Model-Independent Detection of New Physics Signals Using Interpretable Semi-Supervised Classifier Tests

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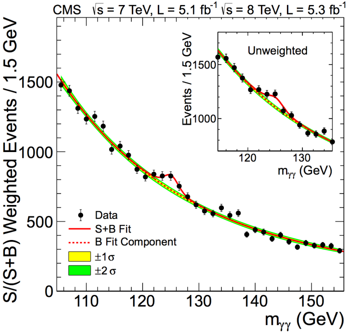
Hypothesis testing for discovery of new physics

Discovery of new phenomena at the LHC usually boils down to testing for the presence of a signal distribution over a background of known SM physics:

- Known physics: $p_b(z)$
- New signal: $p_s(z)$
- Nature: $q(z) = (1 - \lambda)p_b(z) + \lambda p_s(z)$

Want to test $H_0 : \lambda = 0$ vs. $H_1 : \lambda > 0$

If one rejects H_0 at high enough significance level, then one would proceed to claim discovery of new physics



Model-dependent classifier-based tests

Most of these tests are done in the model-dependent mode, where the test statistic is optimized to have power for detecting a specific signal

Relevant datasets:

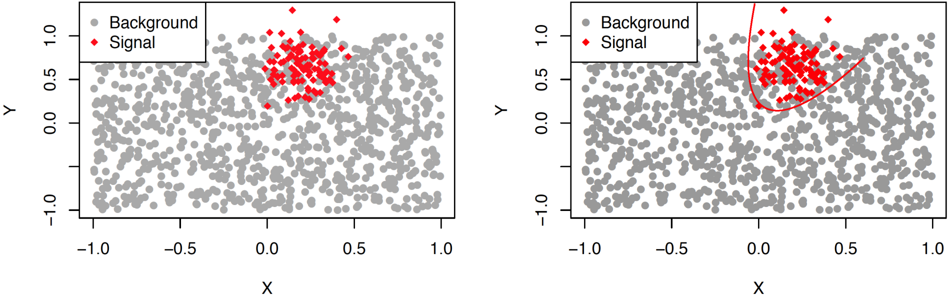
Training background:	$\mathcal{X} = \{X_1, \ldots, X_{m_b}\}$,	$X_i \sim p_b$
Training signal:	$\mathcal{Y} = \{Y_1, \ldots, Y_{m_s}\}$,	$Y_i \sim p_s$
Experimental data:	$\mathcal{W} = \{W_1, \ldots, W_n\}$,	$W_i \sim q = (1-\lambda)p_b + \lambda p_s$

Basic idea: use $\mathcal{X}$ and $\mathcal{Y}$ to find the optimal test for detecting p_s

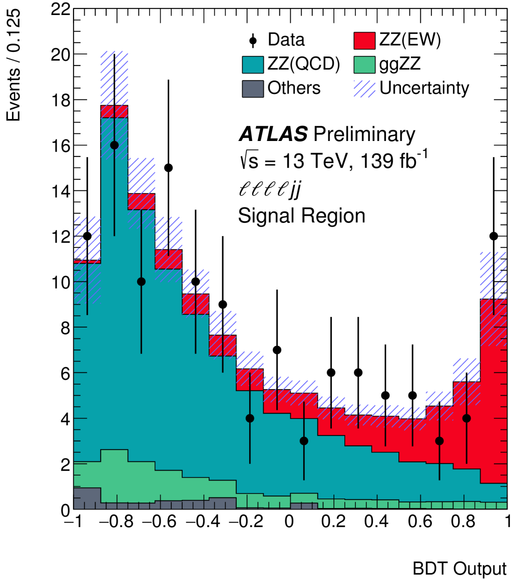
When the data space is high-dimensional, this is usually done using classifiers:

1. Train a supervised classifier to separate $\mathcal{X}$ from $\mathcal{Y}$
2. Use the classifier output to test for the presence of signal in $\mathcal{W}$

Classifier training



Classifier output



Some options for the test:

- Counting experiment in the highest purity output bin
- Cut on the classifier output; test using the resulting signal-enriched sample
- LRT: Use the connection of the classifier output to the likelihood ratio
- ...

