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LECTURES 3 & 4

RTU April 2022

Linear Accelerators

Module 1

Why linear accelerators
Basic linac structure
Acceleration in periodic structures

Definitions

Linear accelerator: a device where charged particles acquire energy moving on a linear path
RF linear accelerator: acceleration is provided by time-varying electric fields (i.e. excludes electrostatic accelerators)

A few definitions:

- CW (Continuous wave) linacs when the beam bunches come continuously out of the linac;
- Pulsed linac when the beam is produced in pulses: τ pulse length, f_r repetition frequency, beam duty cycle $\tau \times f_r$ (%)
- Main parameters:
  - *E* kinetic energy of the particles coming out of the linac [MeV]
  - *I* average current during the beam pulse [mA] (different from *average current* and from *bunch current* !)
  - *P* beam power = electrical power transferred to the beam during acceleration
  - P [W] = $V_{tot} \times I = E$ [eV] $\times I$ [A] $\times$ duty cycle

Variety of linacs

The first and the smallest: Rolf Widerøe thesis (1923)

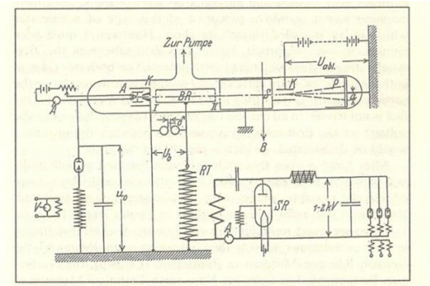
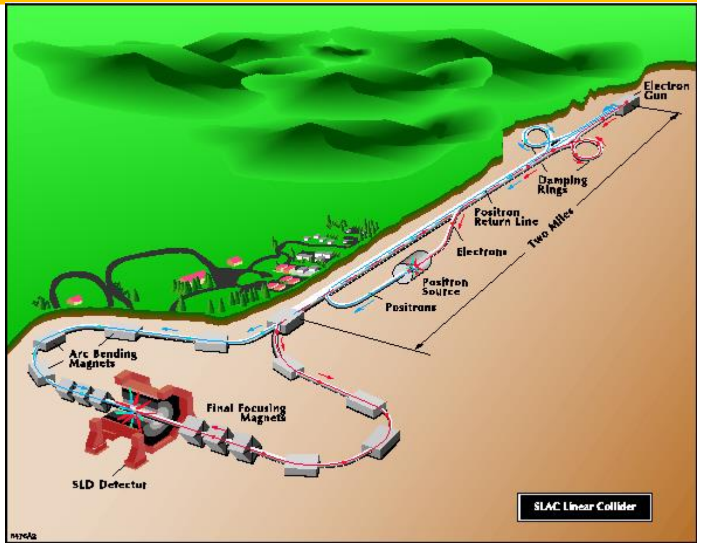
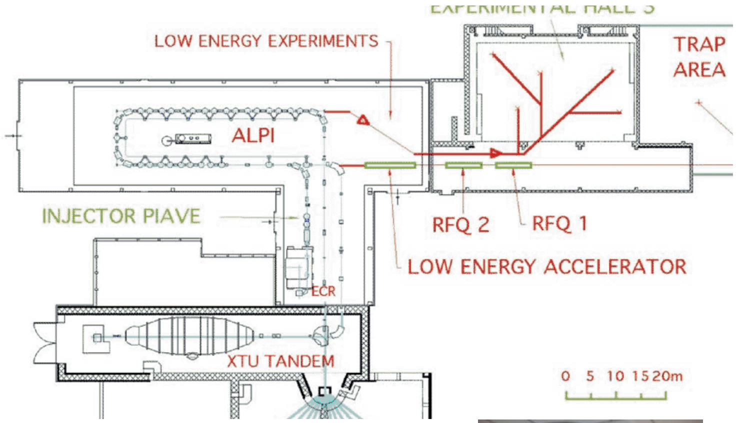


Fig. 3.6: Acceleration tube and switching circuits [Wi28].

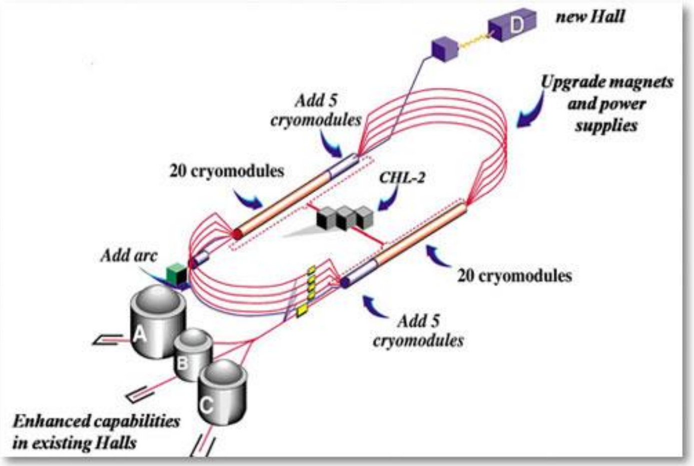
The largest: Stanford Linear Collider (2 miles = 3.2 km) (but CLIC design goes to 48.3 km !)



One of the less linear: ALPI at LNL (Italy)



A limit case, multi-pass linacs: CEBA at JLAB



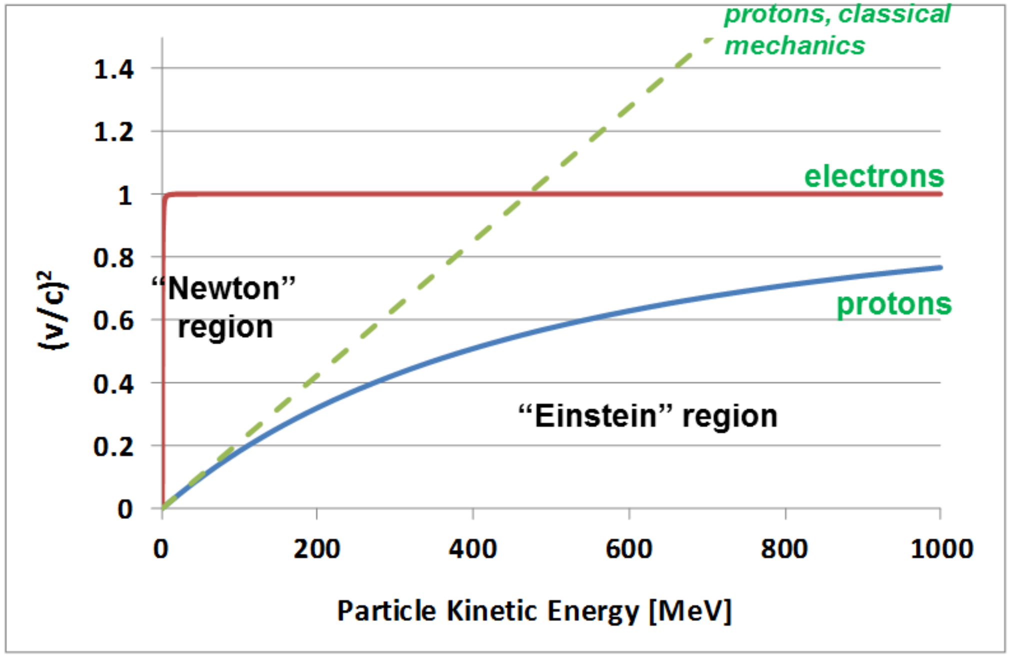
The most common: medical electron linac (more than 7'000 in operation around the world!)

Why Linear Accelerators

	LINACS		SYNCHROTRONS
	Low Energy	**High Energy**	**High Energy**
Protons, Ions	Injectors to synchrotrons, stand alone applications *synchronous with the RF fields in the range where velocity increases with energy.* *Protons : β = v/c =0.51 at 150 MeV, 0.95 at 2 GeV.*	Injection at higher intensity Production of secondary beams (n, ν, RIB, …) *higher cost/MeV than synchrotron but:* *- high average beam current (high repetition rate, less resonances, easier beam loss).* *- higher linac energy allows for higher intensity in the synchrotron.*	*very efficient when velocity is ~constant, (multiple crossings of the RF gaps).* *limited mean current (limited repetition frequency, instabilities)*
Electrons	Injector to synchrotrons, stand-alone applications *simple and compact*	Linear colliders *do not lose energy because of synchrotron radiation – only option for high energy!*	Light sources, factories *can accumulate high beam intensities*

« traditional » linac range ⇨ « new » linac range

Proton and Electron Velocity



$\beta^2=(v/c)^2$ as function of kinetic energy T for protons and electrons.

Classic (Newton) relation:

$$T = m_0 \frac{v^2}{2}, \quad \frac{v^2}{c^2} = \frac{2T}{m_0 c^2}$$

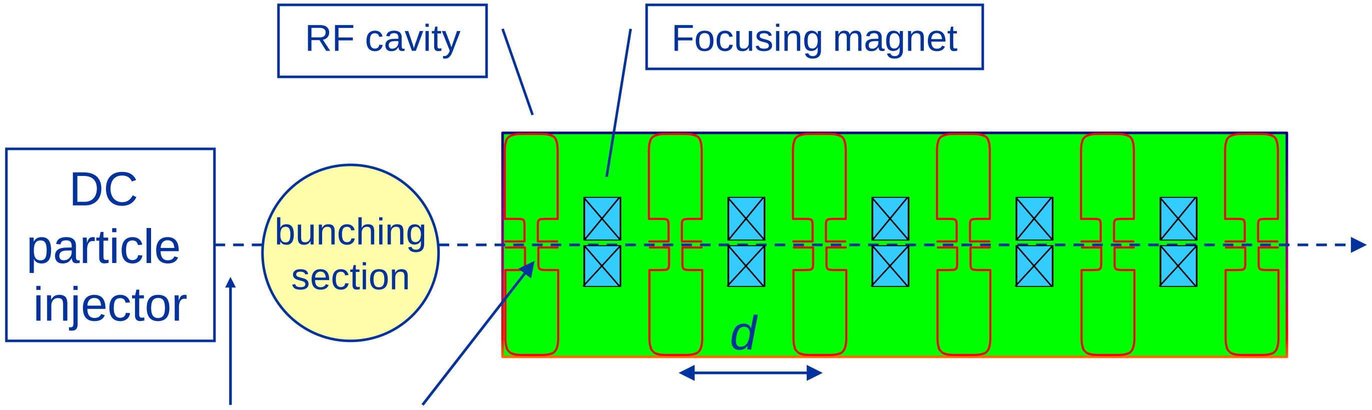
Relativistic (Einstein) relation:

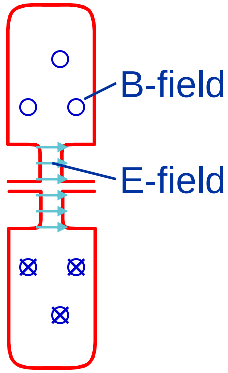
$$\frac{v^2}{c^2} = 1 - \frac{1}{\sqrt{1 + T/m_0 c^2}}$$

→ Protons (rest energy 938.3 MeV): follow "Newton" mechanics up to some tens of MeV ($\Delta v/v < 1\%$ for $W < 15$ MeV) then slowly become relativistic ("Einstein"). From the GeV range velocity is nearly constant (v~0.95c at 2 GeV) → linacs can cope with the increasing particle velocity, synchrotrons cover the range where v nearly constant.

→ Electrons (rest energy 511 keV, 1/1836 of protons): relativistic from the keV range (v~0.1c at 2.5 keV) then increasing velocity up to the MeV range (v~0.95c at 1.1 MeV) → v~c after few meters of acceleration in a linac (typical gradient 10 MeV/m).

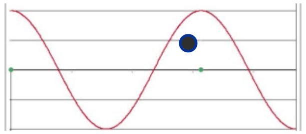
Basic linear accelerator structure





Protons: energy ~100 keV
β= v/c ~ 0.015

Accelerating gap: $E = E_0 \cos(\omega t + \phi)$
en. gain $\Delta W = eV_0 T\cos\phi$



Acceleration $\rightarrow$ the beam has to pass in each cavity on a phase ϕ near the crest of the wave

1. The beam must to be bunched at frequency ω
2. distance between cavities and phase of each cavity must be correlated

Phase change from cavity *i* to *i*+1 is

$$\Delta\phi = \omega\tau = \omega\frac{d}{\beta c} = 2\pi\frac{d}{\beta\lambda}$$

For the beam to be synchronous with the RF wave ("ride on the crest") phase must be related to distance by the relation:

$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

… and on top of acceleration, we need to introduce in our "linac" some focusing elements

… and on top of that, we will couple a number of gaps in an "accelerating structure"

Synchronism condition for the Wideröe

Please note that for the particular case of the Wideröe seen in the previous lectures, $\Delta\phi = \pi$

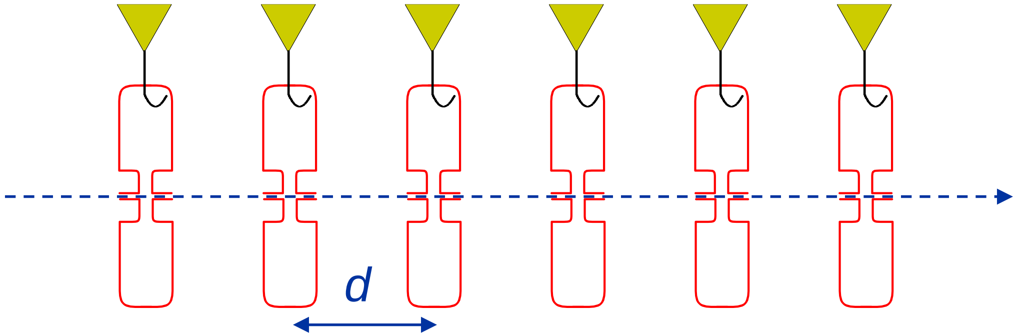
The general relation for synchronicity $$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

becomes $\pi / d = 2\pi / \beta\lambda$ or $d = \beta\lambda / 2$

Accelerating structure architecture

When β increases during acceleration, either the phase difference between cavities $\Delta\phi$ must decrease or their distance *d* must increase.

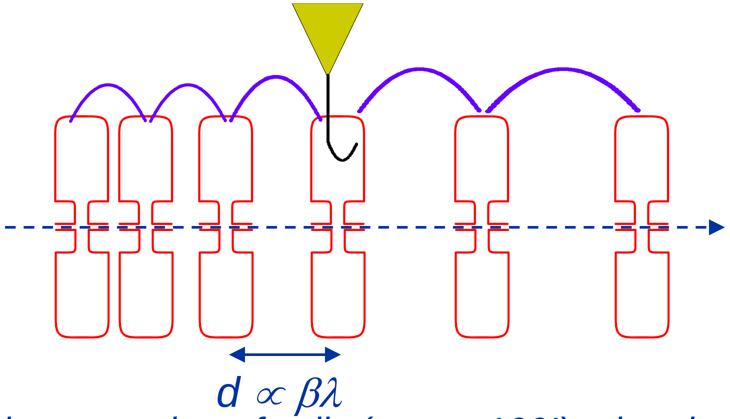
d = const.
ϕ variable



$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

Individual cavities – *distance between cavities constant, each cavity fed by an individual RF source, phase of each cavity adjusted to keep synchronism, used for linacs required to operate with different ions or at different energies. Flexible but expensive!*

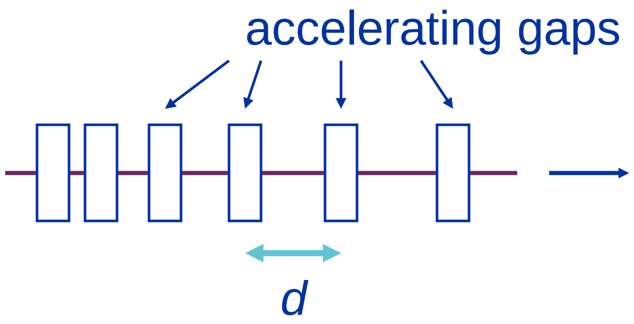
ϕ = const.
d variable



Better, but 2 problems:
1. *create a "coupling";*
2. *create a mechanical and RF structure with increasing cell length.*

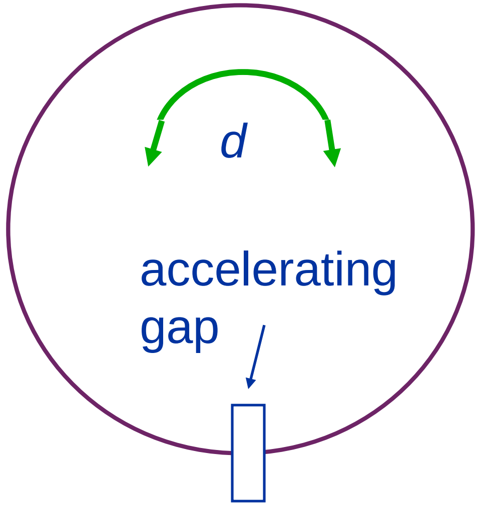
Coupled cell cavities - *a single RF source feeds a large number of cells (up to ~100!) - the phase between adjacent cells is defined by the coupling, distance between cells is adapted to keep synchronism . Once the geometry is defined, it can accelerate only one type of ion for a given energy range. Effective but not flexible.*

Linear and circular accelerators



$d = \beta\lambda/2$ = variable
f = constant

$$d = \frac{\beta c}{2f}\frac{\Delta\varphi}{\pi} = \frac{\beta\lambda}{2}\frac{\Delta\varphi}{\pi}$$



Note that only linacs are real «accelerators», synchrotrons are «mass increaser»!

$d = 2\pi R$ = constant
$f = \beta c/2d$ = variable

Linear accelerator:

Particles accelerated by a sequence of gaps (all at the same RF phase).

Distance between gaps increases proportionally to the particle velocity, to keep synchronicity.

Used in the range where β increases. "Newton" machine

Circular accelerator:

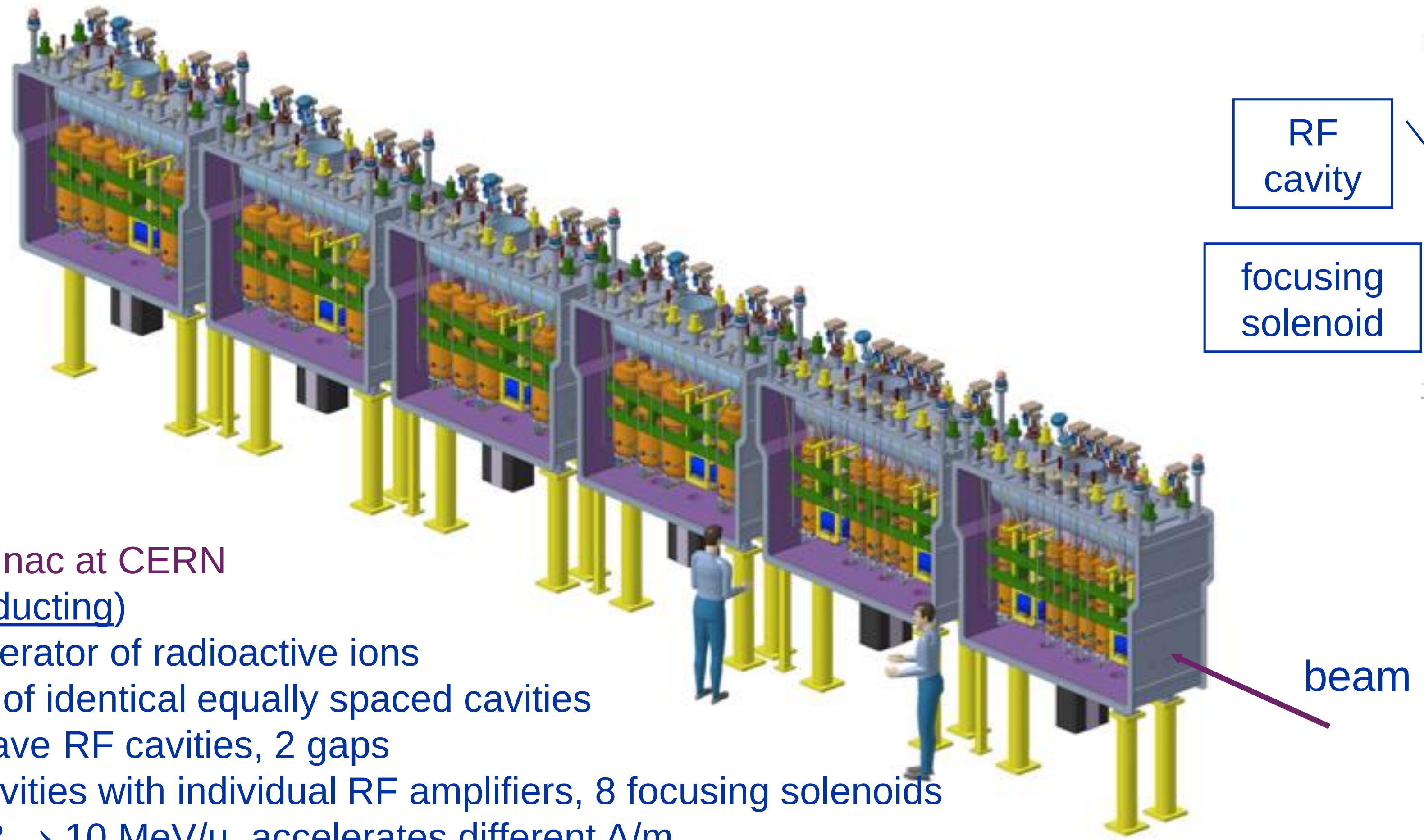
Particles accelerated by one (or more) gaps at given positions in the ring.

