Pseudo and quasi gluon PDF in the BFKL approximation

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Diffraction and Low-x 2022

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Based on

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- Pseudo and quasi quark PDF at Low-x (*in preparation*)

PDF on the Lattice

- Lattice gauge theory is formulated in Euclidean space
    - direct calculation of the PDFs would be impossible for objects that are defined through the light-cone matrix element of gauge-invariant bi-local operators

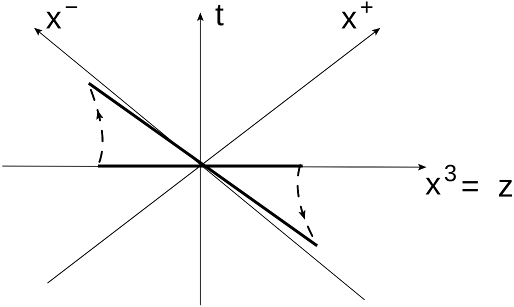
- Idea Consider equal-time correlators and perform the Lattice analysis in coordinate space through the Ioffe-time distributions.

- Fourier transform in momentum space
    - quasi-PDF X. Ji (2013)
    - pseudo-PDF A. Radyushkin (2017)

- Lattice calculations provide values of the Ioffe-time distributions for a limited range of the distance separating the bi-local operators. In order to perform the Fourier transform for the quasi-PDF or the pseudo-PDF, it is then necessary to extrapolate the large-distance behavior.

PDF on the Lattice

$$\langle P|G^{-i}(x^+)[x^+,0]G^{-i}(0)|P\rangle \quad \rightarrow \quad \langle P|G^{3i}(z)[z,0]G^{3i}(0)|P\rangle + \mathcal{O}\left(\frac{\Lambda^2}{(P^3)^2}\right)$$



$x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

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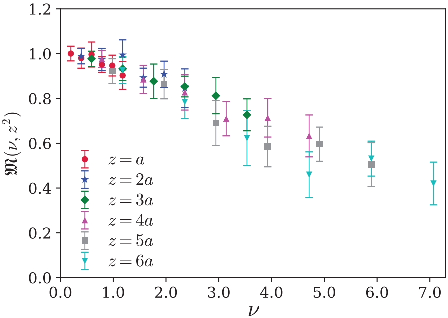


$$z = \{1, 2, 3, 4, 5, 6\} \times a = 0.094\,\text{fm},\ 0.188\,\text{fm},\ 0.282\,\text{fm},\ 0.376\,\text{fm},\ 0.470\,\text{fm},\ 0.564\,\text{fm}$$

Ioffe-time distribution at high energy

$$z_\mu\, z_\nu\, \langle P | G^{ai\mu}(z)[z,0] G_i^{b\,\nu}(0) | P \rangle = 2\, \varrho^2 \mathcal{M}_{pp}(\varrho, z^2)$$

Ioffe-time $\varrho \equiv z \cdot P$ z^μ space-like vector $i = 1, 2$

Tensor decomposition I. Balitsky, W. Morris, A. Radyuskin (2020)

Tensor decomposition over invariant amplitudes of the gluon matrix element

Tensor structures are build from P^μ, x^μ, and $g^{\mu\nu}$

$$\begin{aligned} M_{\mu\alpha;\lambda\beta} &\equiv \langle P|G_{\mu\alpha}(z)[z,0]G_{\lambda\beta}(0)|P\rangle \\ &= I_{1\mu\alpha;\lambda\beta}\mathcal{M}_{pp} + I_{2\mu\alpha;\lambda\beta}\mathcal{M}_{zz} + I_{3\mu\alpha;\lambda\beta}\mathcal{M}_{zp} \\ &\quad + I_{4\mu\alpha;\lambda\beta}\mathcal{M}_{pz} + I_{5\mu\alpha;\lambda\beta}\mathcal{M}_{ppzz} + I_{6\mu\alpha;\lambda\beta}\mathcal{M}_{gg} \end{aligned}$$

the amplitudes $\mathcal{M}$ are functions of the invariants z^2 and $z \cdot P = \varrho$ (Ioffe time)

light-cone Gluon distribution is obtained from

$$g_\perp^{\alpha\beta} M_{+\alpha;\beta_+}(z^+, P) = -2(P^-)^2\mathcal{M}pp$$

The PDF is determined by the $\mathcal{M}_{pp}$ structure

$$M_{+i;+i} = M_{0i;0i} + M_{3i;3i} + M_{0i;3i} + M_{3i;0i}$$

$$M_{0i;i0} + M_{ji;ij} = 2p_0^2\mathcal{M}_{pp} \xrightarrow{\text{high}-\text{energy}} M_{+i;+i}$$

