PDF uncertainties at small x

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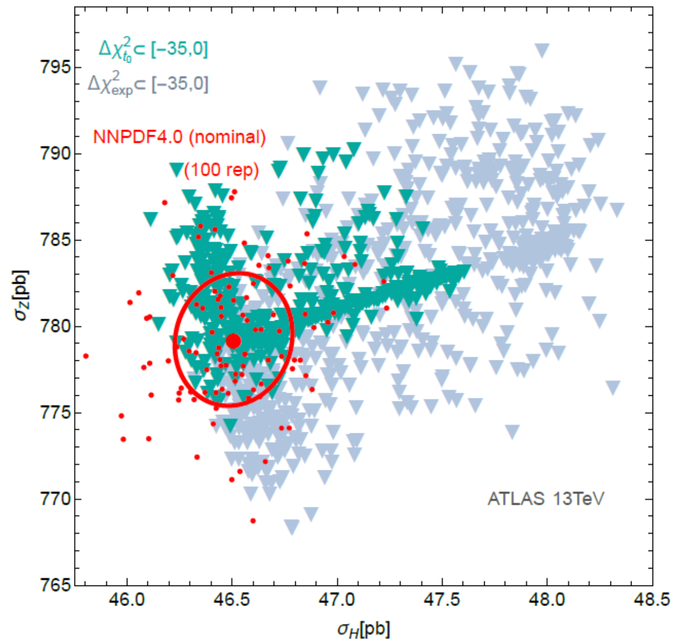
With A. Courtoy, J. Huston, K. Xie, M. Yan, C.-P. Yuan

arXiv: [2205.10444](https://arxiv.org/abs/2205.10444)

[v. 3 – with new figures and discussion]

