + \Delta\bar{q}\right), \qquad \Delta q = q^{\uparrow} - q^{\downarrow} \text{ w.r.t. the proton spin} \tag{2}$$

"The quark spin constitues a rather small fraction of the spin of the nucleon" [PLB, 206, 2, 1988]

$\Rightarrow$ Incompatible with the spin $1/2$ of the proton.
Where is the missing spin of the proton?

Spin sum rule Jaffe Manohar decomposition [Nucl. Phys. B337, 509 (1990)]

Quark OAM

Gluon OAM

$$\frac{1}{2} = \frac{1}{2}\Sigma_q + \Sigma_g + L_q + L_g$$

Gluon spin

Quark spin

Proton spin

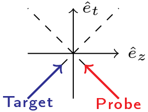
Possibilities:

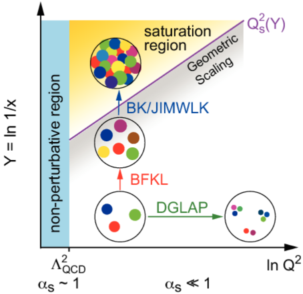
- Gluon spin
- Quark/Gluon angular orbital momentum
- Experiments only access a finite range of x:

A - Large-x
B - Small-x ← Explored here!

Generalities

Using lc coordinates $a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$
Frame choice: $\rightarrow$ Probe minus-moving, target plus-moving.





- Aim: small-x asymptotic. $\longrightarrow$ Evolution in rapidity.
- Approach: Take a TMD. $\longrightarrow$ Simplify / Evolve / Solve.
- Equations in the spirit of BK-evolution. Initiated by [Kovchegov, Pitonyak, and Sievert].
- Usualy working in mixed space $\{k^-, \underline{x}\}$.

Rmk: $\exists$ other frameworks for g_1 at small-x, such as Bartel, Ermolaev, and Ryskin [BER] - (1996).

Def: Wilson Lines for any irreducible representation (irrep) are

$$W_{\underline{x}}^{(R)}[b^-, a^-] \equiv \mathrm{P}\exp\left\{ ig \int_{a^-}^{b^-} dx^- \; t_R^a \; \alpha^a(x^+ = 0, x^-, \underline{x}) \right\}. \tag{3}$$

$\Rightarrow$ Depends only on the background field $\alpha = A^+$ (Lorentz Gauge).

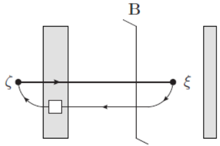
Notation: we use V for fundamental WL, and U for adjoint WL.

Situation prior to this contribution (1/3)

Consider the quark helicity TMD [Kovchegov et al. 2018]

$$g_{1L}^q(x,k_T^2) = \frac{1}{(2\pi)^3}\frac{1}{2}\sum_{S_L} S_L \int \mathrm{d}^2\underline{r}dr^-\ e^{ik\cdot r}\langle p, S_L|\bar{\psi}(0)U[0,r]\frac{\gamma^+\gamma^5}{2}\psi(r)|p,S_L\rangle. \tag{4}$$

- Gauge-link $U[0,r]$ is process dependent, SIDIS $\rightarrow$ forward staple.
- Simplify at small-x, remaining diagram is B.



After some algebra...

$$g_{1L}^q(x,k_T^2) = -\frac{2p^+}{(2\pi)^3}\int \mathrm{d}^2\zeta \mathrm{d}^2 w \frac{\mathrm{d}^2 k_1 dk_1^-}{(2\pi)^3} e^{i(\underline{k}_1+\underline{k})\cdot(\underline{w}-\underline{\zeta})}\theta(k_1^-)\sum_{\sigma_1,\sigma_2} \times \bar{v}_{\sigma_2}(k_2)\frac{1}{2}\gamma^+\gamma^5 v_{\sigma_1}(k_1)2\sqrt{k_1^- k_2^-} \times \left\langle \mathrm{T} V_{\underline{\zeta}}^{ij}\left(\bar{v}_{\sigma_1}(k_1)\hat{V}_{\underline{w}}^{\dagger ji} v_{\sigma_2}(k_2)\right)\right\rangle \times \frac{1}{[2k_1^- xP^+ + \underline{k}_1 - i\epsilon k_1^-][2k_1^- xP^+ + \underline{k}^2 + i\epsilon k_1^-]}\Bigg|_{k_2^-=k_1^-,\underline{k}_2=-\underline{k}} + c.c. \tag{5}$$