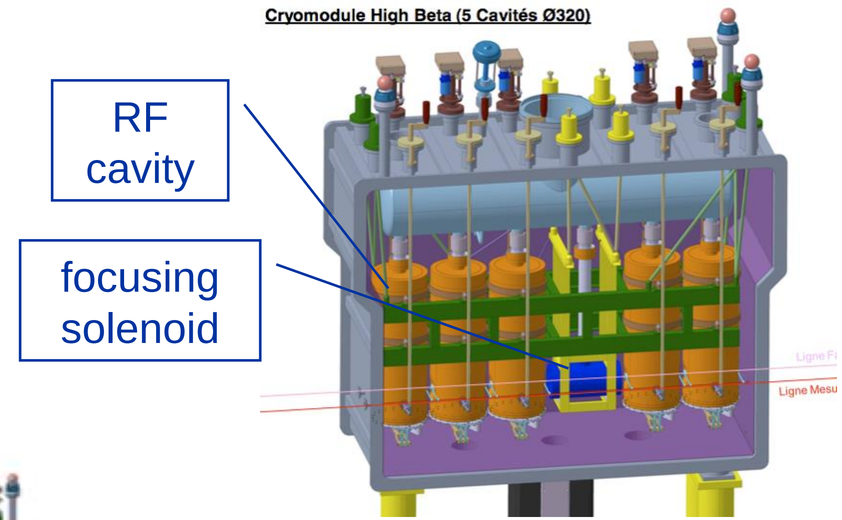
Distance between gaps is fixed. Synchronicity only for β~const, or varying (in a limited range!) the RF frequency.

Used in the range where β is nearly constant. "Einstein" machine

Case 1: a single-cavities linac

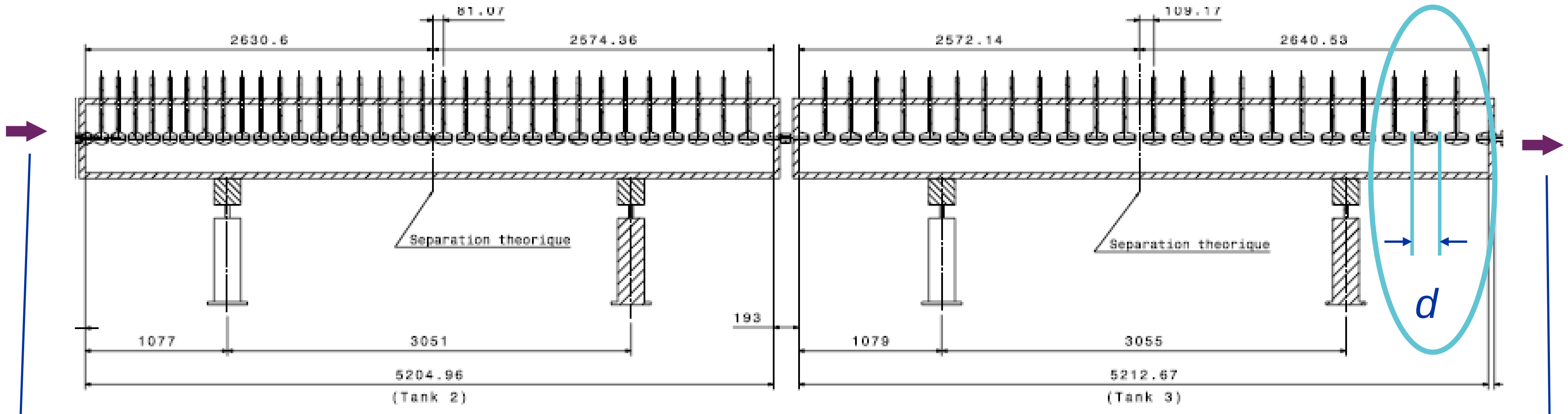
The goal is flexibility: acceleration of different ions (e/m) at different energies → need to change phase relation for each ion-energy





REX-HIE linac at CERN
(superconducting)
Post-accelerator of radioactive ions
2 sections of identical equally spaced cavities
Quarter-wave RF cavities, 2 gaps
12 + 20 cavities with individual RF amplifiers, 8 focusing solenoids
Energy 1.2 → 10 MeV/u, accelerates different A/m

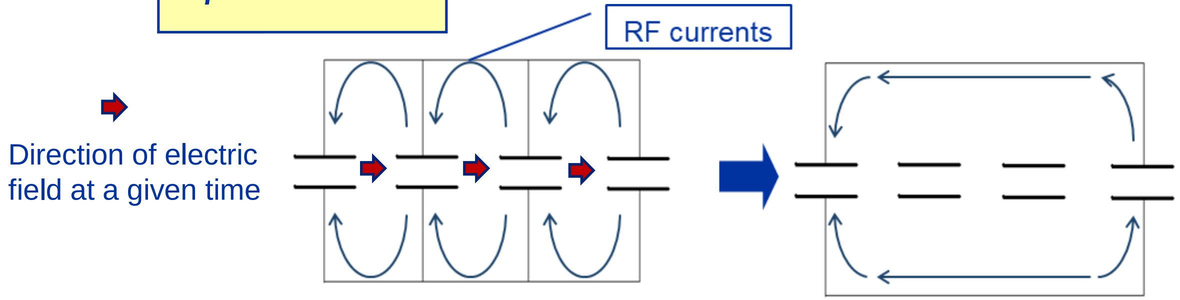
Case 2 : a Drift Tube Linac

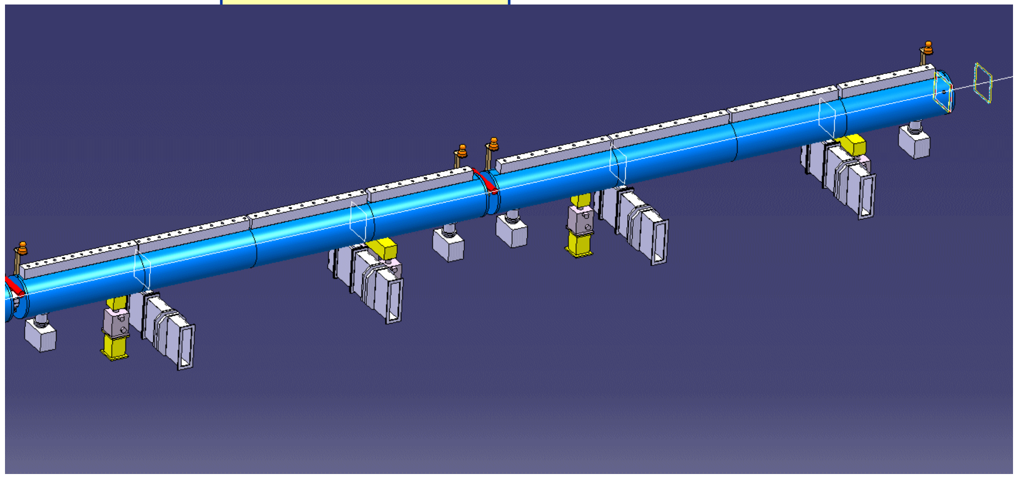


Tank 2 and 3 of the new Linac4 at CERN:
57 coupled accelerating gaps
Frequency 352.2 MHz, λ = 85 cm
Cell length ($d=\beta\lambda$) from 12.3 cm to 26.4 cm (factor 2 !).

10 MeV,
β = 0.145

50 MeV,
β = 0.31





To achieve the maximum possible coupling, in a DTL the walls between cells are absent – possible thanks to the particular DTL operating mode with 2π phase difference between cells

Intermediate cases

But:

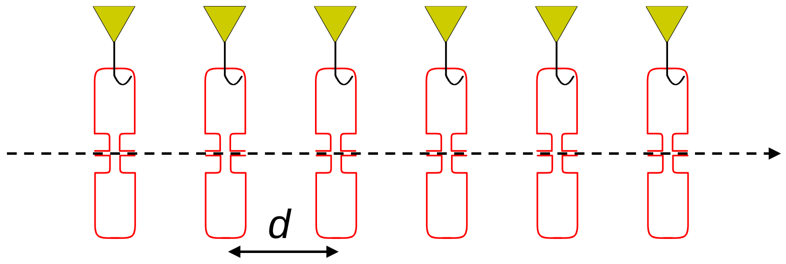
Between the 2 “extremes” there are many “intermediate” cases, because:

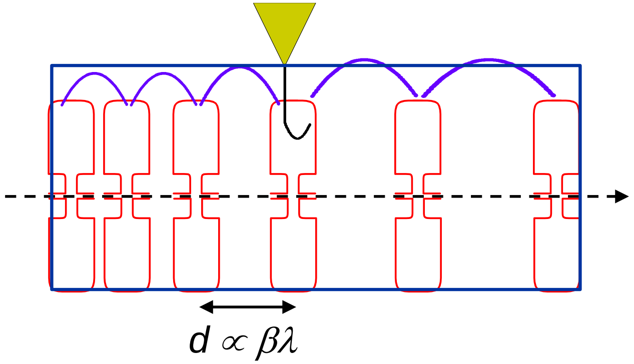
a. Single-gap cavities are expensive (both cavity and RF source!).

b. Structures with each cell matched to the beta profile are mechanically complicated and expensive.

→ as soon as the increase of beta with energy becomes small ($\Delta\beta/\Delta W$) we can accept a small error and:

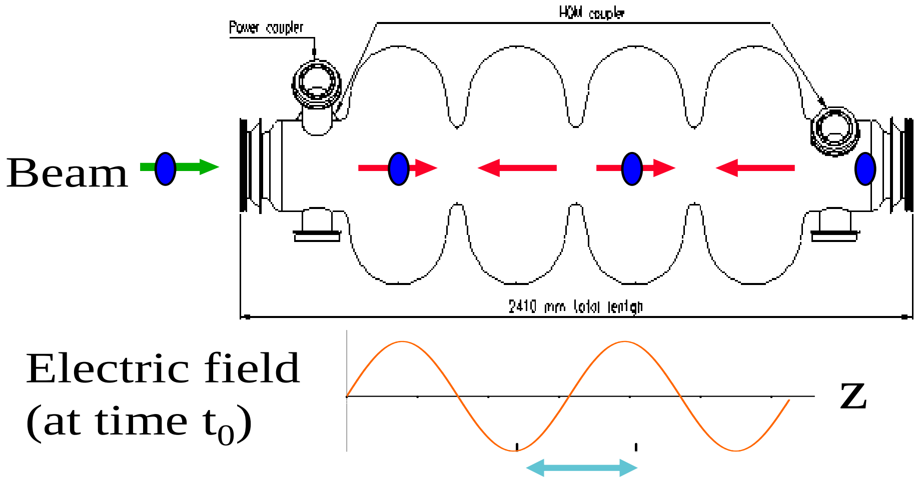
1. Use multi-gap cavities with constant distance between gaps.
2. Use series of identical cavities (standardised design and construction).





Phase slippage (asynchronicity) in a multicell cavity

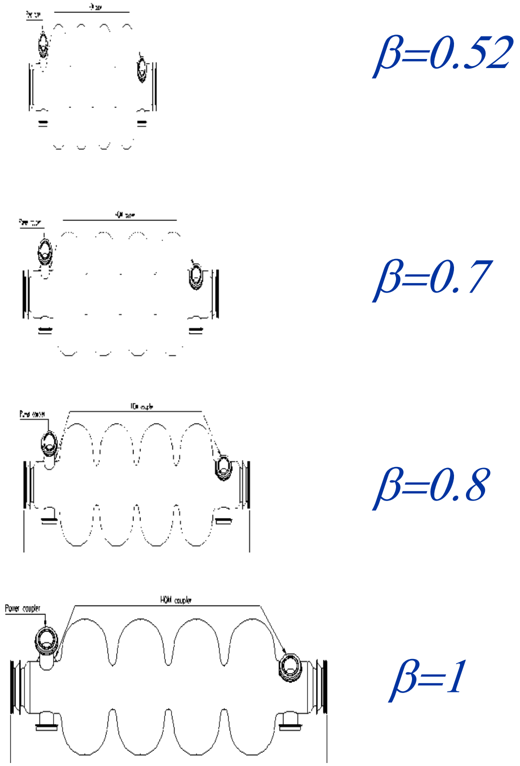
Multi-cell superconducting cavities can be only produced in batches of cavities with cells of identical length (technological reason, cannot afford the complications of increasing cell length).
The cell length is calculated with the usual formula for the beta at the centre of the cavity, but the phase of the beam (with respect to the crest of the wave) will not be "correct" in all other cells.

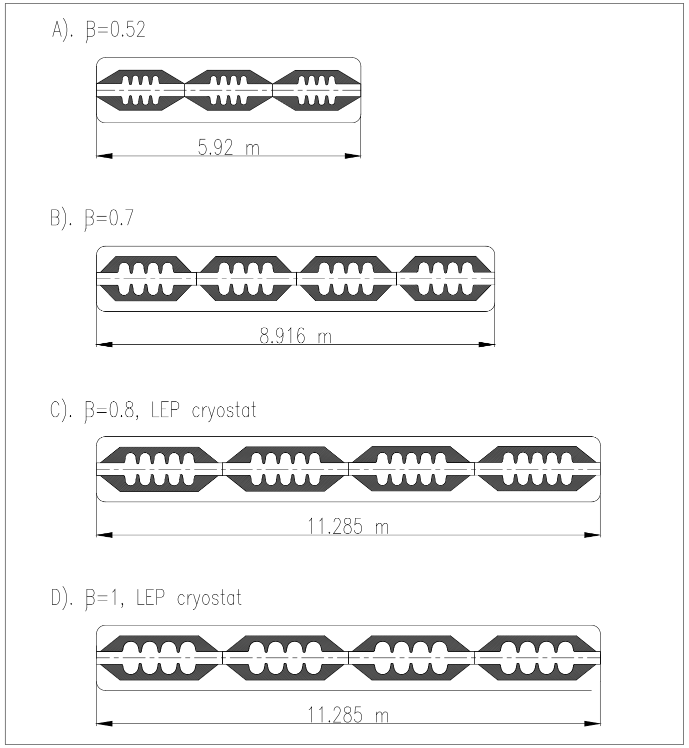


d=*distance between centres of consecutive cells*

superconducting 4-cell accelerating structure

$$\frac{d}{\beta c} = \frac{1}{2f} \Rightarrow d = \frac{\beta c}{2f} = \frac{\beta \lambda}{2}$$





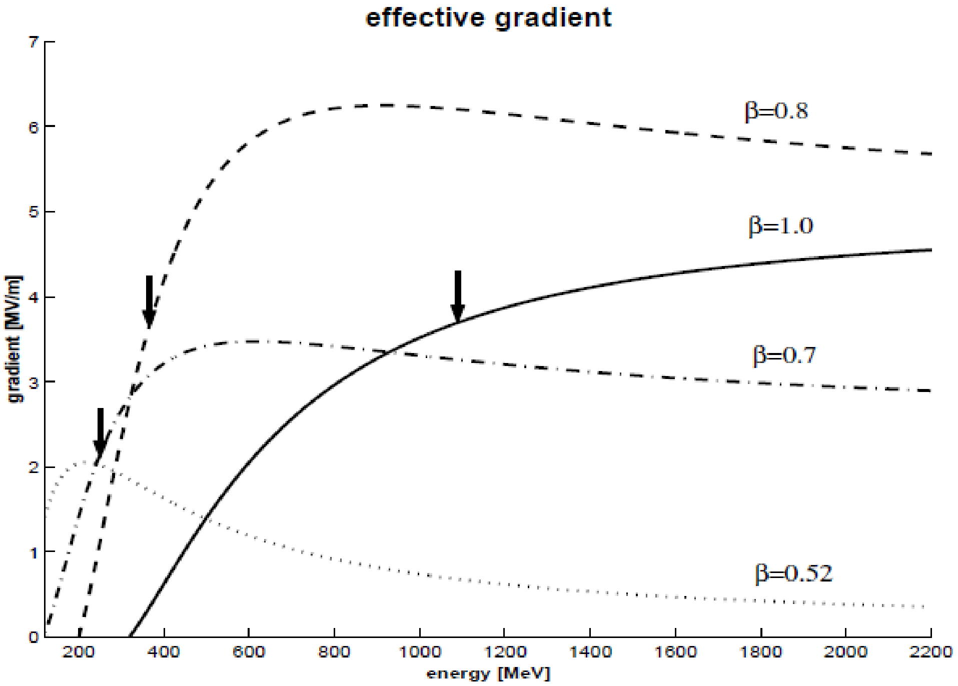
The linac is made of «sections» of identical cavities
For each cavity we accept a «slippage» of the phase around the design phase

Effects of phase slippage

When sequences of cells are not matched to the particle beta → phase slippage

$$\Delta\phi = \omega\Delta t = \pi\frac{\Delta\beta}{\beta}$$

1. The effective gradient seen by the particle is lower.
2. The phase of the bunch centre moves away from the synchronous phase → can go (more) into the non-linear region, with possible longitudinal emittance growth and beam loss.