And with

CTEQ-TEA (Tung Et. Al.) working group

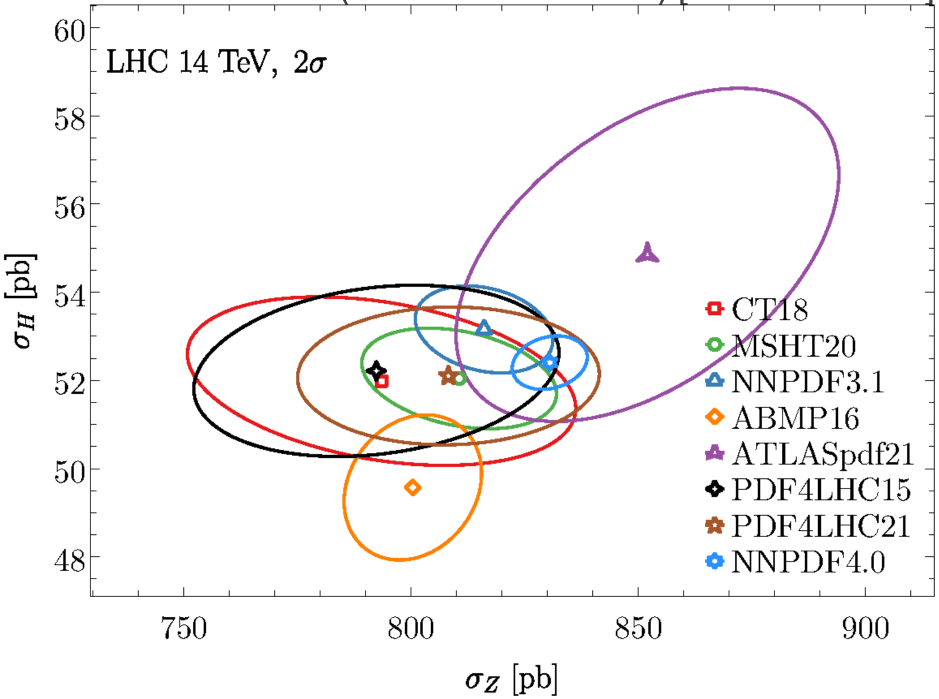


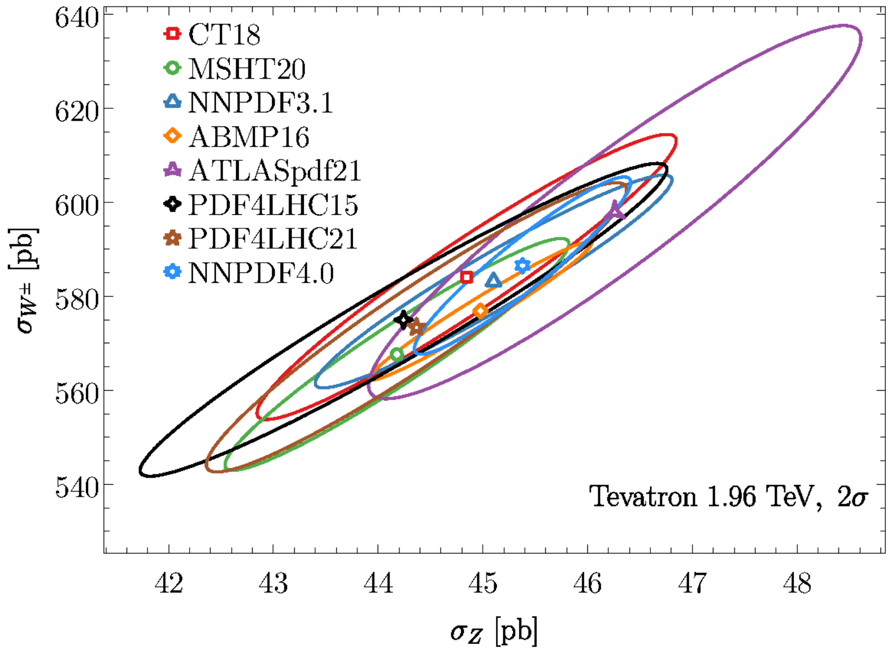
The tolerance puzzle

Courtoy, Huston, Nadolsky, Xie, Yan, Yuan, arXiv: 2205.10444

Why do groups fitting similar data sets obtain different PDF uncertainties?

Precision PDFs (Snowmass 21 WP) [2203.13923v2]





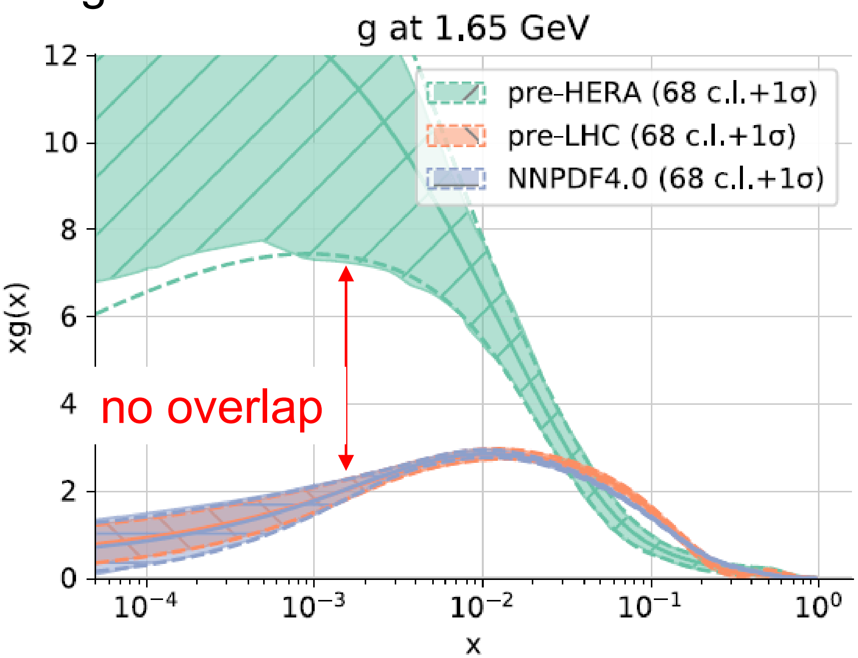
The answer has direct implications for high-stake experiments such as W boson mass measurement, tests of nonperturbative QCD models and lattice QCD, high-mass BSM searches, etc. Especially relevant in forward scattering processes.