At high energy (Regge limit) the transverse components are suppressed and we do not distinguish between the 0-component and the 3-component

Definition of the pseudo and quasi gluon PDF

Ioffe-time distribution at high energy

$$z_\mu\, z_\nu\, \langle P|G^{ai\mu}(z)[z,0]G_i^{b\,\nu}(0)|P\rangle = 2\,\varrho^2 \mathcal{M}_{pp}(\varrho, z^2)$$

Ioffe-time $\varrho \equiv z \cdot P$ z^μ space-like vector $i = 1, 2$

Pseudo-PDF: Fourier transform with respect to P keeping its orientation fixed

$$G_{\rm p}(x_B, z^2) = \int \frac{d\varrho}{2\pi}\, e^{-i\varrho x_B} \mathcal{M}_{pp}(\varrho, z^2)$$

Quasi-PDF: Fourier transform with respect to z keeping its orientation fixed

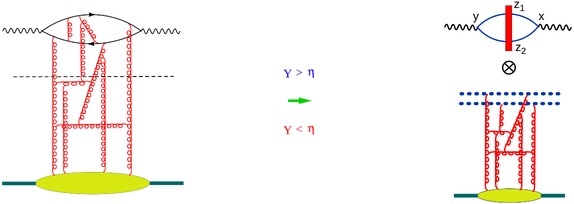
$$G_q(x_B, P_\xi) = P_\xi \int \frac{d\varsigma}{2\pi}\, e^{-i\varsigma\, P_\xi x_B} \mathcal{M}_{pp}(\varsigma P_\xi, \varsigma^2)$$

$\xi^\mu = \frac{z^\mu}{|z|}$ $P_\xi = P \cdot \xi$

DIS at Leading Log Approximation at high-energy

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\, I^{\text{LO}}_{\mu\nu}(z_1,z_2,x,y)\text{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

- Calculate LO Imapct factor: $I^{\text{LO}}_{\mu\nu}(z_1,z_2,x,y)$
- Calculate evolution of matrix element $\text{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$: BK/JIMWLK equation
  - we need only linear terms: BFKL
- Solve the evolution equation with initial condition: GBW/MV model
- Convolute the solution of the evolution equation with the impact factor



High-energy OPE for Ioffe-time distribution

$$\langle P|G^{a\,i-}(x)[x,0]G^{b,i-}(0)|P\rangle = \int d^2z_2 d^2z_z\, I_g(z_1,z_2;x)\langle P|\,\mathrm{Tr}\{U(z_1)U^\dagger(z_2)\}|P\rangle$$

$$\langle P|\bar{\psi}(x)\gamma^-[x,0]\psi(0)|P\rangle = \int d^2z_2 d^2z_z\, I_q(z_1,z_2;x)\langle P|\mathrm{tr}\{U(z_1)U^\dagger(z_2)\}|P\rangle$$

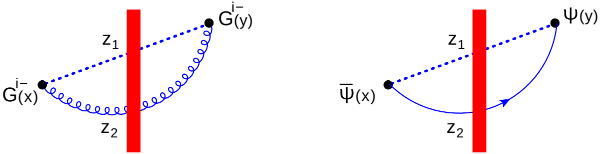
- Calculate coefficient functions (impact factors) I_g and I_q
- Convolute them with the solution of the evolution equation of relative matrix elements

High-energy OPE for Ioffe-time distribution

$$\langle P|G^{a\,i-}(x)[x,0]G^{b,i-}(0)|P\rangle = \int d^2z_2 d^2z_z\, I_g(z_1,z_2;x)\langle P|\,\mathrm{Tr}\{U(z_1)U^\dagger(z_2)\}|P\rangle$$

$$\langle P|\bar\psi(x)\gamma^-[x,0]\psi(0)|P\rangle = \int d^2z_2 d^2z_z\, I_q(z_1,z_2;x)\langle P|\mathrm{tr}\{U(z_1)U^\dagger(z_2)\}|P\rangle$$

Diagrams for the gluon impact factor I_g and quark impact factor I_q respectively

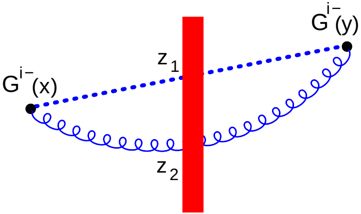


- Gluon: Tr   trace in the adjoint representation;
- Quark: tr   trace in the fundamental representation.