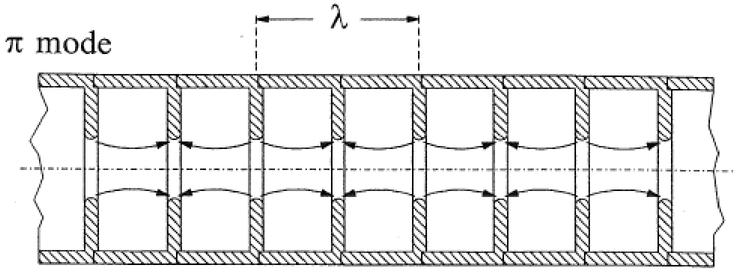


$$\Delta\phi = \pi\frac{\Delta\beta}{\beta} = \pi\frac{1}{\gamma(\gamma-1)}\frac{\Delta W}{W}$$

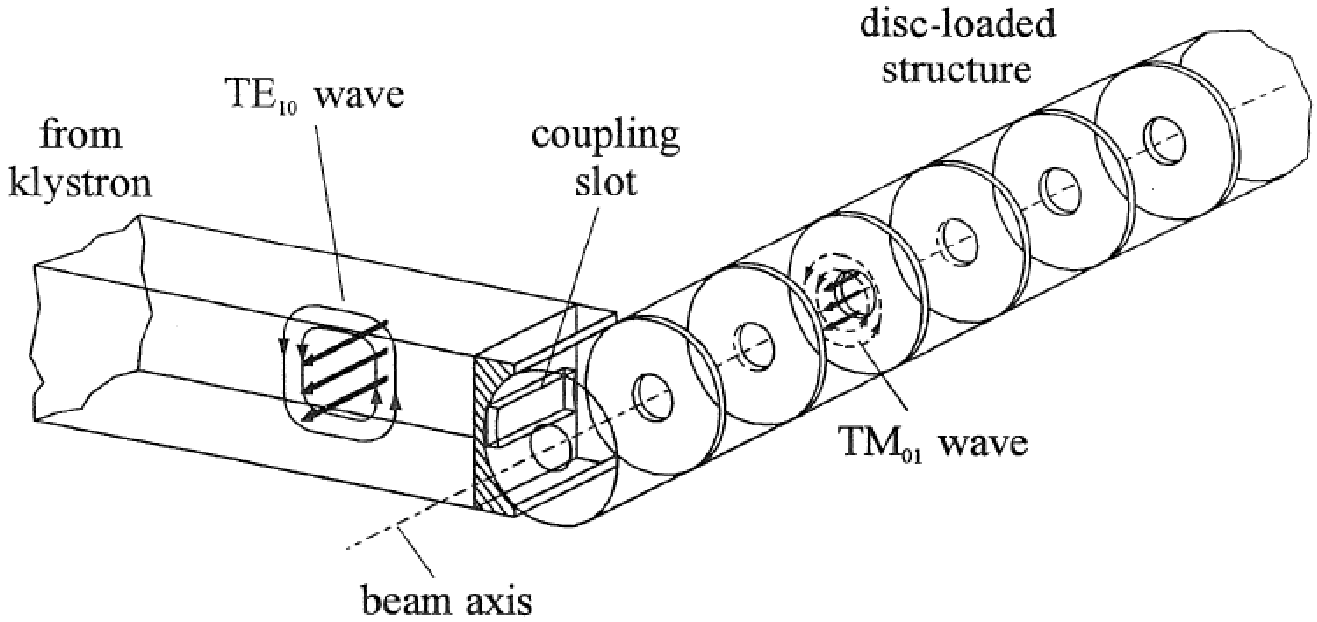
Very large at small energy (γ~1) becomes negligible at high energy (~2.5 °/m for γ~1.5, W=500 MeV).

Curves of effective gradient (gradient seen by the beam for a constant gradient in the cavity) for the previous case (4 sections of beta 0.52, 0.7, 0.8 and 1.0).

Electron linacs

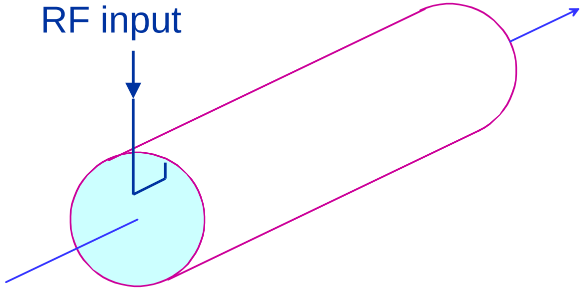


1. In an electron linac velocity is ~ constant. To use the fundamental accelerating mode cell length must be d = βλ / 2.
2. the linac structure will be made of a sequence of identical cells. Because of the limits of the RF source, the cells will be grouped in cavities operating in travelling wave mode.

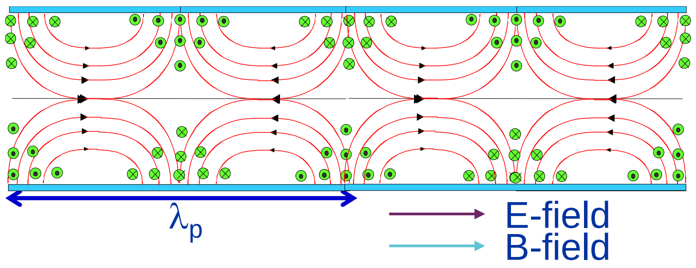


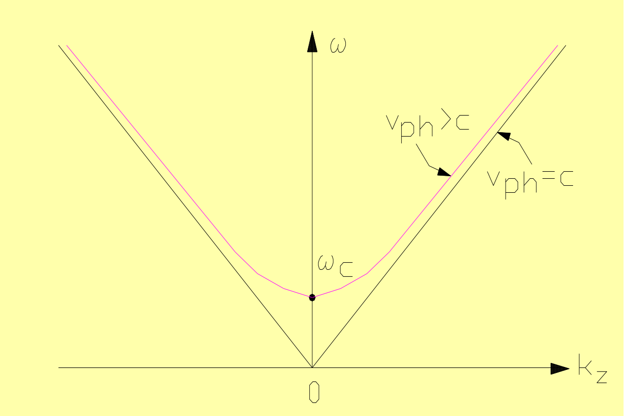
Pictures from K. Wille, The Physics of Particle Accelerators

Acceleration in periodic structures - wave propagation



TM01 field configuration





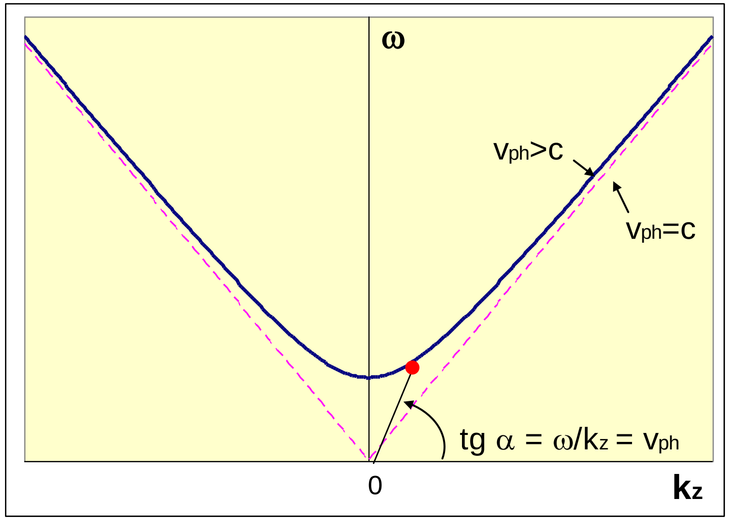
- In a cylindrical waveguide different modes can propagate (=Electromagnetic field distributions, transmitting power and/or information). The field is the superposition of waves reflected by the metallic walls of the pipe → velocity and wavelength of the modes will be different from free space (c, λ)
- To accelerate particles, we need a mode with longitudinal E-field component on axis: a TM mode (Transverse Magnetic, B_z=0). The simplest is TM01.
- We inject RF power at a frequency exciting the TM01 mode: sinusoidal E-field on axis, wavelength λ_p depending on frequency and on cylinder radius. Wave velocity (called "phase velocity") is $v_{ph} = \lambda_p/T = \lambda_p f = \omega/k_z$ with $k_z = 2\pi/\lambda_p$
- The relation between frequency ω and propagation constant k is the DISPERSION RELATION (red curve on plot), a fundamental property of waveguides.

Brillouin diagram

Wave velocity: the dispersion relation

The dispersion relation $\omega(k)$ can be calculated from the theory of waveguides:

$\omega^2 = k^2c^2 + \omega_c^2$ Plotting this curve (hyperbola), we see that:



$k=2\pi/\lambda_p$

$v_{ph}=\omega/k = (c^2+\omega_c^2/k^2)^{1/2}$

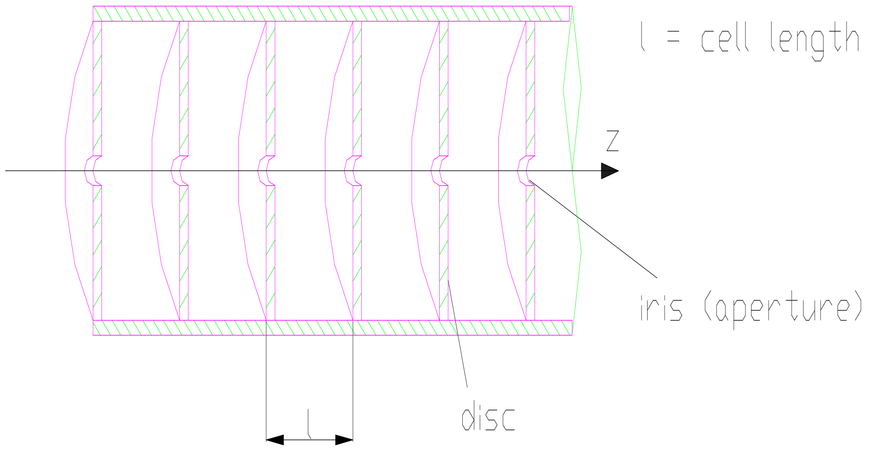
$v_g=d\omega/dk$

1) There is a "cut-off frequency", below which a wave will not propagate. It depends on dimensions (λ_c=2.61a for the TM01 mode in the cylindrical waveguide).
2) At each excitation frequency is associated a phase velocity, the velocity at which a certain phase travels in the waveguide. $v_p=\infty$ at k=0, $\omega=\omega_c$ and then decreases towards $v_p=c$ for $k,\omega\to\infty$.
3) To see at all times an accelerating E-field a particle traveling inside our cylinder has to travel at $v = v_{ph} \rightarrow v > c$!!!

Are we violating relativity? No, energy (and information) travel at group velocity $d\omega/dk$, always between 0 and c.

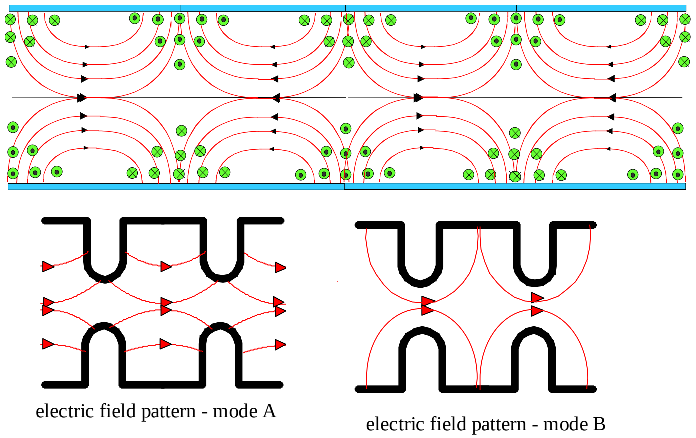
To use the waveguide to accelerate particles, we need a "trick" to slow down the wave.

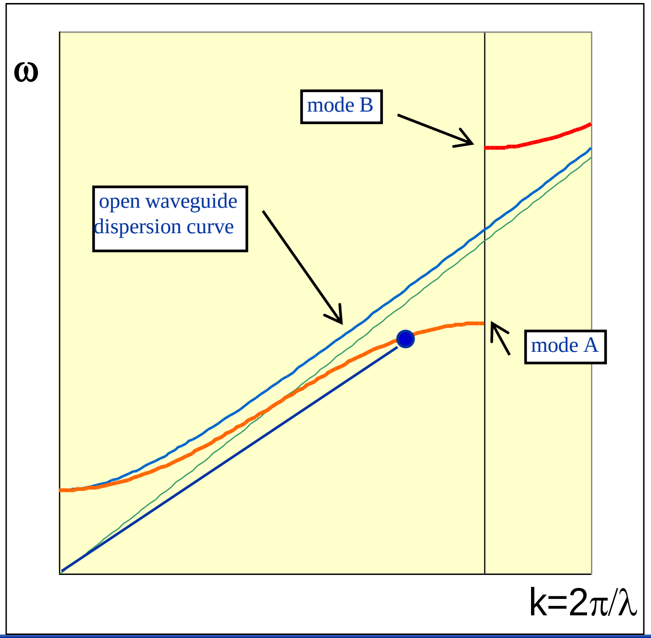
Slowing down waves: the disc- loaded waveguide



Discs inside the cylindrical waveguide, spaced by a distance ℓ , will induce multiple reflections between the discs.

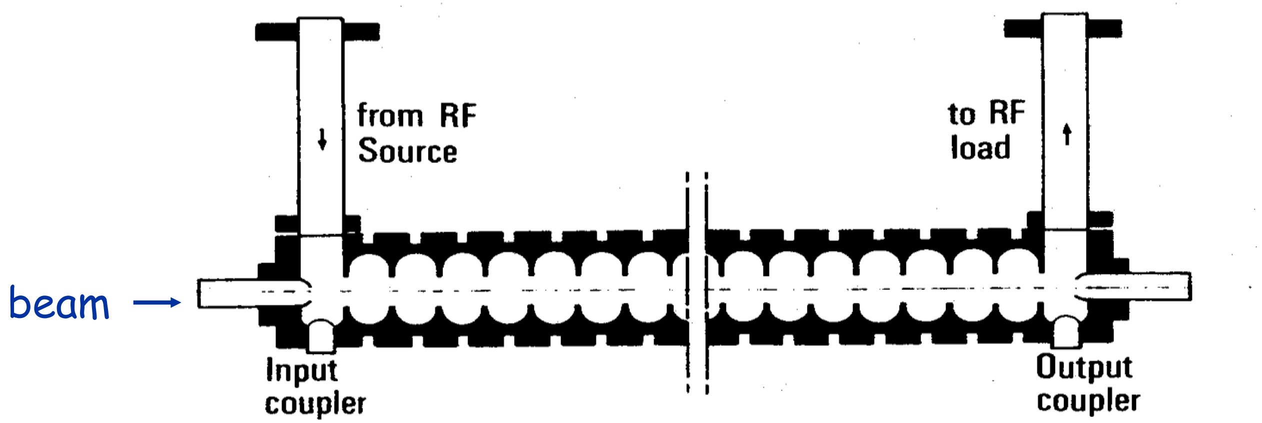
Dispersion relation for the disc-loaded waveguide





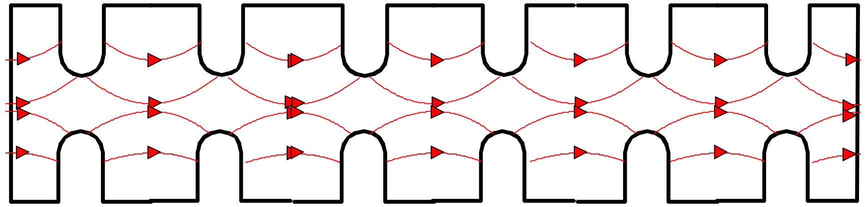
- Wavelengths with $\lambda_p/2\sim \ell$ will be most affected by the discs. On the contrary, for λ_p=0 and $\lambda_p=\infty$ the wave does not see the discs → the dispersion curve remains that of the empty cylinder.
- At $\lambda_p/2=\ell$, the wave will be confined between the discs, and present 2 "polarizations" (mode A and B in the figure), 2 modes with same wavelength but different frequencies → the dispersion curve splits into 2 branches, separated by a stop band.
- In the disc-loaded waveguide, the lower branch of the dispersion curve is now "distorted" in such a way that we can find a range of frequencies with $v_{ph} = c$ → we can use it to accelerate a particle beam!
- We have built a linac for v~c → a TRAVELING WAVE (TW) ELECTRON LINAC

Traveling wave linac structures



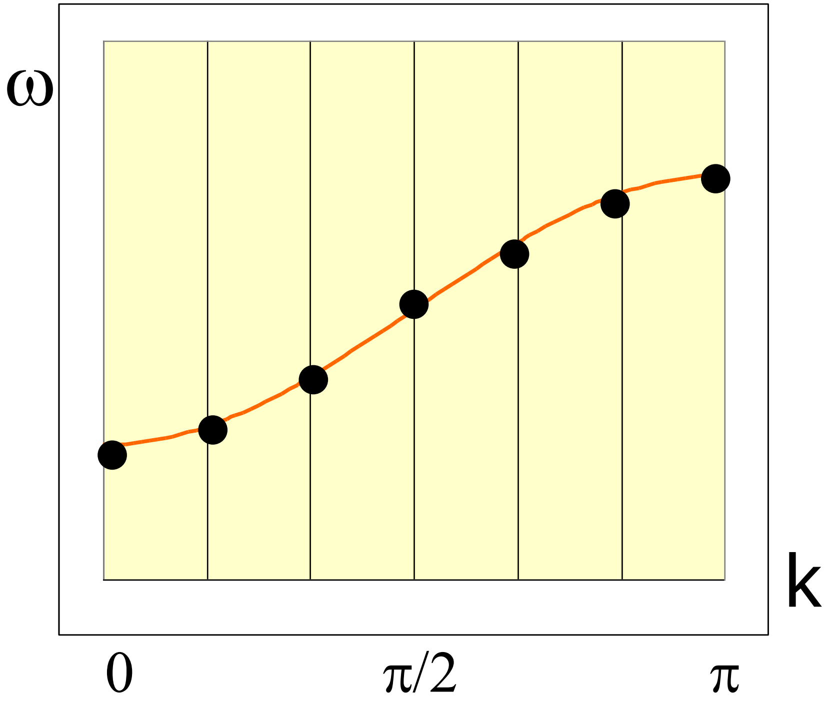
- → Disc-loaded waveguide designed for v_{ph}=c at a given frequency, equipped with an input and an output coupler.
- → RF power is introduced via the input coupler. Part of the power is dissipated in the structure, part is taken by the beam (beam loading) and the rest is absorbed in a matched load at the end of the structure. Usually, structure length is such that ~30% of power goes to the load.
- → The "traveling wave" structure is the standard linac for electrons from β~1.
- → Can not be used for protons at v<c:
  1. constant cell length does not allow synchronism
  2. structures are long, without space for transverse focusing

Standing wave linac structures



To obtain an accelerating structure for protons we close our disc-loaded structure at both ends with metallic walls → multiple reflections of the waves.

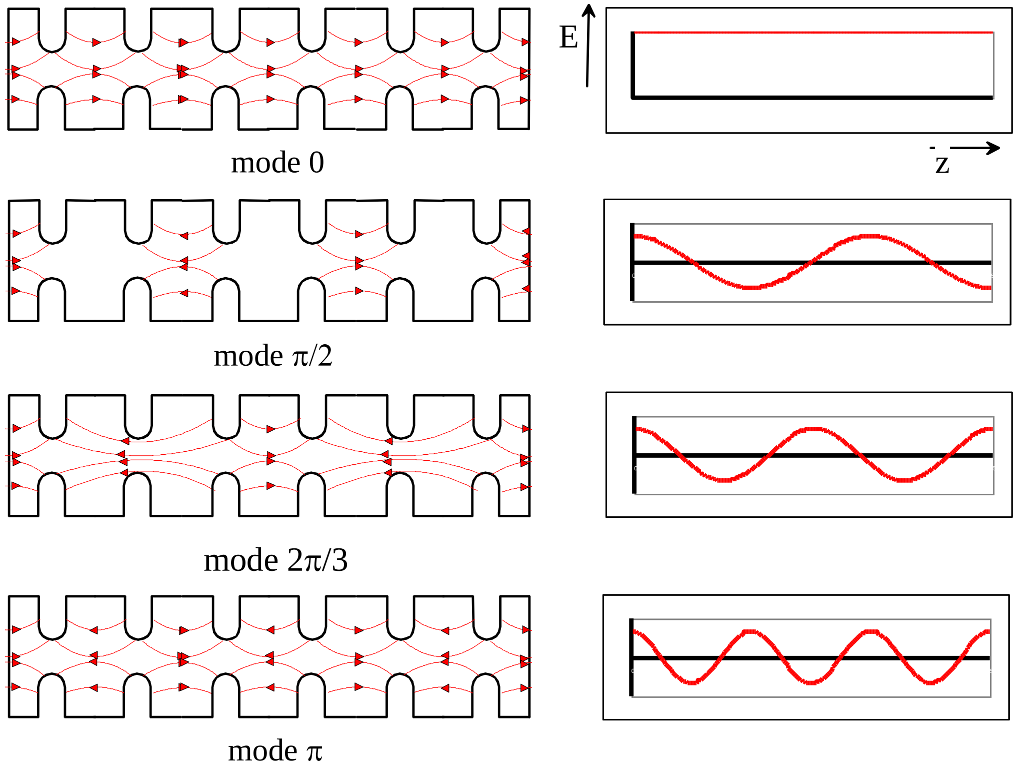
Boundary condition at both ends is that electric field must be perpendicular to the cover → Only some modes on the disc-loaded dispersion curve are allowed → only some frequencies on the dispersion curve are permitted.



