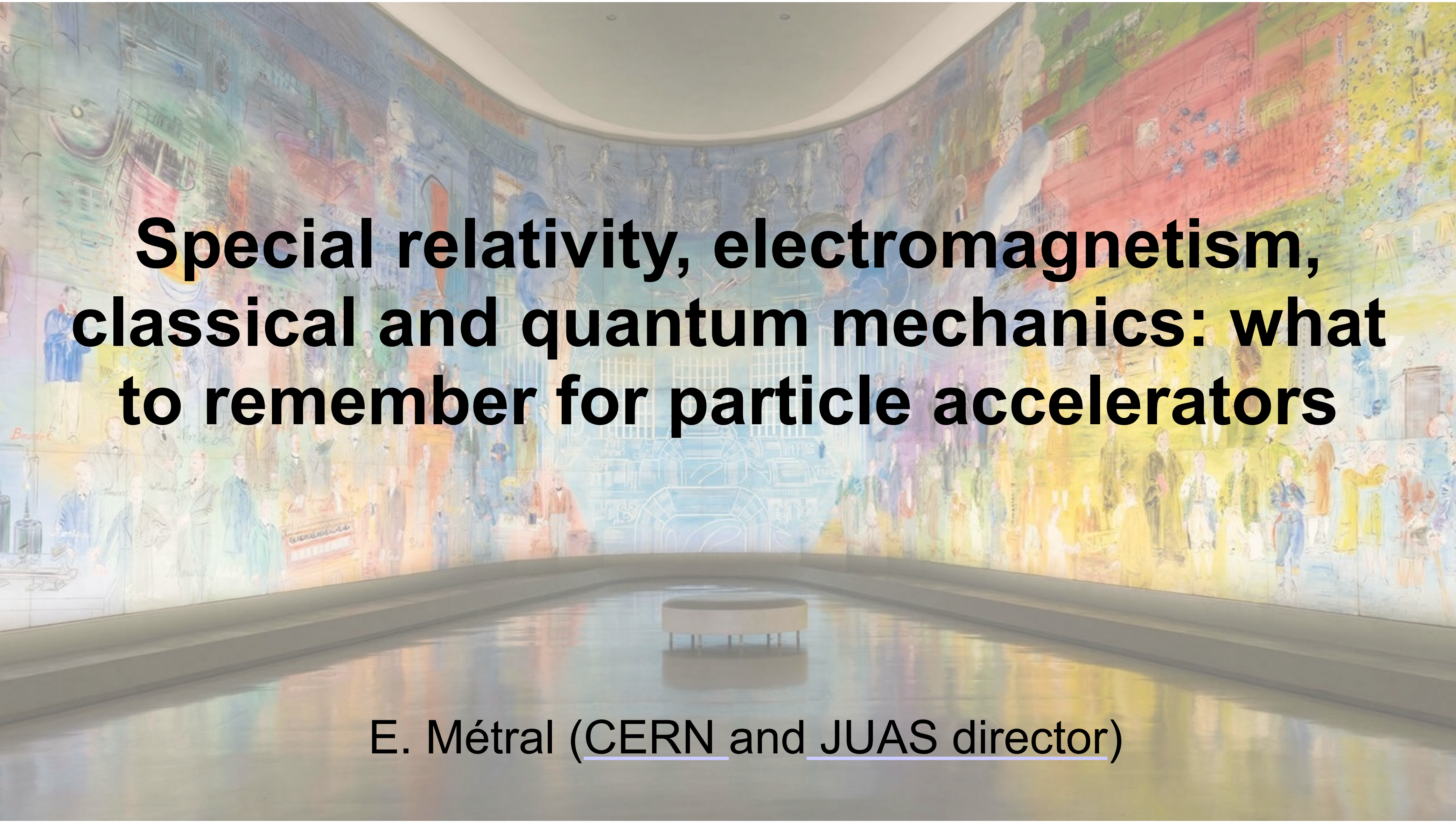

Special relativity, electromagnetism, classical and quantum mechanics: what to remember for particle accelerators

E. Métral (CERN and JUAS director)

What is the link between…?

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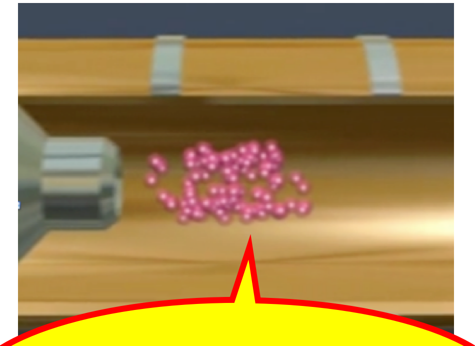
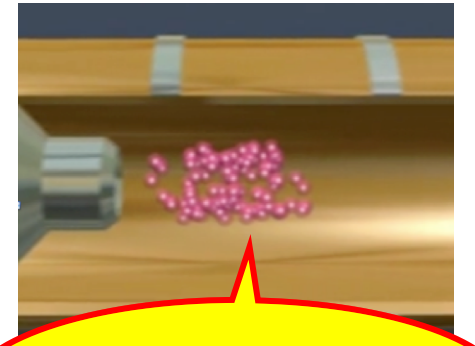


Group ("bunch") of particles (e.g.: p+)

What is the link between…?



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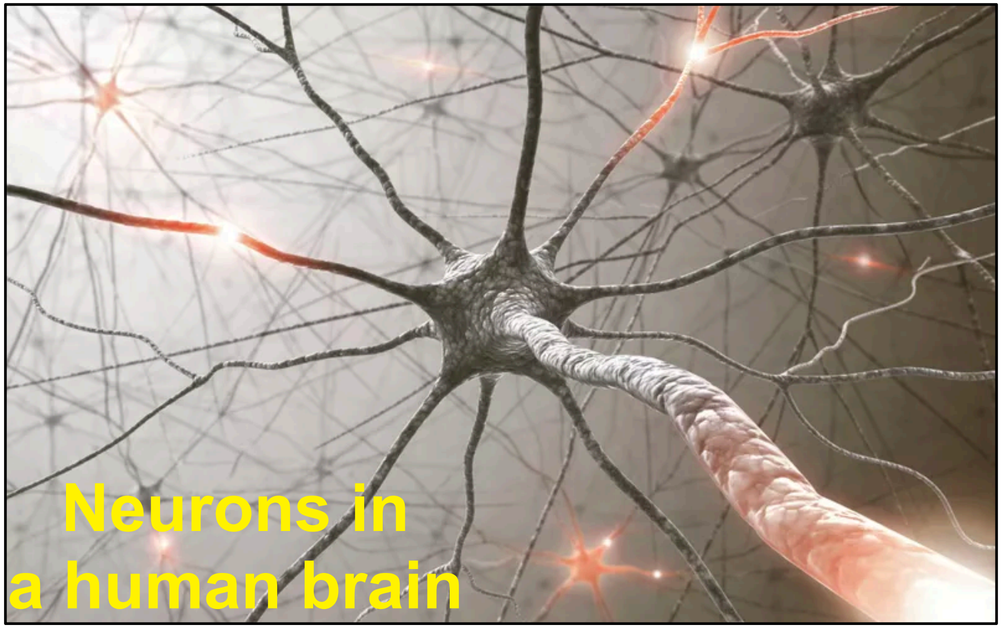

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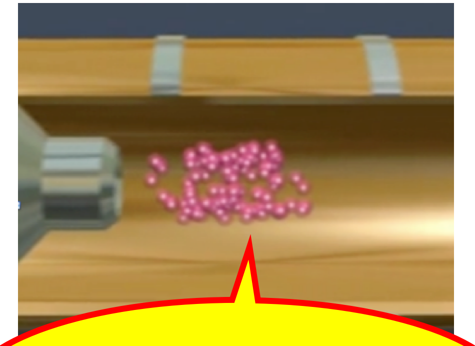

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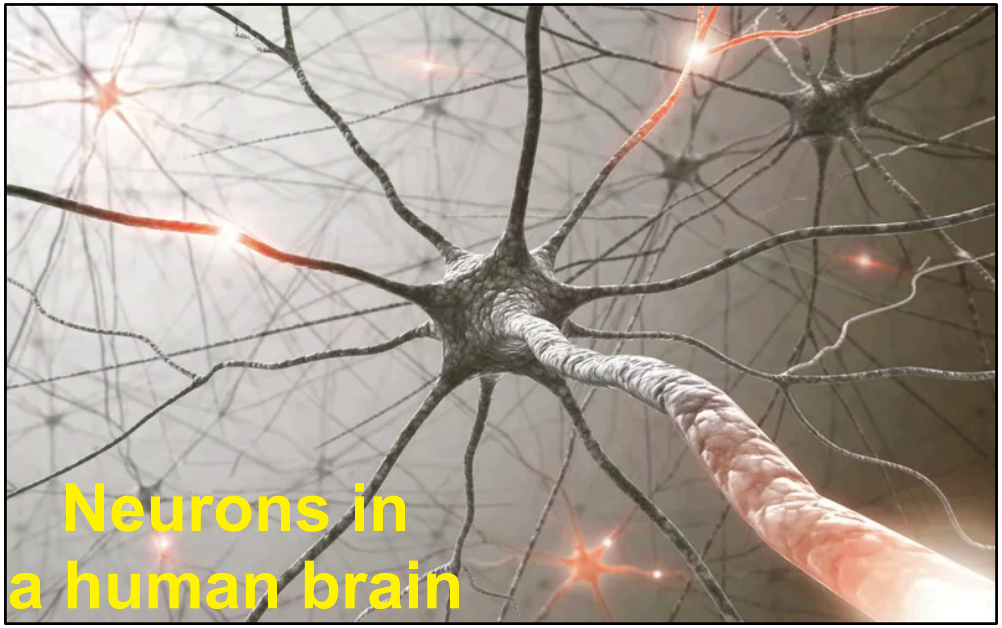

Neurons in a human brain

Stars in our galaxy (Milky Way)

=> The number!

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- 10^6
- 10^9
- 10^{11}
- 10^{13}
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=> How can we keep under control

- 1 (charged) particle?
- $\sim 10^{11}$ particles grouped together in 1 bunch?
- ~ 5500 bunches?

Date and time	Subject	Lecturer	Remarks
11 April 10.00-12.00	Introduction to accelerators, overview of accelerator types – part 1/2	Dr. Maurizio VRETENAR (CERN)	2 x 45 min + breaks and questions
	History of accelerators – part 1/2		
11 April 14.00-16.00	Special relativity, electromagnetism, classical and quantum mechanics: What to remember for particle accelerators": Part 1/3	Dr. Elias METRAL (CERN)	2 x 45 min + breaks and questions
12 April 10.00-12.00	Introduction to accelerators, overview of accelerator types – part 2/2	Dr. Maurizio VRETENAR (CERN)	2 x 45 min + breaks and questions
	History of accelerators – part 2/2		
12 April 14.00-16.00	Special relativity, electromagnetism, classical and quantum mechanics: What to remember for particle accelerators": Part 2/3	Dr. Elias METRAL (CERN)	2 x 45 min + breaks and questions
13 April 10.00-12.00	Linear accelerators – part 1/2	Dr. Maurizio VRETENAR (CERN)	2 x 45 min + breaks and questions
13 April 14.00-16.00	Special relativity, electromagnetism, classical and quantum mechanics: What to remember for particle accelerators": Part 3/3	Dr. Elias METRAL (CERN)	2 x 45 min + breaks and questions
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=> 6 × 45 min

(https://indico.cern.ch/event/1149120/)

We need a force…

We need a force…

- Do you know the number of the currently known fundamental forces in the universe?

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- Do you know the number of the currently known fundamental forces in the universe?
  - 1
  - 2
  - 4
  - 5
  - 10
  - Infinity

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- Do you know the number of the currently known fundamental forces in the universe?
    - 1
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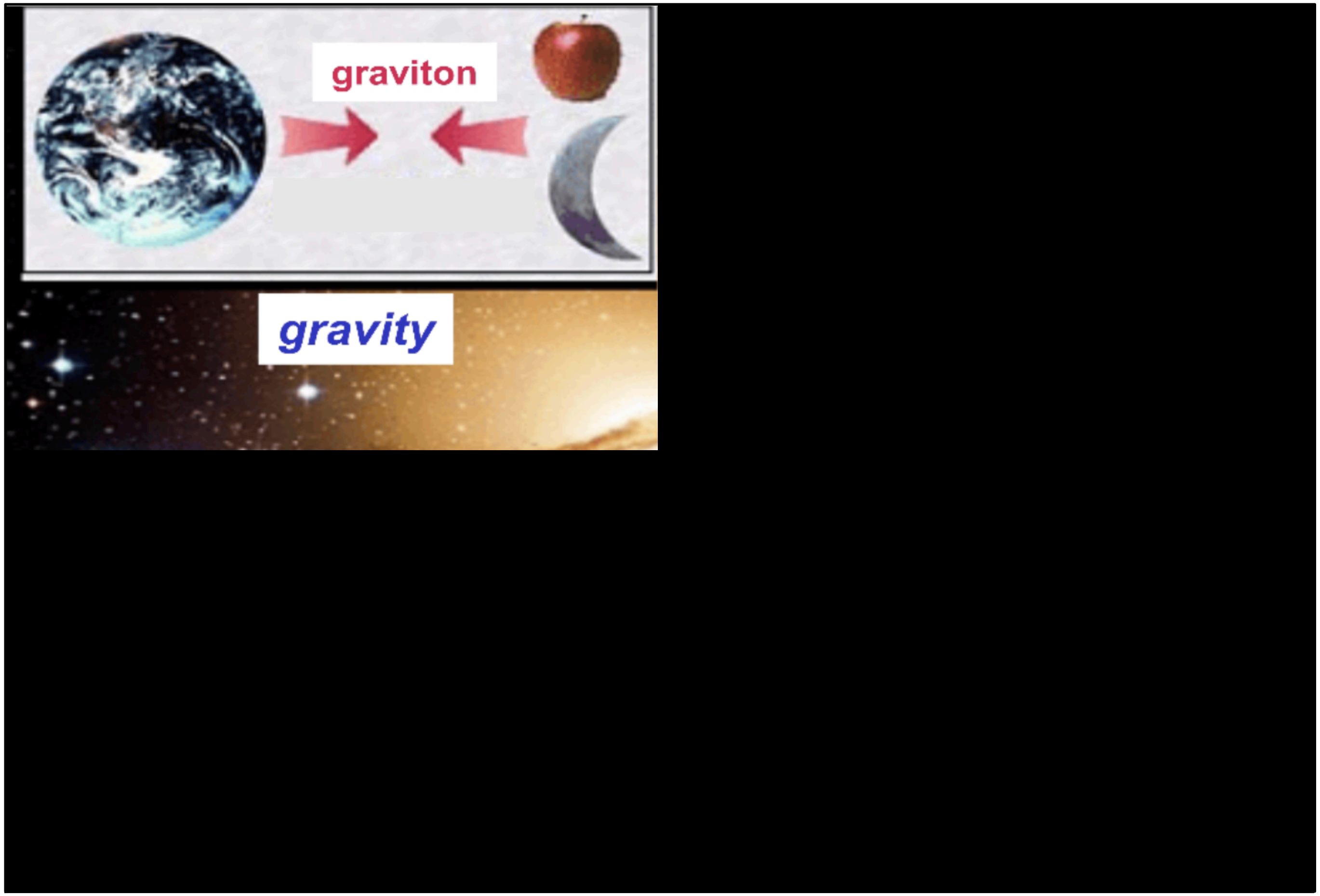

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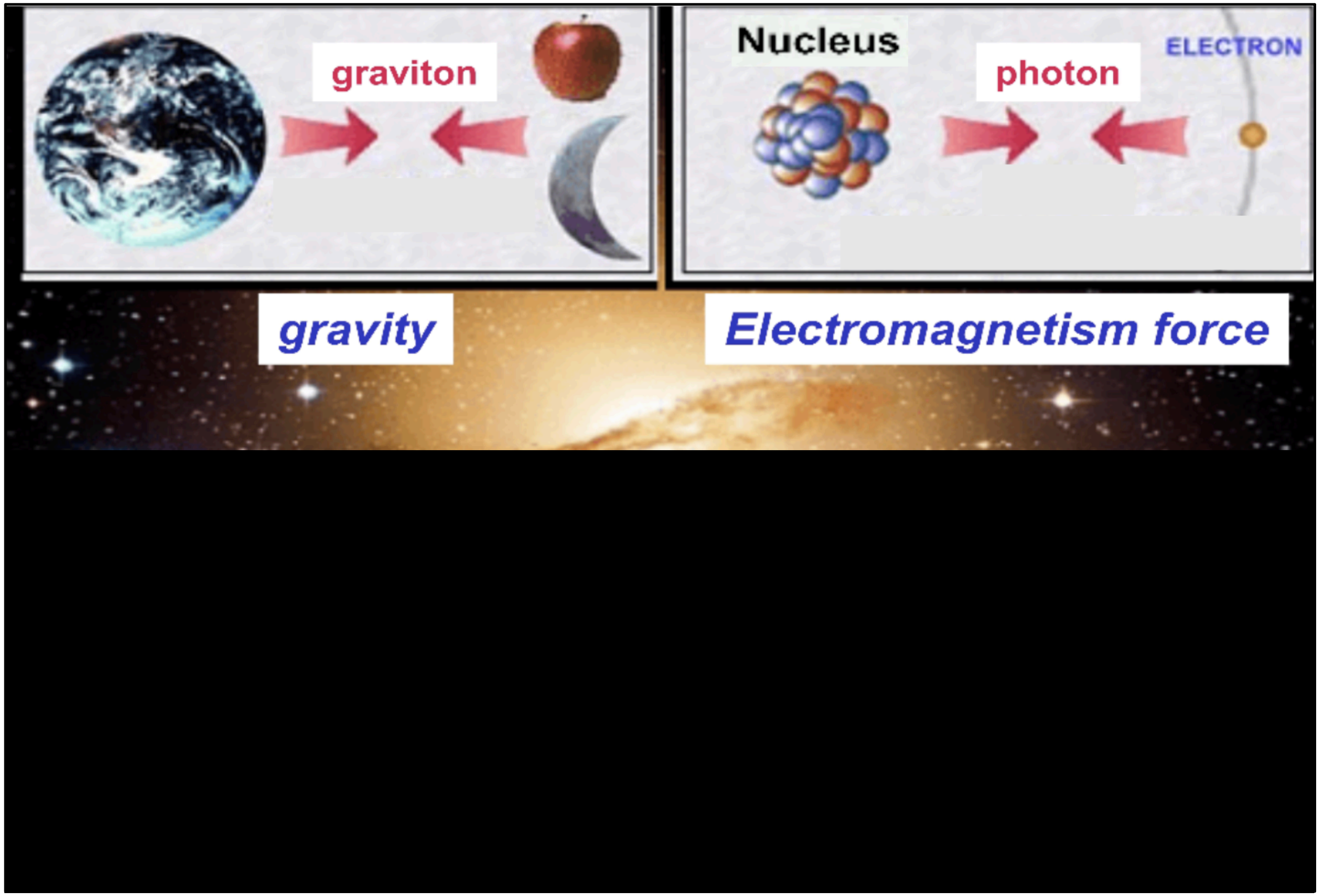

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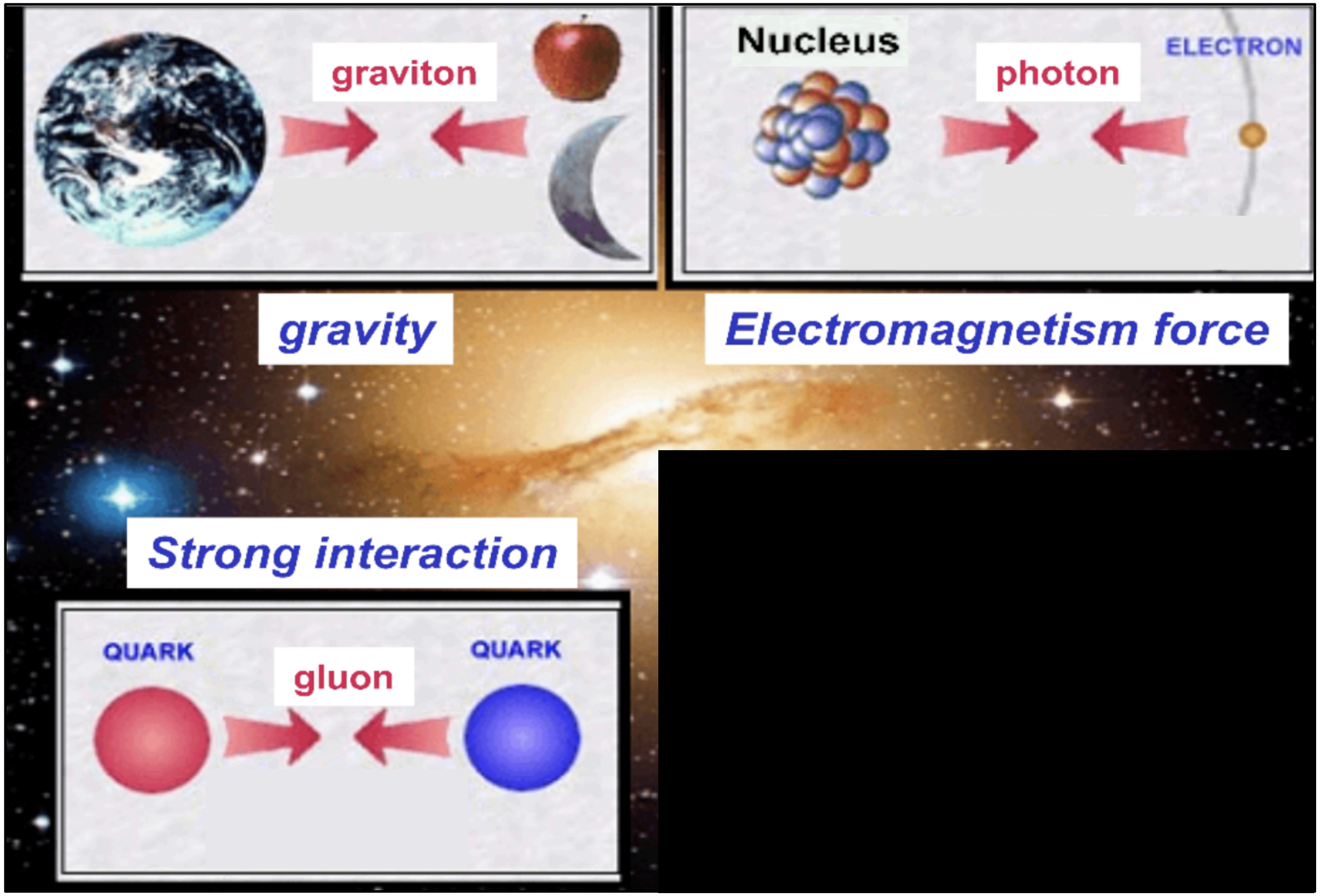
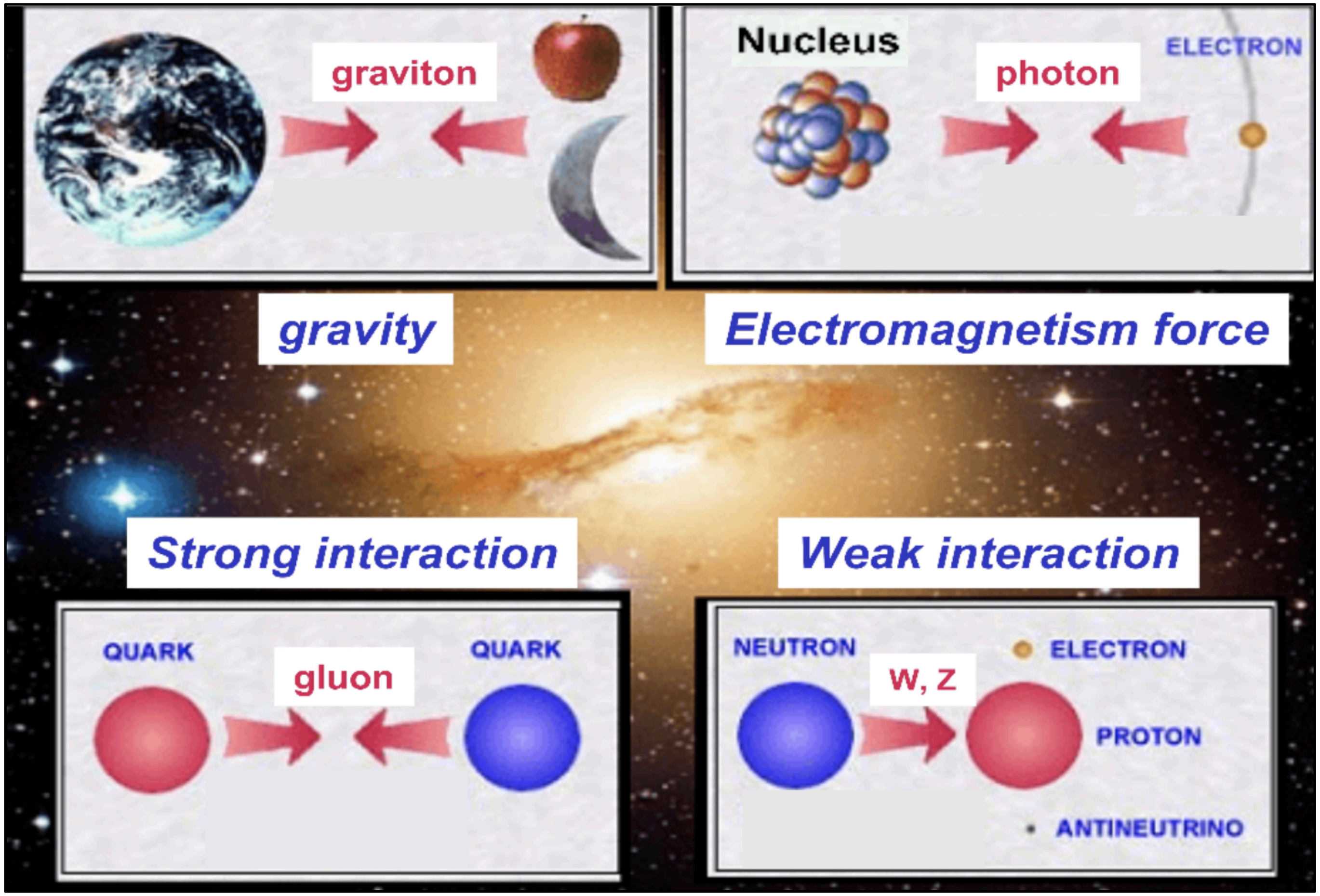


[redacted]

We need a force…



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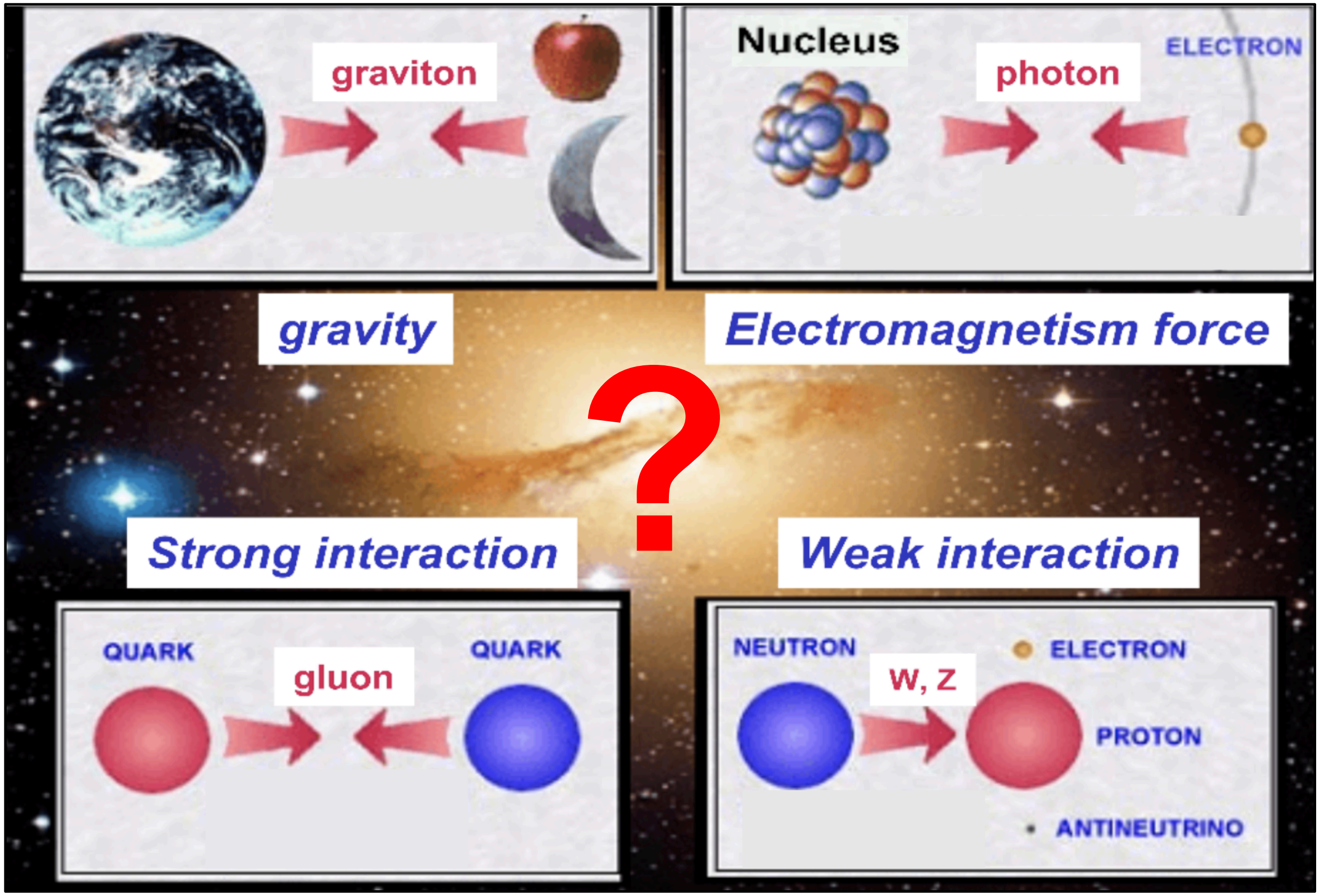

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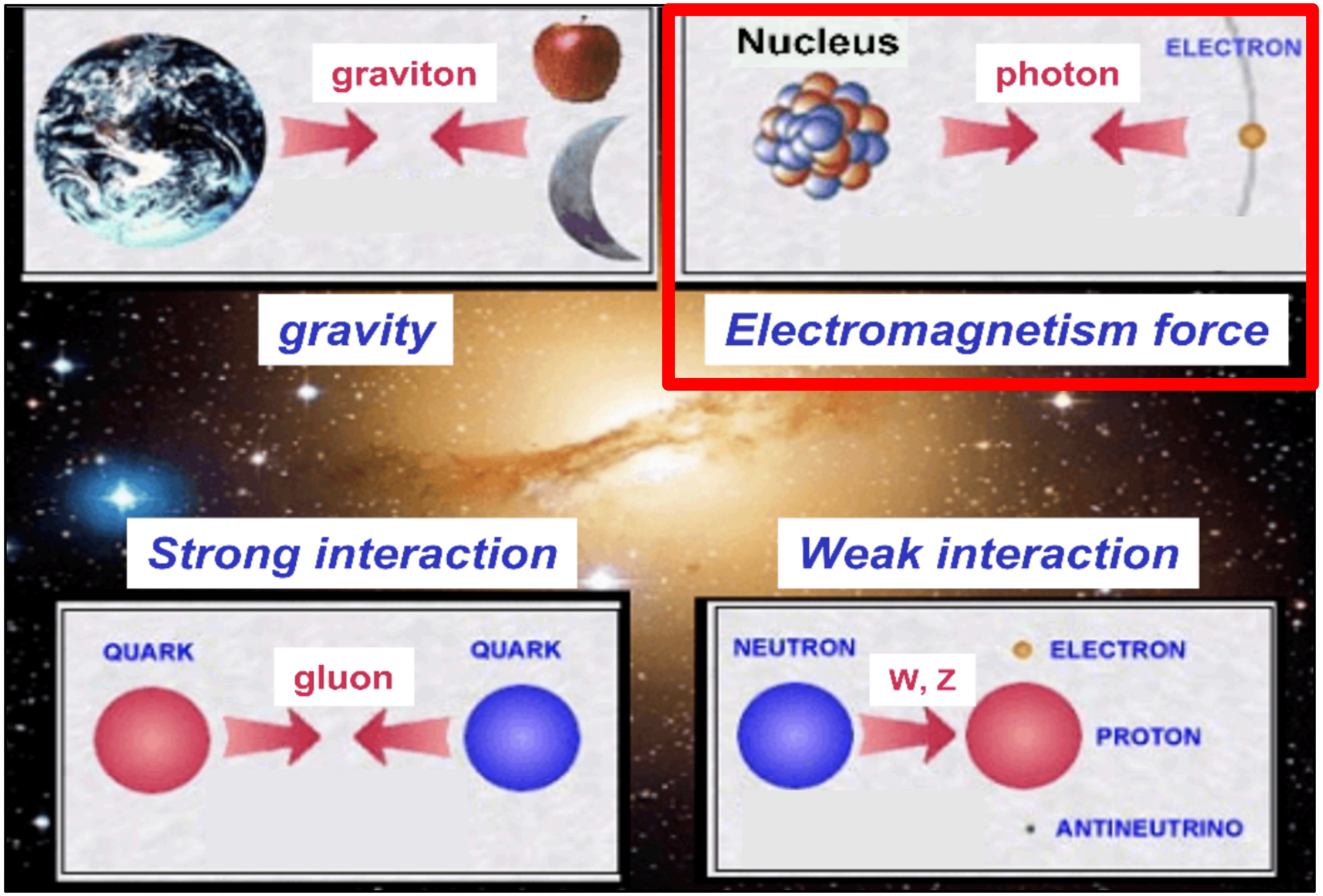
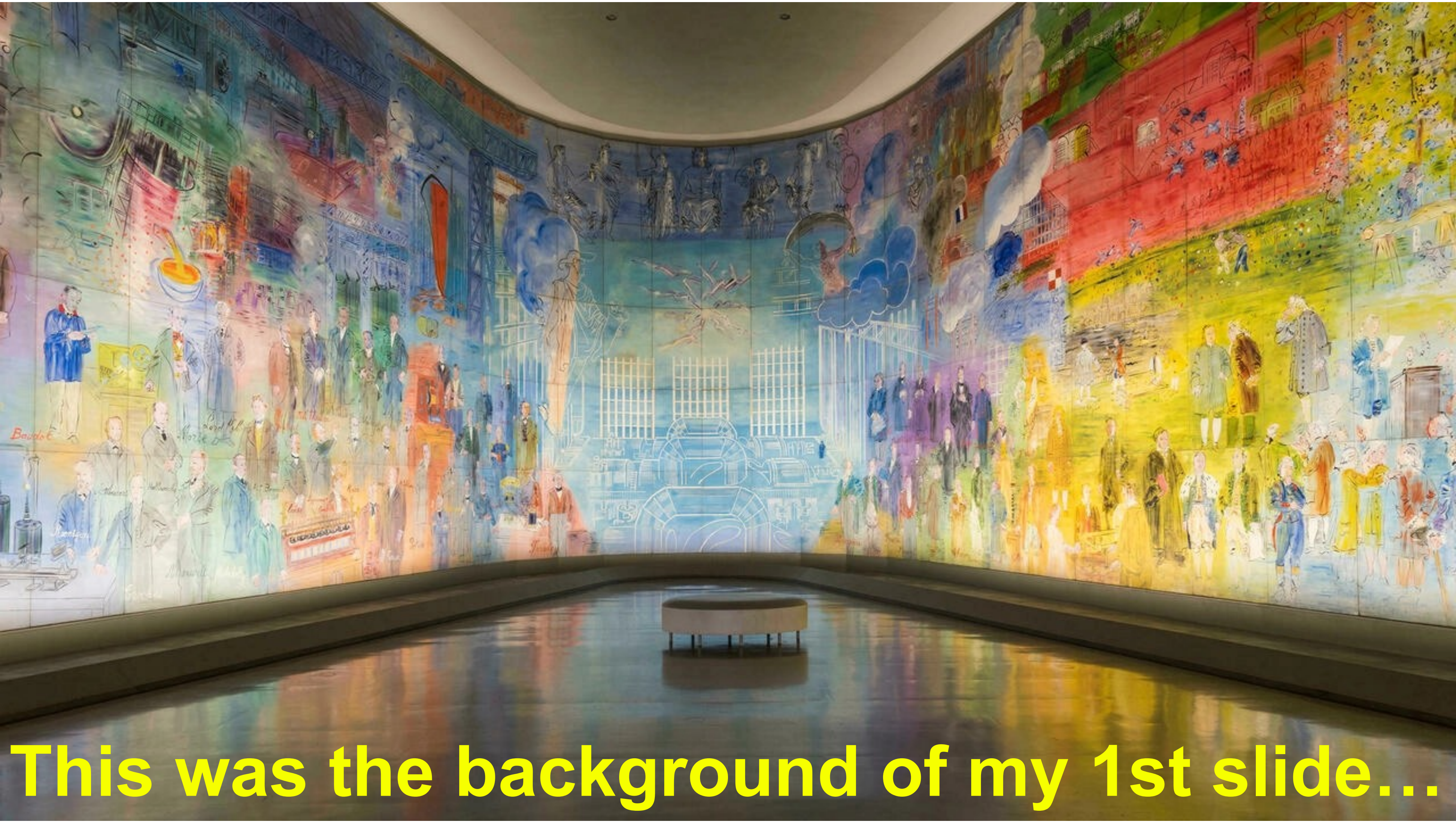




This was the background of my 1st slide…

Do you know what it is?

=> It’s the world’s largest painting (600 m^2)…

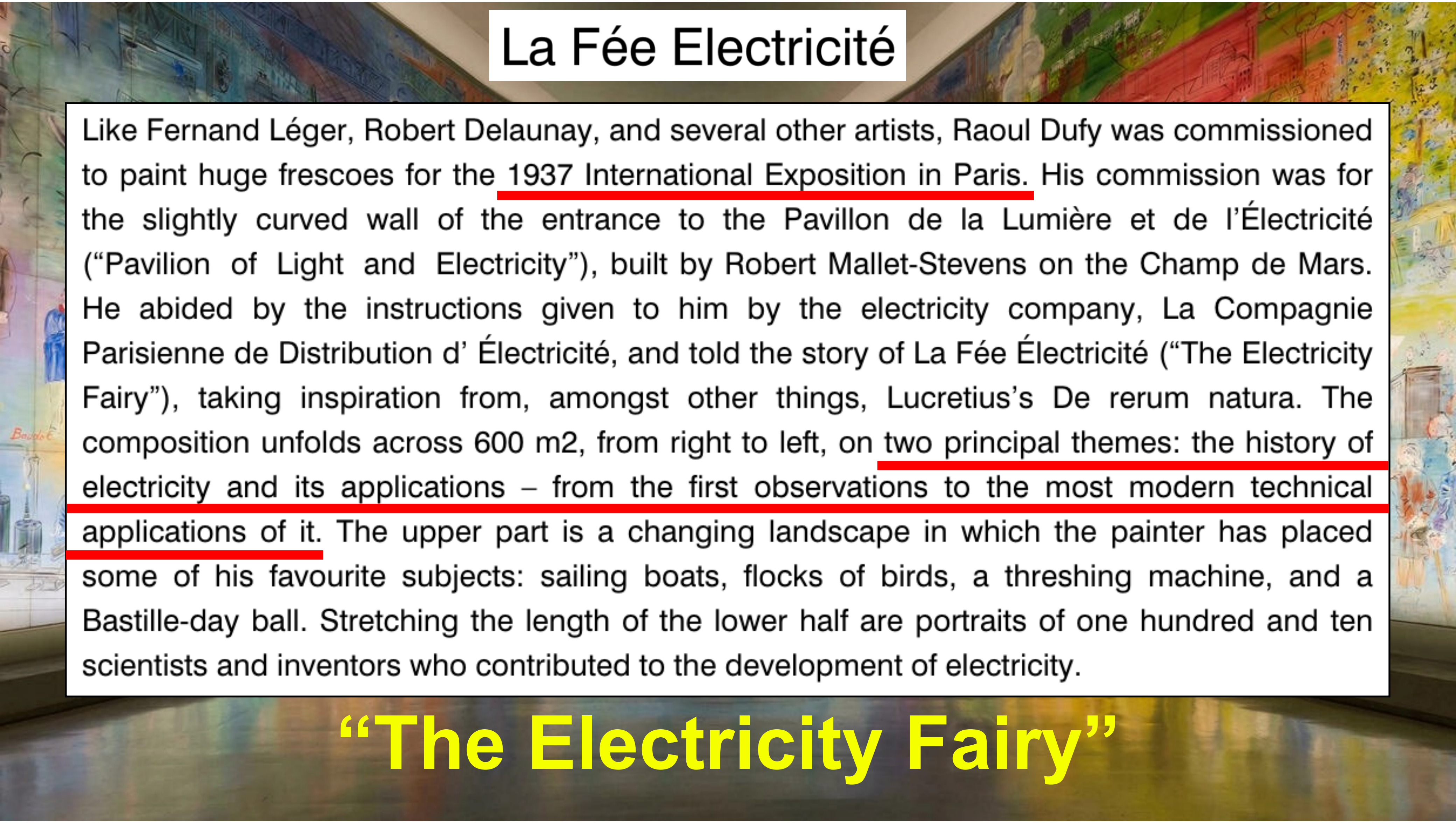
=> It's the world's largest painting (600 m^2)… from Raoul Dufy in Paris's Museum of Modern Art…

“The Electricity Fairy”

La Fée Electricité

“The Electricity Fairy”

La Fée Electricité



Like Fernand Léger, Robert Delaunay, and several other artists, Raoul Dufy was commissioned to paint huge frescoes for the 1937 International Exposition in Paris. His commission was for the slightly curved wall of the entrance to the Pavillon de la Lumière et de l'Électricité ("Pavilion of Light and Electricity"), built by Robert Mallet-Stevens on the Champ de Mars. He abided by the instructions given to him by the electricity company, La Compagnie Parisienne de Distribution d' Électricité, and told the story of La Fée Électricité ("The Electricity Fairy"), taking inspiration from, amongst other things, Lucretius's De rerum natura. The composition unfolds across 600 m2, from right to left, on two principal themes: the history of electricity and its applications – from the first observations to the most modern technical applications of it. The upper part is a changing landscape in which the painter has placed some of his favourite subjects: sailing boats, flocks of birds, a threshing machine, and a Bastille-day ball. Stretching the length of the lower half are portraits of one hundred and ten scientists and inventors who contributed to the development of electricity.

"The Electricity Fairy"

=> Electricity (and Magnetism),
i.e. ElectroMagnetism (EM), is the (only) force
which is used for particle accelerators!

Lorentz force

The motion of a charged particle (proton) in a beam transport channel or a circular accelerator is governed by the LORENTZ FORCE

$$\vec{F} = e\left(\vec{E} + \vec{\upsilon} \times \vec{B}\right)$$

Lorentz force

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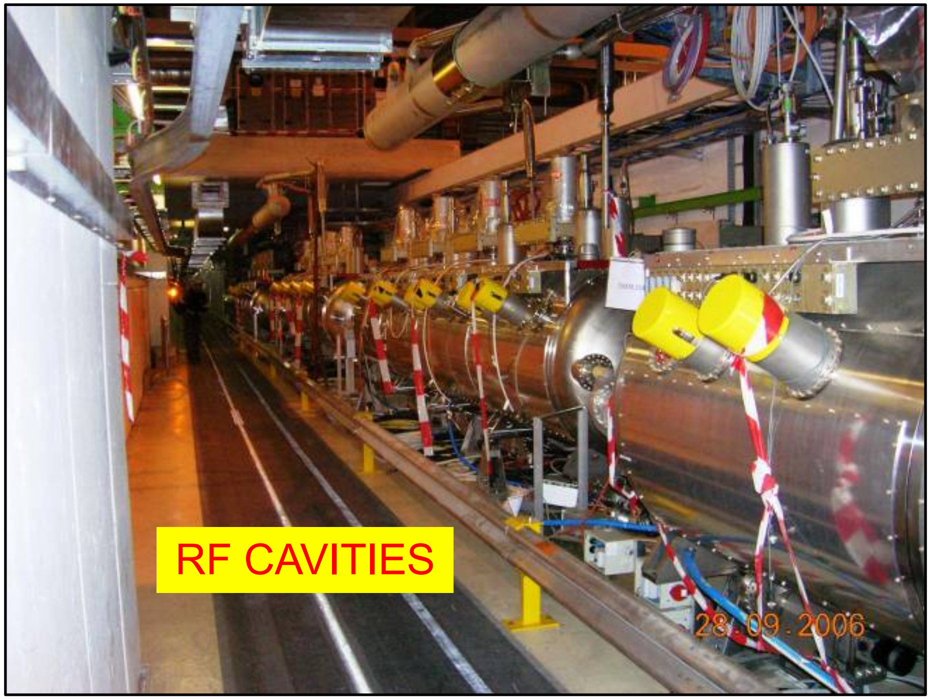

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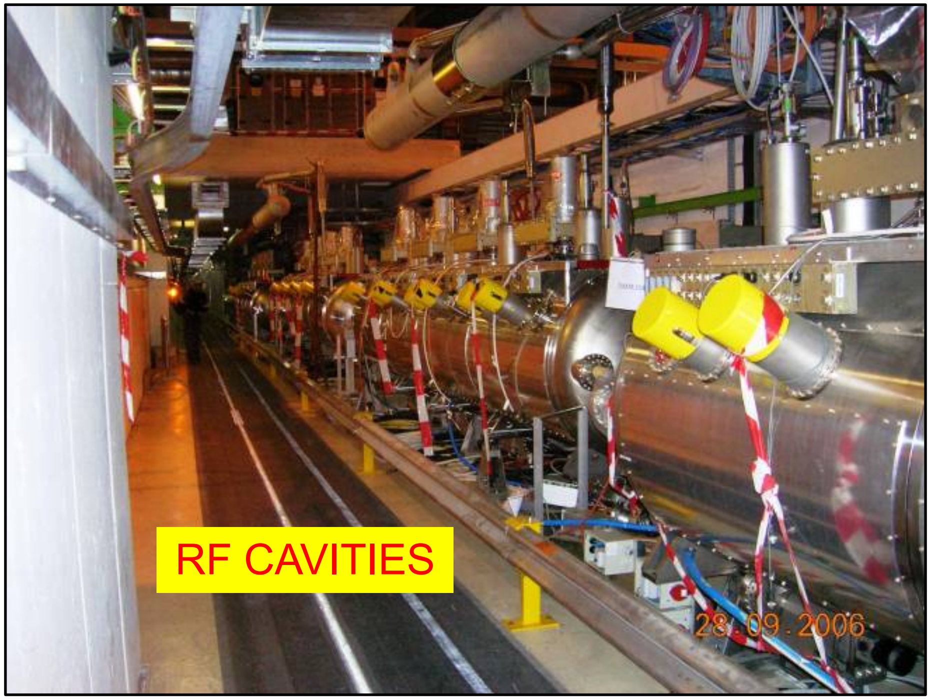
RF CAVITIES


MAGNETS

Lorentz force

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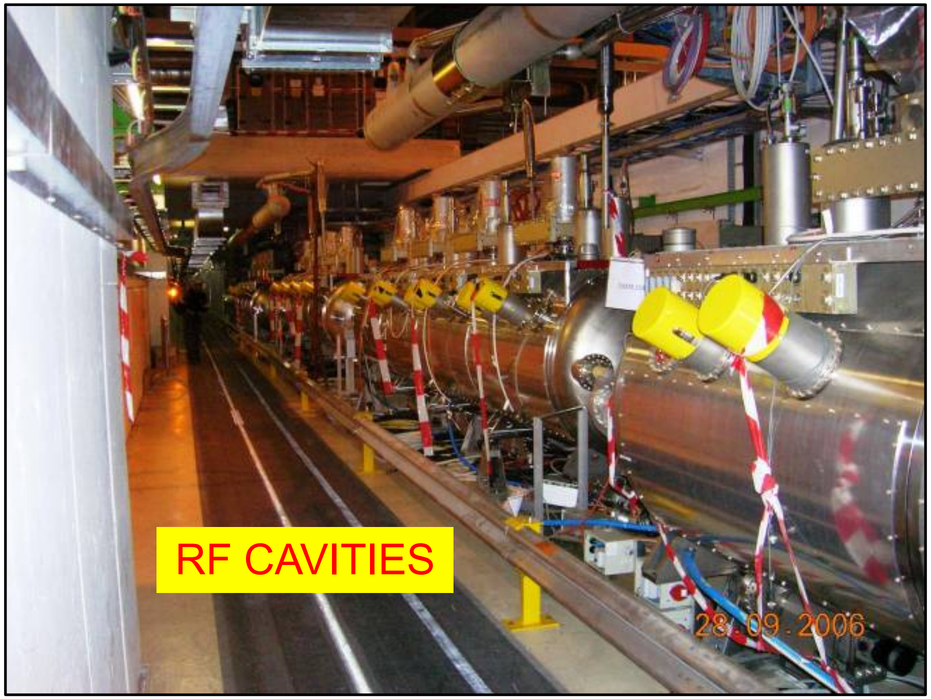
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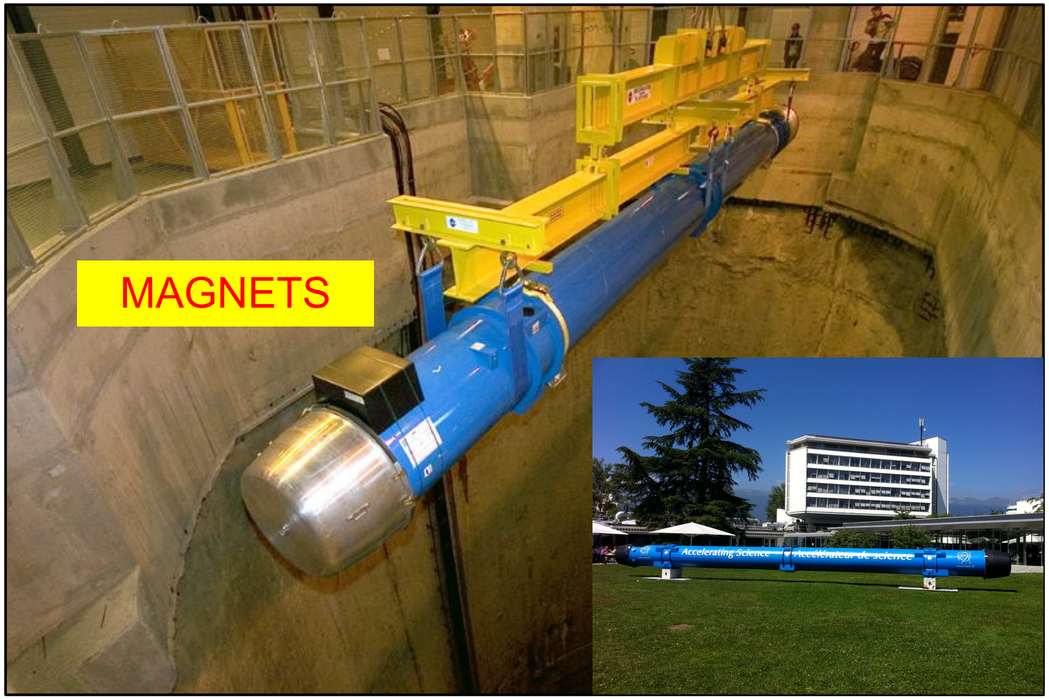
MAGNETS

Lorentz force

$$\vec{F} = e\left(\vec{E} + \vec{\upsilon} \times \vec{B}\right)$$

- **Cartesian (x,y,s)**

$$F_x = e\left(E_x - \upsilon\, B_y\right)$$

$$F_y = e\left(E_y + \upsilon\, B_x\right)$$

$$F_s = e\, E_s$$

- **Cylindrical (r,θ,s)**

$$F_r = e\left(E_r - \upsilon\, B_\vartheta\right)$$

$$F_\vartheta = e\left(E_\vartheta + \upsilon\, B_r\right)$$

$$F_s = e\, E_s$$

Lorentz force

$$\vec{F} = e\left(\vec{E} + \vec{\upsilon} \times \vec{B}\right)$$

- **Cartesian (x,y,s)**

Transverse magnetic field in MAGNETS to guide and confine the particles

- **Cylindrical (r,θ,s)**

$$F_x = e\left(E_x - \upsilon\, B_y\right)$$

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Lorentz force

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Transverse magnetic field in MAGNETS to guide and confine the particles

Longitudinal electric field in RF CAVITIES to accelerate (or decelerate) the particles

- **Cylindrical (r,θ,s)**

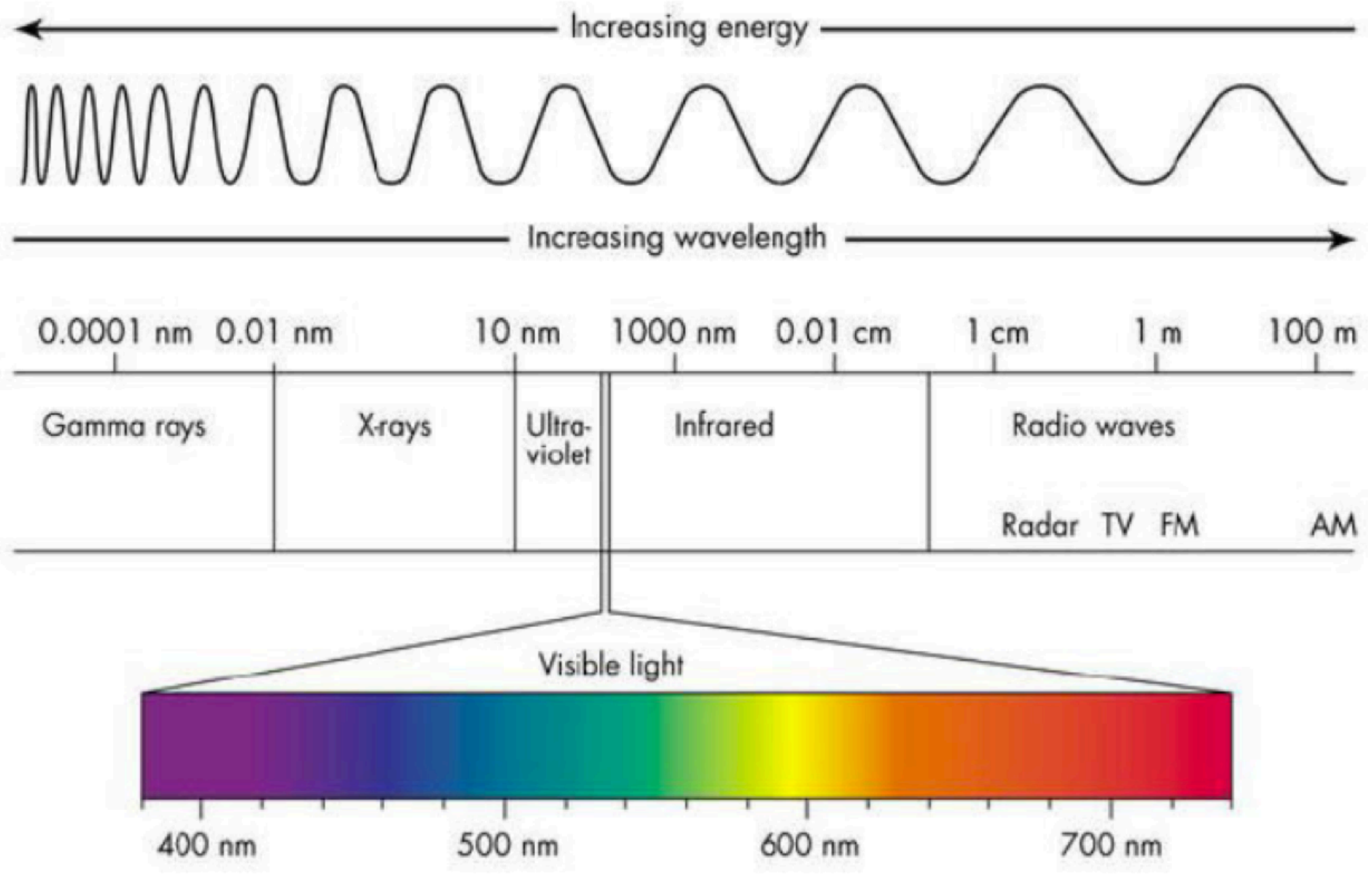
$$F_r = e \left(E_r - \upsilon \, B_\vartheta \right)$$

$$F_\vartheta = e \left(E_\vartheta + \upsilon \, B_r \right)$$

$$F_s = e \, E_s$$

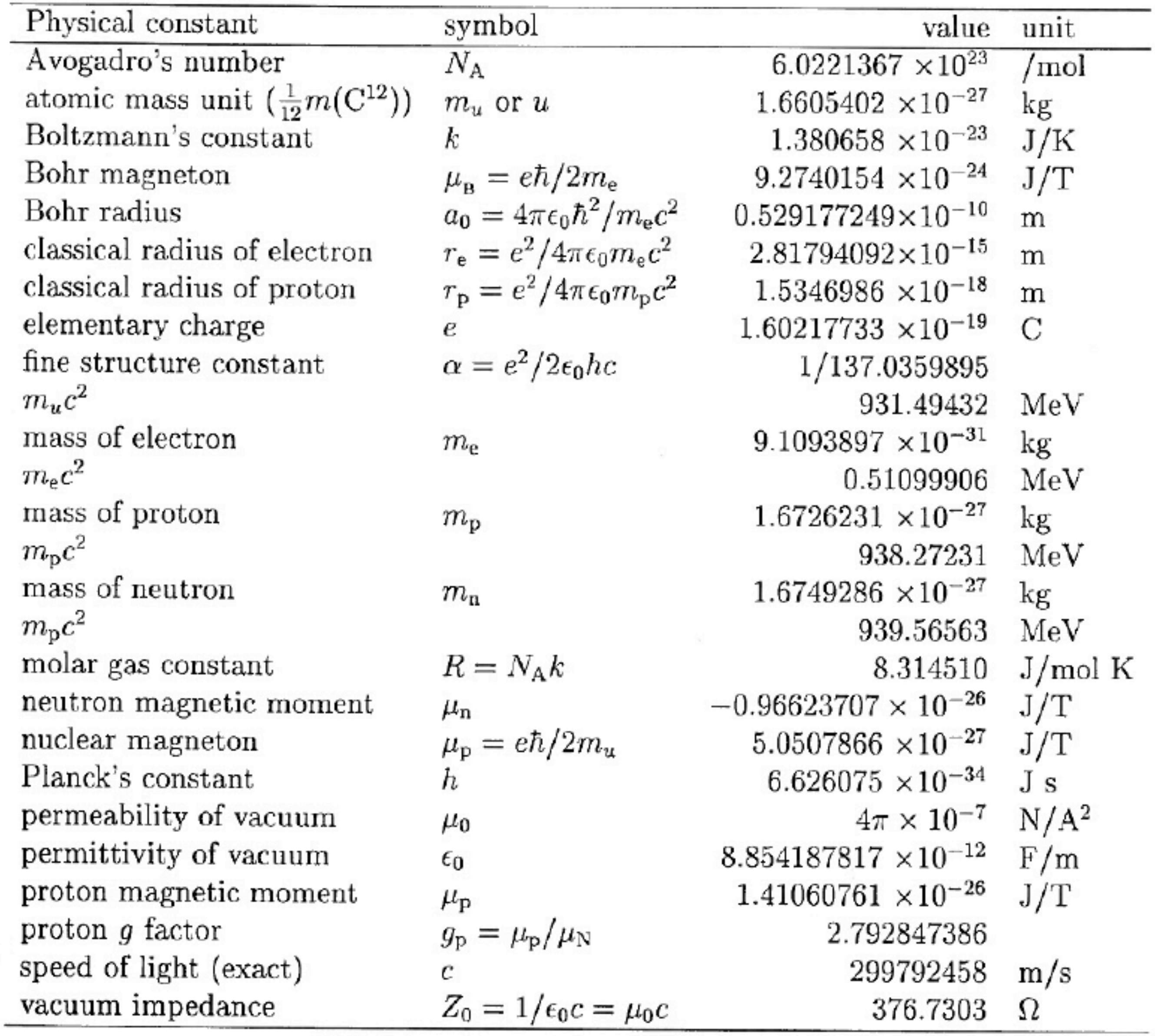
Electromagnetic spectrum

$$E = \frac{h\,c}{\lambda_{wl}}$$



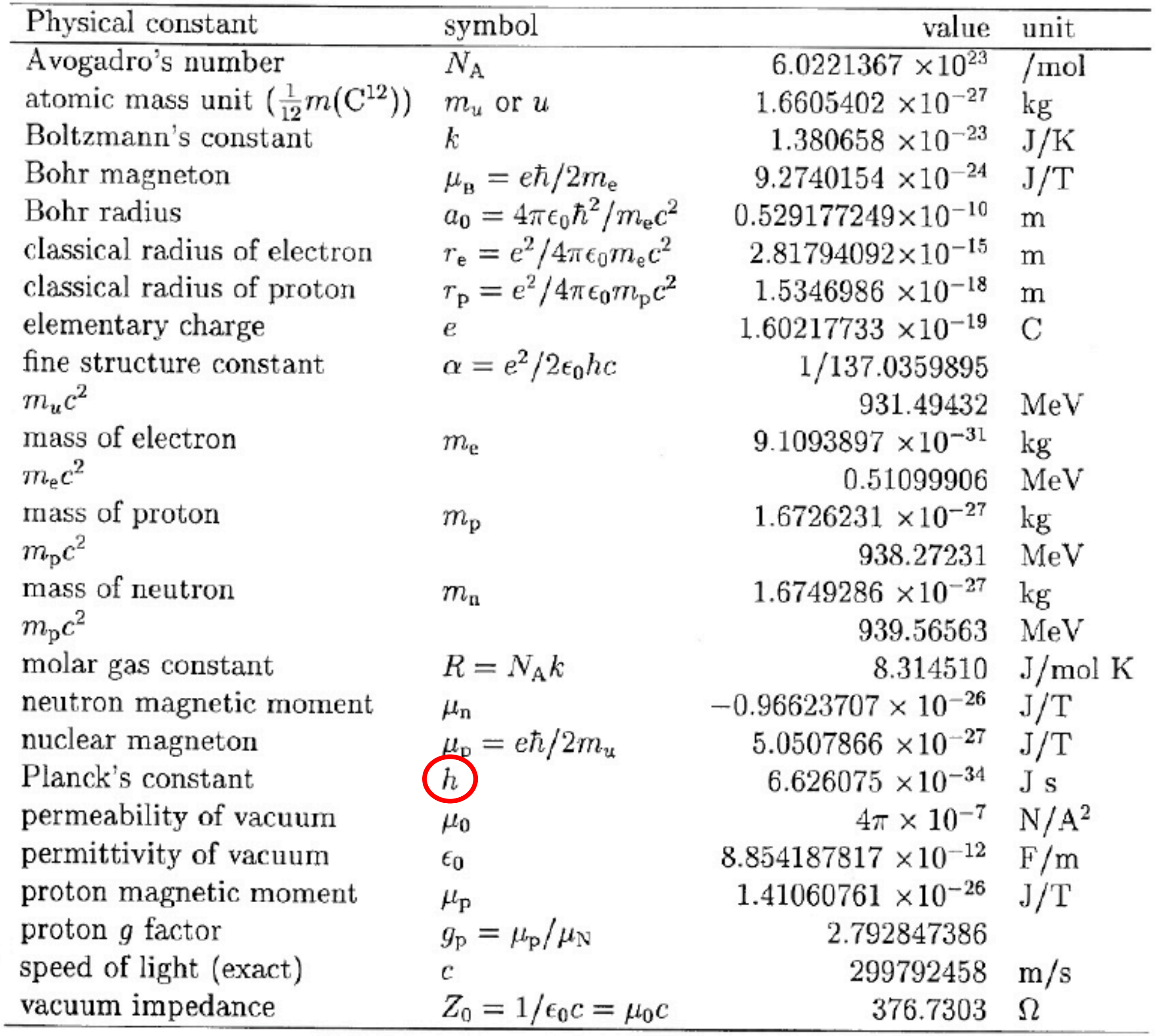
$$\lambda_{wl} = \frac{c}{f}$$

Reminder: Fundamental physical constants



Physical constant	symbol	value	unit
Avogadro's number	N_A	6.0221367×10^{23}	/mol
atomic mass unit ($\frac{1}{12}m(C^{12})$)	m_u or u	$1.6605402 \times 10^{-27}$	kg
Boltzmann's constant	k	1.380658×10^{-23}	J/K
Bohr magneton	$\mu_B = e\hbar/2m_e$	$9.2740154 \times 10^{-24}$	J/T
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/m_e c^2$	$0.529177249 \times 10^{-10}$	m
classical radius of electron	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.81794092 \times 10^{-15}$	m
classical radius of proton	$r_p = e^2/4\pi\epsilon_0 m_p c^2$	$1.5346986 \times 10^{-18}$	m
elementary charge	e	$1.60217733 \times 10^{-19}$	C
fine structure constant	$\alpha = e^2/2\epsilon_0 hc$	$1/137.0359895$	
$m_u c^2$		931.49432	MeV
mass of electron	m_e	$9.1093897 \times 10^{-31}$	kg
$m_e c^2$		0.51099906	MeV
mass of proton	m_p	$1.6726231 \times 10^{-27}$	kg
$m_p c^2$		938.27231	MeV
mass of neutron	m_n	$1.6749286 \times 10^{-27}$	kg
$m_p c^2$		939.56563	MeV
molar gas constant	$R = N_A k$	8.314510	J/mol K
neutron magnetic moment	μ_n	$-0.96623707 \times 10^{-26}$	J/T
nuclear magneton	$\mu_p = e\hbar/2m_u$	$5.0507866 \times 10^{-27}$	J/T
Planck's constant	h	6.626075×10^{-34}	J s
permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
permittivity of vacuum	ϵ_0	$8.854187817 \times 10^{-12}$	F/m
proton magnetic moment	μ_p	$1.41060761 \times 10^{-26}$	J/T
proton g factor	$g_p = \mu_p/\mu_N$	2.792847386	
speed of light (exact)	c	299792458	m/s
vacuum impedance	$Z_0 = 1/\epsilon_0 c = \mu_0 c$	376.7303	Ω

Reminder: Fundamental physical constants



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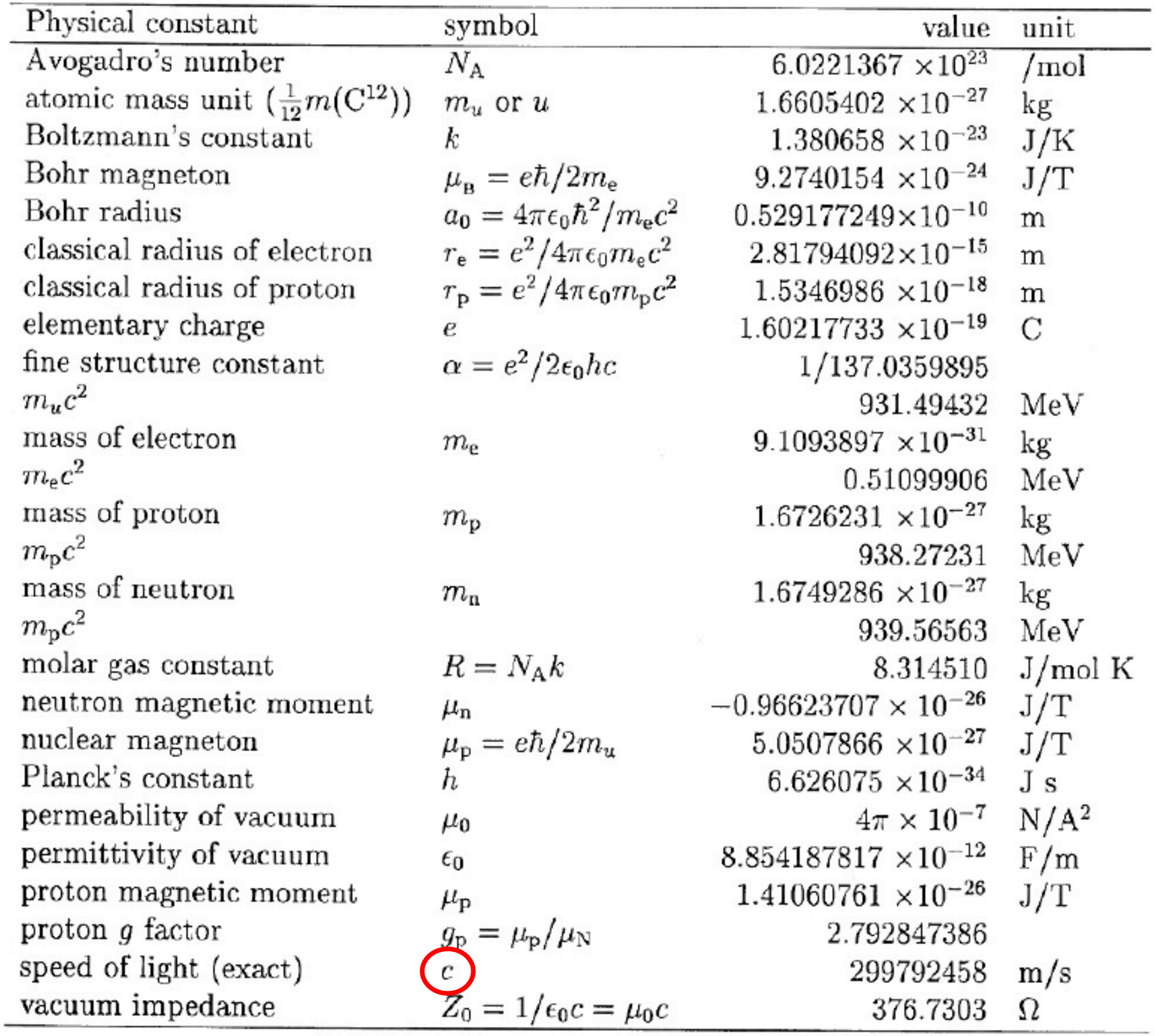
speed of light (exact)	c	299792458	m/s
vacuum impedance	$Z_0 = 1/\epsilon_0 c = \mu_0 c$	376.7303	Ω