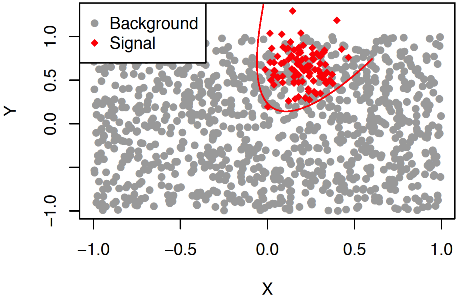
Testing when the signal is misspecified

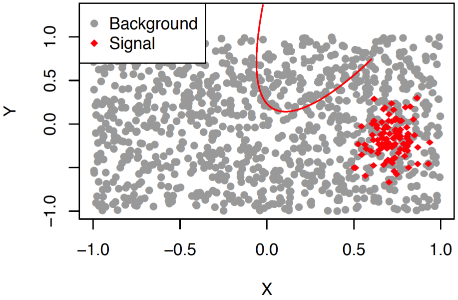
To perform these tests, we need to assume that we can reliably simulate data from both p_b and p_s

However, when either or both of these simulators are systematically misspecified, the test may not behave as desired

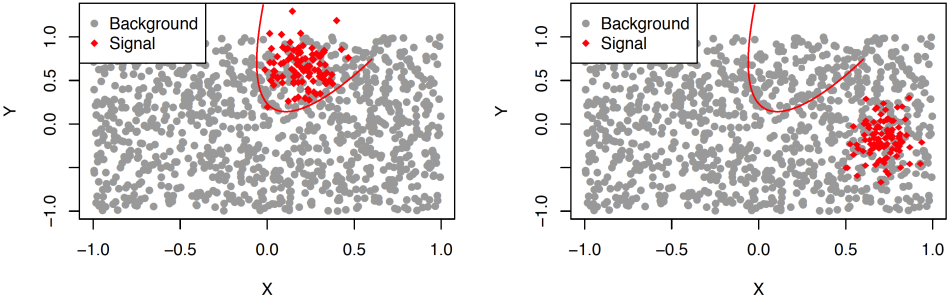
Specifically, if the test is optimized for a misspecified p_s, it may have little to no power for an actual signal

Systematically misspecified signal





Systematically misspecified signal



$\Rightarrow$ How to obtain an omnibus test that would have power for a wide range of signals, even in high-dimensional situations?

Model-independent searches

In model-independent searches of new physics, we assume that we have a reliable sample from p_b but we do not assume access to a training sample from p_s

$\rightarrow$ Provides sensitivity for unexpected or misspecified signals

Available datasets:

$$\begin{aligned} \text{Training background:} \quad & \mathcal{X} = \{X_1, \ldots, X_{m_b}\}, \qquad && X_i \sim p_b \\ \text{Experimental data:} \quad & \mathcal{W} = \{W_1, \ldots, W_n\}, \qquad && W_i \sim q = (1-\lambda)p_b + \lambda p_s \end{aligned}$$

We only have access to $\mathcal{X}$ and $\mathcal{W}$; i.e., no direct access to p_b, q, p_s or λ

Task 1: We want to understand if $\mathcal{W}$ shows evidence for the presence of p_s

Task 2: We want to understand what λ and p_s look like

Related problems in statistics and ML

The model-independent search problem is closely related to a number of problems studied in statistics and machine learning

Specifically, it can be seen as an example of:

1. Two-sample testing (e.g., Kim et al. (2019, 2021)):
   $X_i \overset{\text{iid}}{\sim} p_1,\ Y_i \overset{\text{iid}}{\sim} p_2$, is $p_1 = p_2$?
2. Collective anomaly detection (e.g., Chandola et al. (2009)):
   Is there a collection of data points which taken together deviate from the anticipated data?