Situation prior to this contribution (2/3)

The previous green operator reads

$$\left(\bar{v}_\sigma(p)\hat{V}^\dagger_{\underline{x}} v_{\sigma'}(p')\right) = 2\sqrt{p^- p'^-}\delta_{\sigma\sigma'}\left(V^\dagger_{\underline{x}} - \sigma V^{pol\dagger}_{\underline{x}} + \cdots\right). \tag{6}$$

The flavor-singlet contribution simplified at small-x gives

$$g^S_{1L}(x, k_T^2) = \frac{8N_c i}{(2\pi)^5}\int \mathrm{d}^2\zeta \mathrm{d}^2\underline{w}\; e^{-i\underline{k}\cdot(\underline{\zeta}-\underline{w})}\int_{\Lambda^2/s}^{1}\frac{dz}{z}\frac{\underline{\zeta}-\underline{w}}{(\underline{\zeta}-\underline{w})^2)}\cdot\frac{\underline{k}}{\underline{k}^2}G_{\underline{w},\underline{y}}(zs). \tag{7}$$

The dipole operator $G_{\underline{w},\underline{y}}(zs)$ is

$$G_{\underline{w},\underline{y}}(zs) = \frac{k_1^- p^+}{N_c}\mathrm{Re}\left\langle \mathrm{T\,tr}\left[V_{\underline{x}} V^{pol\dagger}_{\underline{w}}\right] + \mathrm{T\,tr}\left[V^{pol}_{\underline{w}} V^\dagger_{\underline{x}}\right]\right\rangle, \tag{8}$$

where the polarized Wilson line reads

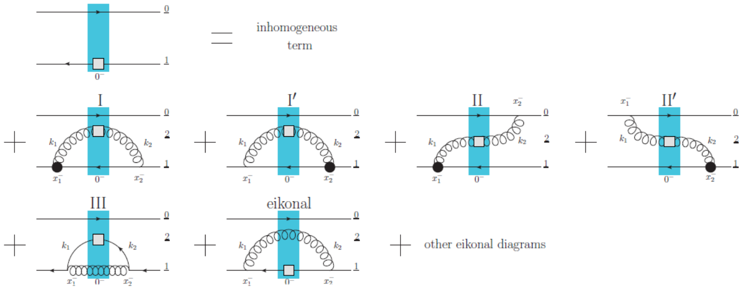
$$\begin{aligned} V^{pol}_{\underline{x}} &= ig\frac{p^+}{s}\int dx^- V_{\underline{x}}[\infty, x^-]F^{12}V_{\underline{x}}[x^-, -\infty] \\ &- g^2\frac{p^+}{s}\int dx_1^- \int_{x_1^-} dx_2^- V_{\underline{x}}[\infty, x_2^-]t^b\psi_\beta(x_2^-, \underline{x}U^{ba}_{\underline{x}}[x_2^-, x_1^-]\left[\tfrac{1}{2}\gamma^+\gamma^5\right]_{\alpha\beta}\bar{\psi}_\alpha(x_1^-, \underline{x})t^a V_{\underline{x}}[x_1^-, -\infty]. \end{aligned} \tag{9}$$

Situation prior to this contribution (3/3)

Remarks

- "Dressed dipoles" involve polarized WL. Obtained as sub-eikonal corrections to the scattering of a quark on a target.
- Corrections are proportional to $\sigma\delta_{\sigma\sigma'}$ in helicity basis (Brodsky-Lepage spinors in the minus direction).

Evolution (Involves the same WL at different coordinates $\longrightarrow \sigma\delta_{\sigma\sigma'}$)



Solve

Intercept in the pure glue case is $\alpha_h^q \sim 2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}$.