What, exactly, did HERA do for us?

Evidence for non-trivial small-x dynamics depends on the uncertainty definitions

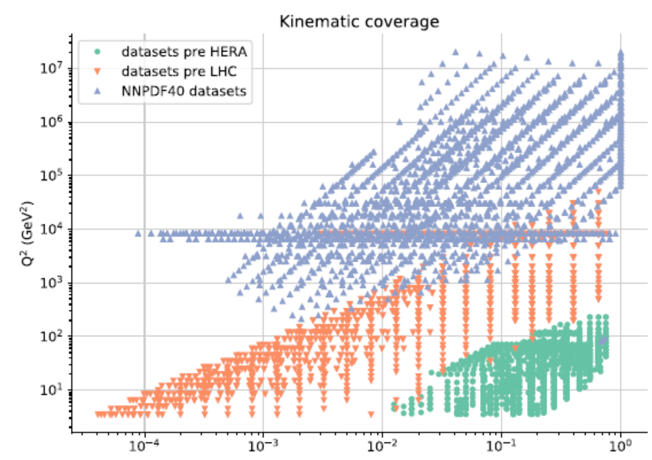
Example: a future test in NNPDF4.0

Fig. 28 in 2109.02653



Historically, HERA was credited for establishing the fast small-x growth of the gluon (hard pomeron), not reducing the growth.

Which view is right?

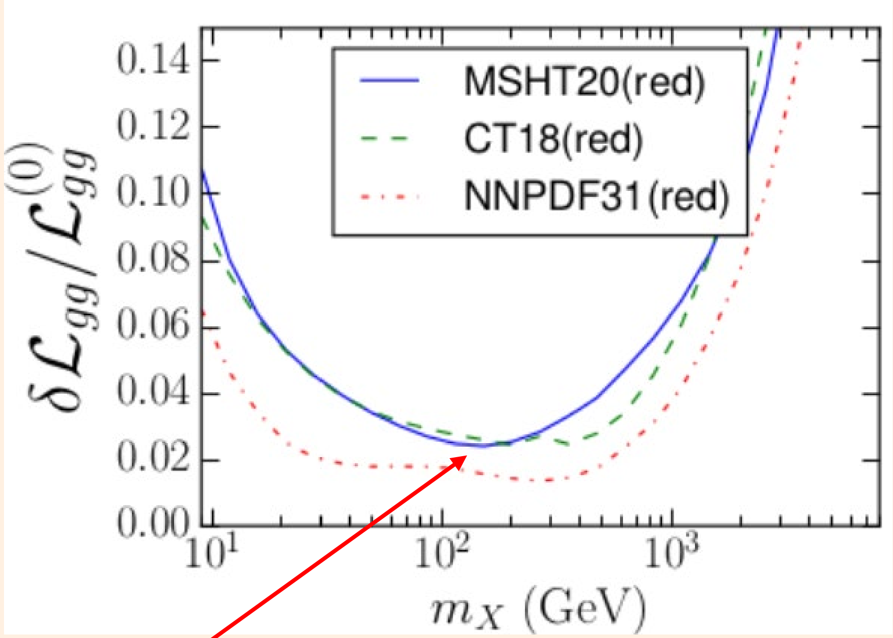


A fit only to the pre-HERA DIS & DY data prefers fast growth of the gluon at $x \to 0$, possibly reflecting a tension of BCDMS and NMC data. The growth is **reduced** by including the HERA data.