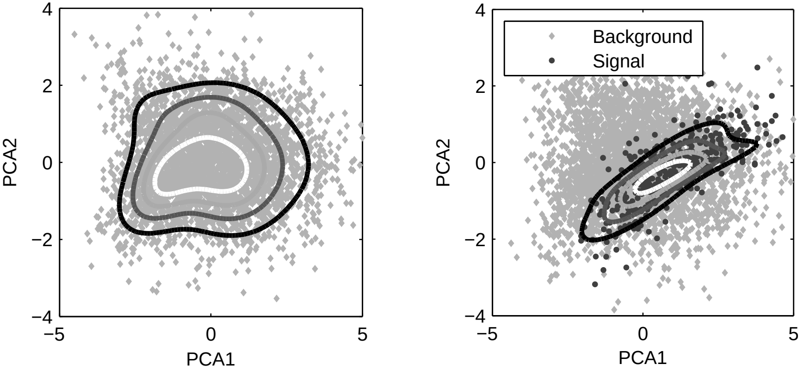
Notice that

$$\text{model independent search} \neq \text{outlier detection}$$

Each signal event is typically indistinguishable from the background on its own; it is the collection of many signal events that defines the excess

Model-independent searches in low-dimensional spaces

In Kuusela et al. (2012) and Vatanen et al. (2012), we used Gaussian mixture models to first fit the background sample and then, given the background model, fit any anomalous signal present in the experimental sample



(a) Background model $p_b(z)$ (b) Signal model $p_s(z)$

This approach works fine in 2–3 dimensions but does not really scale to higher dimensions

Model-independent search using a semi-supervised classifier

What to do when the data space has more than just a couple of dimensions?

$\rightarrow$ Use classifiers!

Basic idea: Train a classifier h to separate background $\mathcal{X}$ from the experimental data $\mathcal{W}$

- Under H_0, the classifier should not be able to separate $\mathcal{X}$ from $\mathcal{W}$
- So if the classifier is able to differentiate between these two samples, then that provides evidence against H_0

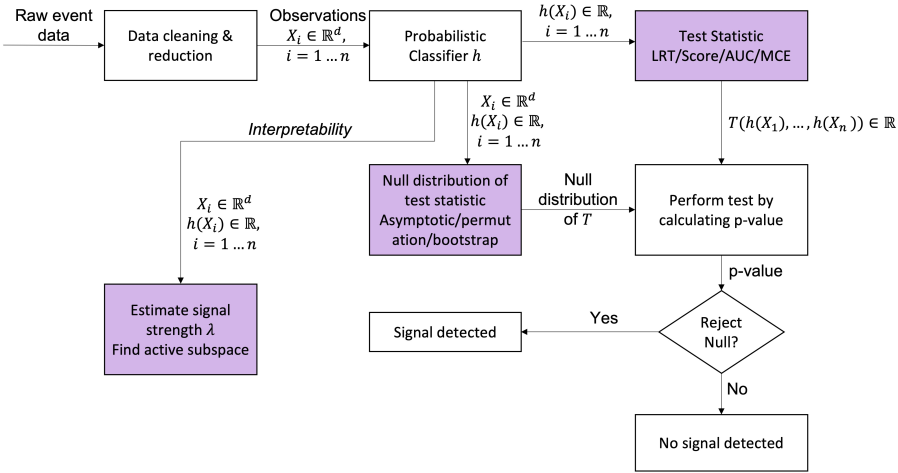
This basic strategy is similar to recent work by D'Agnolo and Wulzer (2019) and D'Agnolo et al. (2021); see also Kim et al. (2019, 2021)

Our contributions

Our work (Chakravarti et al., 2021) has the following new contributions:

1. We investigate various ways of obtaining a test statistic from the trained classifier $\widehat{h}$ as well as various ways of calibrating the tests
2. We propose a way to estimate the signal strength λ based on $\widehat{h}$
3. We propose a way to interpret $\widehat{h}$ using active subspaces

Overview of the approach



Classifier-based test statistics

Test statistics based on a classifier $\widehat{h}$ that is trained to separate experimental data from background data:

1. Likelihood Ratio Test Statistic:

$$\mathrm{LRT} = 2\sum_i \log \widehat{\psi}(W_i),$$

where $\widehat{\psi}(z) = \frac{m_b}{n}\frac{\widehat{h}(z)}{1-\widehat{h}(z)}$ is a classifier-based estimate of the density ratio $\psi = q/p_b$

2. Area Under the Curve (AUC) Test Statistic:

$$\widehat{\theta} = \frac{1}{m_b\ n}\sum_i\sum_j \mathbb{I}\left\{\widehat{h}(W_j) > \widehat{h}(X_i)\right\}$$

Test $H_0 : \theta = 0.5$ versus $H_1 : 0.5 < \theta < 1$.

3. Misclassification Error (MCE) Test Statistic:

$$\widehat{\mathrm{MCE}} = \frac{1}{2}\left[\frac{1}{m_b}\sum_i \mathbb{I}\left\{\widehat{h}(X_i) > \pi\right\} + \frac{1}{n}\sum_j \mathbb{I}\left\{\widehat{h}(W_j) < \pi\right\}\right],\ \pi = n/(n+m_b)$$

Test $H_0 : \mathrm{MCE} = 0.5$ versus $H_1 : \mathrm{MCE} < 0.5$.

Calibration of the tests

In order to control the Type I error, we need to obtain the distribution of the test statistics under the null $H_0 : \lambda = 0$

Notice that under the null both $\mathcal{X}$ and $\mathcal{W}$ are samples from p_b

Three approaches:

1. Asymptotics: Can derive the asymptotic distribution for each of the test statistics; for example, for AUC, Newcombe (2006) showed that

$$\frac{\widehat{\theta} - 0.5}{\sqrt{V_0(\widehat{\theta})}} \rightsquigarrow N(0, 1),$$

   for certain $V_0(\widehat{\theta})$ under the null
2. Nonparametric bootstrap: Sample with replacement from $\mathcal{X} \cup \mathcal{W}$ and randomly label as either X's or W's
3. Permutation: Randomly permute the class labels in $\mathcal{X} \cup \mathcal{W}$

In-sample vs. out-of-sample evaluations

In practice, we need to be careful with in-sample vs. out-of-sample evaluation of the classifier $\widehat{h}$

- For each calibration method, we use half of the data to train the classifier and the other half to evaluate and calibrate the test statistics (sample splitting)
- For the permuation method, we also consider a variant where the classifier is evaluated in-sample, which requires retraining the classifier for each permutation cycle

Kaggle Higgs boson data

We explore the performance of these methods using the Kaggle Higgs boson challenge dataset[1]

- Simulated $H \to \tau\tau$ events in ATLAS
- Select events with two jets and only consider primitive features (transverse momenta, MET, angles,...)
- 15 variables after accounting for rotational symmetry in ϕ
- 80,806 background events; 84,221 signal events
- Generate 50 "replicates" by sampling without replacement $m_b = 40{,}403$ background events, $m_s = 20{,}403$ signal events and $n = 40{,}403$ experimental events from the original samples
- We use Random Forest as the classifier h throughout

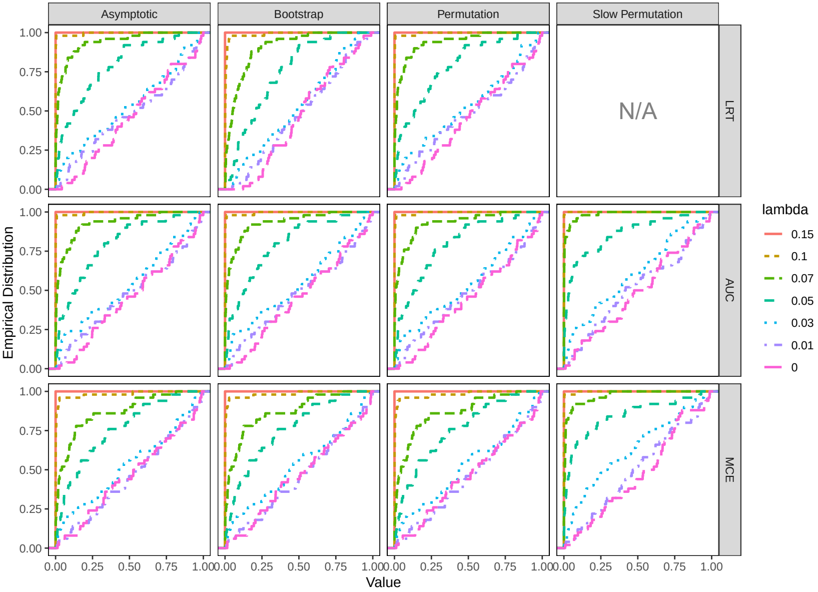
[1] https://www.kaggle.com/c/higgs-boson