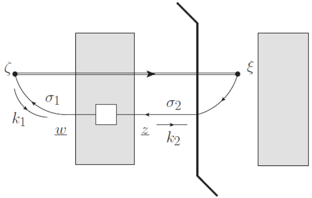
✗ Disagreement with BER pure glue intercept $\alpha_h^q \sim 3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}$

Let us start again from the quark helicity TMD:

$$
\begin{aligned}
g_{1L}^{q}(x,k_T^2) = & -\frac{2p^+}{(2\pi)^3}\int \mathrm{d}^2\zeta \mathrm{d}^2 w \mathrm{d}^2 z \frac{\mathrm{d}^2 k dk^-}{(2\pi)^3} e^{i\underline{k}_1\cdot(\underline{w}-\underline{\zeta})+i\underline{k}\cdot(\underline{z}-\underline{\zeta})}\theta(k_1^-)\sum_{\sigma_1,\sigma_2} \\
& \times \bar{v}_{\sigma_2}(k_2)\tfrac{1}{2}\gamma^+\gamma^5 v_{\sigma_1}(k_1) 2\sqrt{k_1^- k_2^-} \times \left\langle \mathrm{Ttr}\left[V_{\underline{\zeta}} V^{\dagger}_{\underline{z},\underline{w};\sigma_2,\sigma_1}\right]\right\rangle \\
& \times \left.\frac{1}{[2k_1^- xP^+ + \underline{k}_1 - i\epsilon k_1^-][2k_1^- xP^+ + \underline{k}^2 + i\epsilon k_1^-]}\right|_{k_2^-=k_1^-,\,\underline{k}_2=-\underline{k}} + c.c. \qquad (10)
\end{aligned}
$$

Remarks

- $V_{\underline{z},\underline{w};\sigma',\sigma}$ is the quark S-matrix for a quark-target scattering in helicity-basis.
- Allows for non locality before and after the shock wave.



Wilson lines and eikonal expansion

At sub-eikonal order:

$$V_{\underline{x},\underline{y};\sigma',\sigma} = V_{\underline{x}}\, \delta^2(\underline{x}-\underline{y})\, \delta_{\sigma,\sigma'} \tag{11}$$
$$+ \frac{i\,P^+}{s} \int_{-\infty}^{\infty} dz^- d^2z\; V_{\underline{x}}[\infty, z^-]\, \delta^2(\underline{x}-\underline{z}) \left[-\delta_{\sigma,\sigma'} \overleftarrow{D}^i D^i + g\,\sigma\, \delta_{\sigma,\sigma'}\, F^{12}\right](z^-, \underline{z})\, V_{\underline{y}}[z^-, -\infty]\, \delta^2(\underline{y}-\underline{z})$$
$$- \frac{g^2 P^+}{2\,s} \delta^2(\underline{x}-\underline{y}) \int_{-\infty}^{\infty} dz_1^- \int_{z_1^-}^{\infty} dz_2^-\, V_{\underline{x}}[\infty, z_2^-]\, t^b\, \psi_\beta(z_2^-, \underline{x})\, U_{\underline{x}}^{ba}[z_2^-, z_1^-] \left[\delta_{\sigma,\sigma'}\, \gamma^+ - \sigma\, \delta_{\sigma,\sigma'}\, \gamma^+\gamma^5\right]_{\alpha\beta}$$
$$\times\, \bar{\psi}_\alpha(z_1^-, \underline{x})\, t^a\, V_{\underline{x}}[z_1^-, -\infty],$$

Remarks

- Blue $\longrightarrow$ Already used in previous V^{pol}. Label *of the first kind*; notation $V^{pol\,[1]}$. Proportional to $\sigma\delta_{\sigma\sigma'}$.
- Red $\longrightarrow$ "NEW" (in our framework). Label *of the second kind*; notation $V^{pol\,[2]}$. Proportional to $\delta_{\sigma\sigma'}$.

Picture?

For the quark S-matrix at sub eikonal order, see also:

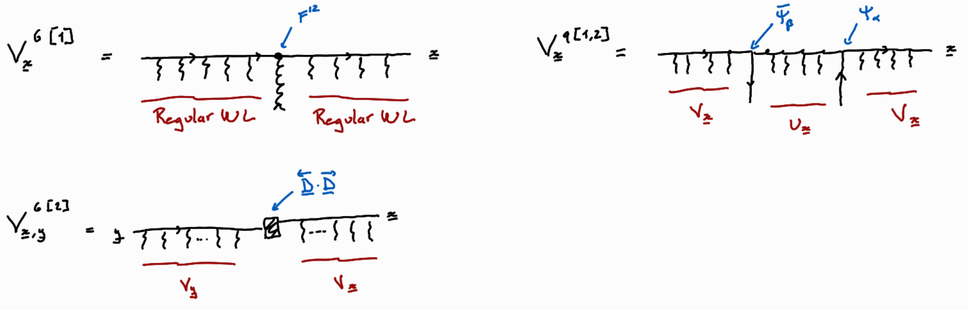
- Balitsky and Tarasov, e.g. [1505.02151]
- Chirilli, e.g. [1807.11435]
- Altinoluk et al., e.g. [2012.03886]
- Kovchegov et al., e.g. [1808.09010] [2108.03667]

Wilson lines and eikonal expansion - Pictures!