The tolerance puzzle

Relative PDF uncertainties on the gg luminosity at 14 TeV in three PDF4LHC21 fits to the **identical** reduced global data set

arXiv:2203.05506



× 1.5 – 2 difference

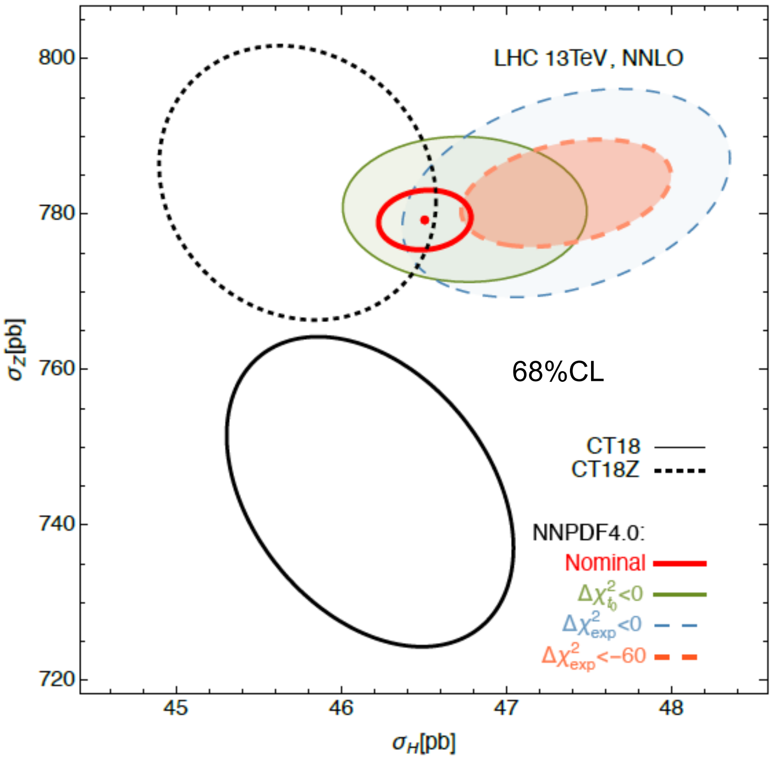
While the fitted data sets are identical or similar in several such analyses, the resulting PDF sets may differ because of methodological choices adopted by the PDF fitting groups.

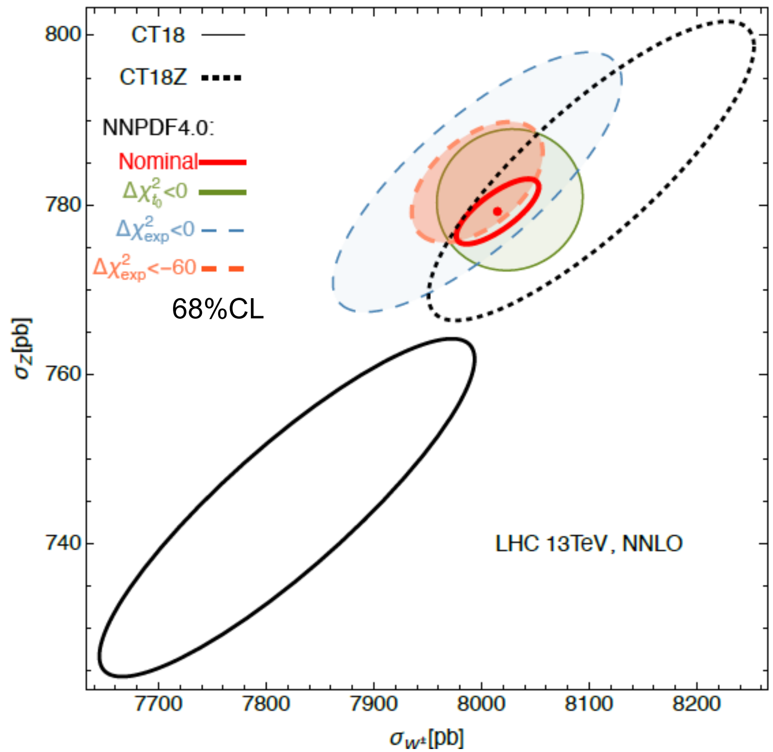
A hard task (harder than class P):
find all good PDF sets by fitting