Power of detecting a signal

Power of detecting a well-specified signal in the Kaggle Higgs boson data

		Signal Strength (λ)						
Model	Method	0.15	0.1	0.07	0.05	0.03	0.01	0
Supervised LRT	Asymptotic	100	100	96	62	18	18	6
	Bootstrap	100	96	78	58	6	0	0
	Permutation	100	98	98	86	28	6	0
Supervised Score	Bootstrap	64	66	74	50	18	0	0
	Permutation	94	92	100	92	80	24	12
Semi-Supervised LRT	Asymptotic	100	98	74	38	16	6	2
	Bootstrap	100	98	48	10	2	2	0
	Permutation	100	98	72	38	16	6	2
	Slow Perm	82	8	0	4	2	0	4
Semi-Supervised AUC	Asymptotic	100	96	78	32	14	4	2
	Bootstrap	100	98	70	32	20	6	2
	Permutation	100	98	68	32	20	4	2
	Slow Perm	100	100	94	56	20	8	4
Semi-Supervised MCE	Asymptotic	100	92	60	28	14	2	2
	Bootstrap	100	96	52	28	16	6	4
	Permutation	100	96	52	30	14	6	6
	Slow Perm	100	98	86	58	16	6	2

p-value distributions for the semi-supervised tests



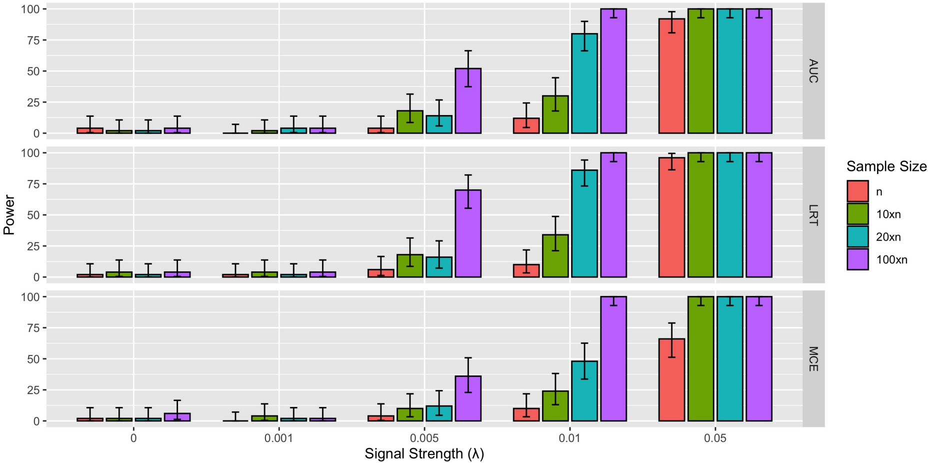
Power of detecting a signal

Power of detecting a misspecified signal in the Kaggle Higgs boson data

		Signal Strength (λ)						
Model	Method	0.15	0.1	0.07	0.05	0.03	0.01	0
Supervised LRT	Asymptotic	2	10	2	8	8	6	4
	Bootstrap	0	0	0	0	0	0	0
	Permutation	0	0	0	0	0	2	0
Supervised Score	Bootstrap	0	0	0	0	0	0	0
	Permutation	0	0	0	0	0	2	8
Semi-Supervised LRT	Asymptotic	100	100	100	82	18	4	4
	Bootstrap	100	100	100	60	4	2	0
	Permutation	100	100	100	82	18	4	2
	Slow Perm	100	100	78	22	2	4	6
Semi-Supervised AUC	Asymptotic	100	100	100	78	16	8	4
	Bootstrap	100	100	100	82	20	10	0
	Permutation	100	100	100	80	20	8	2
	Slow Perm	100	100	100	100	34	10	4
Semi-Supervised MCE	Asymptotic	100	100	100	66	24	6	4
	Bootstrap	100	100	100	62	16	6	4
	Permutation	100	100	100	62	14	6	4