High-energy operator product expansion

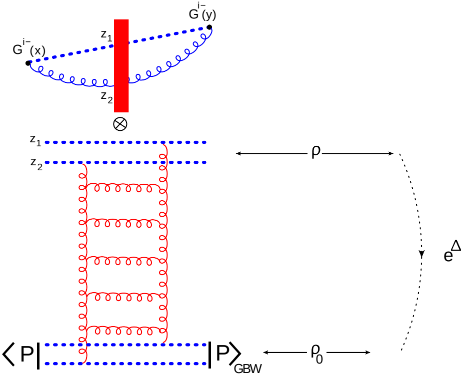
At high-energy we do not distinguish between the 0 and 3 components

$$\langle P|G^{ai-}(z)[z,0]G_i^{b\,-}(0)|P\rangle = 2(P^-)^2\mathcal{M}_{pp}(\varrho, z^2)$$

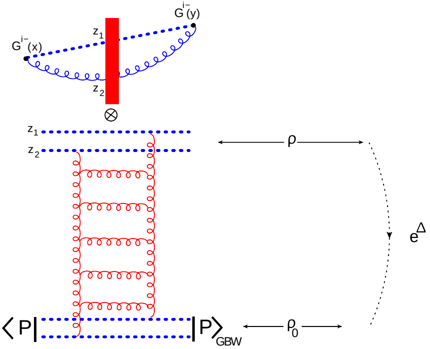


High-energy operator product expansion formalism is formulated in coordinate space ⇒ is suitable to reach our goal.

High-energy operator product expansion



High-energy operator product expansion

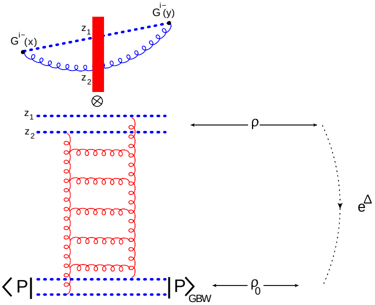


Resum $\alpha_s \ln \varrho$ with BFKL eq. $\quad a = -\frac{2x^+y^+}{(x-y)^2 a_0}$

$$2a\frac{d}{da}\,\mathcal{V}_a(z_\perp) = \frac{\alpha_s N_c}{\pi^2}\int d^2z'\Big[\frac{\mathcal{V}_a(z'_\perp)}{(z-z')^2_\perp} - \frac{(z,z')_\perp \mathcal{V}_a(z_\perp)}{z'^2_\perp (z-z')^2_\perp}\Big]$$

$\frac{1}{z^2_\perp}\mathcal{U}(z_\perp) \equiv \mathcal{V}(z_\perp) \qquad \mathcal{U}(x_\perp, y_\perp) = 1 - \frac{1}{N_c}\mathrm{tr}\{U(x_\perp)U^\dagger(y_\perp)\}$

High-energy operator product expansion

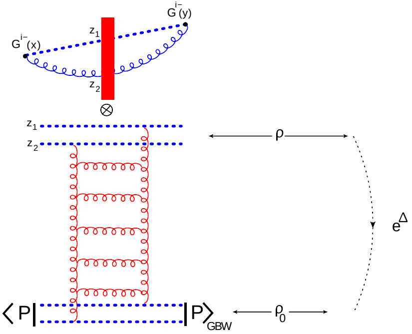


Resum $\alpha_s \ln \varrho$ with BFKL eq. $a = -\frac{2x^+y^+}{(x-y)^2 a_0}$

$$2a\frac{d}{da}\,\mathcal{V}_a(z_\perp) = \frac{\alpha_s N_c}{\pi^2}\int d^2z'\Big[\frac{\mathcal{V}_a(z'_\perp)}{(z-z')^2_\perp} - \frac{(z,z')_\perp \mathcal{V}_a(z_\perp)}{z'^2_\perp (z-z')^2_\perp}\Big]$$

solution $$\mathcal{V}^a(z_{12}) = \int \frac{d\nu}{2\pi^2}(z_{12}^2)^{-\frac{1}{2}+i\nu}\left(\frac{a}{a_0}\right)^{\frac{\aleph(\gamma)}{2}} \int d^2\omega(\omega_\perp^2)^{-\frac{1}{2}-i\nu}\mathcal{V}^{a_0}(\omega_\perp)$$