In general:

1. the modes allowed will be equally spaced in k
2. The number of modes will be identical to the number of cells (N cells → N modes)
3. k represents the phase difference between the field in adjacent cells.

More on standing wave structures



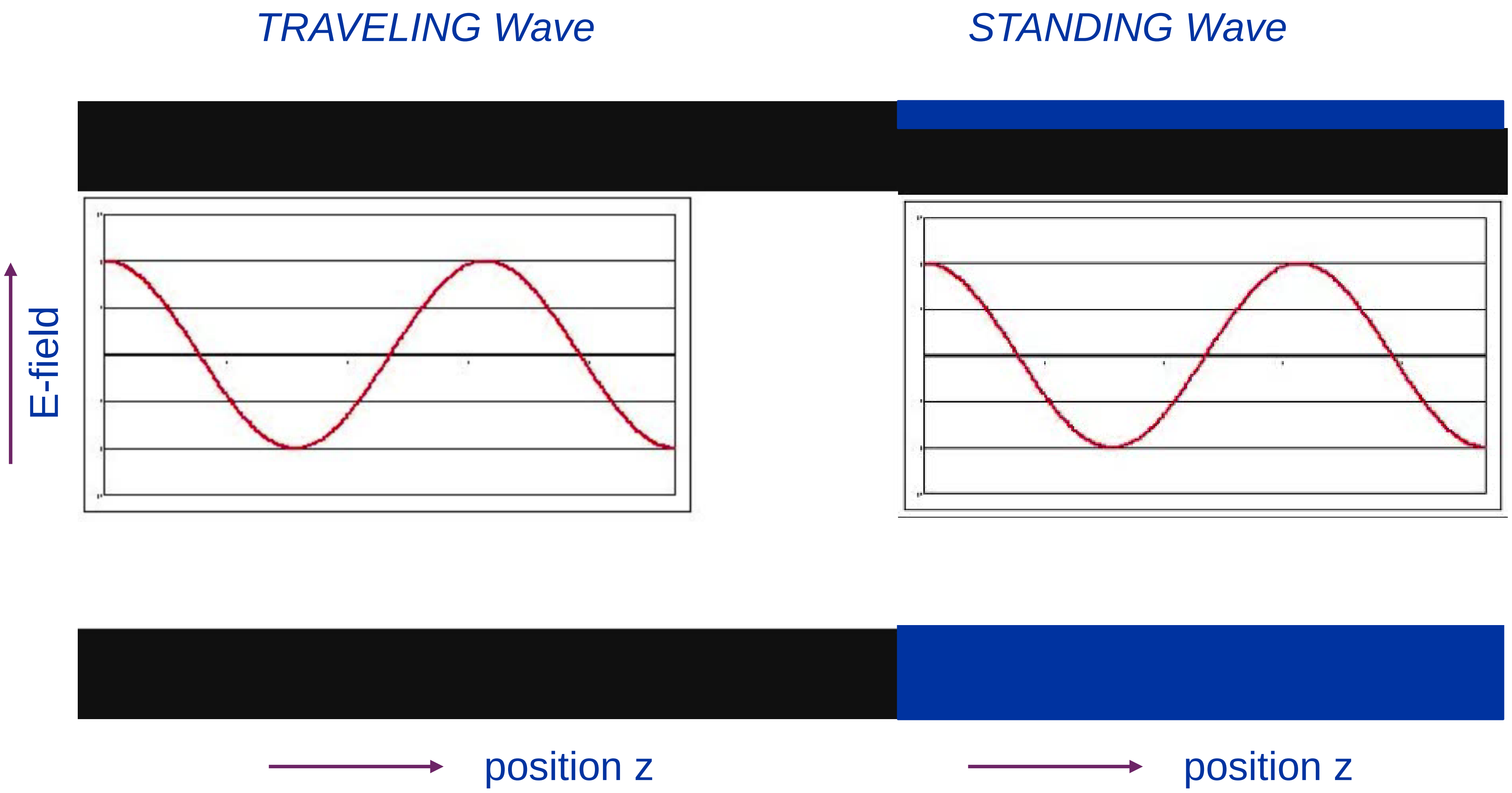
Standing wave modes are named from the phase difference between adjacent cells: in the example above, mode 0, π/2, 2π/3, π.

In standing wave structures, cell length can be matched to the particle velocity !

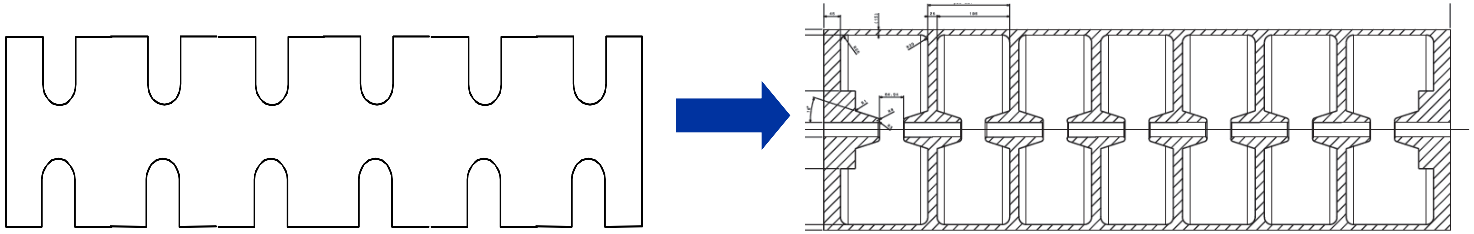
- → STANDING WAVE MODES are generated by the sum of 2 waves traveling in opposite directions, adding up in the different cells.
- → For acceleration, the particles must be in phase with the E-field on axis. We have already seen the π mode: synchronism condition for cell length $\ell = \beta\lambda/2$.
- → Standing wave structures can be used for any β (→ ions and electrons) and their cell length can increase, to follow the increase in β of the ions.

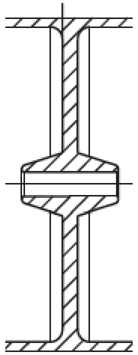
Synchronism conditions:
0-mode : $\ell = \beta\lambda$
π/2 mode: $2\ell = \beta\lambda/2$
π mode: $\ell = \beta\lambda/2$

Acceleration on traveling and standing waves



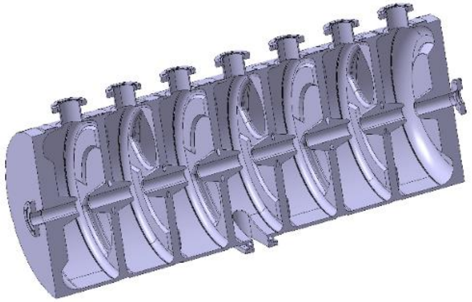
Practical standing wave structures





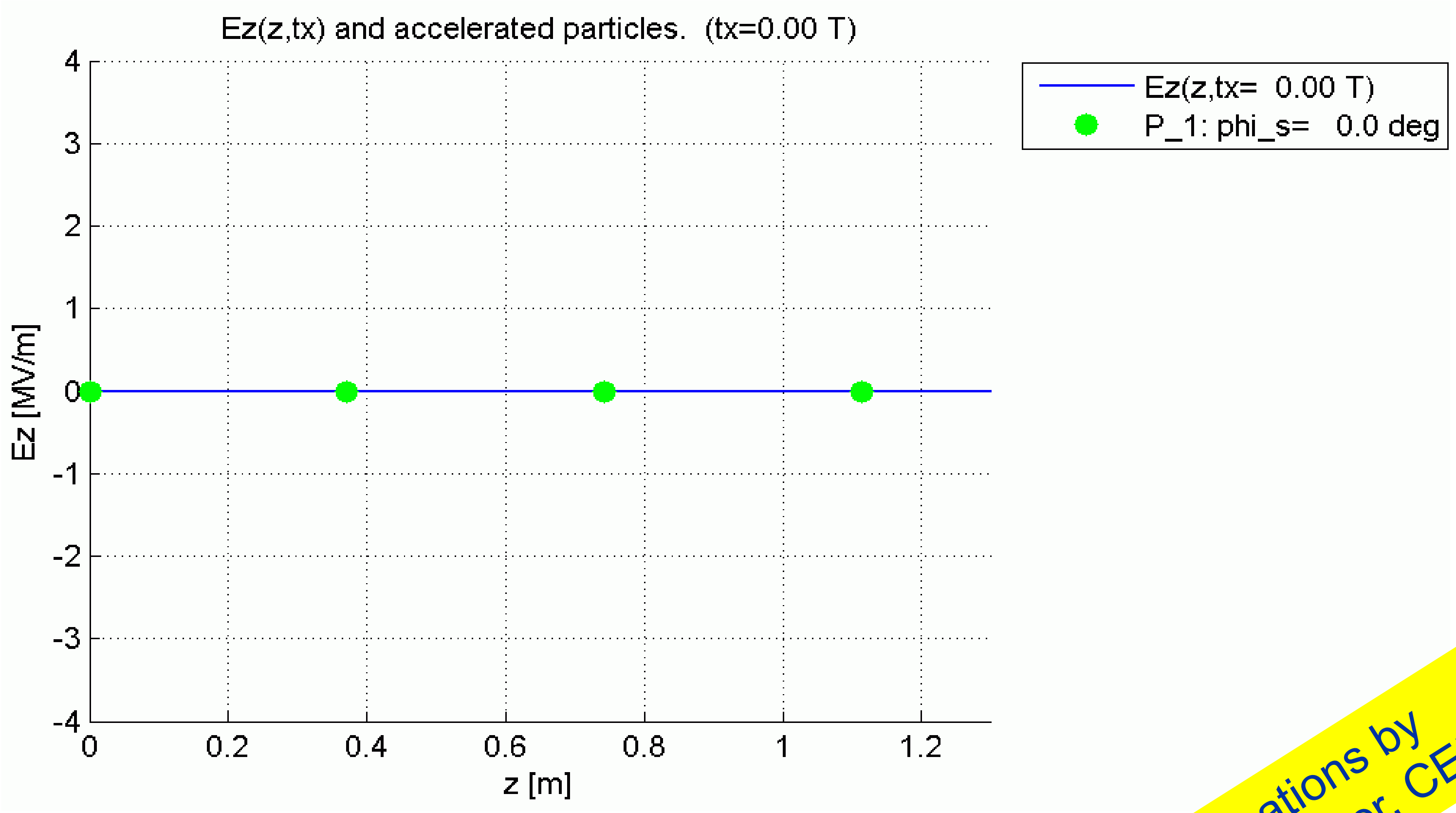
From disc-loaded structure to a real cavity (Linac4 PIMS, Pi-Mode Structure)

1. To increase acceleration efficiency (=shunt impedance ZT^2!) we need to concentrate electric field on axis (Z↑) and to shorten the gap (T↑) → introduction of “noses” on the openings.
2. The smaller opening would not allow the wave to propagate → introduction of “coupling slots” between cells.
3. The RF wave has to be coupled into the cavity from one point, usually in the center.



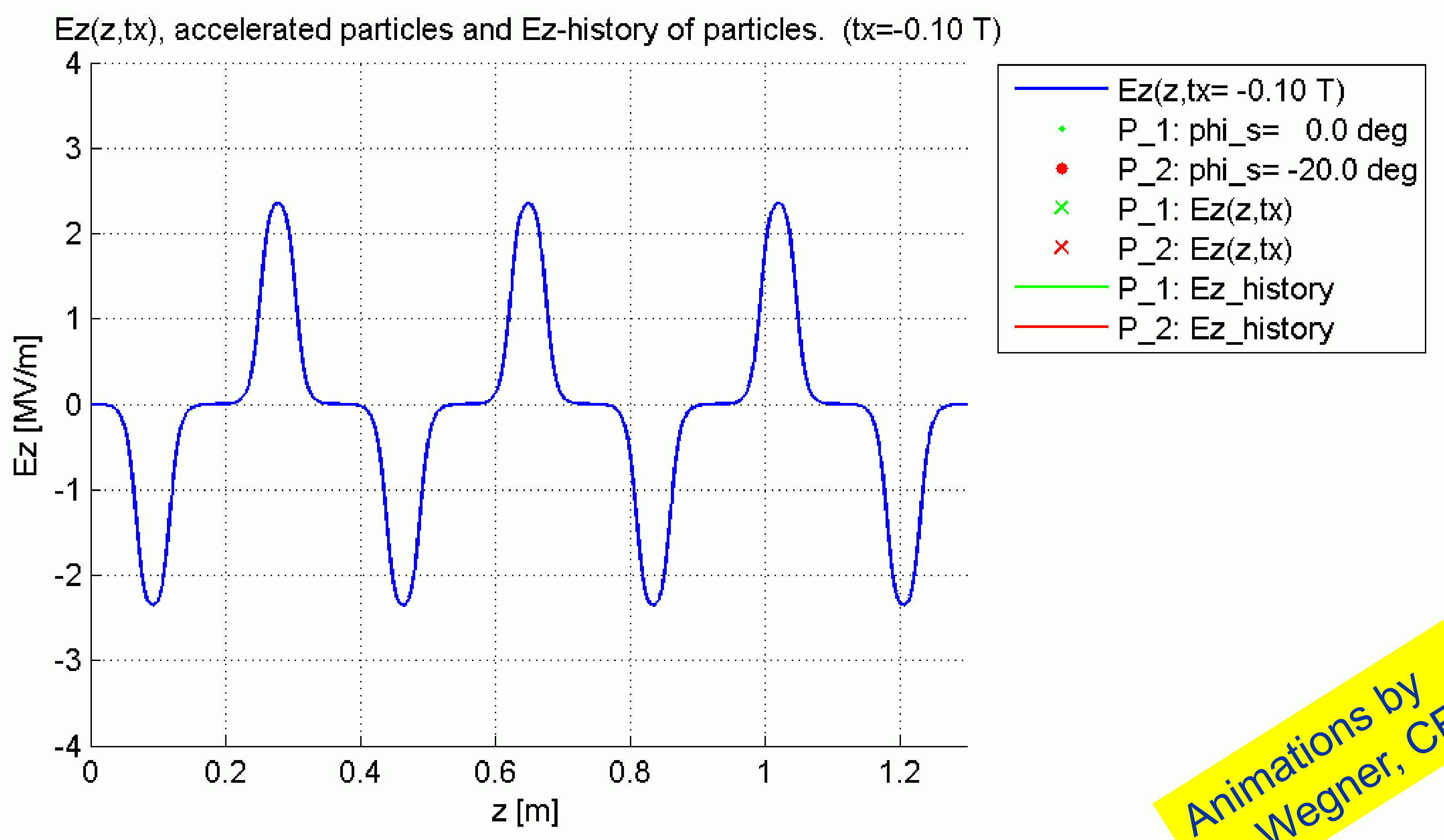
PIMS Prototype

Synchronism in the PIMS



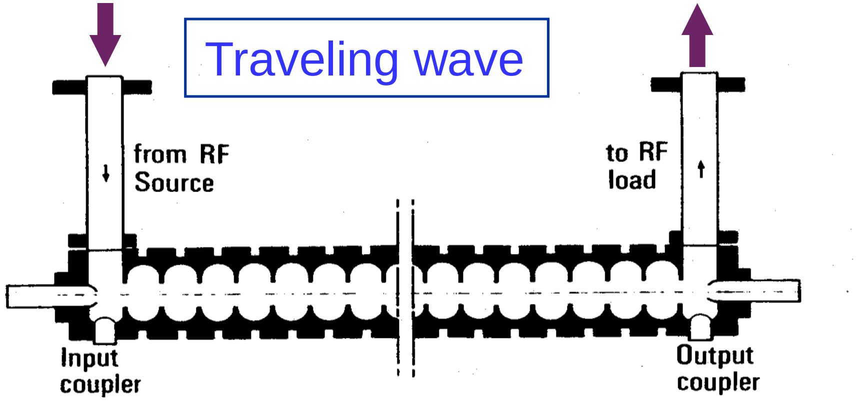
Animations by
R. Wegner, CERN

Acceleration in the PIMS



Animations by R. Wegner, CERN

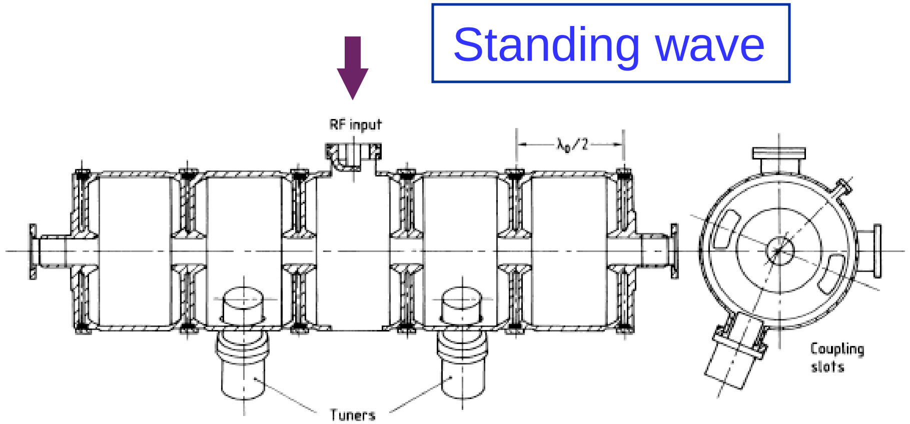
Comparing traveling and standing wave structures



Chain of coupled cells in TW mode
Coupling bw. cells from on-axis aperture.
RF power from input coupler at one end, dissipated in the structure and on a load.

Short pulses, High frequency (≥ 3 GHz).
Gradients 10-20 MeV/m

Used for Electrons at v~c



Chain of coupled cells in SW mode.
Coupling (bw. cells) by slots (or open). On-axis aperture reduced, higher E-field on axis and power efficiency.
RF power from a coupling port, dissipated in the structure (ohmic loss on walls).