	Slow Perm	100	100	100	98	22	8	2

Signal misspecified by transforming $\texttt{tau\_pt}^* = \texttt{tau\_pt} - 0.7\,(\texttt{tau\_pt} - \min(\texttt{tau\_pt}))$

Power as a function of sample size



Power of the asymptotic model-independent tests for increasing sample sizes

Interpreting the semi-supervised classifier

We may want to be able to analyze the trained semi-supervised classifier $\widehat{h}$ to learn about the properties of the potential signal

Signal strength

We estimate the signal strength λ from the classifier $\widehat{h}$ using the Neyman–Pearson quantile transform

Variable importance

We use the *active subspace* of the classifier to identify variable combinations that help separate the signal from the background

Estimating the signal strength

Given a trained semi-supervised classifier $\widehat{h}$, how can we estimate the signal strength λ?

If we know that $p_s(z) = 0$ for some known z, then this is simple

Since

$$\psi(z) = \frac{q(z)}{p_b(z)} = \left(\frac{1-\pi}{\pi}\right)\left(\frac{h(z)}{1-h(z)}\right),$$

we obtain

$$\widehat{\lambda} = 1 - \left(\frac{1-\pi}{\pi}\right)\left(\frac{\widehat{h}(z)}{1-\widehat{h}(z)}\right),$$

for any z with $p_s(z) = 0$

However, in the model-independent setting, we may not know when $p_s(z) = 0 \rightarrow$ What to do?

Estimating the signal strength

Need to assume $\inf_z p_s(z)/p_b(z) = 0$ for identifiability; assume also $p_b, q > 0$ everywhere, for simplicity

Define the Neyman–Pearson Quantile Transform of z as:

$$\rho(z) = P_{X\sim p_b}\left(\frac{q(X)}{p_b(X)} \geq \frac{q(z)}{p_b(z)}\right) = P_{X\sim p_b}\left(\psi(X) \geq \psi(z)\right) = P_{X\sim p_b}\left(h(X) \geq h(z)\right)$$