Reminder: Fundamental physical constants



Physical constant	symbol	value	unit
Avogadro's number	N_{A}	6.0221367×10^{23}	/mol
atomic mass unit ($\frac{1}{12}m(\mathrm{C}^{12})$)	m_u or u	$1.6605402 \times 10^{-27}$	kg
Boltzmann's constant	k	1.380658×10^{-23}	J/K
Bohr magneton	$\mu_{\mathrm{B}} = e\hbar/2m_{\mathrm{e}}$	$9.2740154 \times 10^{-24}$	J/T
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/m_{\mathrm{e}}c^2$	$0.529177249 \times 10^{-10}$	m
classical radius of electron	$r_{\mathrm{e}} = e^2/4\pi\epsilon_0 m_{\mathrm{e}}c^2$	$2.81794092 \times 10^{-15}$	m
classical radius of proton	$r_{\mathrm{p}} = e^2/4\pi\epsilon_0 m_{\mathrm{p}}c^2$	$1.5346986 \times 10^{-18}$	m
elementary charge	e	$1.60217733 \times 10^{-19}$	C
fine structure constant	$\alpha = e^2/2\epsilon_0 hc$	$1/137.0359895$	
$m_u c^2$		931.49432	MeV
mass of electron	m_{e}	$9.1093897 \times 10^{-31}$	kg
$m_{\mathrm{e}}c^2$		0.51099906	MeV
mass of proton	m_{p}	$1.6726231 \times 10^{-27}$	kg
$m_{\mathrm{p}}c^2$		938.27231	MeV
mass of neutron	m_{n}	$1.6749286 \times 10^{-27}$	kg
$m_{\mathrm{p}}c^2$		939.56563	MeV
molar gas constant	$R = N_{\mathrm{A}}k$	8.314510	J/mol K
neutron magnetic moment	μ_{n}	$-0.96623707 \times 10^{-26}$	J/T
nuclear magneton	$\mu_{\mathrm{p}} = e\hbar/2m_u$	$5.0507866 \times 10^{-27}$	J/T
Planck's constant	h	6.626075×10^{-34}	J s
permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
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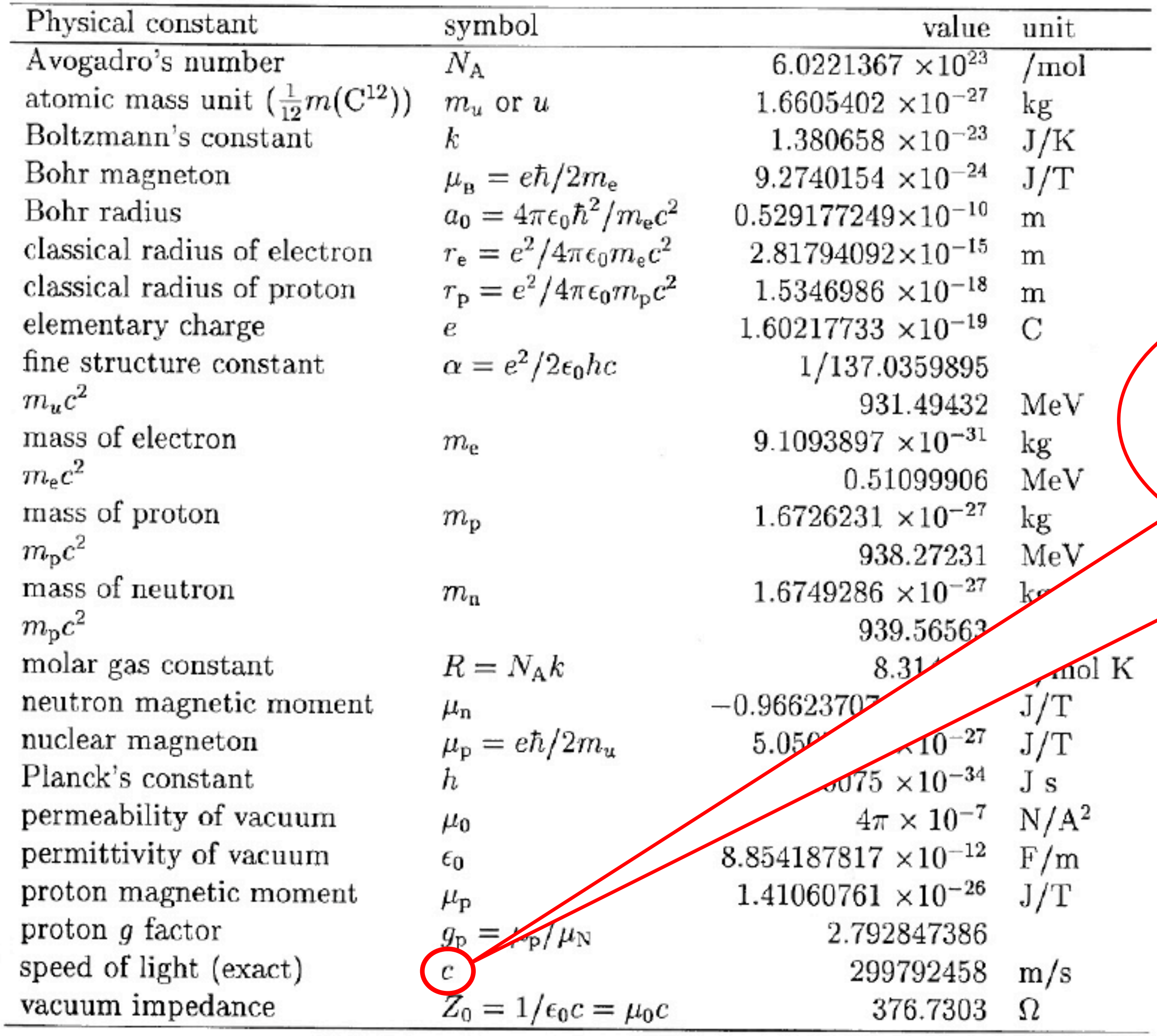

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$m_p c^2$		939.56563	
molar gas constant	$R = N_A k$	8.31	mol K
neutron magnetic moment	μ_n	−0.96623707	J/T
nuclear magneton	$\mu_p = e\hbar/2m_u$	5.050 $\times 10^{-27}$	J/T
Planck's constant	h	075 $\times 10^{-34}$	J s
permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
permittivity of vacuum	ϵ_0	$8.854187817 \times 10^{-12}$	F/m
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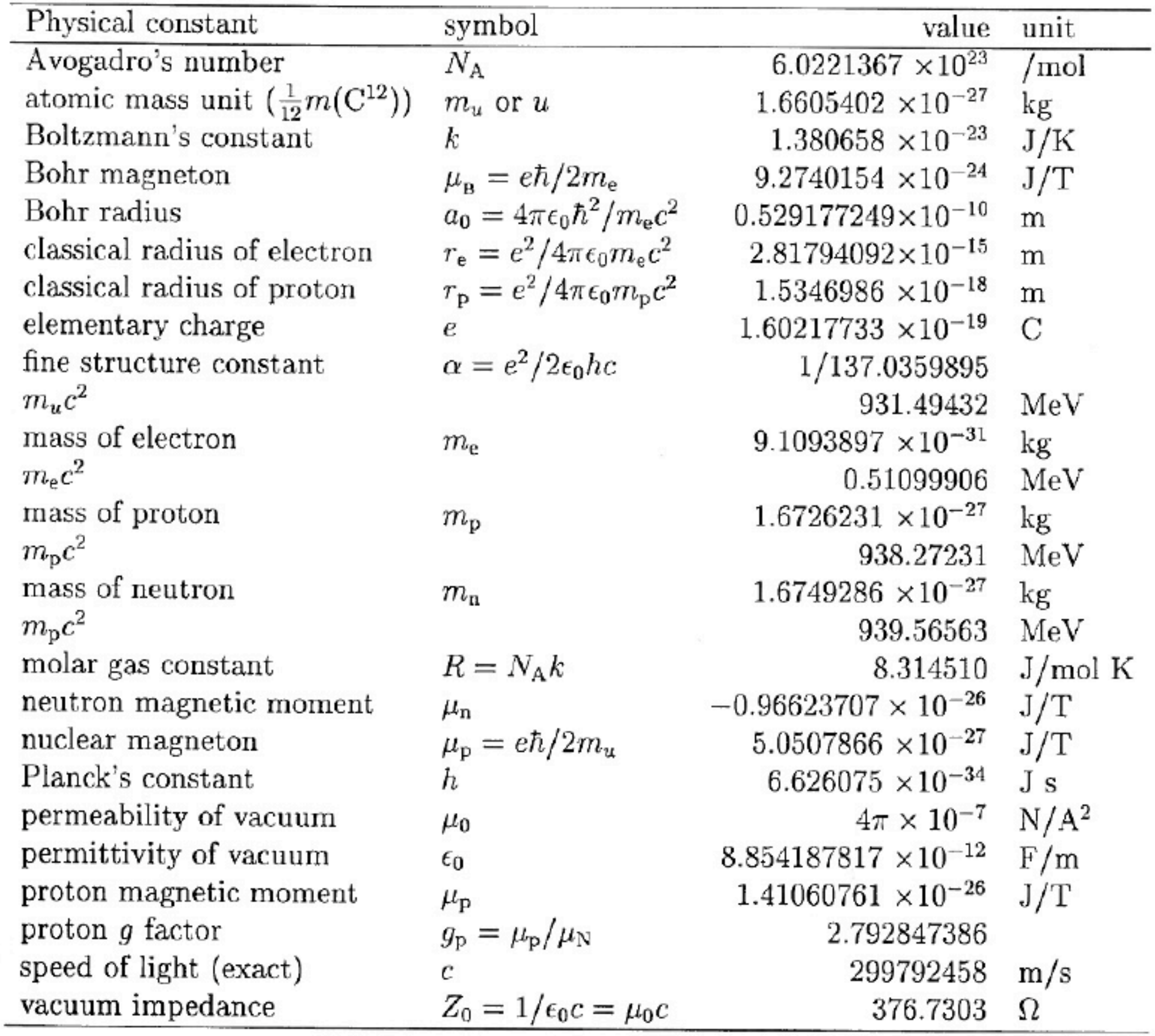
How can we express the speed of light c as a function of some parameters of this table?

Reminder: Fundamental physical constants



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$$c = \frac{1}{\sqrt{\epsilon_0\,\mu_0}} \approx 300\,000 \text{ km/s}$$

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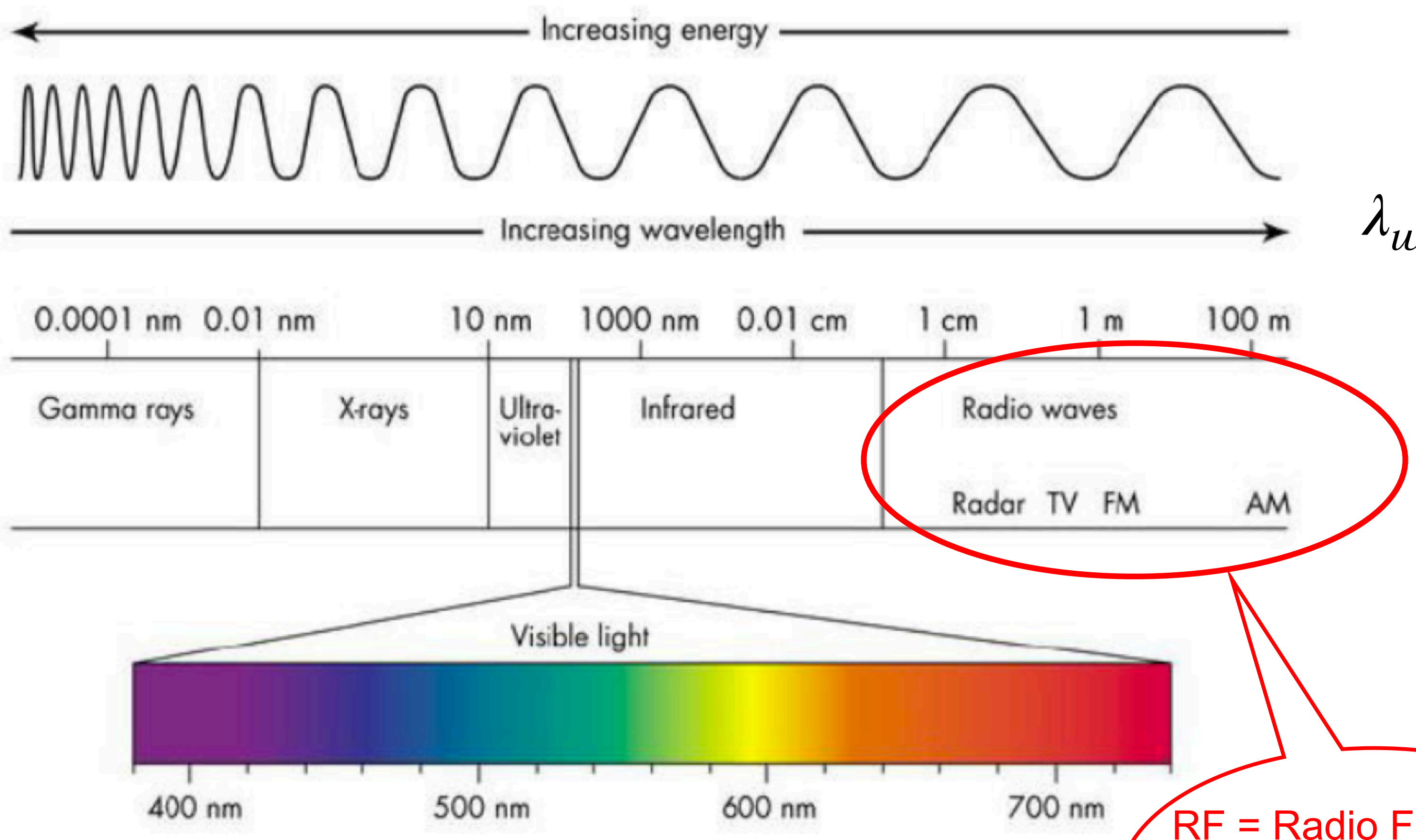
The identification of light with an EM wave (with phase velocity related to the electric permittivity and magnetic permeability) was one of the great achievements of 19th century physics

$$\boxed{c = \frac{1}{\sqrt{\epsilon_0\,\mu_0}} \approx 300\,000 \text{ km/s}}$$

Electromagnetic spectrum

$$E = \frac{h\,c}{\lambda_{wl}}$$



$$\lambda_{wl} = \frac{c}{f}$$

RF = Radio Frequency (from few kHz to hundreds of GHz) used for particle accelerators

Electromagnetic spectrum

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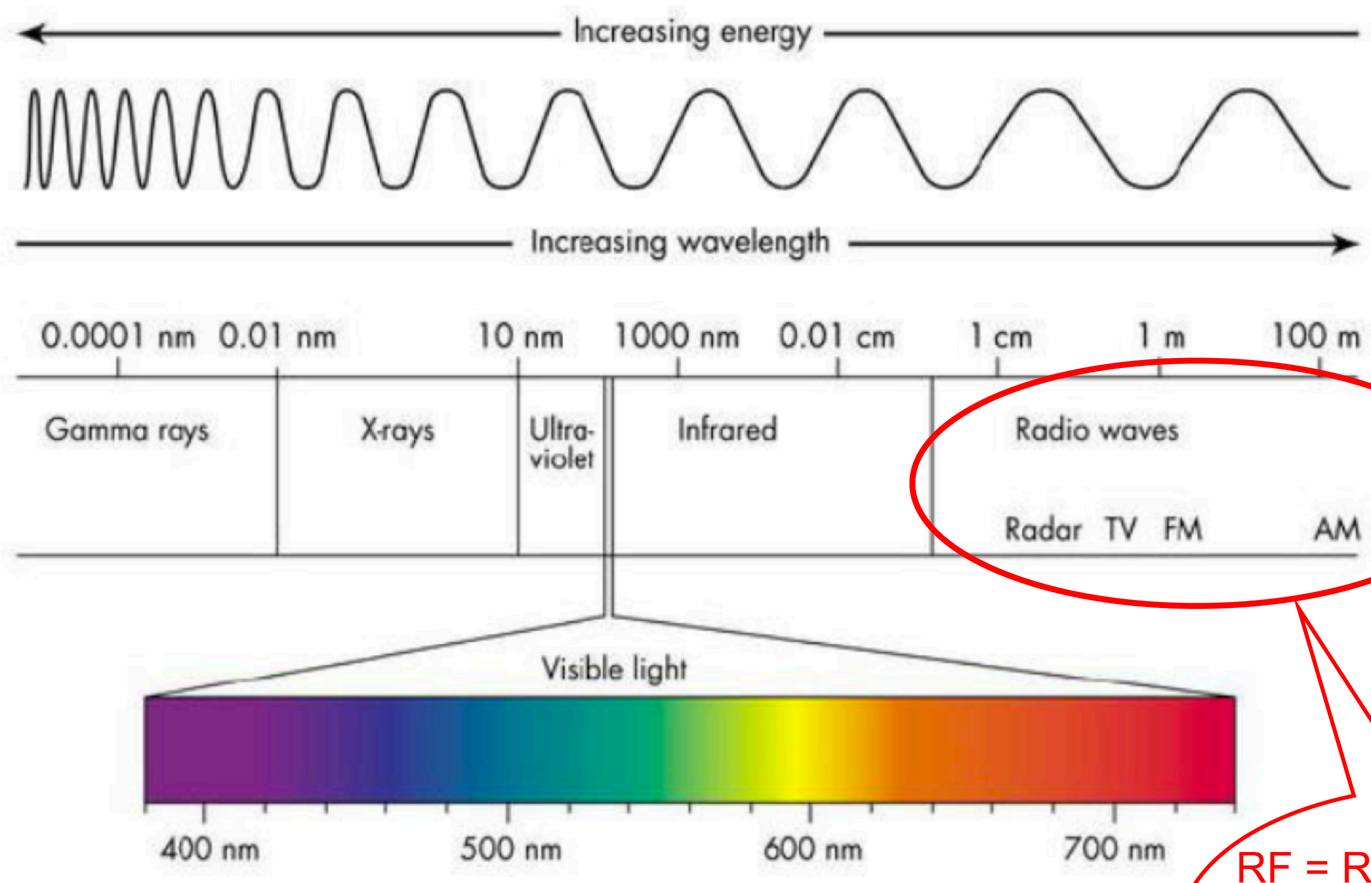

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What is the frequency of your preferred radio?

Electromagnetic spectrum

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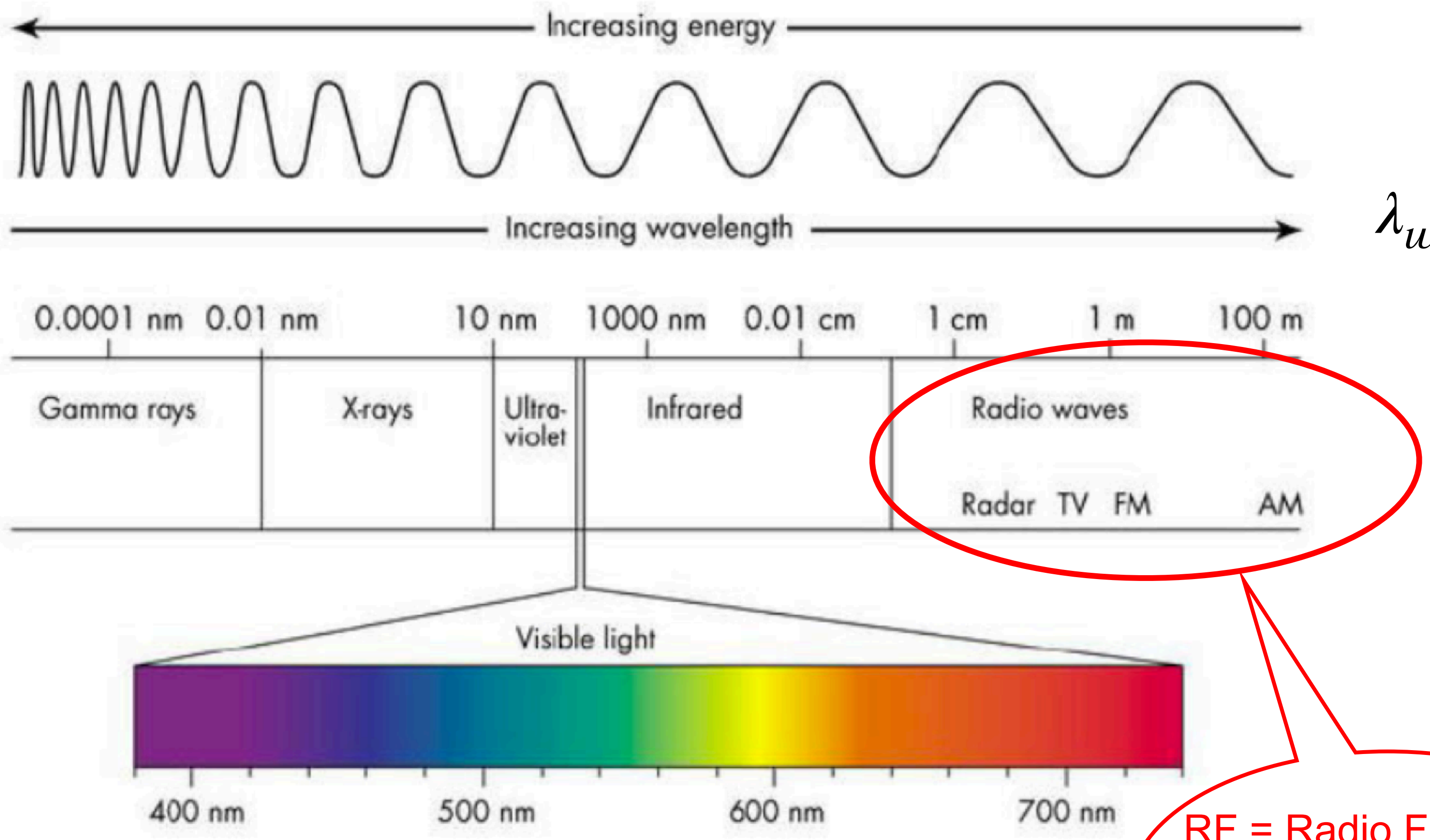

$$\lambda_{wl} = \frac{c}{f}$$

=> For me it is 93.5 MHz (France Inter)

RF = Radio Frequency (from few kHz to hundreds of GHz) used for particle accelerators

Relationship between the force on an object and the motion of this object?

Classical mechanics

Classical mechanics

- Do the Newtonian, Lagrangian and Hamiltonian mechanics describe the same physical mechanisms?
  - Yes
  - No

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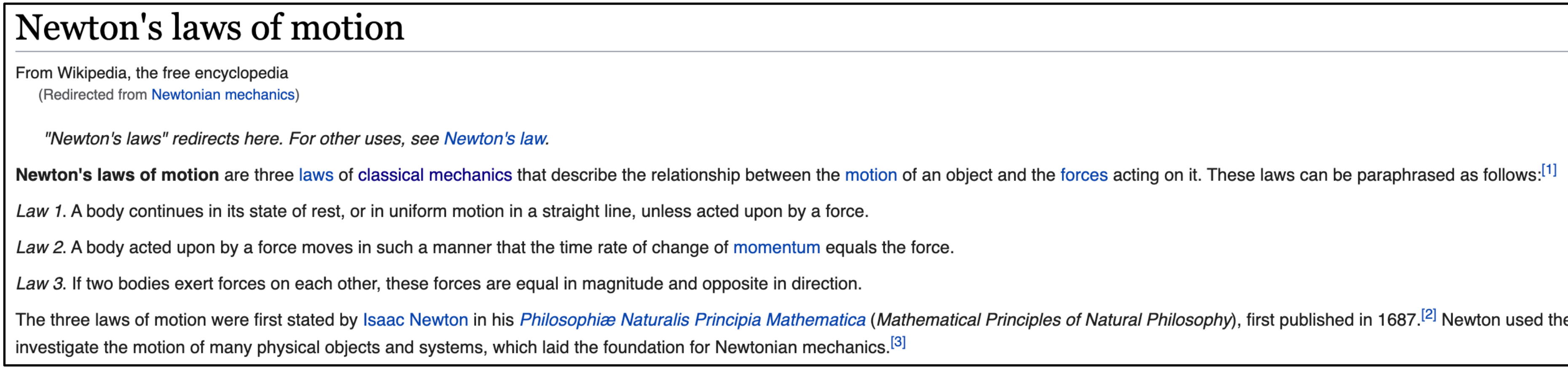

Newton's laws of motion

From Wikipedia, the free encyclopedia
(Redirected from Newtonian mechanics)

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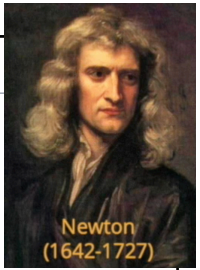
The three laws of motion were first stated by Isaac Newton in his *Philosophiæ Naturalis Principia Mathematica* (*Mathematical Principles of Natural Philosophy*), first published in 1687.[2] Newton used them to explain and investigate the motion of many physical objects and systems, which laid the foundation for Newtonian mechanics.[3]



Newton
(1642-1727)

Classical mechanics

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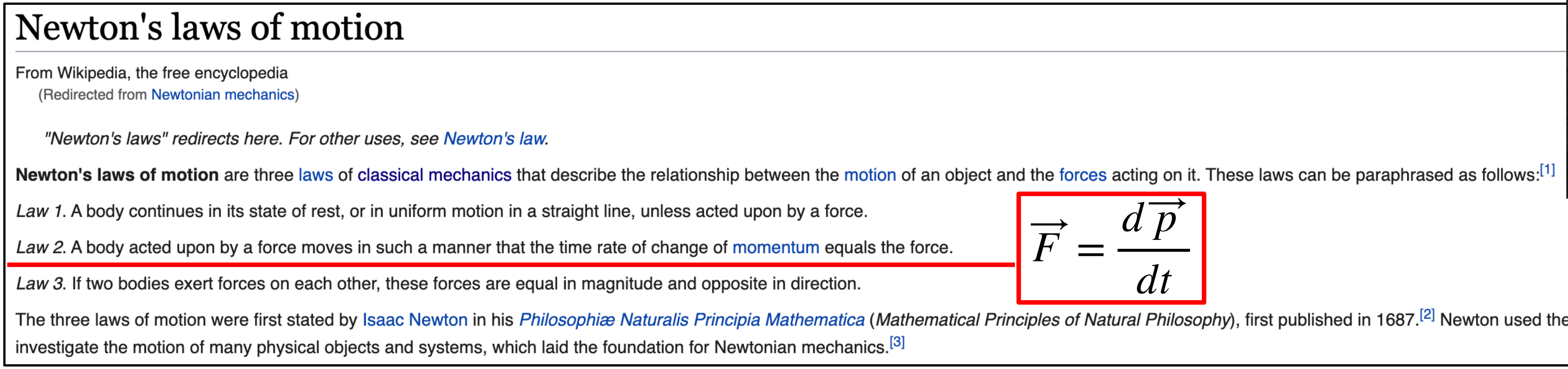

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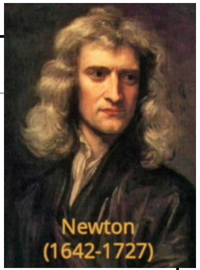
$$\boxed{\vec{F} = \frac{d\,\vec{p}}{dt}}$$



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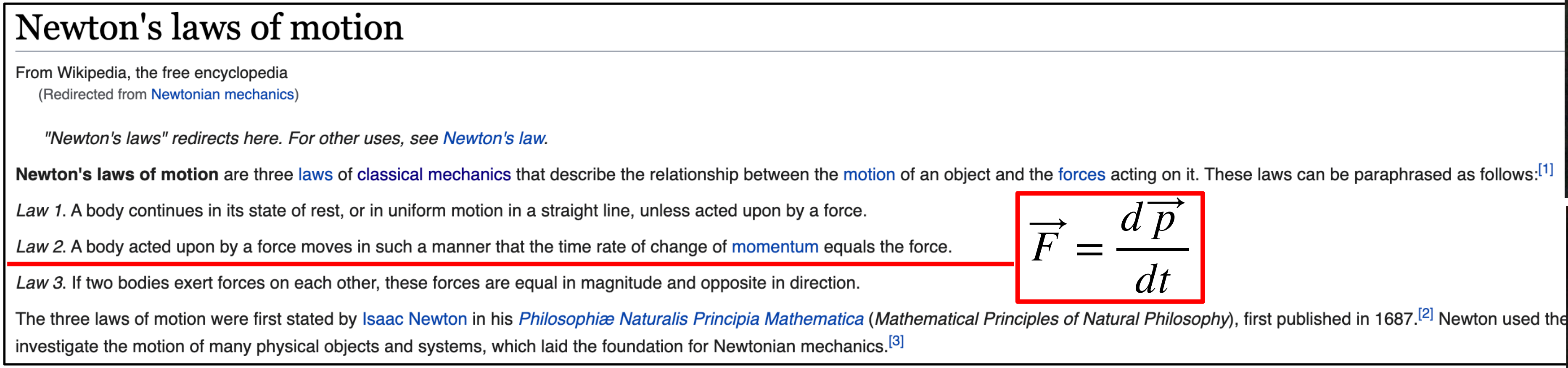

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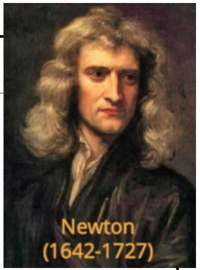

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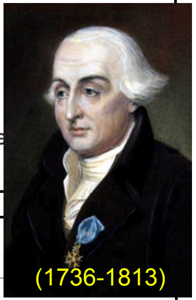
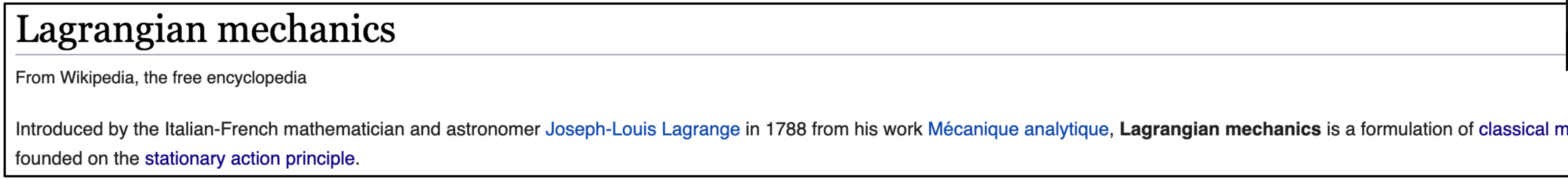


(1736-1813)



Lagrangian mechanics

From Wikipedia, the free encyclopedia

Introduced by the Italian-French mathematician and astronomer Joseph-Louis Lagrange in 1788 from his work Mécanique analytique, **Lagrangian mechanics** is a formulation of classical mechanics and is founded on the stationary action principle.

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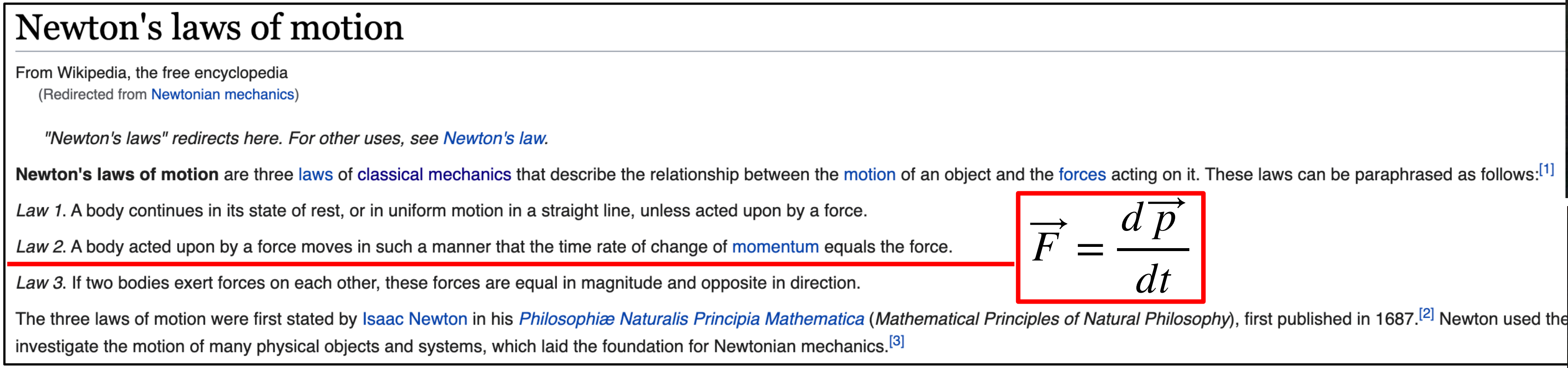

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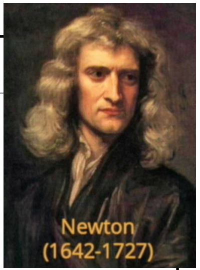
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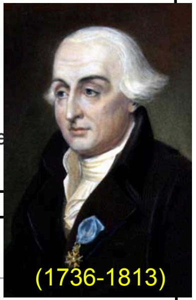
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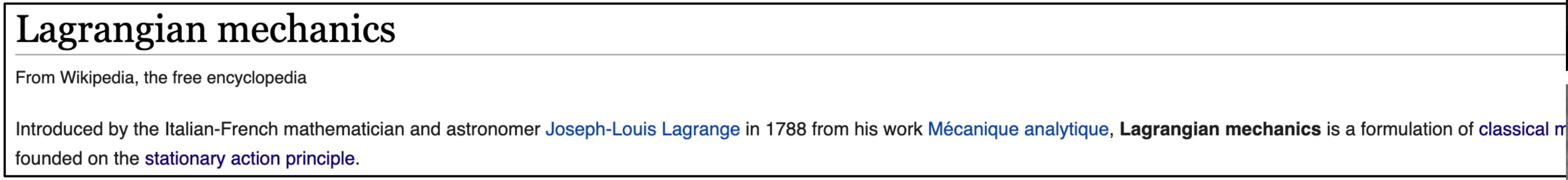

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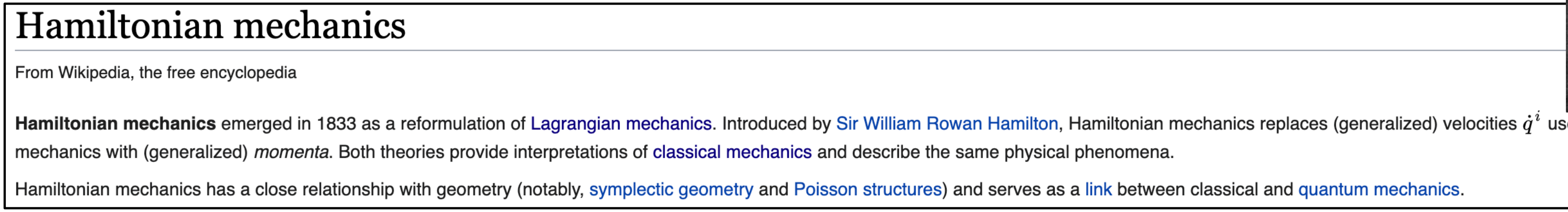
(1805-1865)



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Hamiltonian mechanics has a close relationship with geometry (notably, symplectic geometry and Poisson structures) and serves as a link between classical and quantum mechanics.

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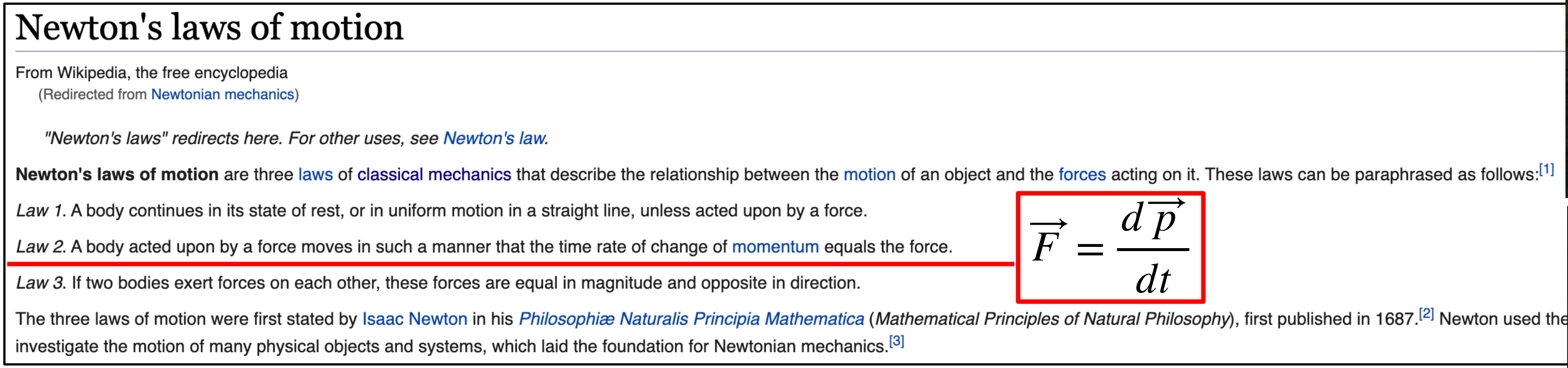

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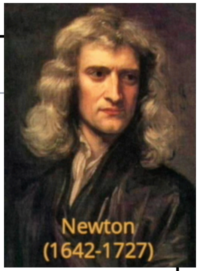

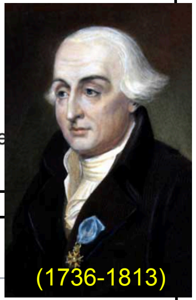

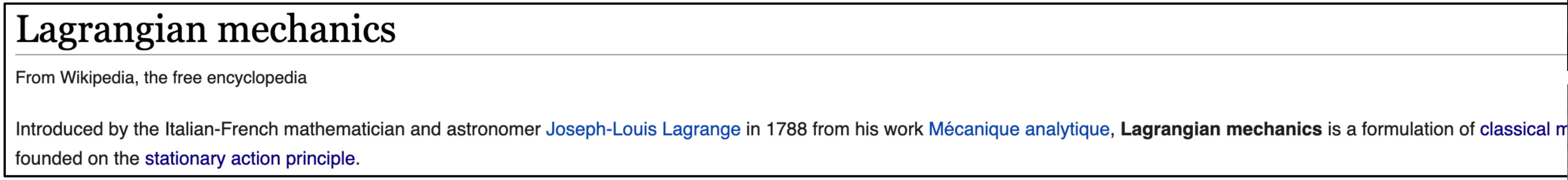

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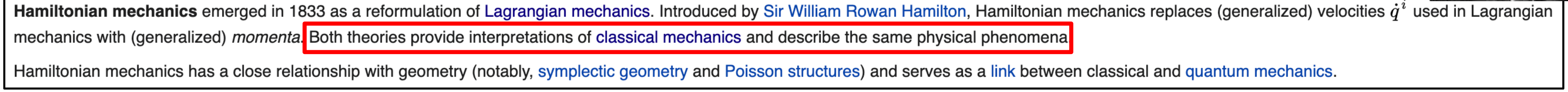

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Classical mechanics

- For particle accelerators, which one(s) of the following major sub-field of mechanics need to be included?
  - Quantum mechanics mainly and sometimes special relativity
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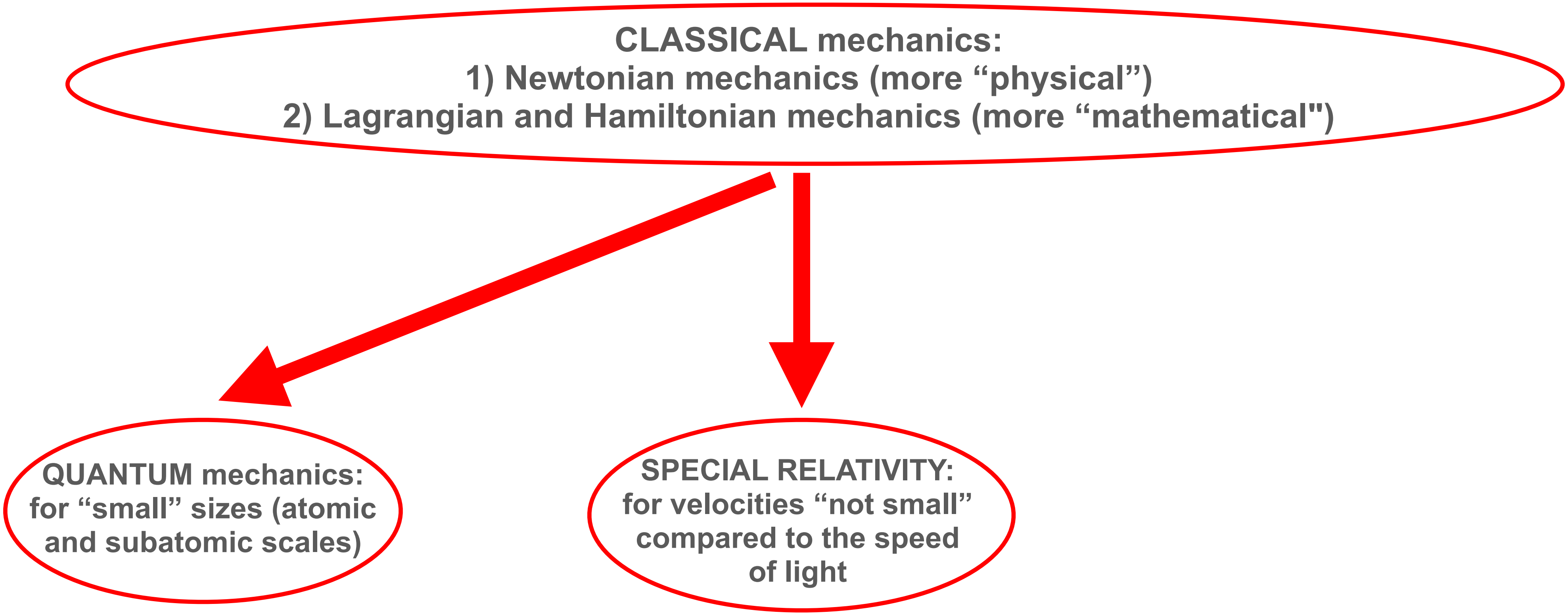
Classical mechanics



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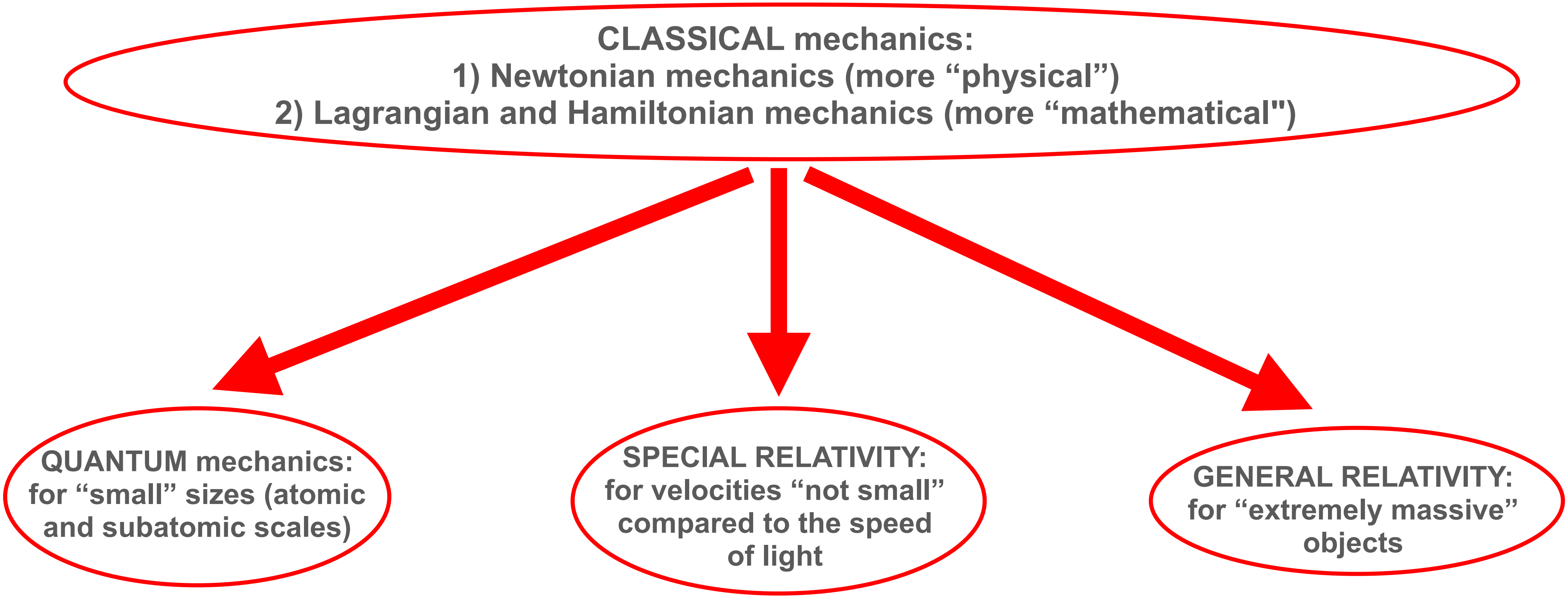

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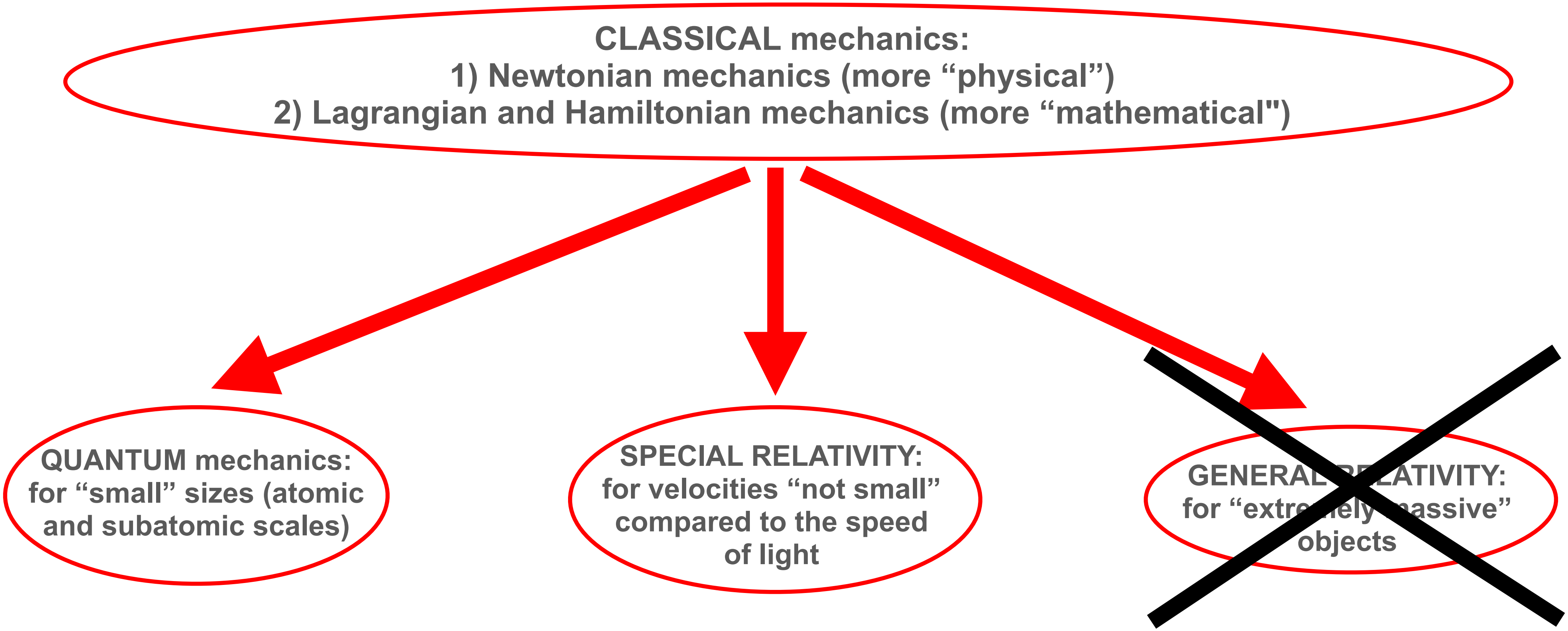

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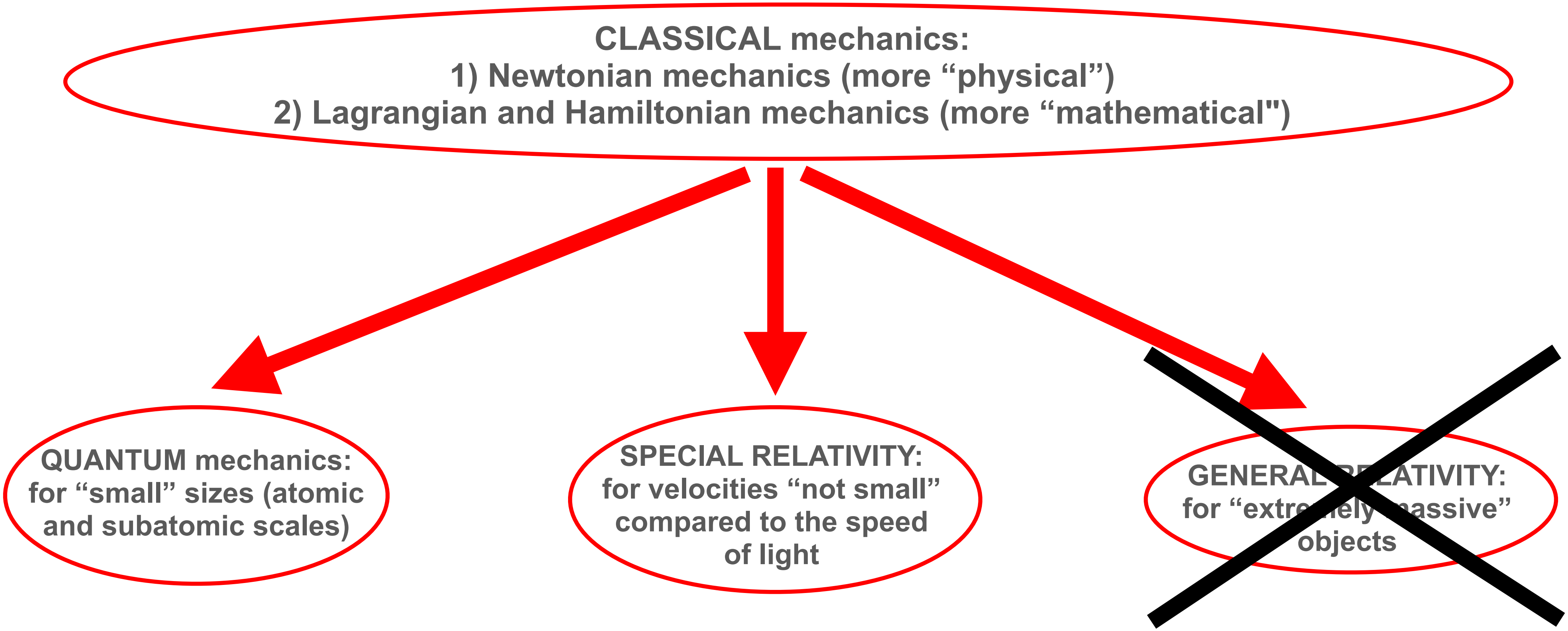

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Note that since some time some people want to test general relativity on particle accelerators

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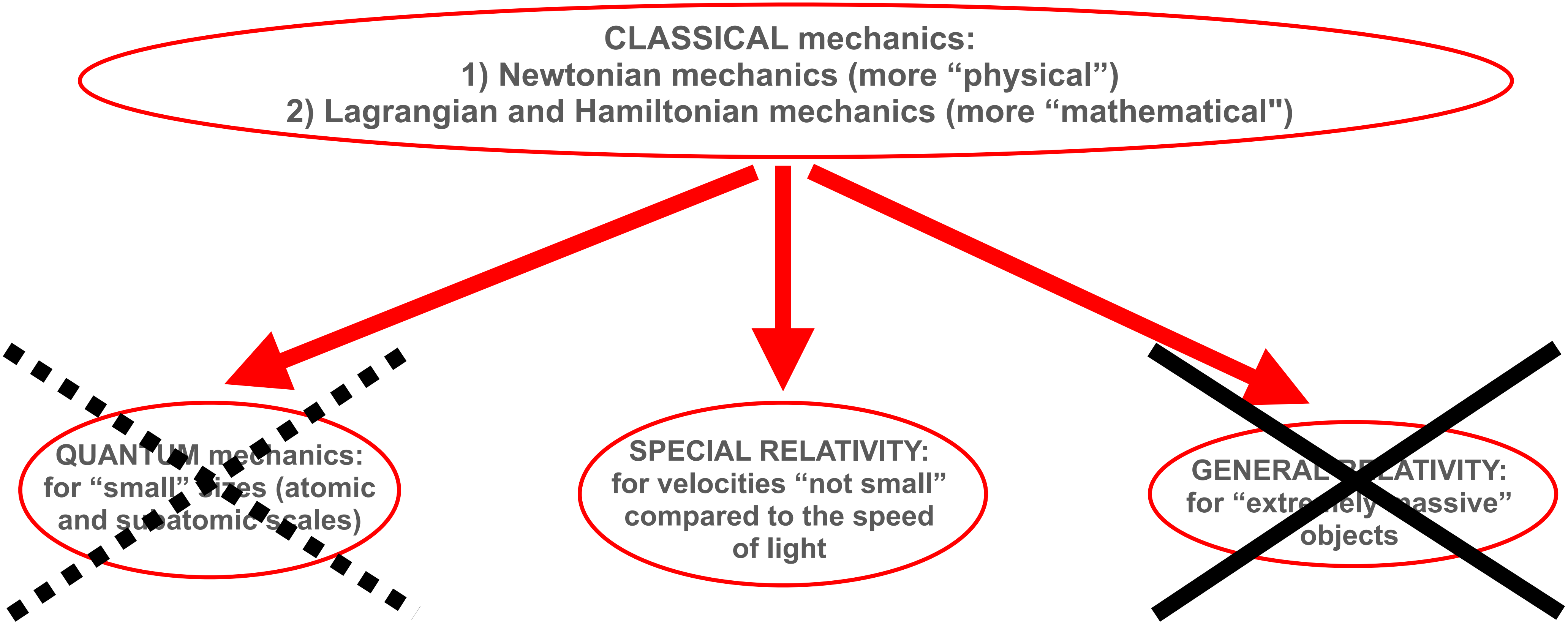

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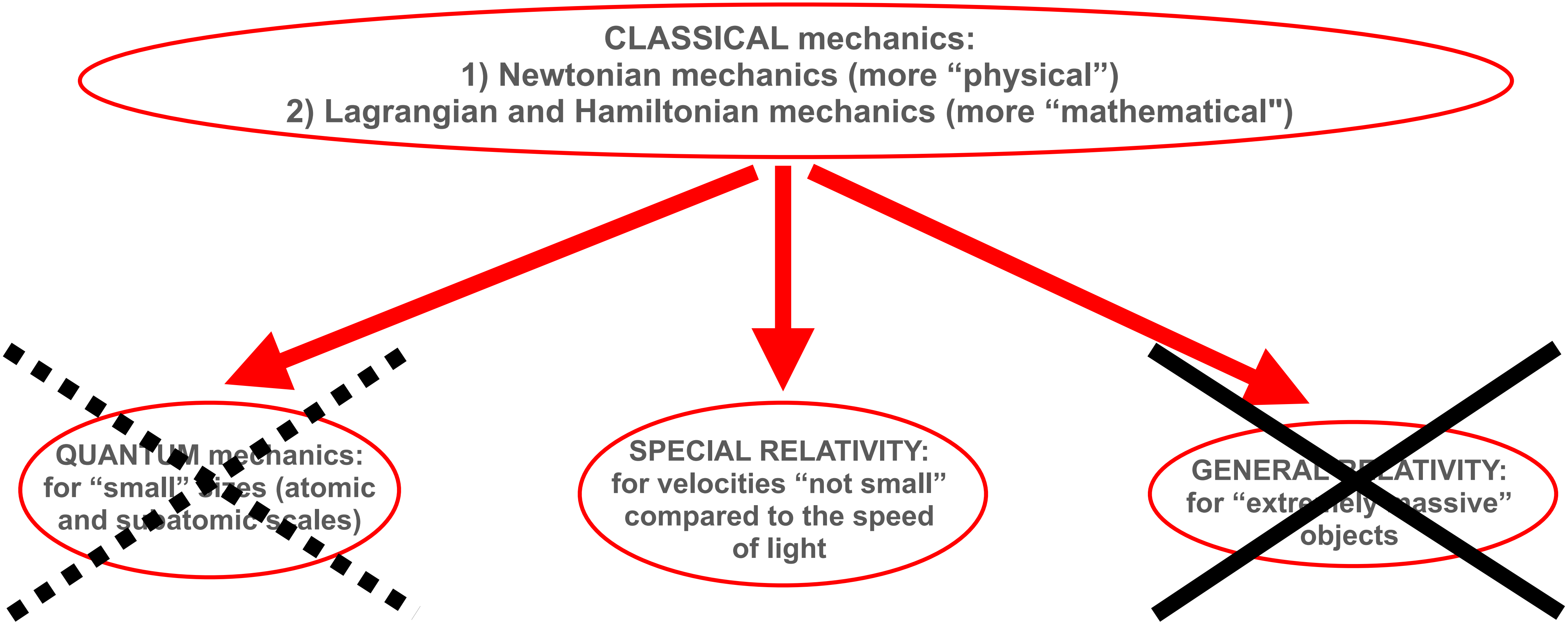

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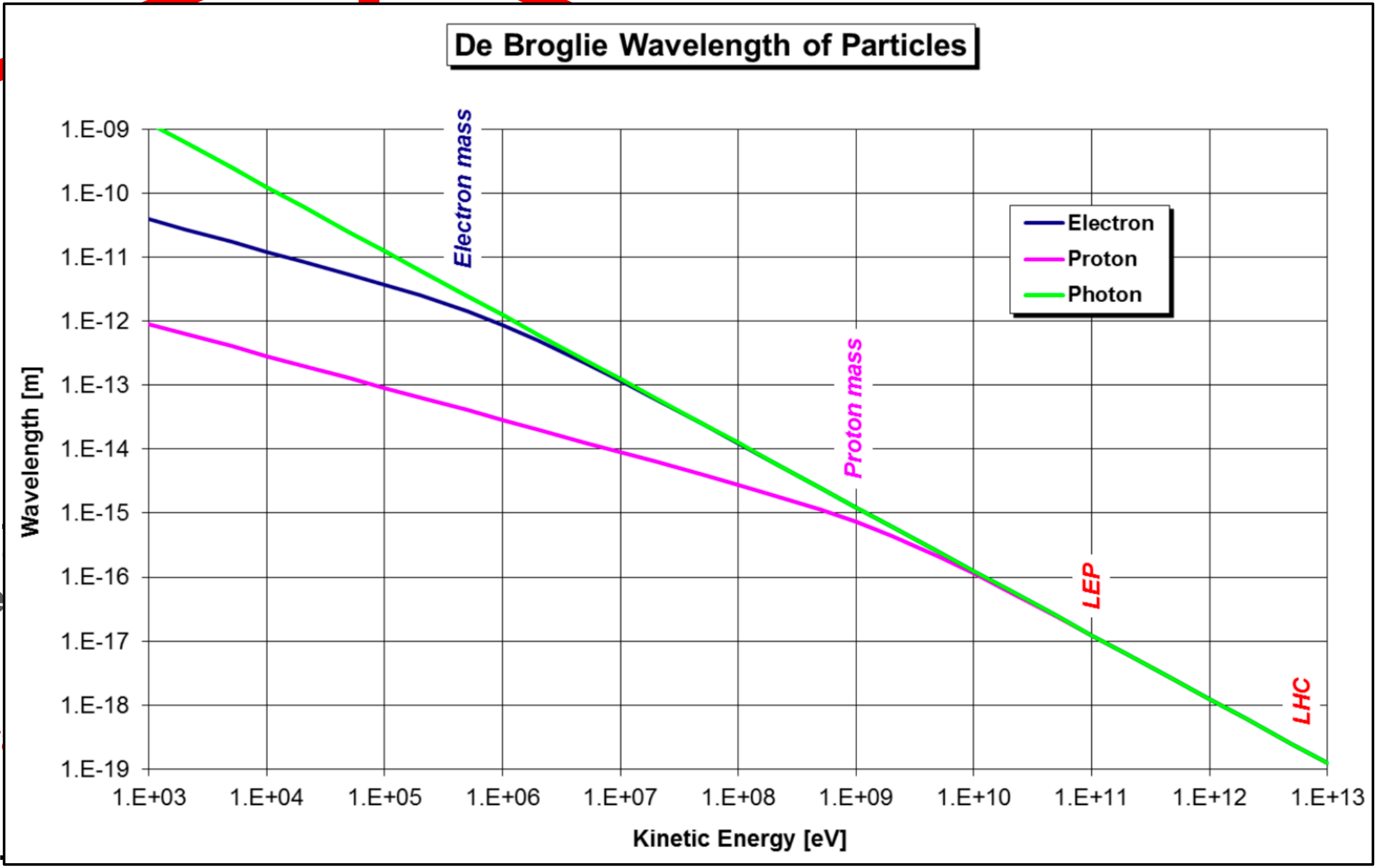

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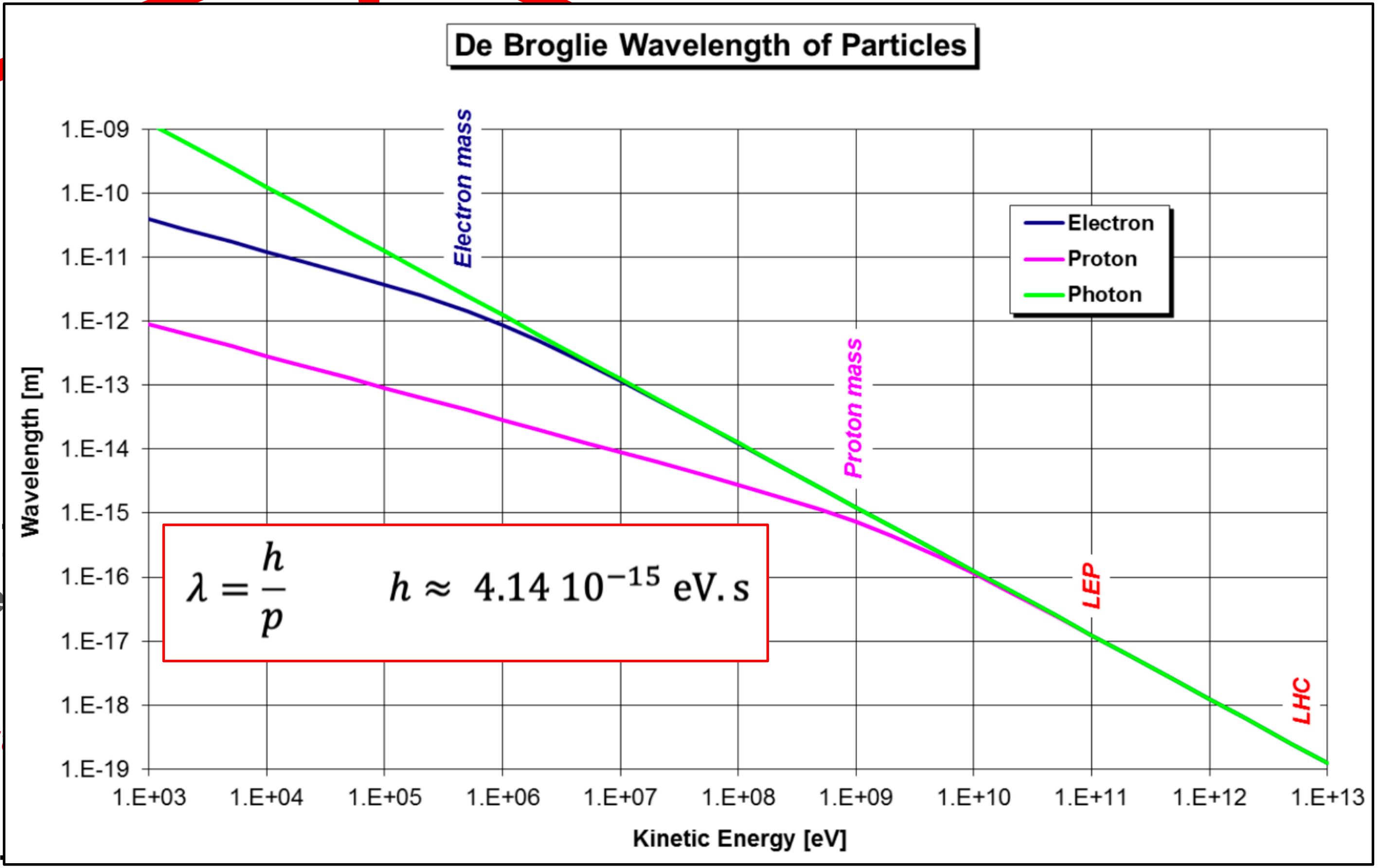

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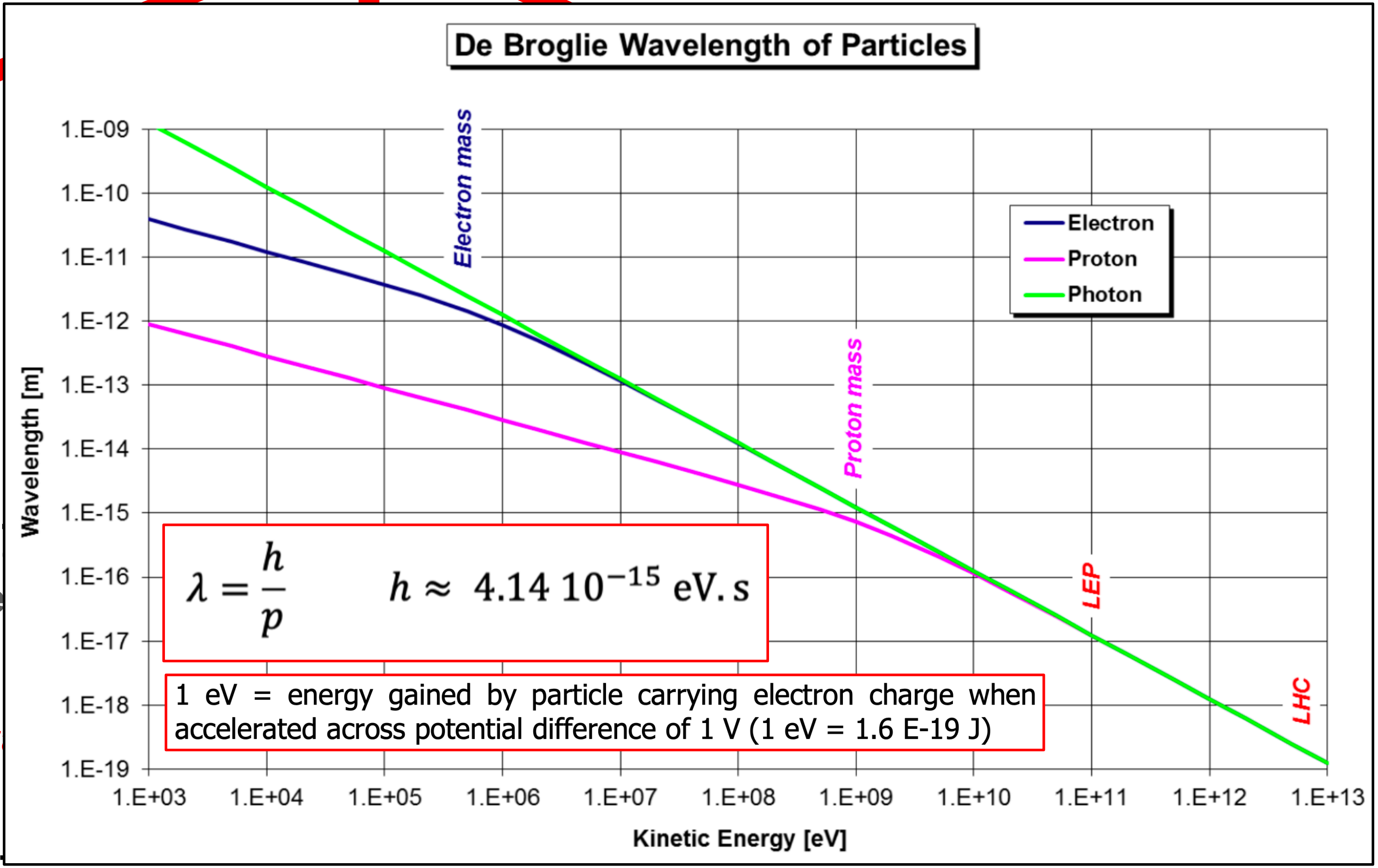