Polarized WL,

$$V_{\underline{x}}^{\mathrm{pol}[1]} = \underbrace{V_{\underline{x}}^{\mathrm{G}[1]} + V_{\underline{x}}^{\mathrm{q}[1]}}_{\sigma\,\delta_{\sigma\sigma'}}, \quad V_{\underline{x},\underline{y}}^{\mathrm{pol}[2]} = \underbrace{V_{\underline{x},\underline{y}}^{\mathrm{G}[2]} + V_{\underline{x}}^{\mathrm{q}[2]}\,\delta^2(\underline{x}-\underline{y})}_{\delta_{\sigma\sigma'}}.$$

can be represented as



Contraction with $(\gamma^+\gamma^5)_{\alpha\beta} \times \sigma\delta_{\sigma\sigma'}$ or $\gamma^+_{\alpha\beta} \times \delta_{\sigma\sigma'}$

Quark flavor-singlet helicity TMD - New dipole (2/2)

Simplified at small-x, the quark flavor-singlet helicity TMD reads

$$g_{1L}^{S}(x,k_T^2) = \frac{8\,N_c\,N_f}{(2\pi)^5}\int\limits_{\Lambda^2/s}^{1}\frac{dz}{z}\int d^2x_{10}\,e^{i\underline{k}\cdot\underline{x}_{10}}\left[i\,\frac{\underline{x}_{10}}{x_{10}^2}\cdot\frac{\underline{k}}{\underline{k}^2}\,\left[Q(x_{10}^2,zs)+G_2(x_{10}^2,zs)\right]\right.$$
$$\left.-\frac{(\underline{k}\times\underline{x}_{10})^2}{\underline{k}^2\,x_{10}^2}\,G_2(x_{10}^2,zs)\right], \qquad (12)$$

The new dipole G_2 is defined with

$$G_{10}^{j}(zs) \equiv \frac{1}{2N_c}\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{\zeta}}^{\dagger}\,V_{\underline{\xi}}^{j\;\mathrm{G}[2]} + \left(V_{\underline{\xi}}^{j\;\mathrm{G}[2]}\right)^{\dagger}\,V_{\underline{\zeta}}\right]\right\rangle\!\!\right\rangle \qquad (13)$$

$$\int d^2\left(\frac{x_1+x_0}{2}\right)\,G_{10}^{i}(zs) = (x_{10})_{\perp}^{i}\,G_1(x_{10}^2,zs) + \epsilon^{ij}\,(x_{10})_{\perp}^{j}\,G_2(x_{10}^2,zs). \qquad (14)$$

Remarks

- Dependence on previously used dipole $Q(x_{10}^2,zs)$.
- The previously missing dependence is proportional to $G_2(x_{10}^2,zs)$.

Evolution, revised and updated

One step of evolution reads the formal form

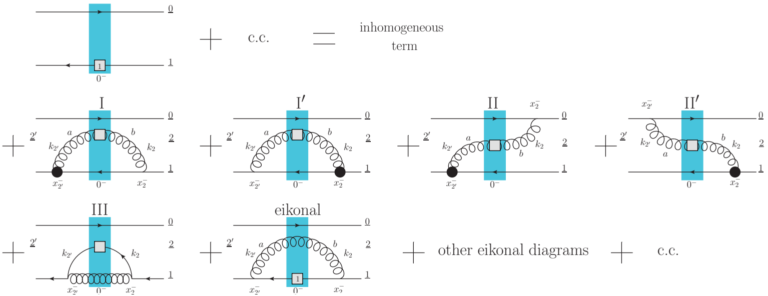
$$\hat{\mathcal{O}}_i = \hat{\mathcal{O}}_i^{(0)} + \sum_j \mathcal{K}_{ij} \otimes \hat{\mathcal{O}}_j \tag{15}$$

- Mixing to operators involving Wilson lines of first and/or second kind.
- Kernel involves transverse and longitudinal logarithmic integrals. The evolution is DLA, as opposed to the unpolarized one being SLA.
- Lifetime ordering is explicit $\theta(z\underline{x}_{10}^2 - z'\underline{x}_{21}^2)$.
- Similar to the Balitsky hierarchy, equation are not closed.
- Can be closed in the 't Hooft large N_c-limit or Veneziano large $N_c \& N_f$-limit.