Let g_q be the density function of $\rho(Z)$ when $Z \sim q$

Then it can be shown that g_q is monotonically decreasing and

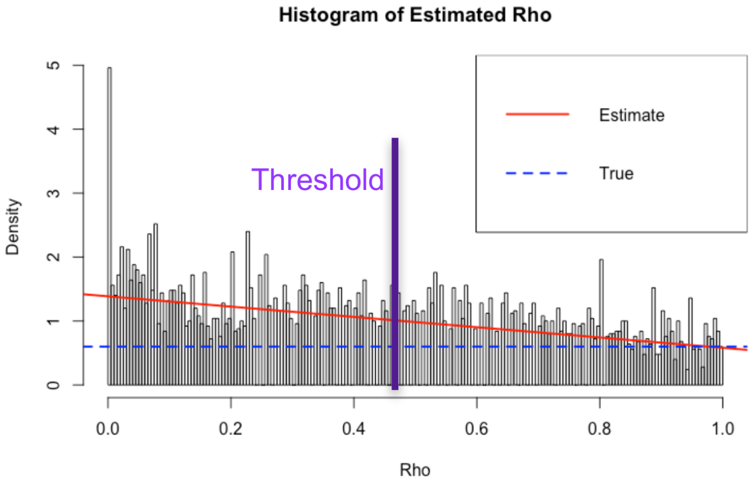
$$g_q(1) = 1 - \lambda$$

which allows us to estimate λ using $\widehat{\lambda} = 1 - \widehat{g_q}(1)$

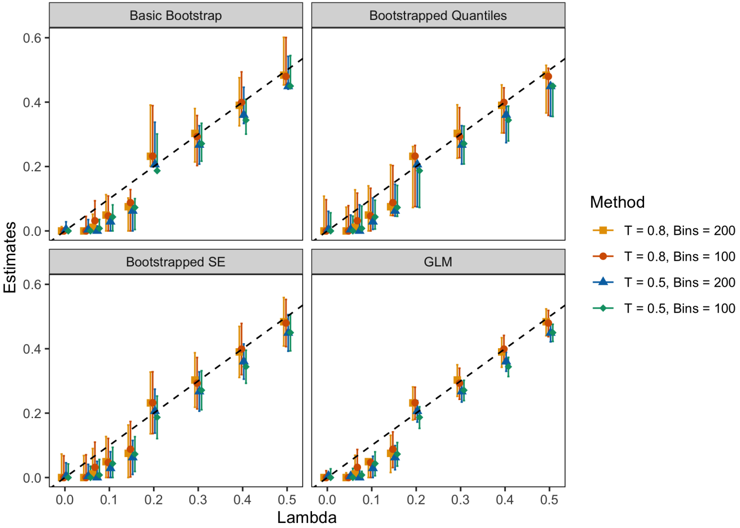
$\rightarrow$ We need to estimate a monotone density at its boundary

Estimating the signal strength

In practice, we form a histogram of $\rho(W_i)$ and estimate $g_q(1)$ using a Poisson regression on bins close to 1



Estimating the signal strength



Estimated λ vs. true λ with various uncertainty estimates

Active subspaces for interpreting the classifier

The fitted classifier surface $\widehat{h}$ contains information about how the experimental data $\mathcal{W}$ differs from the background data $\mathcal{X}$

How do we extract this information from $\widehat{h}$?

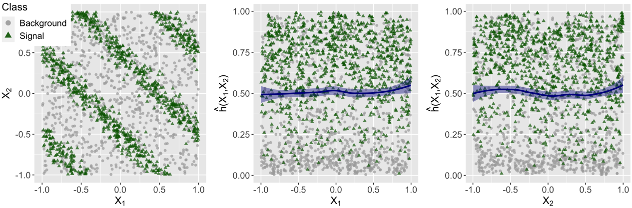
Could look at $\widehat{h}$ as a function of each input variable

But this might not reveal information contained in variable dependencies

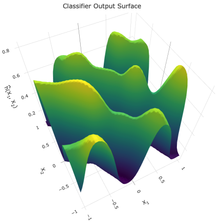
We propose to look at the *active subspace* of $\widehat{h}$ instead

Basic idea: perform PCA on the gradients $\nabla\widehat{h}(z)$ to reveal those directions in which the classifier surface changes the most

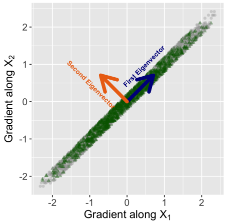
Active subspaces for interpreting the classifier



(a) X_1 versus X_2, $\widehat{h}(X_1, X_2)$ versus X_1 and $\widehat{h}(X_1, X_2)$ versus X_2



(b) Smoothed Classifier Surface



(c) PCA of the Standardized Gradients

Active subspaces for interpreting the classifier

In practice, we look at the gradients of

$$H(z) := \mathrm{logit}(\widehat{h}(z)) = \log\left(\widehat{h}(z)/(1-\widehat{h}(z))\right)$$

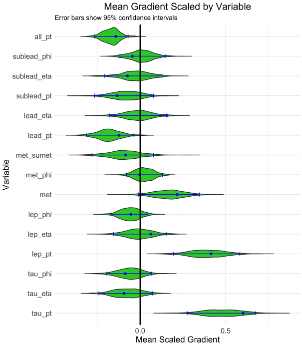
which are estimated by fitting a local linear regression on $H(Z_i)$ where $Z_i \in \mathcal{X} \cup \mathcal{W}$