Ioffe-time distribution in the saddle-point approximation



Saddle point approximation

$$\mathcal{M}_{pp}(\varrho, z^2) = \frac{3N_c^2}{128\,\varrho}\,\frac{Q_s\,\sigma_0}{|z|}\left(\frac{2\,\varrho^2}{z^2M_N^2}+i\epsilon\right)^{\bar{\alpha}_s 2\ln 2}\frac{e^{-\frac{\ln^2\frac{Q_s|z|}{2}}{7\zeta(3)\bar{\alpha}_s\ln\left(\frac{2\,\varrho^2}{z^2M_N^2}+i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s\ln\left(\frac{2\,\varrho^2}{z^2M_N^2}+i\epsilon\right)}}$$

saturation scale Q_s , $\sigma_0 = 29.1$ mb , M_N mass of the nucleon

Pseudo-PDF vs. Quasi-PDF in the Saddle point approximation

$$G_{\rm p}(x_B,z^2)\simeq -i\frac{3N_c^2}{256}\frac{Q_s\sigma_0}{|z|}\,\frac{\text{sign}(x_B)\,e^{\frac{-\ln^2\frac{Q_s|z|}{2}}{7\zeta(3)\bar{\alpha}_s\ln\left(\frac{2}{x_B^2z^2M_N^2}+i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s\ln\left(\frac{2}{x_B^2z^2M_N^2}+i\epsilon\right)}}\left(\frac{2}{x_B^2z^2M_N^2}+i\epsilon\right)^{\bar{\alpha}_s 2\ln 2}$$

$$G_{\rm q}(x_B,P_\xi)\simeq -\frac{3N_c^2}{256}Q_s\sigma_0P_\xi|x_B|\,\frac{e^{-\frac{\ln^2\frac{Q_s}{2P_\xi|x_B|}}{7\bar{\alpha}_s\zeta(3)\ln\left(-\frac{2P_\xi^2}{M_N^2}+i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s\ln\left(-\frac{2P_\xi^2}{M_N^2}+i\epsilon\right)}}\left(-\frac{2P_\xi^2}{M_N^2}+i\epsilon\right)^{\bar{\alpha}_s 2\ln 2}$$

Light-ray operator

Analytic continuation of light-ray operators at $j = 1$

$$F^a_{\xi +}(x)\nabla_+^{j-2}F^{a\ \xi}_{+}(x)\Big|_{x=0} = \frac{\sin[\pi(j-1)]}{\pi[\Gamma(j-1)]^{-1}}\int_0^{+\infty} du\ u^{1-j}\, F^a_{\xi +}(0)[0,un]^{ab}F^{b\ \xi}_{+}(un)$$

OPE in light-ray operators in QCD (Balitsky, Braun (1989))

2-point function in BFKL limit (Balitsky; Balitsky, Kazakov, Sobkov (2013-2018))

2-point function in triple Regge limit (Balitsky 2018)

A lot of activity on light-ray operators in CFT (e.g. Kravchuk, Simmons-Duffin (2018))

Leading and next-to-leading twist

Analytic continuation of local-operator $\Rightarrow$ light-ray operators

$$F^{a}_{p_1\xi}(x)\nabla^{j-2}F^{a\;\xi}_{p_1}(x)\Big|_{x=0} \overset{\text{forw.}}{=} \frac{1}{\Gamma(2-j)}\int_0^\infty dv\, v^{1-j}\, F^{a}_{p_1\xi}(0)[0,vp_1]^{ab}F^{b\;\xi}_{p_1}(vp_1)$$

$\omega = j-1 \to 0 \;\; \Leftrightarrow \;\; x_B \to 0$ at $\frac{\alpha_s}{\omega} \sim 1 \;\; \Rightarrow$ resummation: BFKL eq.

To get the leading and next-to-leading residues we need to approach the DGLAP limit $\alpha_s \ll \omega \ll 1$

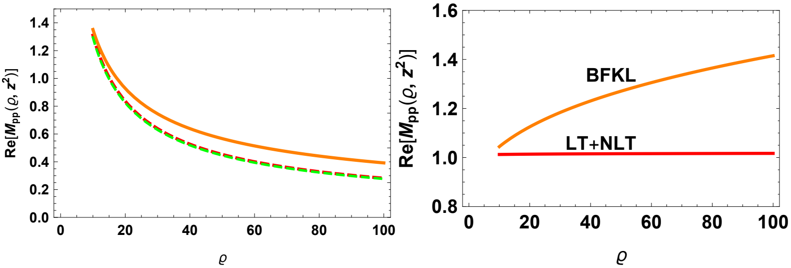
Leading and next-to-leading twist for the Ioffe-time-distribution