An easy task, class NP:
check if a given PDF set passes goodness-of-fit criteria

⇒ **hopscotch scans**

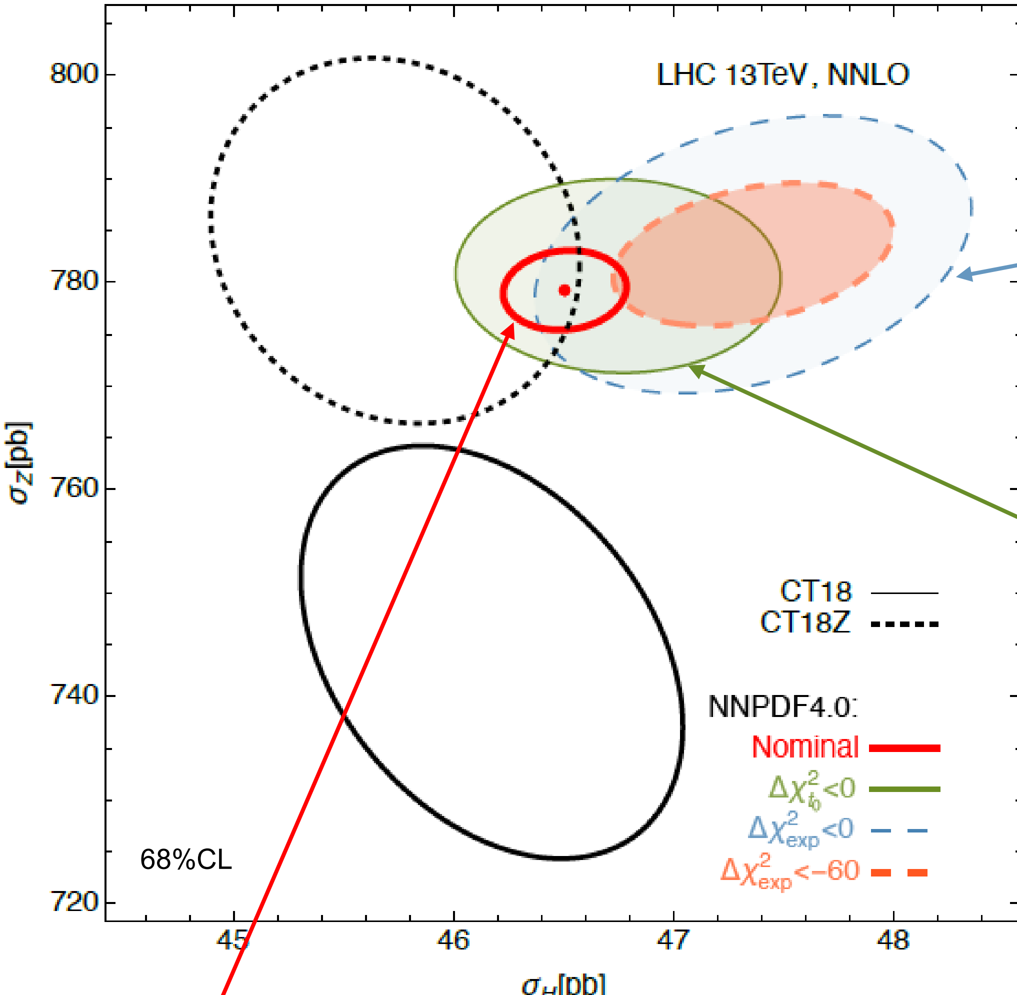
Monte-Carlo sampling of PDF parametrizations





Using the public NNPDF4.0 fitting code, we find well-behaving PDF solutions to the NN4.0 fit that have better χ^2 with respect to central data values (by as much as 35-80 units depending on the χ^2 definition) than the published replica 0. These replicas are outside of nominal (red) NN4.0 uncertainties.