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1 eV = energy gained by particle carrying electron charge when accelerated across potential difference of 1 V (1 eV = 1.6 E-19 J)

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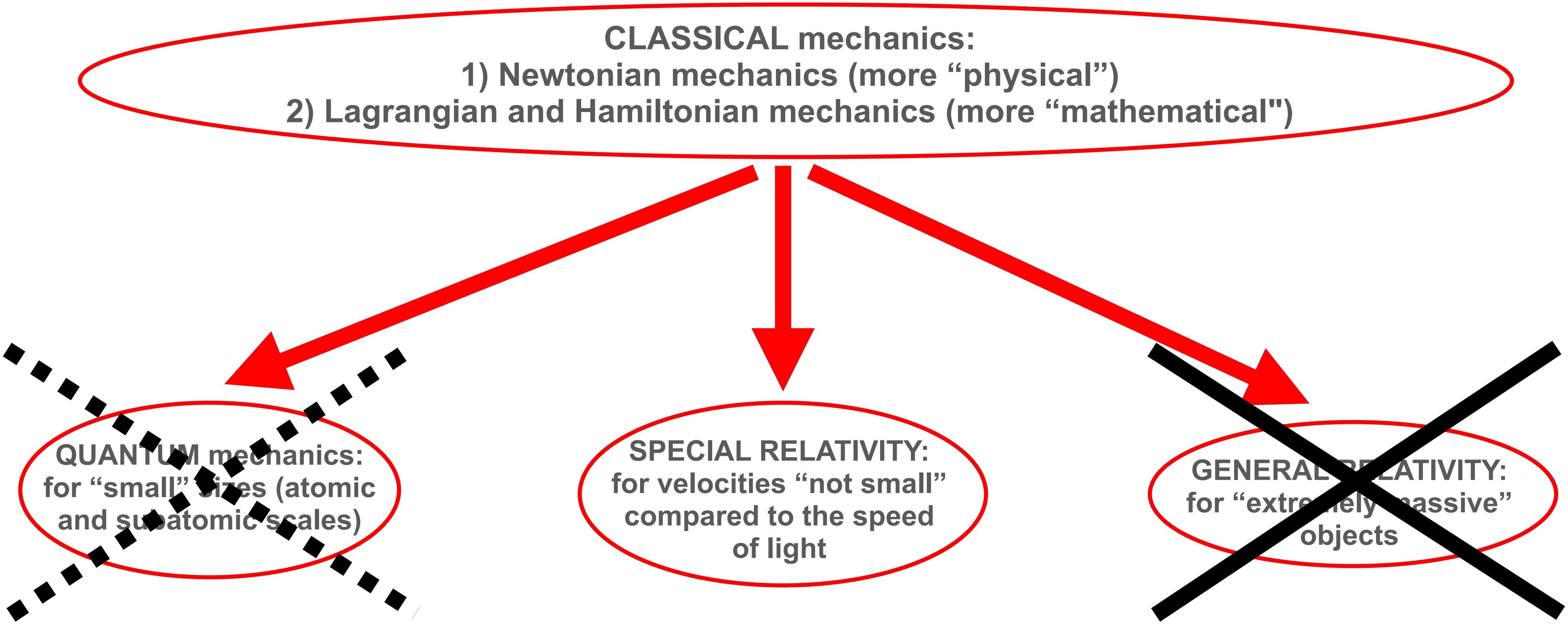

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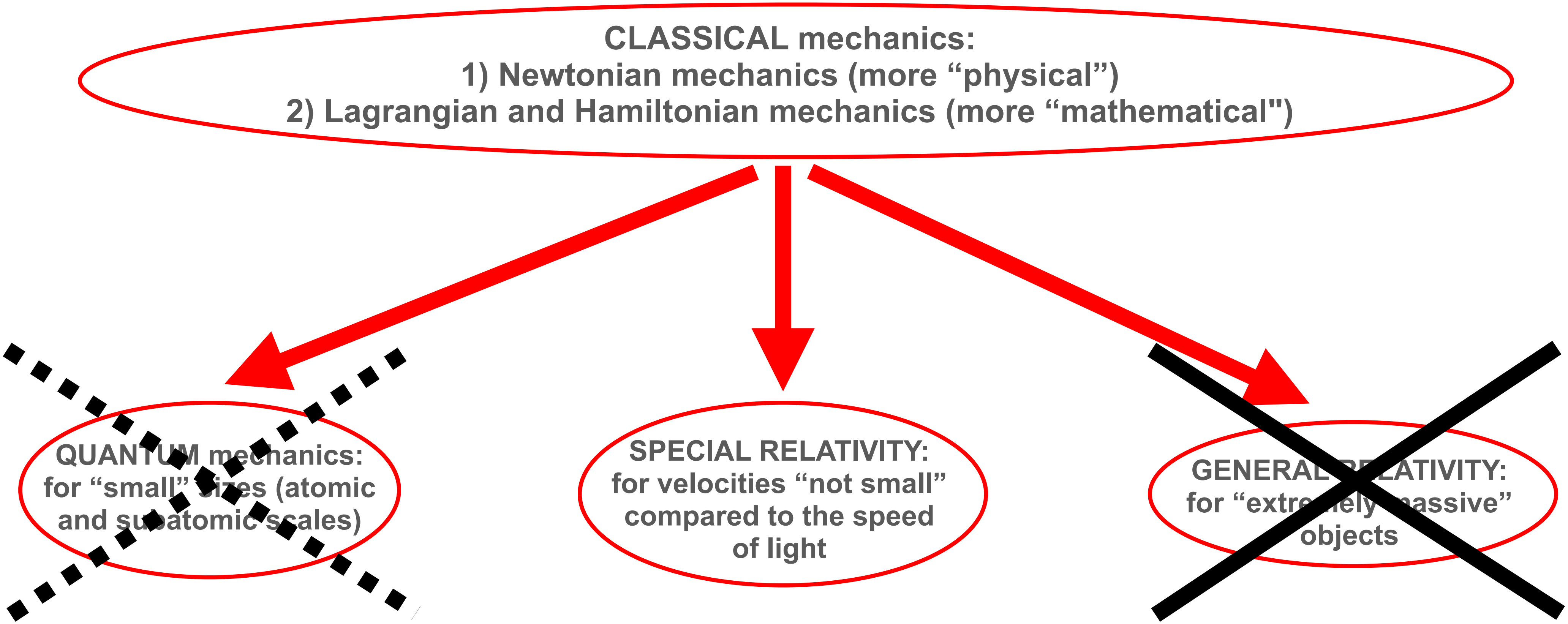

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=> See for instance "Quantum aspects of beam physics" from 1999 (https://accelconf.web.cern.ch/p99/PAPERS/TUCR1.PDF)

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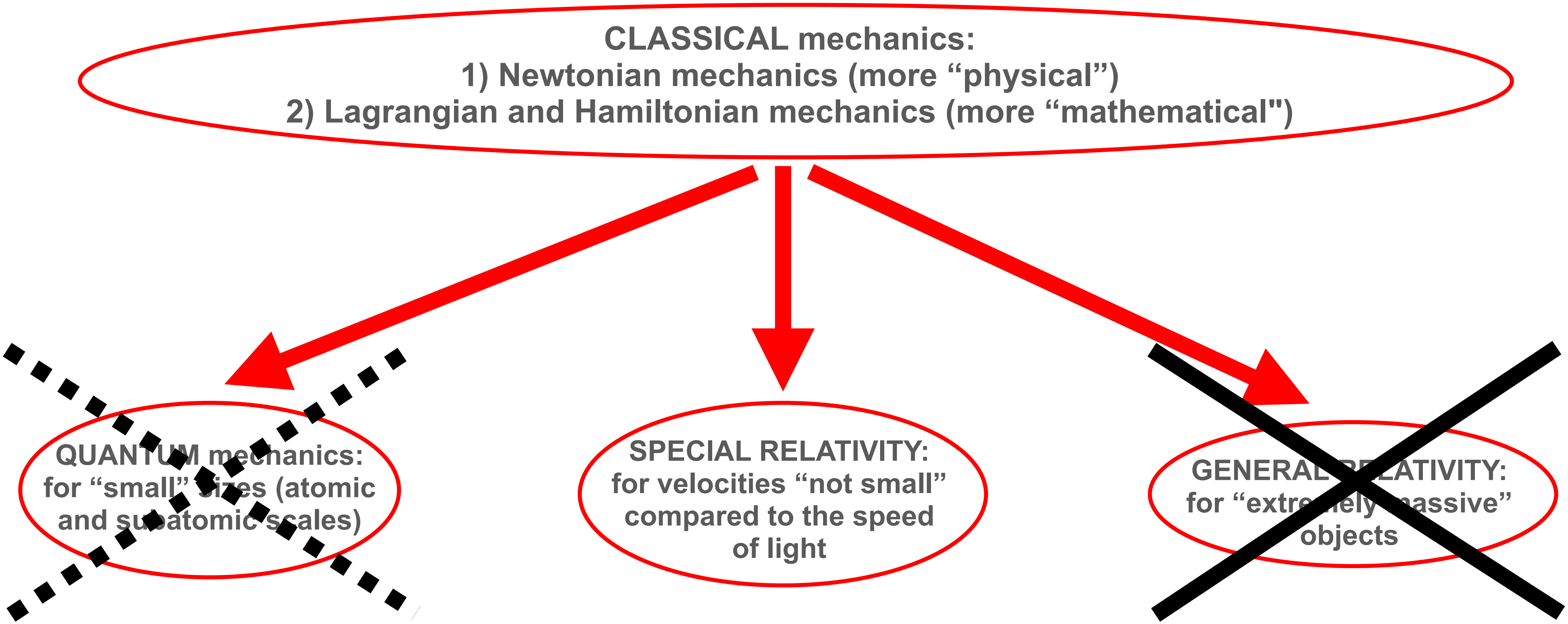

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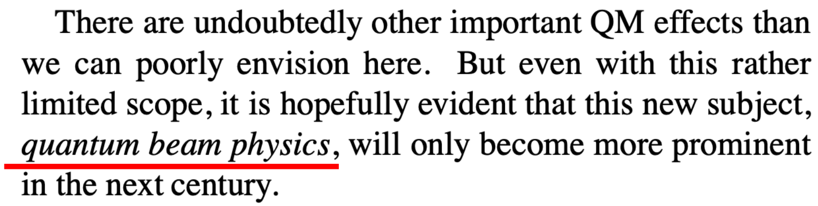
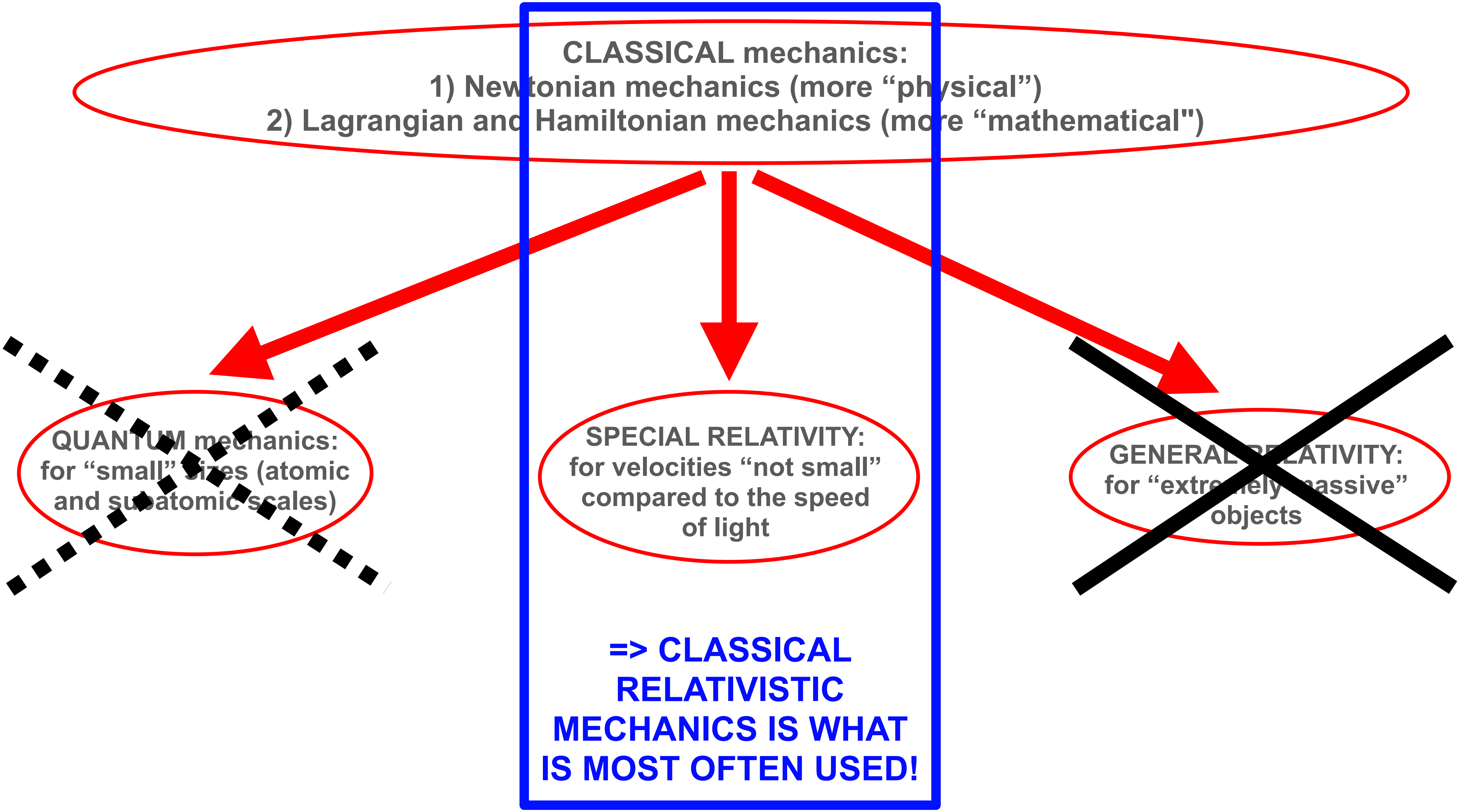


There are undoubtedly other important QM effects than we can poorly envision here. But even with this rather limited scope, it is hopefully evident that this new subject, *quantum beam physics*, will only become more prominent in the next century.

Classical mechanics



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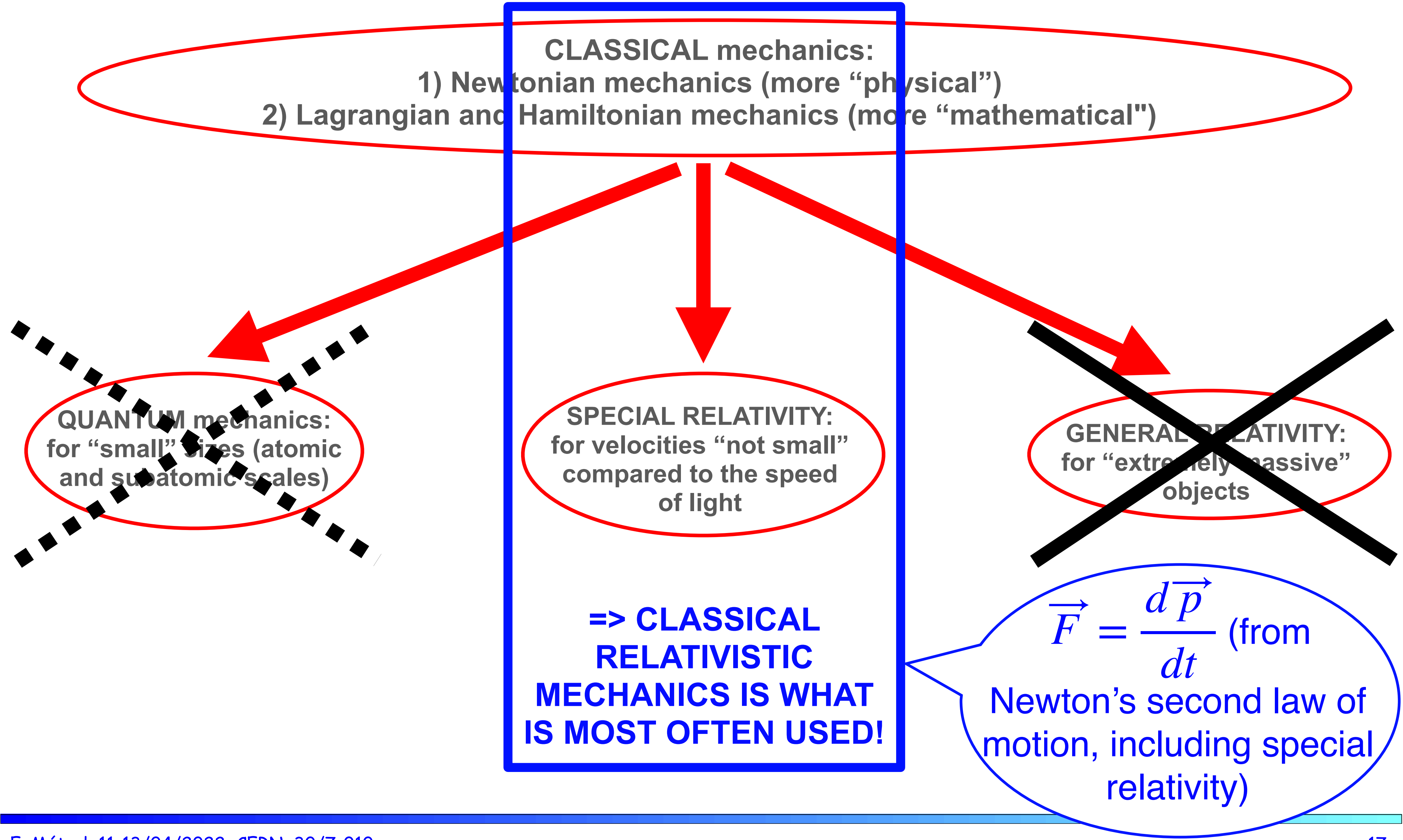

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$\vec{F} = \frac{d\vec{p}}{dt}$ (from Newton’s second law of motion, including special relativity)

Particle accelerators

- Particle accelerators are devices that handle the motion of particles by means of **EM fields**

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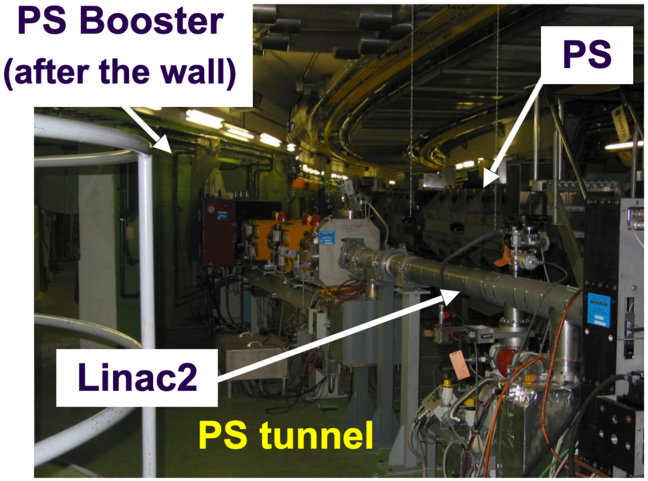
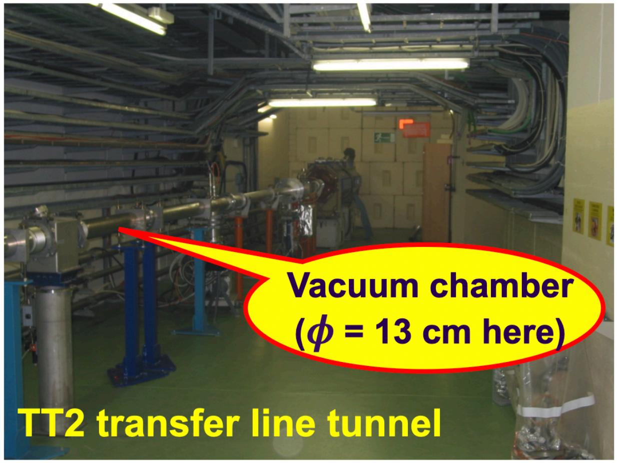
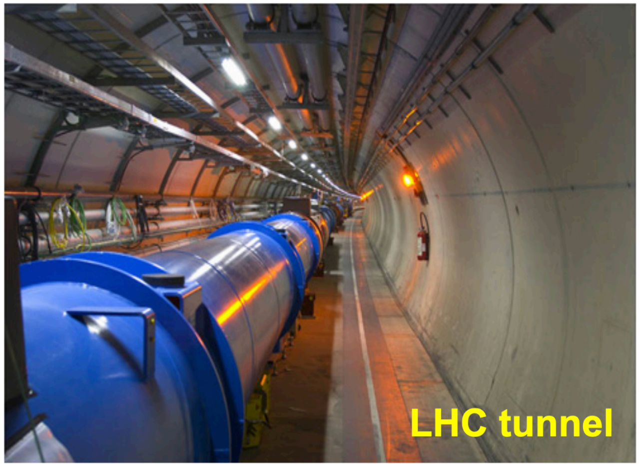
Example of some particle accelerators from CERN







Particle accelerators

- 3 conditions must be satisfied: **which ones?**

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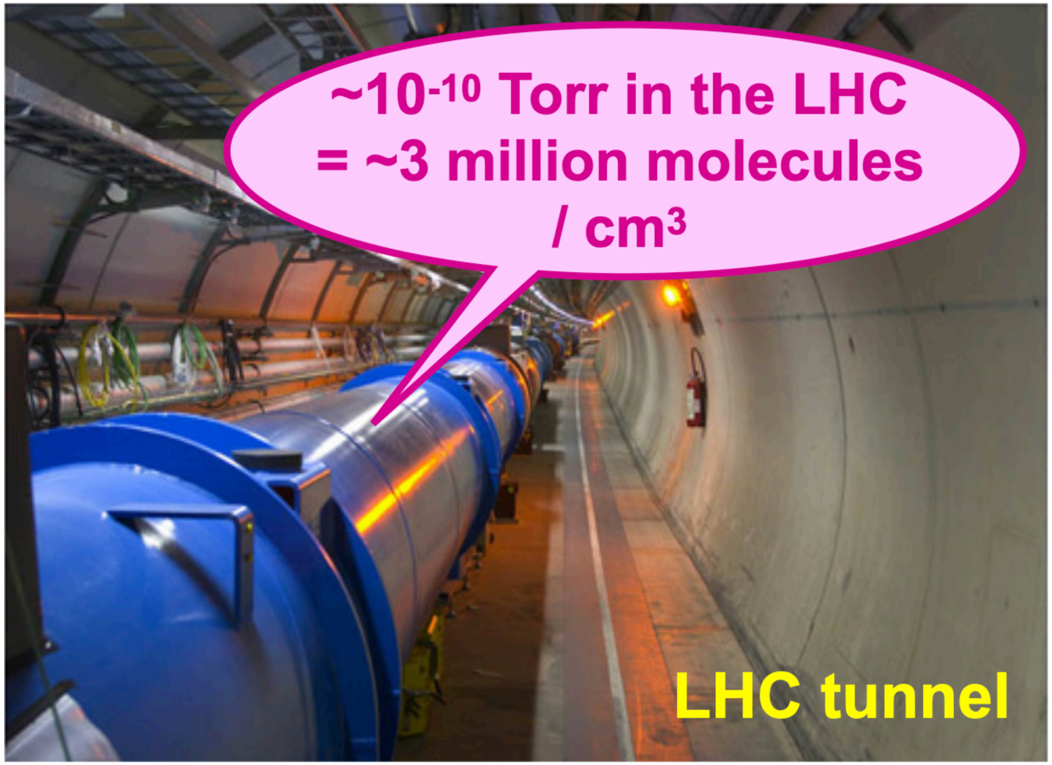

~10^{-10} Torr in the LHC = ~3 million molecules / cm^3

LHC tunnel

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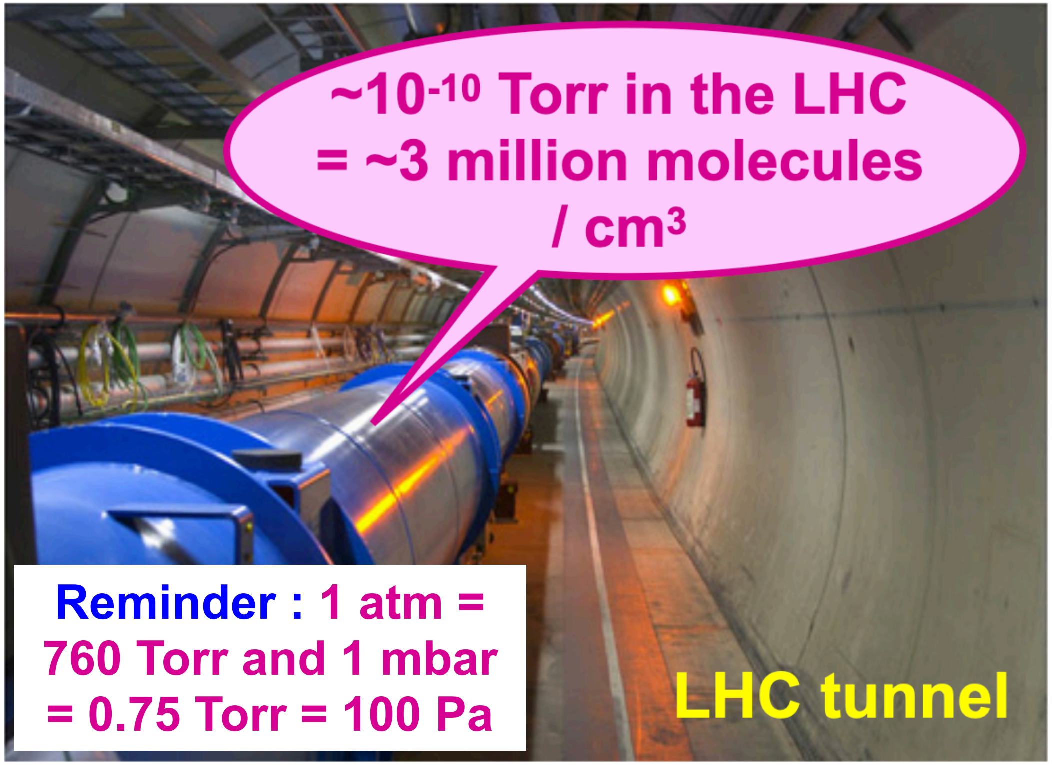

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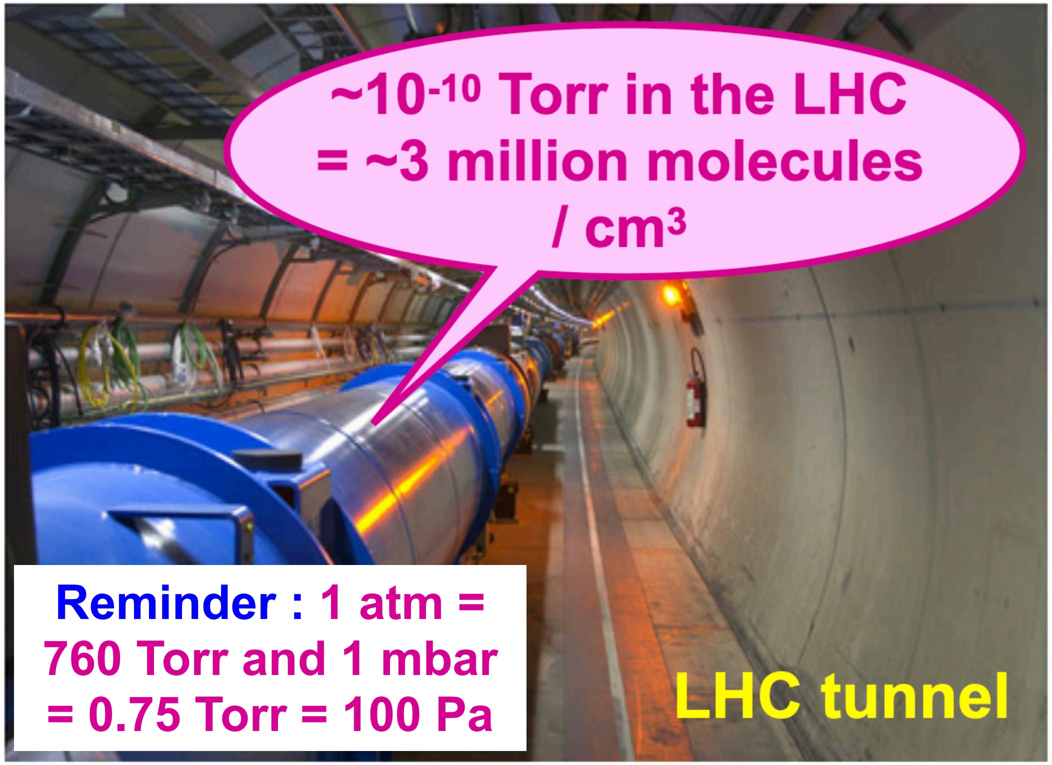

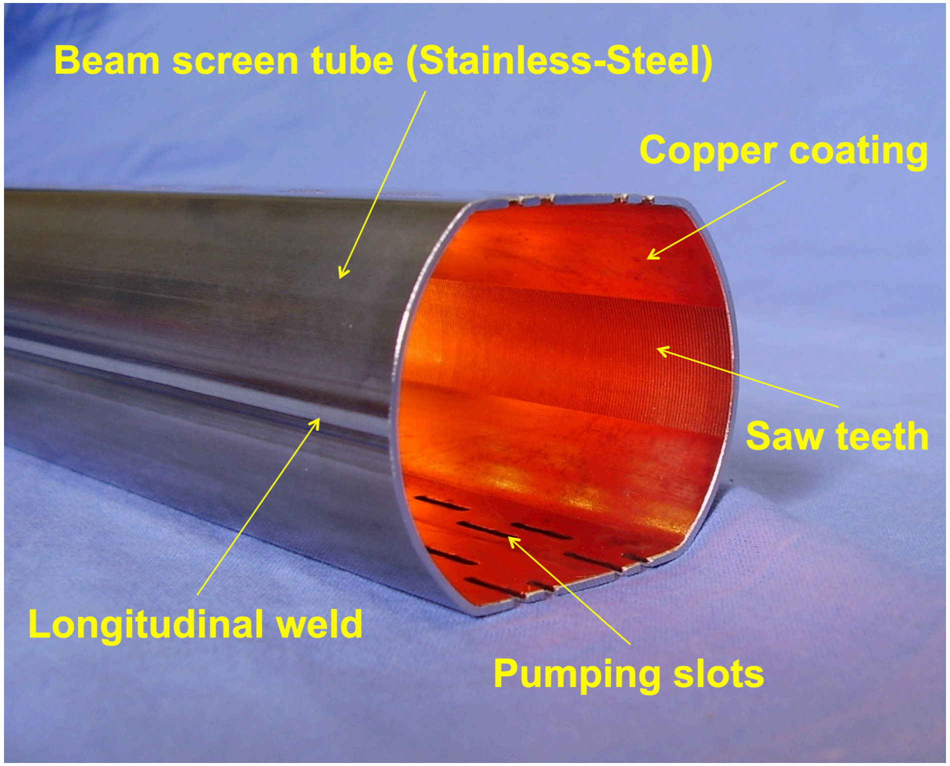

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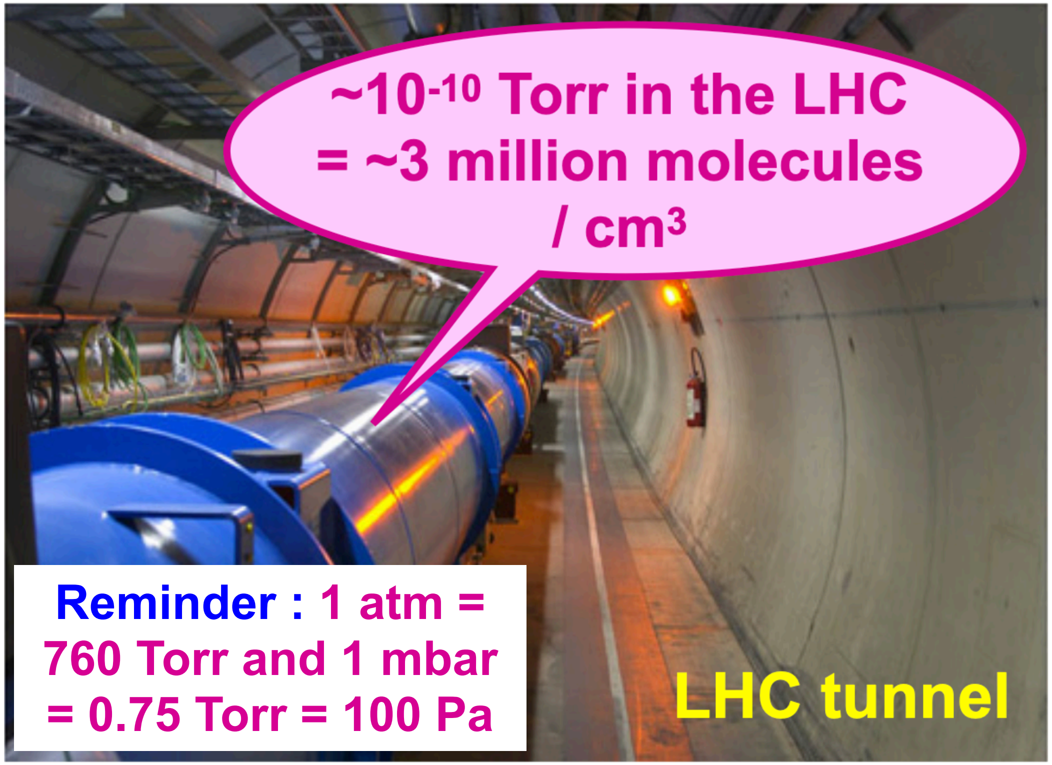

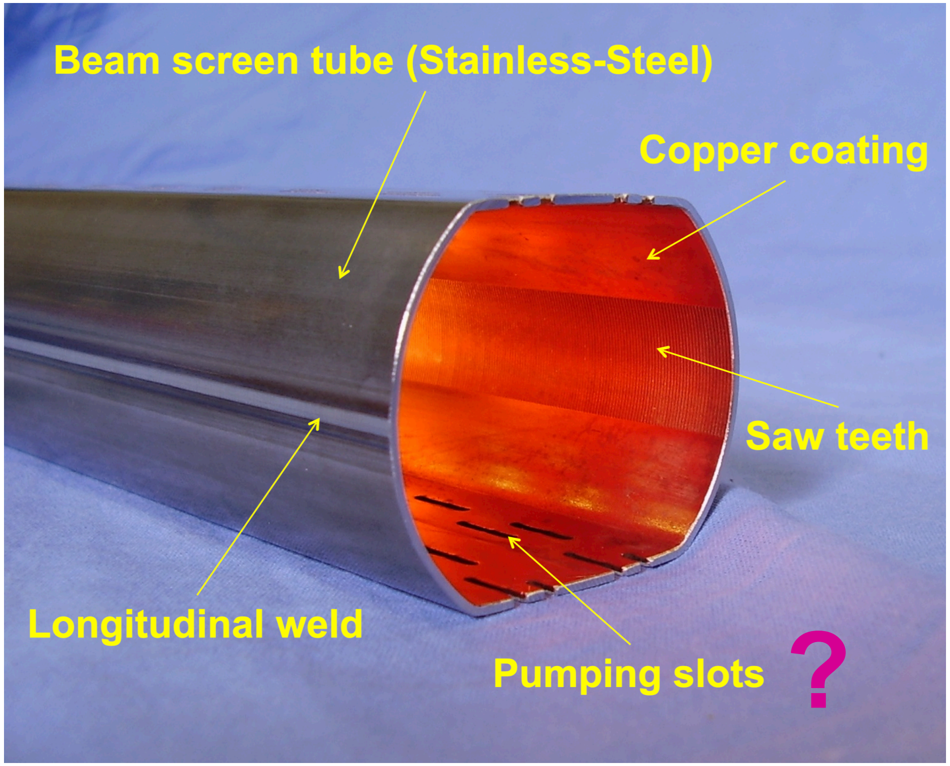

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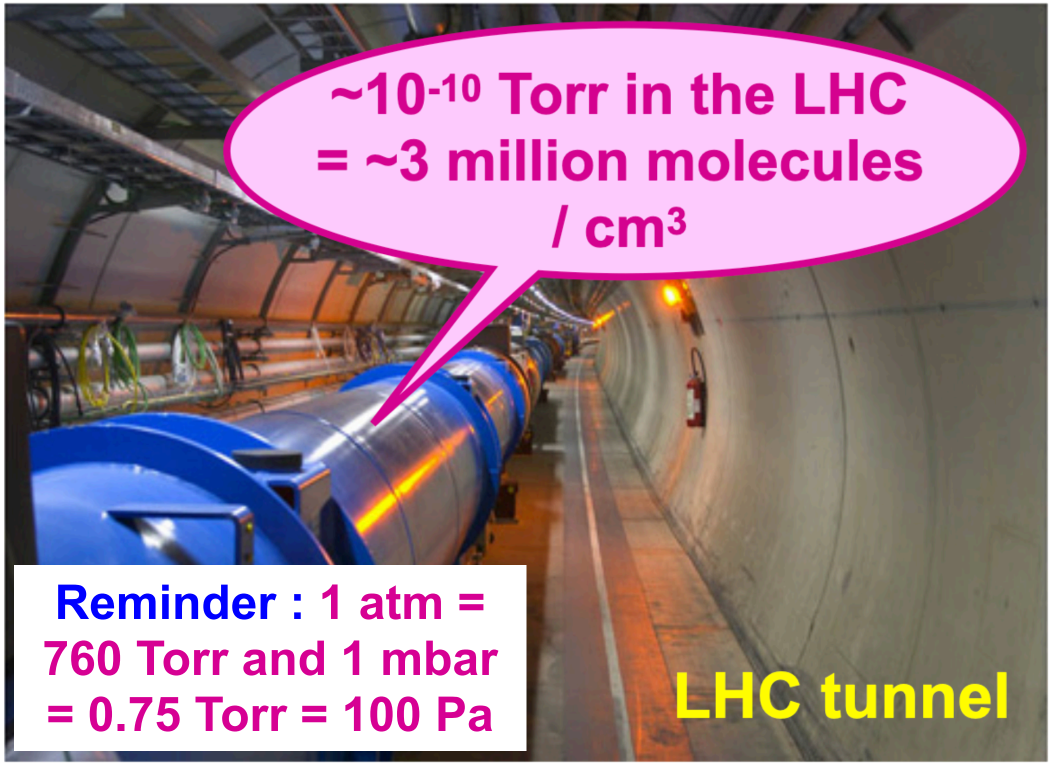

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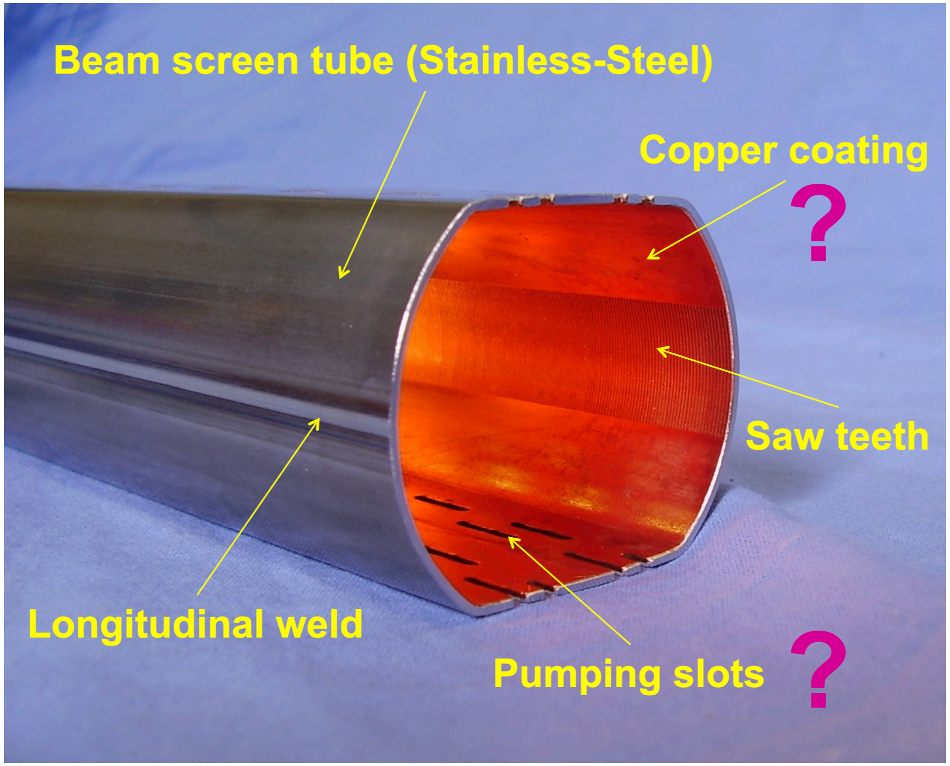

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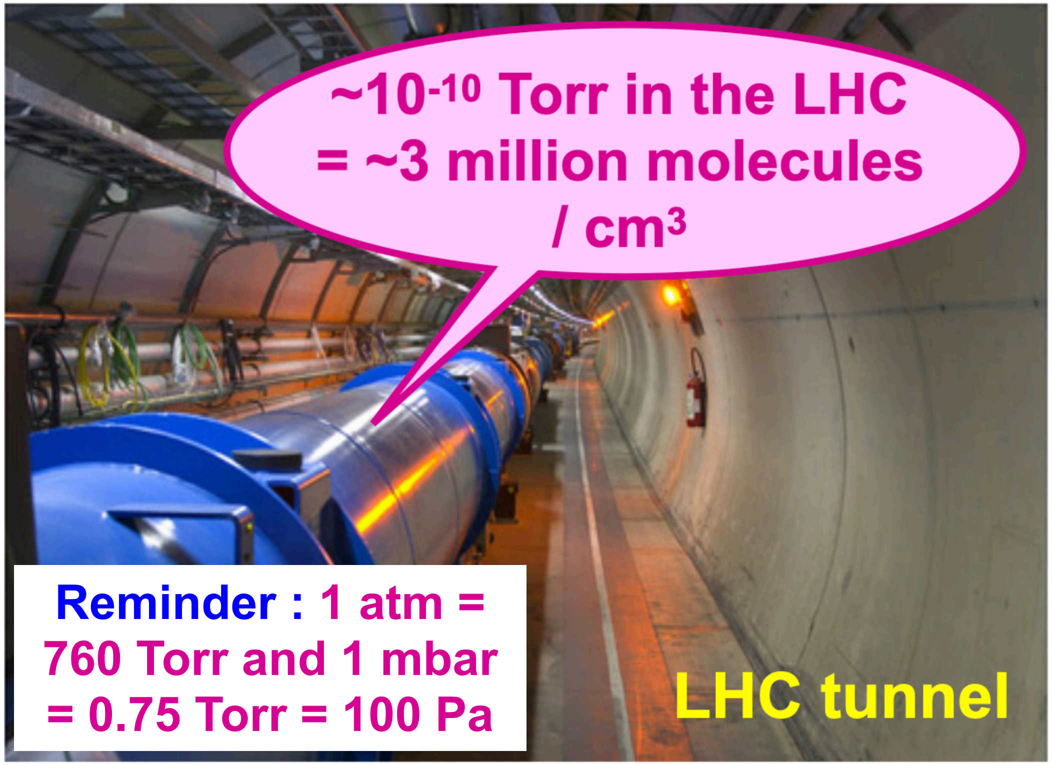
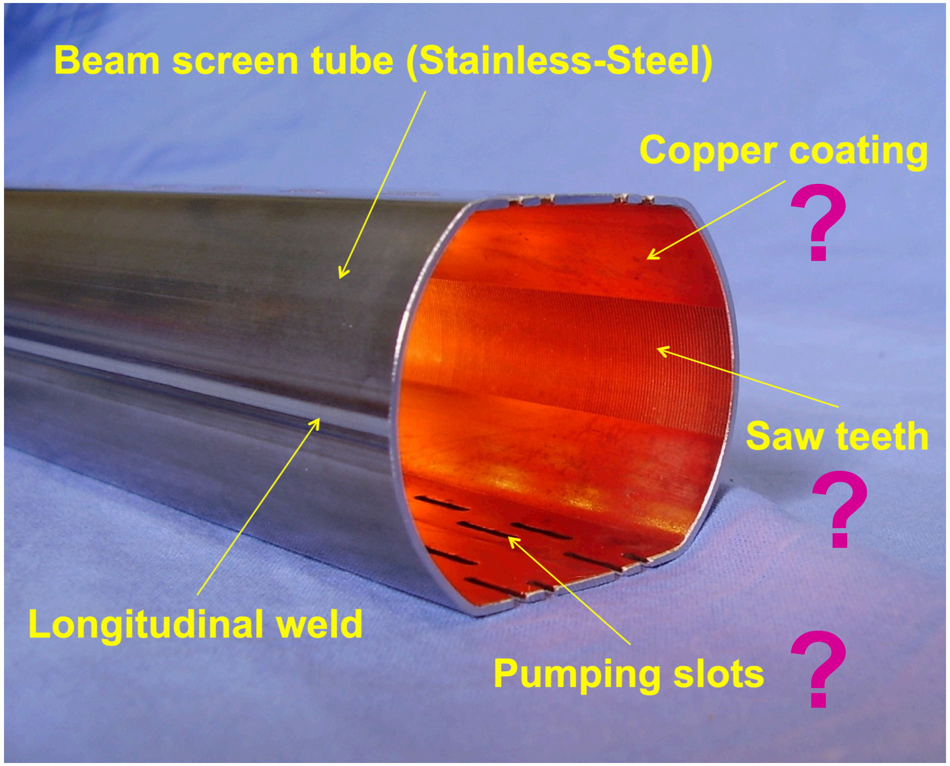


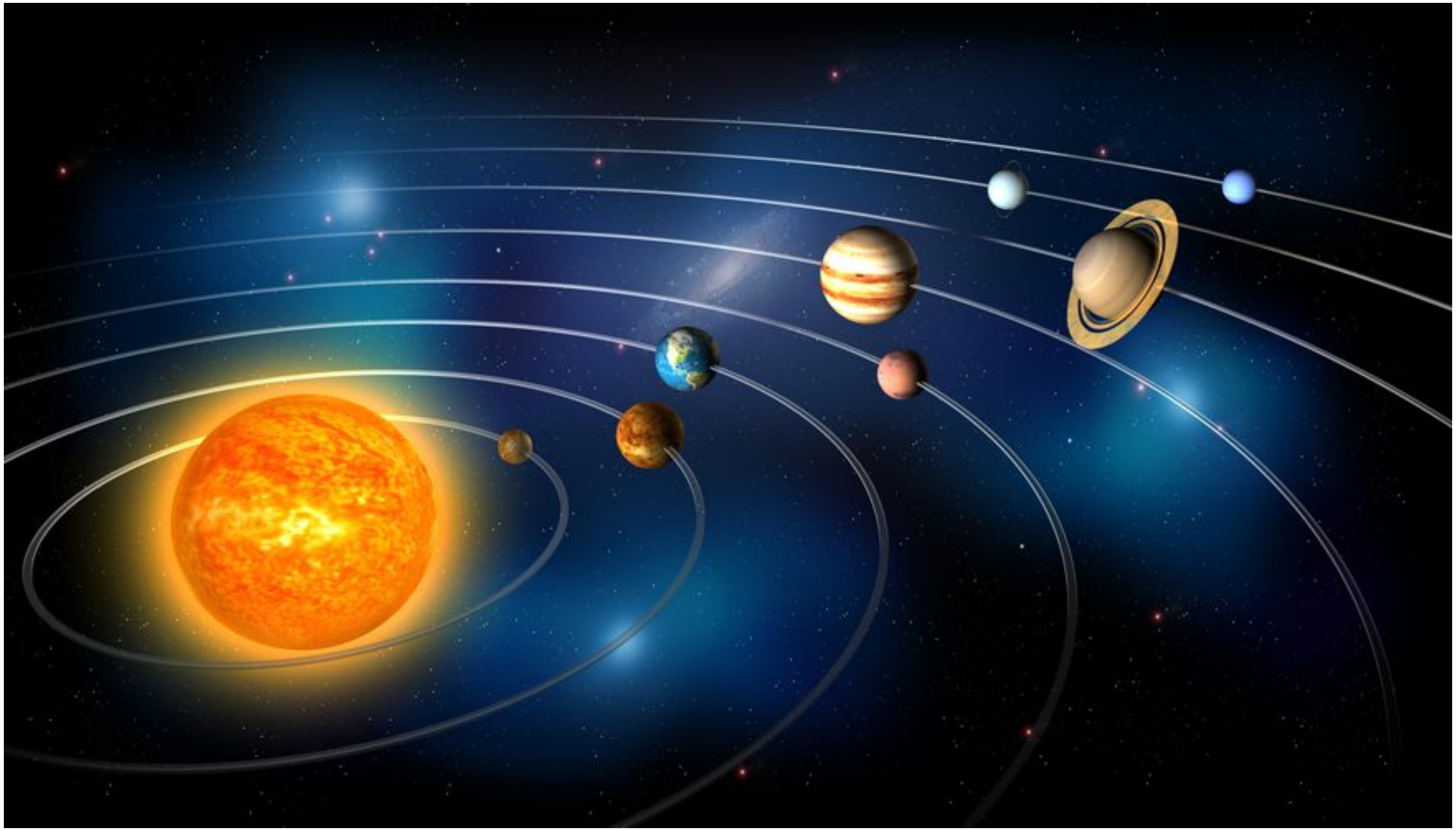


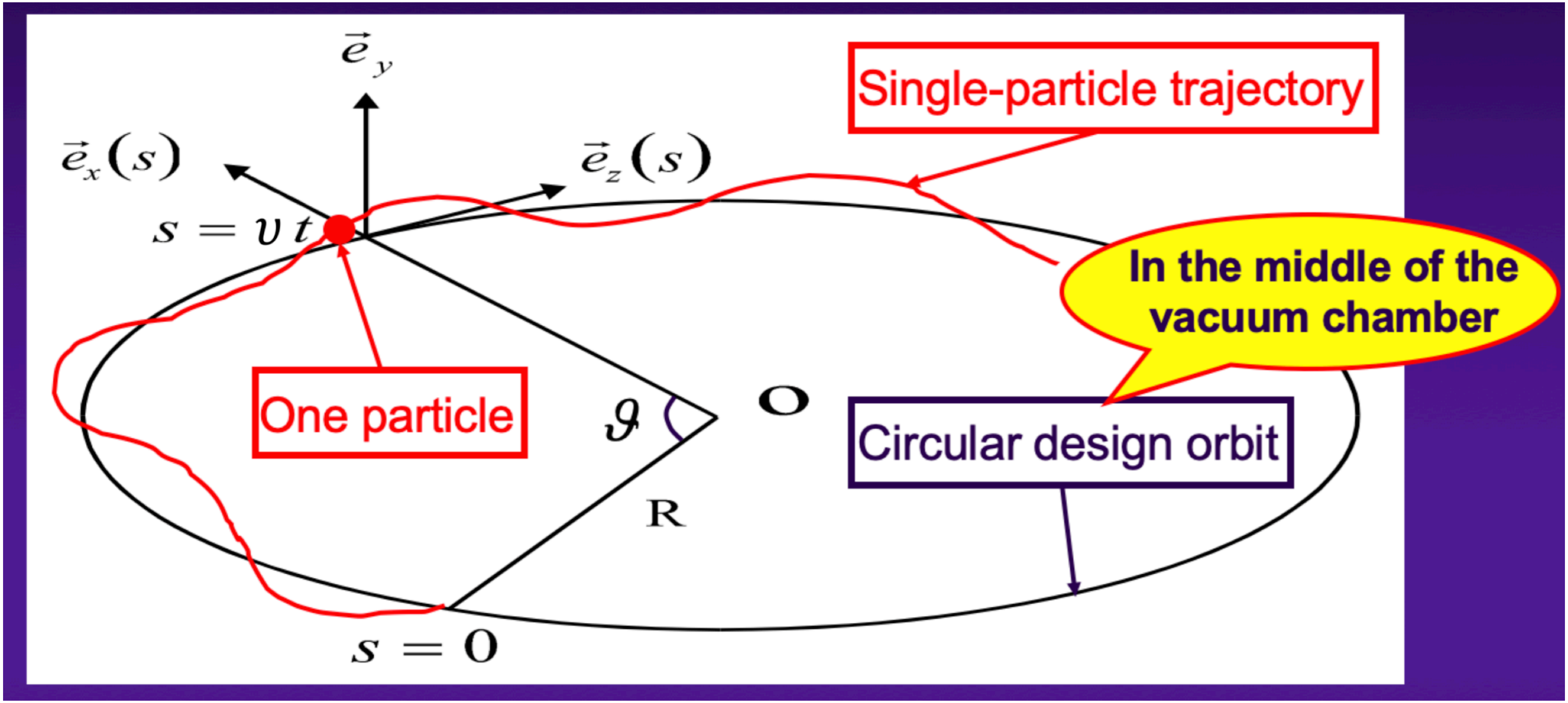
Particle accelerators

- TRICK of particle accelerators: ?

Particle accelerators

- **TRICK of particle accelerators**: the best way to keep something (here particles) under control (i.e. stable) is to **make it oscillate**! And this is what we are doing…in the 3 planes





Case here of a “synchrotron”

LHC proton beam in the injector chain

$$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}$$

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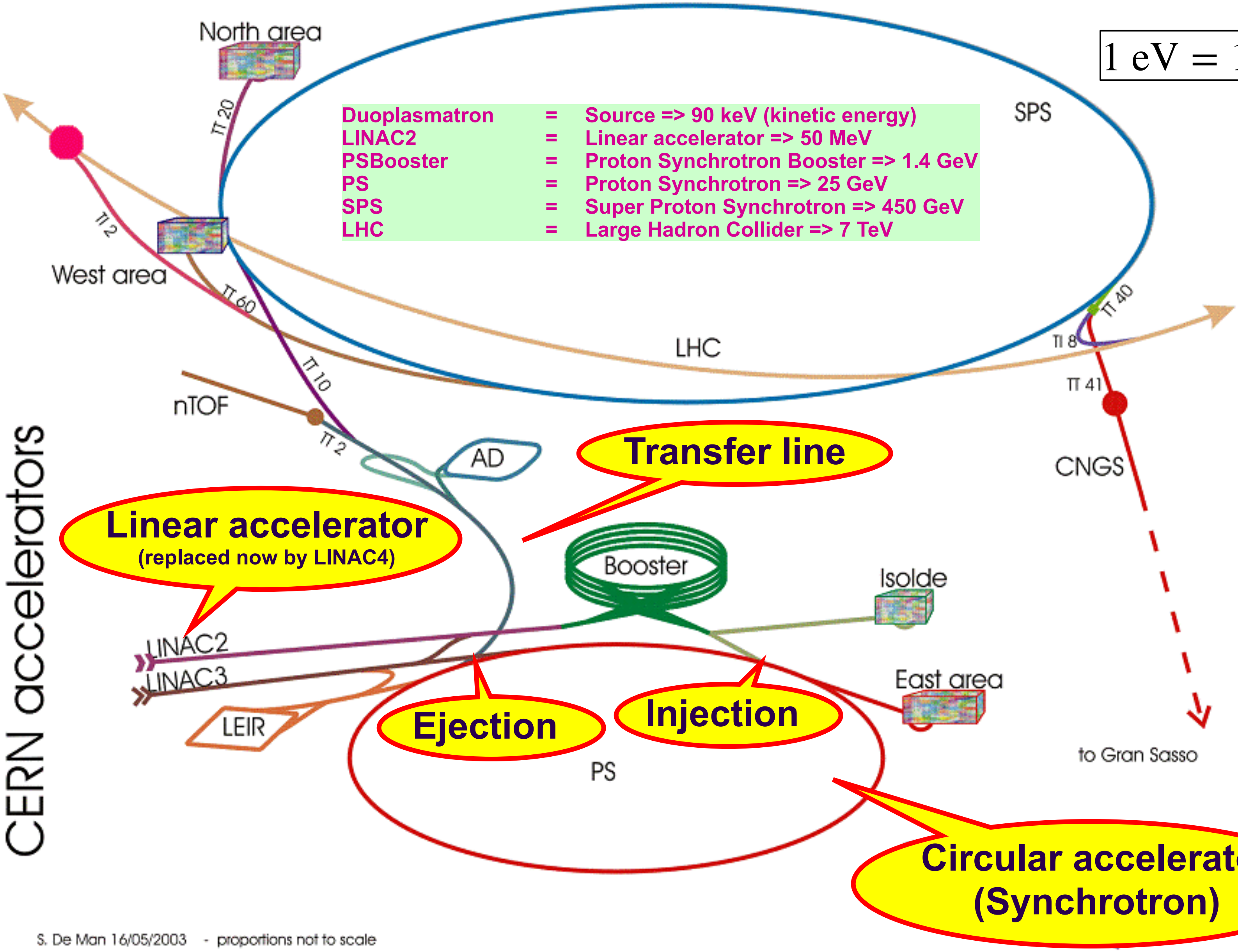

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LINAC2 = Linear accelerator => 50 MeV

PSBooster = Proton Synchrotron Booster => 1.4 GeV

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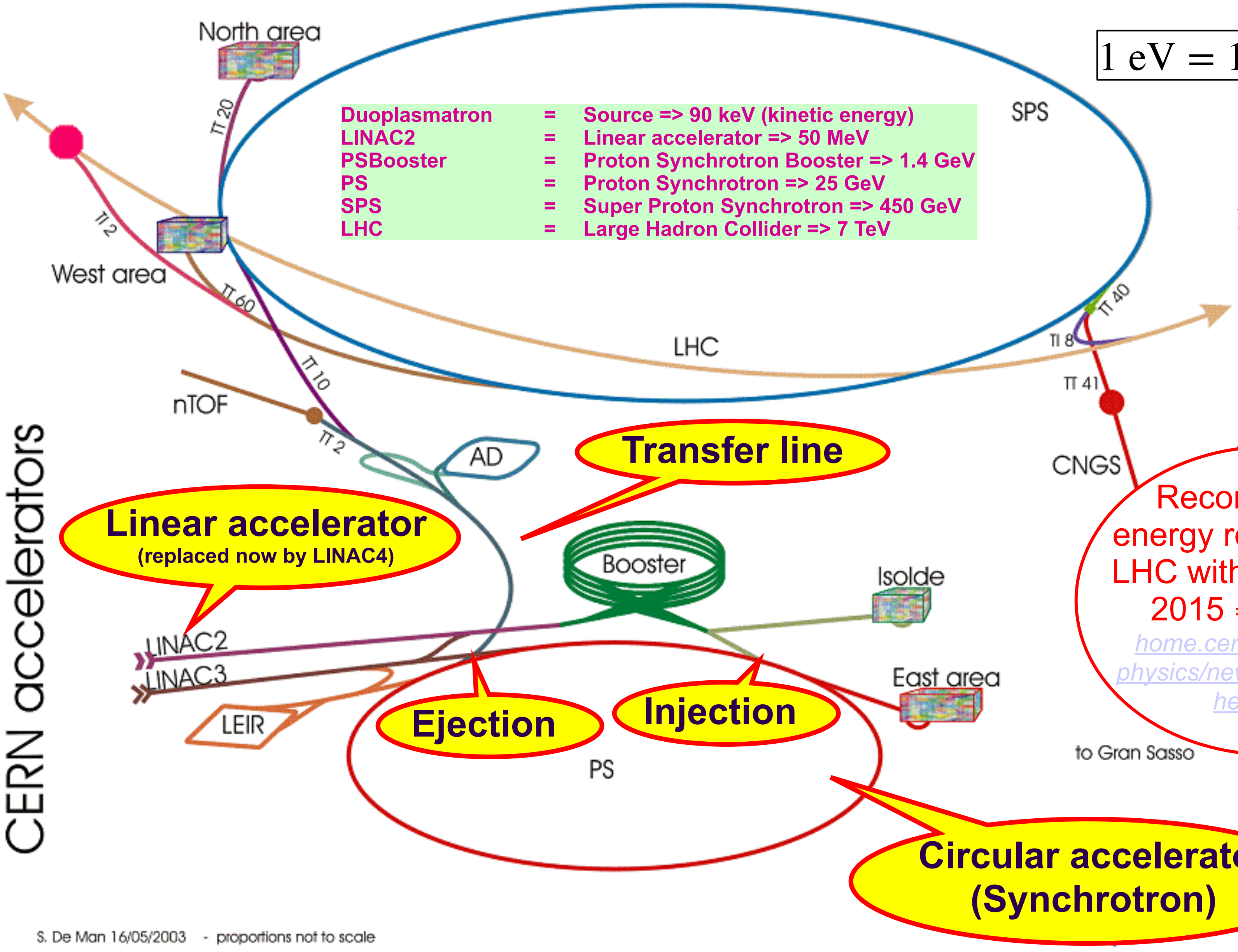
Transfer line

Linear accelerator
(replaced now by LINAC4)

Ejection

Injection

Circular accelerator (Synchrotron)



S. De Man 16/05/2003 - proportions not to scale

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Record collision energy reached in the LHC with Lead ions in 2015 => *See https://home.cern/news/opinion/physics/new-energy-frontier-heavy-ions*

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$$\frac{dp_x}{dt} = - \frac{\partial H}{\partial x} \quad => \quad \frac{dp_x}{dt} = - \omega x$$

Notion of phase space (instead of real space)

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=> $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$

=> $x(t) \propto \cos(\omega t)$ or $x(t) \propto \sin(\omega t)$

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=> $$\frac{d^2 x}{dt^2} + \omega^2 x = 0$$

=> $x(t) \propto \cos(\omega t)$, $p_x(t) \propto \sin(\omega t)$ or $x(t) \propto \sin(\omega t)$, $p_x(t) \propto \cos(\omega t)$

Notion of phase space (instead of real space)

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$$\Rightarrow \quad \frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\Rightarrow \quad x(t) \propto \cos(\omega t) \quad \text{or} \quad x(t) \propto \sin(\omega t)$$

$$p_x(t) \propto \sin(\omega t) \qquad p_x(t) \propto \cos(\omega t)$$

=> Circular motion in phase space

Notion of phase space (instead of real space)

- Let's have a look to the simple case of a harmonic oscillator: $H = \omega \dfrac{x^2 + p_x^2}{2}$

$$\frac{dx}{dt} = \frac{\partial H}{\partial p_x} \quad => \quad \frac{dx}{dt} = \omega p_x$$

$$\frac{dp_x}{dt} = -\frac{\partial H}{\partial x} \quad => \quad \frac{dp_x}{dt} = -\,\omega x$$

$$=> \quad \frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$=> \quad \begin{matrix} x(t) \propto \cos(\omega t) \\ p_x(t) \propto \sin(\omega t) \end{matrix} \quad \text{or} \quad \begin{matrix} x(t) \propto \sin(\omega t) \\ p_x(t) \propto \cos(\omega t) \end{matrix}$$

=> Circular motion in phase space

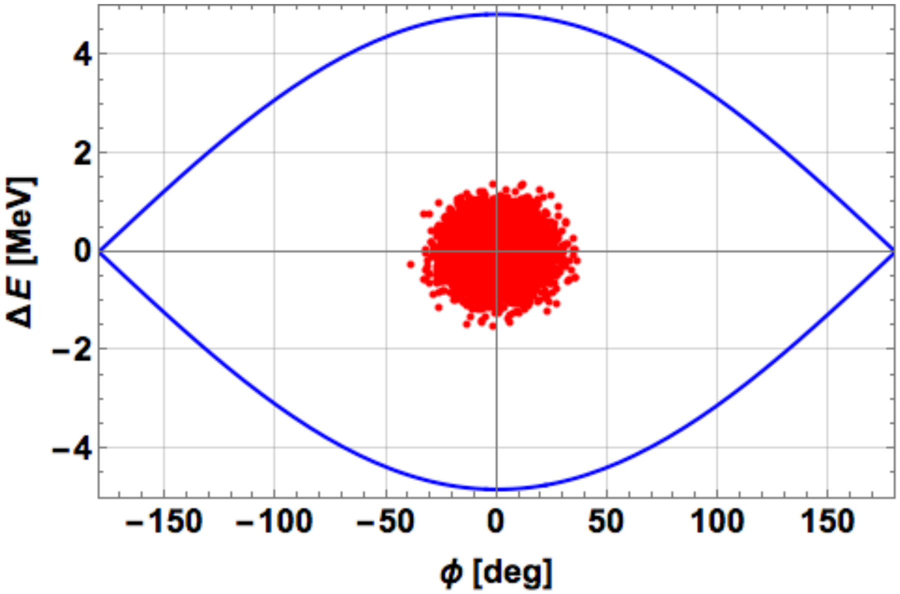
- And similarly for the other directions y and z => The motion of a particle in the 3D real space is studied and described in a 6D phase space

Notion of phase space (instead of real space)

- Let's have a look, for instance, to the **motion of a bunch of particles, turn after turn, in the longitudinal phase space** (z, p_z)

Notion of phase space (instead of real space)