$$\mathcal{M}_{pp}(\varrho,z^2) = \frac{N_c^2}{8\pi^2\bar\alpha_s}\frac{Q_s^2\sigma_0}{\varrho}\left(\frac{4\bar\alpha_s\left|\ln\frac{Q_s|z|}{2}\right|}{\ln\left(\frac{2\,\varrho^2}{z^2M_N^2}+i\epsilon\right)}\right)^{\frac{1}{2}} I_1(\tilde t)\left(1+\frac{Q_s^2|z|^2}{5}\right) + O\left(\frac{Q_s^4|z|^4}{16}\right)$$

with

$$\tilde t = \left[4\bar\alpha_s\left|\ln\frac{Q_s|z|}{2}\right|\ln\left(\frac{2\,\varrho^2}{z^2M_N^2}+i\epsilon\right)\right]^{\frac{1}{2}}$$

Ioffe-time distribution at large-longitudinal distances



- Left panel
  - Orange curve is the BFKL resummation
  - Green-dash and red-dash are the LT and LT+NLT respectively.

- Right panel: BFKL resummation (Orange) and LT+NLT (red) both normalized to the LT.

Pseudo gluon PDF

BFKL resummation

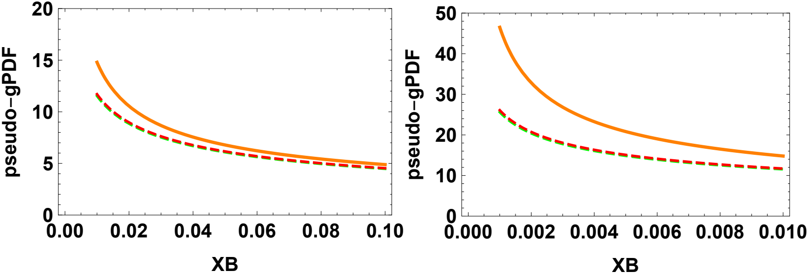
$$G_{\mathrm{p}}(x_B, z^2) = -i\frac{3N_c^2}{256}\frac{Q_s\sigma_0}{|z|}\frac{\mathrm{sign}(x_B)\, e^{\frac{-\ln^2\frac{Q_s|z|}{2}}{7\zeta(3)\bar{\alpha}_s\ln\left(\frac{2}{x_B^2 z^2 M_N^2}+i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s\ln\left(\frac{2}{x_B^2 z^2 M_N^2}+i\epsilon\right)}}\left(\frac{2}{x_B^2 z^2 M_N^2}+i\epsilon\right)^{\bar{\alpha}_s 2\ln 2}$$

Leading and next-to-leading twist

$$G_{\mathrm{p}}(x_B, z^2) = \frac{N_c^2 Q_s^2\sigma_0}{16\pi^3\bar{\alpha}_s}\left(1+\frac{Q_s^2|z|^2}{5}\right)I_0(h) + O\left(\frac{Q_s^4|z|^4}{16}\right)$$

$$h = \left[2\bar{\alpha}_s\left|\ln\frac{4}{|z|^2 Q_s^2}\right|\ln\frac{2}{x_B^2|z^2|M_N^2}\right]^{\frac{1}{2}}$$

Pseudo gluon PDF



Pseudo-PDF have typical behavior of gluon distribution at low-x_B.

quasi gluon PDF

BFKL resumation $\aleph(\gamma) \equiv \frac{\alpha_s N_c}{\pi}\Big(2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\Big) \qquad \gamma = \frac{1}{2} + i\nu$

$$G_{\rm q}(x_B, P_\xi) \simeq -\frac{3N_c^2}{256} Q_s \sigma_0 P_\xi |x_B| \frac{e^{-\frac{\ln^2 \frac{Q_s}{2P_\xi |x_B|}}{7\bar\alpha_s \zeta(3) \ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar\alpha_s \ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)}} \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)^{\bar\alpha_s 2\ln 2}$$

Leading + next-to-leading twist

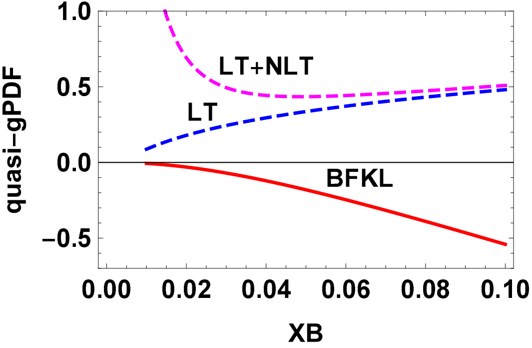
$$G_{\rm q}(x_B, P_\xi) \simeq -\frac{N_c^2\, Q_s^2\, \sigma_0}{16\bar\alpha_s^2 \pi^3} \frac{1}{2\pi i} \int_{1-i\infty}^{1+i\infty} d\omega \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)^{\frac{\omega}{2}} \left(-\frac{4P_\xi^2 x_B^2}{Q_s^2} + i\epsilon\right)^{\frac{\bar\alpha_s}{\omega}} \left(\omega + \frac{2\bar\alpha_s Q_s^2}{5} \frac{1}{P_\xi^2 x_B^2}\right)$$