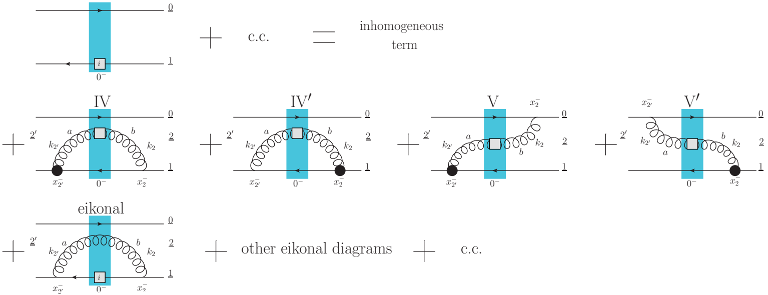
Results

- In the pure glue sector, the intercept becomes $\alpha_h^q \sim 3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}$. In complete agreement with BER result.
- Iterating this kernel, one recover the small-x spin-dependent DGLAP kernel.

Evolution, revised and updated - What is really looks like...



$$
\begin{aligned}
&\frac{1}{2N_c}\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{c.c.}\right\rangle\!\!\right\rangle(zs) = \frac{1}{2N_c}\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{c.c.}\right\rangle\!\!\right\rangle_0(zs) \qquad (95)\\
&+\frac{\alpha_s N_c}{2\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int d^2x_2\Bigg\{\left[\frac{1}{x_{21}^2}-\frac{\underline{x}_{21}}{x_{21}^2}\cdot\frac{\underline{x}_{20}}{x_{20}^2}\right]\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b V_{\underline{0}} t^a V_{\underline{1}}^{\dagger}\right]\left(U_{\underline{2}}^{\mathrm{pol}[1]}\right)^{ba} + \mathrm{c.c.}\right\rangle\!\!\right\rangle(z's)\\
&+\left[2\frac{\epsilon^{ij}\, x_{21}^j}{x_{21}^4}-\frac{\epsilon^{ij}\left(x_{20}^j+x_{21}^j\right)}{x_{20}^2\, x_{21}^2}-\frac{2\,\underline{x}_{20}\times\underline{x}_{21}}{x_{20}^2\, x_{21}^2}\left(\frac{x_{21}^i}{x_{21}^2}-\frac{x_{20}^i}{x_{20}^2}\right)\right]\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b V_{\underline{0}} t^a V_{\underline{1}}^{\dagger}\right]\left(U_{\underline{2}}^{i\,\mathrm{G}[2]}\right)^{ba} + \mathrm{c.c.}\right\rangle\!\!\right\rangle(z's)\Bigg\}\\
&+\frac{\alpha_s N_c}{4\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int \frac{d^2x_2}{x_{21}^2}\Bigg\{\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{2}}^{\mathrm{pol}[1]\,\dagger}\right] U_{\underline{1}}^{ba}\right\rangle\!\!\right\rangle(z's)+2\,\frac{\epsilon^{ij}\,x_{21}^j}{x_{21}^2}\,\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{2}}^{i\,\mathrm{G}[2]\,\dagger}\right] U_{\underline{1}}^{ba}\right\rangle\!\!\right\rangle(z's)+\mathrm{c.c.}\Bigg\}\\
&+\frac{\alpha_s N_c}{2\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int d^2x_2\,\frac{x_{10}^2}{x_{21}^2\, x_{20}^2}\Bigg\{\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] U_{\underline{2}}^{ba}\right\rangle\!\!\right\rangle(z's)-\frac{C_F}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right]\right\rangle\!\!\right\rangle(z's)+\mathrm{c.c.}\Bigg\}.
\end{aligned}
$$