Monte-Carlo sampling of PDF parametrizations



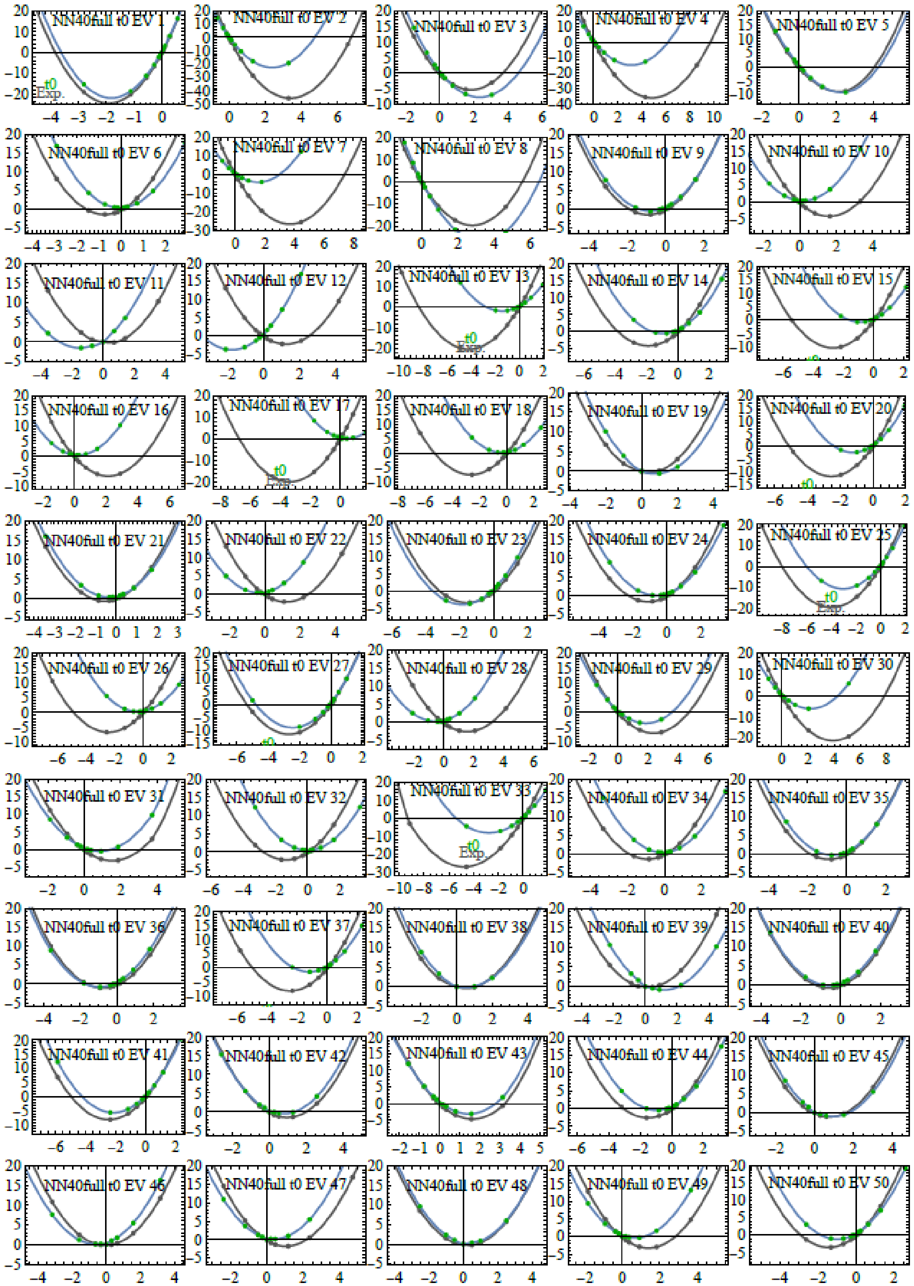
Regions containing (very) good solutions according to the experimental form of χ^2 (is used in χ^2 summary tables of the NN4.0 article, was a default in the NN4.0 public code)

Region containing good solutions according to the t_0 form of χ^2 (used to train NN replicas)