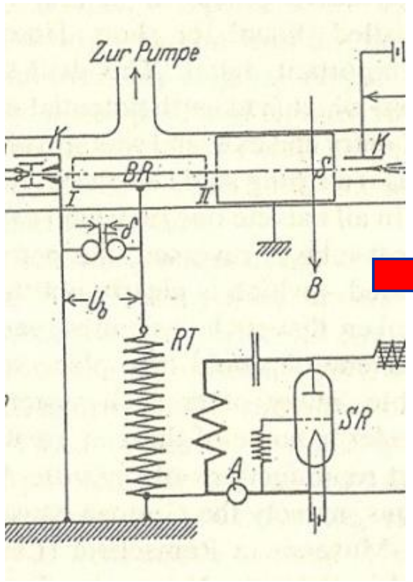
Long pulses. Gradients 2-5 MeV/m

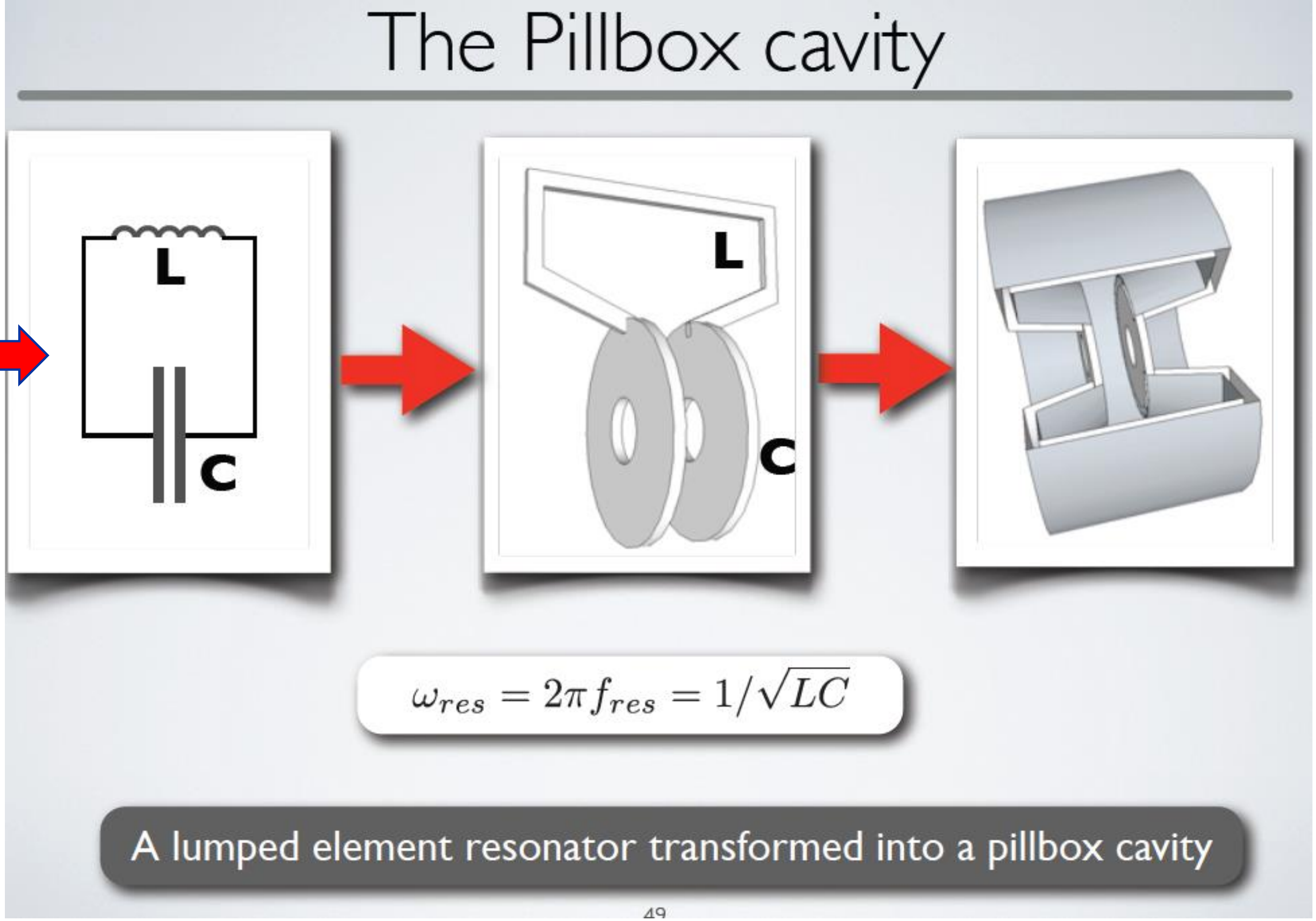
Used for Ions and electrons, all energies

Module 2

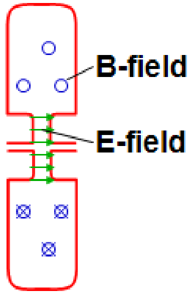
Coupled resonator chains
Stability and stabilization
Acceleration in periodic structures
Special accelerating structures
Superconducting linac structures

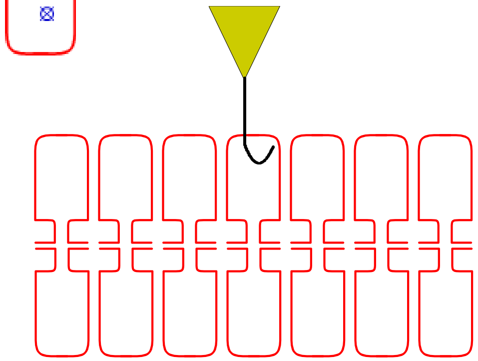
From the Wideröe gap to the linac cell



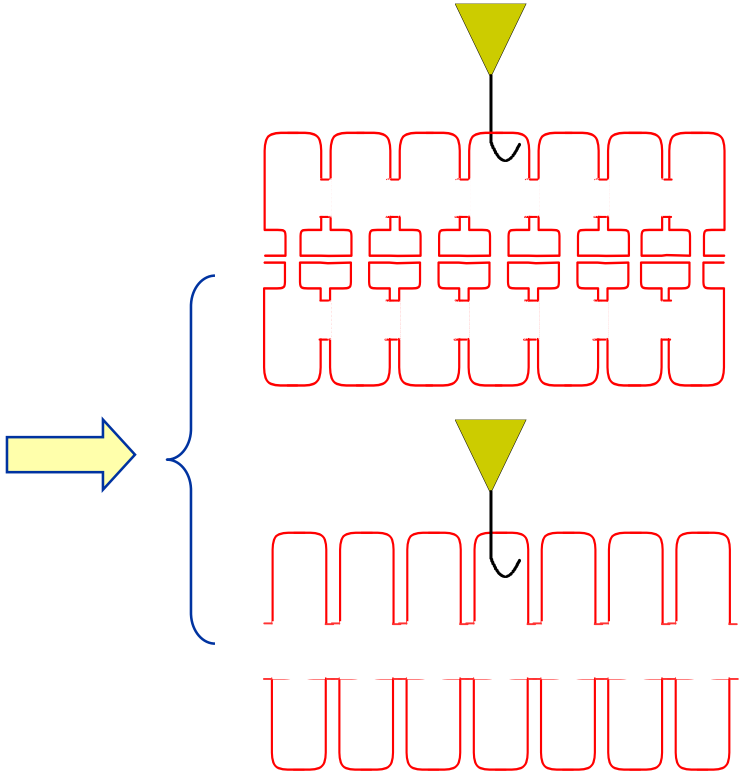


Coupling cavities





How can we couple together a chain of n reentrant (=loaded pillbox) accelerating cavities ?



1. Magnetic coupling:

open “slots” in regions of high magnetic field → B-field can couple from one cell to the next

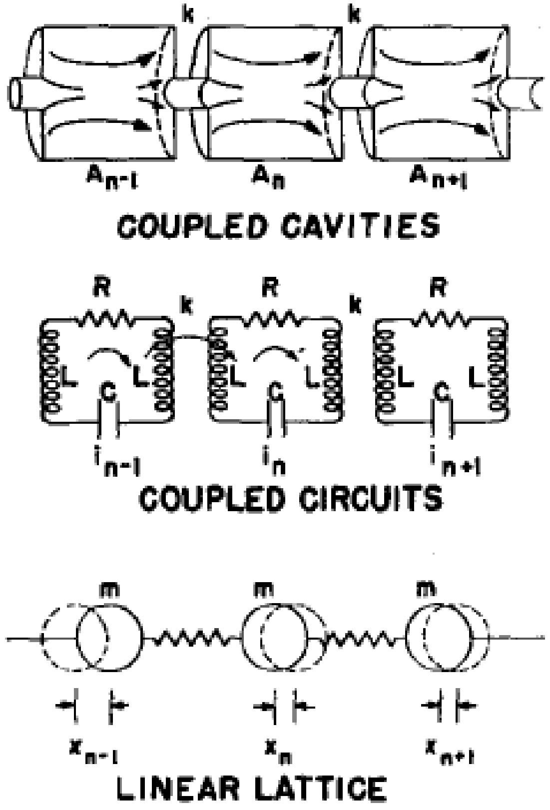
2. Electric coupling:

enlarge the beam aperture → E-field can couple from one cell to the next

The effect of the coupling is that the cells no longer resonate independently, but will have common resonances with well defined field patterns.

Chains of coupled resonators

What is the relative phase and amplitude between cells in a chain of coupled cavities?

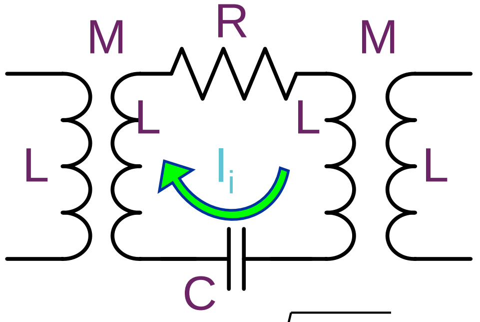


A linear chain of accelerating cells can be represented as a sequence of resonant circuits magnetically coupled.

Individual cavity resonating at $\omega_0 \rightarrow$ frequenci(es) of the coupled system ?

Resonant circuit equation for circuit *i* (neglecting the losses, R≅0):

$$I_i\left(2j\omega L + \frac{1}{j\omega C}\right) + j\omega k L\left(I_{i-1} + I_{i+1}\right) = 0$$



$$\omega_0 = 1/\sqrt{2LC}$$

$$M = k\sqrt{L_1 L_2} = kL$$

Dividing both terms by *2jωL:*

$$X_i\left(1 - \frac{\omega_0^2}{\omega^2}\right) + \frac{k}{2}\left(X_{i-1} + X_{i+1}\right) = 0$$

General response term, ∝ (stored energy)$^{1/2}$, can be voltage, E-field, B-field, etc.

General resonance term

Contribution from adjacent oscillators

The Coupled-system Matrix

A chain of N+1 resonators is described by a (N+1)x(N+1) matrix:

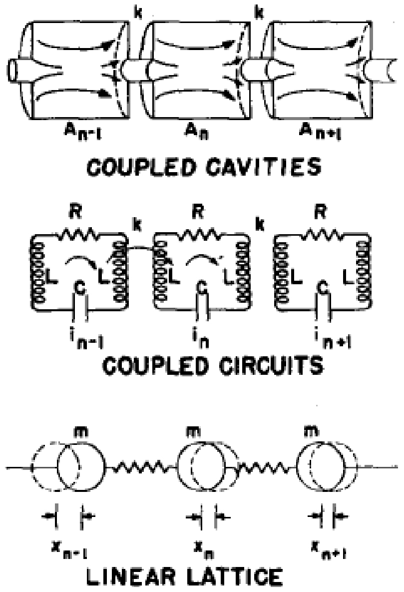
$$X_i\left(1-\frac{\omega_0^2}{\omega^2}\right)+\frac{k}{2}\left(X_{i-1}+X_{i+1}\right)=0$$

$$i=0,..,N$$

$$\begin{vmatrix} 1-\frac{\omega_0^2}{\omega^2} & \frac{k}{2} & 0 & \dots \\ \frac{k}{2} & 1-\frac{\omega_0^2}{\omega^2} & \frac{k}{2} & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \frac{k}{2} & 1-\frac{\omega_0^2}{\omega^2} \end{vmatrix} \begin{vmatrix} X_0 \\ X_2 \\ \dots \\ X_N \end{vmatrix} = 0$$

or

$$M\,X=0$$



This matrix equation has solutions only if $\det M = 0$

Eigenvalue problem!

1. System of order (N+1) in $\omega \rightarrow$ only N+1 frequencies will be solution of the problem (“eigenvalues”, corresponding to the resonances) $\rightarrow$ a system of N coupled oscillators has N resonance frequencies $\rightarrow$ an *individual resonance opens up into a band of frequencies*.
2. At each frequency ω_i will correspond a set of relative amplitudes in the different cells (X_0, X_2, …, X_N): the “eigenmodes” or “modes”.

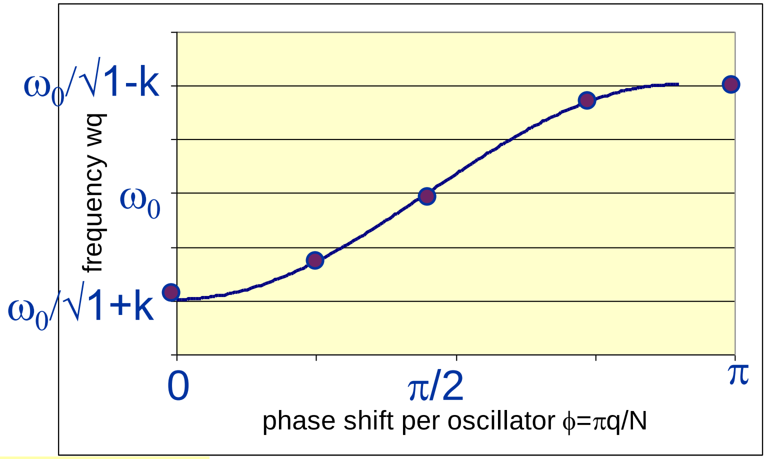
Modes in a linear chain of oscillators

We can find an analytical expression for eigenvalues (frequencies) and eigenvectors (modes):

Frequencies of the coupled system :

$$\omega_q^2 = \frac{\omega_0^2}{1 + k\cos\frac{\pi q}{N}}, \qquad q = 0,.., N$$

the index q defines the number of the solution → is the "mode index"



→ Each mode is characterized by a phase πq/N. Frequency vs. phase of each mode can be plotted as a "dispersion curve" ω=f(ϕ):

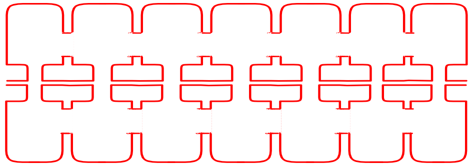
1. each mode is a point on a sinusoidal curve.
2. modes are equally spaced in phase.

The "eigenvectors = relative amplitude of the field in the cells are:

$$X_i^{(q)} = (const)\cos\frac{\pi q i}{N} e^{j\omega_q t} \qquad q = 0,..., N$$

STANDING WAVE MODES, defined by a phase πq/N corresponding to the phase shift between an oscillator and the next one → πq/N=Φ is the phase difference between adjacent cells that we have introduces in the 1st part of the lecture.

Acceleration on the normal modes of a 7-cell structure



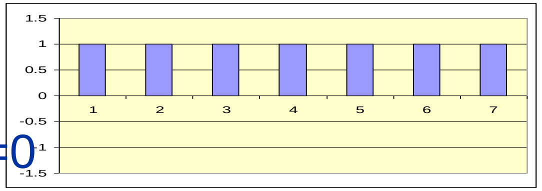
$$X_i^{(q)} = (const)\cos\frac{\pi q i}{N} e^{j\omega_q t} \qquad q = 0,\dots,N$$

$$\Delta\phi = 2\pi\frac{d}{\beta\lambda}$$

Remember the phase relation!

0

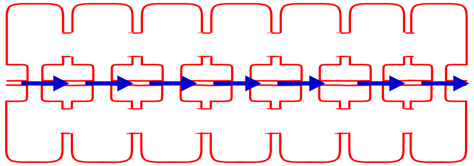
q=0

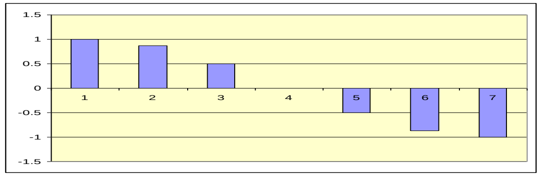


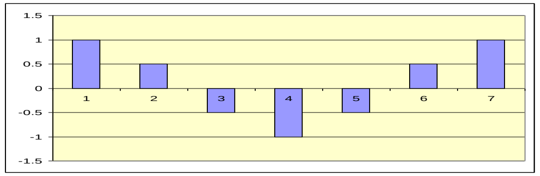
$$\Phi = 2\pi, \quad 2\pi\frac{d}{\beta\lambda} = 2\pi, \quad d = \beta\lambda$$

$\omega = \omega_0/\sqrt{1+k}$

0 (or 2π) mode, acceleration if d = $\beta\lambda$



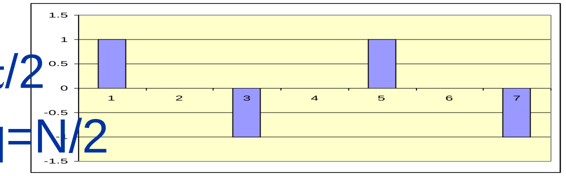




Intermediate modes

$\pi/2$

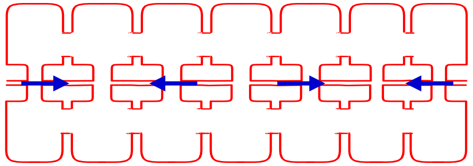
q=N/2



$$\Phi = \frac{\pi}{2}, \quad 2\pi\frac{d}{\beta\lambda} = \frac{\pi}{2}, \quad d = \frac{\beta\lambda}{4}$$

$\omega = \omega_0$

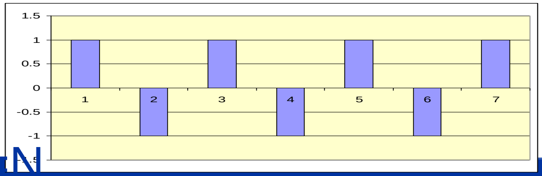
$\pi/2$ mode, acceleration if d = $\beta\lambda/4$



…

π

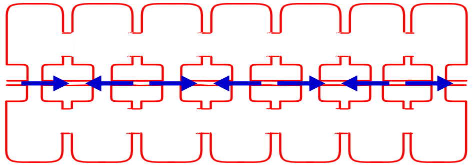
q=N



$$\Phi = \pi, \quad \pi\frac{d}{\beta\lambda} = 2\pi, \quad d = \frac{\beta\lambda}{2}$$

$\omega = \omega_0/\sqrt{1-k}$

π mode, acceleration if d = $\beta\lambda/2$,



Note: Field always maximum in first and last cell!

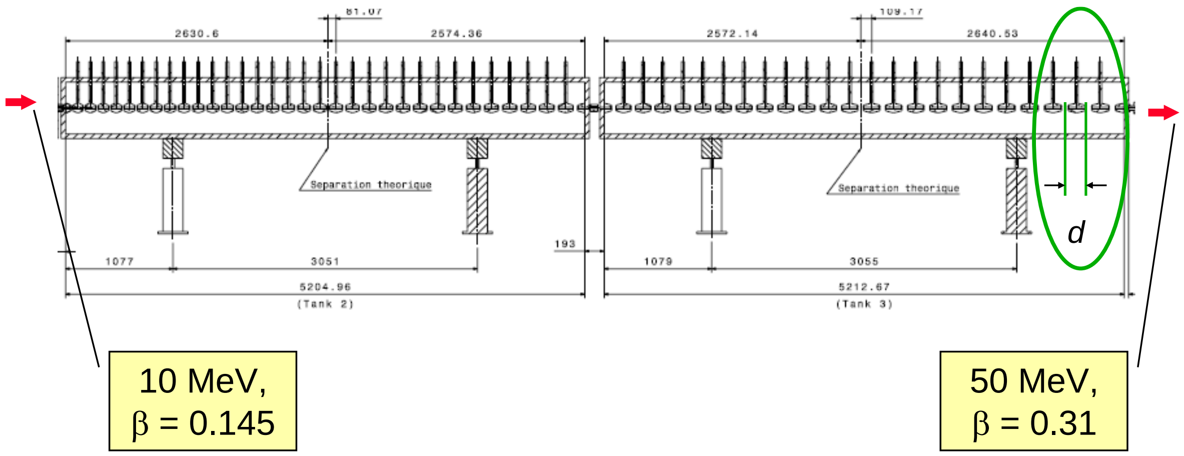
Practical linac accelerating structures

Note: our equations depend only on the cell frequency ω, not on the cell length d !!!

$$\omega_q^2 = \frac{\omega_0^2}{1 + k\cos\frac{\pi q}{N}}, \qquad q = 0,\dots,N$$

$$X_n^{(q)} = (const)\cos\frac{\pi q n}{N} e^{j\omega_q t} \qquad q = 0,\dots,N$$

$\rightarrow$ As soon as we keep the frequency of each cell constant, we can change the cell length following any acceleration (β) profile!