- Let's have a look, for instance, to the **motion of a bunch of particles, turn after turn, in the longitudinal phase space** (z, p_z) => Using here some other normalised parameters proportional to z (for the horizontal axis) and p_z (for the vertical axis)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ΔE [MeV]
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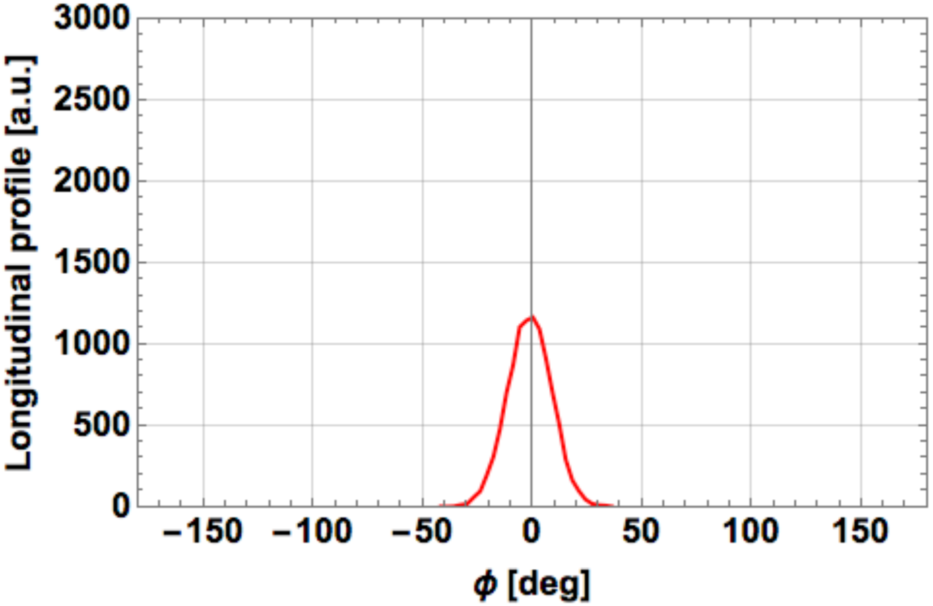
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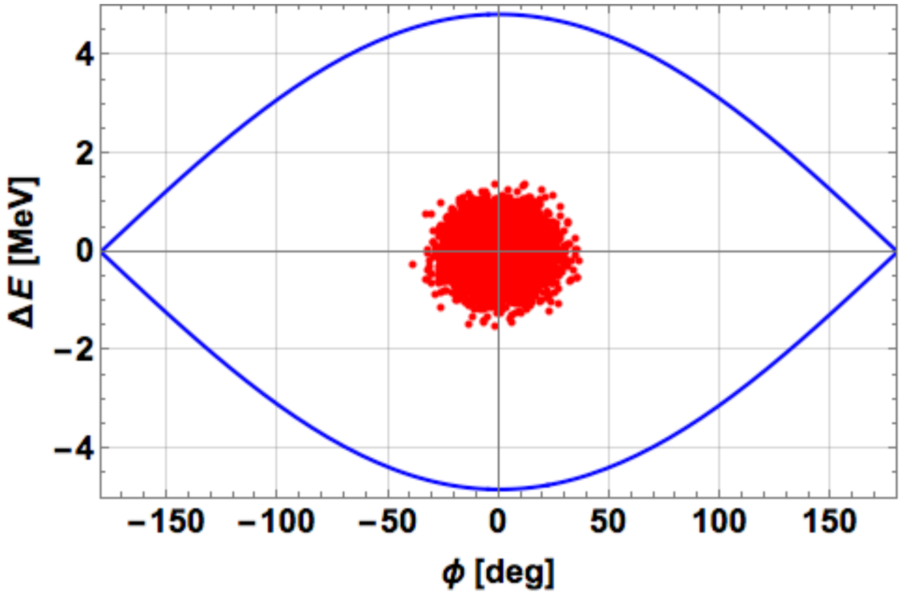
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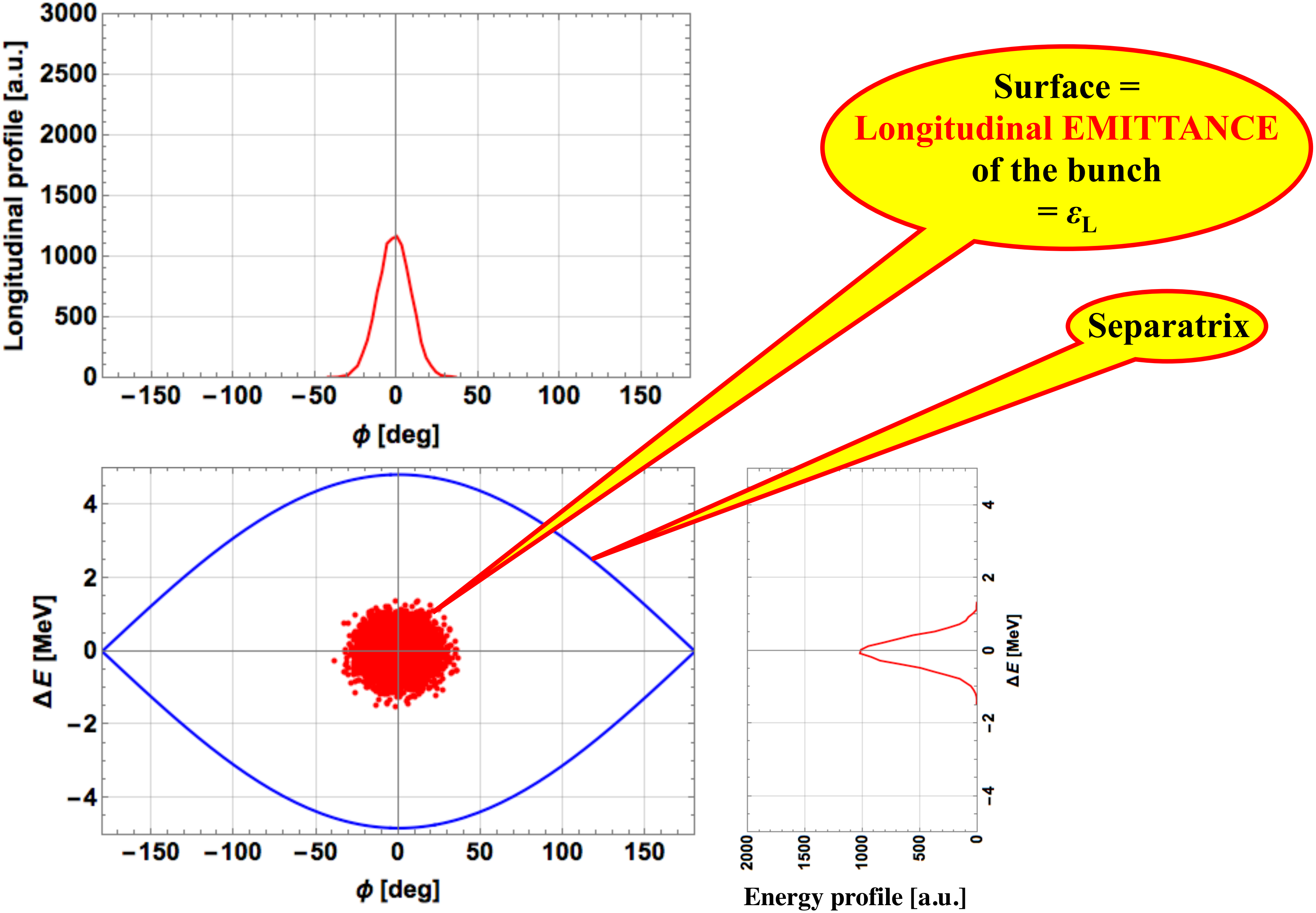
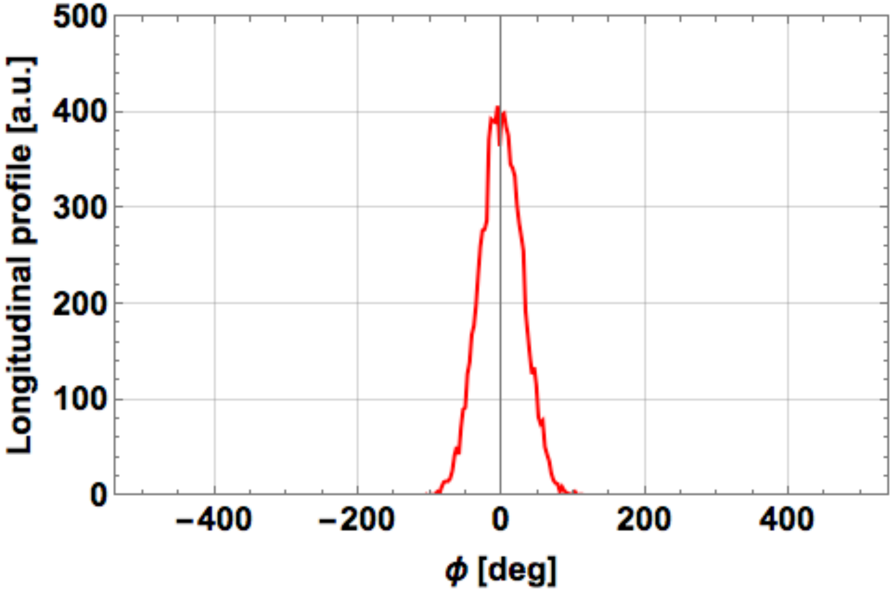
MATCHED BUNCH



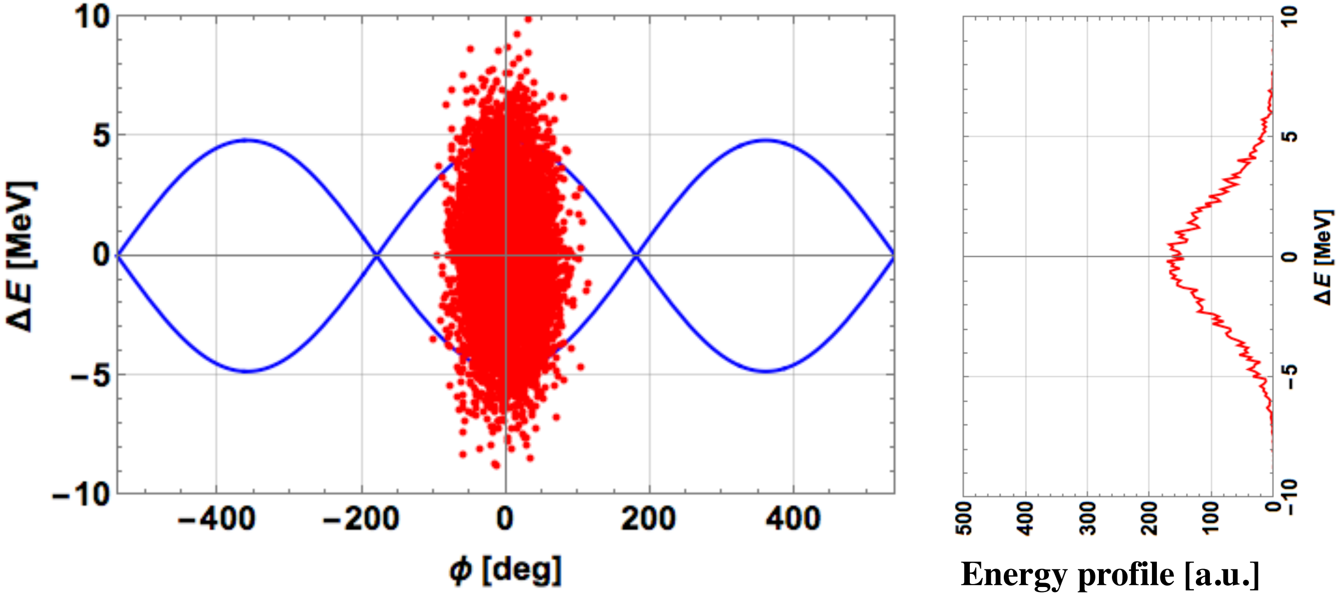
Surface =
Longitudinal EMITTANCE
of the bunch
= ε_L

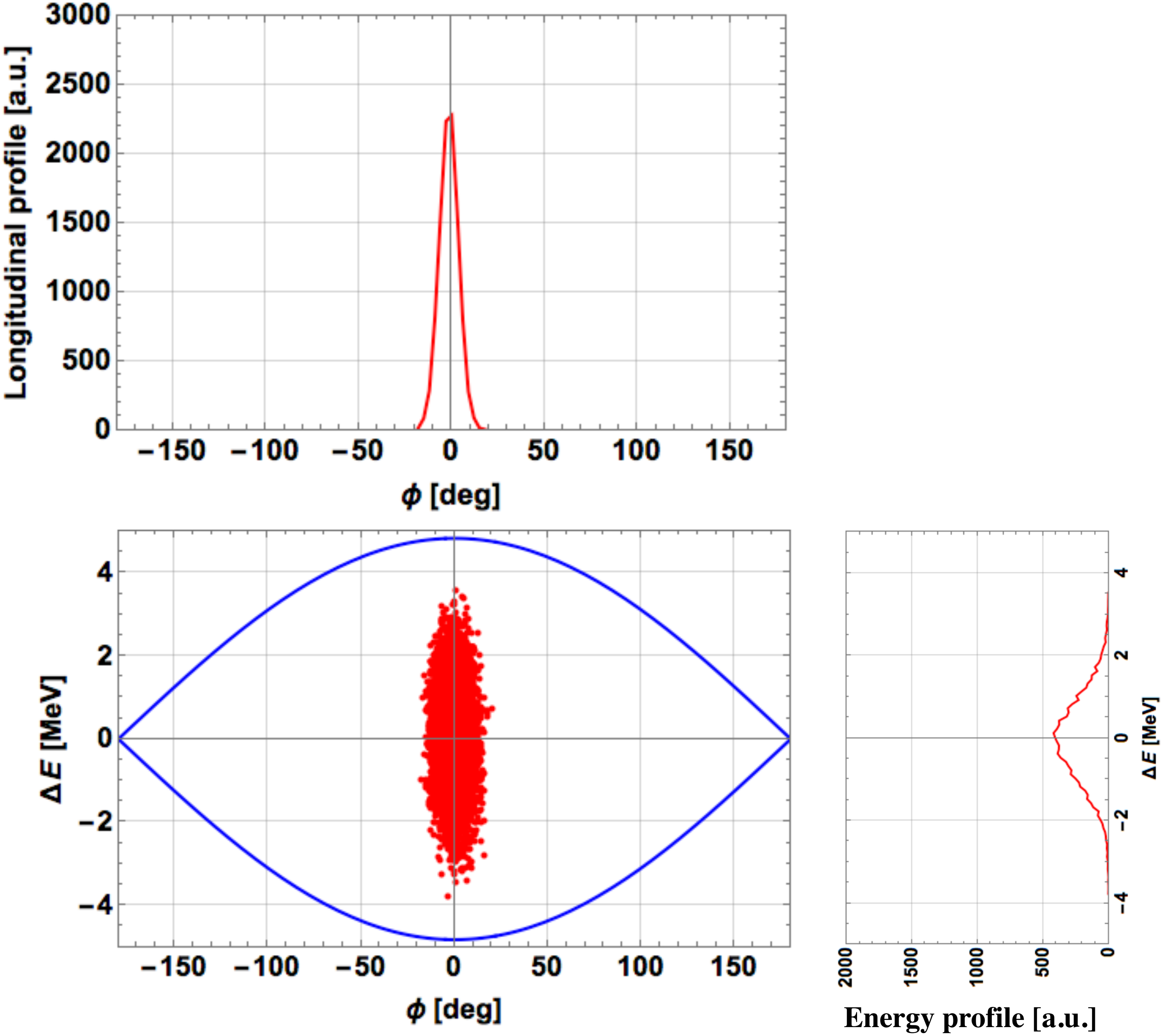
Separatrix

MISMATCHED BUNCH



Leads to an increase of the longitudinal emittance and/or particle losses





Longitudinal profile [a.u.]
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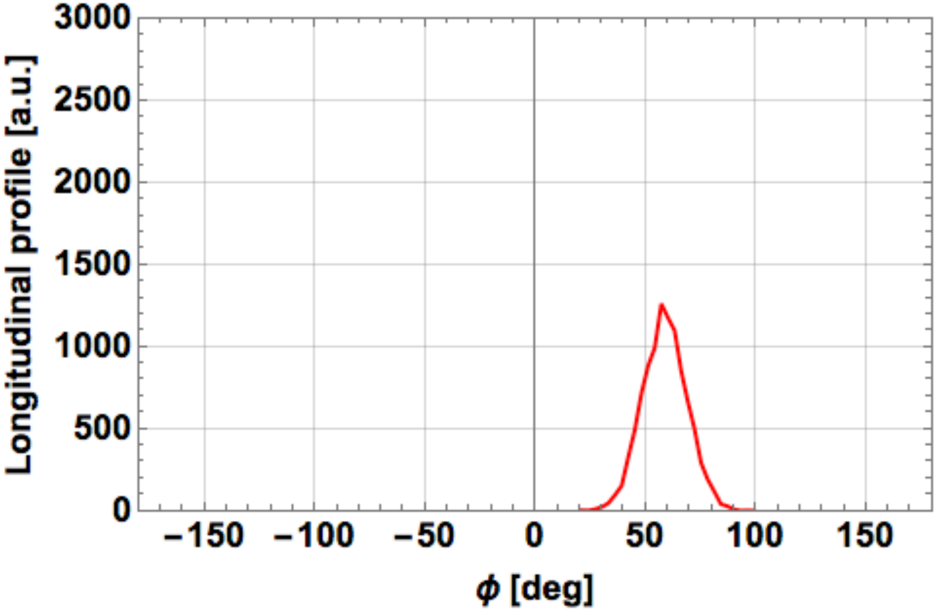
ΔE [MeV]
Energy profile [a.u.]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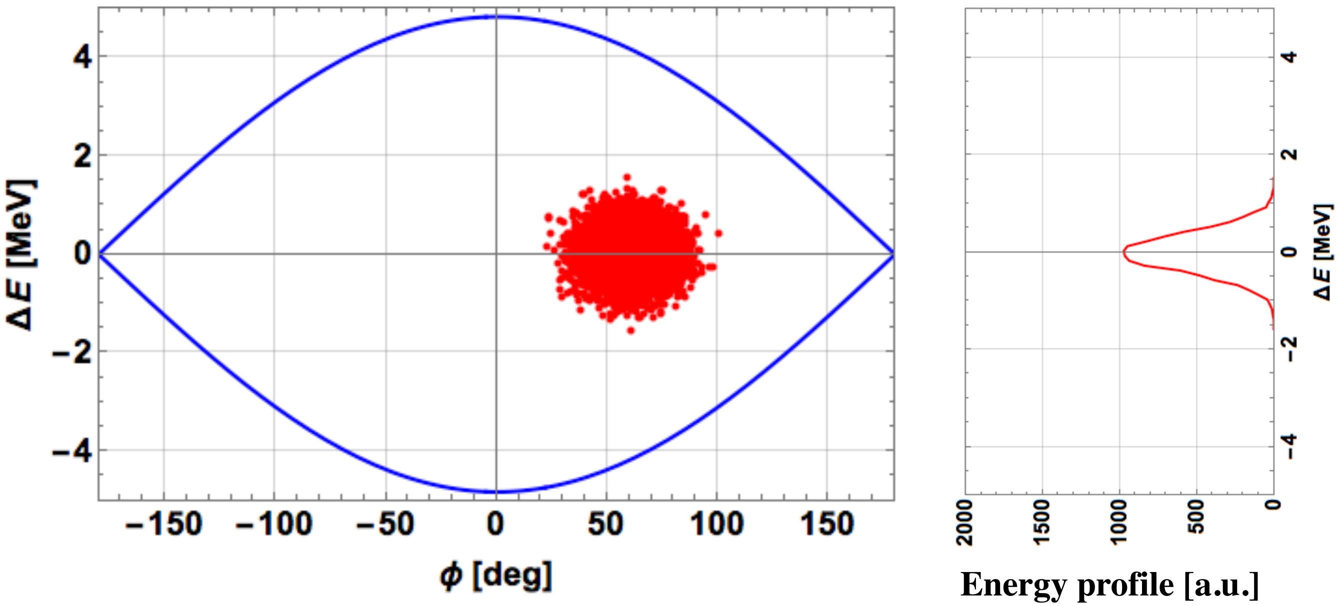
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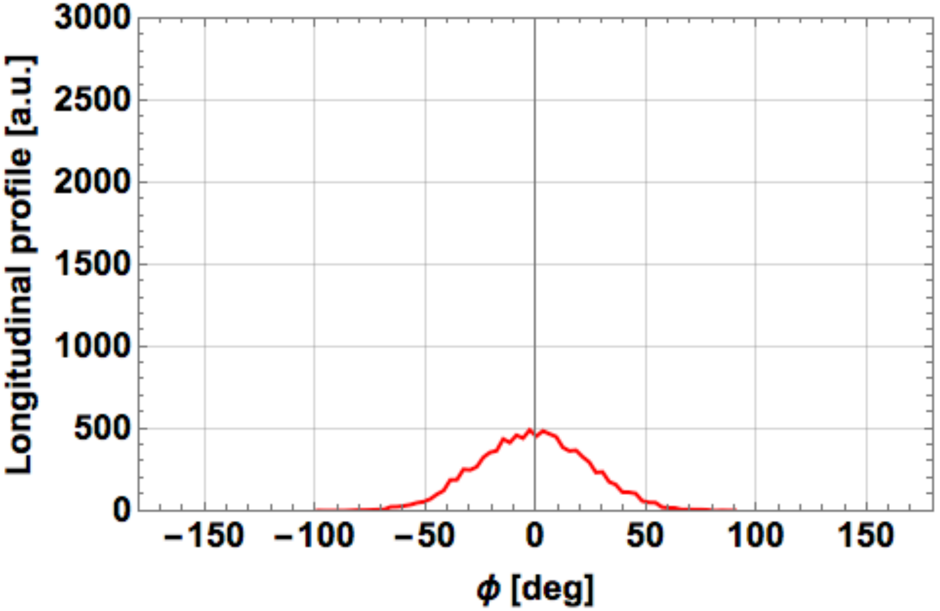
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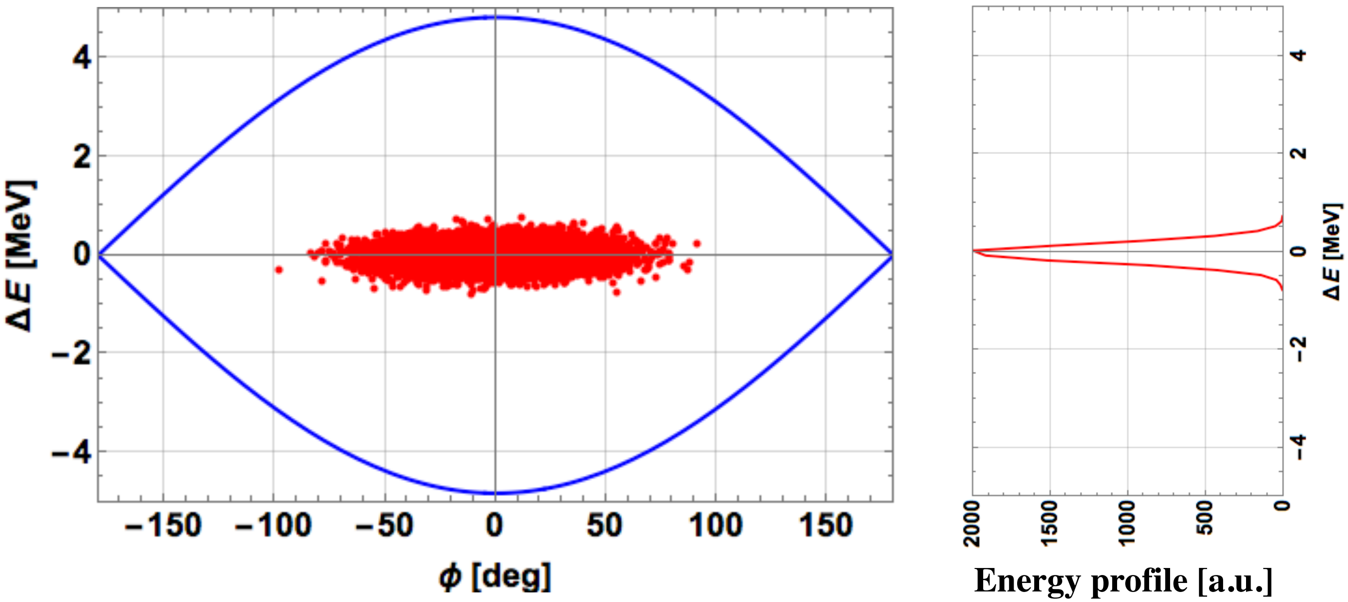
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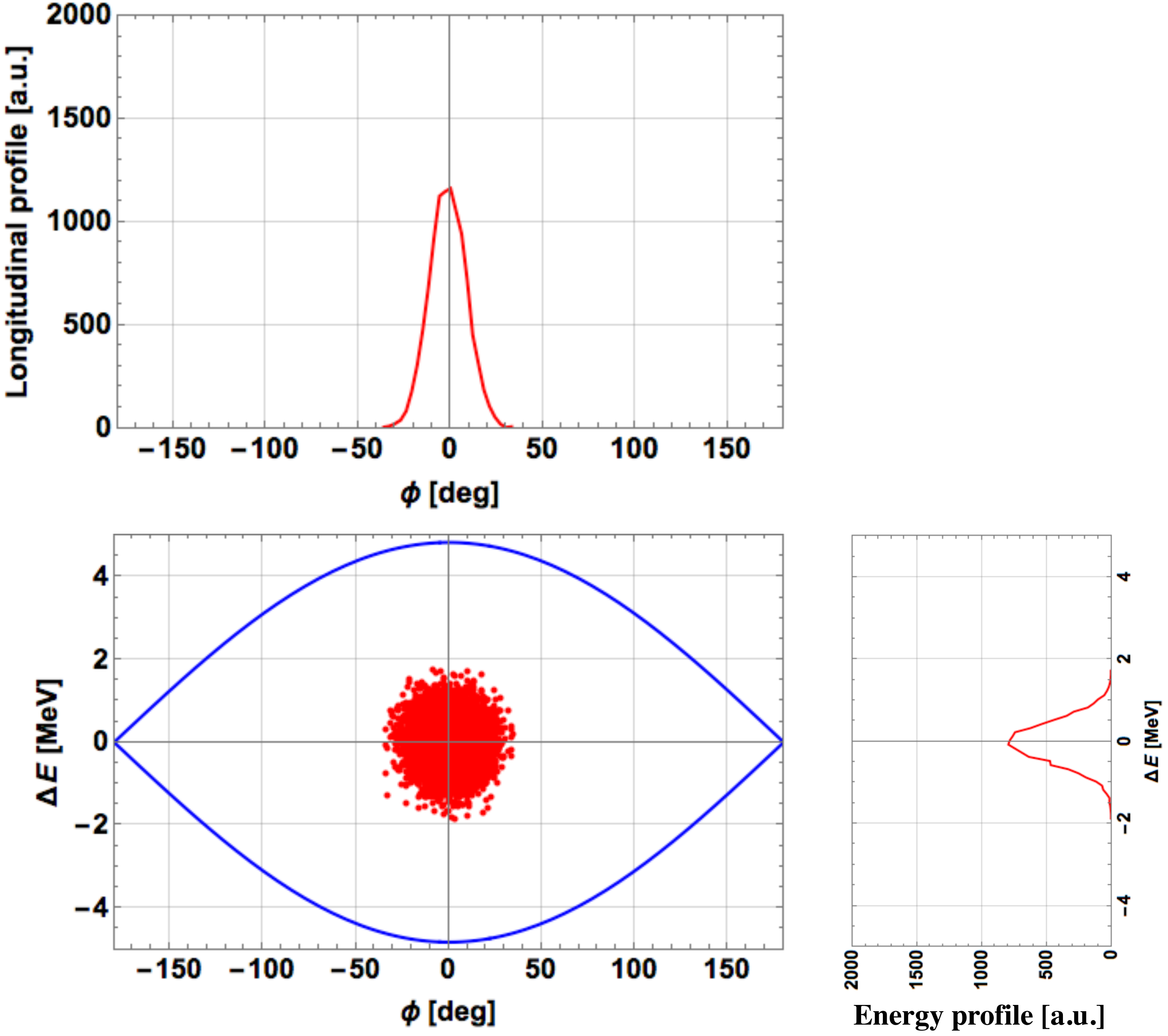
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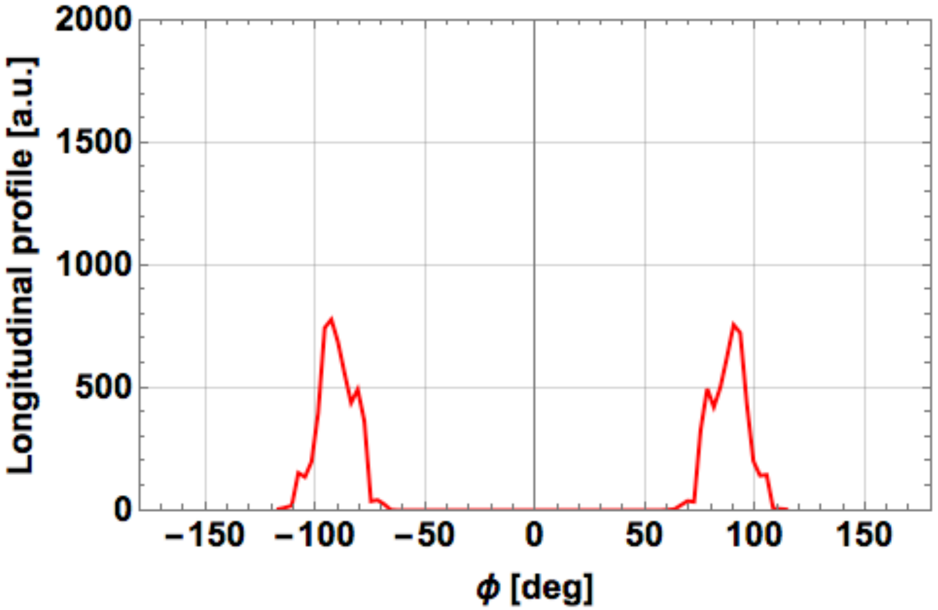
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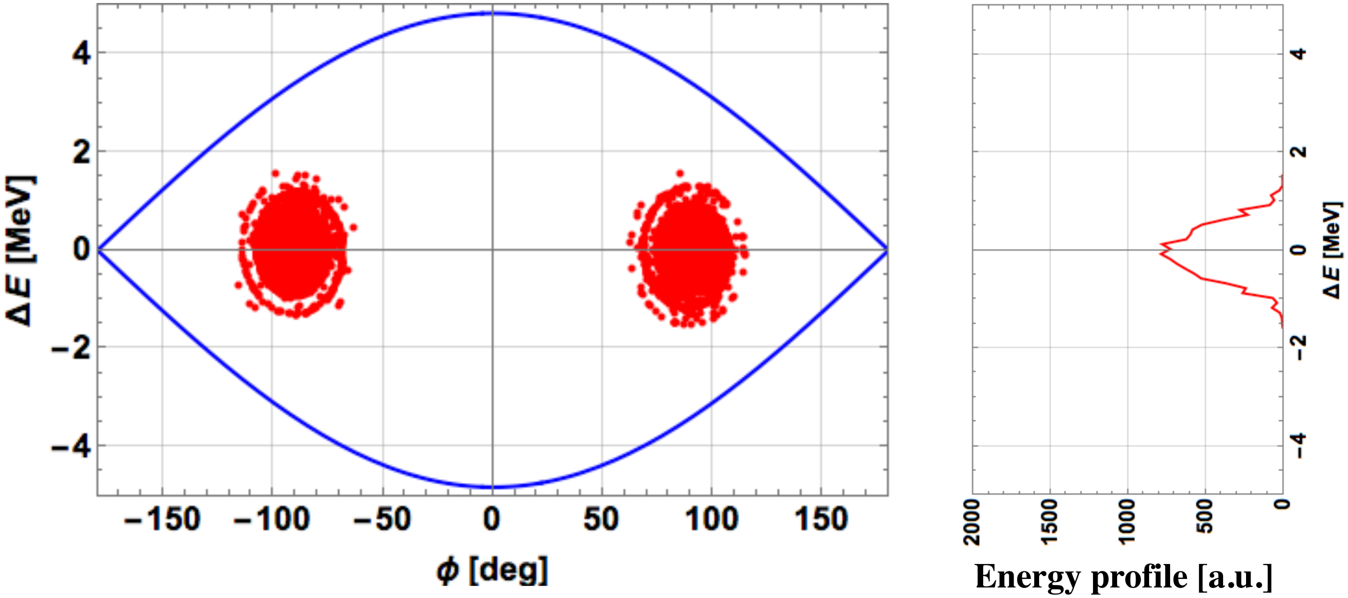
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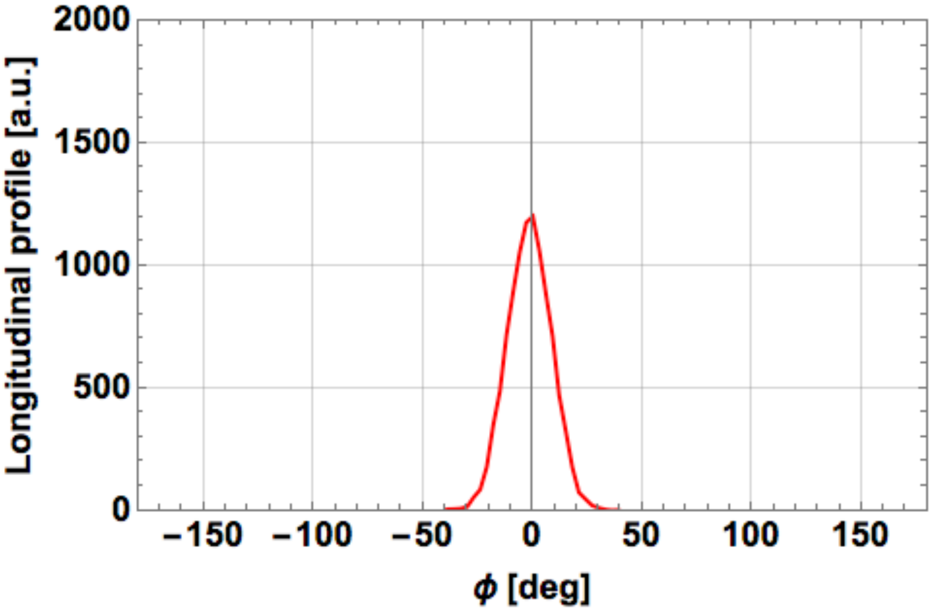
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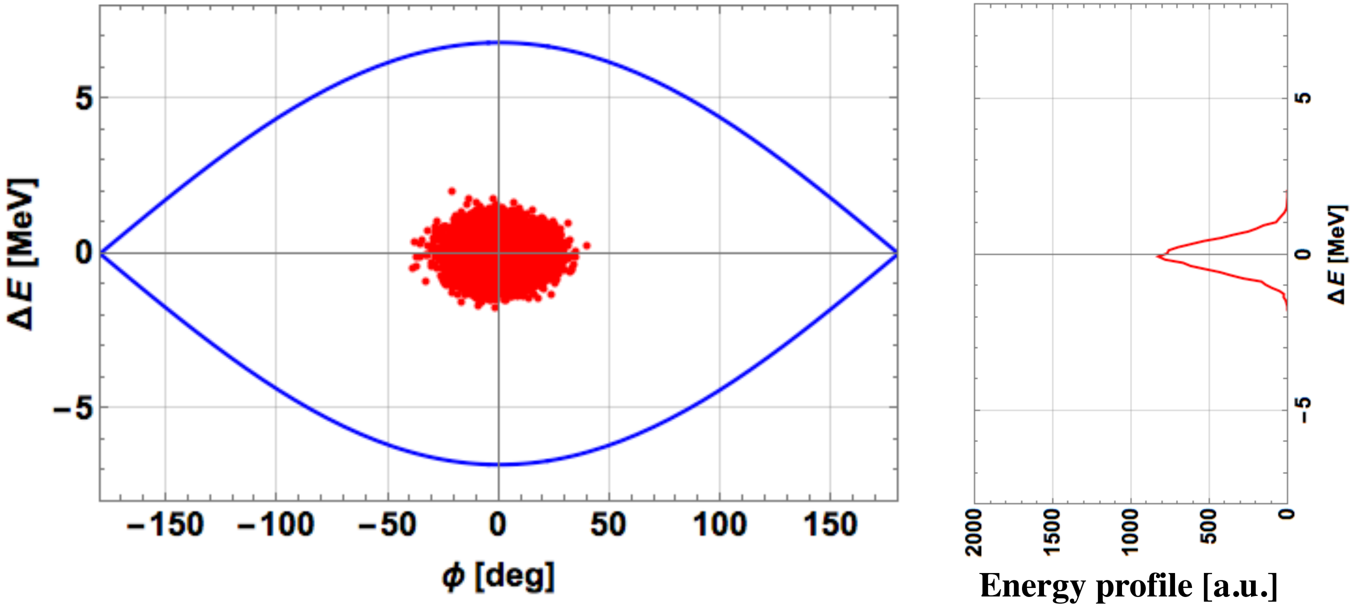
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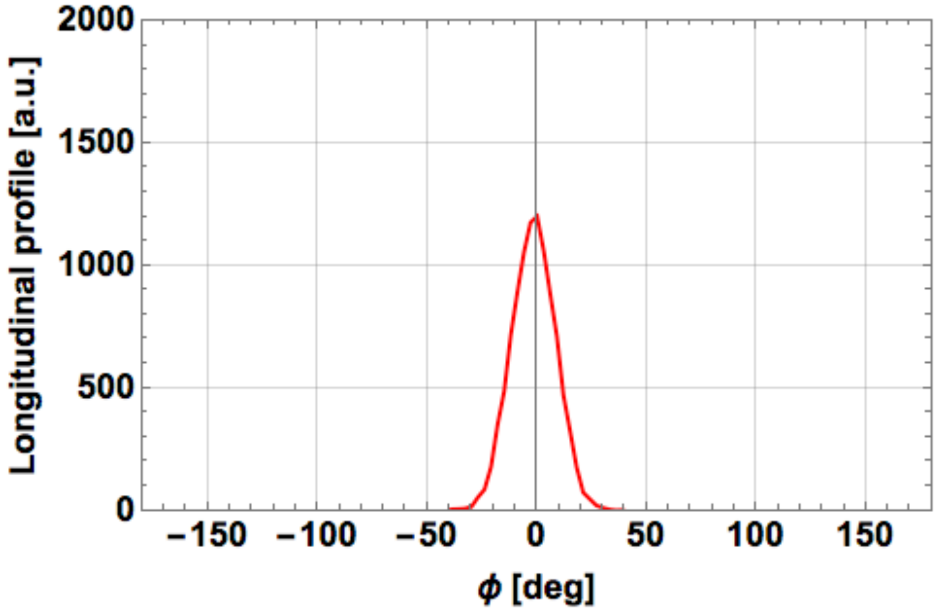
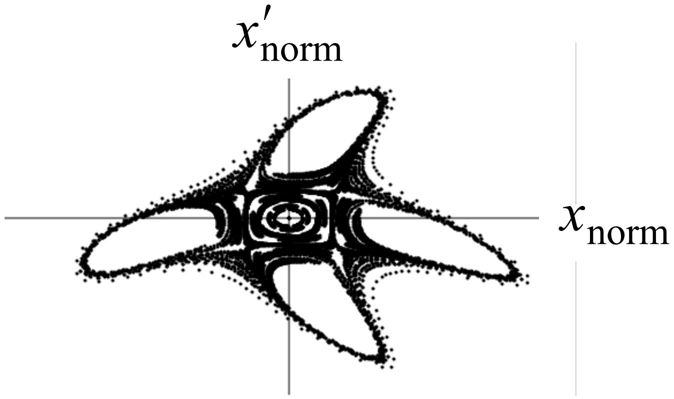
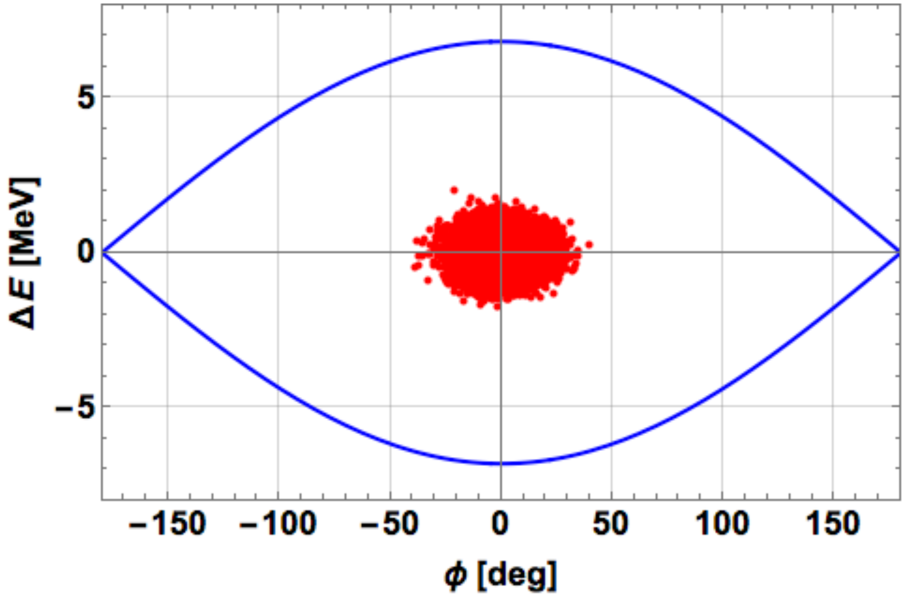
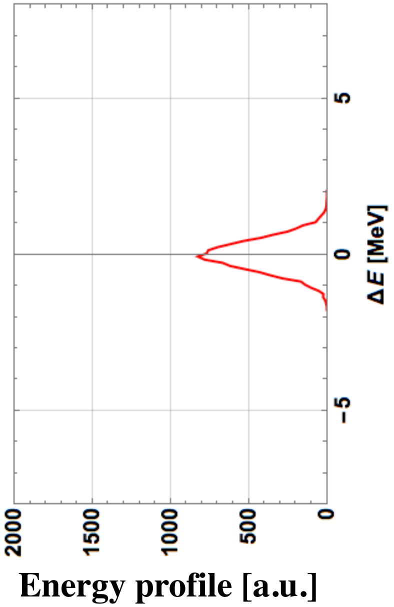


- Similarly, in the transverse (e.g. x) plane, linear and nonlinear motions can also be observed depending on the amplitude







Beam emittance

Beam emittance

- Similarly, in the transverse planes (x or y), these definitions are usually used

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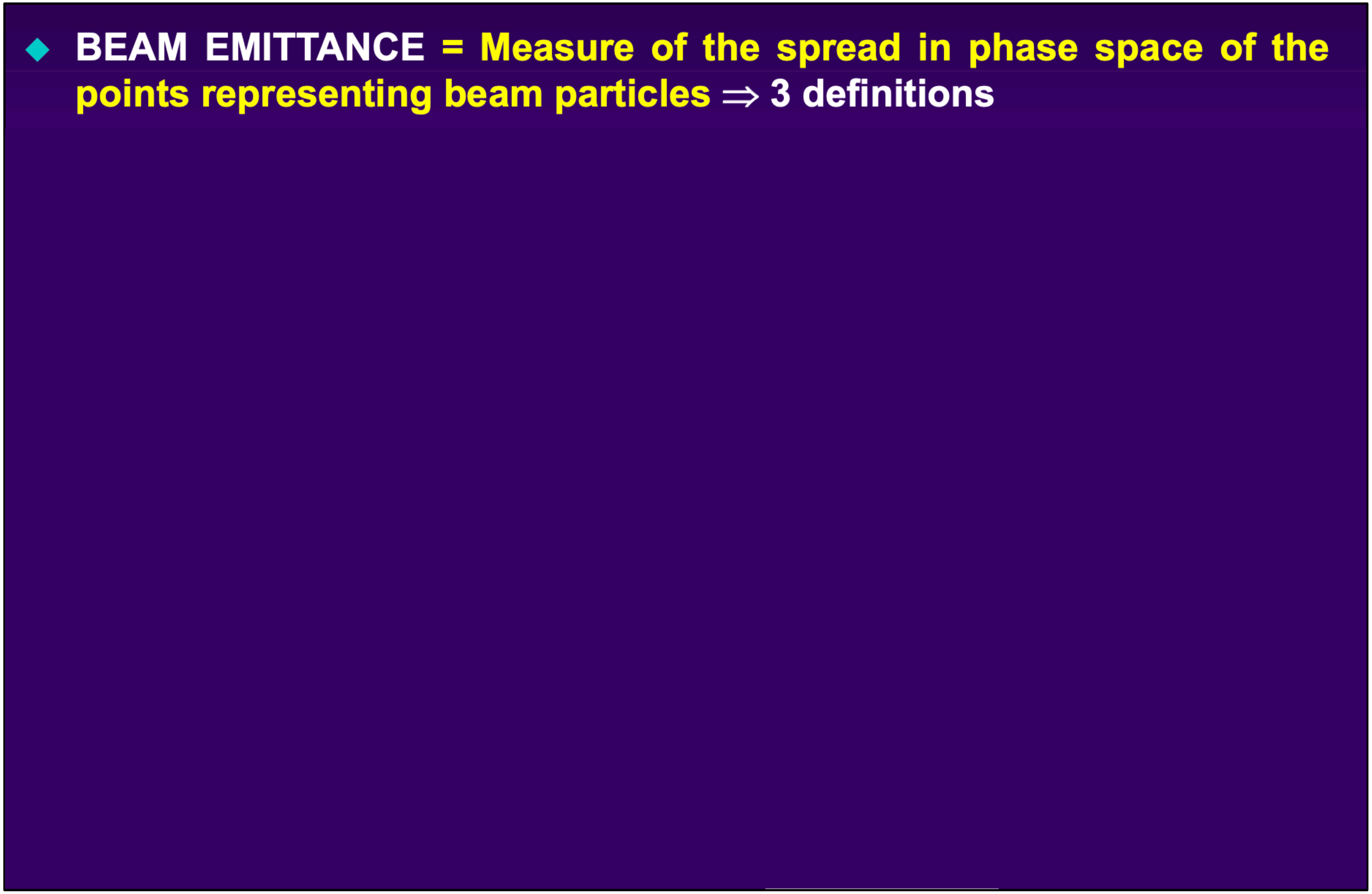

- **BEAM EMITTANCE = Measure of the spread in phase space of the points representing beam particles ⇒ 3 definitions**

Beam emittance

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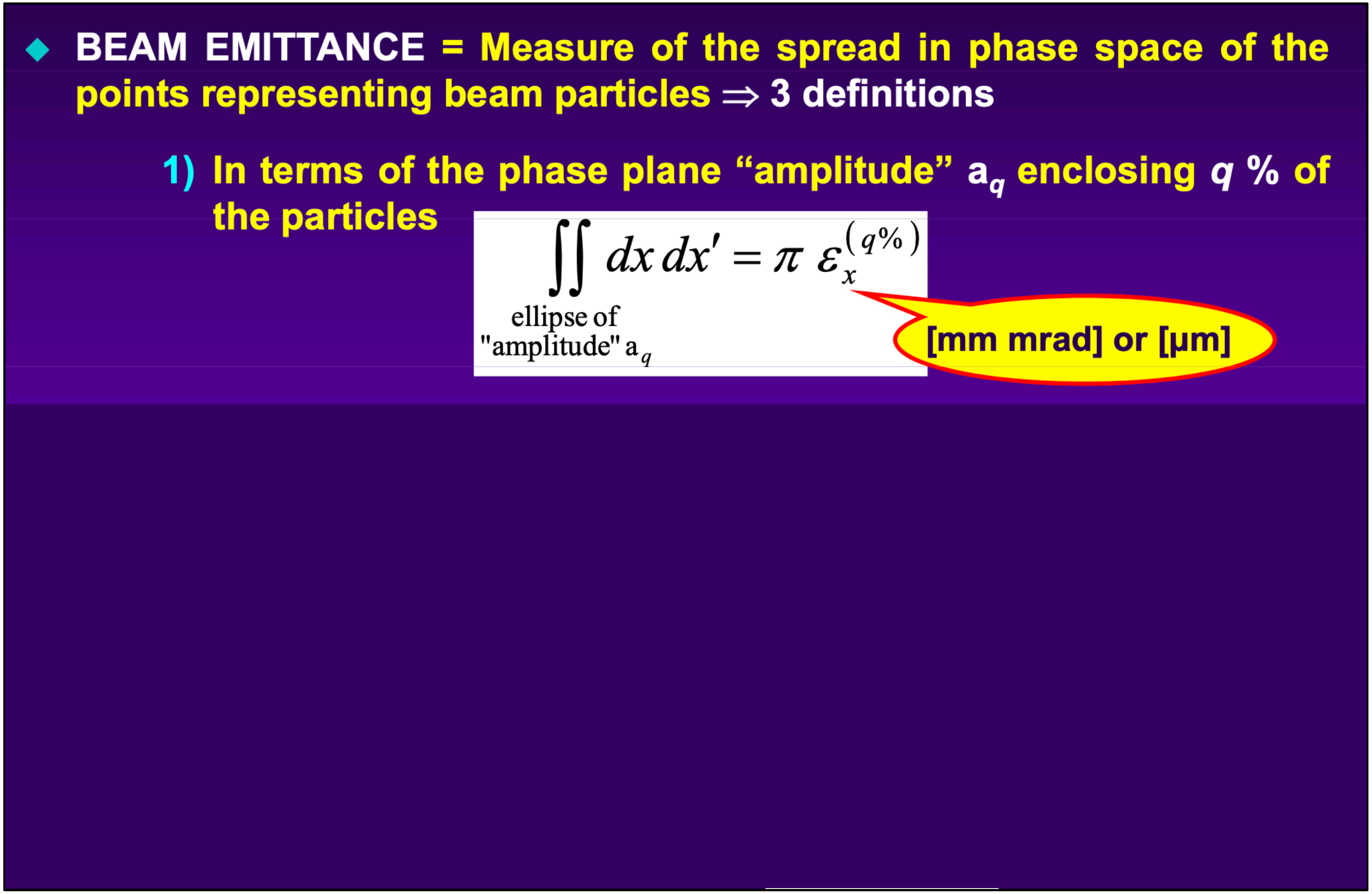


- **BEAM EMITTANCE = Measure of the spread in phase space of the points representing beam particles ⇒ 3 definitions**

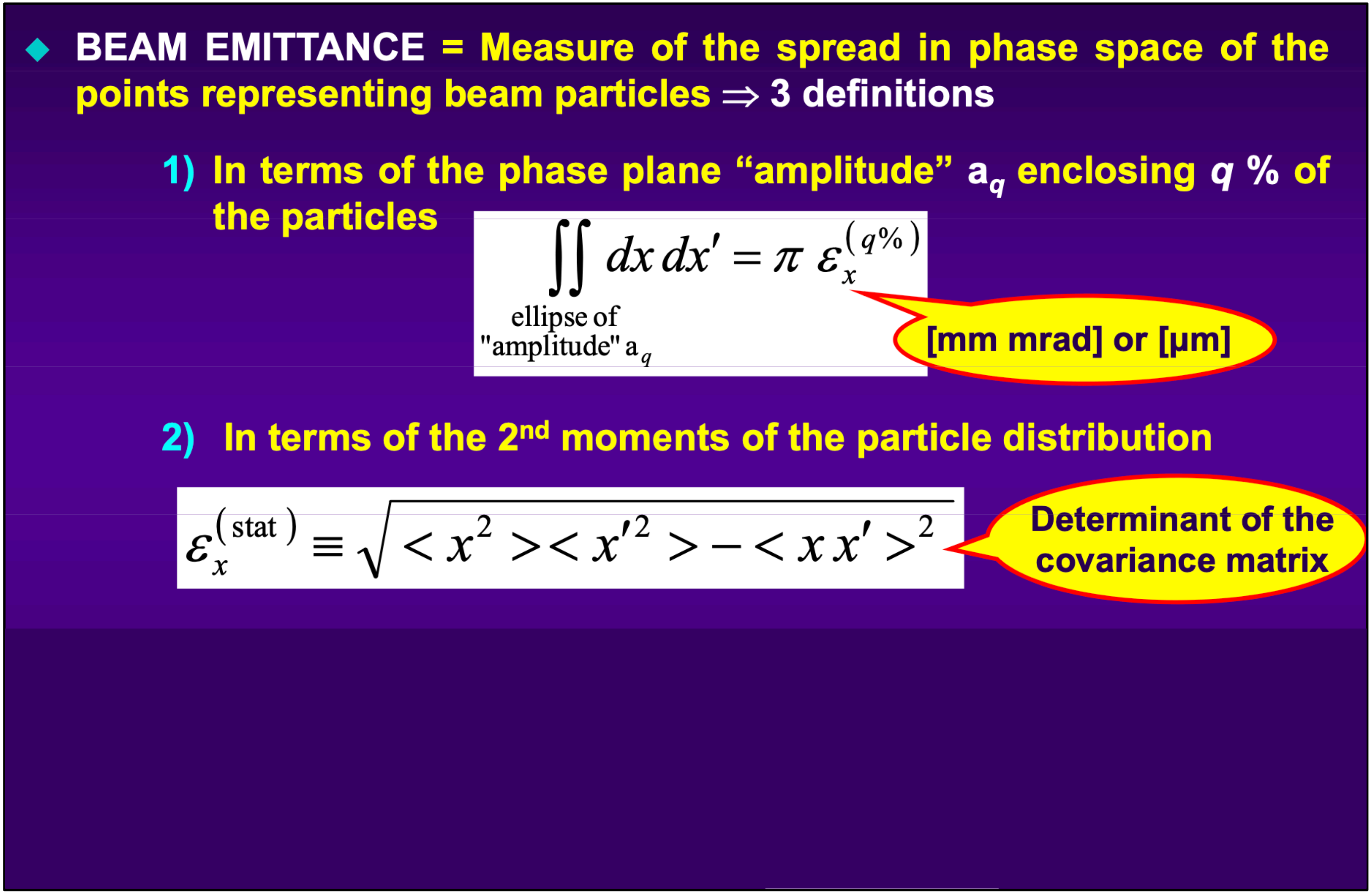
1) In terms of the phase plane "amplitude" a_q enclosing q % of the particles

$$\iint_{\substack{\text{ellipse of} \\ \text{"amplitude" } a_q}} dx\,dx' = \pi\,\varepsilon_x^{(q\%)}$$

[mm mrad] or [µm]

Beam emittance

- Similarly, in the transverse planes (x or y), these definitions are usually used



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2) In terms of the 2nd moments of the particle distribution

$$\varepsilon_x^{(\text{stat})} \equiv \sqrt{< x^2 >< x'^2 > - < x\, x' >^2}$$

Determinant of the covariance matrix

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$$\varepsilon_x^{(\sigma_x)} \equiv \frac{\sigma_x^2}{\beta_x}$$

Beam emittance

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The β-function reflects the size of the beam and depends only on the lattice

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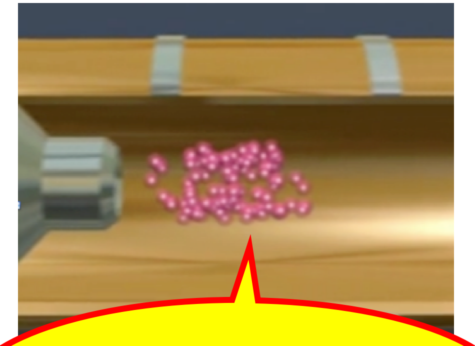
LATTICE = Arrangement of magnets along the design orbit

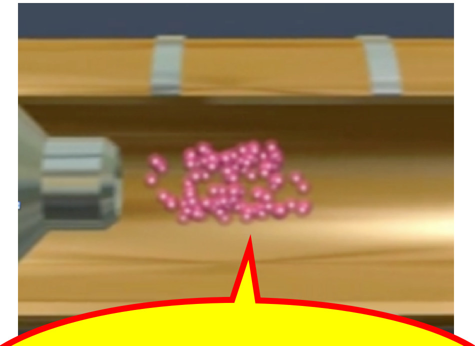
Special relativity will help…

Special relativity will help…



Group ("bunch") of particles (e.g.: p^+)

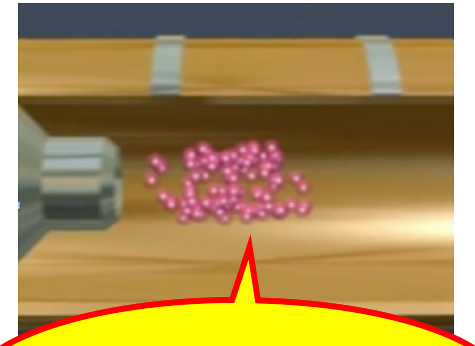
Special relativity will help…



Group (“bunch”) of particles (e.g.: p^+)

- With the Coulomb repulsion

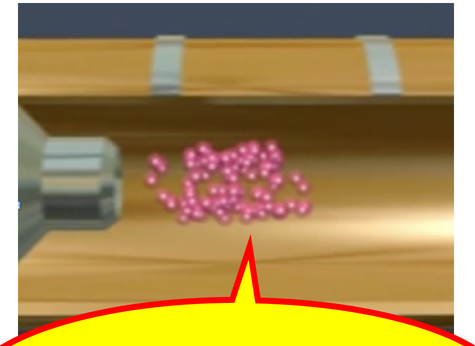
Special relativity will help…



Group ("bunch") of particles (e.g.: p^{+})

- With the Coulomb repulsion
- The short muon lifetime (~ 2.2 μs at rest) for a possible future muon collider

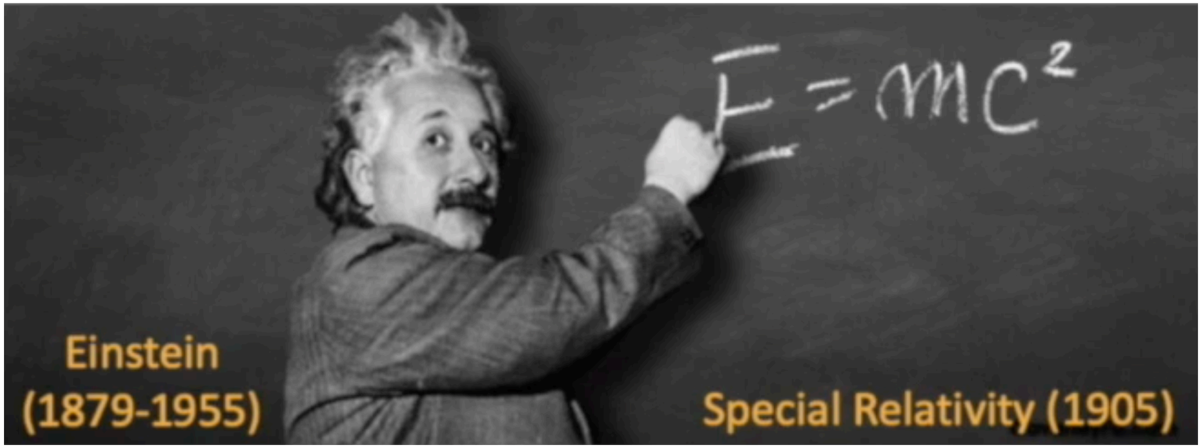
Special relativity will help…



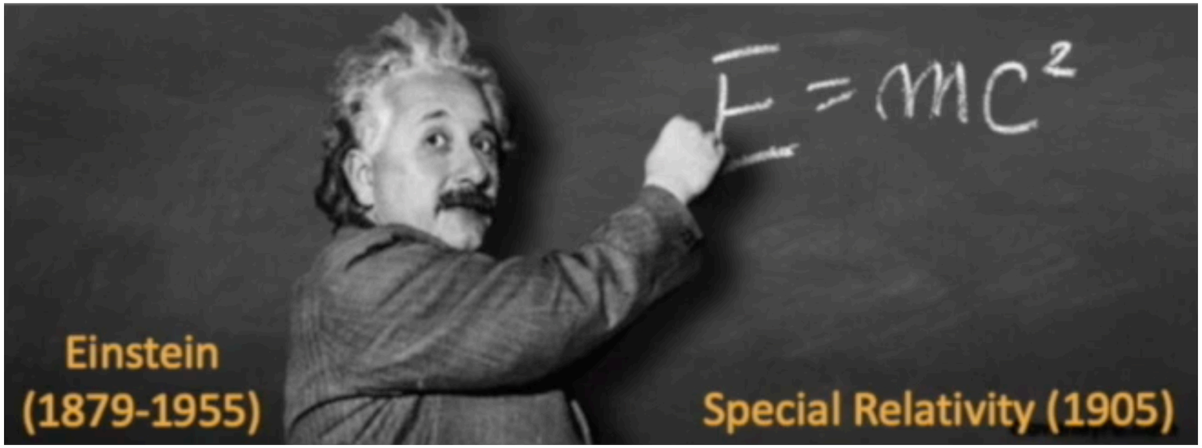
Group (“bunch”) of particles (e.g.: p⁺)

- With the Coulomb repulsion
- The short muon lifetime (~ 2.2 μs at rest) for a possible future muon collider
- Etc.

Special relativity

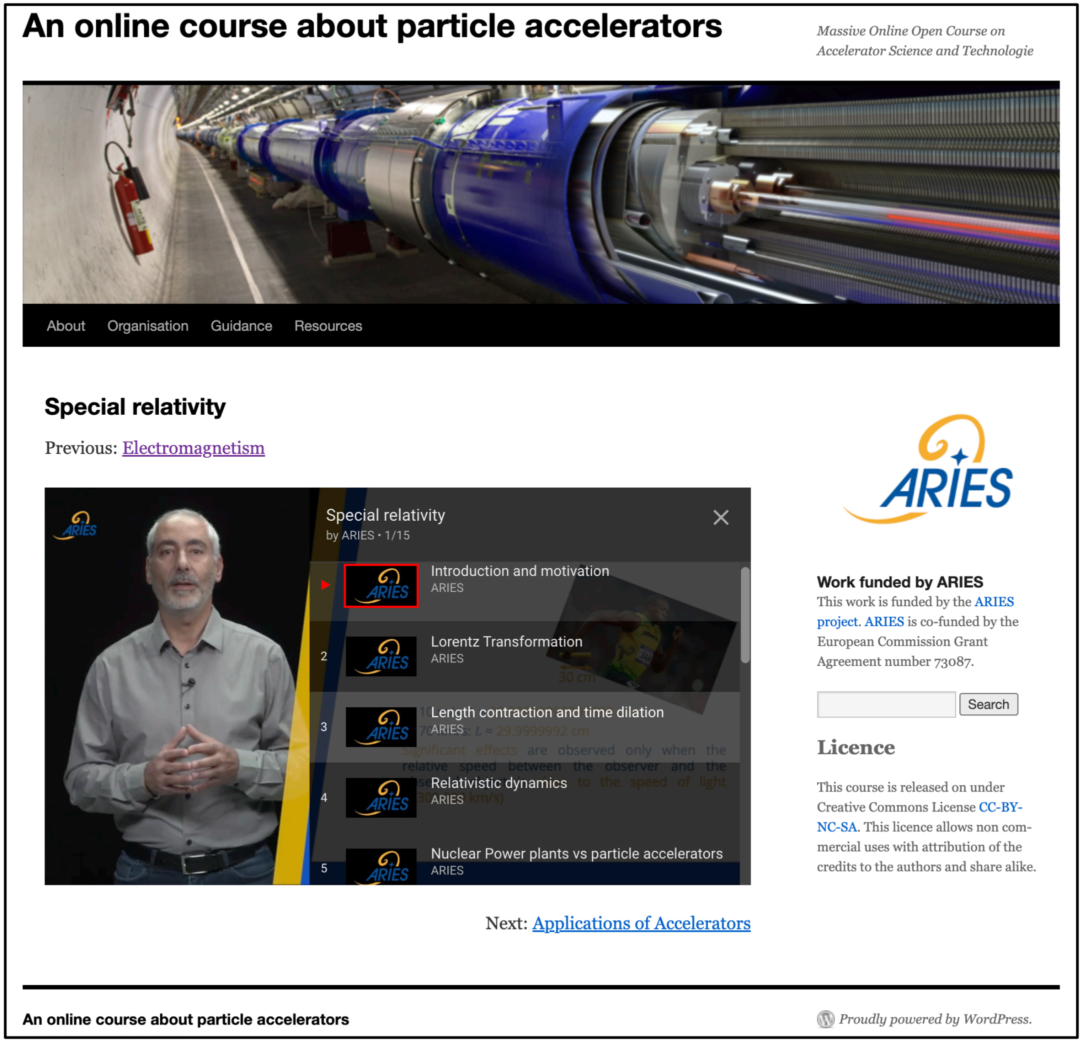


Special relativity



=> See **MOOC** (Massive Open Online Course) **on Special Relativity (SR)**: http://mooc.particle-accelerators.eu/special-relativity/

Special relativity



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Special relativity

Previous: Electromagnetism

Special relativity
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1. Introduction and motivation
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2. Lorentz Transformation
ARIES
3. Length contraction and time dilation
ARIES
4. Relativistic dynamics
ARIES
5. Nuclear Power plants vs particle accelerators
ARIES

Next: Applications of Accelerators

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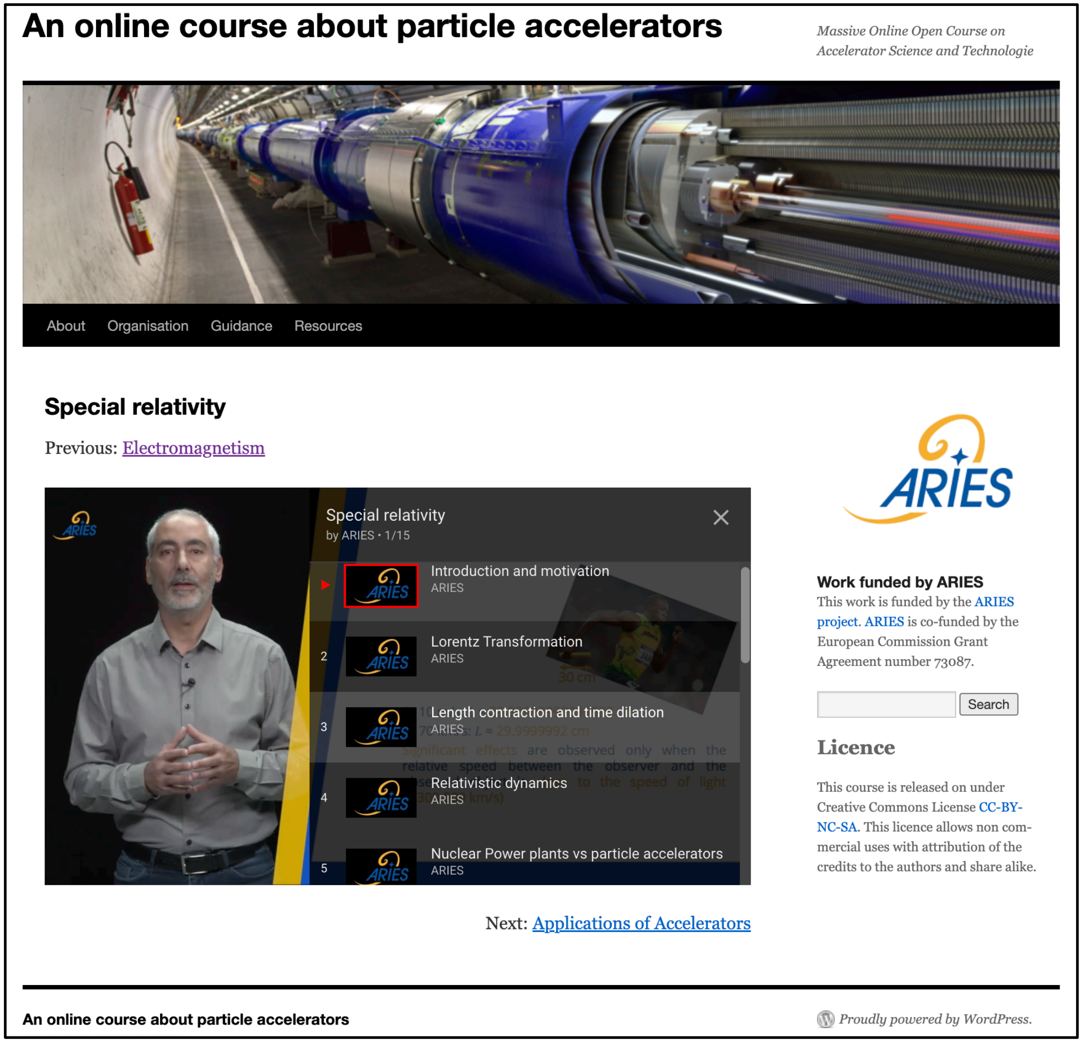
Special relativity



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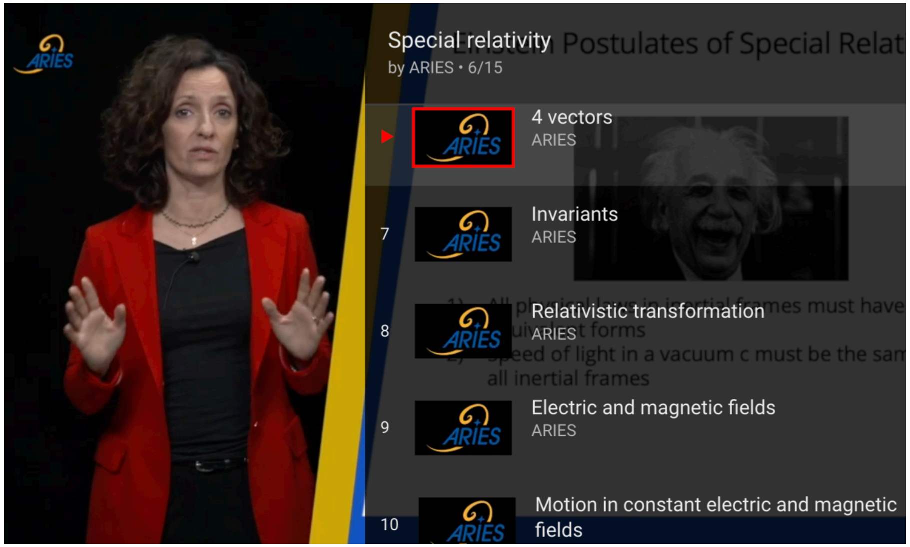

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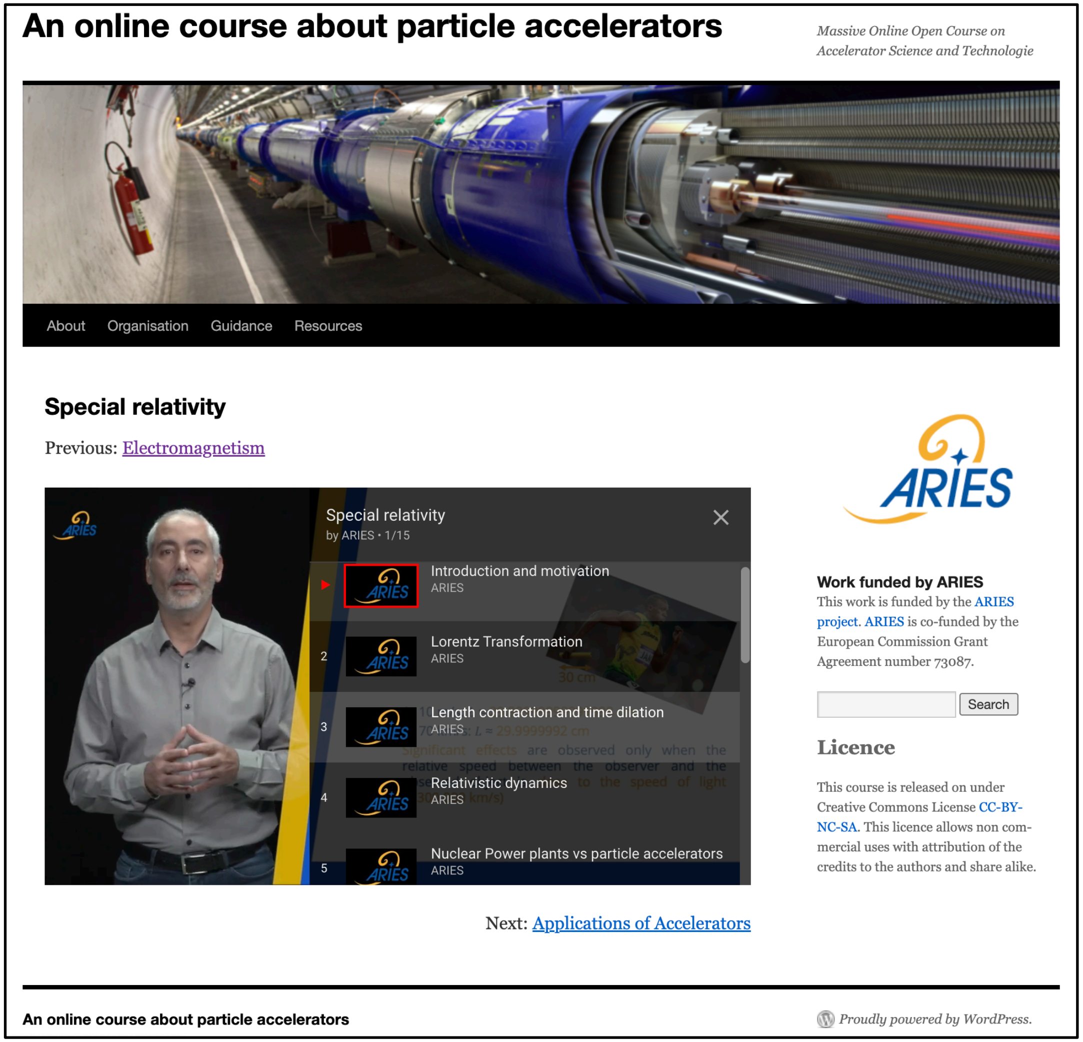
6. 4 vectors — ARIES
7. Invariants — ARIES
8. Relativistic transformation — ARIES
9. Electric and magnetic fields — ARIES
10. Motion in constant electric and magnetic fields

Special relativity



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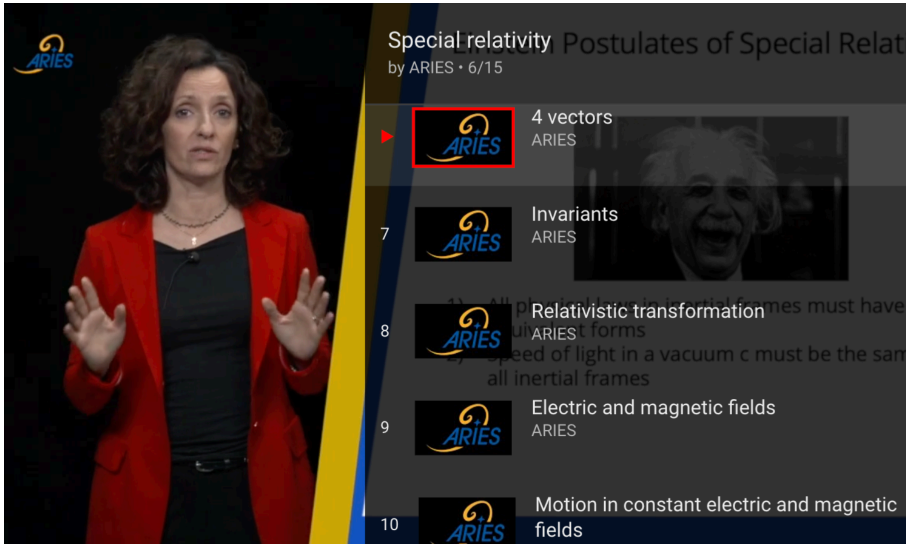
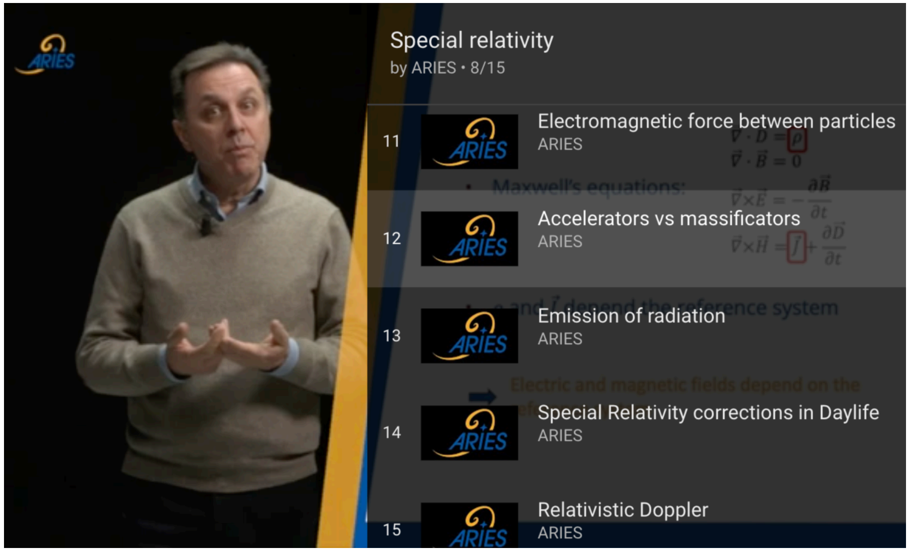


Special relativity
by ARIES • 6/15

6. 4 vectors – ARIES
7. Invariants – ARIES
8. Relativistic transformation – ARIES
9. Electric and magnetic fields – ARIES
10. Motion in constant electric and magnetic fields



Special relativity
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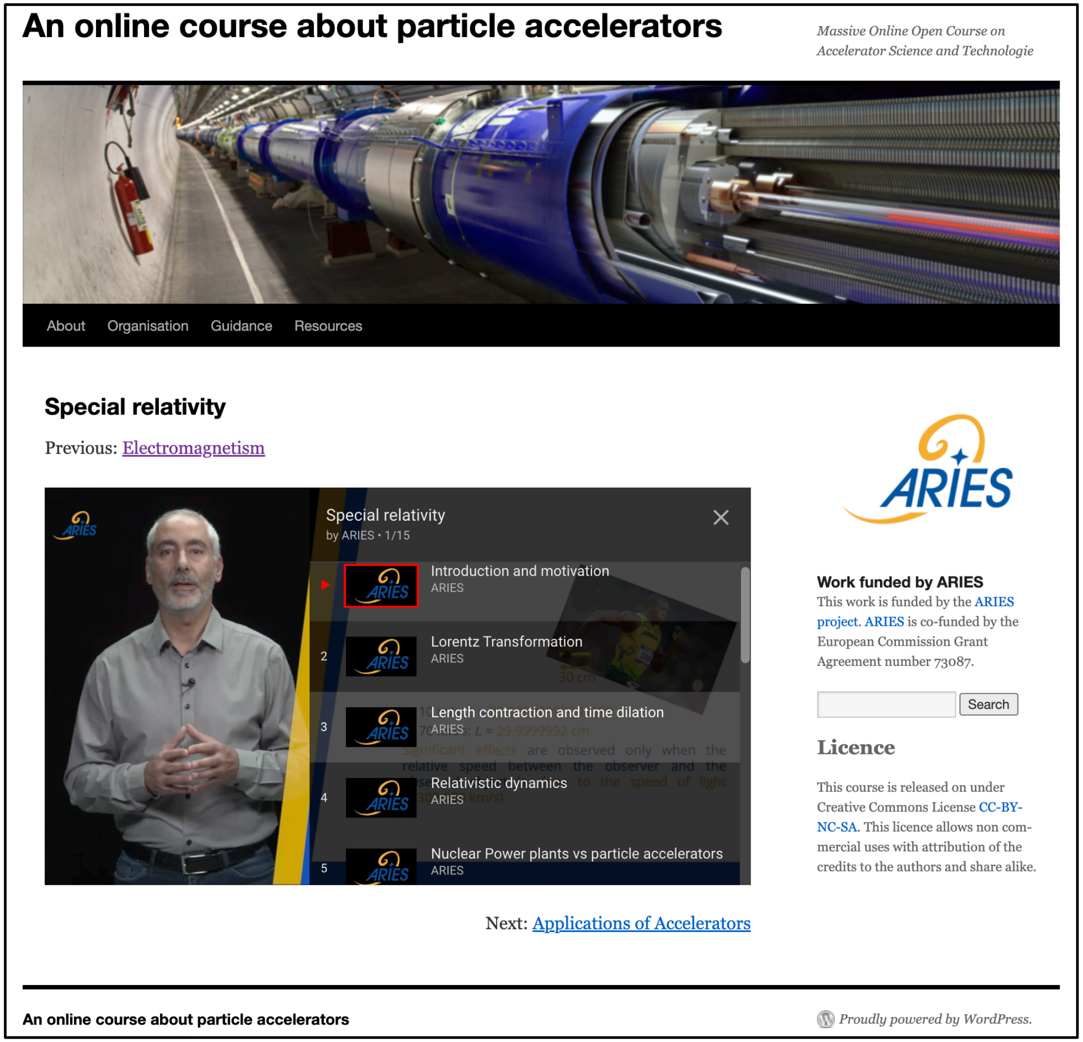
11. Electromagnetic force between particles – ARIES
12. Accelerators vs massificators – ARIES
13. Emission of radiation – ARIES
14. Special Relativity corrections in Daylife – ARIES
15. Relativistic Doppler – ARIES

Special relativity



An online course about particle accelerators

Massive Online Open Course on Accelerator Science and Technologie

Special relativity

Previous: Electromagnetism

Special relativity
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Introduction and motivation
ARIES

2 Lorentz Transformation
ARIES

3 Length contraction and time dilation
ARIES

4 Relativistic dynamics
ARIES

5 Nuclear Power plants vs particle accelerators
ARIES

Next: Applications of Accelerators

Work funded by ARIES

This work is funded by the ARIES project. ARIES is co-funded by the European Commission Grant Agreement number 73087.