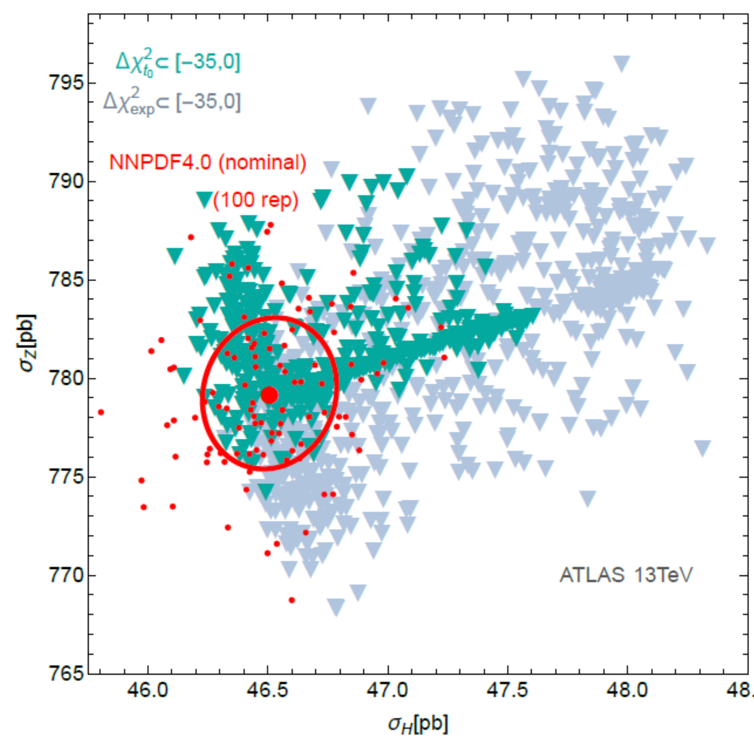
Nominal NN4.0 Hessian or MC 68%cl

These regions are approximate, at least as large as shown

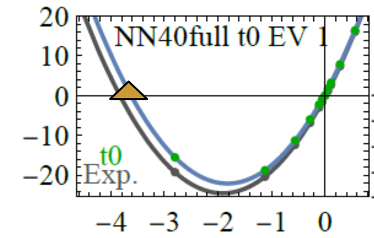
How the hopscotch solutions are found

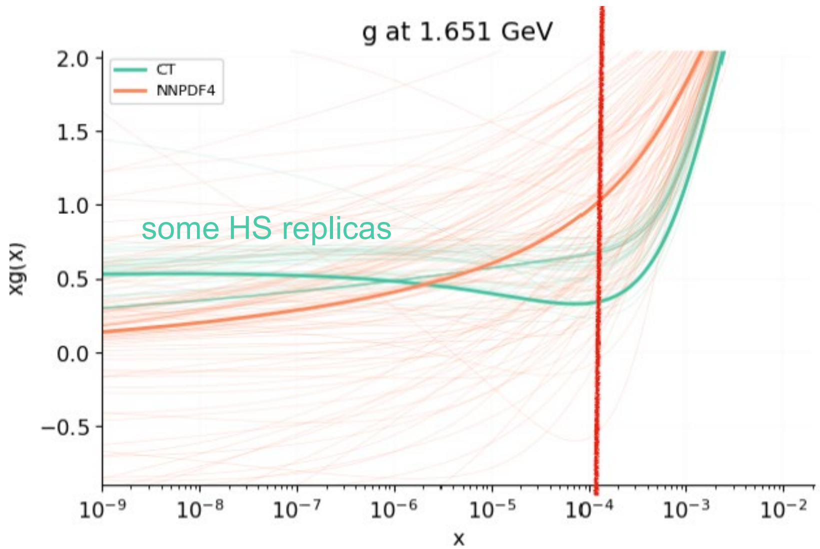


1. Examine the quasi-Gaussian χ^2 dependence along 50 Hessian EV directions
2. Perform high-density MC sampling of a span of a few EV directions that drive the specific PDF uncertainty