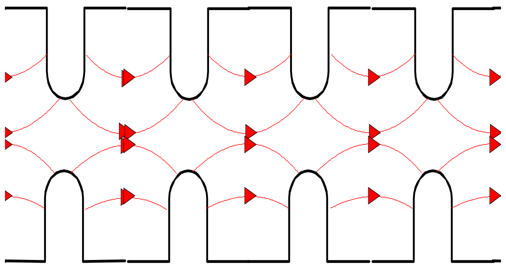
Example:
The Drift Tube Linac (DTL)

Chain of many (up to 100!) accelerating cells operating in the 0 mode. The ultimate coupling slot: no wall between the cells!

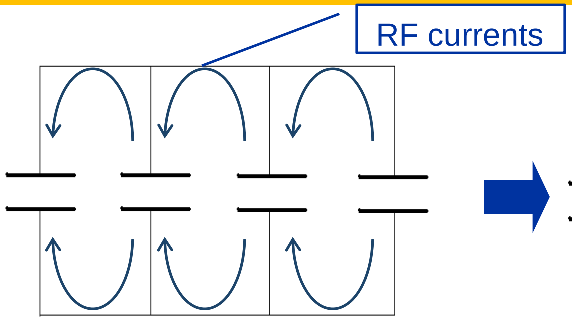
Each cell has a different length, but the cell frequency remains constant $\rightarrow$ *"the EM fields don't see that the cell length is changing!"*

d↑ → (L ↑ , C↓) → LC ~ const → ω ~ const

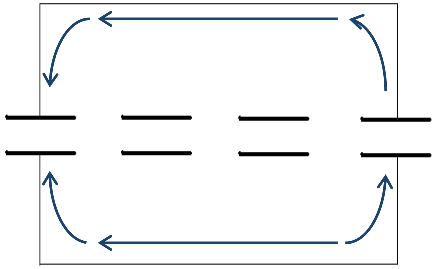
0-mode structures: the Drift Tube Linac ("Alvarez")



Disc-loaded structures operating in 0-mode



Add tubes for high shunt impedance

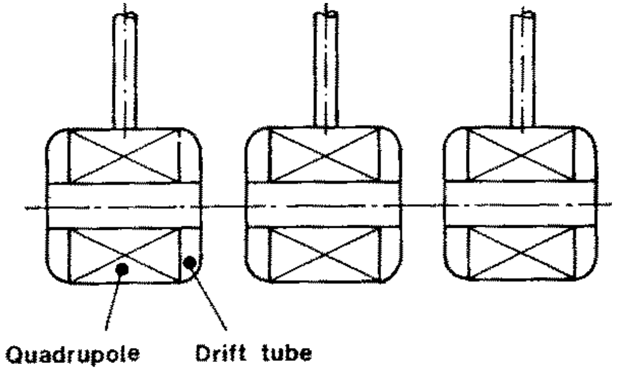


Maximize coupling between cells → remove completely the walls

2 advantages of the 0-mode:

1. the fields are such that if we eliminate the walls between cells the fields are not affected, but we have less RF currents and higher power efficiency ("shunt impedance").
2. The "drift tubes" are long (~0.75 $\beta\lambda$). The particles are inside the tubes when the electric field is decelerating, and we have space to introduce focusing elements (quadrupoles) inside the tubes.

Disadvantage (w.r.t. the π mode): half the number of gaps per unit length!



The Drift Tube Linac

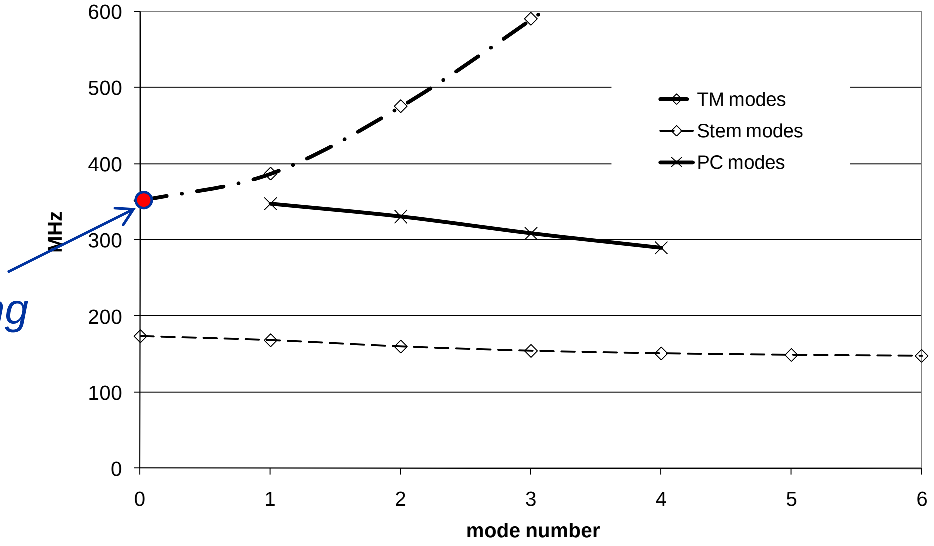
A DTL tank with N drift tubes will have N modes of oscillation.
For acceleration, we choose the 0-mode, the lowest of the band. All cells (gaps) are in phase, then $\Delta\phi=2\pi$

$$\Delta\phi = 2\pi \frac{d}{\beta\lambda} = 2\pi \quad \Rightarrow \quad d = \beta\lambda$$

Distance between gaps must be $\beta\lambda$

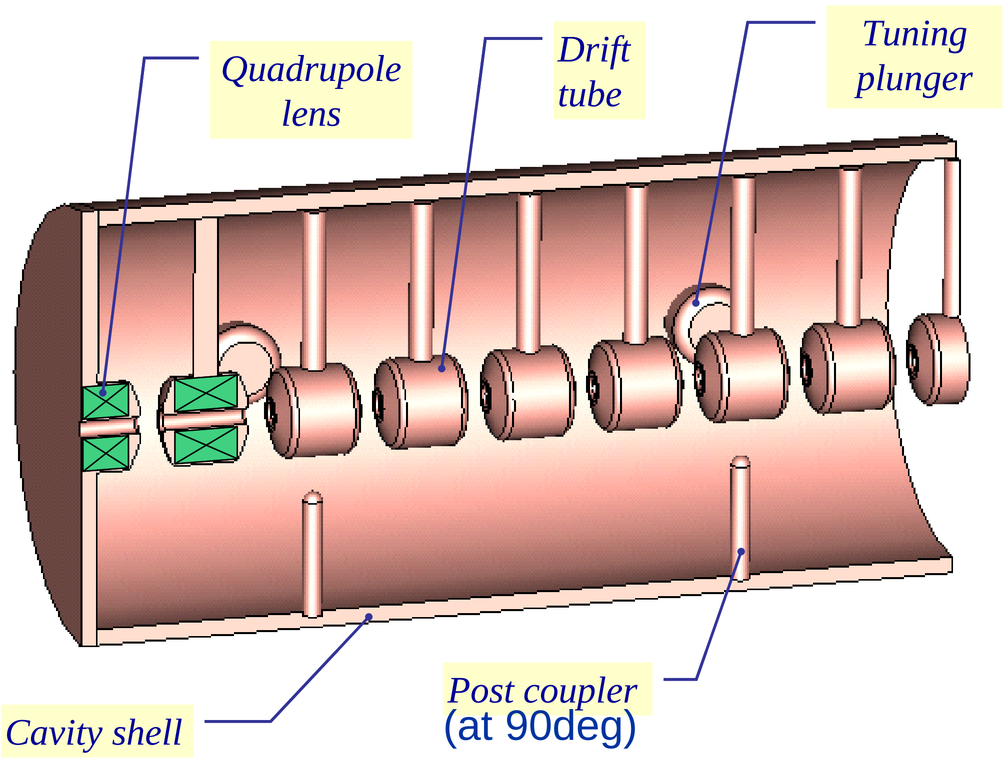
The other modes in the band (and many others!) are still present.
If mode separation >> bandwidth, they are not “visible” at the operating frequency, but they can come out in case of frequency errors between the cells (mechanical errors or others).

operating 0-mode



mode distribution in a DTL tank (operating frequency 352 MHz, are plotted all frequencies < 600 MHz)

DTL construction

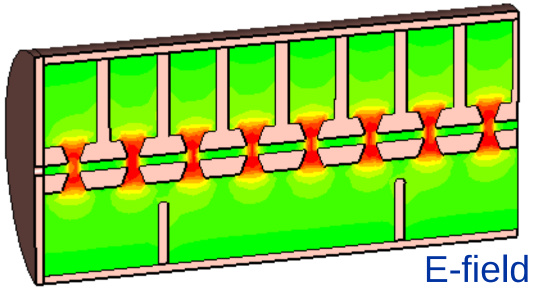


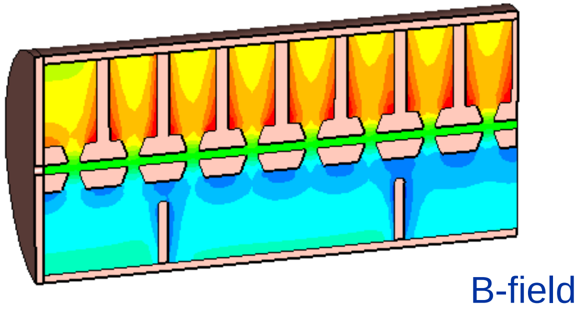
Standing wave linac structure for protons and ions, β=0.1-0.5, f=20-400 MHz

Drift tubes are suspended by stems (no net RF current on stem)

Coupling between cells is maximum (no slot, fully open !)

The 0-mode allows a long enough cell (d=βλ) to house focusing quadrupoles inside the drift tubes!



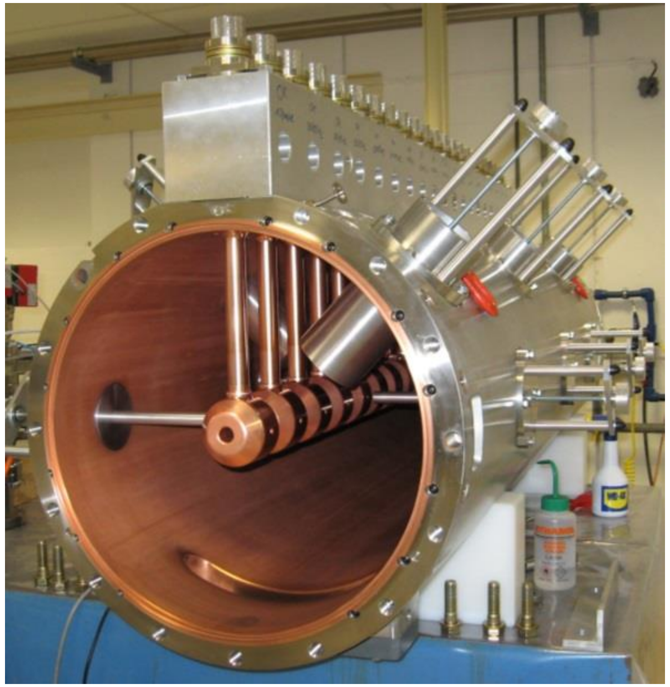


Examples of DTL

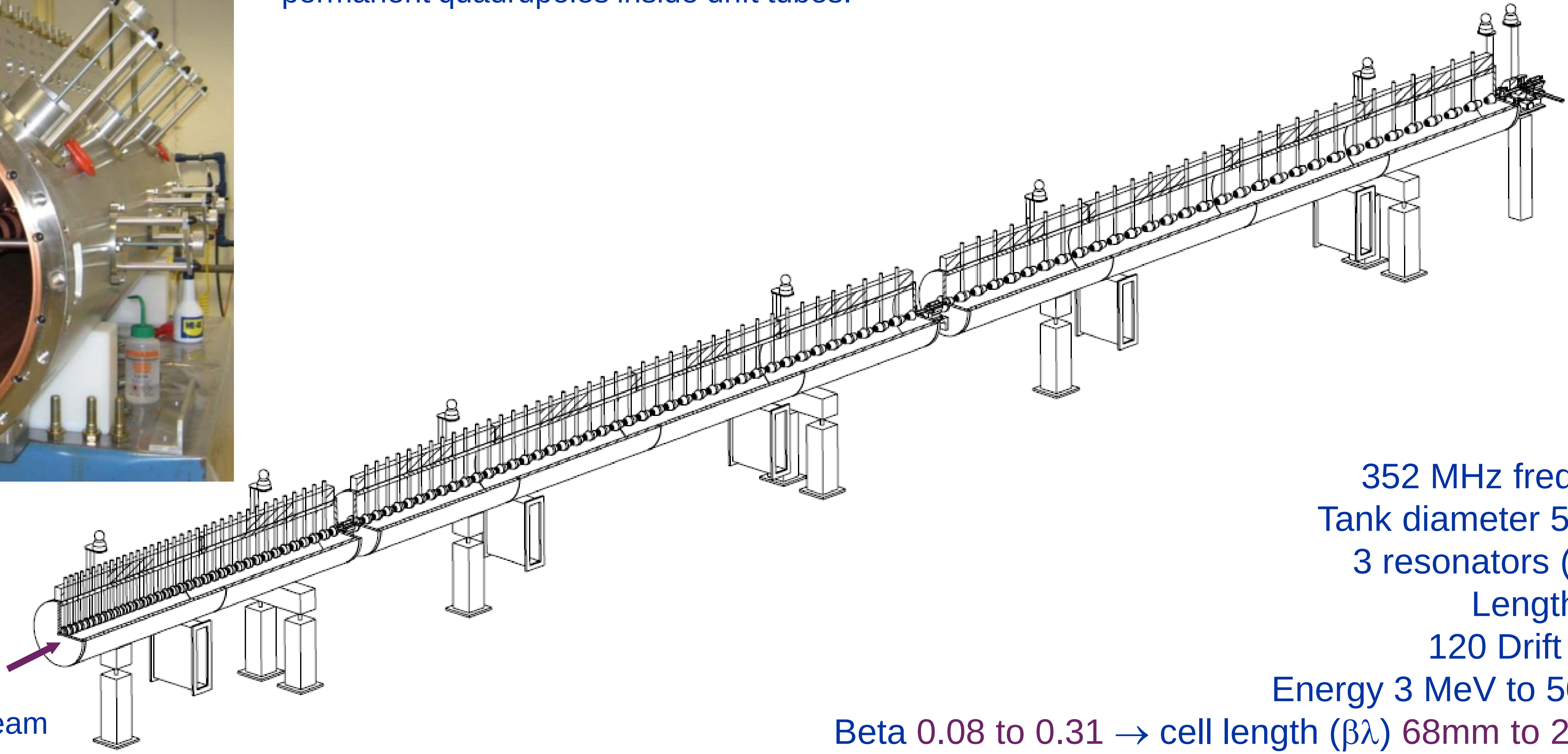
Top; CERN Linac2 Drift Tube Linac: 1978, 202.5 MHz, 3 tanks, final energy 50 MeV, tank diameter 1 meter.

Left: The Drift Tube Linac of the SNS at Oak Ridge (USA): 402.5 MHz, 6 tanks, final energy 87 MeV.

The Linac4 DTL



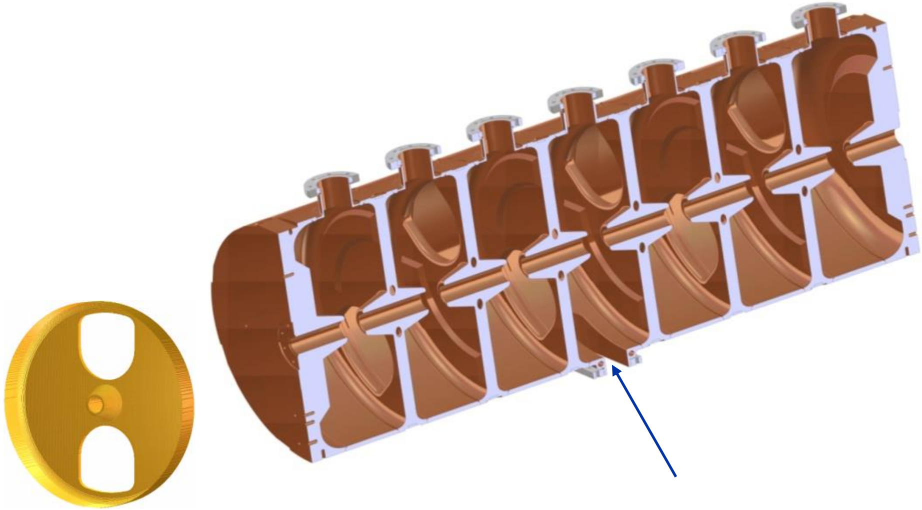
DTL tank 1 fully equipped: focusing by small permanent quadrupoles inside drift tubes.



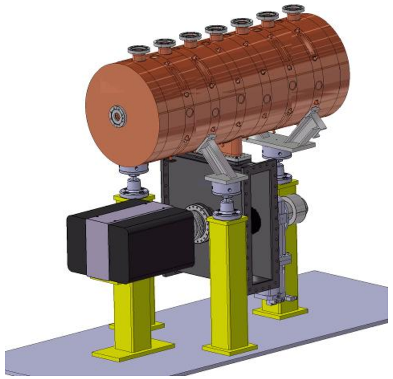
352 MHz frequency
Tank diameter 500mm
3 resonators (tanks)
Length 19 m
120 Drift Tubes
Energy 3 MeV to 50 MeV
Beta 0.08 to 0.31 $\rightarrow$ cell length ($\beta\lambda$) 68mm to 264mm
$\rightarrow$ factor 3.9 increase in cell length

Pi-mode structures: the PIMS

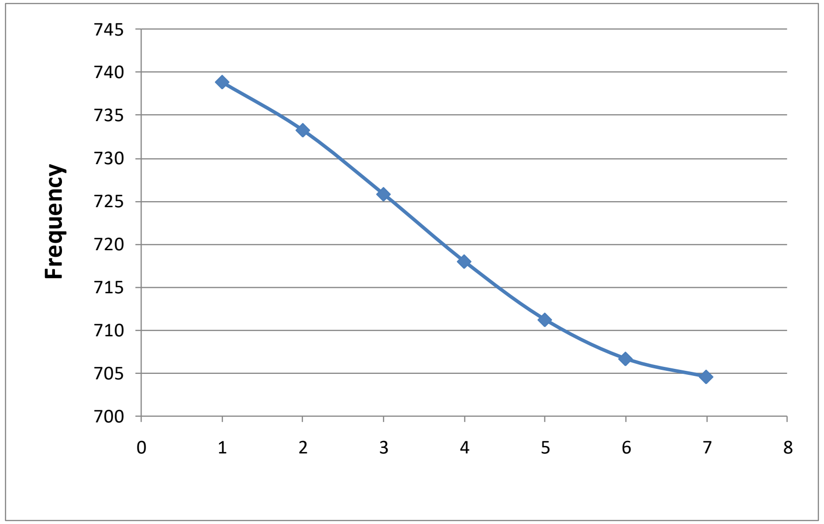
PIMS = Pi-Mode Structure, will be used in Linac4 at CERN to accelerate protons from 100 to 160 MeV (β > 0.4)



RF input



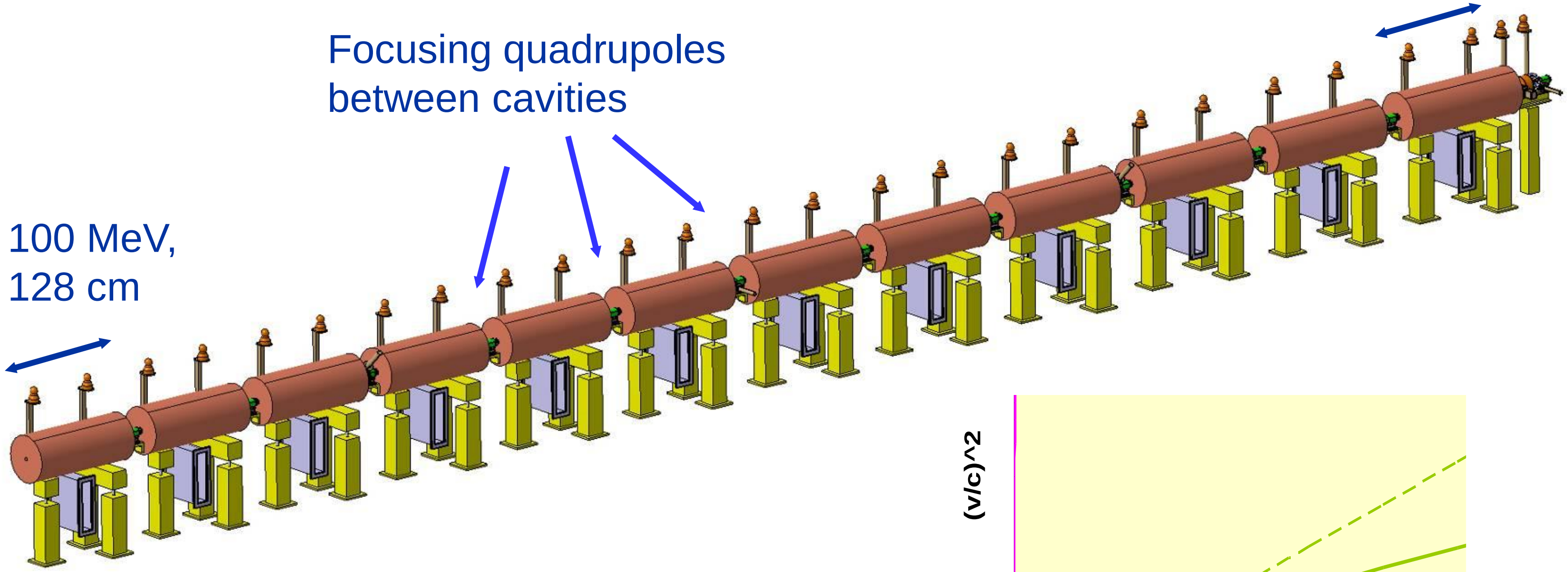
7 cells magnetically coupled, 352 MHz
Operating in π-mode, cell length $\beta\lambda/2$.