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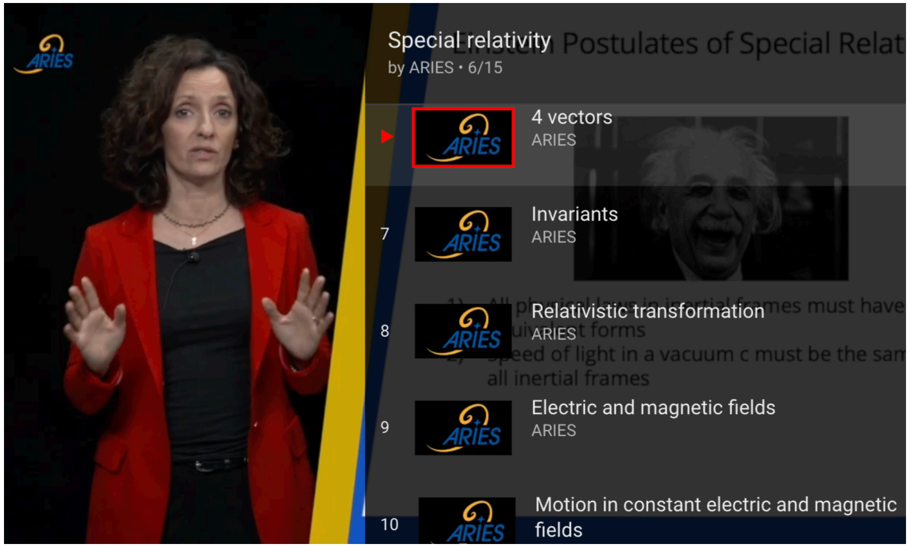

Special relativity
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4 vectors
ARIES

7 Invariants
ARIES

8 Relativistic transformation
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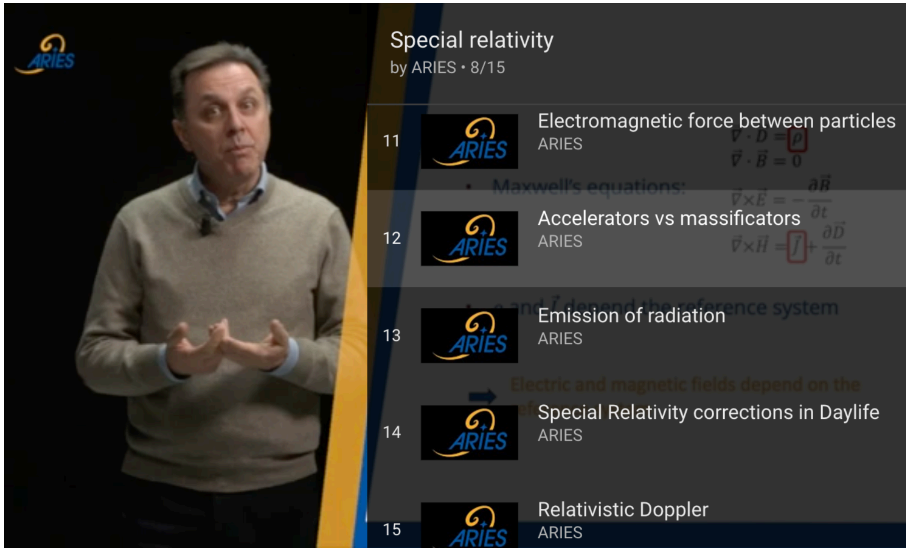


Special relativity
by ARIES • 8/15

11 Electromagnetic force between particles
ARIES

12 Accelerators vs massificators
ARIES

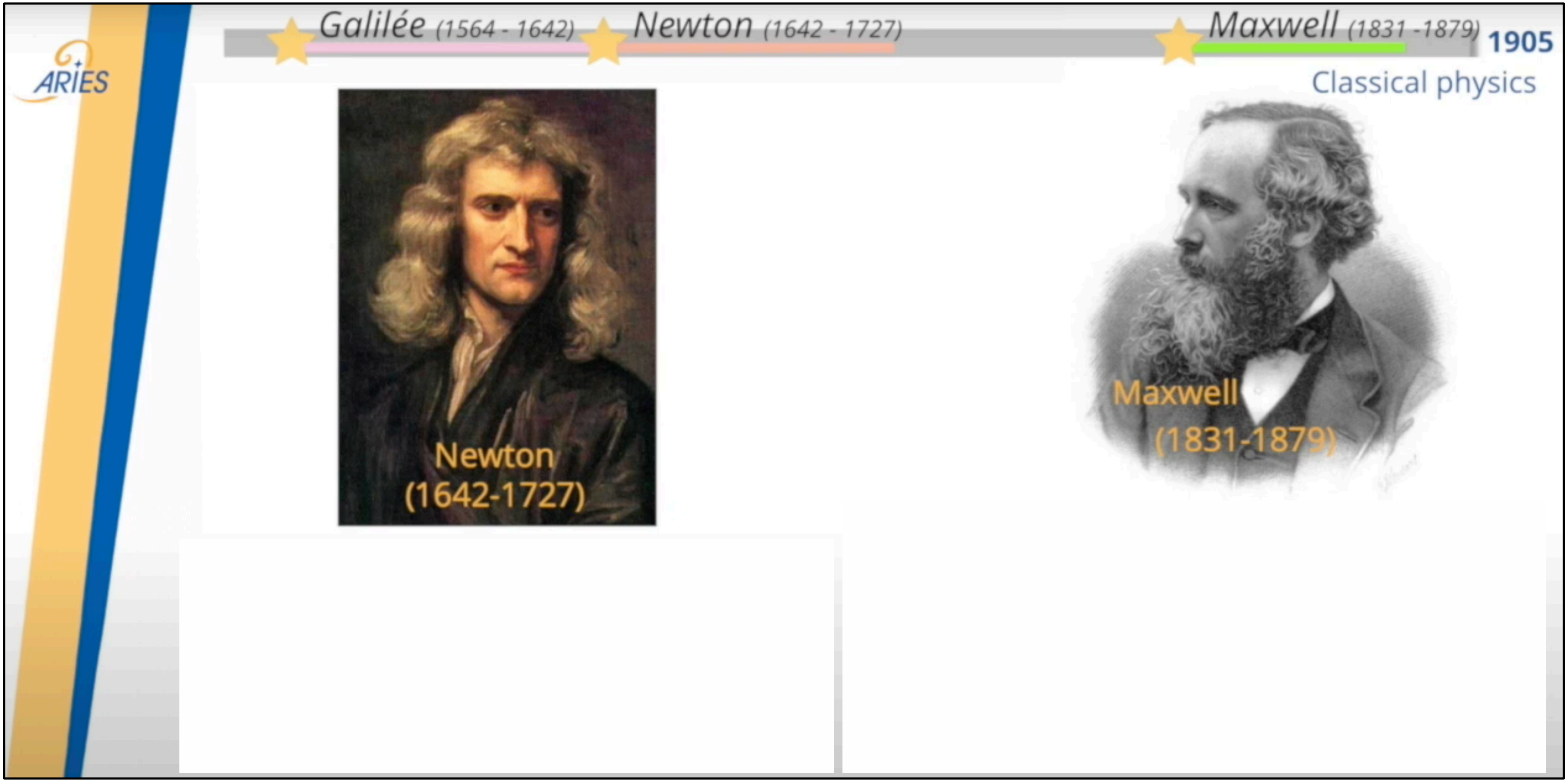
13 Emission of radiation
ARIES

14 Special Relativity corrections in Daylife
ARIES

15 Relativistic Doppler
ARIES

=> Let's have a look to the first 2 minutes…: http://mooc.particle-accelerators.eu/special-relativity/

Special relativity



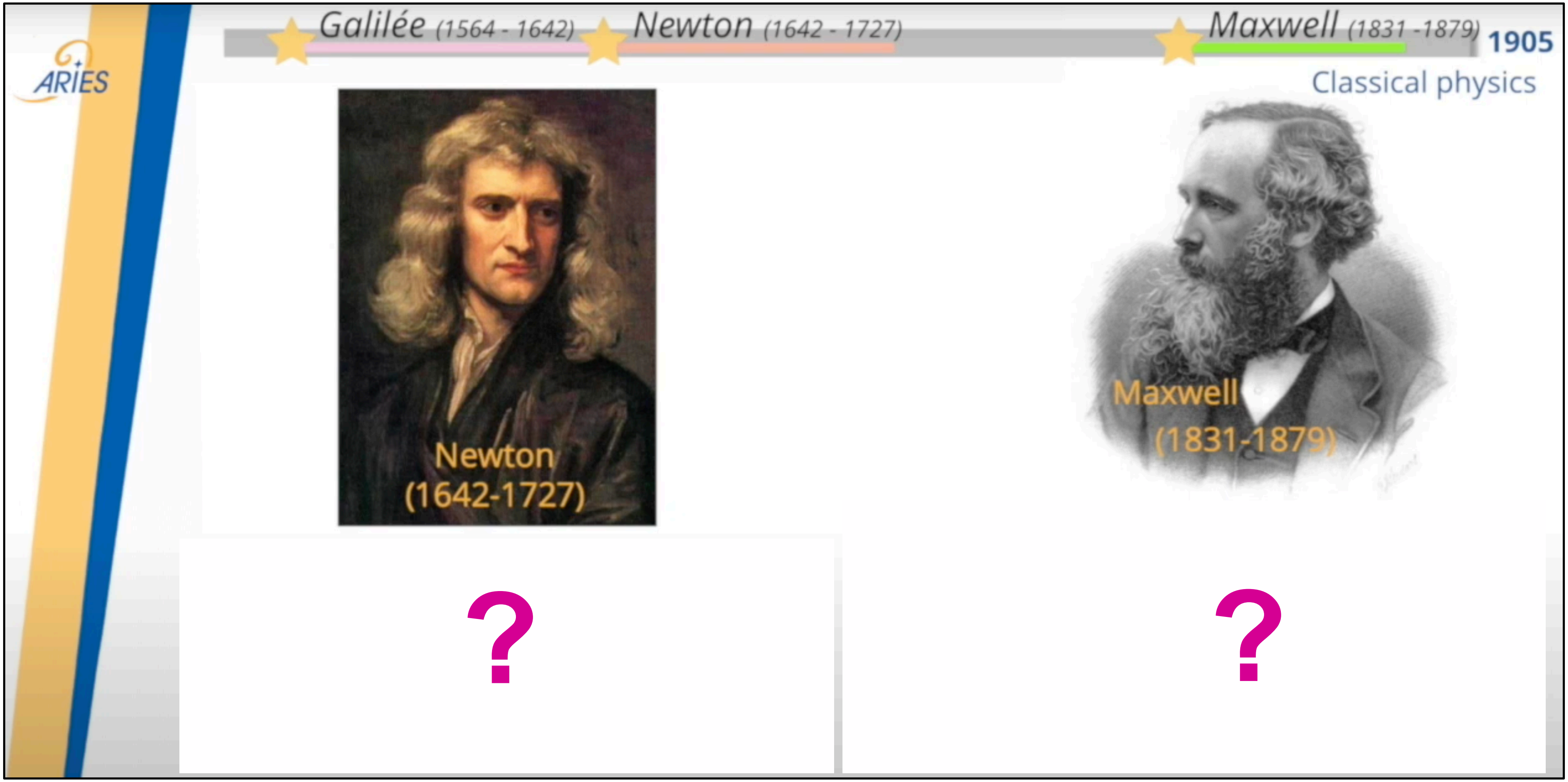
Galilée (1564 - 1642) *Newton* (1642 - 1727) *Maxwell* (1831 -1879) **1905**

Classical physics

Newton
(1642-1727)

Maxwell
(1831-1879)

Special relativity



ARIES

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Newton
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Maxwell
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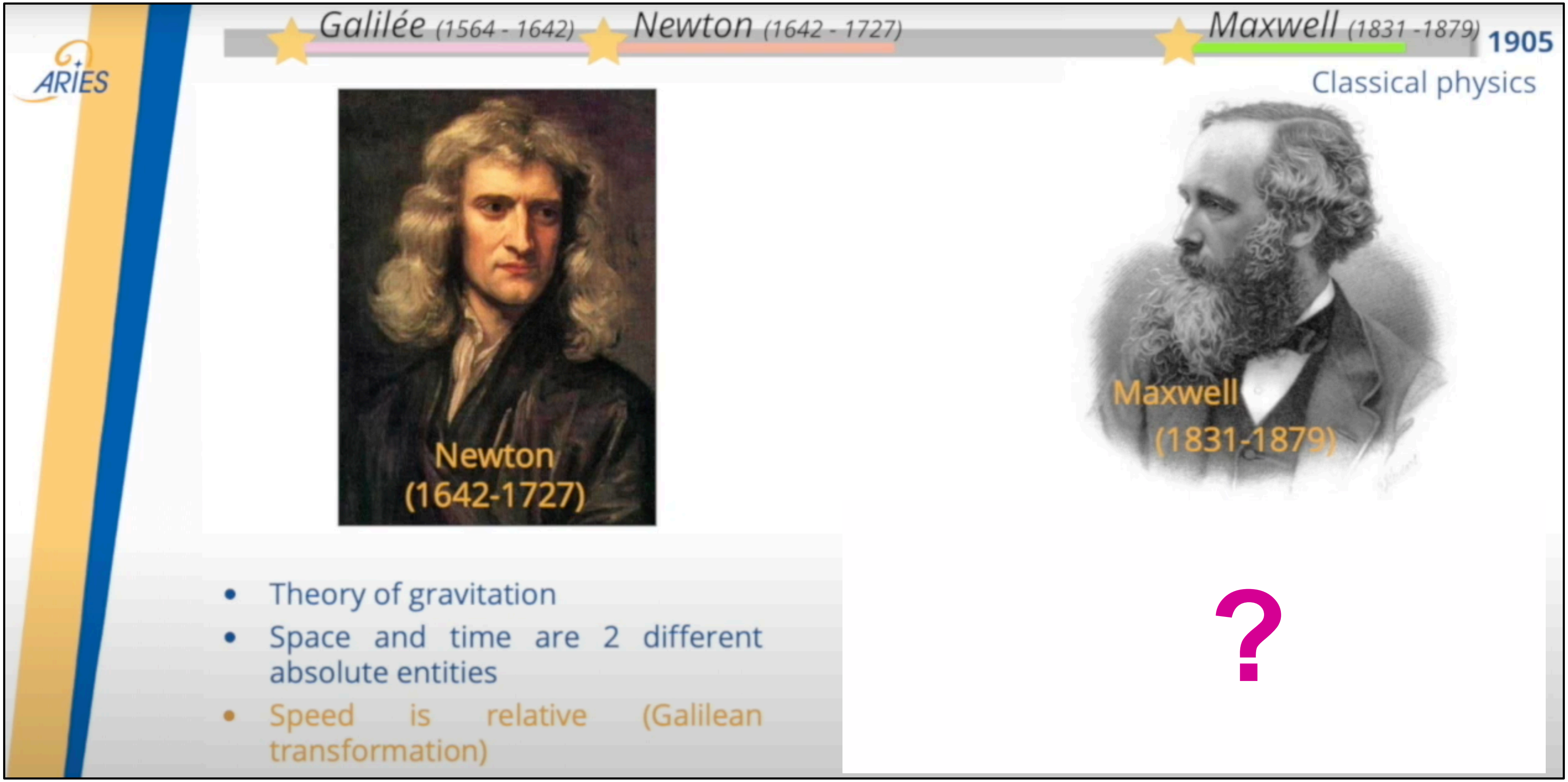
?

?

Special relativity



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- Theory of gravitation
- Space and time are 2 different absolute entities
- Speed is relative (Galilean transformation)

?

Special relativity

Galilée *(1564 - 1642)* *Newton* *(1642 - 1727)* *Maxwell* *(1831 -1879)* **1905**

Classical physics

Newton
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Maxwell
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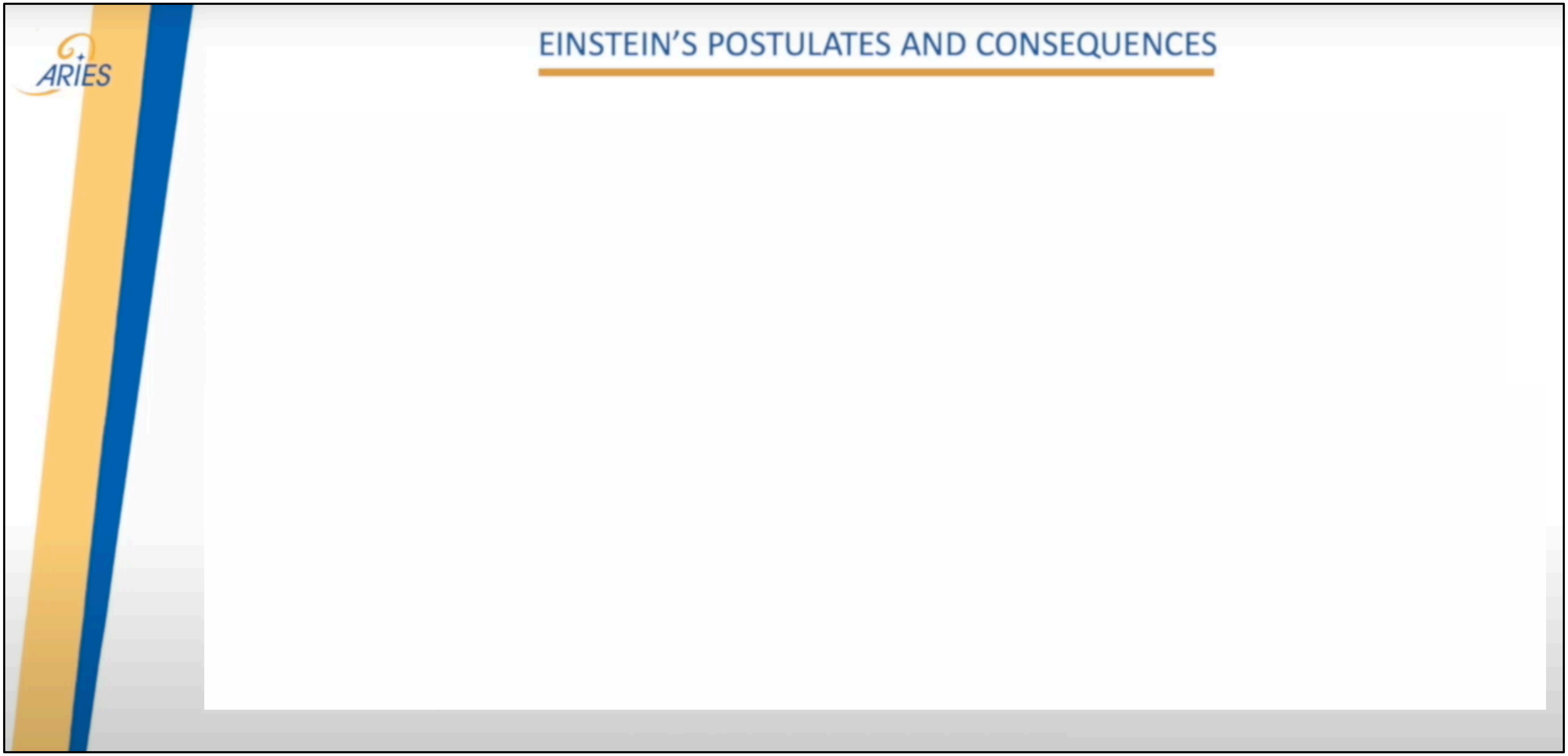
- Theory of gravitation
- Space and time are 2 different absolute entities
- Speed is relative (Galilean transformation)

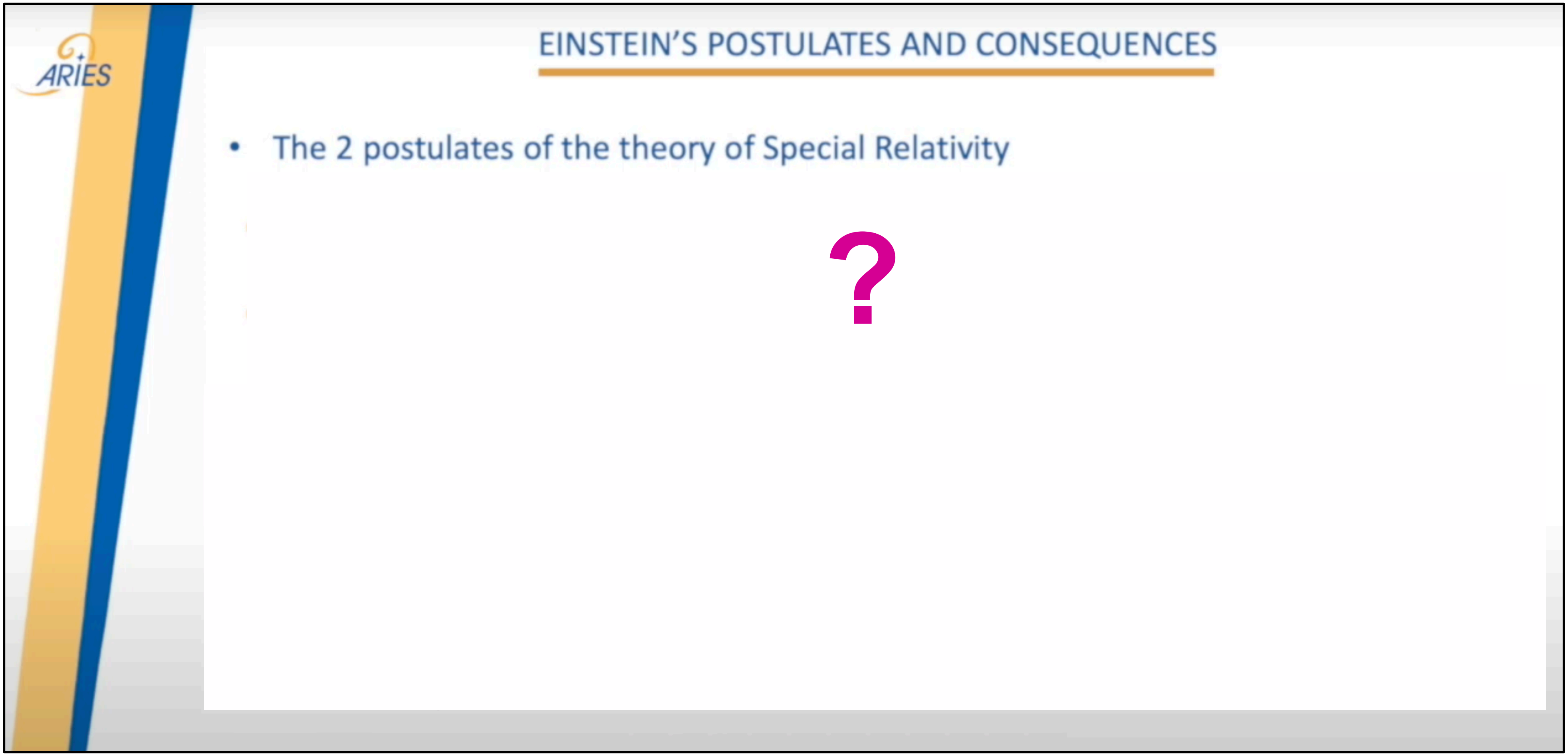
- Theory of electromagnetism
- Light is an electromagnetic wave, propagating through the “ether”
- Speed of light is constant in vacuum in all frames

Special relativity



EINSTEIN'S POSTULATES AND CONSEQUENCES

Special relativity

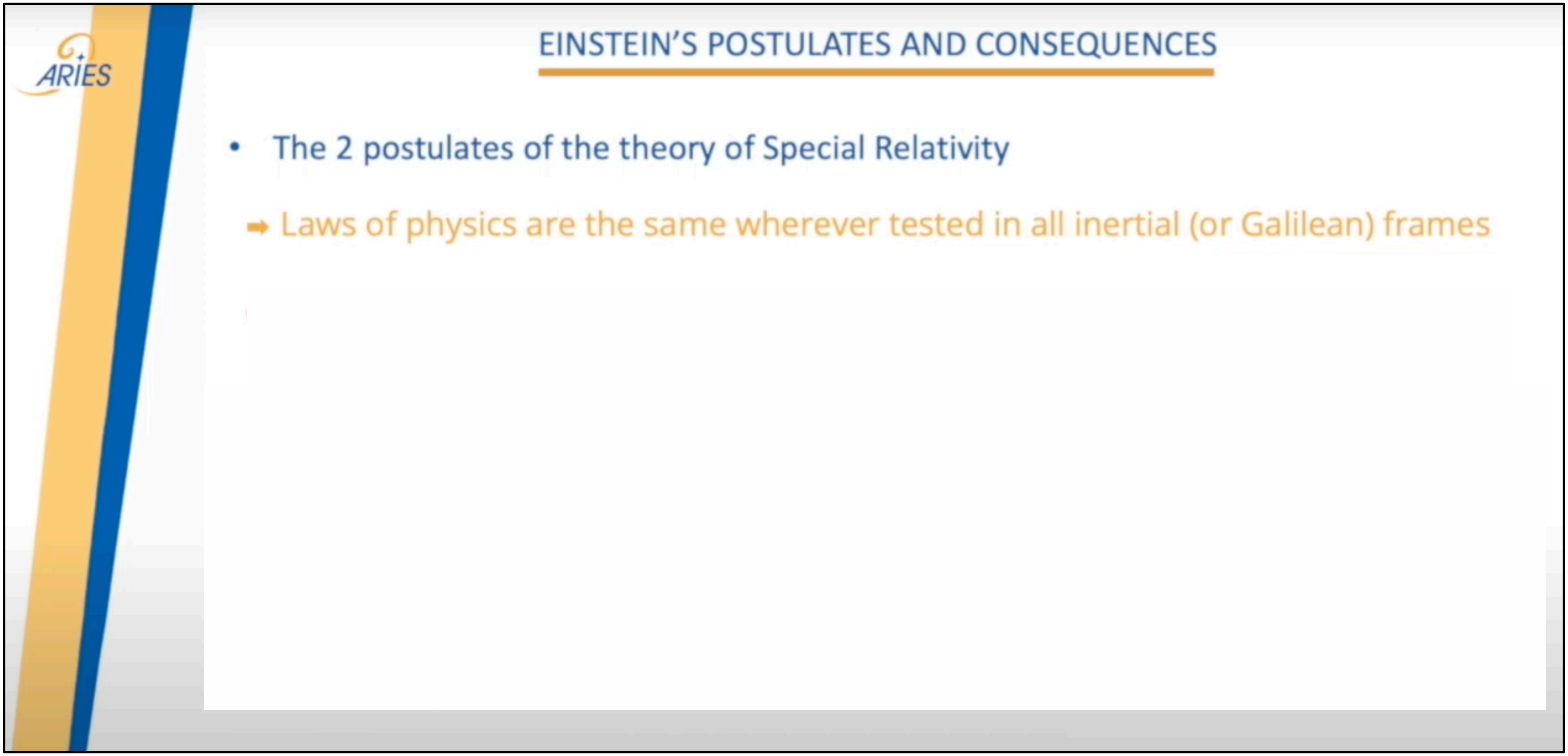


EINSTEIN’S POSTULATES AND CONSEQUENCES

- The 2 postulates of the theory of Special Relativity

?

Special relativity

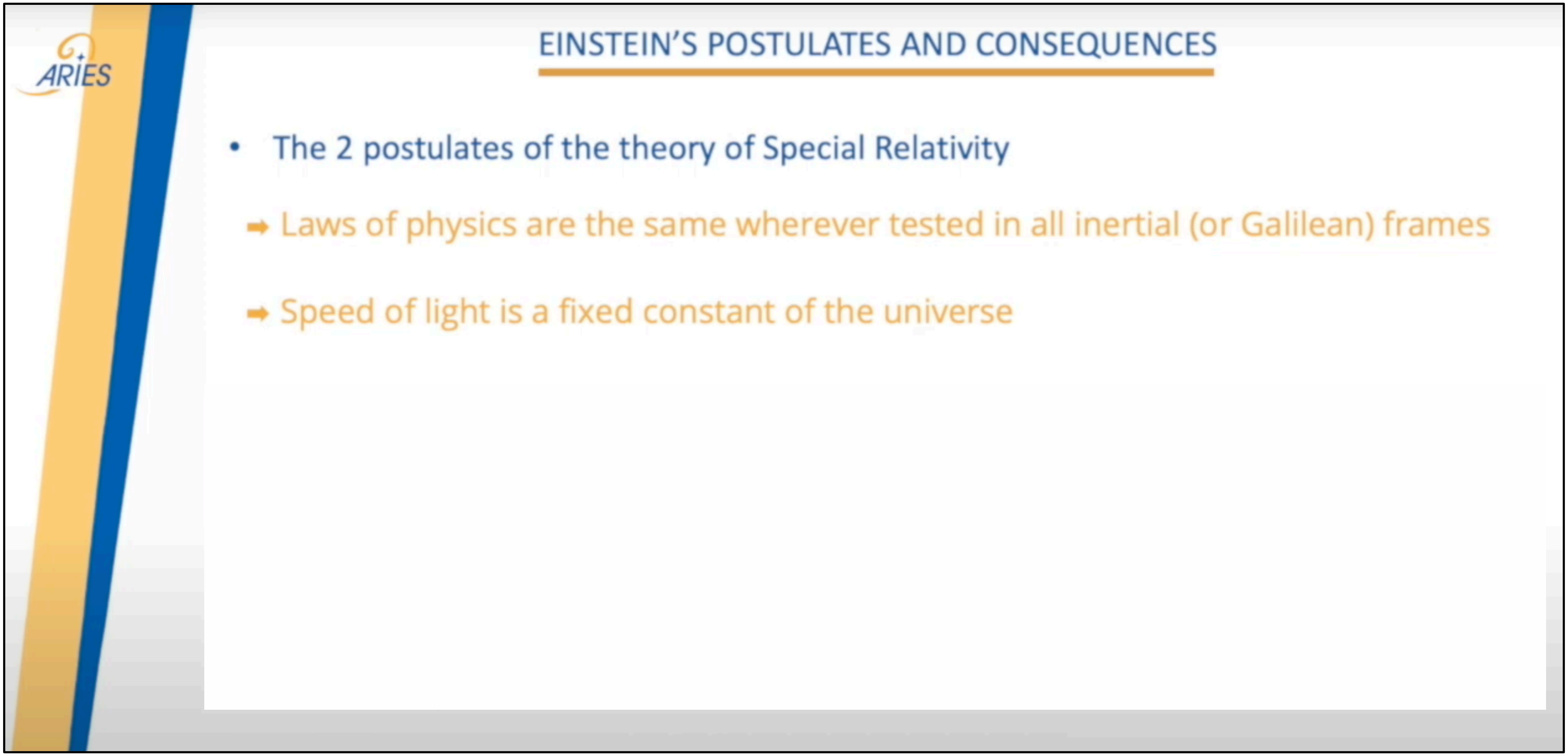


EINSTEIN'S POSTULATES AND CONSEQUENCES

- The 2 postulates of the theory of Special Relativity

➡ Laws of physics are the same wherever tested in all inertial (or Galilean) frames

Special relativity



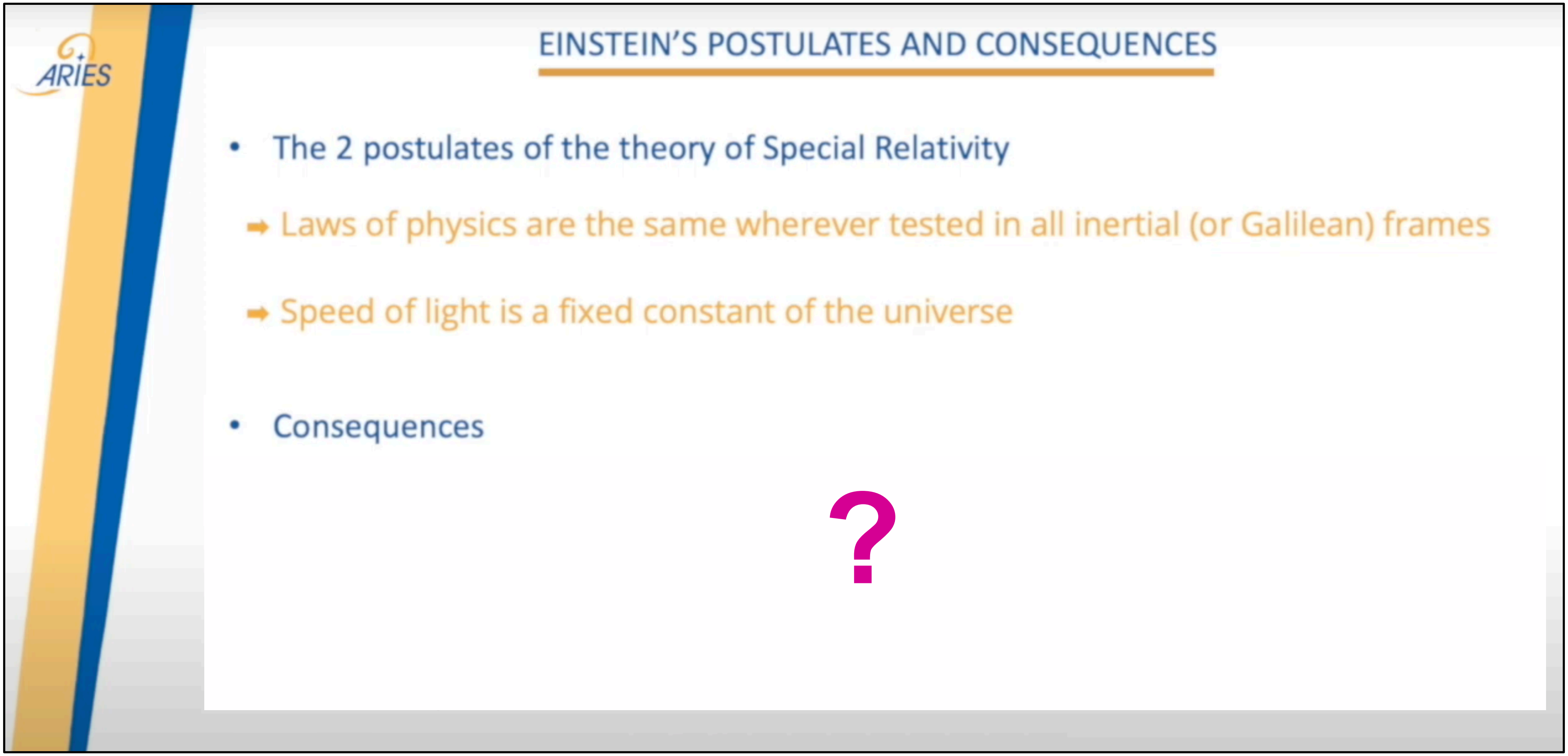
EINSTEIN'S POSTULATES AND CONSEQUENCES

- The 2 postulates of the theory of Special Relativity

➡ Laws of physics are the same wherever tested in all inertial (or Galilean) frames

➡ Speed of light is a fixed constant of the universe

Special relativity



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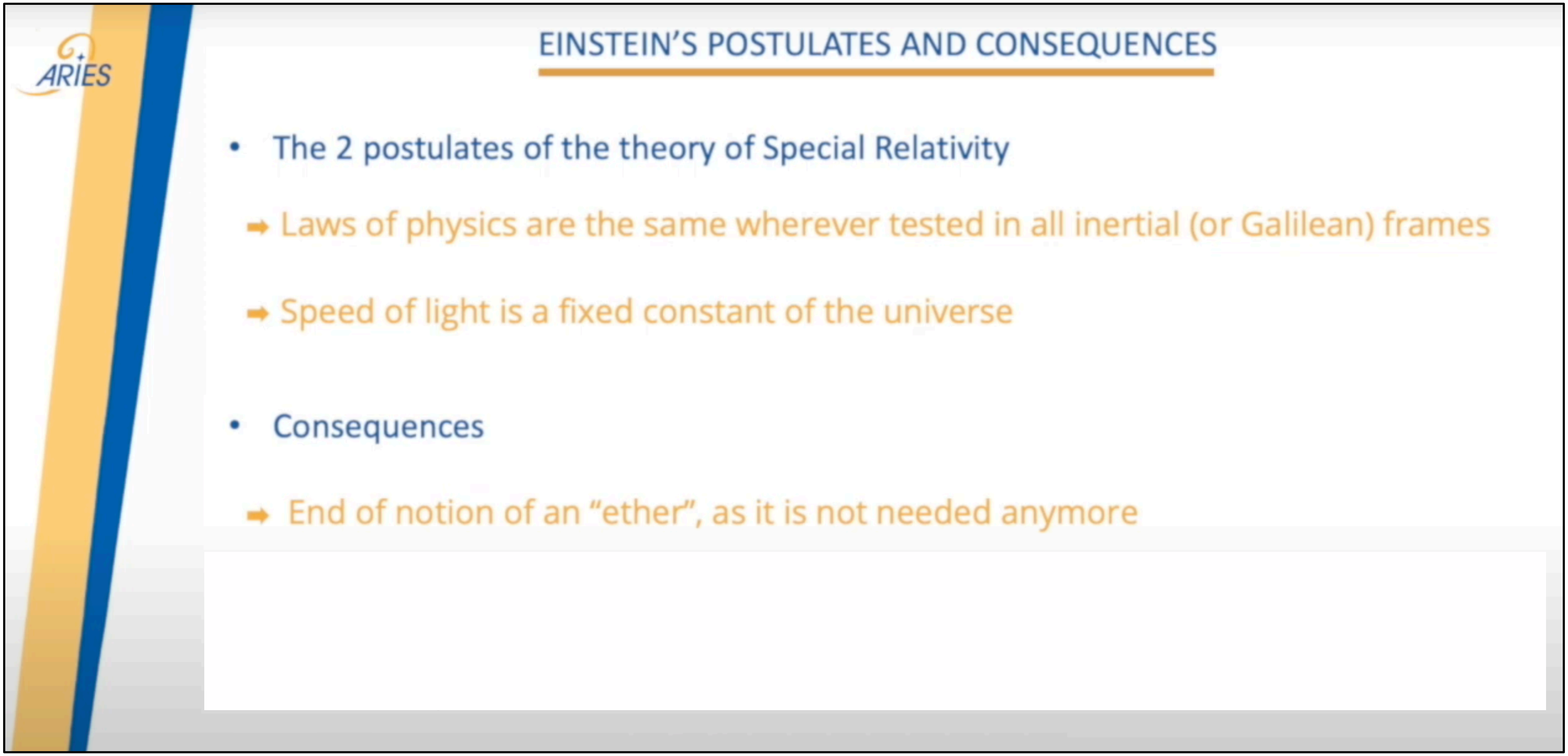
Special relativity



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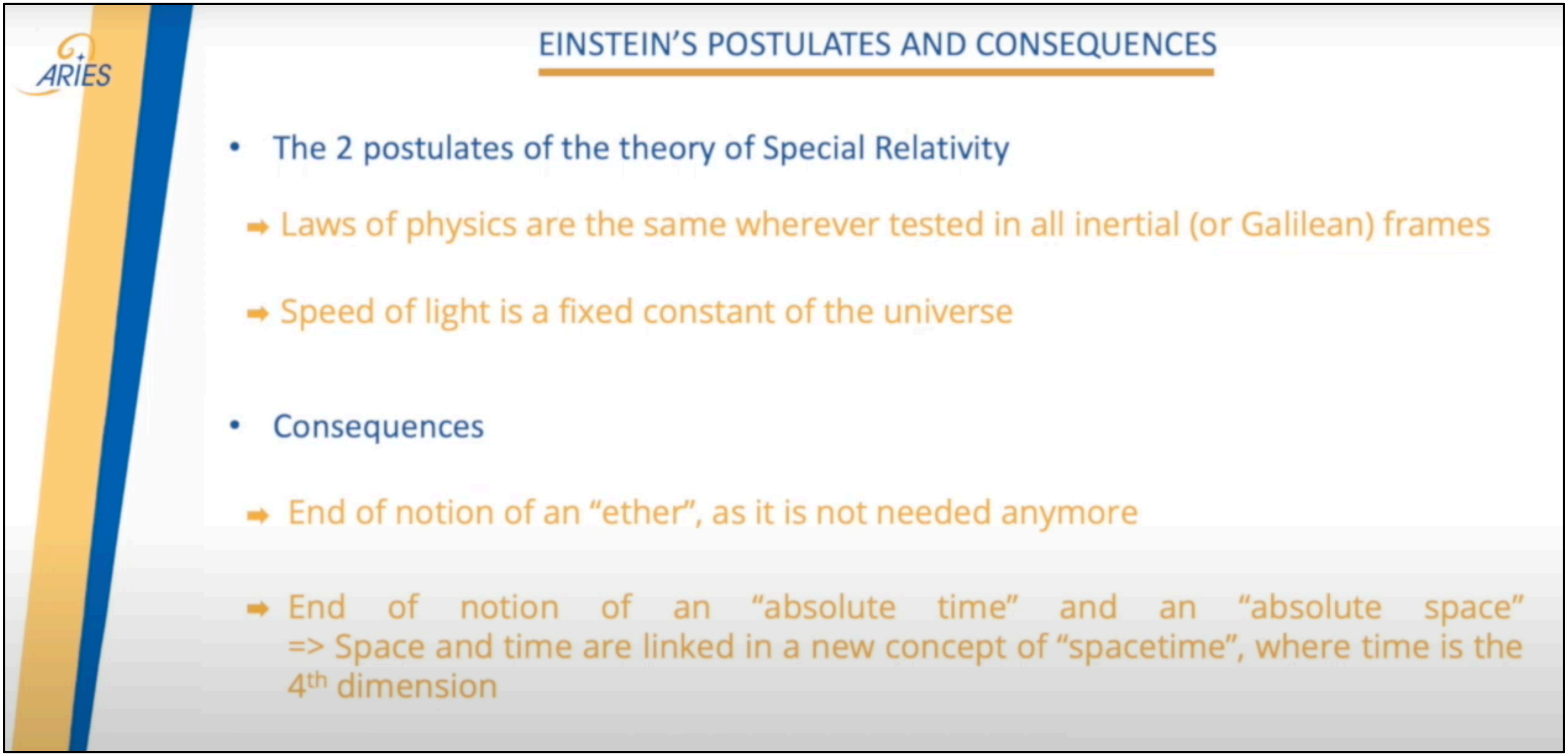
➡ End of notion of an "ether", as it is not needed anymore

Special relativity



EINSTEIN'S POSTULATES AND CONSEQUENCES

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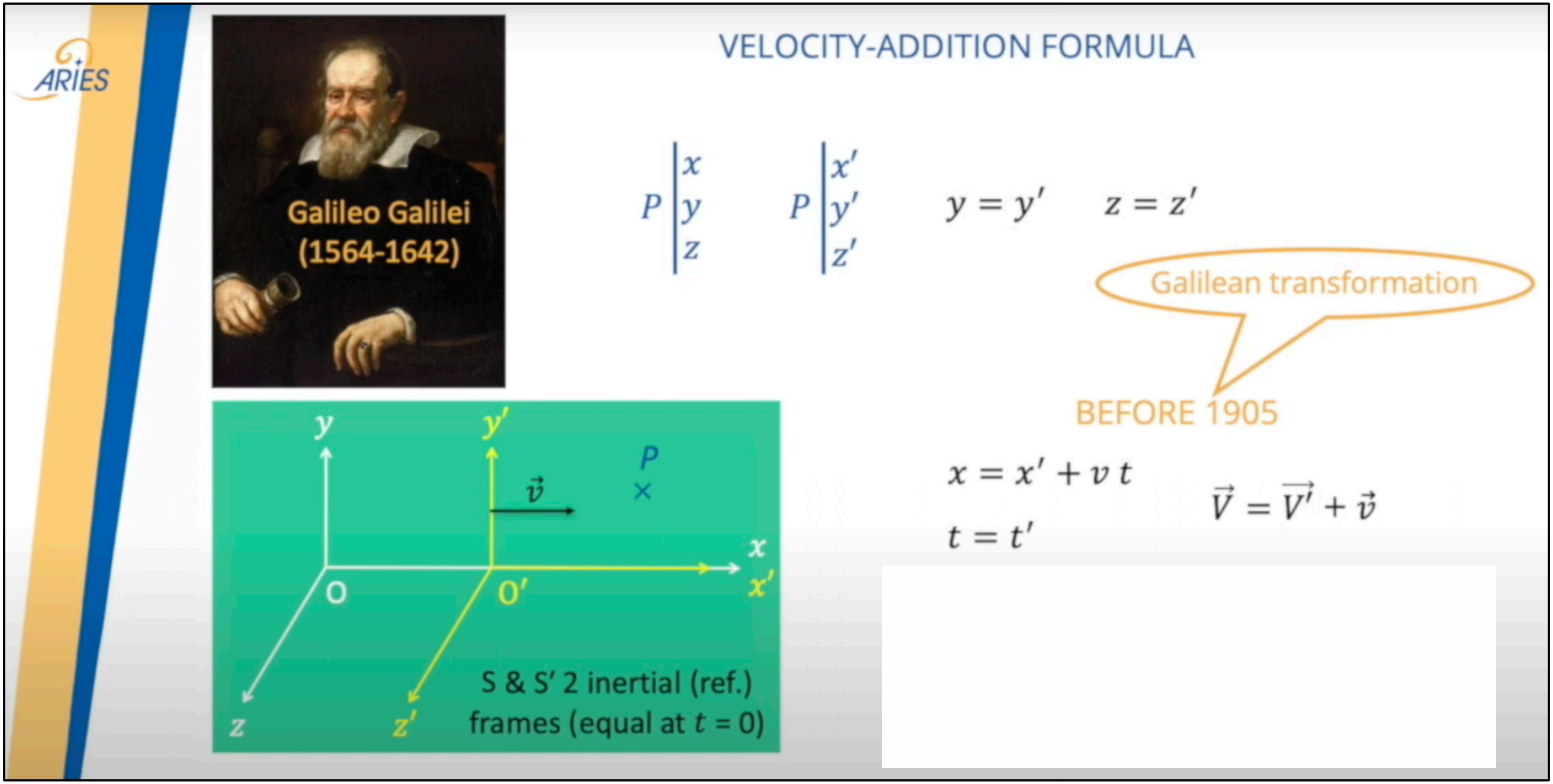
- Consequences

➡ End of notion of an "ether", as it is not needed anymore

➡ End of notion of an "absolute time" and an "absolute space" => Space and time are linked in a new concept of "spacetime", where time is the 4th dimension

Special relativity



ARIES

VELOCITY-ADDITION FORMULA

Galileo Galilei
(1564-1642)

$$P \left| \begin{matrix} x \\ y \\ z \end{matrix} \right. \qquad P \left| \begin{matrix} x' \\ y' \\ z' \end{matrix} \right. \qquad y = y' \qquad z = z'$$

Galilean transformation

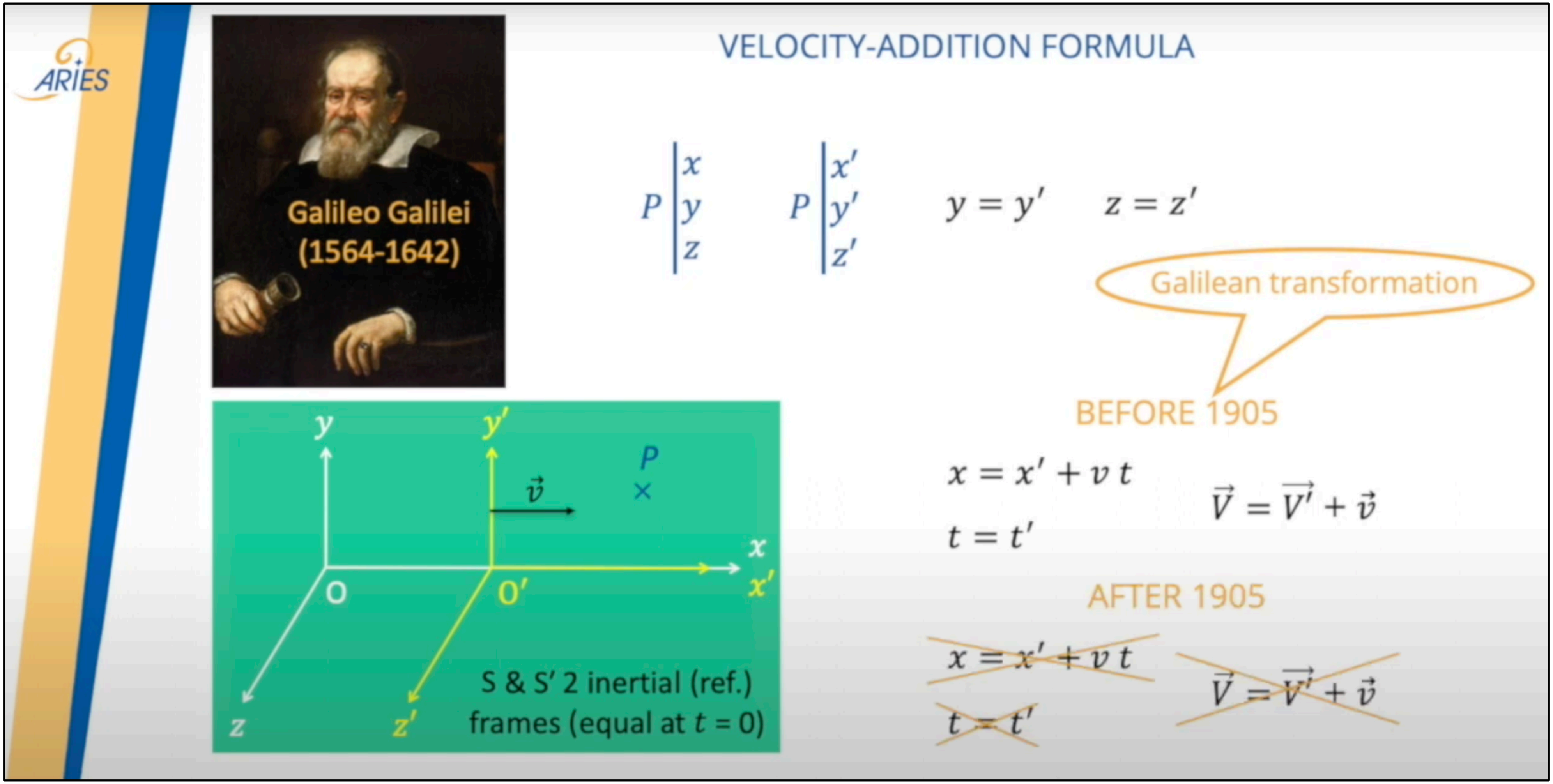
BEFORE 1905

$$x = x' + v\,t \qquad \vec{V} = \vec{V'} + \vec{v}$$

$$t = t'$$

S & S' 2 inertial (ref.) frames (equal at t = 0)

Special relativity



ARIES

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$P \begin{vmatrix} x \\ y \\ z \end{vmatrix}$ $P \begin{vmatrix} x' \\ y' \\ z' \end{vmatrix}$ $y = y'$ $z = z'$

Galilean transformation

BEFORE 1905

$x = x' + v\,t$

$t = t'$

$\vec{V} = \vec{V'} + \vec{v}$

AFTER 1905

$\cancel{x = x' + v\,t}$

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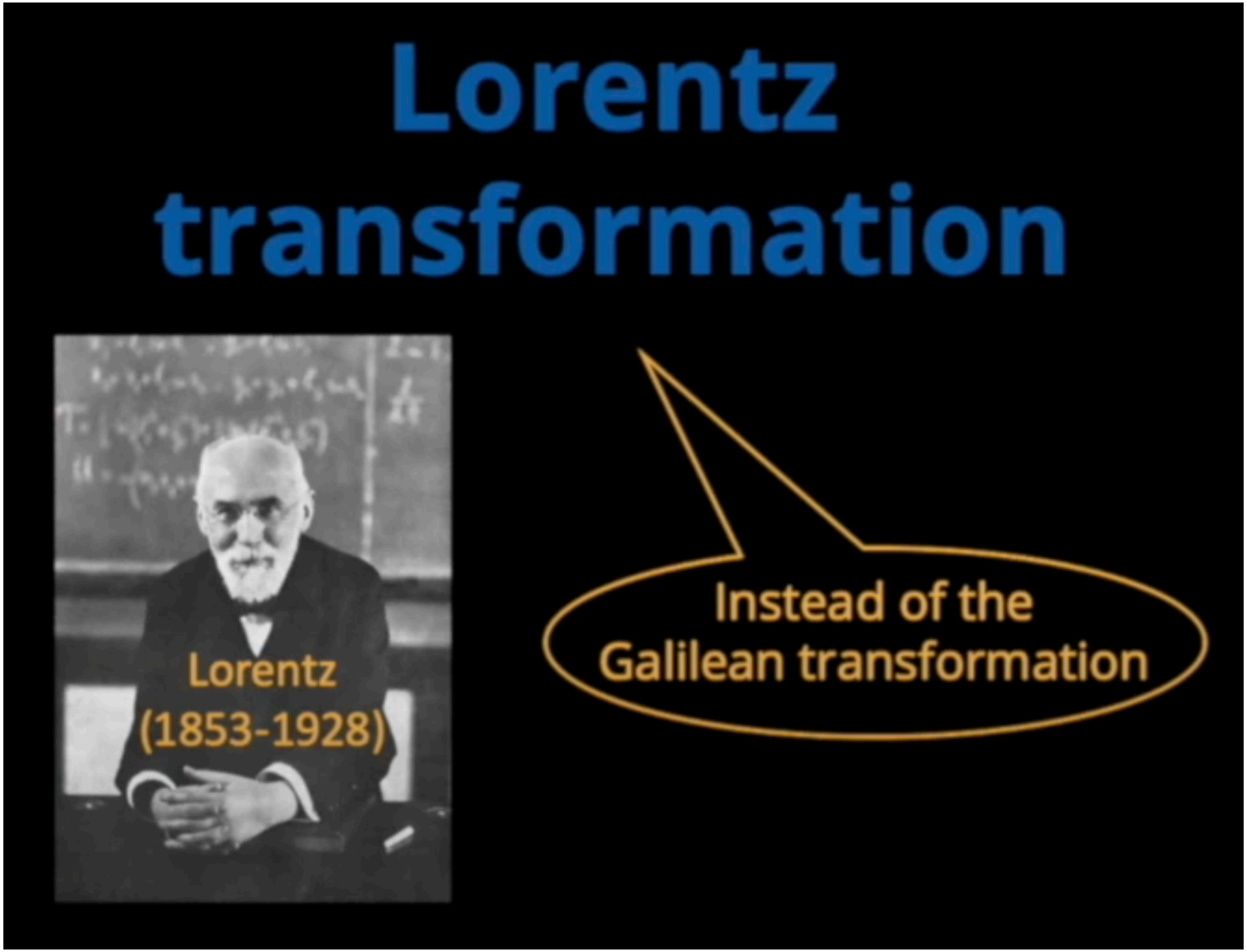
S & S' 2 inertial (ref.) frames (equal at t = 0)

Special relativity



Lorentz transformation

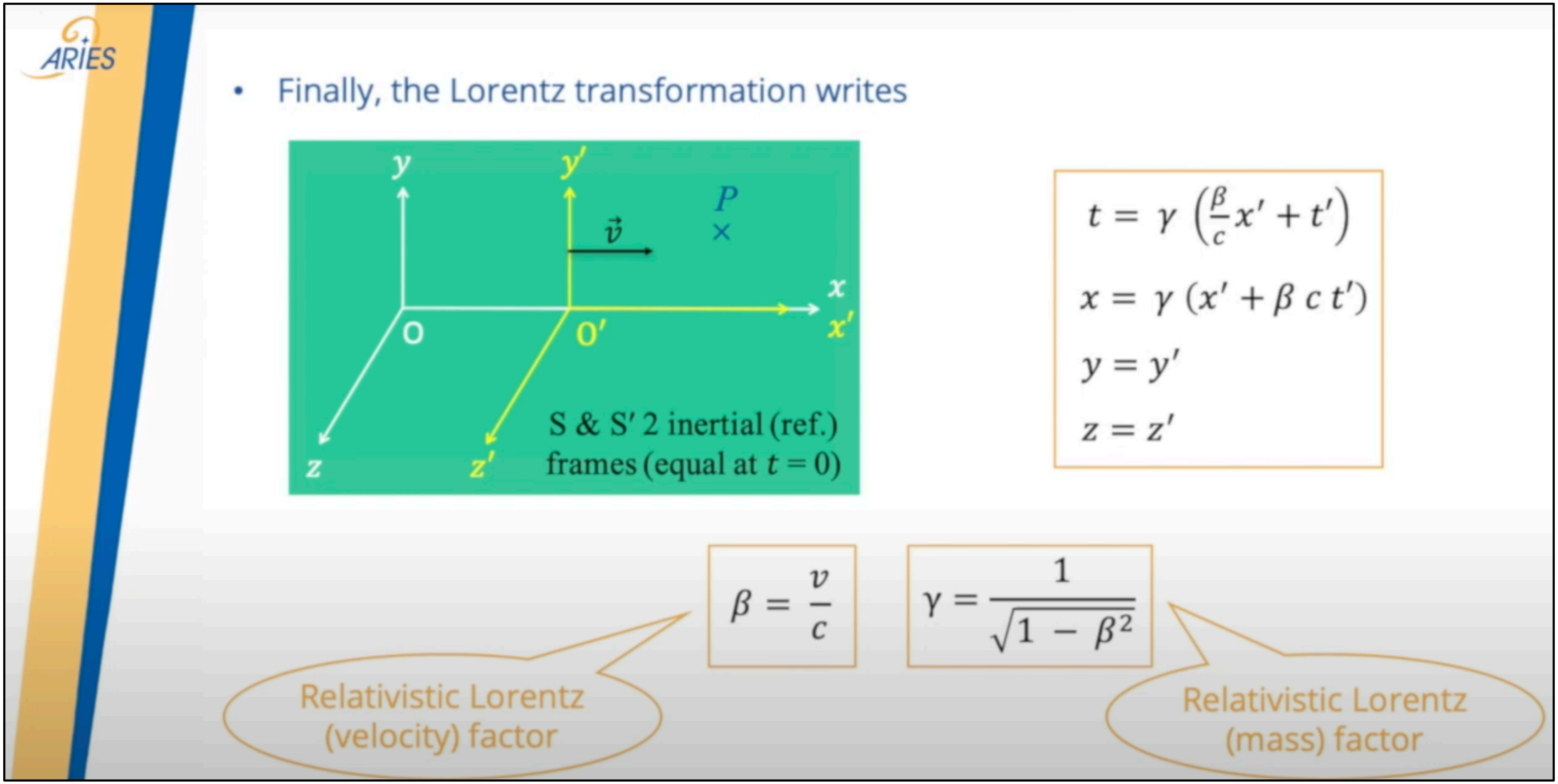
Lorentz

(1853-1928)