Furthermore, we standardize the gradients by their estimated standard errors: $G(z) = \frac{\widehat{\nabla H(z)}}{\sqrt{\widehat{\mathrm{Var}}(\widehat{\nabla H(z)})}}$

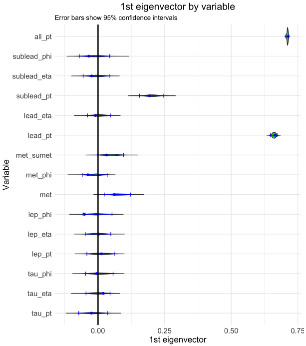
We then perform PCA on $G(Z_i)$: the mean of $G(Z_i)$ describes the slope of $H(z)$ and the principal components of $G(Z_i)$ capture the variation of $H(z)$ around the slope

Uncertainty estimates using bootstrapping

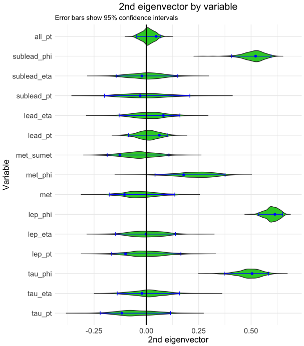
Active subspaces for interpreting the classifier



(a) Mean Gradient



(b) First Eigenvector



(c) Second Eigenvector

Discussion: Background systematics

The aforementioned approaches assume that the training background $\mathcal{X}$ comes from the true background p_b

However, in practice the MC generator for $\mathcal{X}$ is likely to be systematically misspecified

So the "signals" found might simply be due to background mismodeling

That does not necessarily mean that these techniques are not useful:

- Can be used to identify and characterize regions of high-dimensional phase space where background is mismodeled
- Can be used as a pilot analysis to guide dedicated model-dependent searches
- Can serve as a starting point for model-independent analyses accounting for background systematics

Discussion: Background systematics

In principle, there is no reason we couldn't incorporate background systematics into model-independent searches

Can learn from modeling techniques developed for model-dependent searches: template morphing, parameterization using nuisance parameters, two-point systematics,...

Building such systematic variations into the model-independent tests requires developing *new statistical methodology*

D'Agnolo et al. (2022) is a very interesting recent contribution toward this goal

This is one of those areas in HEP where statistical methodology is not yet fully established
$\rightarrow$ There is room for further exciting methods development!

Conclusions

- Model-independent searches may be able to increase the sensitivity of LHC for unexpected or misspecified signals
  - Has received increased attention in recent years due to the absence of major new signals in model-dependent searches
- Recent contributions have used classifiers to extend model-independent searches into high-dimensional spaces
- In our work[2], we have in the classifier-based setting:
  - Explored various test statistics and calibration methods
  - Proposed a way to estimate the signal strength
  - Used active subspaces to analyze the fitted classifier
- Big open question: how to incorporate background systematics?
  - Should be solvable through development of new statistical methodology
- Looking forward to further exciting discussions at Phystat-Anomalies on May 24–25!

[2] P. Chakravarti, M. Kuusela, J. Lei, and L. Wasserman. Model-Independent Detection of New Physics Signals Using Interpretable Semi-Supervised Classifier Tests. Preprint: arXiv:2102.07679 [stat.AP], 2021.

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Backup

Density Ratios and Classifiers

In general, given two densities p and q and samples

$$X_1, \ldots, X_n \sim p$$

$$Y_1, \ldots, Y_n \sim q$$

Give labels:

$$\begin{array}{c|cccccc} & X_1 & \ldots & X_n & Y_1 & \ldots & Y_n \\ Z & 1 & \ldots & 1 & 0 & \ldots & 0 \end{array}$$

Classifier ψ:

$$\psi(u) = P(Z = 1|u) = \frac{p}{p+q}$$

and so

$$\frac{p}{q} = \frac{\psi}{1-\psi}.$$

p-value distributions for the supervised tests