Usual exponentiation of the BFKL pomeron intercept, which resums logarithms of x_B, is missing.

For low values of x_B and fixed values of P these corrections are enhanced rather than suppressed at this regime.

quasi gluon PDF

Here $P_\xi = 4$ GeV.



Behavior of curves will not change even for values of $P_\xi = 100\,$GeV.

Quasi-PDF have rather unusual behavior at low-x_B.

The usual exponentiation of the BFKL pomeron intercept, which resums logarithms of x_B, is missing.

Conclusions

- Large-distance behavior of the gluon Ioffe-time distribution is computed
  - Ioffe-time ϱ acts as rapidity parameter.
    - ★ $\alpha_s \ln \varrho$ resumed by BFKL eq.
  - Ioffe-time distribution is a very slowly varying function at large values of ϱ.
- Pseudo-PDF and quasi-PDF have a very different behavior at low-x_B.
  - pseudo-PDF have typical behavior of gluon distribution at low-x_B.
  - quasi-PDF have rather unusual behavior at low-x_B.
    - ★ usual exponentiation of the BFKL pomeron intercept, which resums logarithms of x_B, is missing.
- The power corrections in the quasi-PDF do not come in as inverse powers of P but as inverse powers of $x_B P$
  - for low values of x_B and fixed values of P these corrections are enhanced rather than suppressed at this regime.

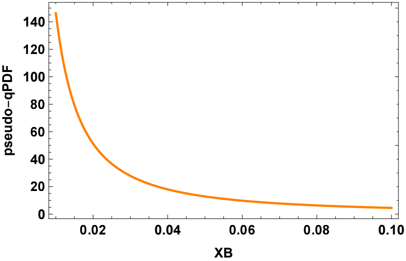
Conclusions

- The physical origin of the difference between the two distributions lay in the two different Fourier transforms under which they are defined.
- pseudo-PDF case
  - the scale is the resolution that is, the square of the length of the gauge link separating the bi-local operator.
- quasi-PDF case
  - the scale is the energy that is, the momentum of the hadronic target (the nucleon) projected along the direction of the gauge link.

Outlook

- Pseudo- and quasi-quark PDF in the BFKL approximation

Preliminary result for pseudo quark-PDF



The quasi quark PDF seems to be even more problematic than the gluon quasi-PDF.

n-th moment of the structure function

The Q^2 behavior of DIS structure function is obtained from the anomalous dimension of twist-two operators

$$\mu\frac{d}{\mu}F^a_{\xi +}\nabla_+^{n-2}F^{a\ \xi}_{+} = \gamma(\alpha_s, n)F^a_{\xi +}\nabla_+^{n-2}F^{a\ \xi}_{+}$$

Dipole DIS cross-section can be written as

$$\sigma^{\gamma^* p}(x_B, Q^2) = \int d\nu\, F(\nu)\, x_B^{-\aleph(\nu)-1}\left(\frac{Q^2}{P^2}\right)^{\frac{1}{2}+i\nu}$$

$-q^2 = Q^2 \gg P^2$, and $s = (P+q)^2 \gg Q^2$

$\aleph(\gamma)$ BFKL pomeron intercept.

The n-th moment of the structure function is

$$\int_0^1 dx_B\, x_B^{n-1}\sigma^{\gamma^* p}(x_B, Q^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\gamma\, \frac{F(\gamma)}{n-1-\aleph(\gamma)}\left(\frac{Q^2}{P^2}\right)^{\gamma}$$

Integrating over γ-parameter we get the anomalous dimensions of the leading and higher twist operators at the *unphysical point* $n = 1$.