Instead of the

Galilean transformation

Special relativity



- Finally, the Lorentz transformation writes

S & S' 2 inertial (ref.) frames (equal at $t = 0$)

$$t = \gamma \left(\frac{\beta}{c} x' + t' \right)$$

$$x = \gamma \left(x' + \beta \, c \, t' \right)$$

$$y = y'$$

$$z = z'$$

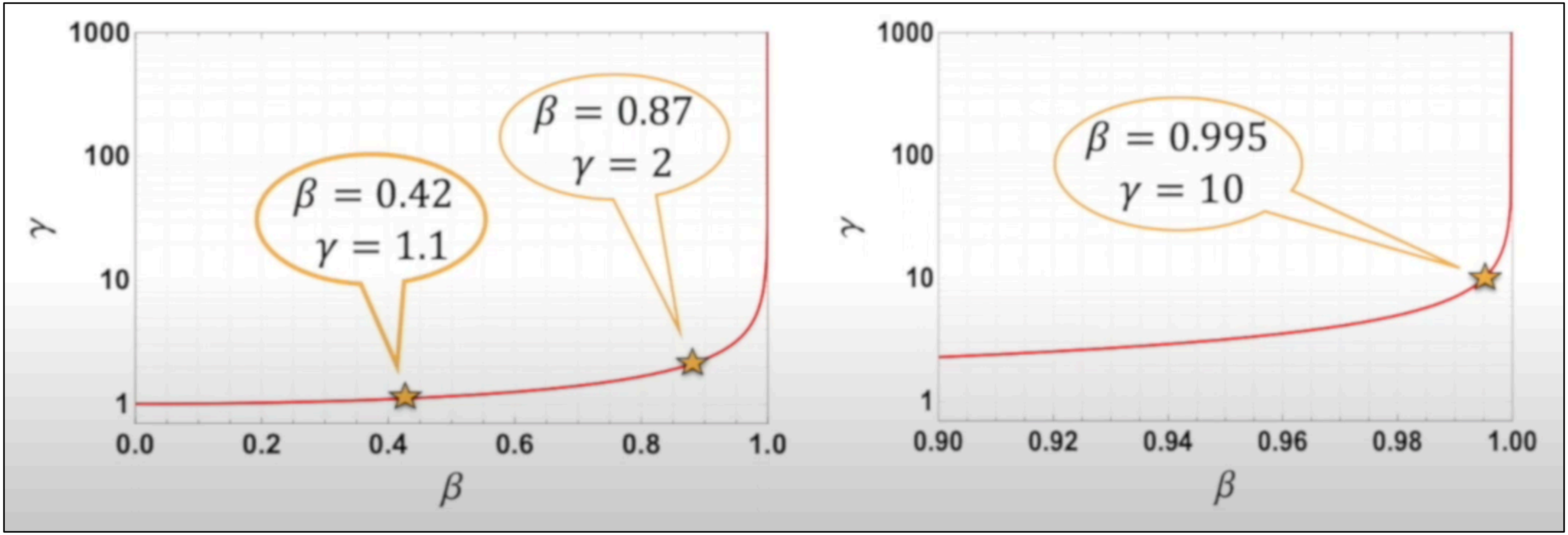
$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

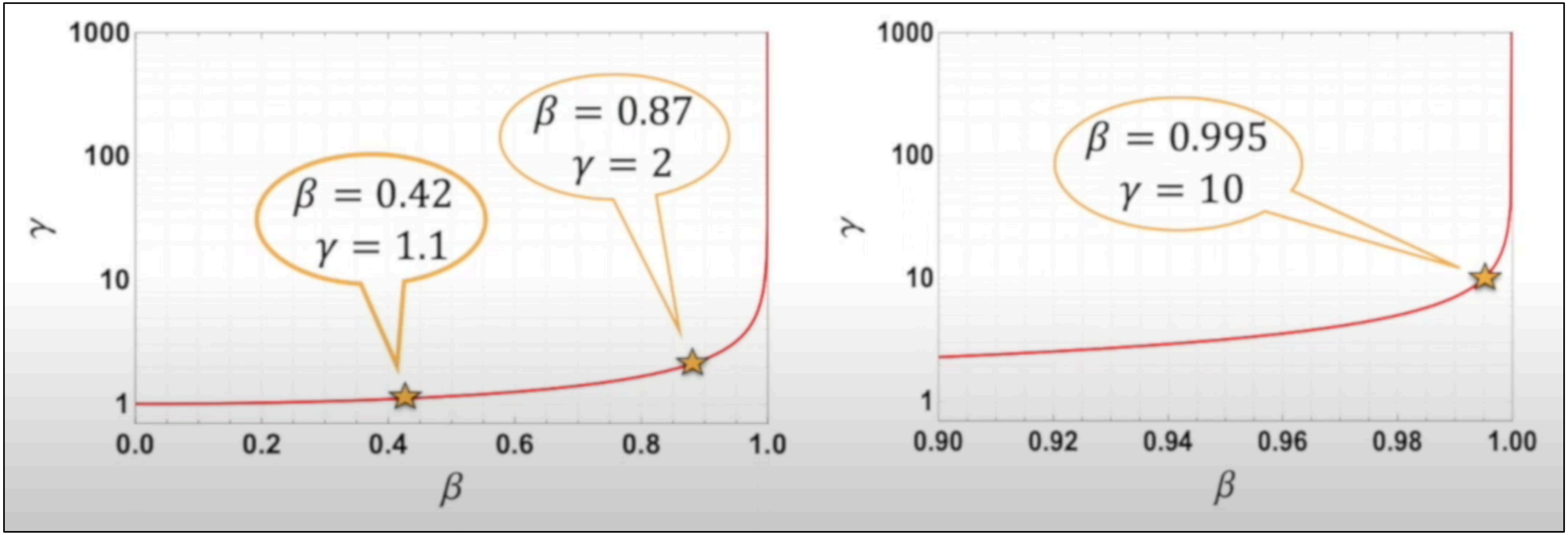
Relativistic Lorentz (velocity) factor

Relativistic Lorentz (mass) factor

Special relativity

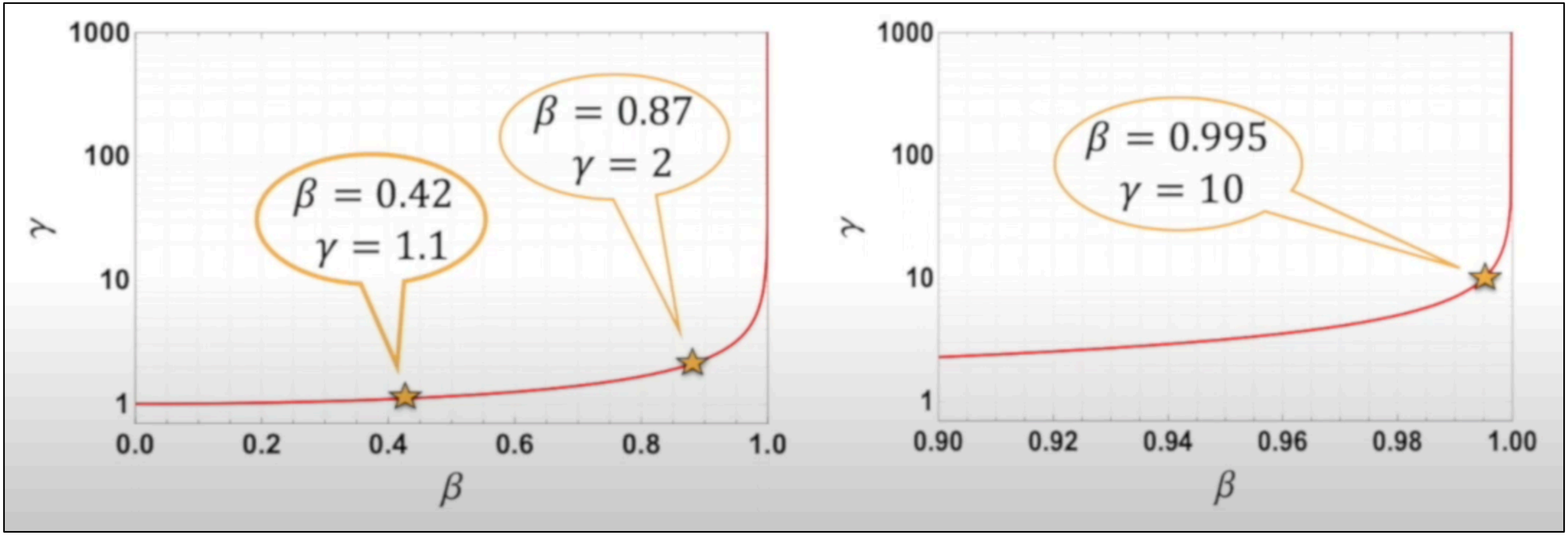


Special relativity



◆ Length contraction $L = \frac{L'}{\gamma}$

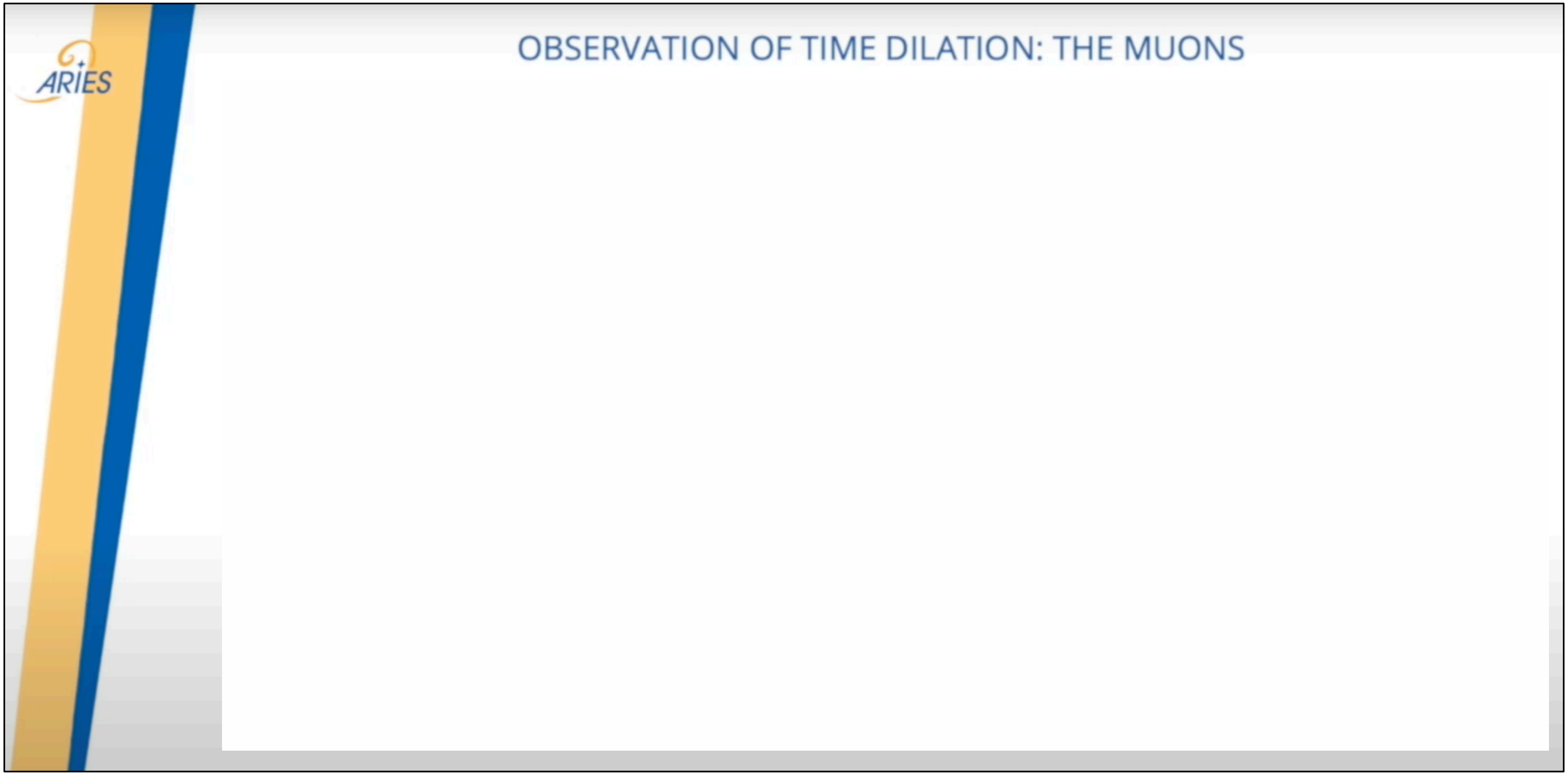
Special relativity



- Length contraction $L = \frac{L'}{\gamma}$

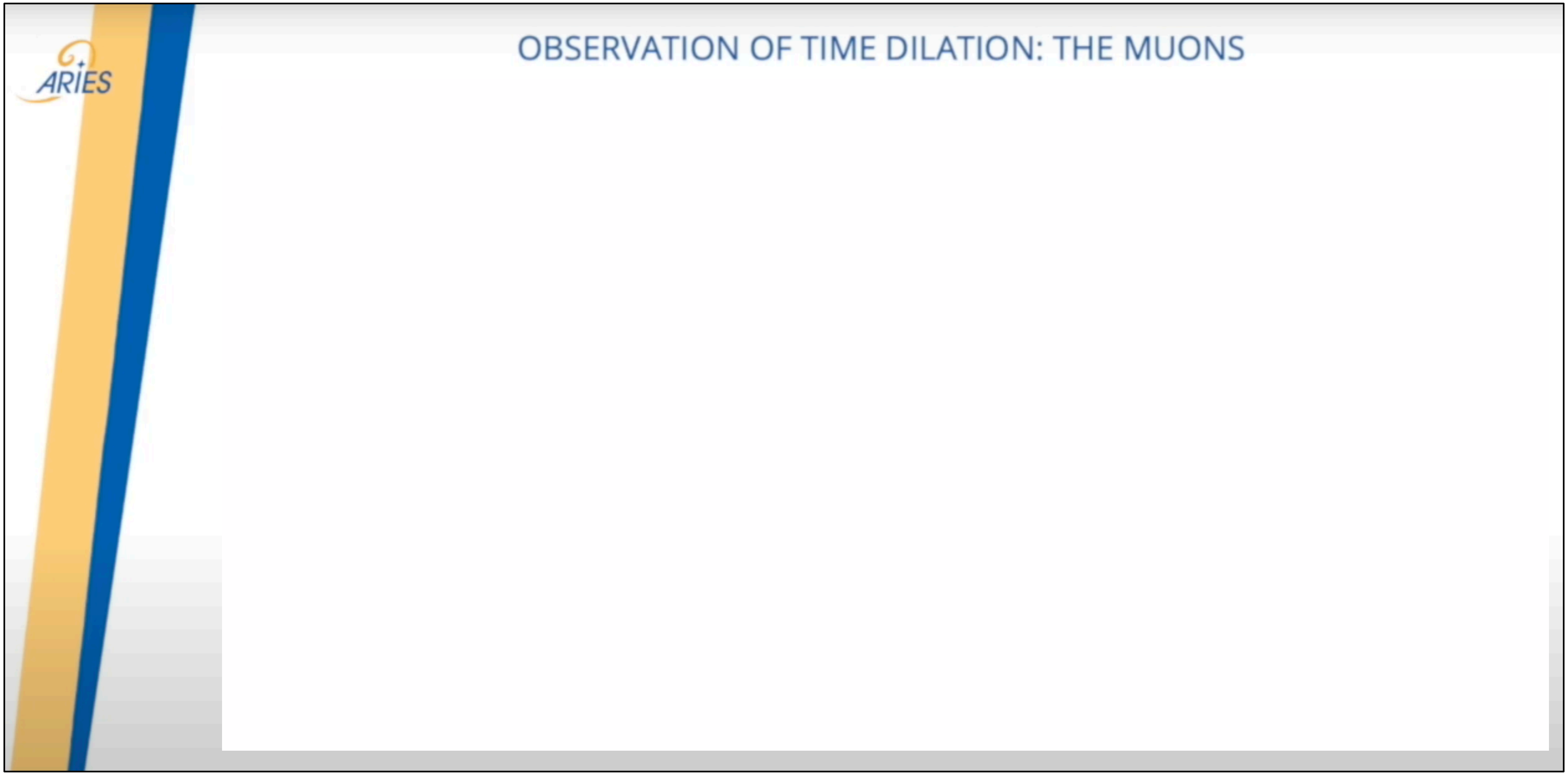
- Time dilation $t = \gamma t'$

Special relativity



OBSERVATION OF TIME DILATION: THE MUONS

Special relativity



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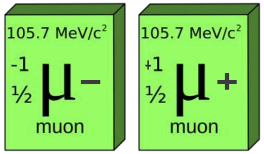
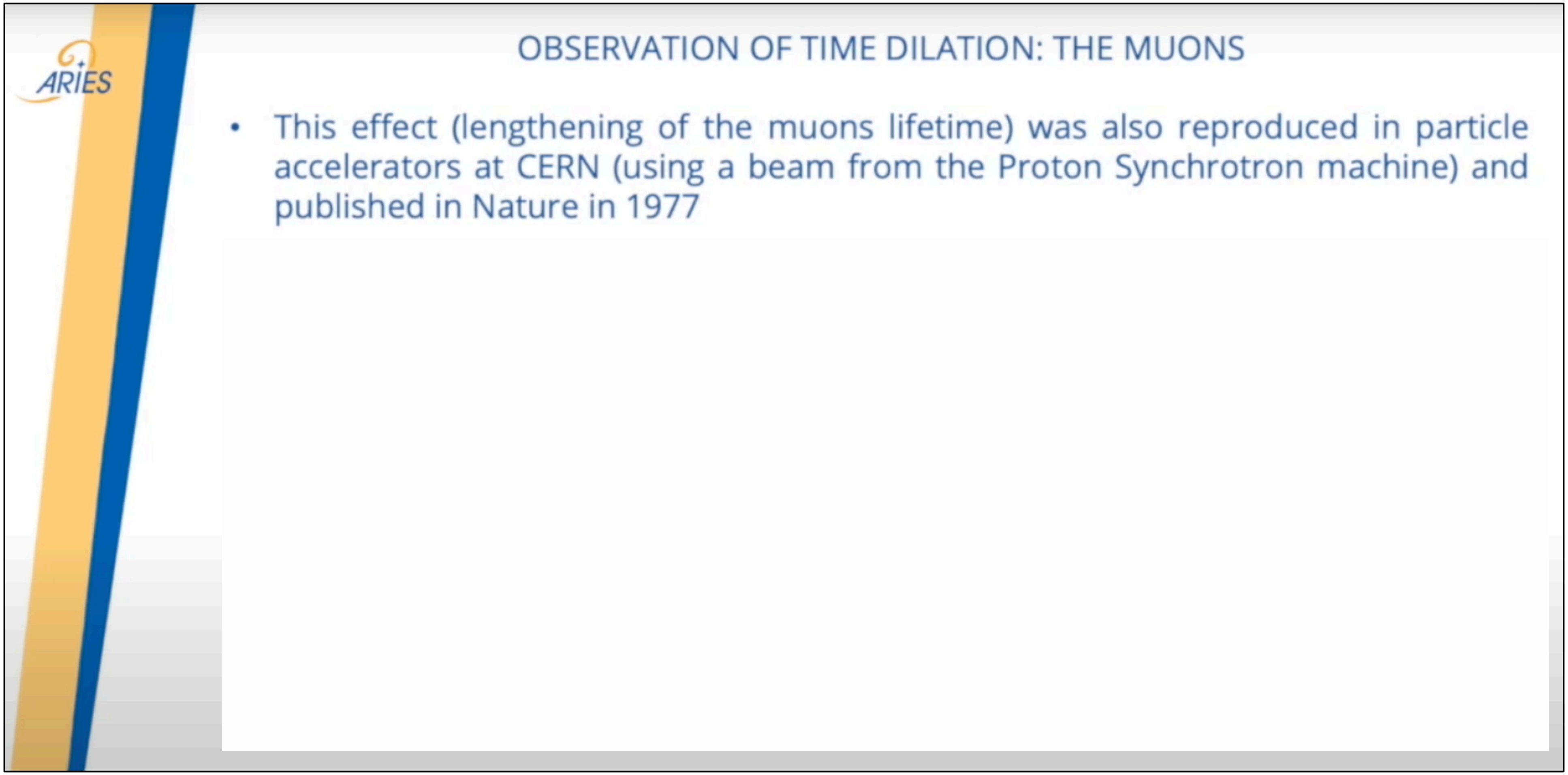


$$m_\mu = 105.7\ MeV/c^2$$
$$\tau_\mu = 2.2\ \mu s$$

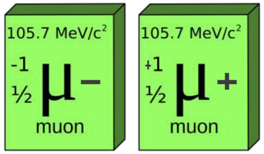
Special relativity



ARIES

OBSERVATION OF TIME DILATION: THE MUONS

- This effect (lengthening of the muons lifetime) was also reproduced in particle accelerators at CERN (using a beam from the Proton Synchrotron machine) and published in Nature in 1977



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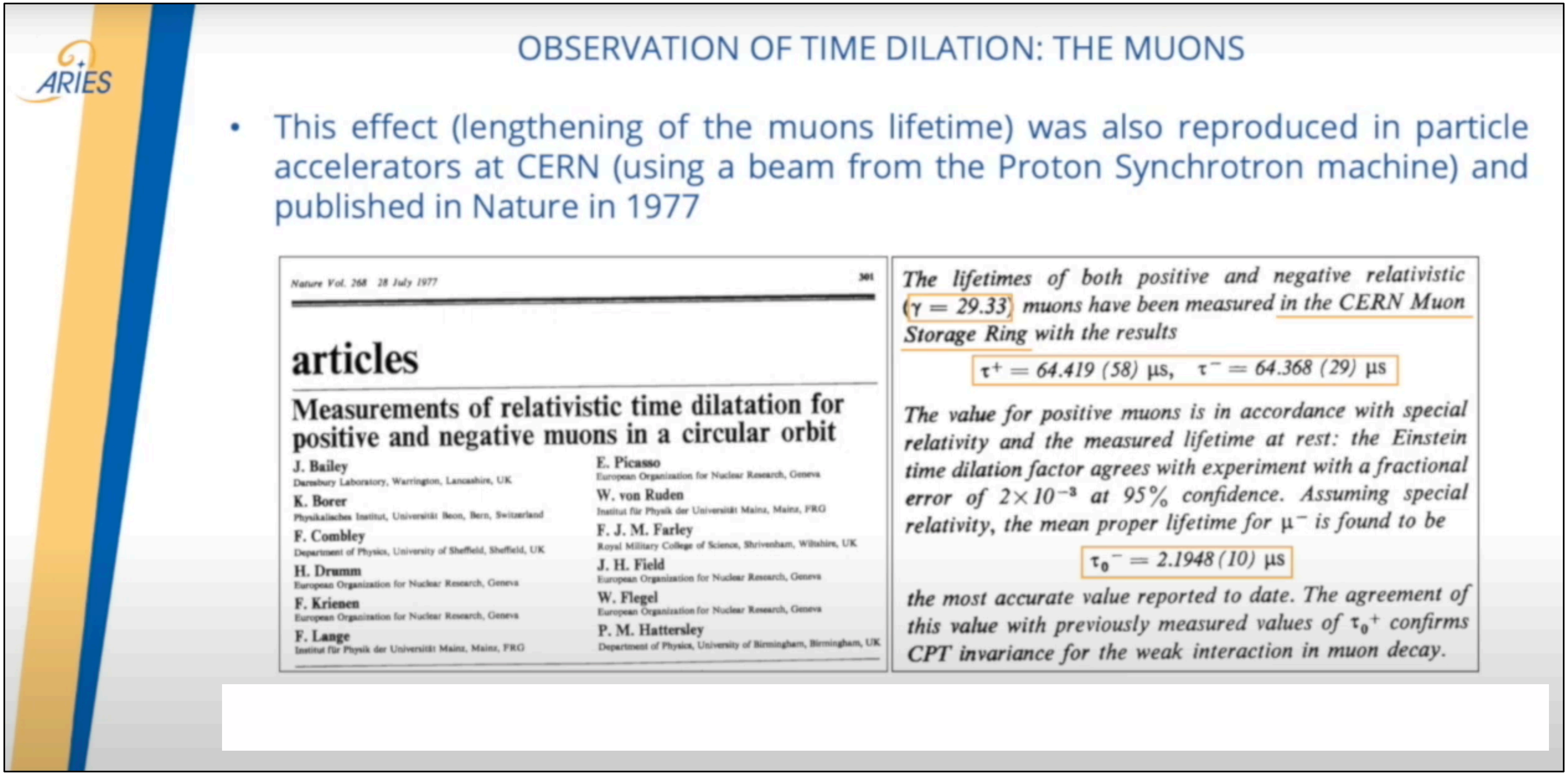

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Nature Vol. 268 28 July 1977 301

articles

Measurements of relativistic time dilatation for positive and negative muons in a circular orbit

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Daresbury Laboratory, Warrington, Lancashire, UK
K. Borer
Physikalisches Institut, Universität Bern, Bern, Switzerland
F. Combley
Department of Physics, University of Sheffield, Sheffield, UK
H. Drumm
European Organization for Nuclear Research, Geneva
F. Krienen
European Organization for Nuclear Research, Geneva
F. Lange
Institut für Physik der Universität Mainz, Mainz, FRG
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F. J. M. Farley
Royal Military College of Science, Shrivenham, Wiltshire, UK
J. H. Field
European Organization for Nuclear Research, Geneva
W. Flegel
European Organization for Nuclear Research, Geneva
P. M. Hattersley
Department of Physics, University of Birmingham, Birmingham, UK

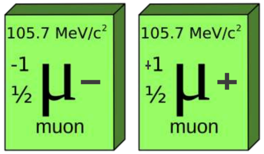
The lifetimes of both positive and negative relativistic ($\gamma = 29.33$) muons have been measured in the CERN Muon Storage Ring with the results

$$\tau^+ = 64.419\ (58)\ \mu s, \quad \tau^- = 64.368\ (29)\ \mu s$$

The value for positive muons is in accordance with special relativity and the measured lifetime at rest: the Einstein time dilation factor agrees with experiment with a fractional error of 2×10^{-3} at 95% confidence. Assuming special relativity, the mean proper lifetime for μ^- is found to be

$$\tau_0^- = 2.1948\ (10)\ \mu s$$

the most accurate value reported to date. The agreement of this value with previously measured values of τ_0^+ confirms CPT invariance for the weak interaction in muon decay.



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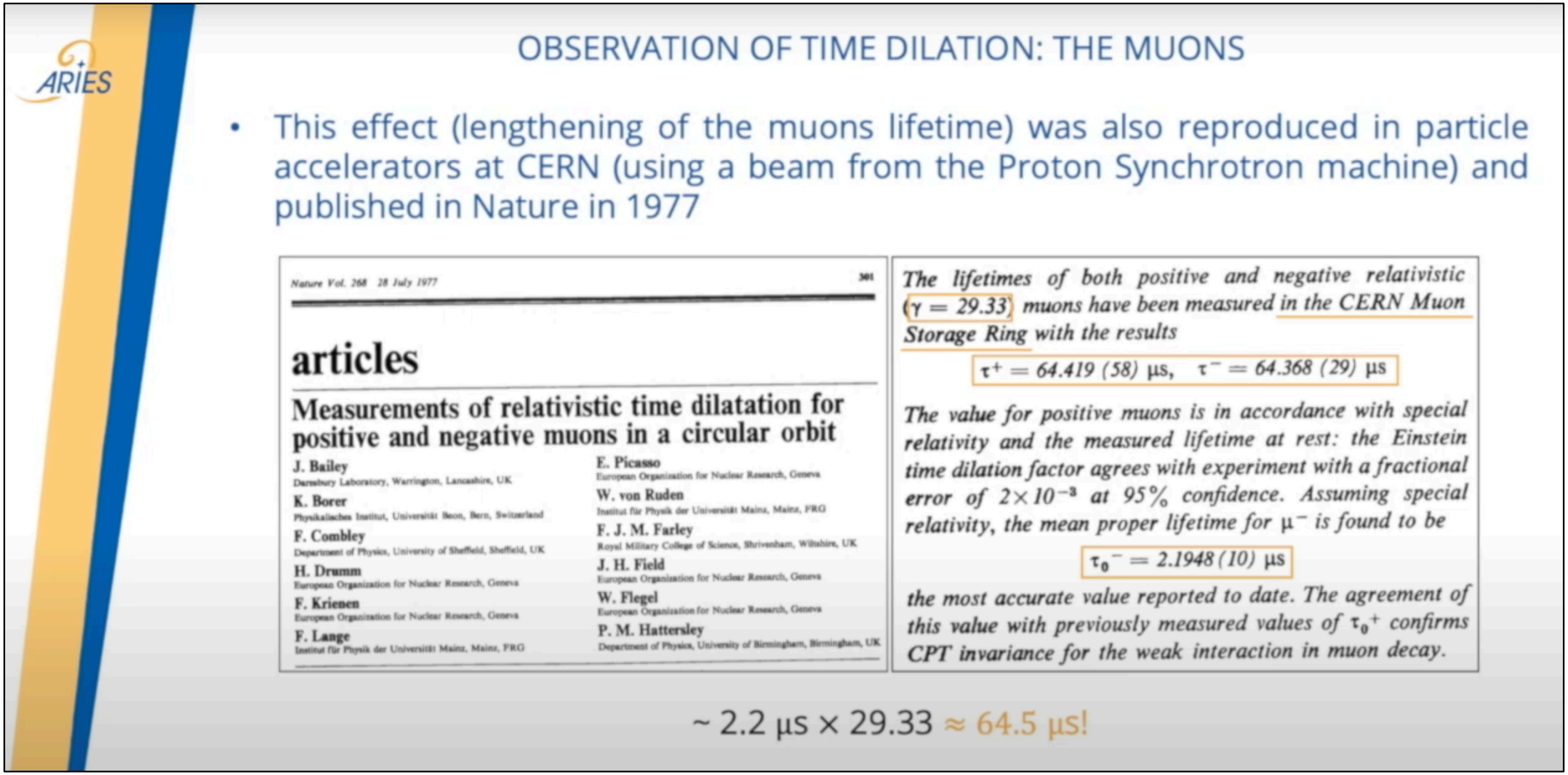

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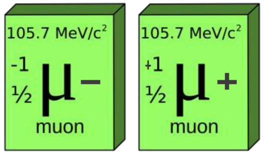
$\sim 2.2\ \mu s \times 29.33 \approx 64.5\ \mu s!$



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Special relativity

- A muon collider has been discussed for some time as the ultimate lepton collider (see https://muoncollider.web.cern.ch/)

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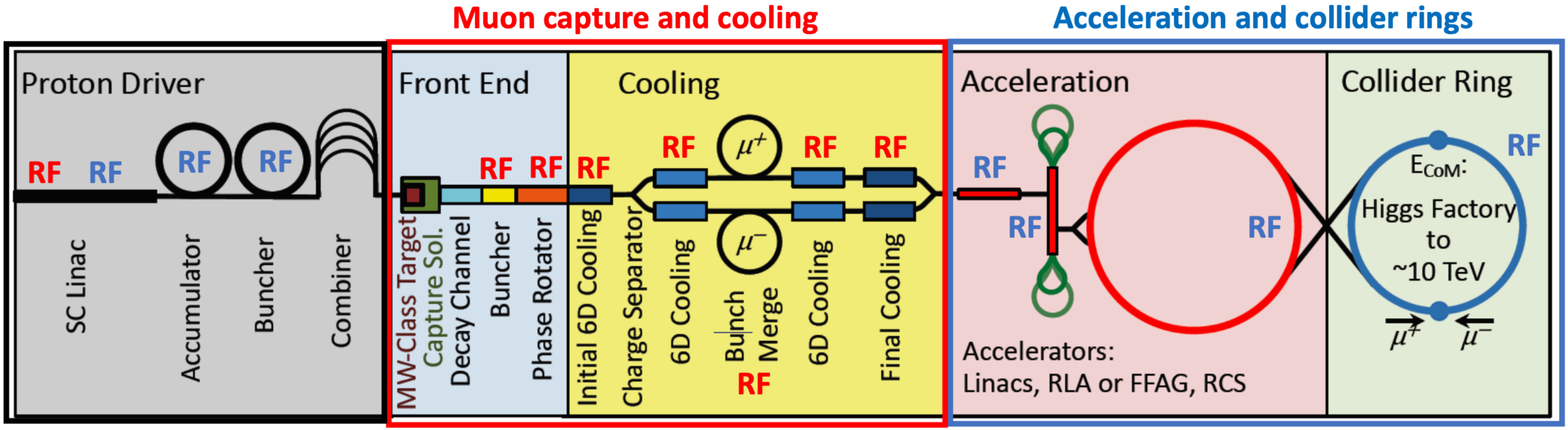

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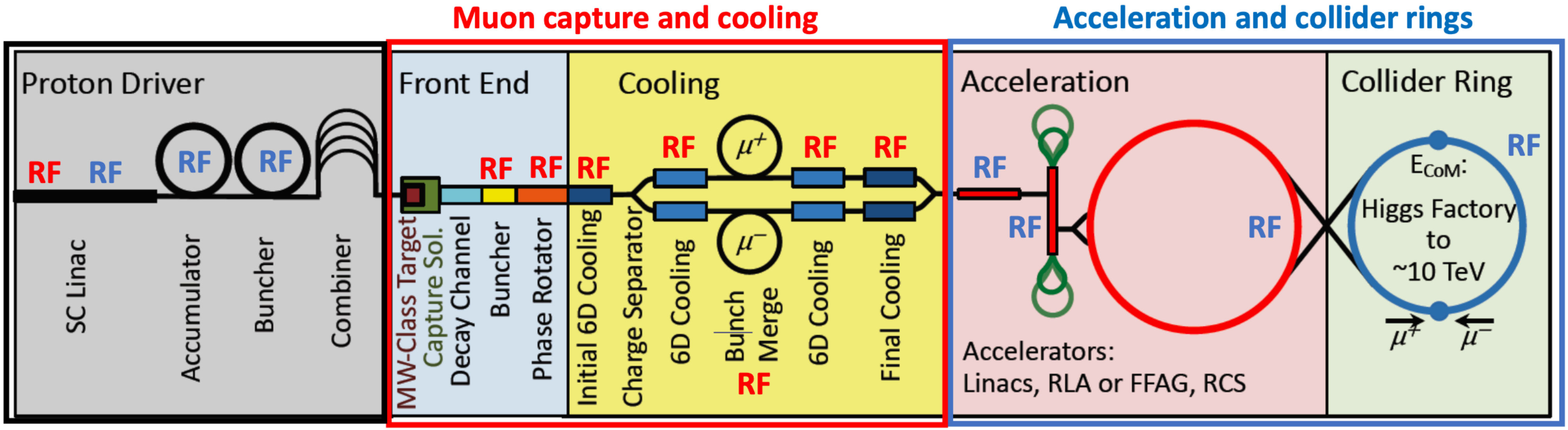

$$\boxed{\tau = \gamma \, \tau_0}$$

~ 150 ms at 7 TeV

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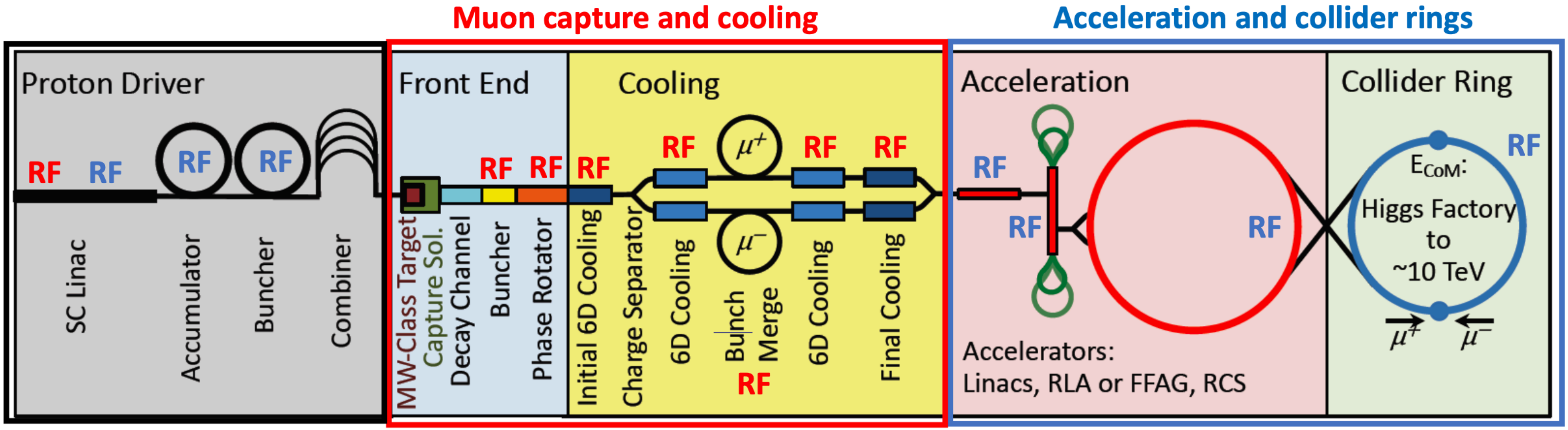
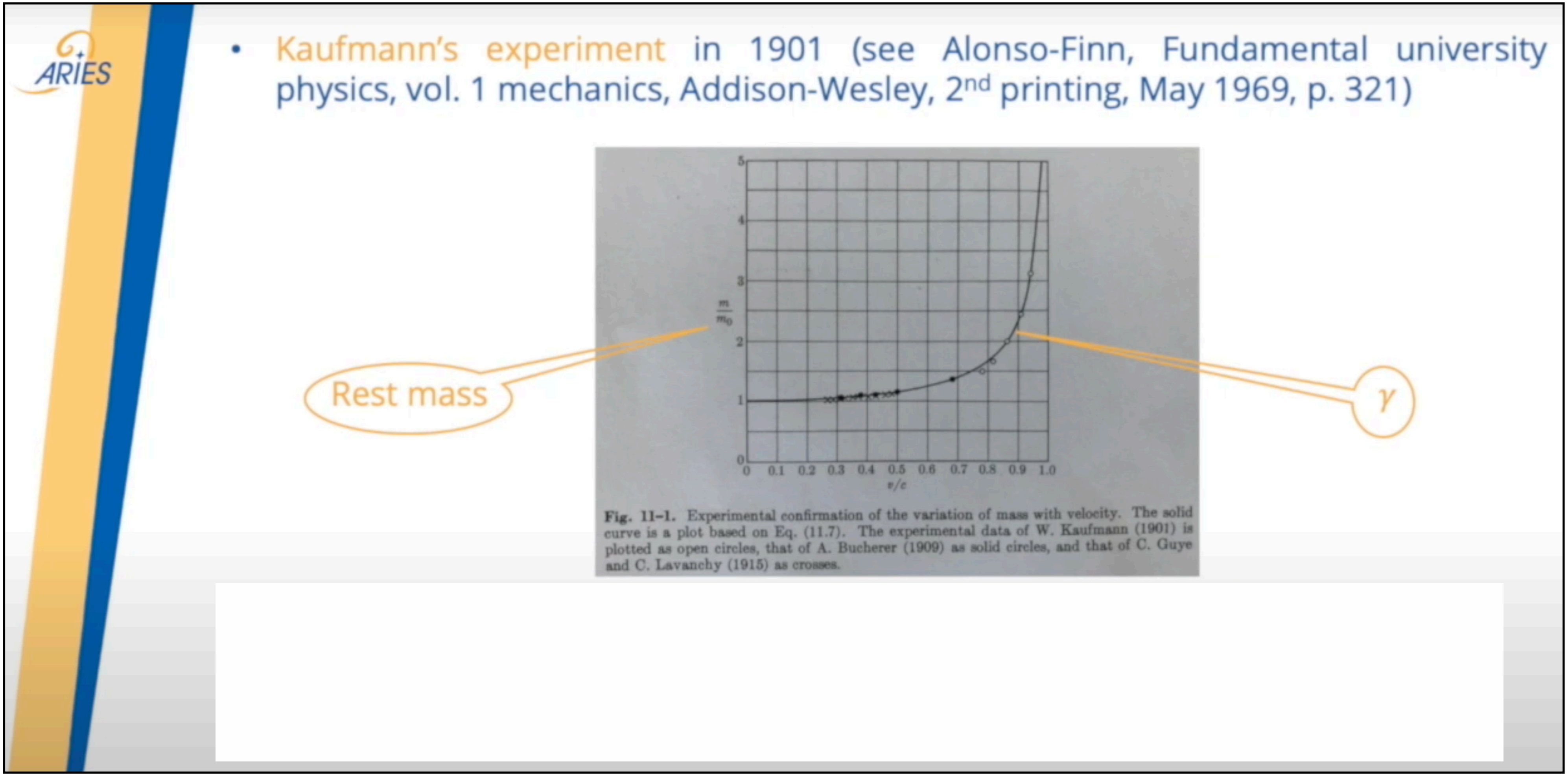


$$\boxed{\tau = \gamma \, \tau_0}$$

~ 150 ms at 7 TeV

=> Everything needs to be done swiftly!

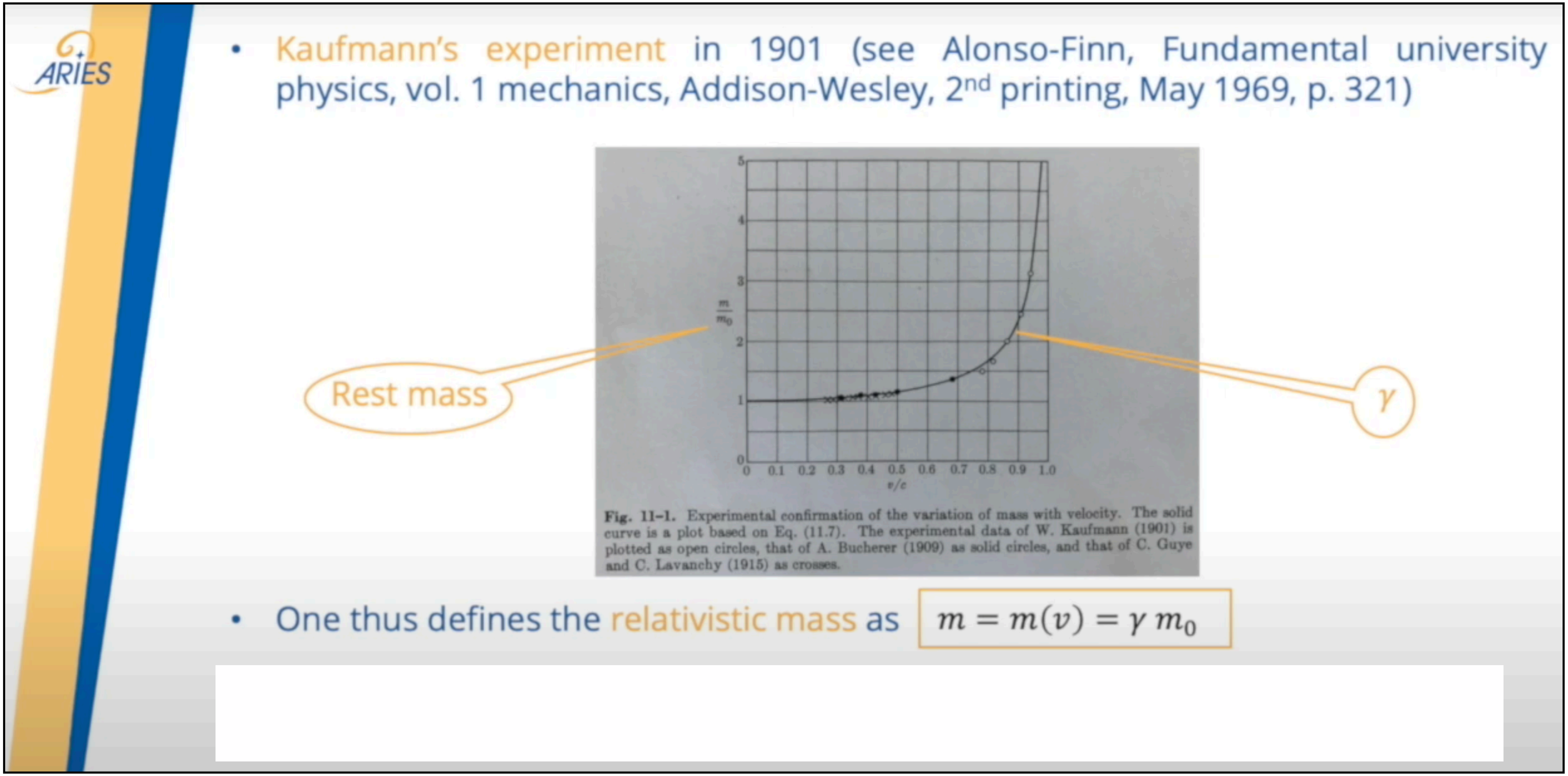
Special relativity



- Kaufmann's experiment in 1901 (see Alonso-Finn, Fundamental university physics, vol. 1 mechanics, Addison-Wesley, 2nd printing, May 1969, p. 321)

Fig. 11–1. Experimental confirmation of the variation of mass with velocity. The solid curve is a plot based on Eq. (11.7). The experimental data of W. Kaufmann (1901) is plotted as open circles, that of A. Bucherer (1909) as solid circles, and that of C. Guye and C. Lavanchy (1915) as crosses.

Special relativity



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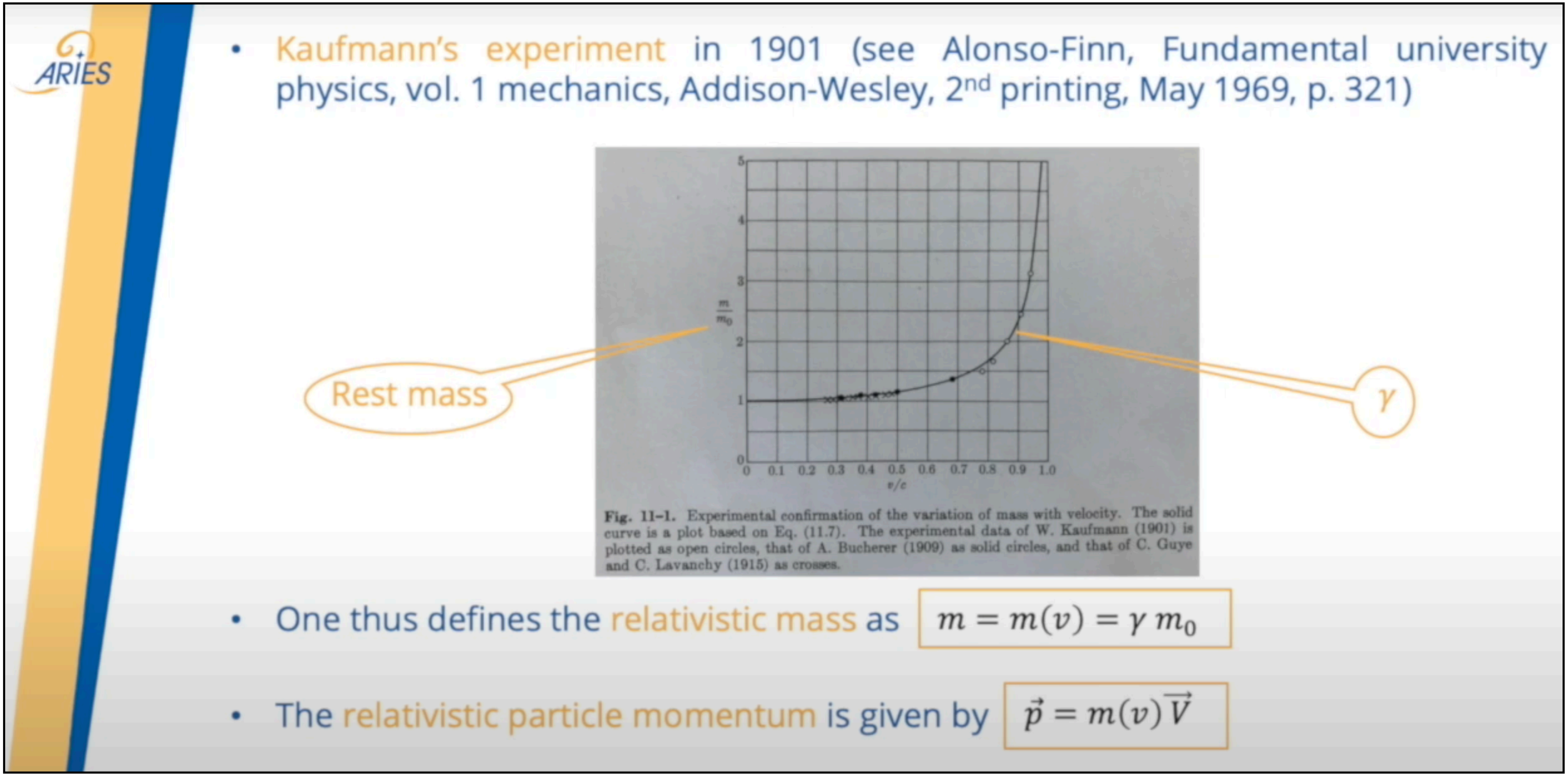
- One thus defines the relativistic mass as $\boxed{m = m(v) = \gamma\, m_0}$

Special relativity



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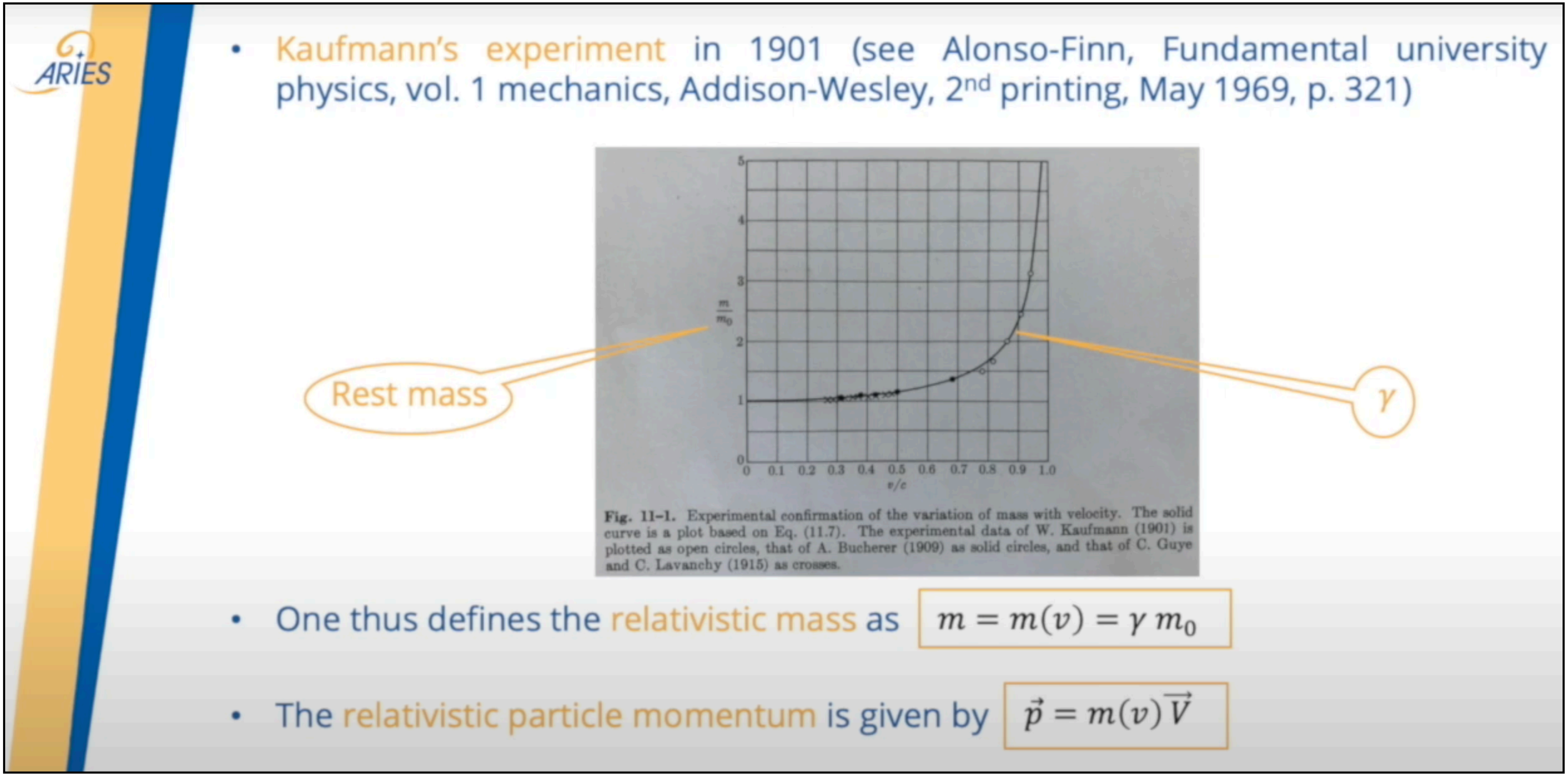
- One thus defines the relativistic mass as $\boxed{m = m(v) = \gamma\, m_0}$
- The relativistic particle momentum is given by $\boxed{\vec{p} = m(v)\,\vec{V}}$

Special relativity



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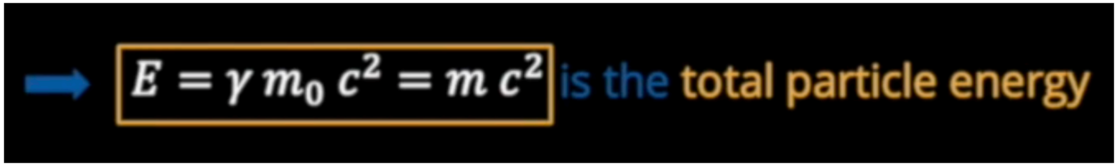
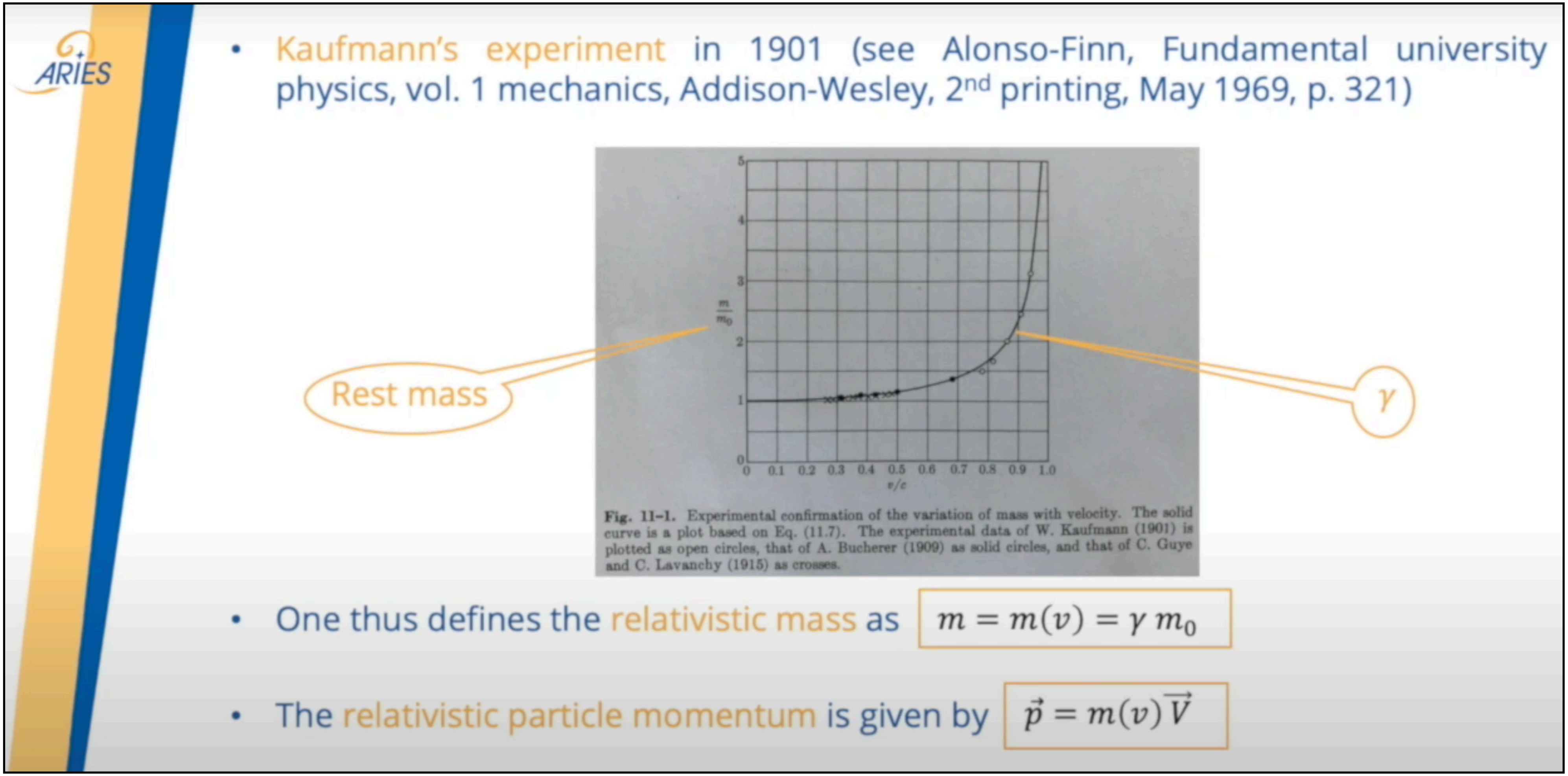
- One thus defines the relativistic mass as $\boxed{m = m(v) = \gamma\, m_0}$
- The relativistic particle momentum is given by $\boxed{\vec{p} = m(v)\,\vec{V}}$



→ $\boxed{E = \gamma\, m_0\, c^2 = m\, c^2}$ is the total particle energy

Special relativity



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Rest mass

γ

Fig. 11-1. Experimental confirmation of the variation of mass with velocity. The solid curve is a plot based on Eq. (11.7). The experimental data of W. Kaufmann (1901) is plotted as open circles, that of A. Bucherer (1909) as solid circles, and that of C. Guye and C. Lavanchy (1915) as crosses.

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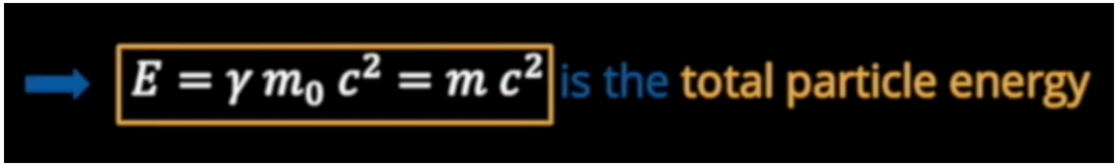
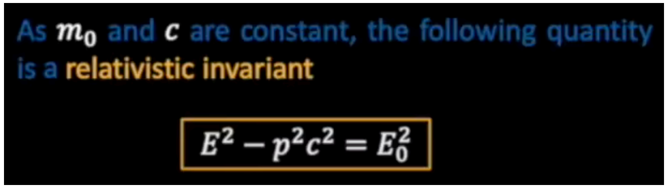


➡ $\boxed{E = \gamma\, m_0\, c^2 = m\, c^2}$ is the **total particle energy**



As m_0 and c are constant, the following quantity is a **relativistic invariant**

$$\boxed{E^2 - p^2 c^2 = E_0^2}$$

Special relativity

$$\boxed{E = m\,c^2}$$

normalized velocity

$$\beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

energy

$$E = E_{kin} + E_0$$

total

kinetic

rest

total energy / rest energy

momentum

$$\gamma = \frac{E}{E_0} = \frac{m}{m_0} = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

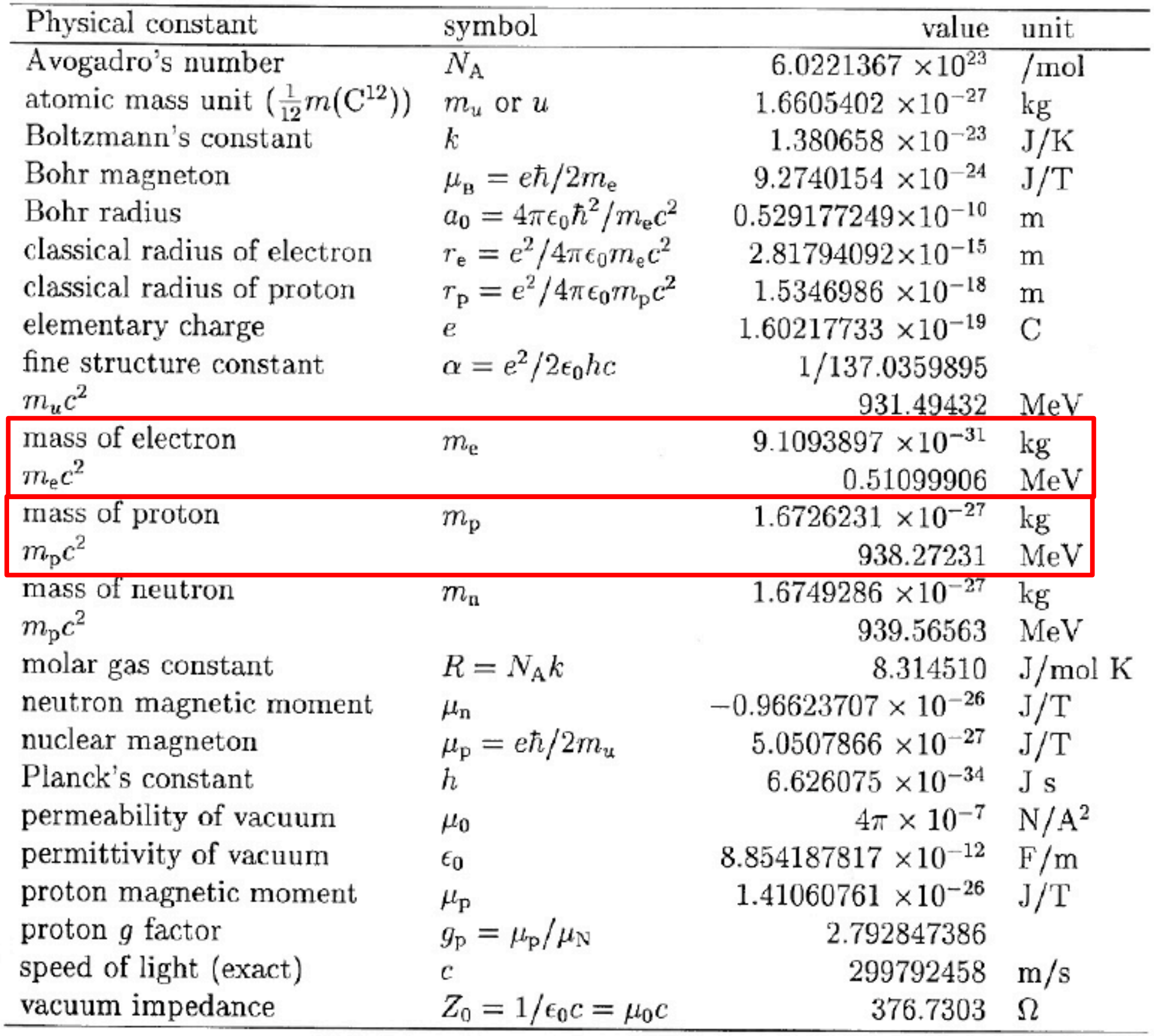
$$p = mv = \beta \frac{E}{c} = \beta\,\gamma\,m_0\,c$$

energy	momentum	mass
eV	eV/c	eV/c^2

$$p^2c^2 = E^2 - E_0{}^2$$

$$\gamma = 1 + \frac{E_{kin}}{E_0}$$

Special relativity

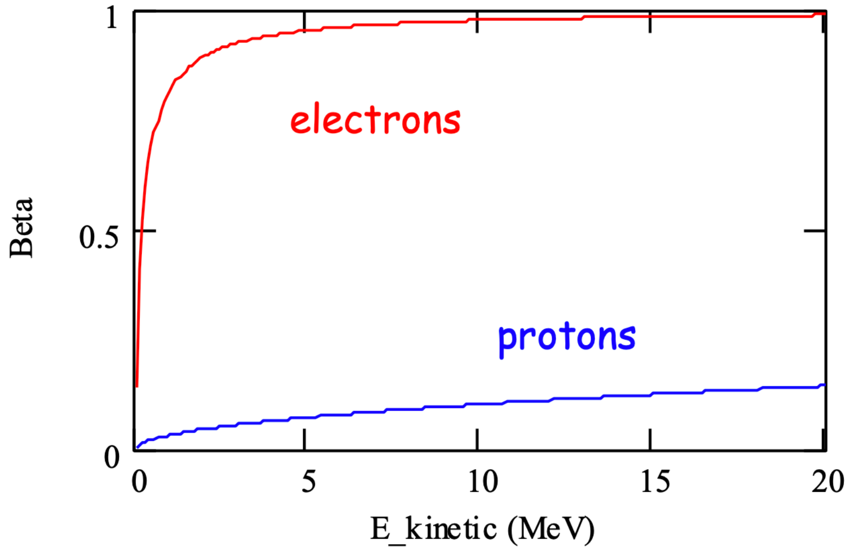


Physical constant	symbol	value	unit
Avogadro's number	N_A	6.0221367×10^{23}	/mol
atomic mass unit ($\frac{1}{12}m(C^{12})$)	m_u or u	$1.6605402 \times 10^{-27}$	kg
Boltzmann's constant	k	1.380658×10^{-23}	J/K
Bohr magneton	$\mu_B = e\hbar/2m_e$	$9.2740154 \times 10^{-24}$	J/T
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/m_ec^2$	$0.529177249 \times 10^{-10}$	m
classical radius of electron	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.81794092 \times 10^{-15}$	m
classical radius of proton	$r_p = e^2/4\pi\epsilon_0 m_p c^2$	$1.5346986 \times 10^{-18}$	m
elementary charge	e	$1.60217733 \times 10^{-19}$	C
fine structure constant	$\alpha = e^2/2\epsilon_0 hc$	1/137.0359895	
$m_u c^2$		931.49432	MeV
mass of electron	m_e	$9.1093897 \times 10^{-31}$	kg
$m_e c^2$		0.51099906	MeV
mass of proton	m_p	$1.6726231 \times 10^{-27}$	kg
$m_p c^2$		938.27231	MeV
mass of neutron	m_n	$1.6749286 \times 10^{-27}$	kg
$m_p c^2$		939.56563	MeV
molar gas constant	$R = N_A k$	8.314510	J/mol K
neutron magnetic moment	μ_n	$-0.96623707 \times 10^{-26}$	J/T
nuclear magneton	$\mu_p = e\hbar/2m_u$	$5.0507866 \times 10^{-27}$	J/T
Planck's constant	h	6.626075×10^{-34}	J s
permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
permittivity of vacuum	ϵ_0	$8.854187817 \times 10^{-12}$	F/m
proton magnetic moment	μ_p	$1.41060761 \times 10^{-26}$	J/T
proton *g* factor	$g_p = \mu_p/\mu_N$	2.792847386	
speed of light (exact)	c	299792458	m/s
vacuum impedance	$Z_0 = 1/\epsilon_0 c = \mu_0 c$	376.7303	Ω

Special relativity

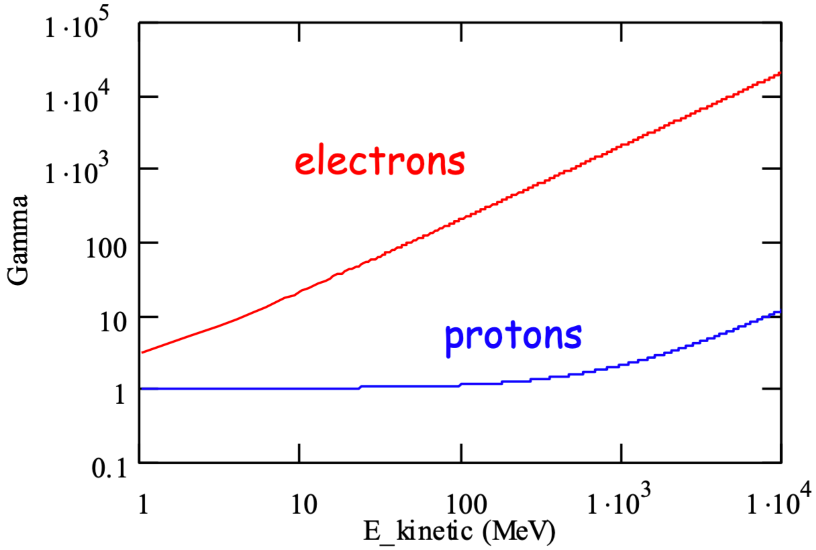
normalized velocity

$$\beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$



$$\frac{\text{total energy}}{\text{rest energy}}$$

$$\gamma = \frac{E}{E_0} = \frac{m}{m_0} = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$



EM: the ? Maxwell equations


Maxwell

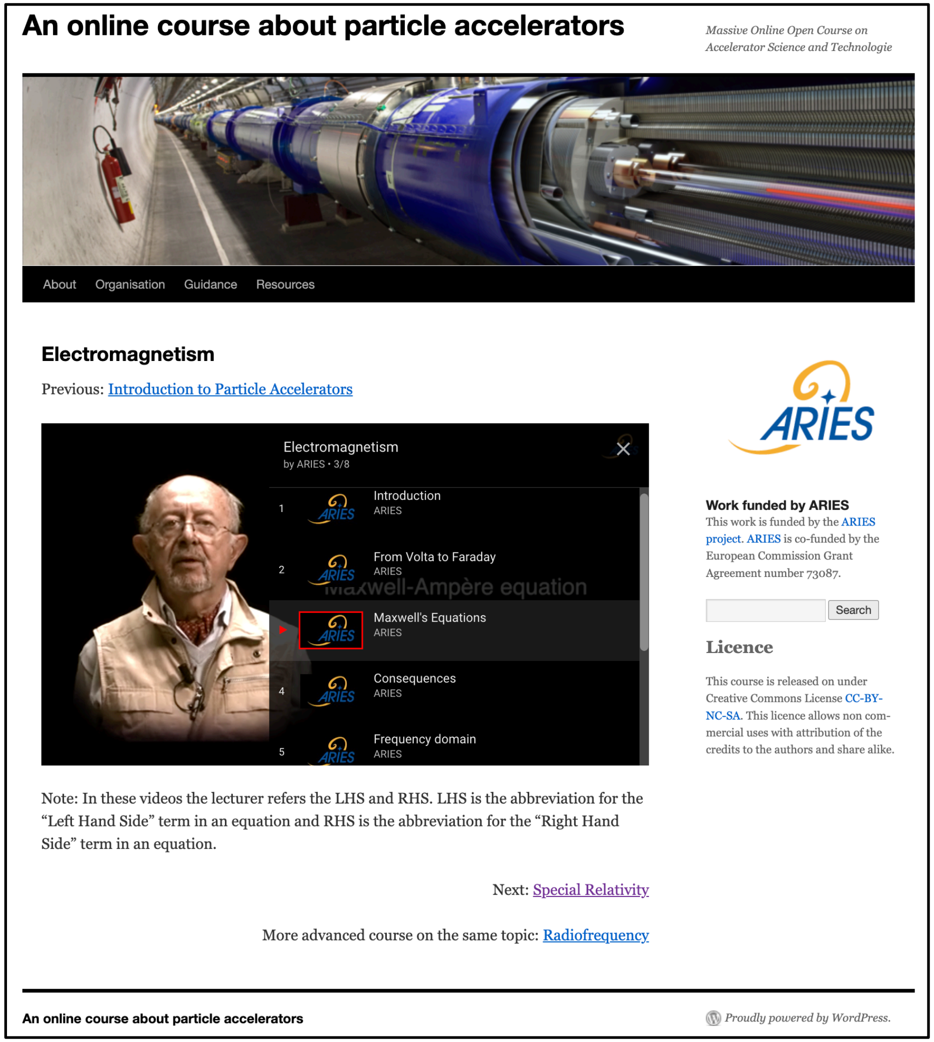
(1831-1879)

EM: the 4 Maxwell equations

EM: the 4 Maxwell equations

=> See also **MOOC on Electromagnetism**: http://mooc.particle-accelerators.eu/electromagnetism/

EM: the 4 Maxwell equations



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Massive Online Open Course on Accelerator Science and Technologie

Electromagnetism

Previous: Introduction to Particle Accelerators

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1 Introduction
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2 From Volta to Faraday
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Maxwell's Equations
ARIES

4 Consequences
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5 Frequency domain
ARIES

Note: In these videos the lecturer refers the LHS and RHS. LHS is the abbreviation for the “Left Hand Side” term in an equation and RHS is the abbreviation for the “Right Hand Side” term in an equation.