$$
\begin{aligned}
&\frac{1}{2N_c}\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{i\,\mathrm{G}[2]\,\dagger}\right] + \mathrm{c.c.}\right\rangle\!\!\right\rangle(zs) = \frac{1}{2N_c}\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{i\,\mathrm{G}[2]\,\dagger}\right] + \mathrm{c.c.}\right\rangle\!\!\right\rangle_0(zs) \qquad (106)\\
&+ \frac{\alpha_s N_c}{4\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int d^2x_2 \Bigg\{ \left[\frac{\epsilon^{ij} x_{21}^j}{x_{21}^2} - \frac{\epsilon^{ij} x_{20}^j}{x_{20}^2} + 2x_{21}^i\,\frac{\underline{x}_{21}\times\underline{x}_{20}}{x_{21}^2\, x_{20}^2}\right]\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b V_{\underline{0}} t^a V_{\underline{1}}^{\dagger}\right]\left(U_{\underline{2}}^{\mathrm{pol}[1]}\right)^{ba} + \mathrm{c.c.}\right\rangle\!\!\right\rangle(z's)\\
&+ \left[\delta^{ij}\left(\frac{3}{x_{21}^2} - 2\frac{\underline{x}_{20}\cdot\underline{x}_{21}}{x_{20}^2\, x_{21}^2} - \frac{1}{x_{20}^2}\right) - 2\frac{x_{21}^i\, x_{20}^j}{x_{21}^2\, x_{20}^2}\left(2\frac{\underline{x}_{20}\cdot\underline{x}_{21}}{x_{20}^2} + 1\right) + 2\frac{x_{21}^i\, x_{21}^j}{x_{21}^2\, x_{20}^2}\left(2\frac{\underline{x}_{20}\cdot\underline{x}_{21}}{x_{21}^2} + 1\right) + 2\frac{x_{20}^i\, x_{20}^j}{x_{20}^4} - 2\frac{x_{21}^i\, x_{21}^j}{x_{21}^4}\right]\\
&\times \frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b V_{\underline{0}} t^a V_{\underline{1}}^{\dagger}\right]\left(U_{\underline{2}}^{j\,\mathrm{G}[2]}\right)^{ba} + \mathrm{c.c.}\right\rangle\!\!\right\rangle(z's)\Bigg\}\\
&+ \frac{\alpha_s N_c}{2\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int d^2x_2\, \frac{x_{10}^2}{x_{21}^2\, x_{20}^2}\left\{\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{1}}^{i\,\mathrm{G}[2]\,\dagger}\right]\left(U_{\underline{2}}\right)^{ba}\right\rangle\!\!\right\rangle(z's) - \frac{C_F}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{i\,\mathrm{G}[2]\,\dagger}\right]\right\rangle\!\!\right\rangle(z's) + \mathrm{c.c.}\right\}.
\end{aligned}
$$

A very last slide

A quick conclusion

- Small-x evolution equations for helicity distributions at DLA.
- Involve G_2 operator.
- Numerical agreement with the intercept found by BER.

Some Prospects

- Solving those equation in the Veneziano limit (oscillations?).
- Going beyond the DLA limit. Resumming IR-log, and thus interfacing with full spin-dependent DGLAP.
- Fixing h-JIMWLK.
- Phenomenology using the JAM framework.

Extra

- Gluon helicity and Lipatov vertex
- Propagator and Shockwave formalism
- Large N_c limit

From the Jaffe-Manohar (JM) gluon helicity PDF

$$\Delta G(x,Q^2) = \int^{Q^2} d^2k\, g_{1L}^{G\,dip}(x,k_T^2) = \frac{-2i}{x\,P^+}\frac{1}{4\pi}\frac{1}{2}\sum_{S_L} S_L \int_{-\infty}^{\infty} d\xi^-\, e^{ixP^+\xi^-}$$

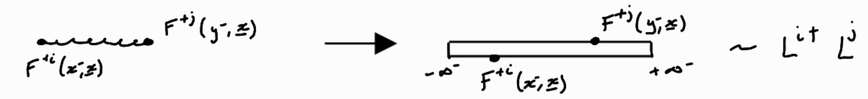
$$\times \langle P,S_L|\,\epsilon^{ij}\, F^{a+i}(0^+,0^-,\underline{0})\, U_{\underline{0}}^{ab}[0,\xi^-]\, F^{b+j}(0^+,\xi^-,\underline{0})\,|P,S_L\rangle\,, \qquad (16)$$