Analytic continuation

$$\int_0^1 dx_B\, x_B^{n-1} \sigma^{\gamma^* p}(x_B, Q^2) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\gamma\, \frac{F(\gamma)}{\omega - \aleph(\gamma)} \left(\frac{Q^2}{P^2}\right)^{\gamma}$$

Analytic continuation: $n-1 \to \omega$ complex continuous variable

$\Rightarrow$ Residues $\omega = \aleph(\gamma)$; expand $\aleph(\gamma)$ for small γ and solve for γ

$$\gamma(\alpha_s, \omega) = \frac{\alpha_s N_c}{\pi\omega} + \mathcal{O}(\alpha_s^2), \quad F(\omega, Q^2) \sim \left(\frac{Q^2}{P^2}\right)^{\frac{\alpha_s N_c}{\pi\omega}}$$

Thus, we get the analytic continuation of anomalous dimension at the *unphysical point* $j \to 1$ of twist-2 gluon operator $F^a_{\xi+} \nabla^{-1} F^{\xi a}{}_{+}$

LT and NLT quasi-PDF: analytic expression

$\ln \frac{4P_\xi^2 x_B^2}{Q_s^2} < 0$

$$G_{\mathrm{q}}(x_B, P_\xi) \simeq \frac{N_c^2\, Q_s^2\, \sigma_0}{16\bar{\alpha}_s \pi^3} \left[\frac{\ln\left(-\frac{Q_s^2}{4P_\xi^2 x_B^2} - i\epsilon\right)}{\ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)} \left(J_0(m) - J_2(m) - \frac{2}{m} J_1(m) \right) \right.$$
$$\left. + \frac{2Q_s^2}{5P_\xi^2 x_B^2} \left(\frac{2\bar{\alpha}_s \ln\left(-\frac{Q_s^2}{4P_\xi^2 x_B^2} - i\epsilon\right)}{\ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)} \right)^{\frac{1}{2}} J_1(m) \right]$$

$$m \equiv \left[2\bar{\alpha}_s \ln\left(-\frac{Q_s^2}{4P_\xi^2 x_B^2} - i\epsilon\right) \ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right) \right]^{\frac{1}{2}}$$

LT and NLT quasi-PDF: analytic expression

$\ln \frac{4P_\xi^2 x_B^2}{Q_s^2} < 0$

$$G_q(x_B, P_\xi) \simeq -\frac{N_c^2 Q_s^2 \sigma_0}{16\bar{\alpha}_s \pi^3}\left[\frac{\ln\left(-\frac{4P_\xi^2 x_B^2}{Q_s^2} + i\epsilon\right)}{\ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)}\left(I_0(\tilde{m}) + I_2(\tilde{m}) - \frac{2}{\tilde{m}} I_1(\tilde{m})\right)\right.$$
$$\left. + \frac{2Q_s^2}{5P_\xi^2 x_B^2}\left(\frac{2\bar{\alpha}_s \ln\left(-\frac{4P_\xi^2 x_B^2}{Q_s^2} + i\epsilon\right)}{\ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)}\right)^{\frac{1}{2}} I_1(\tilde{m})\right]$$

$$\tilde{m} \equiv \left[2\bar{\alpha}_s \ln\left(-\frac{4P_\xi^2 x_B^2}{Q_s^2} + i\epsilon\right)\ln\left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)\right]^{\frac{1}{2}}$$

Light-ray operator

Analytic continuation of light-ray operators at $j = 1$

$$F^a_{\xi+}(x)\nabla_+^{j-2}F^{a\,\xi}_{+}(x)\Big|_{x=0} = \frac{\Gamma(2-j)}{2\pi i}\int_0^{+\infty} du\ u^{1-j}\,F^a_{\xi+}(0)[0,un]^{ab}F^{b\,\xi}_{+}(un)$$

OPE in light-ray operators in QCD (Balitsky, Braun (1989))

2-point function in BFKL limit (Balitsky; Balitsky, Kazakov, Sobkov (2013-2018))

2-point function in triple Regge limit (Balitsky 2018)

A lot of activity on light-ray operators in CFT (e.g. Kravchuk, Simmons-Duffin (2018))

Super-multiplet of local operators in $\mathcal{N}$=4 SYM

$$\mathcal{O}^j_\phi(x_\perp) = \int du\, \bar\phi^a_{AB} \nabla^j_- \phi^{ABa}(up_1 + x_\perp)$$
$$\mathcal{O}^j_\lambda(x_\perp) = \int du\, i\bar\lambda^a_A \nabla^{j-1}_- \lambda^a_A(up_1 + x_\perp)$$
$$\mathcal{O}^j_g(x_\perp) = \int du\, F^{a+}{}_i \nabla^{j-2}_- F^{a+i}(up_1 + x_\perp)$$