Next: Special Relativity

More advanced course on the same topic: Radiofrequency

Work funded by ARIES

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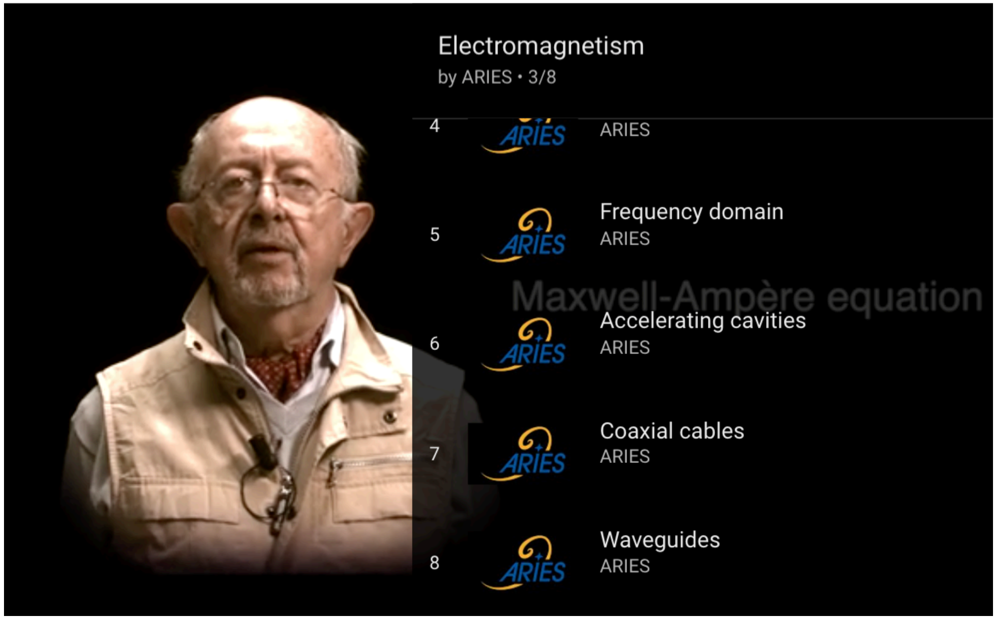

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EM: the 4 Maxwell equations

- 4 “coupled” equations, which combine the work of **Gauss**, **Faraday**, **Lenz** and **Ampere**


Maxwell

(1831-1879)

EM: the 4 Maxwell equations


Maxwell

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- 4 “coupled” equations, which combine the work of **Gauss**, **Faraday**, **Lenz** and **Ampere**

- Apply to all electric and magnetic phenomena and describe the behavior of the electric and magnetic fields, and electric charges and currents (the magnetic charge does not exist) **=> Framework for all calculations involving EM fields**

EM: the 4 Maxwell equations

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- Apply to all electric and magnetic phenomena and describe the behavior of the electric and magnetic fields, and electric charges and currents (the magnetic charge does not exist) **=> Framework for all calculations involving EM fields**
- Predicted **EM waves**
- Led Einstein to discover special relativity (together with the “failed” Michelson-Morley experiment)

EM: the 4 Maxwell equations

- **Differential forms**

(1) $$div\ \vec{E} = \frac{\rho}{\varepsilon}$$

Gauss's law for electric charge

(2) $$div\ \vec{H} = 0$$

Gauss's law for magnetic charge

(3) $$\overrightarrow{rot}\ \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

Faraday's and Lenz law

(4) $$\overrightarrow{rot}\ \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

Ampere's law

- **Integral forms**

$$\iiint div\ \vec{E}\ dV = \iint \vec{E} . d\vec{S} = \frac{1}{\varepsilon} \iiint \rho\ dV$$

$$\iiint div\ \vec{H}\ dV = \iint \vec{H} . d\vec{S} = 0$$

$$\iint \overrightarrow{rot}\, \vec{E} . d\vec{S} = \oint \vec{E} . d\vec{s} = -\mu \iint \frac{\partial \vec{H}}{\partial t} . d\vec{S}$$

$$\iint \overrightarrow{rot}\, \vec{H} . d\vec{S} = \oint \vec{H} . d\vec{s} = \iint \vec{J} . d\vec{S} + \varepsilon \iint \frac{\partial \vec{E}}{\partial t} . d\vec{S}$$

with

$$\vec{B} = \mu\ \vec{H}$$

$$\vec{D} = \varepsilon\ \vec{E}$$

$$\vec{J} = \rho \vec{\upsilon} + \sigma \vec{E}$$

Maxwell equations valid in homogeneous, isotropic, continuous media

EM: the 4 Maxwell equations

permeability

conductivity

$$\vec{B} = \mu \vec{H}$$

$$\mu = \mu_0 \, \mu_1 = \mu_0 \, \mu_r \left(1 - j \tan \vartheta_{\mathrm{M}} \right)$$

$$\vec{D} = \varepsilon \; \vec{E}$$

$$\varepsilon \; = \varepsilon_0 \, \varepsilon_1 = \varepsilon_0 \left(\varepsilon_r' - j \, \varepsilon_r'' \right) = \varepsilon_0 \, \varepsilon_b + \frac{\sigma}{j 2 \pi f}$$

permittivity

Imaginary parts describe the losses

EM: the 4 Maxwell equations

- q: electric charge [C] => $q = e$ for a proton
- ρ: electric charge density [C/m^3]
- I, $\overrightarrow{J}$: electric current [A], electric current density [A/m^2]
- $\overrightarrow{E}$: electric field [V/m]
- $\overrightarrow{H}$: magnetic field [A/m]
- $\overrightarrow{D}$: electric displacement [C/m^2]
- $\overrightarrow{B}$: magnetic induction or magnetic flux density [T] => But, beware: it is often called “magnetic field”

EM: the 4 Maxwell equations

- **Cartesian (x,y,s)**

$$\vec{\nabla} \equiv \left| \begin{array}{c} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial s} \end{array} \right.$$

- **Cylindrical (r,θ,s)**

$$\vec{\nabla} \equiv \left| \begin{array}{c} \dfrac{\partial}{\partial r} \\ \dfrac{1}{r}\left(\dfrac{\partial}{\partial \vartheta}\right) \\ \dfrac{\partial}{\partial s} \end{array} \right.$$

Also noted
$\overrightarrow{curl}\,\vec{\mathrm{E}}$ or $\vec{\nabla} \wedge \vec{\mathrm{E}}$

$$\overrightarrow{grad}\,\rho \equiv \vec{\nabla}\,\rho = \left| \begin{array}{c} \dfrac{\partial \rho}{\partial x} \\ \dfrac{\partial \rho}{\partial y} \\ \dfrac{\partial \rho}{\partial s} \end{array} \right.$$

$$\overrightarrow{rot}\,\vec{E} \equiv \vec{\nabla} \times \vec{E} = \left| \begin{array}{c} \dfrac{\partial E_s}{\partial y} - \dfrac{\partial E_y}{\partial s} \\ \dfrac{\partial E_x}{\partial s} - \dfrac{\partial E_s}{\partial x} \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} \end{array} \right.$$

$$\overrightarrow{grad}\,\rho = \left| \begin{array}{c} \dfrac{\partial \rho}{\partial r} \\ \dfrac{1}{r}\left(\dfrac{\partial \rho}{\partial \vartheta}\right) \\ \dfrac{\partial \rho}{\partial s} \end{array} \right.$$

$$\overrightarrow{rot}\,\vec{E} = \left| \begin{array}{c} \dfrac{1}{r}\left(\dfrac{\partial E_s}{\partial \vartheta}\right) - \dfrac{\partial E_\theta}{\partial s} \\ \dfrac{\partial E_r}{\partial s} - \dfrac{\partial E_s}{\partial r} \\ \dfrac{1}{r}\left[\dfrac{\partial\left(r\,E_\theta\right)}{\partial r} - \dfrac{\partial E_r}{\partial \theta}\right] \end{array} \right.$$

$$div\,\vec{E} \equiv \vec{\nabla}\,.\,\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_s}{\partial s}$$

$$div\,\vec{E} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,E_r\right) + \frac{1}{r}\frac{\partial E_\theta}{\partial \theta} + \frac{\partial E_s}{\partial s}$$

$$\Delta\,\rho \equiv \nabla^2 \rho = \text{Laplacian operator}$$
$$= \frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial s^2}$$

$$\Delta\,\rho = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial \rho}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \rho}{\partial \theta^2} + \frac{\partial^2 \rho}{\partial s^2}$$

EM: the 4 Maxwell equations

- Maxwell Eqs. (2) and (3) are independent of ρ and $\overrightarrow{J}$ => They are referred to as the **"homogenous Maxwell equations"**

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- ρ and $\vec{J}$ may be regarded as **sources of EM fields**

EM: the 4 Maxwell equations

- When ρ and $\vec{J}$ are specified, one can integrate Maxwell equations to find possible electric and magnetic fields in the system

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EM: the 4 Maxwell equations

- When ρ and $\vec{J}$ are specified, one can integrate Maxwell equations to find possible electric and magnetic fields in the system
- The solution one finds by integration is not unique: for example, there are many possible field patterns that may exist in a cavity (or waveguide) of given geometry
- Most realistic situations are sufficiently complicated that solutions to Maxwell equations cannot be obtained analytically **=> A variety of computer codes exist to provide solutions numerically**

EM: the 4 Maxwell equations

- Important feature of Maxwell equations: for systems containing materials **with constant permittivity and permeability** (i.e. permittivity and permeability that are independent of the fields present), the **equations are linear in the fields and sources** => As a consequence, **the principle of superposition applies**

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  - If $\left(\vec{E}_1 , \vec{B}_1\right)$ and $\left(\vec{E}_2 , \vec{B}_2\right)$ are solutions of Maxwell equations with given boundary conditions, then $\left(\vec{E}_1 + \vec{E}_2 , \vec{B}_1 + \vec{B}_2\right)$ will also be solutions of Maxwell equations, with the same boundary conditions

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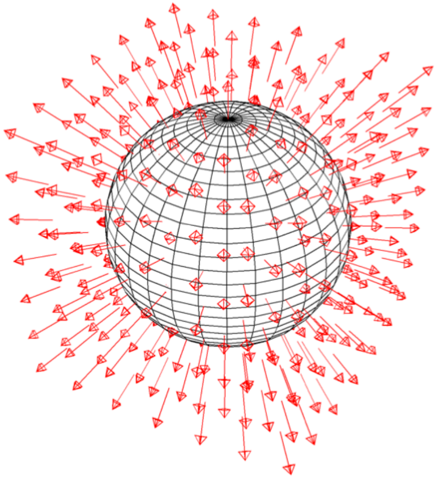
  - An important and widely used **analysis technique** for EM systems, including RF cavities and waveguides, is **to find a set of solutions to Maxwell equations from which more complete and complicated solutions may be constructed**

EM: the 4 Maxwell equations

- From Eq. (1)

$$\iiint div\ \vec{E}\ dV = \iint \vec{E}\,.\,d\vec{S} = \frac{1}{\varepsilon}\boxed{\iiint \rho\ dV}$$

= total charge q



=> Coulomb's law:

$$E = \frac{q}{4\pi\varepsilon r^2}$$

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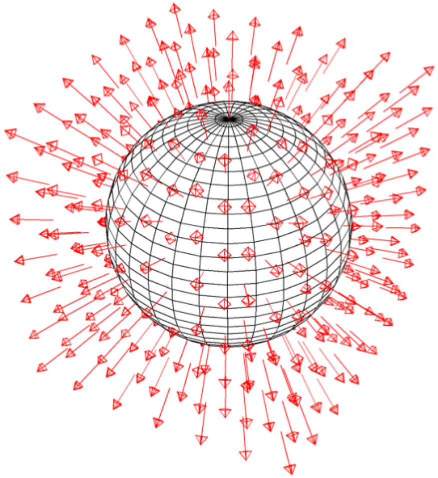


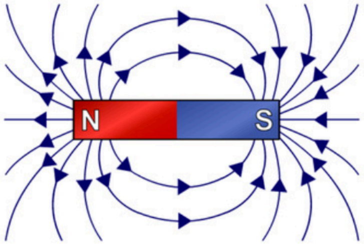
=> Coulomb's law:

$$E = \frac{q}{4\pi\varepsilon r^2}$$

- From Eq. (2)

$$\iiint div\,\vec{H}\,dV = \iint \vec{H}\,.\,d\vec{S} = 0$$

=> Absence of magnetic monopoles (lines of magnetic flux always occur in closed loop)

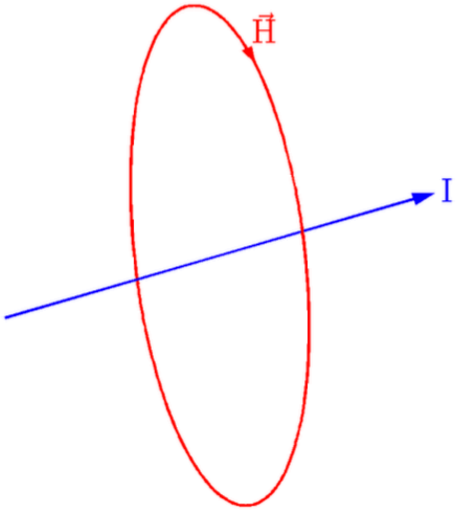


EM: the 4 Maxwell equations

- From Eq. (4)

$$\oint \vec{H} \, . \, d\vec{s} = \iint \vec{J} \, . \, d\vec{S} + \varepsilon \iint \frac{\partial \vec{E}}{\partial t} \, . \, d\vec{S}$$

=> In absence of 2nd term



$$B = \frac{\mu I}{2\pi r}$$

EM: the 4 Maxwell equations

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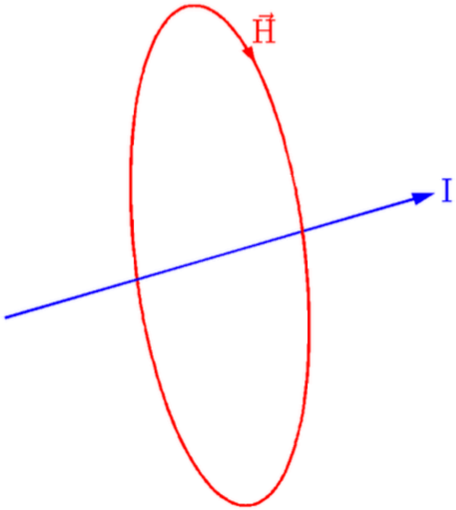

$$B = \frac{\mu I}{2\pi r}$$

- From Eq. (3)

$$\iint \overrightarrow{rot}\, \vec{E} \,.\, d\vec{S} = \oint \vec{E} \,.\, d\vec{s} = -\,\mu \iint \frac{\partial \vec{H}}{\partial t} \,.\, d\vec{S}$$

Eqs. (3) and (4) tell us that a time dependent electric (magnetic) field will induce a magnetic (electric) field => Fields in RF cavities and waveguides always consist of both electric and magnetic fields

EM: the 4 Maxwell equations

- EM fields can be written as derivatives of **scalar and vector potentials** $\phi\,(x,\ y,\ s)$ **and** $\overrightarrow{A}\,(x,\ y,\ s)$

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \mathrm{curl}\vec{A}$$

EM: the 4 Maxwell equations

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$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \mathrm{curl}\vec{A}$$

- The knowledge of the potentials allows the computation of the fields

EM: the 4 Maxwell equations

- While the absolute values of the electric and magnetic fields can be measured, the absolute values of the potentials are not defined. The EM potentials are merely auxiliary “constructions”, although very important ones, in particular, for the relativistic formulation of the EM theory

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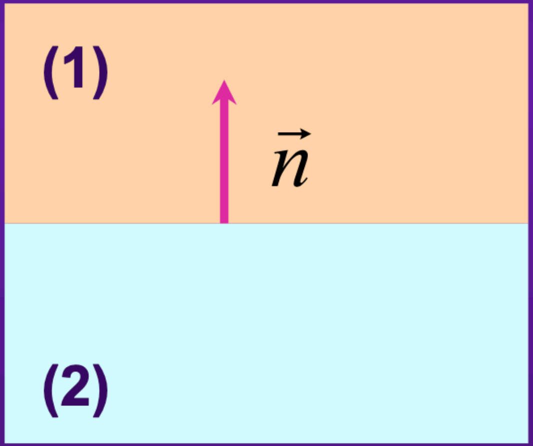
- **The scalar and vector potentials are used in particular if one uses the Hamiltonian formalism** to describe the beam dynamics (which leads to the same results as the ones obtained using the Lorentz force and Newton’s second law of motion)

Field matching

Consider a surface separating two media "1" and "2". The following boundary conditions can be derived from Maxwell equations for the normal (⊥) and parallel (//) components of the fields at the surface



$$\vec{E}^1_{//} = \vec{E}^2_{//}$$

$$\vec{H}^1_{//} - \vec{H}^2_{//} = \vec{K}$$

$$D^1_{\perp} - D^2_{\perp} = \Sigma$$

$$B^1_{\perp} = B^2_{\perp}$$

where Σ is the surface charge density and $\vec{K}$ is the surface current density

Energy of EM waves

- **Poynting vector**: $\overrightarrow{S} = \overrightarrow{E} \times \overrightarrow{H}$

=> It points in the direction of propagation and describes the "energy flux", i.e. the energy crossing a unit area per second

Energy of EM waves

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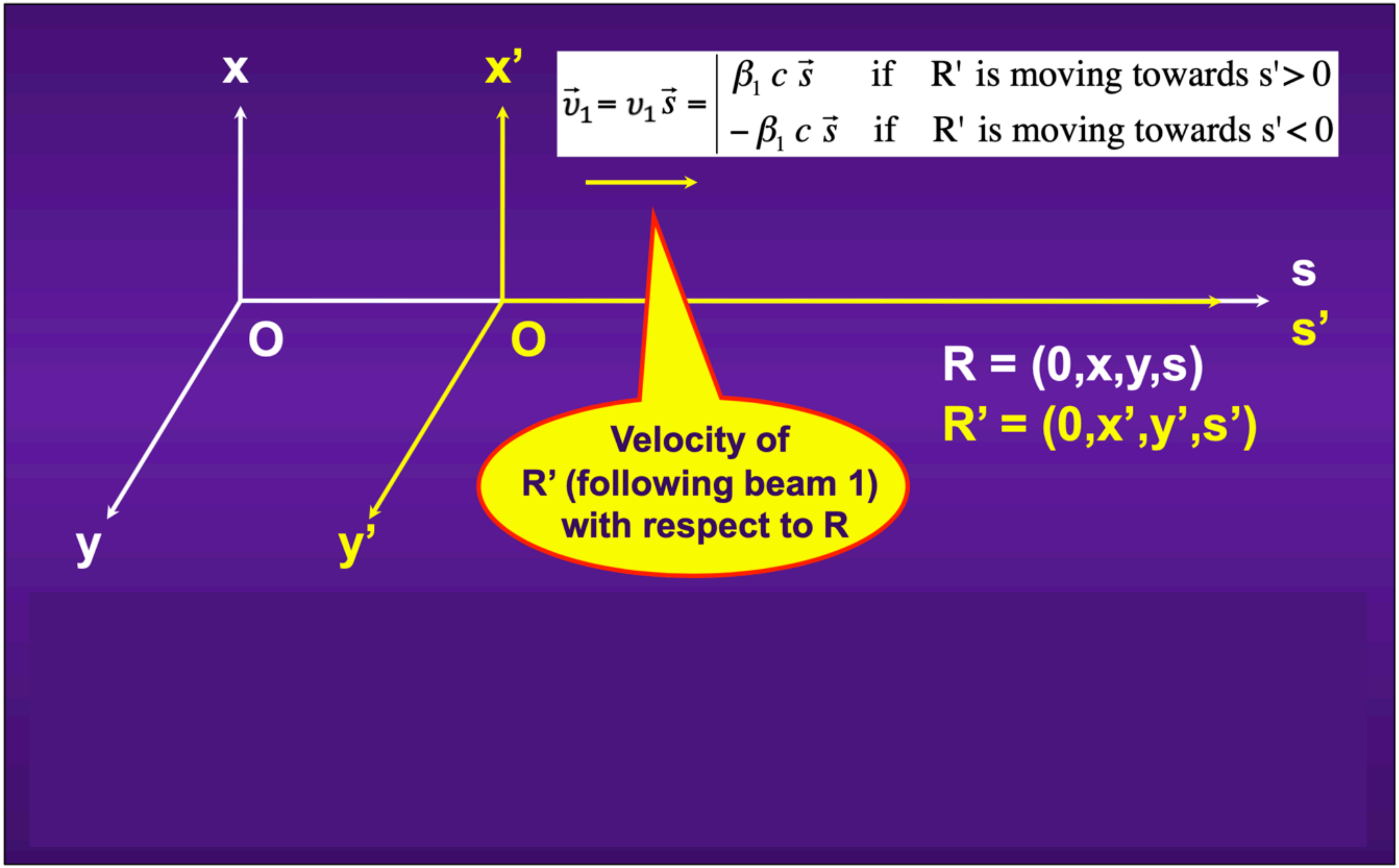
- **Remark on complex notations for vectors**
  - As long as we deal with linear equations, we can carry out all the algebraic manipulations using complex field vectors, where **it is implicit that the physical quantities are obtained by taking the real parts of the complex vectors**
  - However, when using the complex notation, particular care is needed when taking the product of two complex vectors: to be safe, one should always take the real part before multiplying two complex quantities, the real parts of which represent physical quantities

Relativistic transformation of EM fields



$$\vec{v}_1 = v_1\,\vec{s} = \begin{cases} \beta_1\,c\,\vec{s} & \text{if} \quad \text{R' is moving towards s'} > 0 \\ -\beta_1\,c\,\vec{s} & \text{if} \quad \text{R' is moving towards s'} < 0 \end{cases}$$

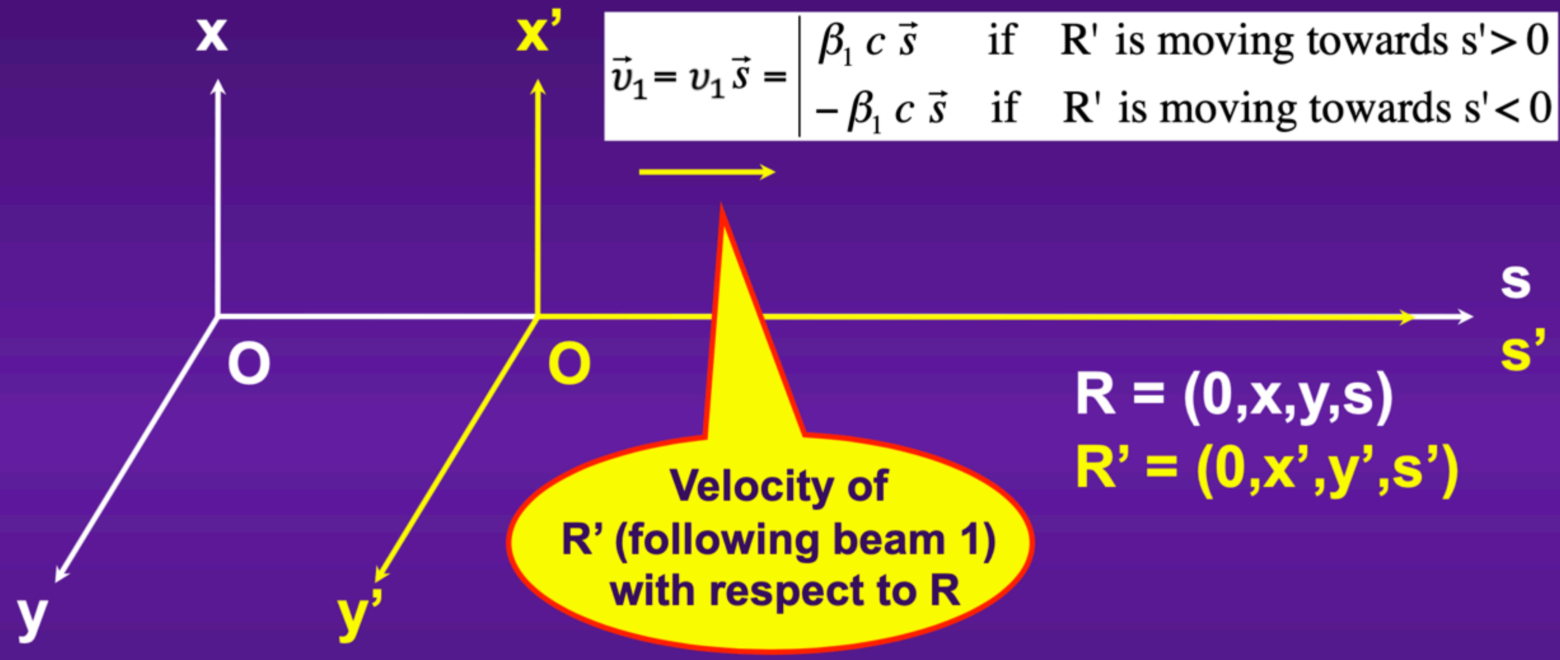
Velocity of R' (following beam 1) with respect to R

R = (0,x,y,s)

R' = (0,x',y',s')

Relativistic transformation of EM fields

$$\vec{v}_1 = v_1 \vec{s} = \left| \begin{array}{lll} \beta_1 \, c \, \vec{s} & \text{if} & \text{R' is moving towards s'} > 0 \\ -\beta_1 \, c \, \vec{s} & \text{if} & \text{R' is moving towards s'} < 0 \end{array} \right.$$



$$E'_x = \gamma_1 \left(E_x - v_1 B_y \right)$$

$$E'_y = \gamma_1 \left(E_y + v_1 B_x \right)$$

$$B'_x = \gamma_1 \left(B_x + \frac{v_1}{c^2} E_y \right)$$

$$B'_y = \gamma_1 \left(B_y - \frac{v_1}{c^2} E_x \right)$$

$$E'_s = E_s$$

$$B'_s = B_s$$

Relativistic transformation of EM fields

- Lorentz force on the particle 2 moving with velocity $\vec{\upsilon}_2 = \upsilon_2 \, \vec{s}$

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=> $B_x = - \frac{\upsilon_1}{c^2} E_y$ $B_y = \frac{\upsilon_1}{c^2} E_x$ $B_s = 0$

Space charge

=> $$F_{x,y} = e \, E_{x,y} \left| \begin{array}{ll} \left(1 - \beta_1 \, \beta_2\right) & \text{if 2 moves in same direction as 1} \\ \left(1 + \beta_1 \, \beta_2\right) & \text{if 2 moves in oppo. direction as 1} \end{array} \right.$$

Beam beam

Relativistic transformation of EM fields

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Electric part

Magnetic part

Relativistic transformation of EM fields

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=> $$F_{x,y} = e\,E_{x,y}\left(\underbrace{1}_{\text{Electric part}} - \underbrace{\beta^2}_{\text{Magnetic part}}\right) = e\,\frac{E_{x,y}}{\gamma^2}$$

and

$$E'_{x,y} = \frac{E_{x,y}}{\gamma} \qquad B_x = -\frac{\beta}{c}\,E_y$$

$$E'_s = E_s \qquad B_y = \frac{\beta}{c}\,E_x$$

$$B'_x = B'_y = B'_s = 0 \qquad B_s = 0$$

(RF) cavities & waveguides

- At the surface of **an ideal (or perfect) conductor** (i.e. with **no energy dissipation**), the normal component of $\overrightarrow{B}$ and the tangential component of $\overrightarrow{E}$ must both vanish => **Standing waves that can persist within the cavity are determined by the shape of the cavity**

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- The effect on the beam is determined by the field pattern. Therefore, it is important to design the shape of the cavity, so that the fields in the cavity interact with the beam in the desired way; and that undesirable interactions (which always occur to some extent) are minimized

(RF) cavities & waveguides

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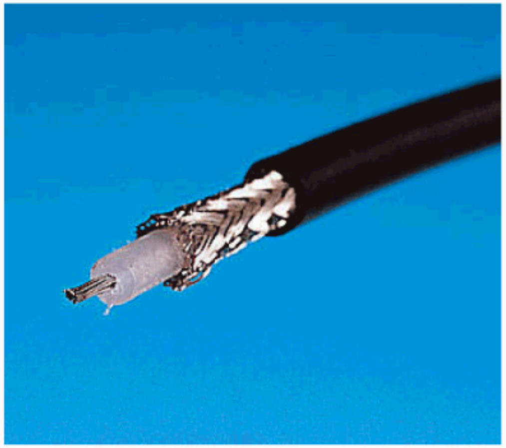

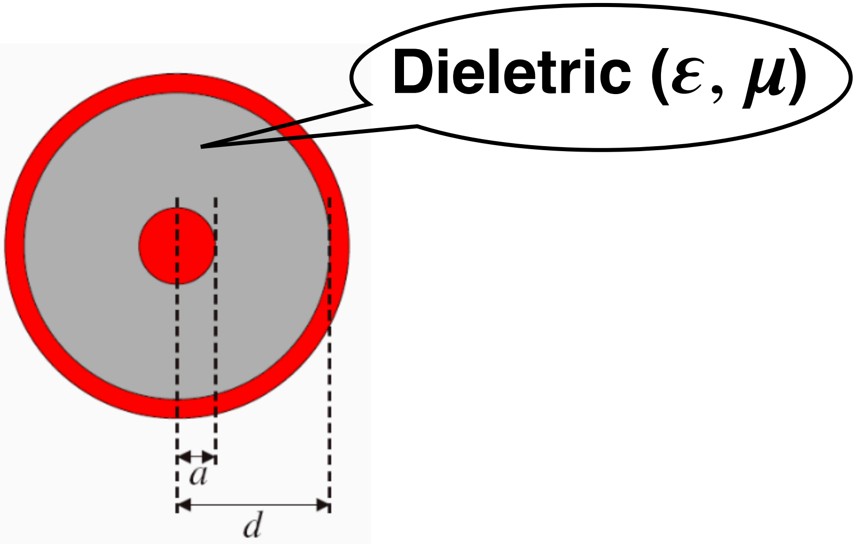

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- As was the case for cavities, the patterns of the fields in the resonant modes are determined by the geometry of the boundary

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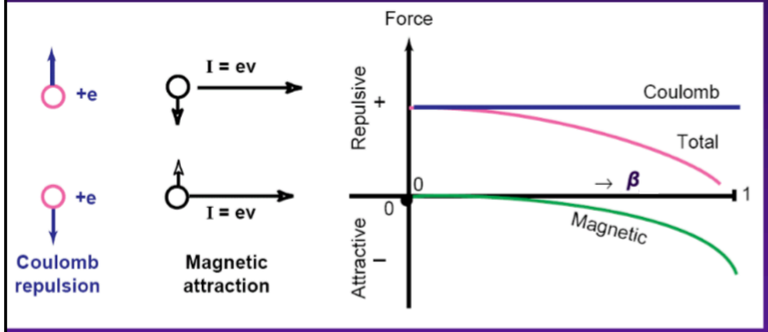
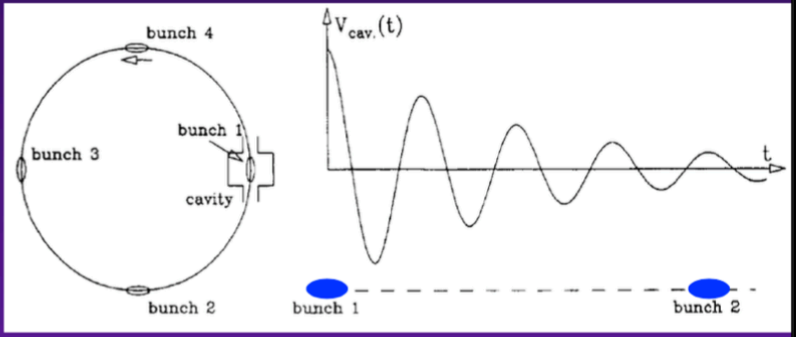
  - Etc. **=> To correctly describe the dynamics of a beam of particles, all the wanted and unwanted EM interactions need to be taken into account!**

Conclusions on EM & SR

SPACE CHARGE

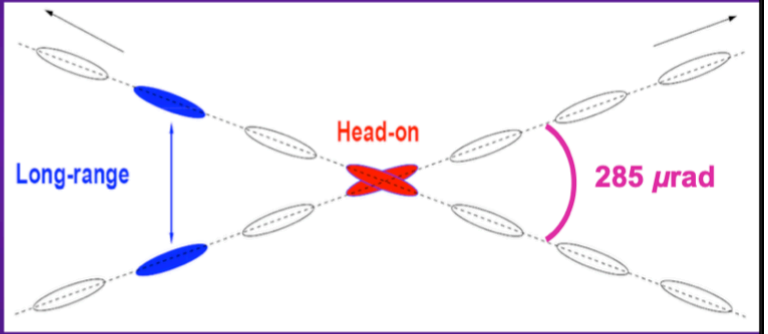


WAKE FIELD (or IMPEDANCE)

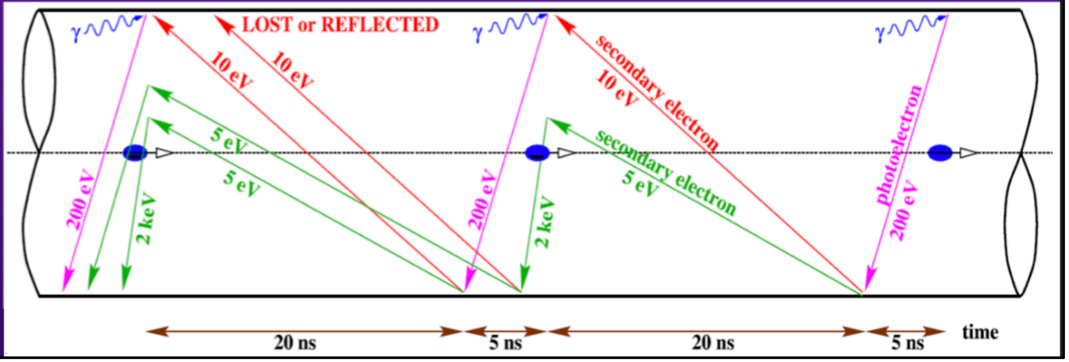


IBS = INTRA-BEAM SCATTERING (due to collisions between the particles), etc.

BEAM-BEAM

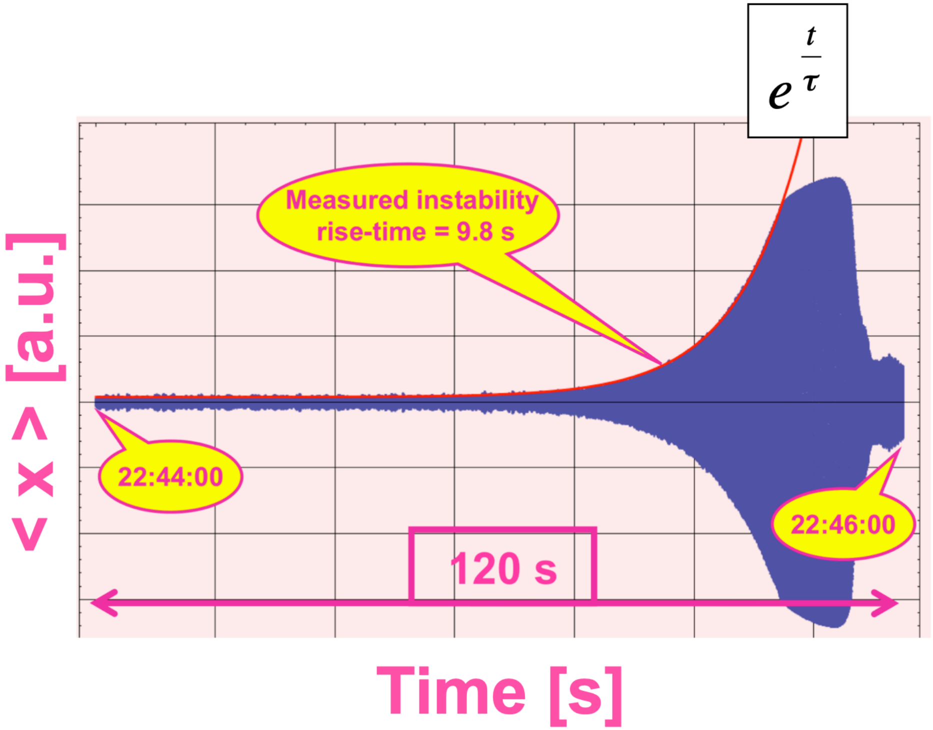


ELECTRON CLOUD



Conclusions on EM & SR

- Example of a coherent instability due to the wake field in the CERN LHC



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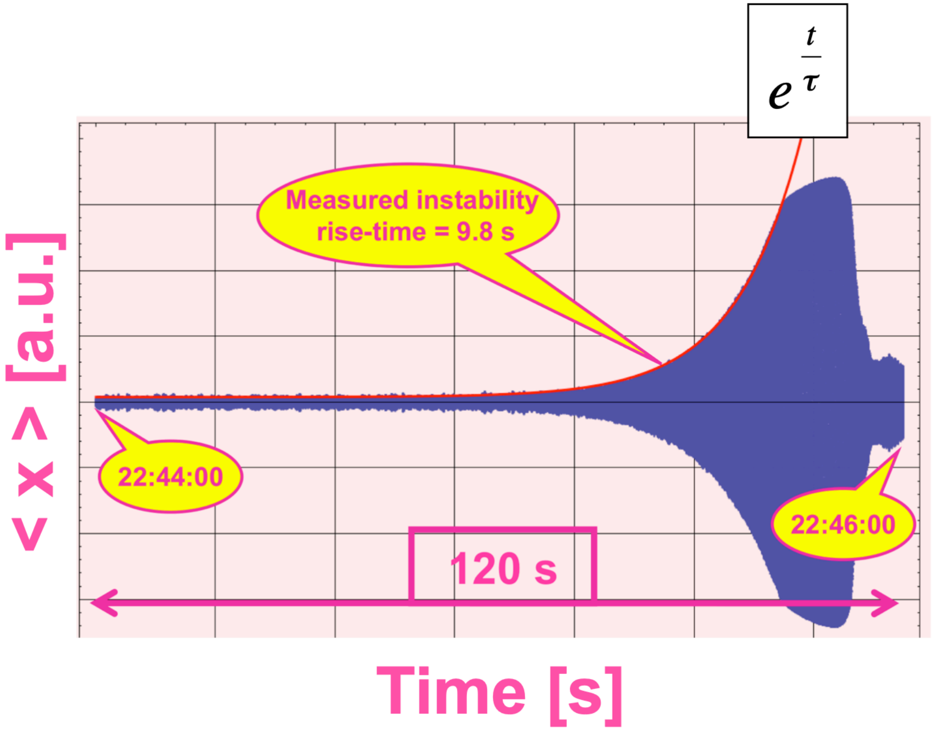

- Many kinds of instabilities exist and several mitigation measures are needed to push the performance of particle accelerators

2 modes of particle accelerators: Fixed-target vs. Collider

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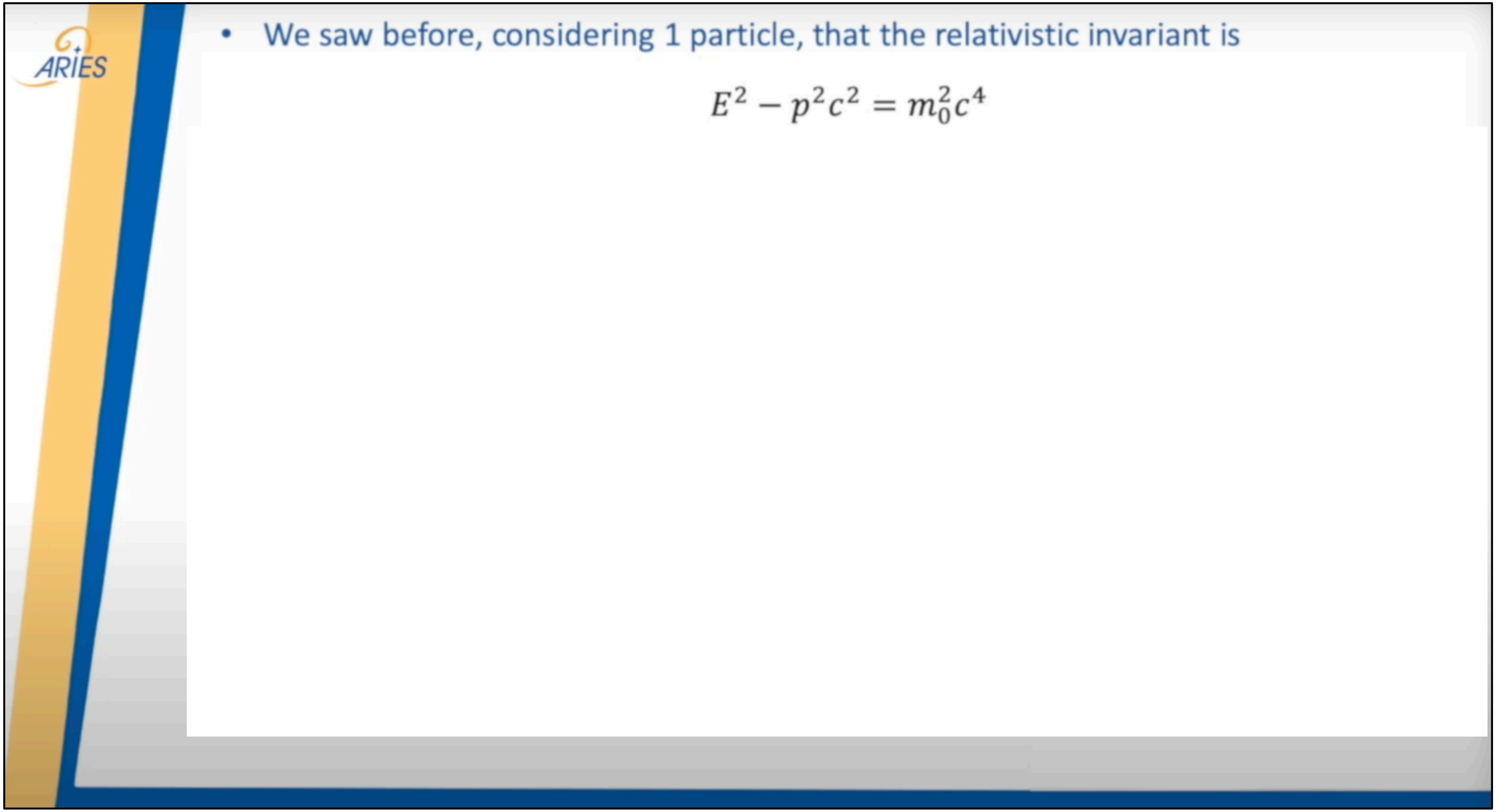

- We saw before, considering 1 particle, that the relativistic invariant is

$$E^2 - p^2c^2 = m_0^2c^4$$

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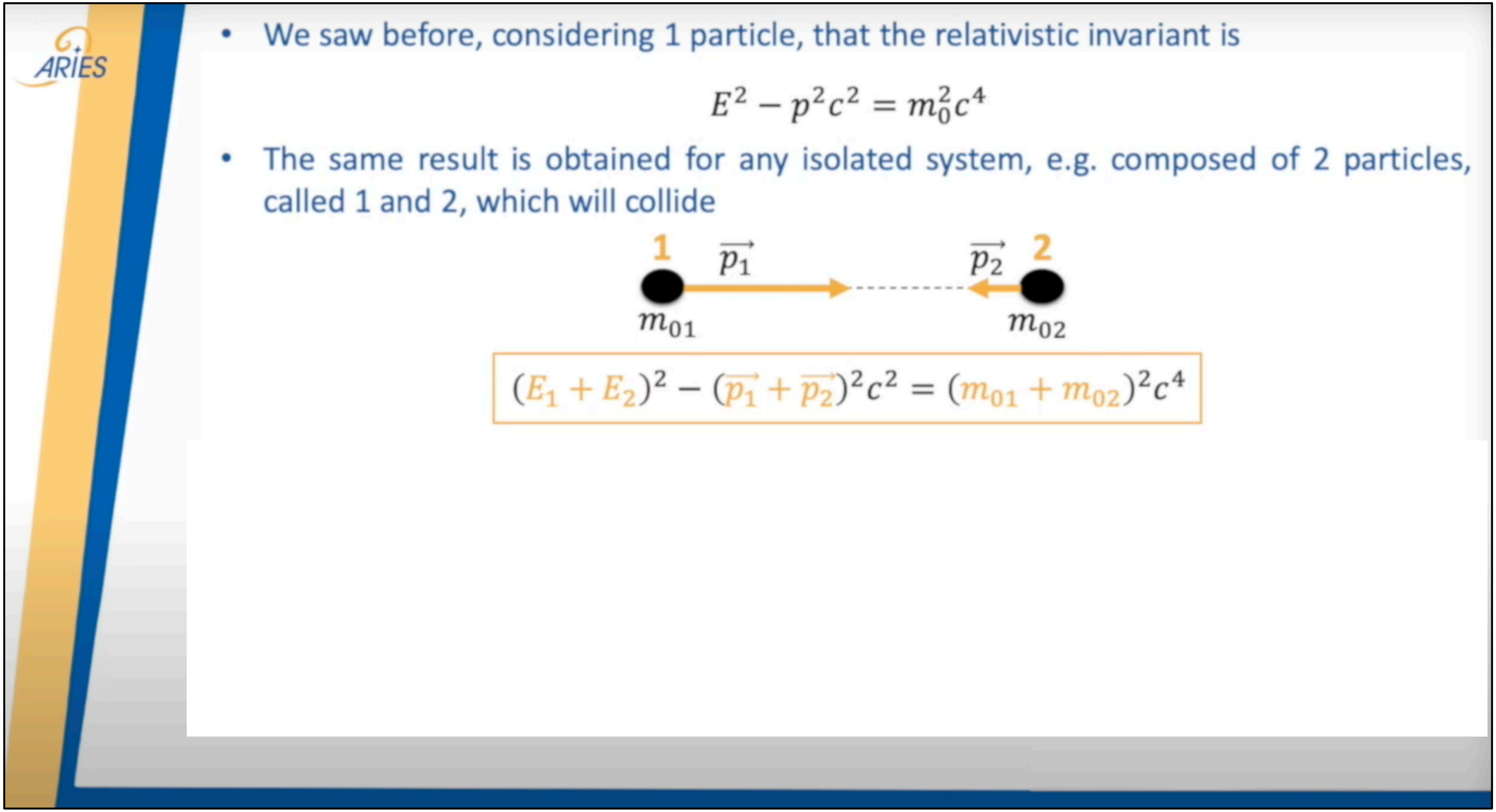

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$$\boxed{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2 c^2 = (m_{01} + m_{02})^2 c^4}$$

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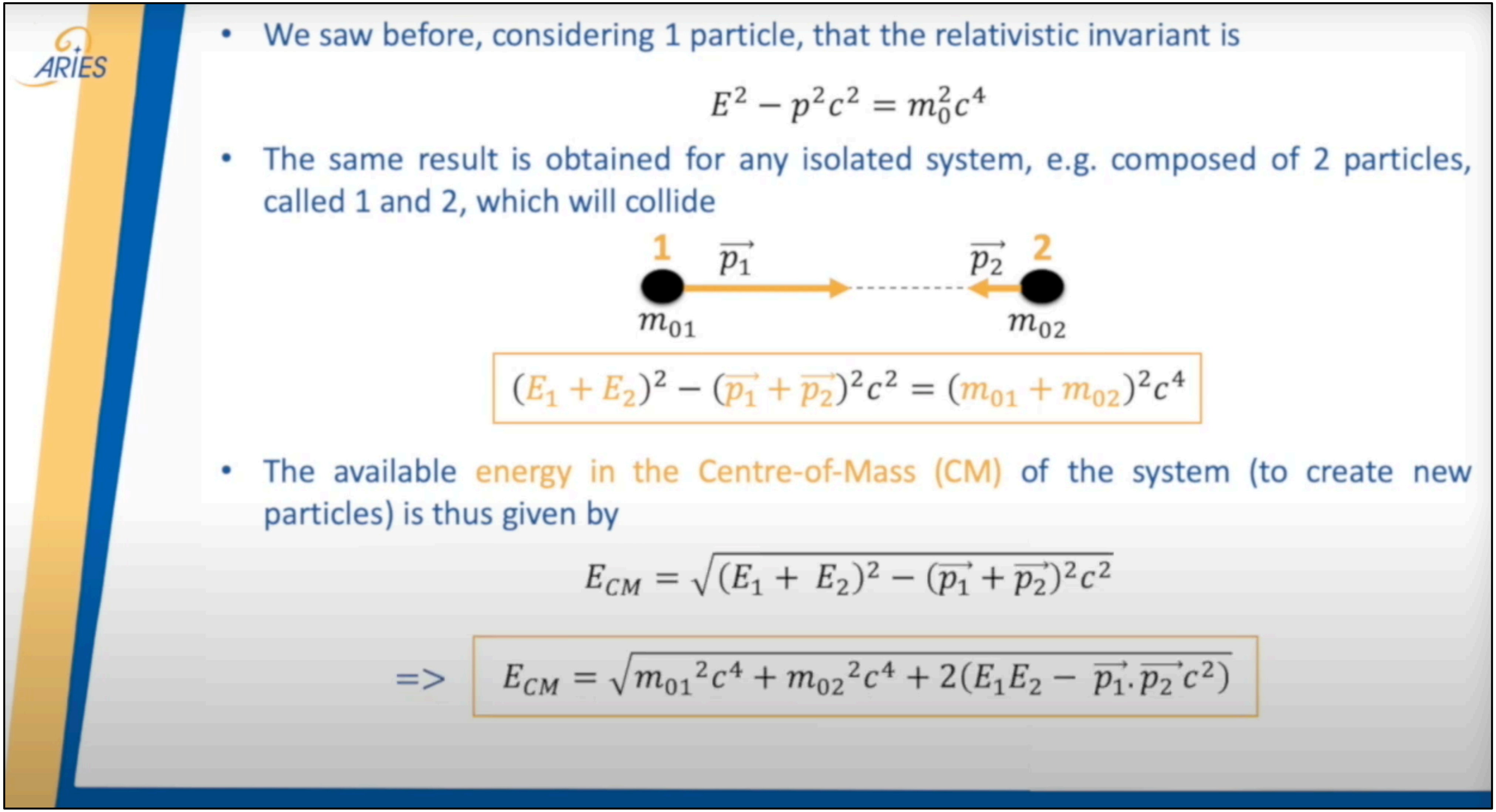

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$$\boxed{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2c^2 = (m_{01} + m_{02})^2c^4}$$

- The available energy in the Centre-of-Mass (CM) of the system (to create new particles) is thus given by

$$E_{CM} = \sqrt{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2c^2}$$

$$=> \quad \boxed{E_{CM} = \sqrt{m_{01}{}^2c^4 + m_{02}{}^2c^4 + 2(E_1E_2 - \vec{p_1}.\vec{p_2}\,c^2)}}$$

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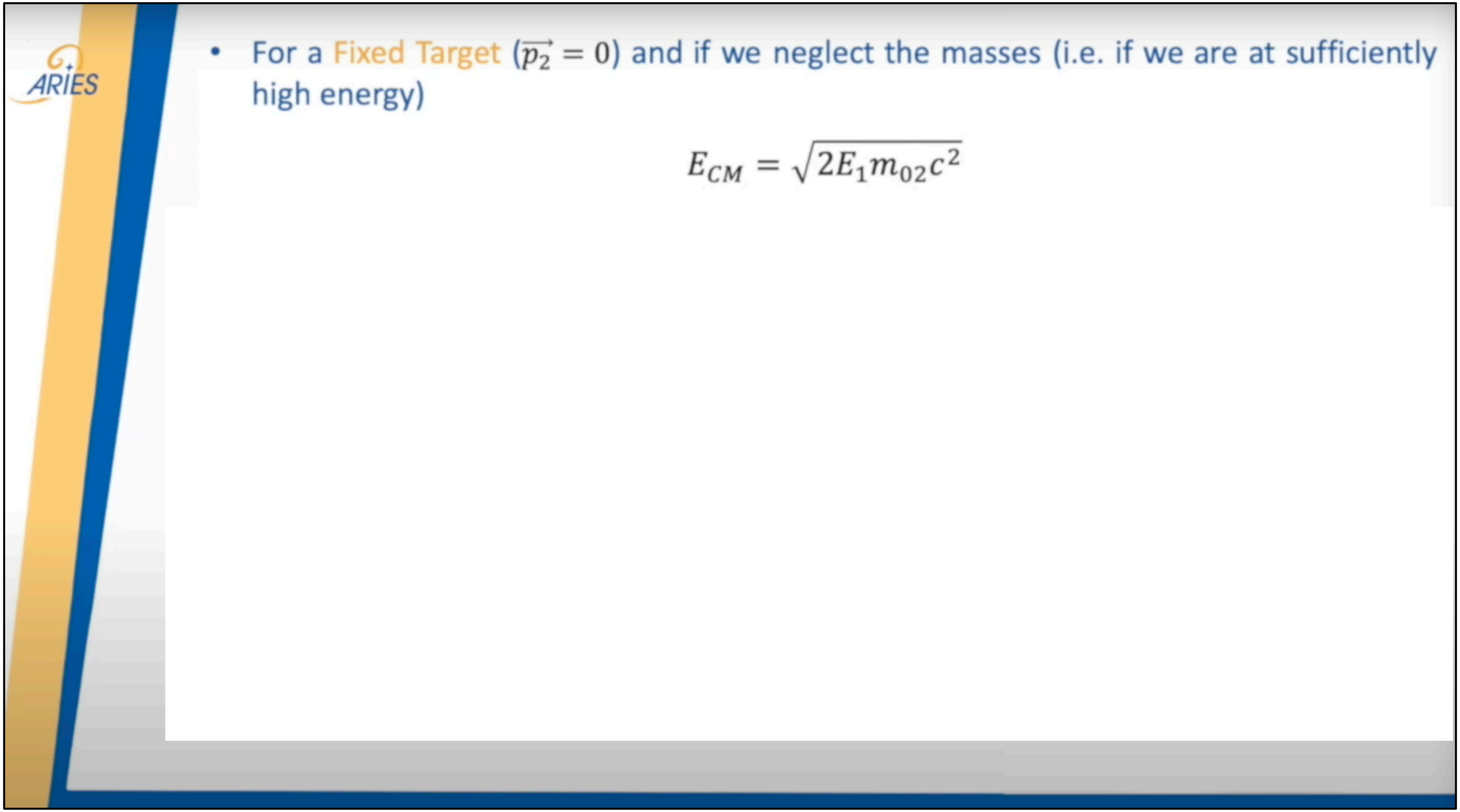

- For a Fixed Target ($\overrightarrow{p_2} = 0$) and if we neglect the masses (i.e. if we are at sufficiently high energy)

$$E_{CM} = \sqrt{2E_1 m_{02} c^2}$$

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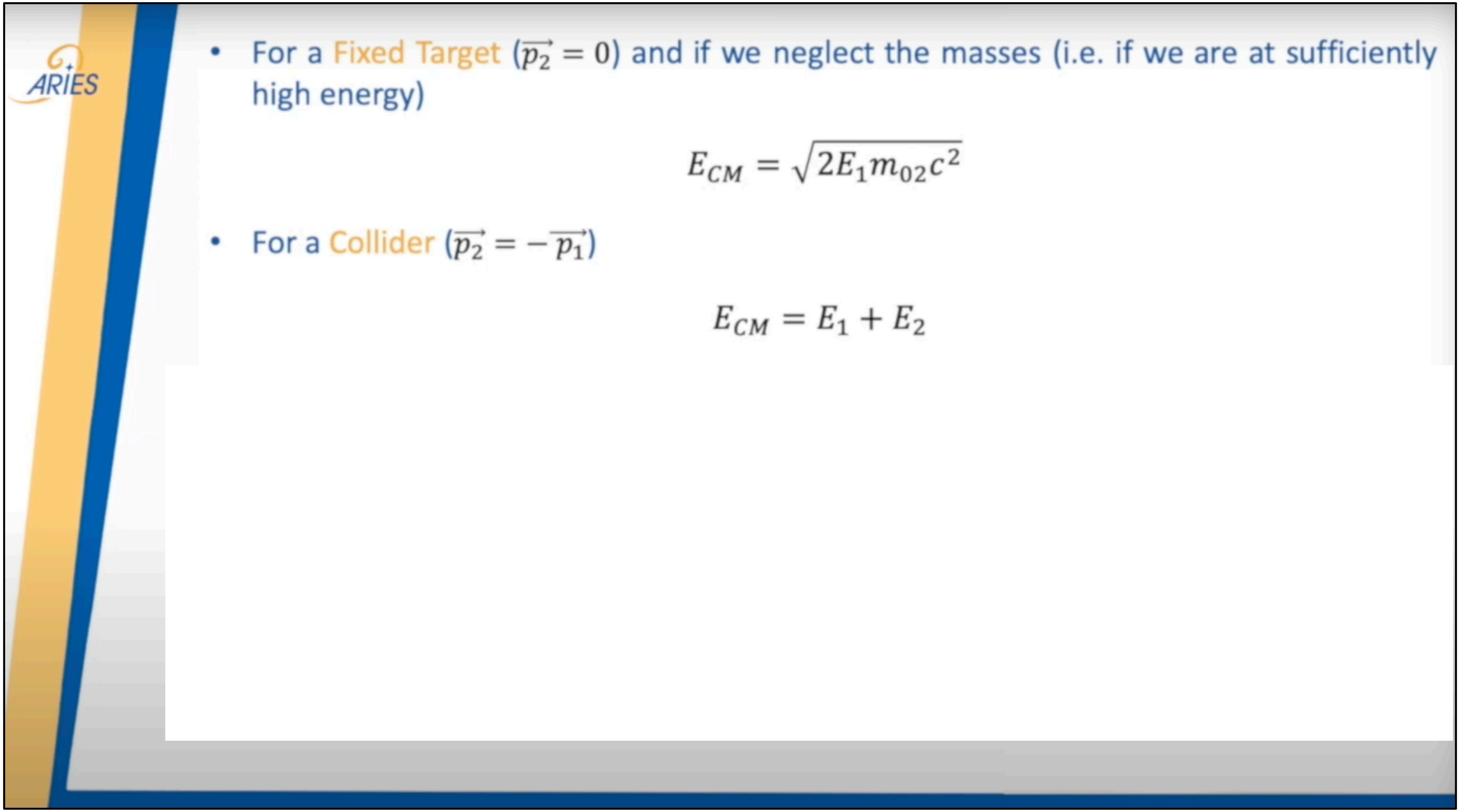

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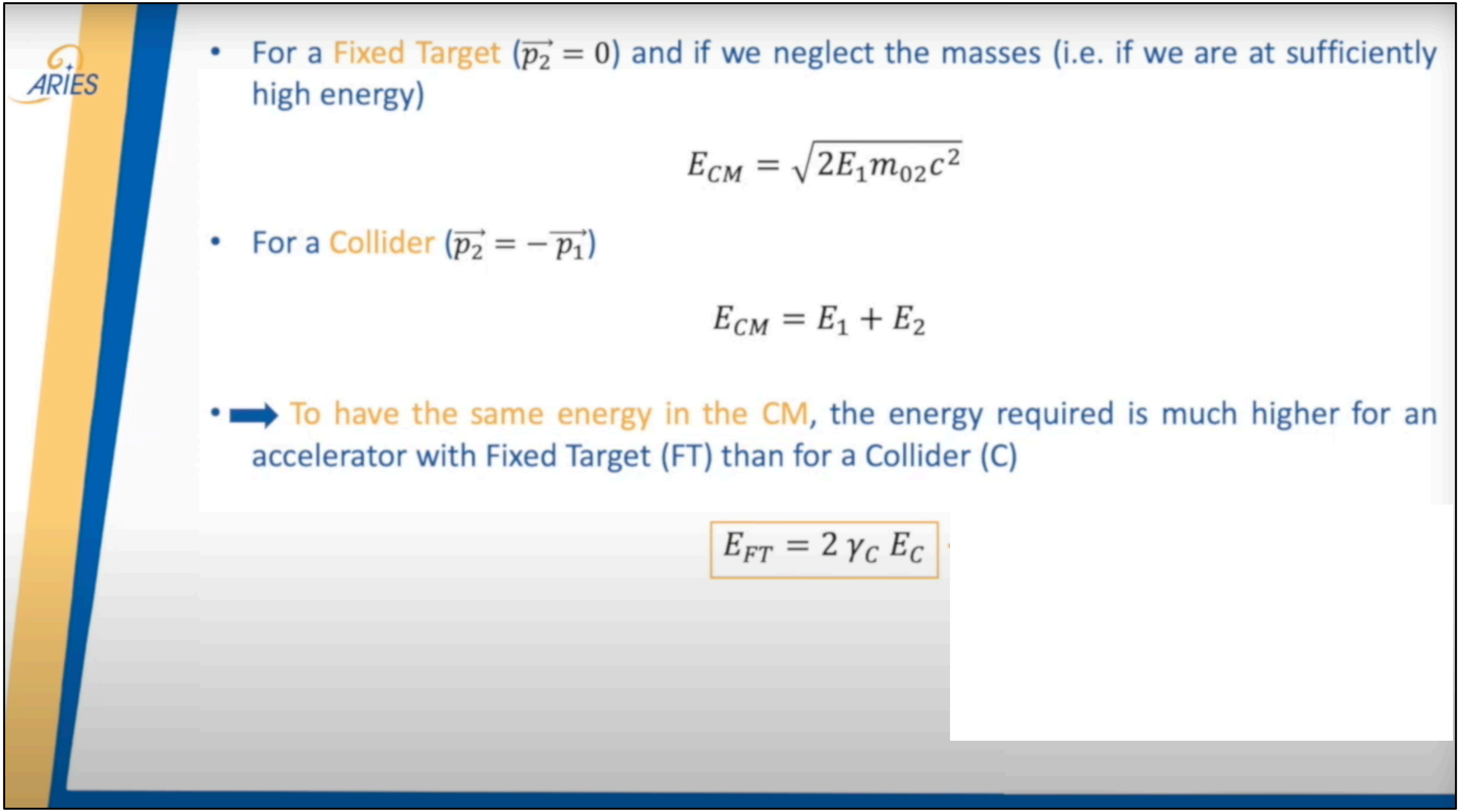

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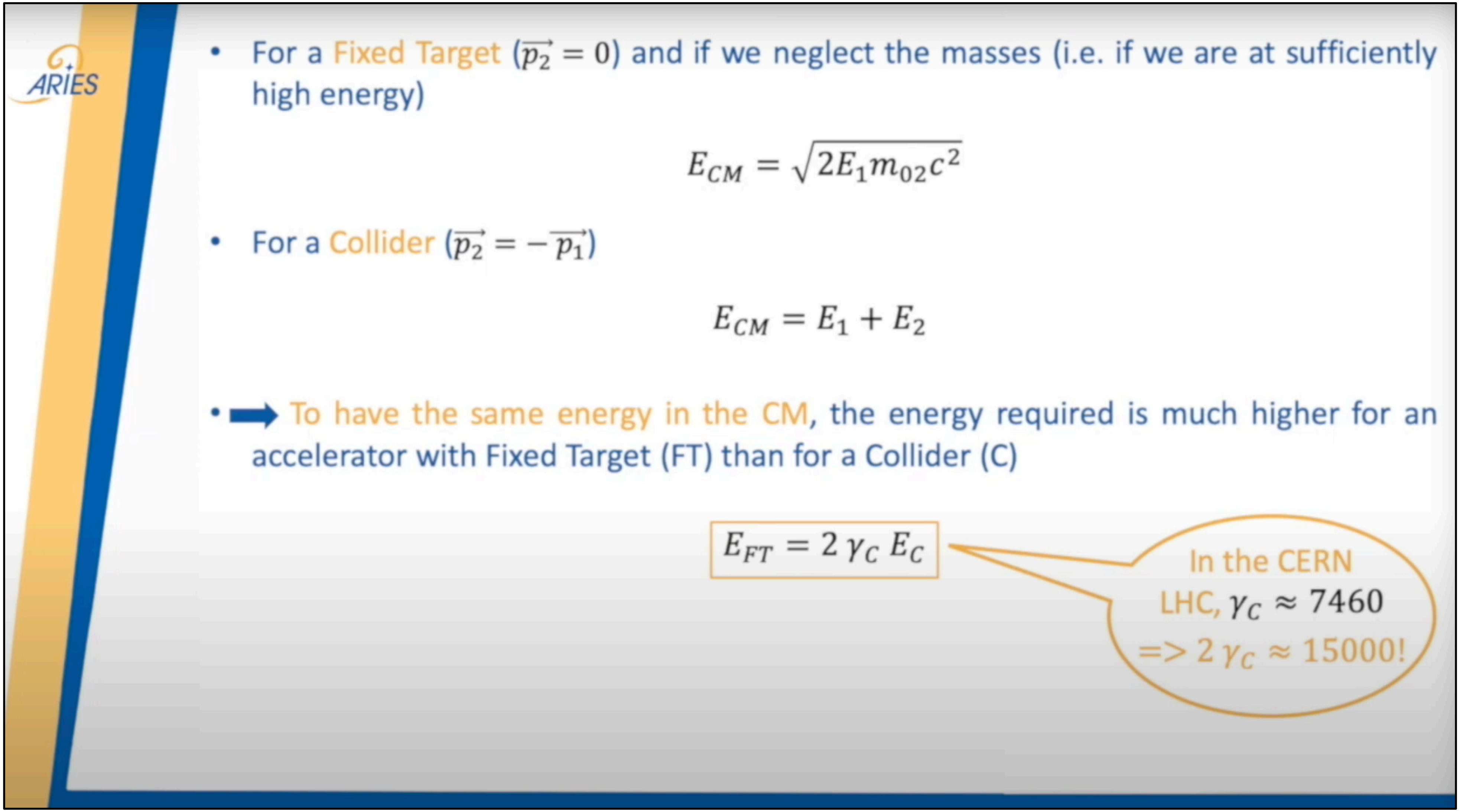
- ➡ To have the same energy in the CM, the energy required is much higher for an accelerator with Fixed Target (FT) than for a Collider (C)

$$\boxed{E_{FT} = 2\,\gamma_C\,E_C}$$

2 modes of particle accelerators: Fixed-target vs. Collider



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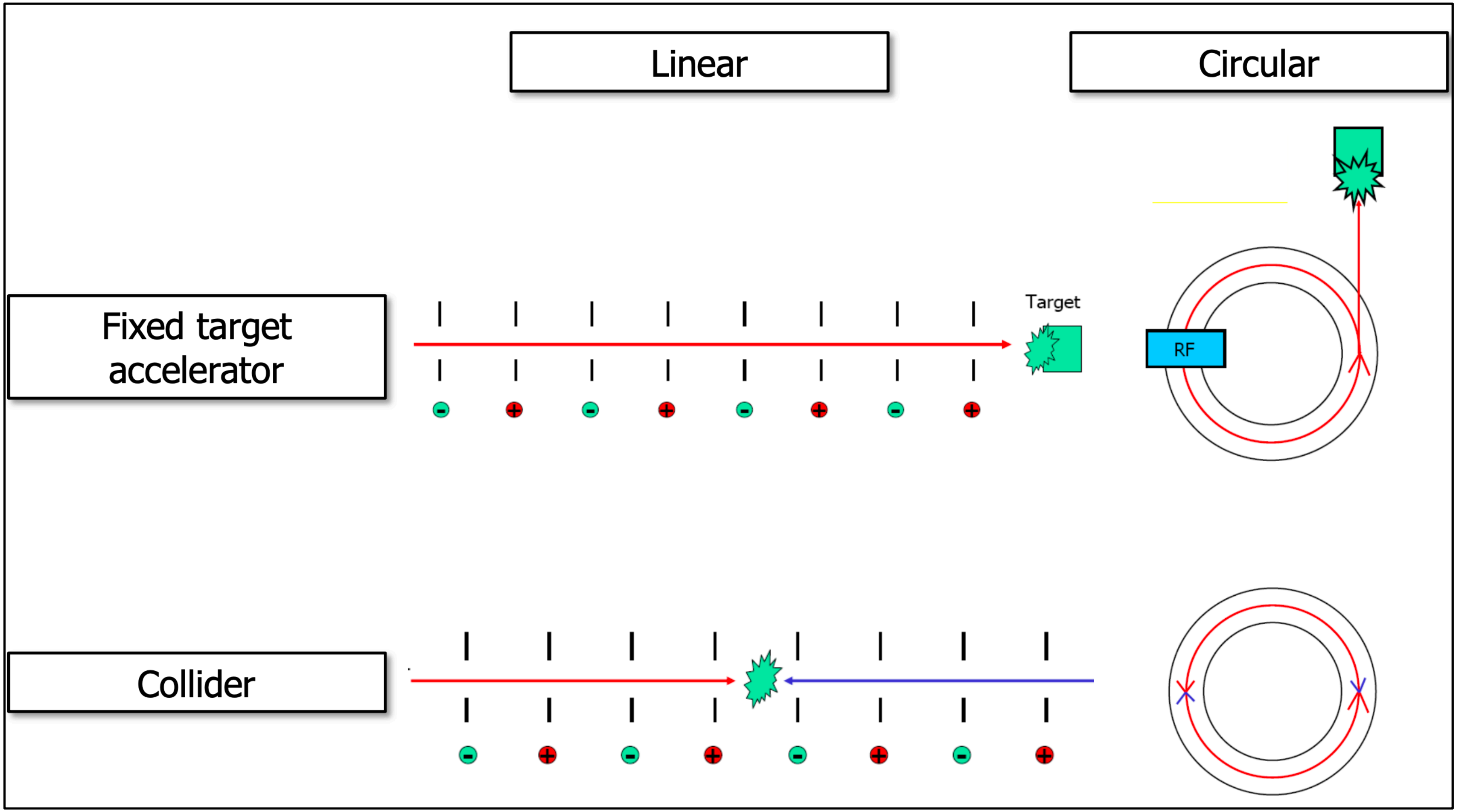
In the CERN LHC, $\gamma_C \approx 7460$ => $2\,\gamma_C \approx 15000$!