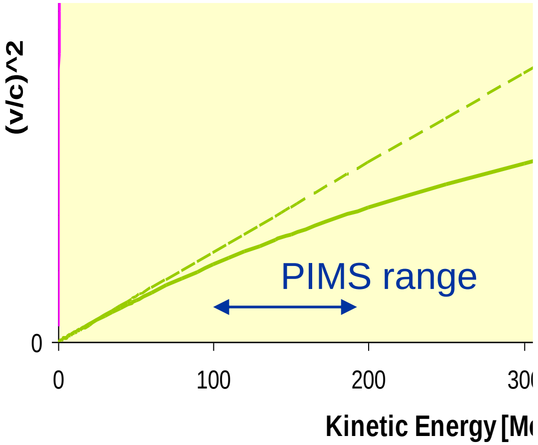


Cells in a cavity have the same length.
When more cavities are used for acceleration, the cells are longer from one cavity to the next, to follow the increase in beam velocity.

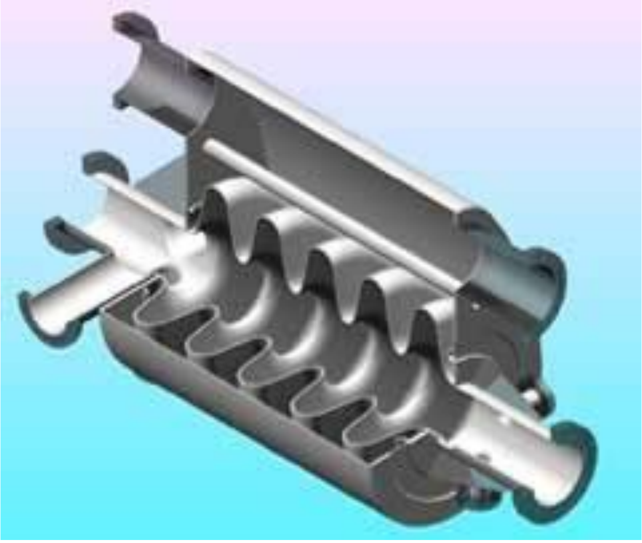
Sequence of PIMS cavities

Cells have same length inside a cavity (7 cells) but increase from one cavity to the next.
At high energy (>100 MeV) beta changes slowly and phase error ("phase slippage") is small.





Pi-mode superconducting structures (elliptical)

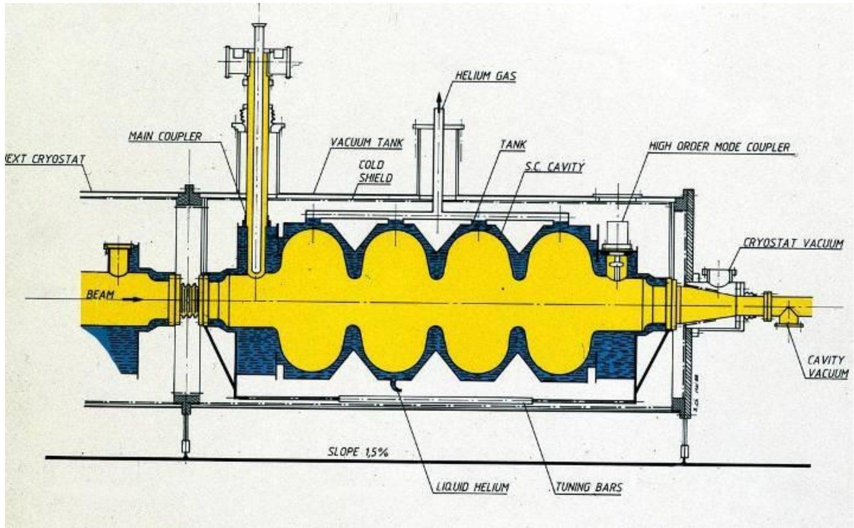


Standing wave structures for particles at β>0.5-0.7, widely used for protons (SNS, etc.) and electrons (ILC, etc.) f=350-700 MHz (protons), f=350 MHz – 3 GHz (electrons)

Chain of cells electrically coupled, large apertures (ZT^2 not a concern).

Operating in π-mode, cell length $\beta\lambda/2$

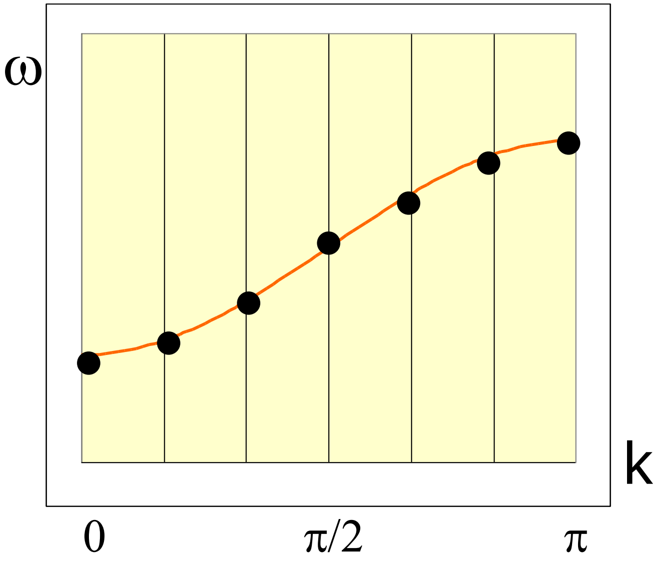
Input coupler placed at one end.

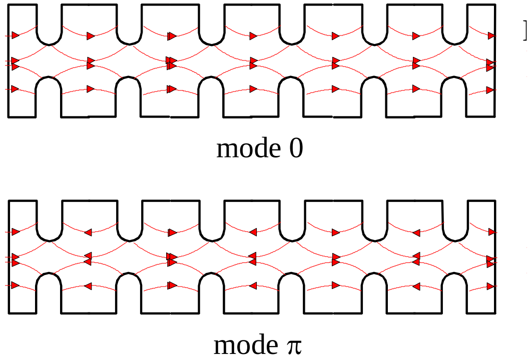


Long chains of linac cells

- To reduce RF cost, linacs use high-power RF sources feeding a large number of coupled cells (DTL: 30-40 cells, other high-frequency structures can have >100 cells).

- But long linac structures (operating in 0 or π mode) become extremely sensitive to mechanical errors: small machining errors in the cells can induce large differences in the accelerating field between cells.





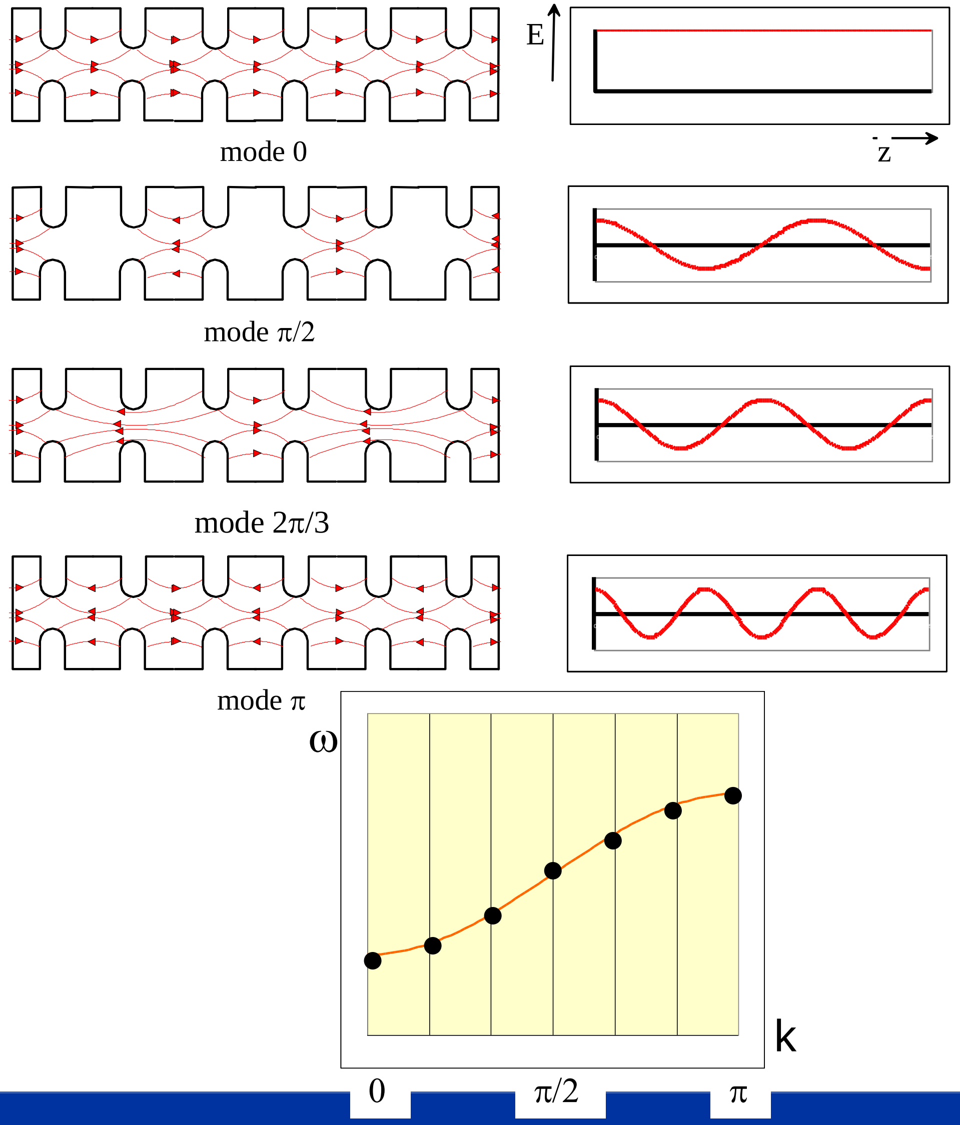
Stability of long chains of coupled resonators

Mechanical errors → differences in frequency between cells → to respect the new boundary conditions the electric field will be a linear combination of all modes, with weight

$$\frac{1}{f^2 - f_0^2}$$

(general case of small perturbation to an eigenmode system, the new solution is a linear combination of all the individual modes)

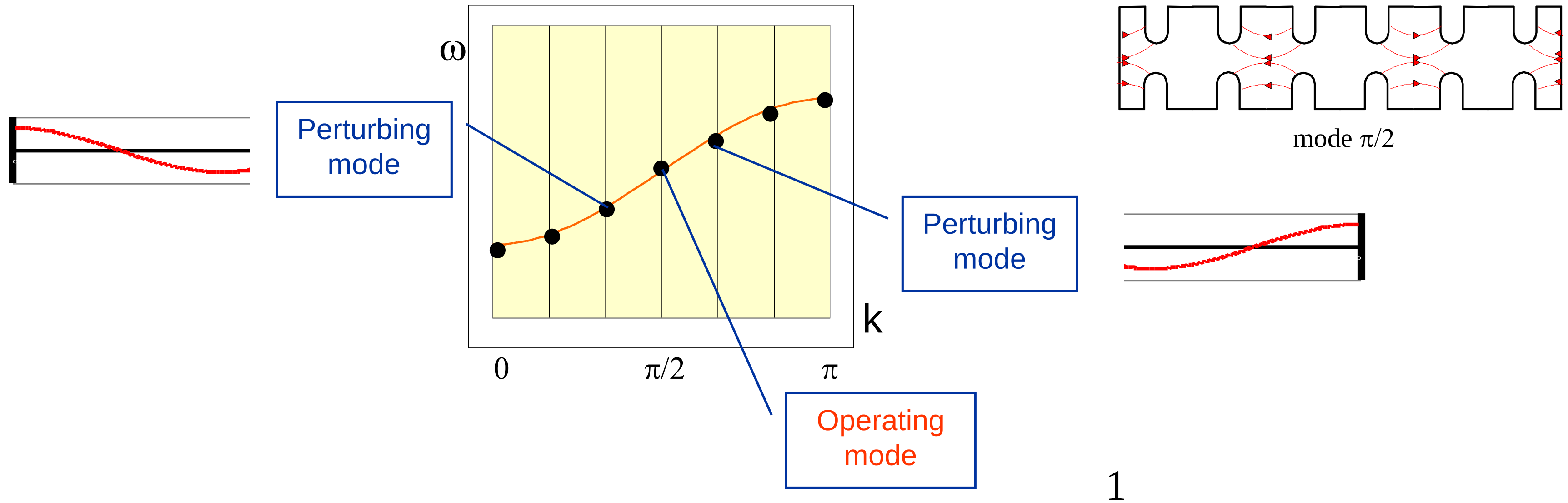
The nearest modes have the highest effect, and when there are many modes on the dispersion curve (number of modes = number of cells, but the total bandwidth is fixed = k !) the difference in E-field between cells can be extremely high.



Stabilization of long chains: the π/2 mode

Long chains of linac cells can be operated in the π/2 mode, which is intrinsically insensitive to mechanical errors = differences in the cell frequencies.

In presence of errors, the E-field will have components from the adjacent modes, with amplitude proportional to the error and to the mode separation.



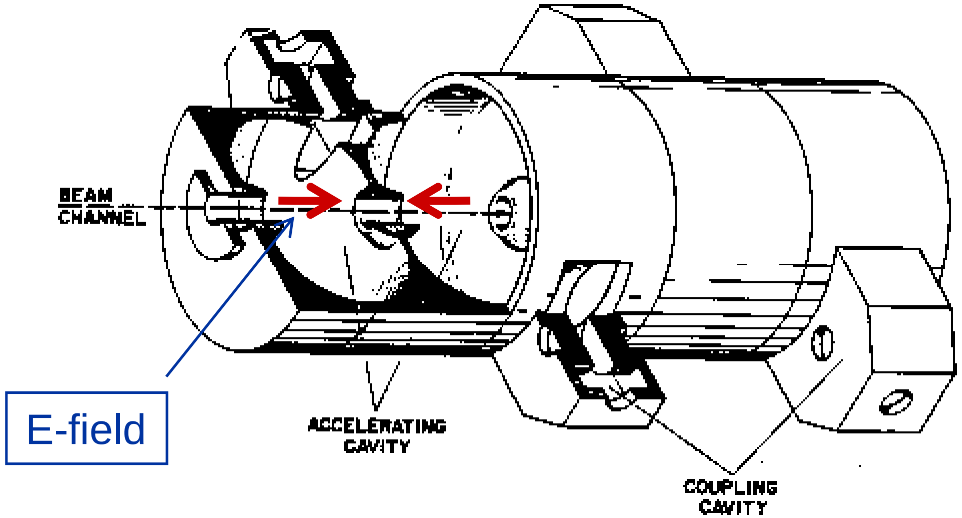
→ Contribution from adjacent modes proportional to $\frac{1}{f^2 - f_0^2}$ with the sign !!!

The perturbation will add a component $\Delta E/(f^2-f_o^2)$ for each of the nearest modes.

Contributions from equally spaced modes in the dispersion curve will cancel each other !!

Pi/2 mode structures: the Side Coupled Linac

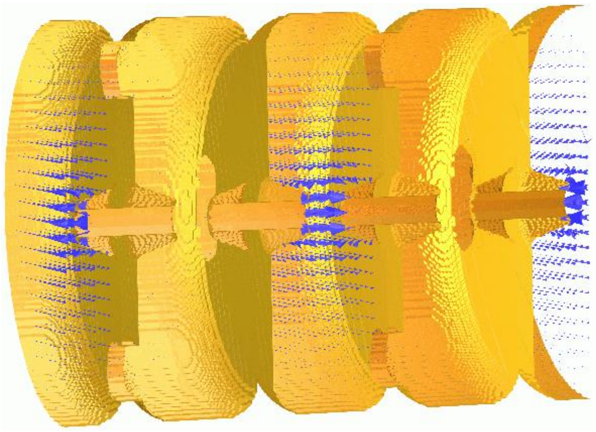
To operate efficiently in the $\pi/2$ mode, the cells that are not excited can be removed from the beam axis → they are called "coupling cells", as for the Side Coupled Structure.