Multiplicatively renormalizable operators

$$S^j_1 = \mathcal{O}^j_g + \frac{1}{4}\mathcal{O}^j_\lambda - \frac{1}{2}\mathcal{O}^j_\phi$$
$$S^j_2 = \mathcal{O}^j_g - \frac{1}{4(j-1)}\mathcal{O}^j_\lambda + \frac{j+1}{6(j-1)}\mathcal{O}^j_\phi$$
$$S^j_3 = \mathcal{O}^j_g - \frac{j+2}{2(j-1)}\mathcal{O}^j_\lambda - \frac{(j+1)(j+2)}{2j(j-1)}\mathcal{O}^j_\phi$$

with anomalous dimensions

$$\gamma^{S_1}_j = 4[\psi(j-1) + \gamma_E] + O(\alpha_s^2), \quad \gamma^{S_2}_j = \gamma_{j+2}, \quad \gamma^{S_3}_j = \gamma^{S_1}_{j+4}$$

A. V. Belitsky, *et al* (2004)

Analytical continuation of local super-multiplet

$$\mathcal{F}^j(x_\perp) = \int_0^\infty du\, u^{1-j}\mathcal{F}(up_1 + x_\perp)\,,$$
$$\Lambda^j(x_\perp) = \int_0^\infty du\, u^{-j}\Lambda(up_1 + x_\perp)\,,$$
$$\Phi^j(x_\perp) = \int_0^\infty du\, u^{-1-j}\Phi(up_1 + x_\perp)$$

with

$$\mathcal{F}(up_1, x_\perp) = \int dv\, F^{a-}{}_{\mu}(up_1 + vp_1 + x_\perp)[u+v, v]_x^{ab} F^{b-\mu}(vp_1 + x_\perp)\,,$$
$$\Lambda(up_1, x_\perp) = \frac{i}{2}\int dv\Big(- \bar{\lambda}_A^a(up_1 + vp_1 + x_\perp)[u+v, v]_x^{ab}\sigma_-\lambda_A^b(vp_1 + x_\perp)$$
$$+\bar{\lambda}_A^a(vp_1 + x_\perp)[v, u+v]_x^{ab}\sigma_-\lambda_A^b(up_1 + vp_1 + x_\perp)\Big)\,,$$
$$\Phi(u, x_\perp) = \int dv\, \phi_I^a(up_1 + vp_1 + x_\perp)[u+v, v]_x^{ab}\phi_I^b(vp_1 + x_\perp)$$

I. Balitsky, V. Kazakov, and E. Sobko (2013)

multiplicatively renorm. light-ray operators

Forward matrix elements

$$\begin{aligned}
\mathcal{S}_1^j &= \mathcal{F}^j + \frac{j-1}{4}\Lambda^j - j(j-1)\frac{1}{2}\Phi^j\,,\\
\mathcal{S}_2^j &= \mathcal{F}^j - \frac{1}{4}\Lambda^j + \frac{j(j+1)}{6}\Phi^j\,,\\
\mathcal{S}_3^j &= \mathcal{F}^j - \frac{j+2}{2}\Lambda^j - \frac{(j+1)(j+2)}{2}\Phi^j\,.
\end{aligned}$$

Notice the different coefficients between the S-operators and the $\mathcal{S}$-operators.

Correlation function in CFT at high-energy, $j \to 1$

$$\langle \mathcal{F}^j(x_\perp)\mathcal{F}^{j'}(y_\perp)\rangle = \langle \mathcal{S}_1^j(x_\perp)\mathcal{S}_1^{j'}(y_\perp)\rangle \overset{\text{CFT}}{=} \delta(j-j')\frac{C(\Delta,j)s^{j-1}}{[(x-y)_\perp^2]^{\Delta-1}}\mu^{-2\gamma_a}$$

Δ: canonical dimension d plus anomalous dim. γ_a
μ: normalization point.
$C(\Delta,j)$: unknown structure constant. Calculate it in the BFKL limit.

Wilson frame vs quasi-pdf frame

In the BFKL limit the two-point correlation function is UV divergent.

Regularization: point splitting ⇒

- Wilson frame   Balitsky (2013, 2019), Balitsky, Kazhakov, Sobko (20013-2018)
  - Motivation: Give an example of actual calculation of correlation function; goal: understanding full dynamics of $\mathcal{N}= 4$ SYM.

- quasi-pdf frame   G.A.C. *Quark and Gluon quasi-pdf at low-x*
  - Motivation: check of the calculation comparing with expected CFT general result; goal: calculate the behavior of the quasi-pdf at small-x$_B$.