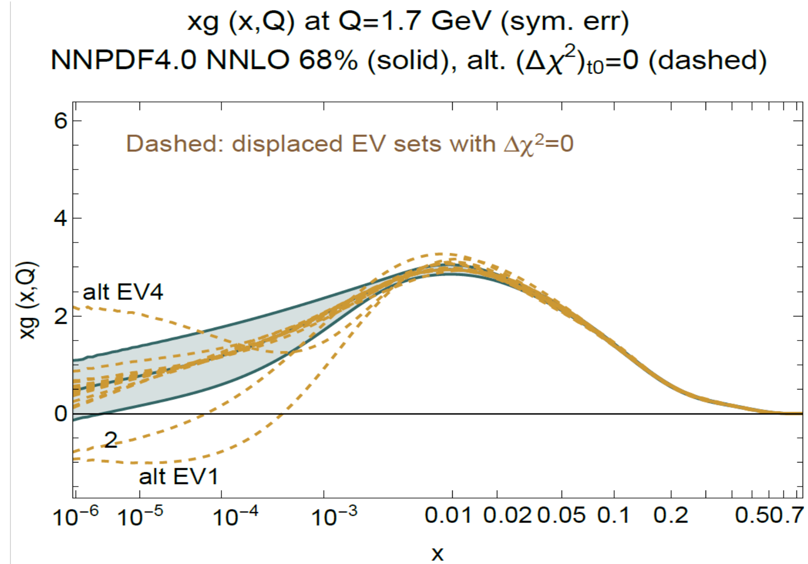
Why doesn't NNPDF4.0 find HS solutions?





NNPDF authors find that some HS replicas fail the initial-stage overfitting test

(M. Ubiali, HP2 2022 workshop, Durham, 2022-09-22)



HS solutions have much lower χ^2 than NN MC replicas. HS PDFs are outside the 50-dim neighborhood of NN replica 0. We do not see evidence of "overfitting" according to CT18 criteria.

What is the faithful PDF uncertainty on QCD cross sections?

Such studies of CT, NNPDF, also MSHT fits show that the stated (as in CT18) or unstated (as in NN4.0) *uncertainty due to methodology* (parametrization/NN architecture, smoothness, data tensions, model for syst. errors, ...) exceeds the impact of most recent data sets in global fits

PDF uncertainties in high-stake measurements (Higgs cross sections, W mass…) thus should be examined for *robustness of sampling over acceptable methodologies* and demonstrate *absence of biases* in this sampling.

Big data paradox: "the bigger the data, the surer we may fool ourselves".
Data analysis and (quasi-) MC integration with many (> 20) parameters are often at a risk of hard-to-detect, but dangerous sampling biases that take over the law of large numbers.

An undetected sampling bias may result in a wrong prediction with a low nominal uncertainty.
[X.-L. Meng, "Statistical paradises and paradoxes in big data (I): Law of large populations, big data paradox, and the 2016 US presidential election," The Annals of Applied Statistics 12, (2018) 685.]

Experience with statistics of big surveys and (quasi-) Monte-Carlo integration shows how to quantify such effects on QCD parameters or cross sections. *[arXiv: 2205.10444, Sec. 2.]*

Hopscotch scans illustrated for the NNPDF4.0 —thanks to the publicly available code.
Impact on the uncertainties at small x, in other regions.
Applicable to other analyses using a large parameter space — CT/MSHT tolerance, polarized PDFs, etc.

From arXiv: 2205.10444 v.3 , Sec. 3D

If the hopscotch solutions are acceptable, a natural question to raise is why they are not covered by the nominal NNPDF set. … As a possible hint, any hopscotch solution can be represented by a neural network in accord with the universal approximation theorems. The challenge of representative sampling in a high-dimensional space must therefore be also present in the NN approach. The nominal NNPDF replicas only resample the fitted data points while using a fixed methodology, with specific choices made on the NN architecture, the cost function, stopping and smoothness conditions. Finding a hopscotch solution in an NN approach may require variations in the training methodology, … which may thus constitute an unstated part of the uncertainty, together with the uncertainty due to the prescription for experimental systematic errors. The closure test … checks for the agreement of the PDFs with the pseudodata within the uncertainties. Yet it does not establish the full size of uncertainties in all directions, and neither it rules out potential subtle biases with the real data…