Identify after some algebra the dipole gluon helicity TMD

$$g_{1L}^{G\,dip}(x,k_T^2) = \frac{-2i}{x\,P^+\,V^-}\frac{1}{(2\pi)^3}\frac{1}{2}\sum_{S_L} S_L\ \langle P,S_L|\,\epsilon^{ij}\,\mathrm{tr}\Big[L^{i\,\dagger}(x,\underline{k})\,L^j(x,\underline{k})\Big]\,|P,S_L\rangle \qquad (17)$$

where we define the Lipatov vertex:

$$L^j(x,\underline{k}) \equiv \int_{-\infty}^{\infty} d\xi^-\, d^2\xi\, e^{ixP^+\xi^- - i\underline{k}\cdot\underline{\xi}}\, V_{\underline{\xi}}[\infty,\xi^-]\,(\partial^j A^+ + ixP^+A^j)\,V_{\underline{\xi}}[\xi^-,-\infty] \qquad (18)$$



Gluon helicity

Expanding the Lipatov vertex in eikonality (i.e. Bjorken x)

$$L^j(x,\underline{k}) = \int_{-\infty}^{\infty} d\xi^- \, d^2\xi \, e^{-i\underline{k}\cdot\underline{\xi}} \, V_{\underline{\xi}}[\infty,\xi^-] \left[\partial^j A^+ + ixP^+ \left(\xi^- \partial^j A^+ + A^j\right) + \mathcal{O}(x^2)\right] V_{\underline{\xi}}[\xi^-,-\infty], \tag{19}$$

which we can write

$$L^j(x,\underline{k}) = -\frac{k^j}{g} \int d^2\xi \, e^{-i\underline{k}\cdot\underline{\xi}} \, V_{\underline{\xi}} - \frac{xP^+}{2g} \int d^2\xi \, e^{-i\underline{k}\cdot\underline{\xi}} \int_{-\infty}^{\infty} dz^- \, V_{\underline{\xi}}[\infty,z^-] \left[D^j - \overleftarrow{D}^j\right] V_{\underline{\xi}}[z^-,-\infty] \tag{20}$$

Performing the helicity dependent "CGC average"

$$g_{1L}^{G\,dip}(x,k_T^2) = \frac{-4i}{g^2\,(2\pi)^3} \, \epsilon^{ij} \, k^i \int d^2\zeta \, d^2\xi \, e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} \underbrace{\left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{\zeta}}^\dagger \, V_{\underline{\xi}}^{j\;\mathrm{G}[2]} + \left(V_{\underline{\xi}}^{j\;\mathrm{G}[2]}\right)^\dagger V_{\underline{\zeta}}\right]\right\rangle\!\!\right\rangle}_{=2N_c G^j_{\underline{\xi},\underline{\zeta}}(zs)}, \tag{21}$$

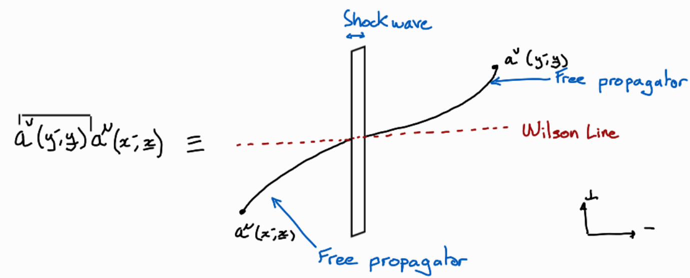
with a polarized Wilson line of the second kind (more on this soon)

$$V_{\underline{z}}^{i\;\mathrm{G}[2]} \equiv \frac{P^+}{2s} \int_{-\infty}^{\infty} dz^- \, V_{\underline{z}}[\infty,z^-] \left[D^i(z^-,\underline{z}) - \overleftarrow{D}^i(z^-,\underline{z})\right] V_{\underline{z}}[z^-,-\infty]. \tag{22}$$

$\Longrightarrow$ We call $G^j_{\underline{\xi},\underline{\zeta}}(zs)$ a Polarized dipole amplitude of the second kind.

Recipe:

- Split the background field A^μ into a new background A^μ and a quantum field a^μ (according to their longitudinal momentum fraction)
- Integrate out quantum fields a^μ.
- Require the propagator in the new background. Use shockwave approximation.
- Pull out the corresponding kernel for one step of evolution.