2 modes of particle accelerators: Fixed-target vs. Collider



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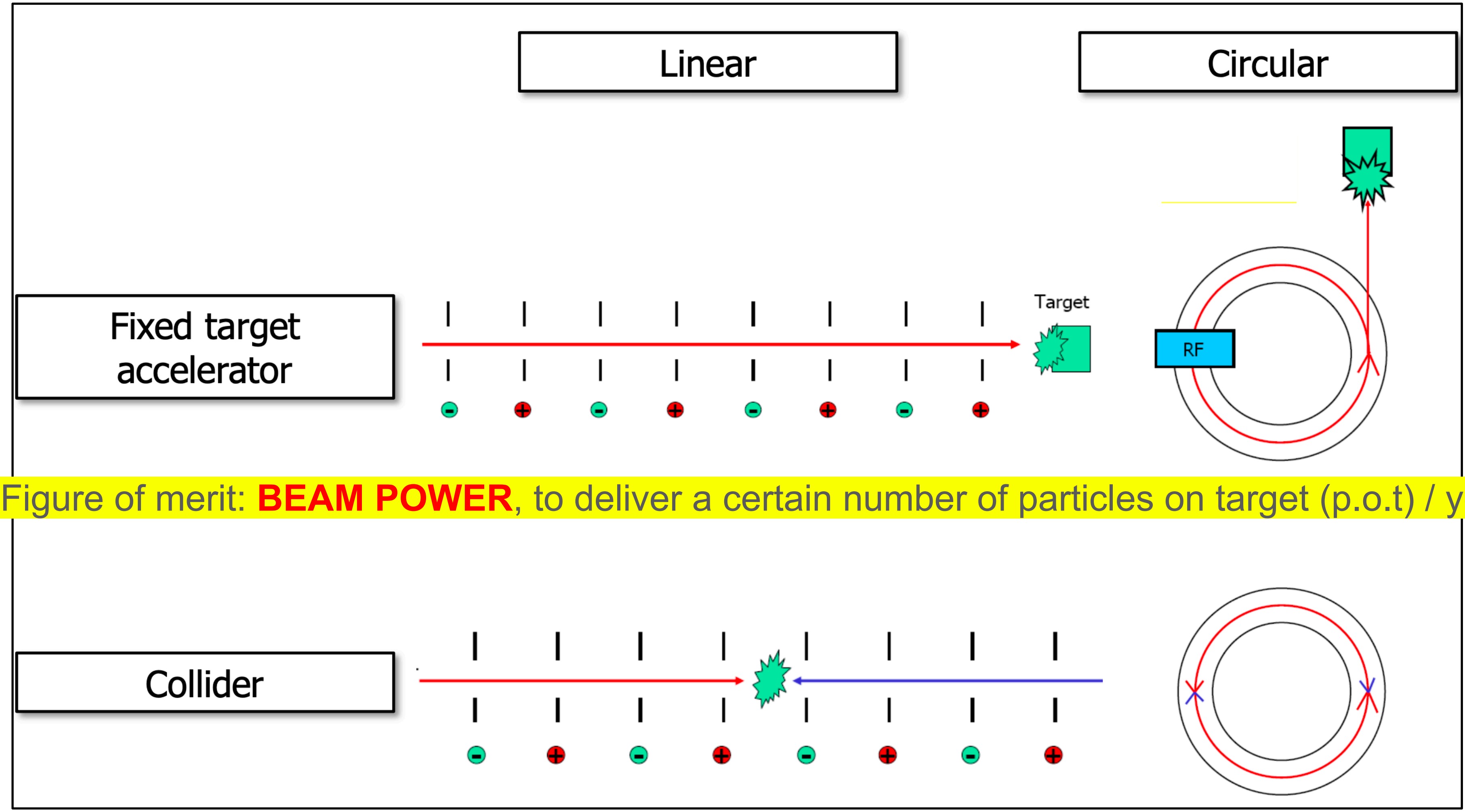
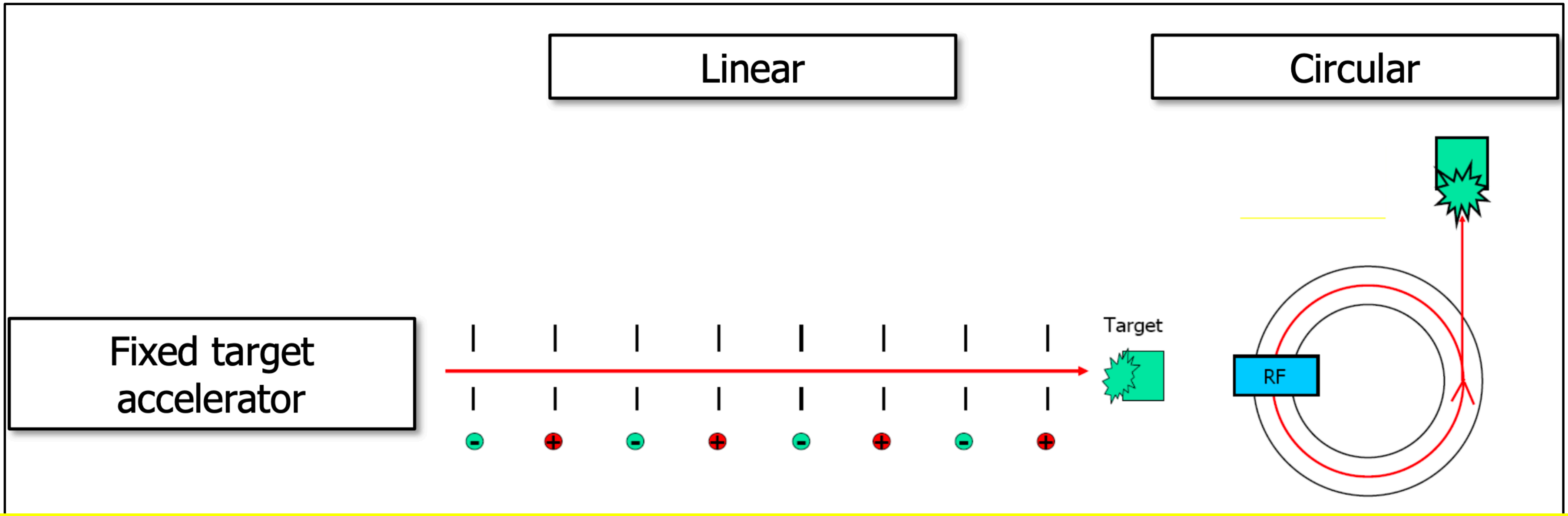


=> Figure of merit: **BEAM POWER**, to deliver a certain number of particles on target (p.o.t) / year

2 modes of particle accelerators: Fixed-target vs. Collider



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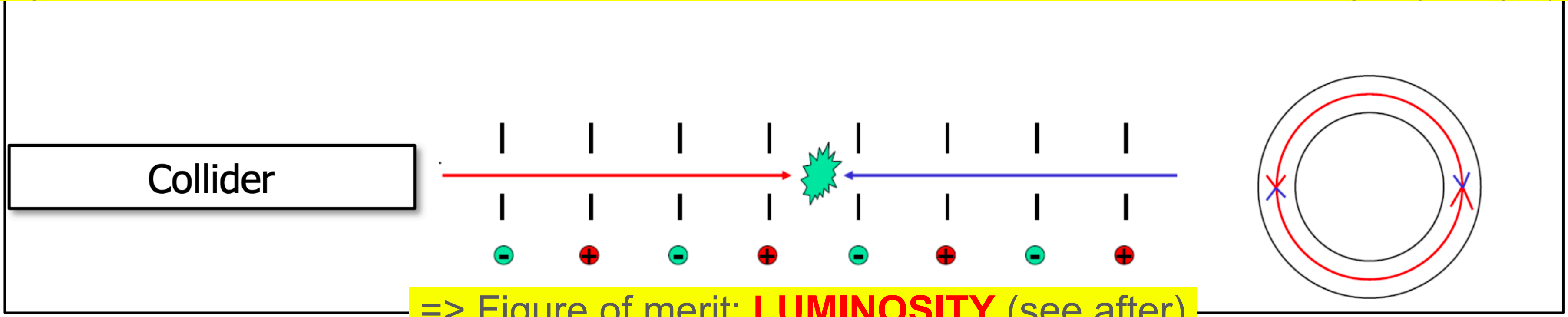
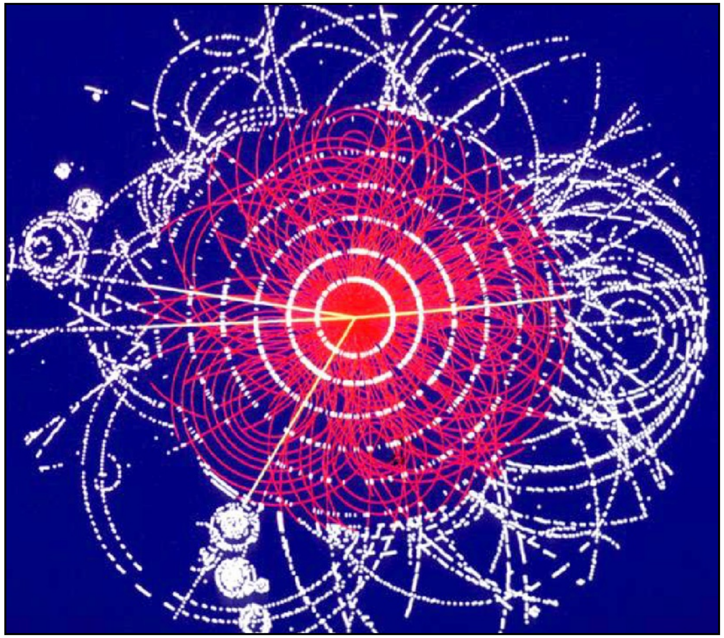


=> Figure of merit: **LUMINOSITY** (see after)

Short intro to colliders (luminosity and pile-up)



Why colliders?

Why colliders? => Particle discoveries and precision measurements

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Accelerators contributed to 26 Nobel Prizes in physics since 1939

Courtesy of P. Lebrun

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- 1939 Ernest O. Lawrence
- 1951 John D. Cockcroft & Ernest Walton
- 1952 Felix Bloch
- 1957 Tsung-Dao Lee & Chen Ning Yang
- 1959 Emilio G. Segrè & Owen Chamberlain
- 1960 Donald A. Glaser
- 1961 Robert Hofstadter
- 1963 Maria Goeppert Mayer
- 1967 Hans A. Bethe
- 1968 Luis W. Alvarez
- 1976 Burton Richter & Samuel C.C. Ting
- 1979 Sheldon L. Glashow, Abdus Salam & Steven Weinberg
- 1980 James W. Cronin & Val L. Fitch
- 1981 Kai M. Siegbahn
- 1983 William A. Fowler
- 1984 Carlo Rubbia & Simon van der Meer
- 1986 Ernst Ruska
- 1988 Leon M. Lederman, Melvin Schwartz & Jack Steinberger
- 1989 Wolfgang Paul
- 1990 Jerome I. Friedman, Henry W. Kendall & Richard E. Taylor
- 1992 Georges Charpak
- 1995 Martin L. Perl
- 2004 David J. Gross, Frank Wilczek & H. David Politzer
- 2008 Makoto Kobayashi & Toshihide Maskawa

Higgs boson in the CERN LHC (2012)

- 2013 François Englert & Peter Higgs
- 2015 Takaaki Kajita & Arthur B. MacDonald

Courtesy of P. Lebrun

Short history of colliders

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- 1943, R. Widerøe patents the concept of colliding beams in storage rings
- 1961, the first electron-positron storage ring AdA is built in Frascati
- 1971, CERN starts operating the ISR, first proton-proton collider
- 1982, the CERN SPS is converted into a proton-antiproton collider
- 1987, the TeVatron at Fermilab is converted into a proton-antiproton collider
- 1987, the SSC, a 40 TeV proton-proton collider, is approved for construction in the USA. The project was subsequently cancelled in 1993.

- 1989, CERN starts operating the 26.7 km, high-energy electron-positron collider LEP
- 1989, SLAC starts operating the SLC, first linear collider converted from the linac
- 1991, HERA at DESY becomes the first proton-electron collider
- 1999, RHIC at BNL becomes the first heavy-ion collider
- 2008, CERN starts operation of the LHC, 14 TeV proton-proton collider
- 2012, design studies are published for electron-positron linear colliders, ILC and CLIC
- 2014, CERN launches design study for Future Circular Colliders (100 km circumference)

Courtesy of P. Lebrun

Hadrons vs. Leptons in circular colliders

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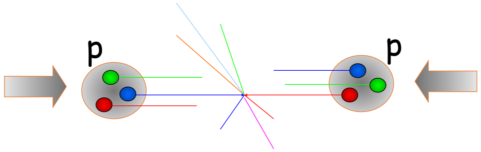

6 quarks

hadron collider => frontier of physics

- discovery machine
- collisions of quarks
- not all nucleon energy available in collision
- huge background

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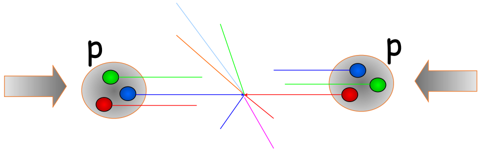

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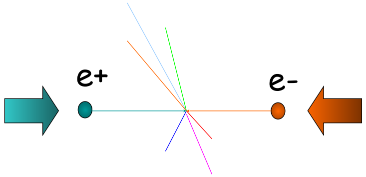
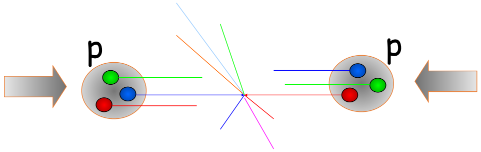


2 leptons

lepton collider => precision physics

- study machine
- elementary particles collisions
- well defined CM energy
- polarization possible

Hadrons vs. Leptons in circular colliders



6 quarks

Limited by the dipole field available
and the ring size

$$p[\mathrm{GeV/c}] \simeq 0.3B[\mathrm{T}]\rho[\mathrm{m}]$$

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- collisions of quarks
- not all nucleon energy available in collision
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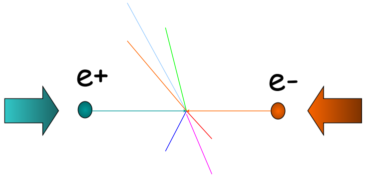

2 leptons

lepton collider => precision physics

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- elementary particles collisions
- well defined CM energy
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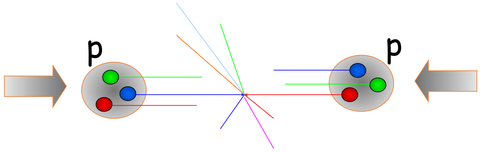
Hadrons vs. Leptons in circular colliders



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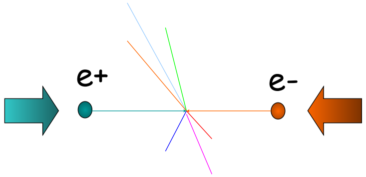
⬇

Go to higher magnetic fields (=> Superconducting) or/and large circumferences (=> ten's km)



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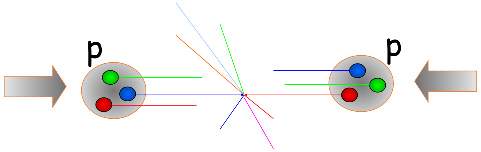

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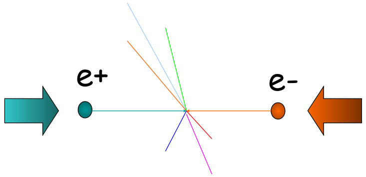

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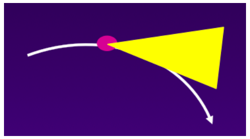

$$U_{lost} \propto \frac{E^4}{\rho E_0^4}$$

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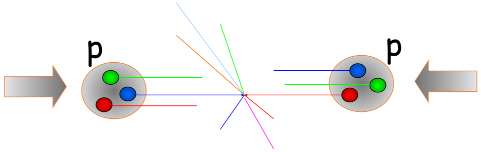

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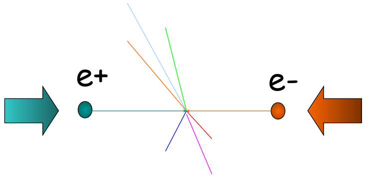

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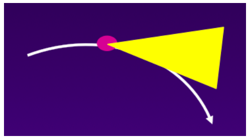

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Go to linear colliders or heavier leptons

Luminosity: figure of merit of a collider

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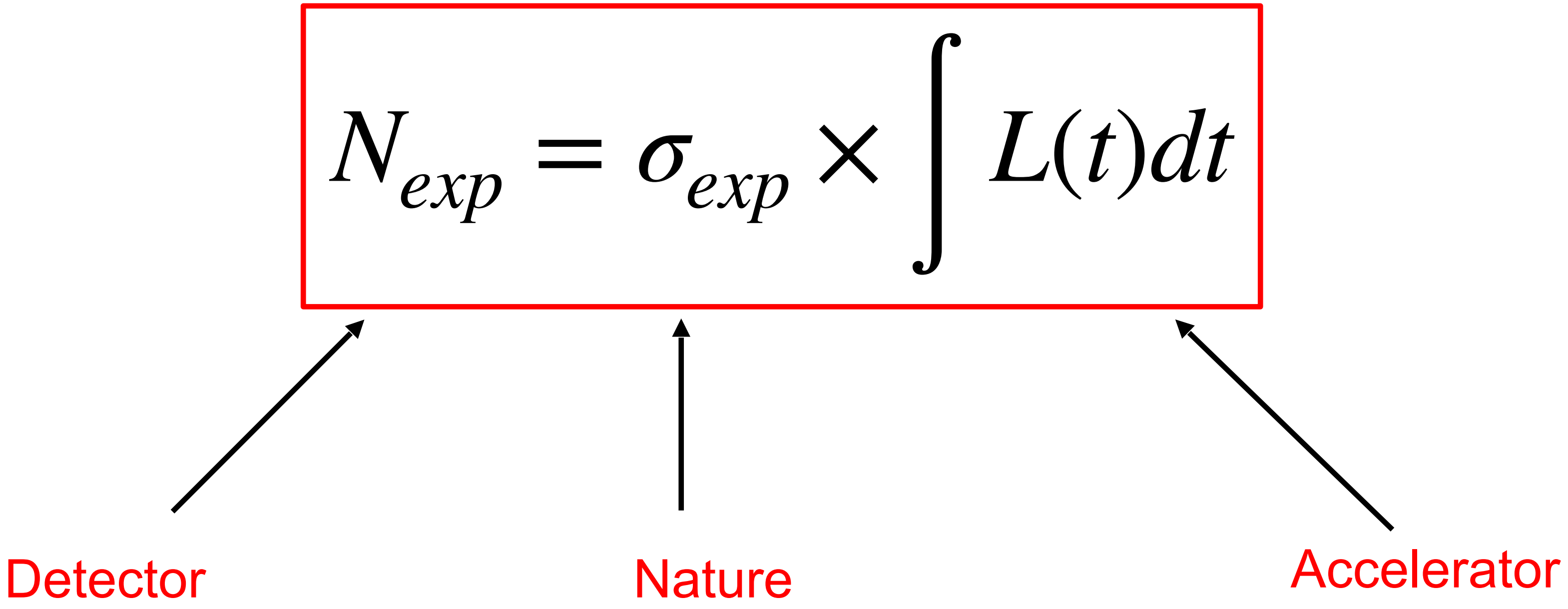

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Accelerator

Luminosity for the **SIMPLEST case**

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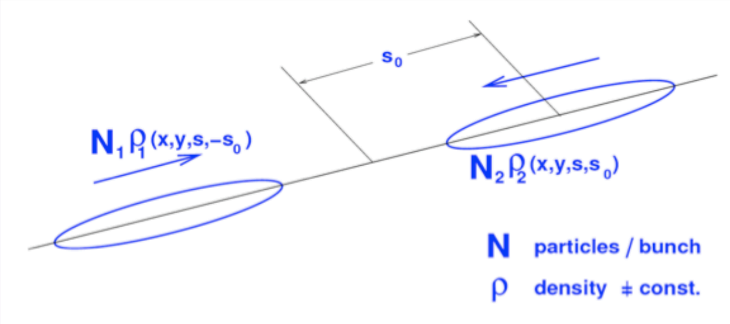

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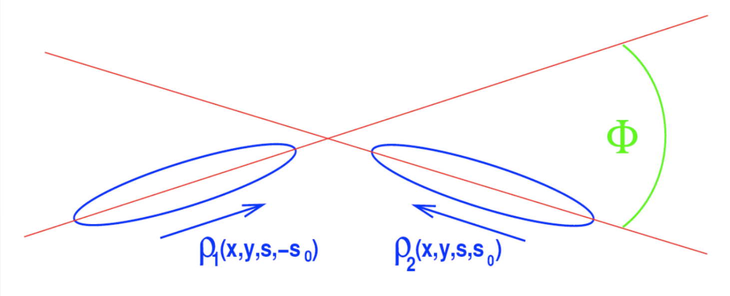

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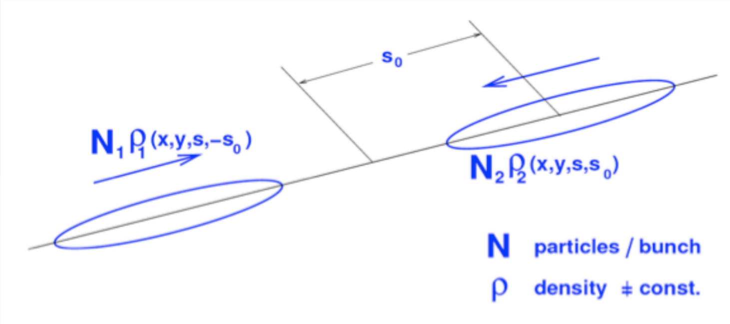

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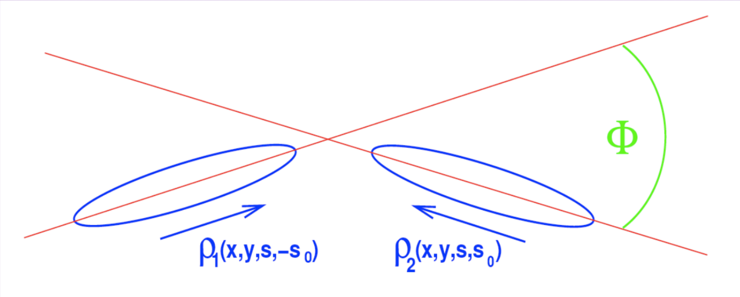

◆ Luminosity in the absence of crossing angle (and transverse beam offset and hourglass effect => See later)

$$\boxed{M f_{rev} = f_{coll}}$$

Number of bunches

$$\boxed{L = M\, N_1\, N_2\, f_{rev}\; 2 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho_1(x\,,y\,,s\,,-s_0)\,\rho_2(x\,,y\,,s\,,s_0)\,dx\,dy\,ds\,ds_0}$$

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the simplest formula for the **peak luminosity** is obtained

$$\boxed{L = \frac{M \, N_1 \, N_2 \, f_{rev}}{4 \, \pi \, \sigma_x \, \sigma_y}}$$

Let's call it L_0

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using $\varepsilon_n = \beta\,\gamma\,\varepsilon = \beta\,\gamma\,\frac{\sigma^2}{\beta^*}$

Normalized transverse beam emittance

β-function at the collision point

Luminosity for the GENERAL case

- In the general case: $\boxed{L = L_0 \times F \text{ with } 0 \leq F \leq 1}$

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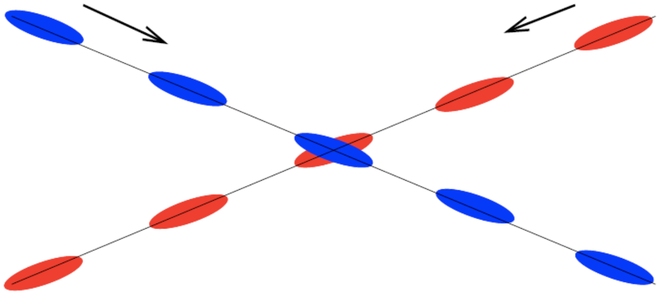

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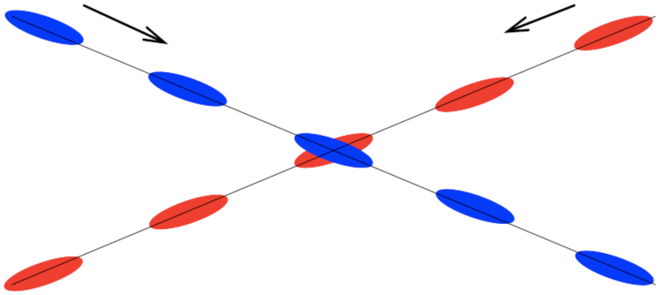
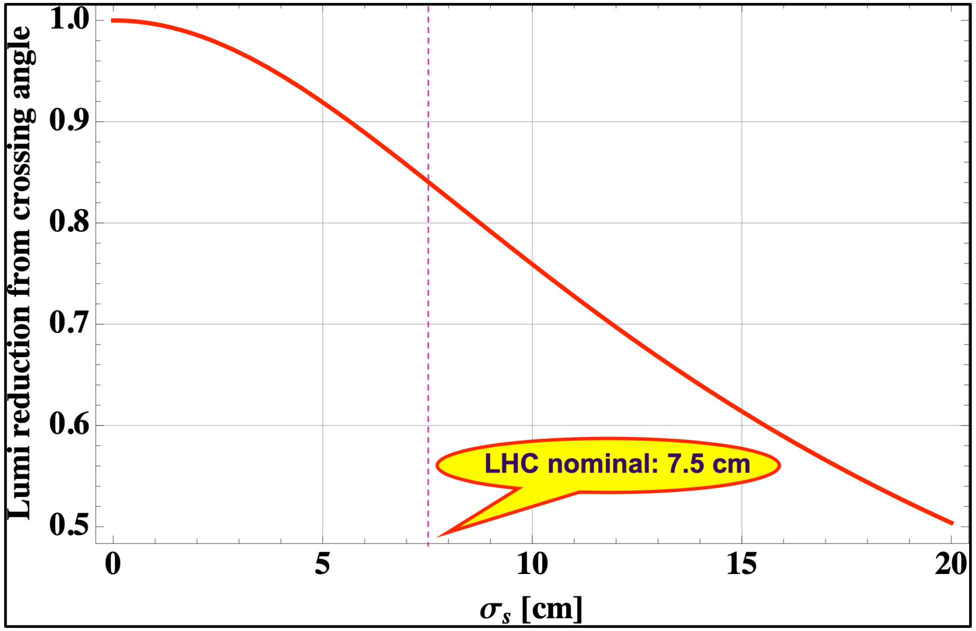


$$F_{CA} = \frac{1}{\sqrt{1 + \left(\frac{\sigma_s}{\sigma_x} \tan \frac{\Phi}{2} \right)^2}}$$

tan Φ / 2 ~ Φ / 2



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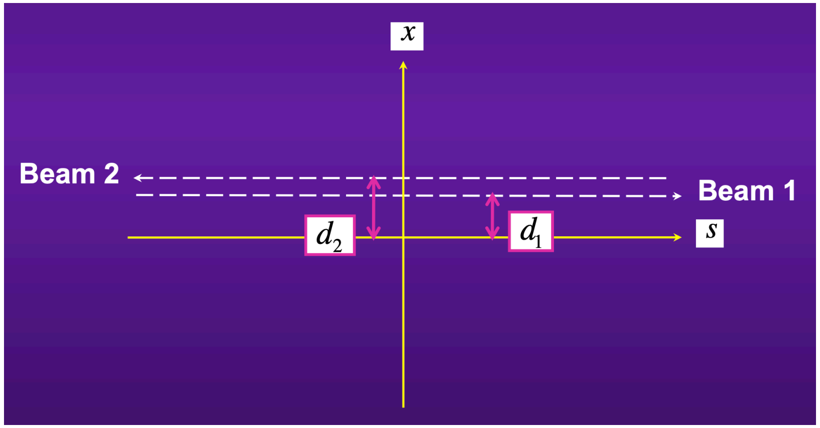

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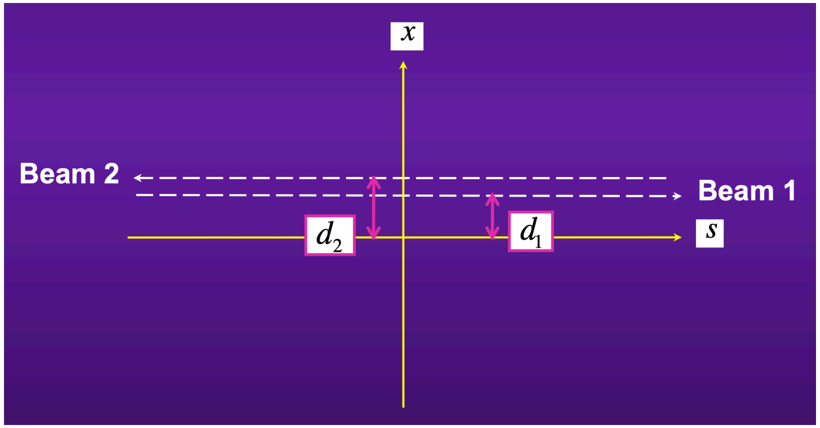
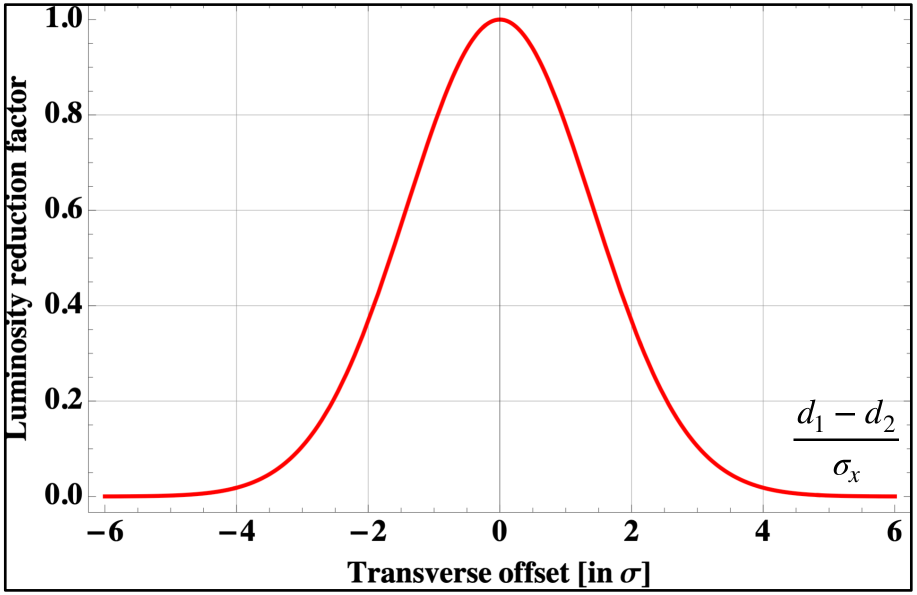


$$\boxed{F_{TO} = e^{-\left(\frac{d_1 - d_2}{2\sigma_x}\right)^2}}$$



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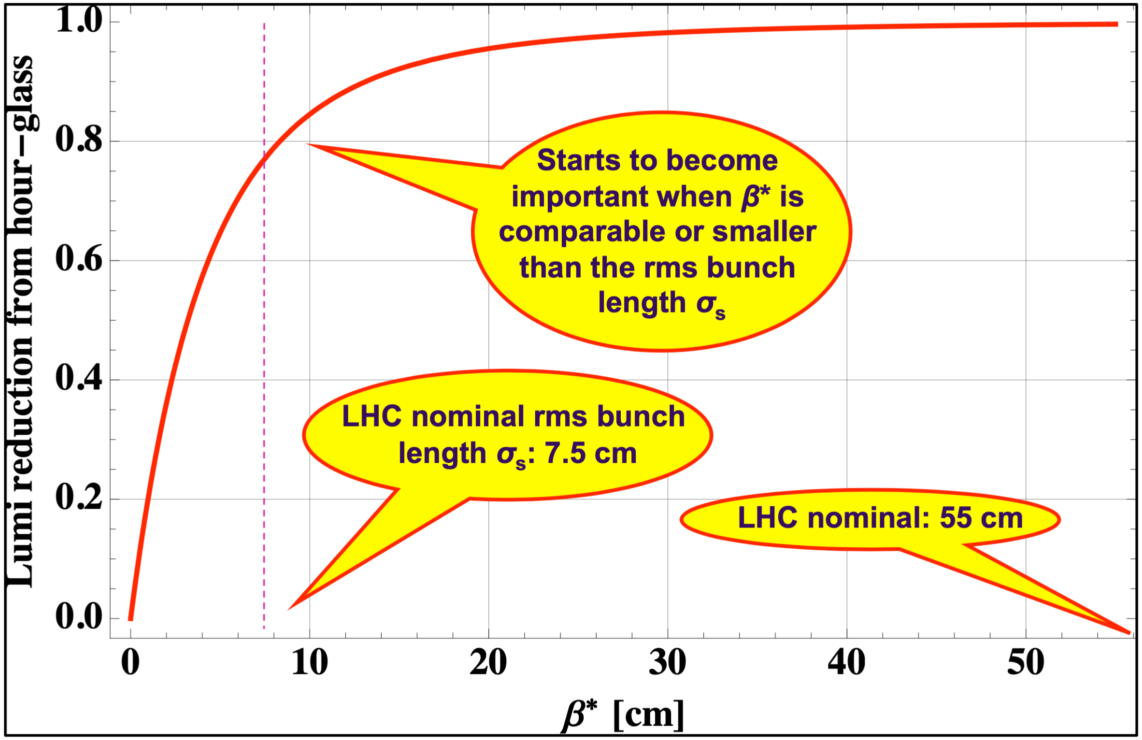

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- Thus if a detector has accumulated 100 fb^{-1} of integrated luminosity, one expects to find 100 events per femtobarn of cross-section within these data

Pile-up

- Pile-Up (PU) = Number of events / crossing for a given luminosity

$$\boxed{\mathrm{PU} = \frac{L\sigma_{exp}}{M f_{rev}}}$$

  - This is a limit coming from the experiments' detectors => Better to have larger number of bunches (for the same beam intensity)

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PU = 19 from
LHC Design Report
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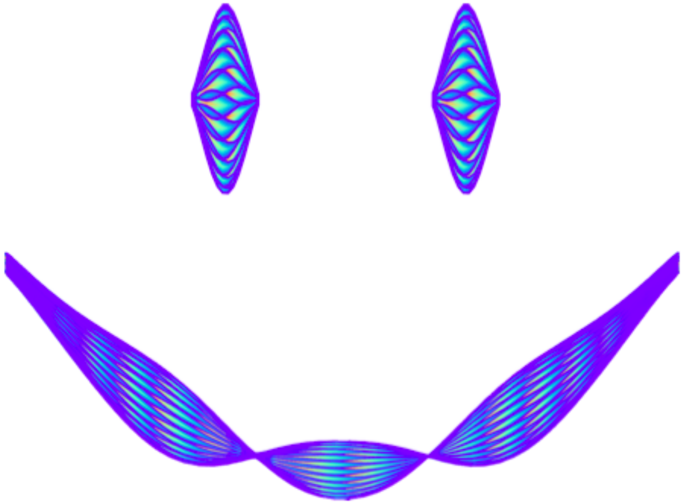
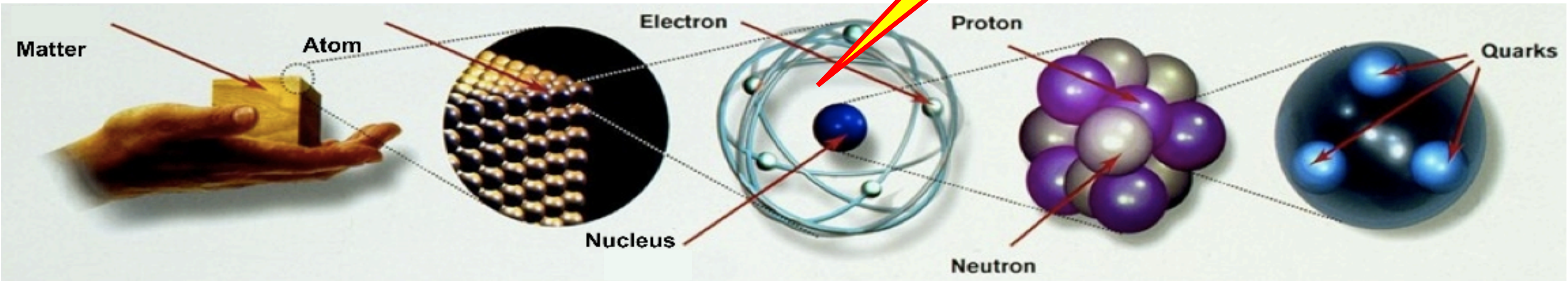
Many thanks for your attention and welcome to the fascinating world of particle accelerators!



APPENDIX

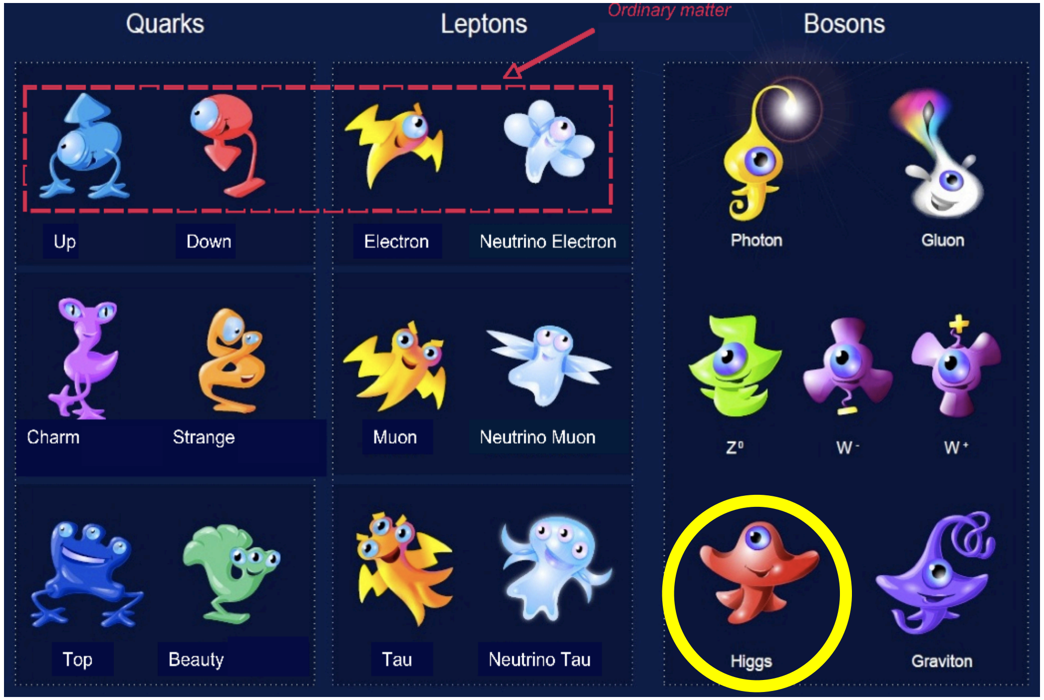
Standard Model

- After a century of discoveries and measurements, the particle physicists have developed the Standard Model, explaining almost all the components of matter and the forces between them



> 99.99% is empty space…

Standard Model



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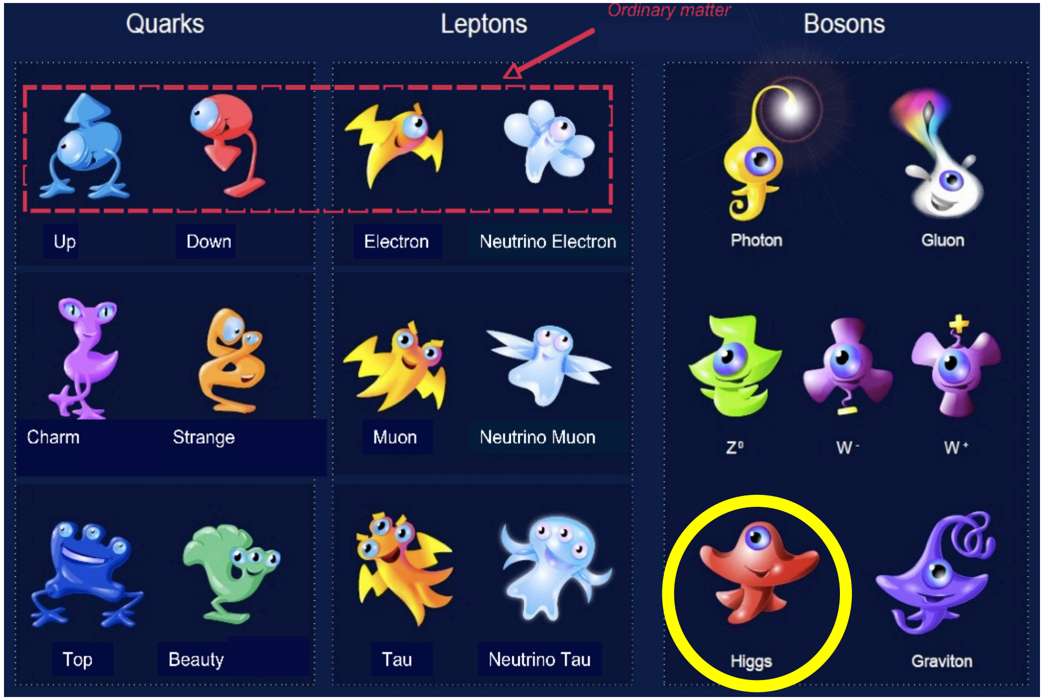

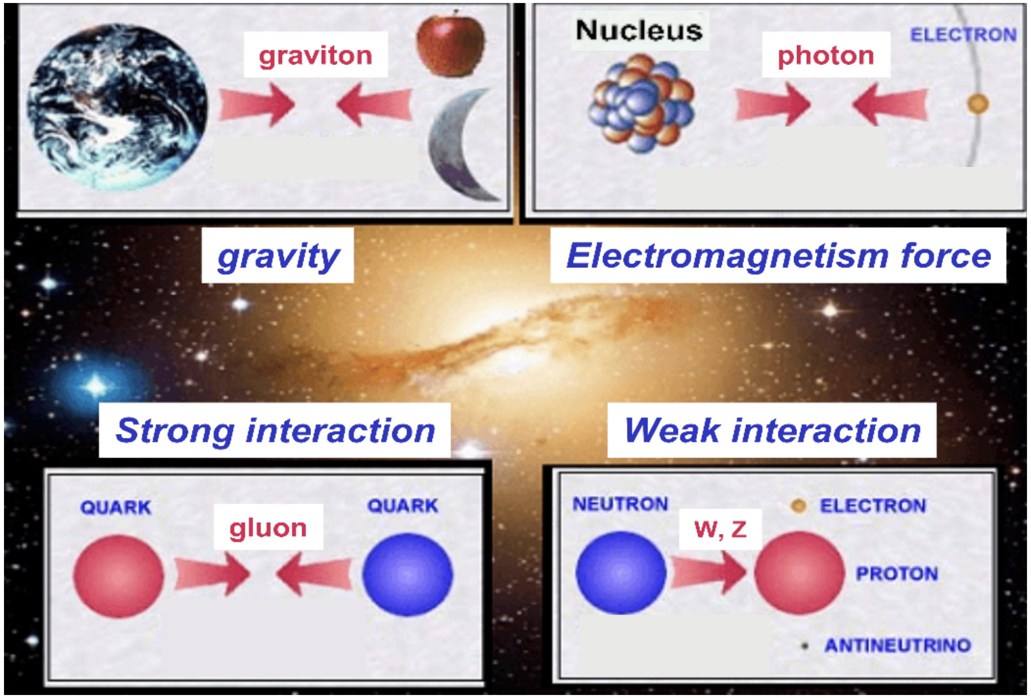

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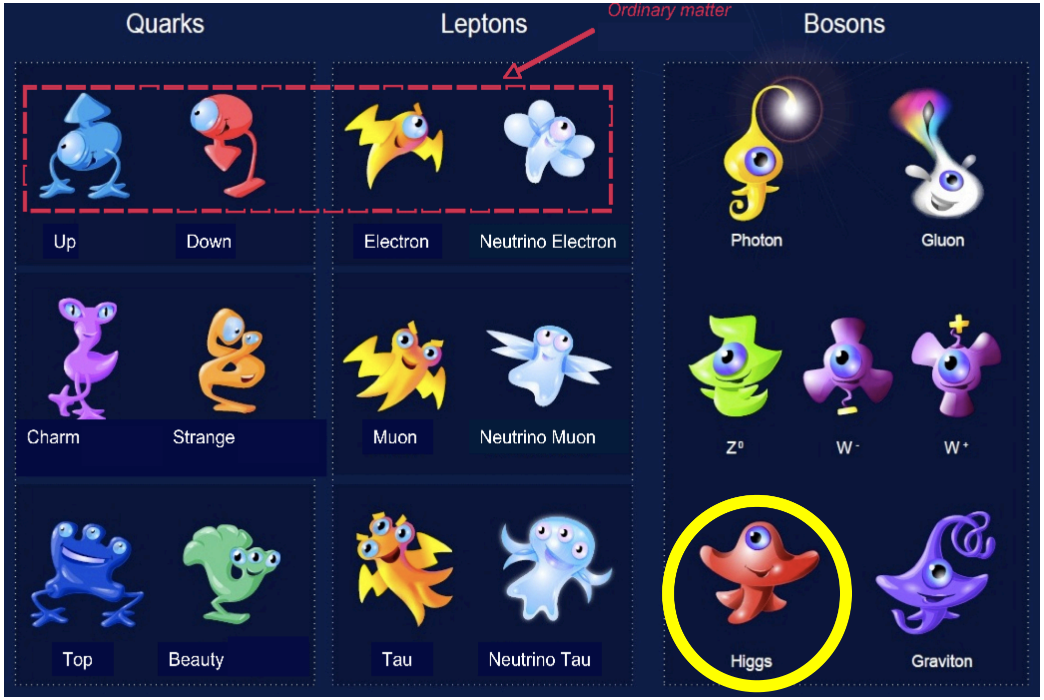

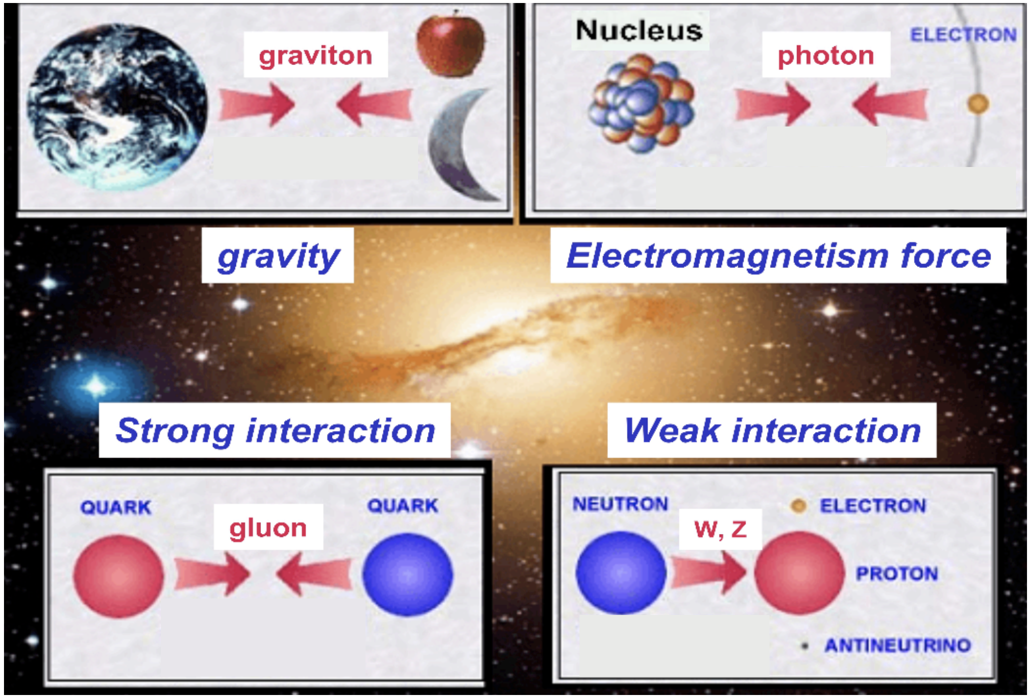

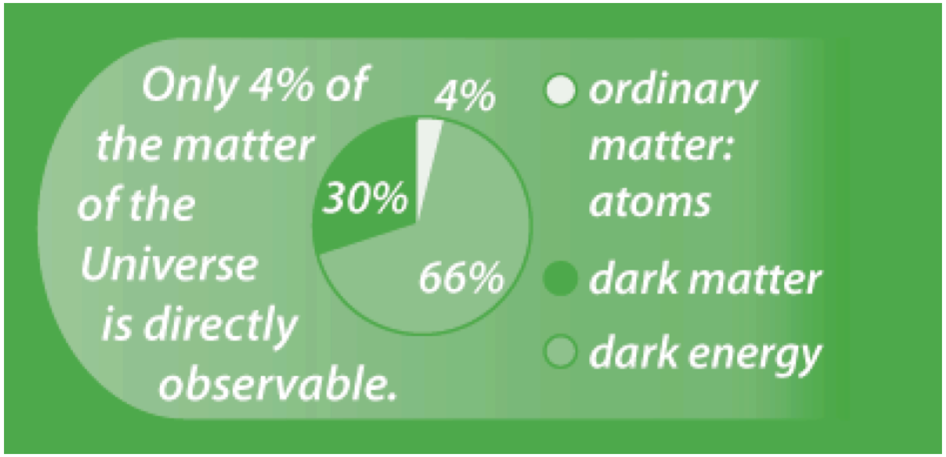

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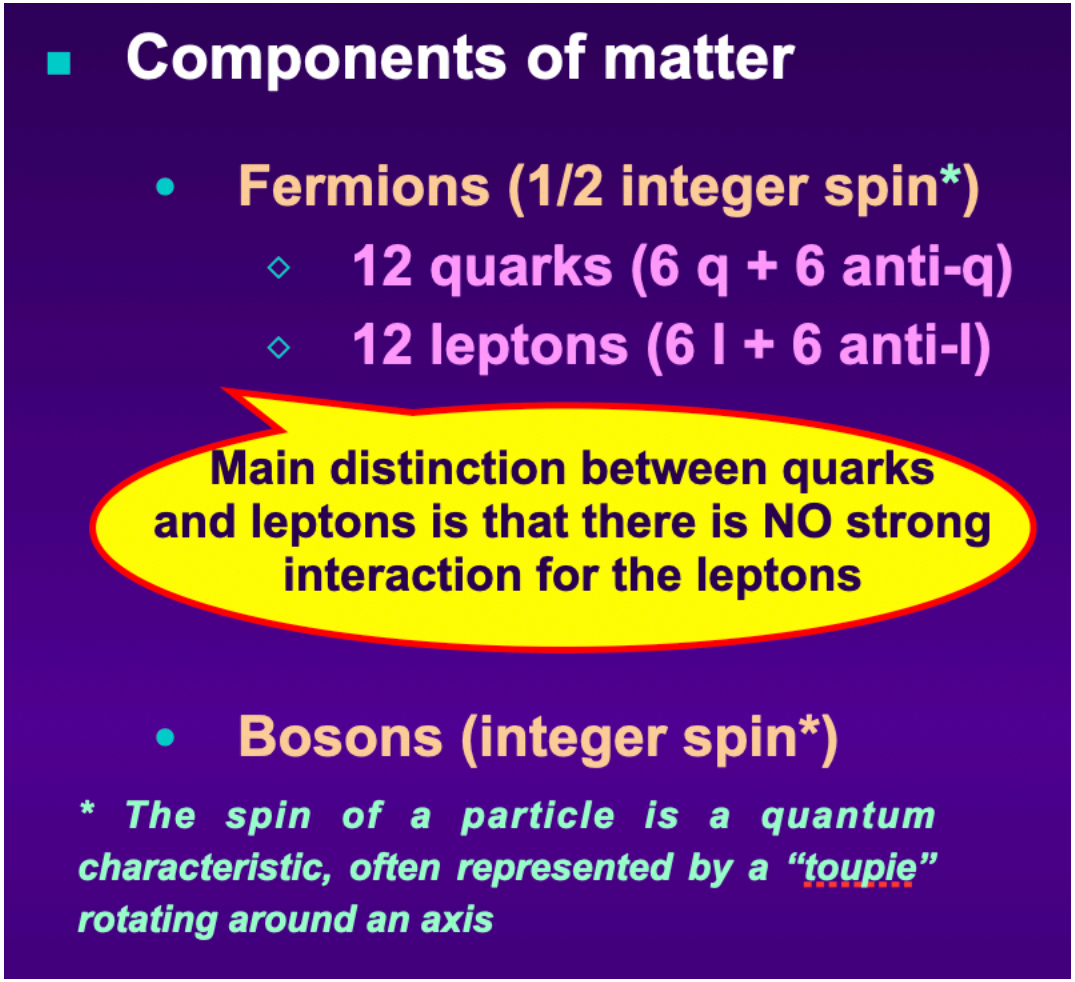


- **Components of matter**
  - Fermions (1/2 integer spin*)
    - 12 quarks (6 q + 6 anti-q)
    - 12 leptons (6 l + 6 anti-l)

Main distinction between quarks and leptons is that there is NO strong interaction for the leptons

  - Bosons (integer spin*)

** The spin of a particle is a quantum characteristic, often represented by a "toupie" rotating around an axis*

- **By assembling quarks we create HADRONS (= Heavy in Greek) => 2 families**
  - **BARYONS (odd number of quarks => ½ integer spin)**
    Ex: p^+, n
  - **MESONS (even number of quarks => Integer spin)**
    Ex: pion

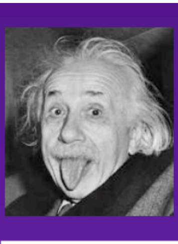
Leptons => Light in Greek

Energy

- **The 2 roles of energy**

  1) Producing new particles (see before)

  2) Resolving the inner structure of matter



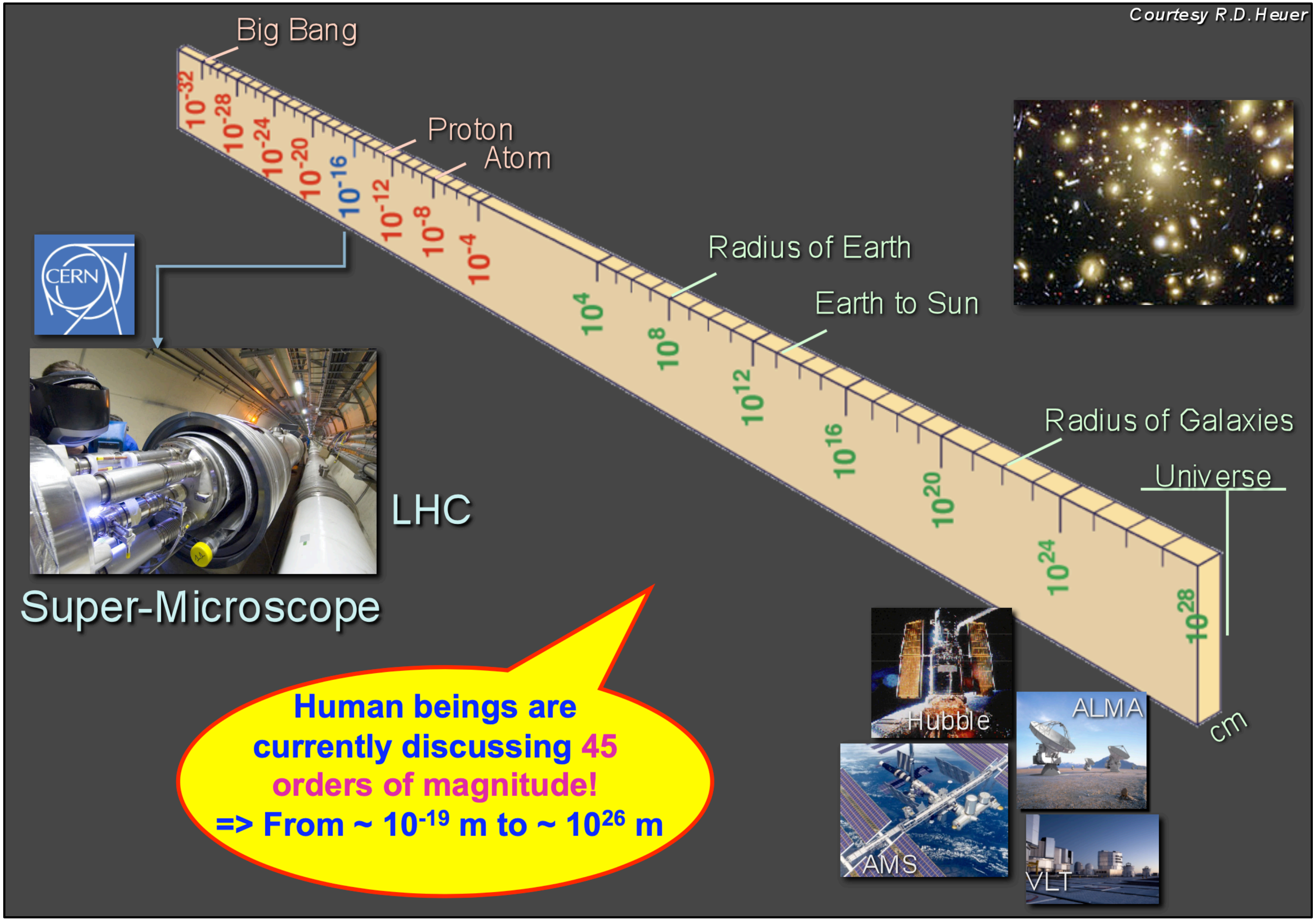
$$E = \frac{h\,c}{\lambda}$$

Object	Size [m]	Energy needed [GeV]
Atom	10^{-10}	$\sim 10^{-5}$
Nucleus	10^{-14}	~ 0.1
Nucleon	10^{-15}	~ 1
Quark	$\sim 10^{-19}$	$\sim 10^{4}$

Wavelength => Should be < object to be resolved

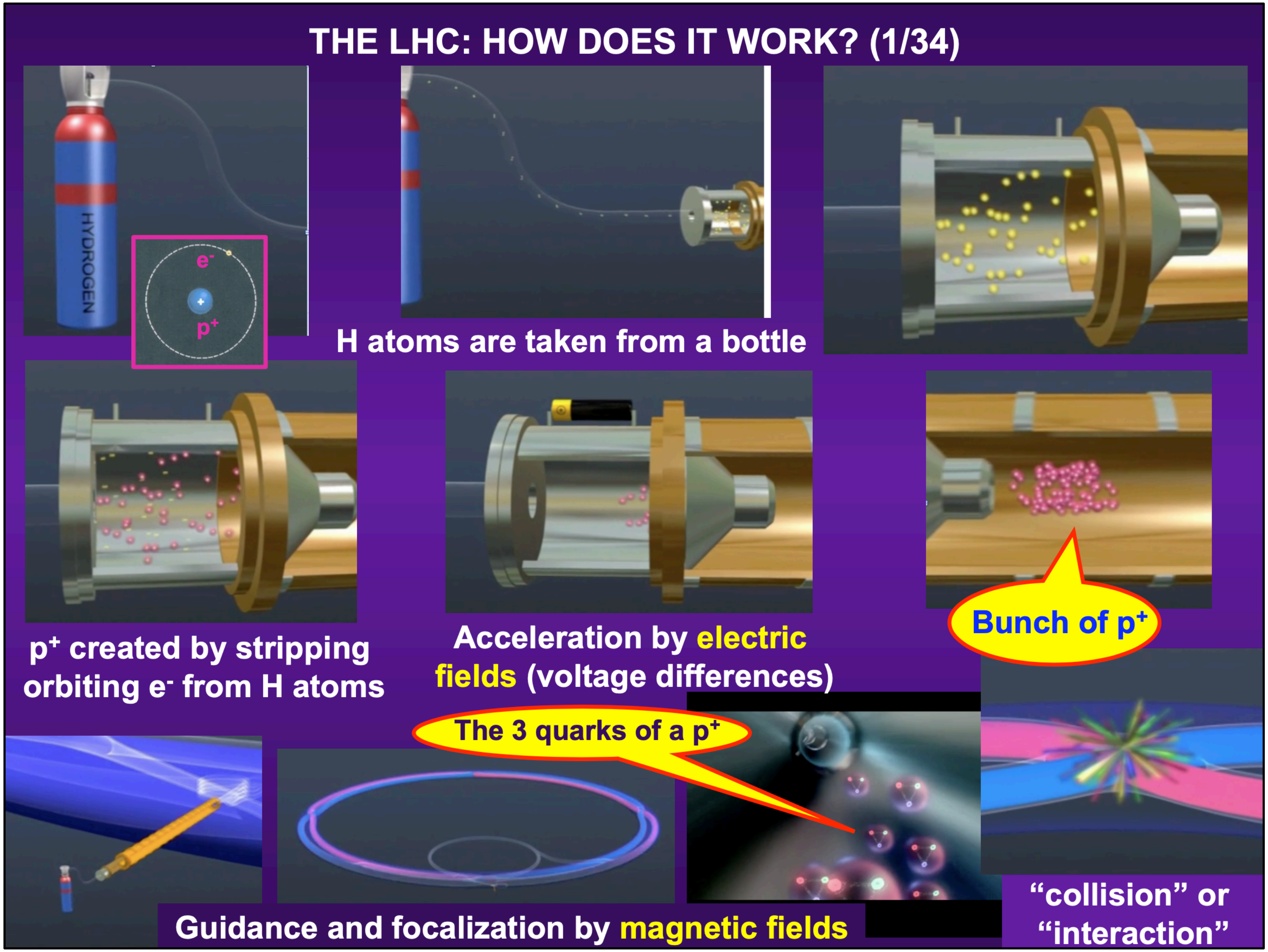
Planck constant $\approx 6.62\ 10^{-34}$ Js

Energy



Human beings are currently discussing 45 orders of magnitude!
=> From ~ 10^{-19} m to ~ 10^{26} m

The CERN LHC



THE LHC: HOW DOES IT WORK? (1/34)

H atoms are taken from a bottle

p^+ created by stripping orbiting e^- from H atoms

Acceleration by **electric fields** (voltage differences)

Bunch of p^+

The 3 quarks of a p^+

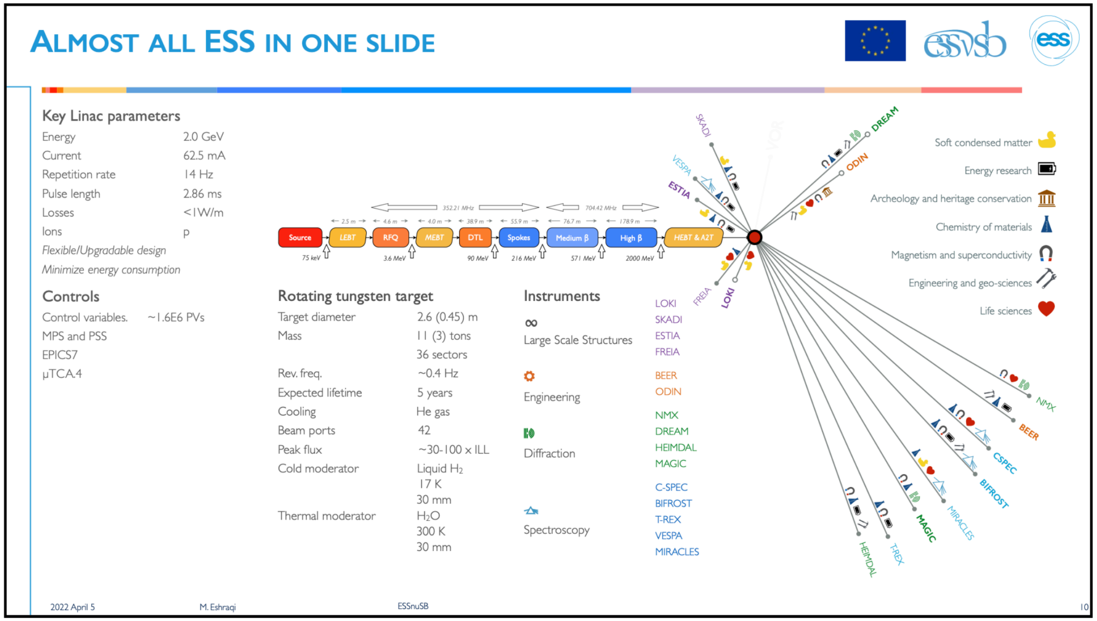
Guidance and focalization by **magnetic fields**

“collision” or “interaction”

Beam power for fixed-target experiments

- The European Spallation Source (ESS) in Lund (Sweden) is a multi-disciplinary research facility based on the world's brightest pulsed neutron source driven by **the most powerful proton linac (5 MW)**
- ESS will start the scientific user programme in 2025

$$P_{beam} = \frac{Energy}{e} \times Current \times RepRate \times PulseLength = 5\ \text{MW.}$$



ALMOST ALL ESS IN ONE SLIDE

Key Linac parameters

Energy	2.0 GeV
Current	62.5 mA
Repetition rate	14 Hz
Pulse length	2.86 ms
Losses	<1W/m
Ions	p

Flexible/Upgradable design
Minimize energy consumption

Controls

Control variables.	~1.6E6 PVs
MPS and PSS	
EPICS7	
µTCA.4	

Rotating tungsten target

Target diameter	2.6 (0.45) m
Mass	11 (3) tons 36 sectors
Rev. freq.	~0.4 Hz
Expected lifetime	5 years
Cooling	He gas
Beam ports	42
Peak flux	~30-100 x ILL
Cold moderator	Liquid H_2 17 K 30 mm
Thermal moderator	H_2O 300 K 30 mm

Instruments

∞ Large Scale Structures	LOKI SKADI ESTIA FREIA
Engineering	BEER ODIN
Diffraction	NMX DREAM HEIMDAL MAGIC
Spectroscopy	C-SPEC BIFROST T-REX VESPA MIRACLES

Soft condensed matter
Energy research
Archeology and heritage conservation
Chemistry of materials
Magnetism and superconductivity
Engineering and geo-sciences
Life sciences

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