Remarks

- In our case, we go beyond eikonal approximation since helicity-dependence is a genuine subeikonal effect.
- Introduce Wilson line and polarized Wilson lines, up to subeikonal level.

Solving, 't Hooft limit - Color

In the large N_c-limit (drop quarks t-channel exchanges)

$$U_{\underline{x}}^{\mathrm{pol}[1]} \to U_{\underline{x}}^{\mathrm{G}[1]} \tag{23}$$

Replace adjoint WL using:

$$(U_{\underline{x}})^{ba} = 2\,\mathrm{tr}[t^b V_{\underline{x}} t^a V_{\underline{x}}^{\dagger}]. \tag{24}$$

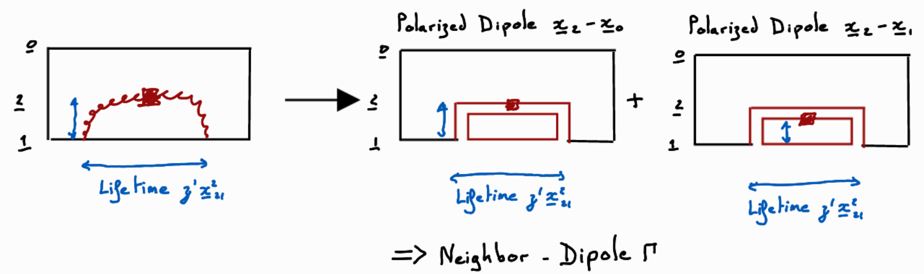
and

$$\left(U_{\underline{x}}^{\mathrm{G}[1]}\right)^{ba} = 2 \times \left\{2\,\mathrm{tr}\left[t^b\, V_{\underline{x}}\, t^a\, V_{\underline{x}}^{\mathrm{G}[1]\,\dagger}\right] + 2\,\mathrm{tr}\left[t^b\, V_{\underline{x}}^{\mathrm{G}[1]}\, t^a\, V_{\underline{x}}^{\dagger}\right]\right\}. \tag{25}$$

$$\left(U_{\underline{x}}^{i\,\mathrm{G}[2]}\right)^{ba} = 2\,\mathrm{tr}\left[t^b\, V_{\underline{x}}\, t^a\, V_{\underline{x}}^{i\,\mathrm{G}[2]\,\dagger}\right] + 2\,\mathrm{tr}\left[t^b\, V_{\underline{x}}^{i\,\mathrm{G}[2]}\, t^a\, V_{\underline{x}}^{\dagger}\right] \tag{26}$$

Notice the factor 2 in the former. A gluon has twice the spin of a quark.

Solving, 't Hooft limit - Lifetime



After Fiertzing arround, introduce neighbor dipole amplitude Γ to enforce lifetime ordering at each step of the evolution.

Solving, 't Hooft limit - Equation and intercept

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[\Gamma(x_{10}^2, x_{21}^2, z' s) + 3\, G(x_{21}^2, z' s) + 2\, G_2(x_{21}^2, z' s) + 2\, \Gamma_2(x_{10}^2, x_{21}^2, z' s) \Big], \tag{27a}$$

$$\Gamma(x_{10}^2, x_{21}^2, z' s) = G^{(0)}(x_{10}^2, z' s) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \Big[\Gamma(x_{10}^2, x_{32}^2, z'' s) + 3\, G(x_{32}^2, z'' s) + 2\, G_2(x_{32}^2, z'' s) + 2\, \Gamma_2(x_{10}^2, x_{32}^2, z'' s) \Big], \tag{27b}$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z' s}\right]}^{\min\left[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \Big[G(x_{21}^2, z' s) + 2\, G_2(x_{21}^2, z' s) \Big], \tag{27c}$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z' s) = G_2^{(0)}(x_{10}^2, z' s) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max\left[x_{10}^2, \frac{1}{z'' s}\right]}^{\min\left[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{32}^2}{x_{32}^2} \Big[G(x_{32}^2, z'' s) + 2\, G_2(x_{32}^2, z'' s) \Big]. \tag{27d}$$

Numerical solution for the intercept:

$$\Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim g_1(x, Q^2) \sim (1/x)^{3.66 \sqrt{\frac{\alpha_s N_c}{2\pi}}}. \tag{28}$$