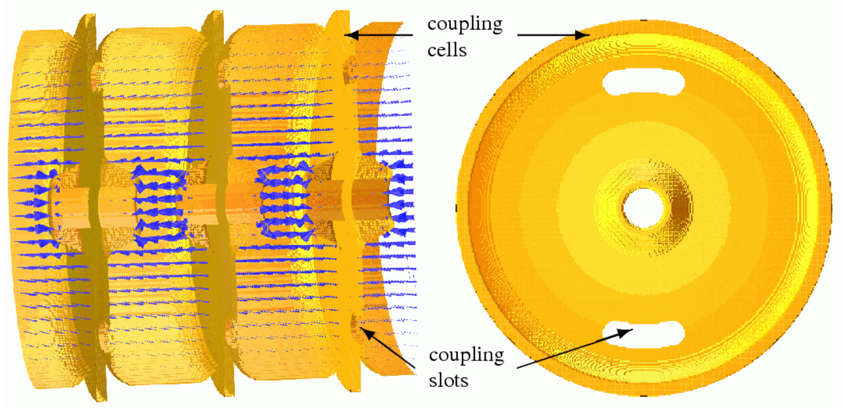
Example: the Cell-Coupled Linac at the SNS linac, 805 MHz, 100-200 MeV, >100 cells/module

Examples of $\pi/2$ structures

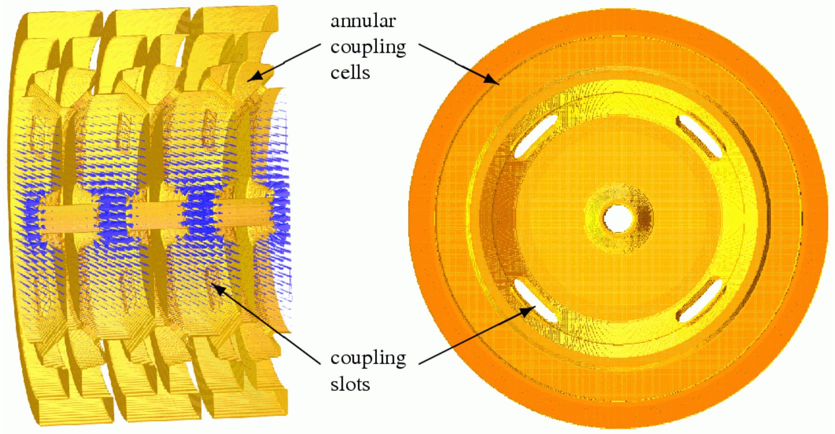
π/2-mode in a coupled-cell structure



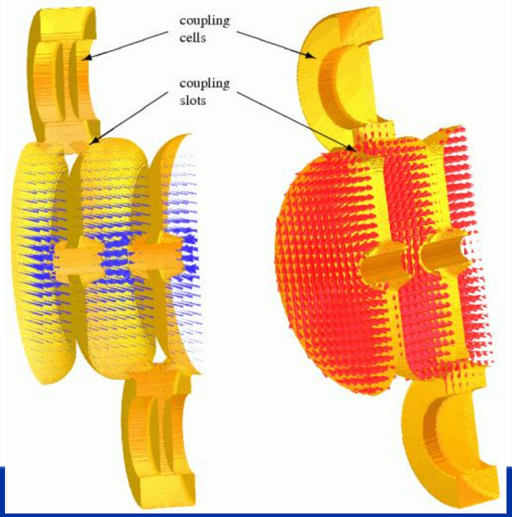
On axis Coupled Structure (OCS)



Annular ring Coupled Structure (ACS)

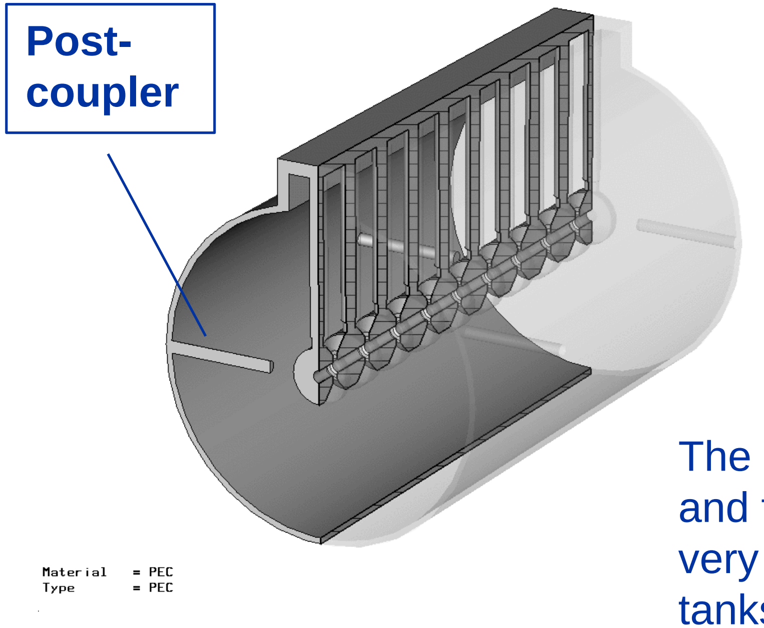


Side Coupled Structure (SCS)

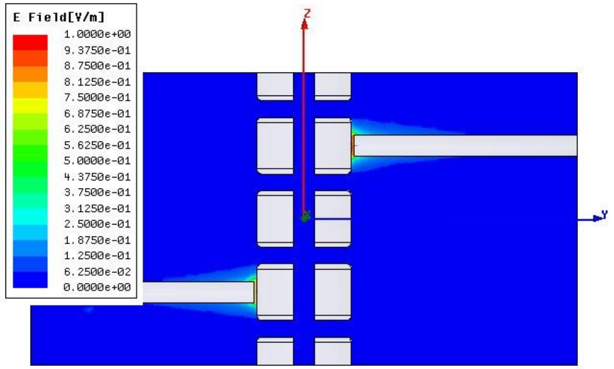


Bonus slide 1: Stabilisation of Drift Tube Linac

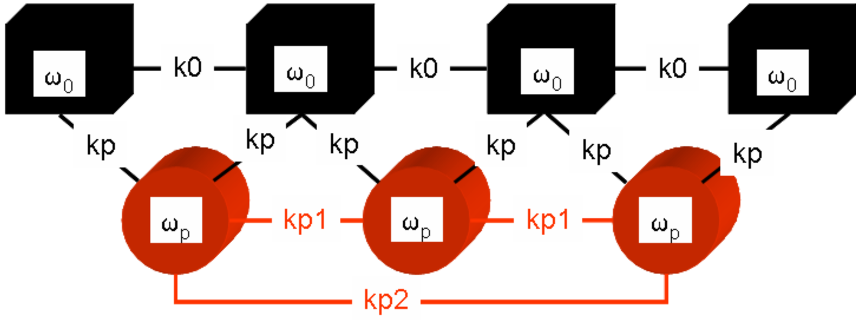
Post-coupler

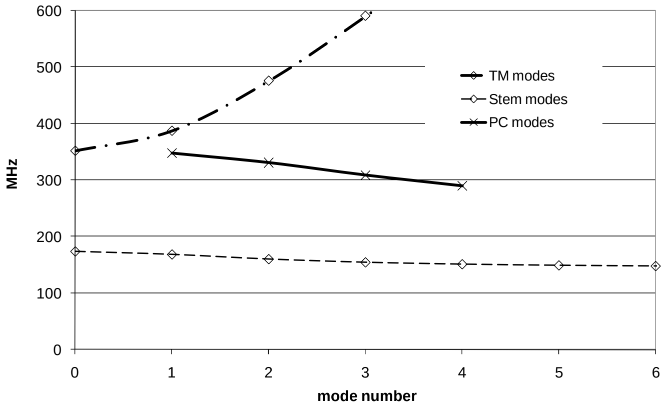


In a DTL, can be added "post-couplers" on a plane perpendicular to the stems.
Each post is a resonator that can be tuned to the same frequency as the main 0-mode and coupled to this mode to double the chain of resonators allowing operation in stabilised $\pi/2$-like mode!



The equivalent circuit becomes very complex and tuning is an issue, but $\pi/2$ stabilization is very effective and allows having long DTL tanks!





Bonus slide 2: Ion sources

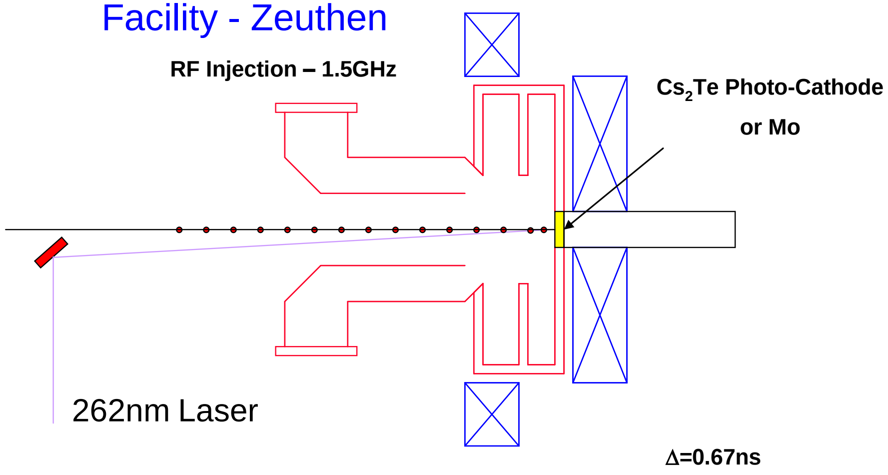
Electron sources:

give energy to the free electrons inside a metal to overcome the potential barrier at the boundary.

Used for electron production:

- thermoionic effect
- laser pulses
- surface plasma

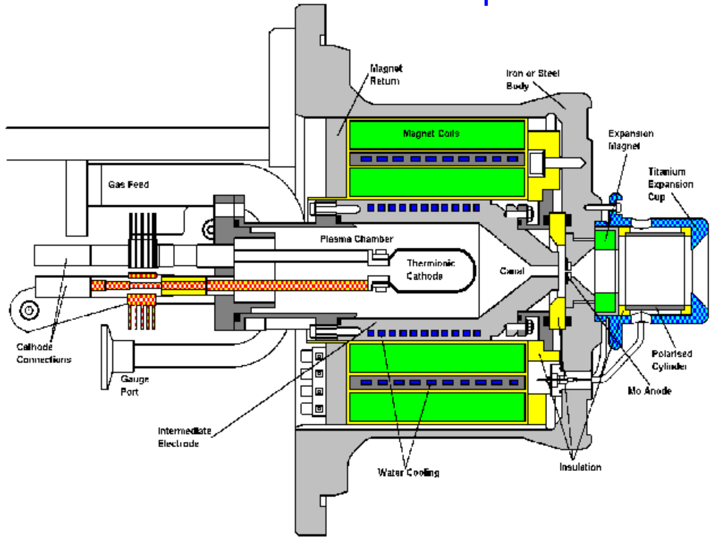
Photo Injector Test Facility - Zeuthen



Ion sources:

create a plasma and optimise its conditions (heating, confinement and loss mechanisms) to produce the desired ion type. Remove ions from the plasma via an aperture and a strong electric field.

CERN Duoplasmatron proton Source



Module 3

A quick overview of beam dynamics in linear accelerators

Introduction

A linear accelerator requires an accurate beam optics design, in order to:

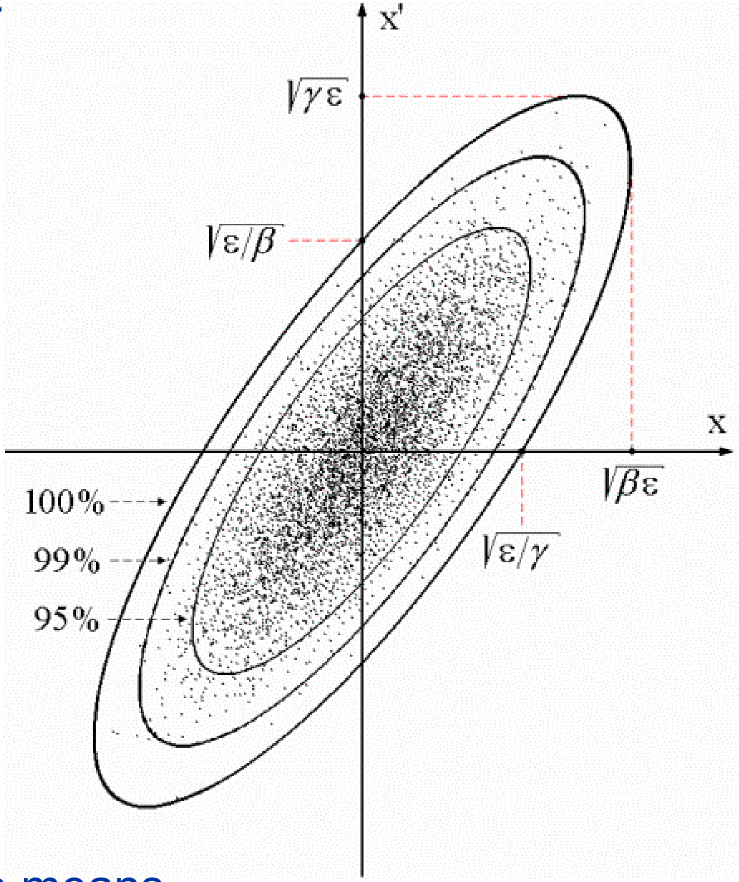
- ❑ Minimize emittance growth (remember Liouville: emittance can only increase!);
- ❑ Minimize beam loss, to: a) avoid activation of the accelerator (of the linac and of the following machine!) and b) reduce the requirements on the ion source.

Note that the operating regimes in linacs can be very different, between

- μa peak currents for heavy ion linacs (~ 10^5 particles / bunch)
- mA peak currents for proton linacs (~ 10^8 particles / bunch)
- 100's mA peak currents for high-power proton linacs (~ 10^{11} particles / bunch)
- A's peak currents for electron linacs

Beam emittance

Recall some basic definitions that we will need in the following:

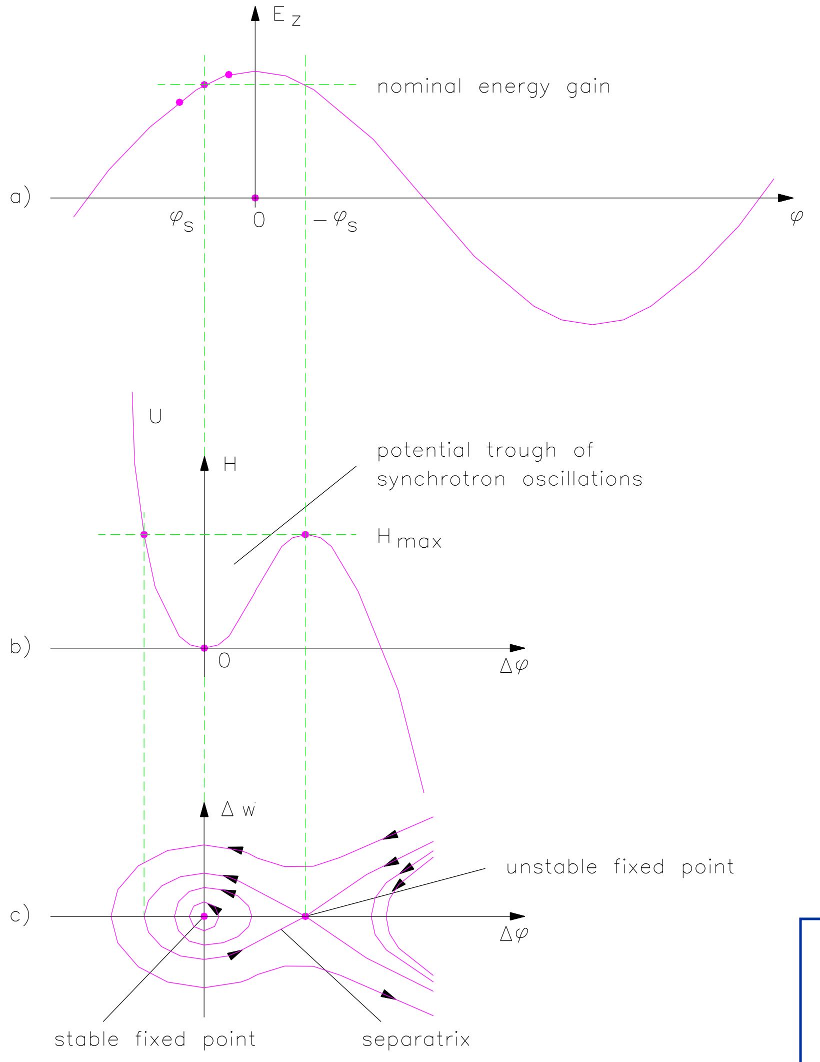


Beam emittance: is the volume in the phase space occupied by the particle beam. The space is 6-dimensional (x,x',y,y',W,ϕ) but are important the projections in the 3 dimensions: transverse emittance in x and y (ε_x, ε_y) and longitudinal emittance (ΔW, ϕ).

Liouville theorem: under the action of linear forces, the beam emittance is constant. (BUT: in presence of non-linear forces, it increases). Consequence: in an accelerator, where non-linear forces are often present, the emittance will progressively increase.

Beam brightness: beam current/transverse emittance.
Corresponds to the "density" in phase space. Preserving brightness means minimizing emittance growth.

Longitudinal dynamics



→ Ions are accelerated around a (negative = linac definition) synchronous phase.

→ Particles around the synchronous one perform oscillations in the longitudinal phase space.

→ Frequency of small oscillations:

$$\omega_l^2 = \omega_0^2 \frac{qE_0T\sin(-\varphi)\lambda}{2\pi\, mc^2\beta\gamma^3}$$

→ Tends to zero for relativistic particles γ>>1.

→ Note phase damping of oscillations:

$$\Delta\varphi = \frac{const}{(\beta\gamma)^{3/4}} \qquad \Delta W = const \times (\beta\gamma)^{3/4}$$

At relativistic velocities phase oscillations stop, and the beam is compressed in phase around the initial phase. The crest of the wave can be used for acceleration.

Longitudinal dynamics - electrons

→ Electrons at v=c remain at the injection phase.

→ Electrons at v<c injected into a TW structure designed for v=c will move from injection phase φ_0 to an asymptotic phase φ, which depends only on gradient and β_0 at injection.

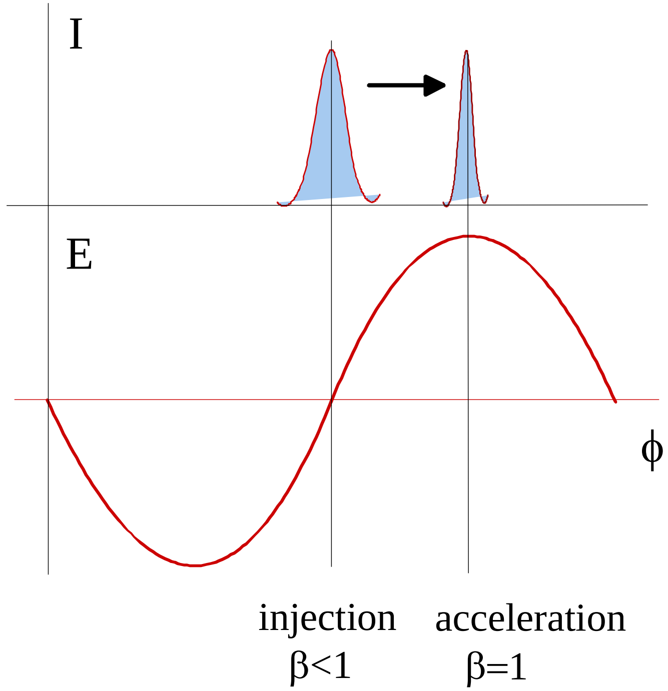
→ The beam can be injected with an offset in phase, to reach the crest of the wave at β=1

→ Capture condition, relating E_0 and β_0 :

$$\frac{2\pi}{\lambda_g}\frac{mc^2}{qE_0}\left[\sqrt{\frac{1-\beta_0}{1+\beta_0}}\right]=1$$

Example: λ=10cm, W_{in}=150 keV and E_0=8 MV/m.

$$\sin\varphi=\sin\varphi_0+\frac{2\pi}{\lambda_g}\frac{mc^2}{qE_0}\left[\sqrt{\frac{1-\beta_0}{1+\beta_0}}-\sqrt{\frac{1-\beta}{1+\beta}}\right]$$



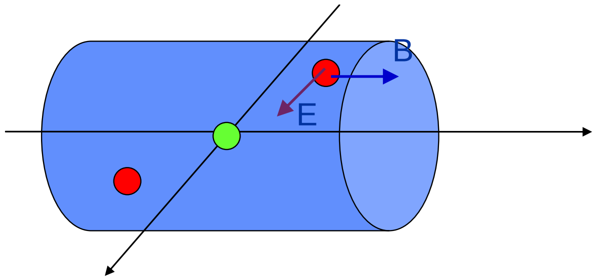
In high current linacs, a bunching and pre-acceleration sections up to 4-10 MeV prepares the injection in the TW structure that occurs already on the crest

Transverse dynamics - Space charge

→ Large numbers of particles per bunch (~10^{10}).

→ Coulomb repulsion between particles (space charge) plays an important role and is the main limitation to the maximum current in a linac.

→ But space charge forces ~ $1/\gamma^2$ disappear at relativistic velocity

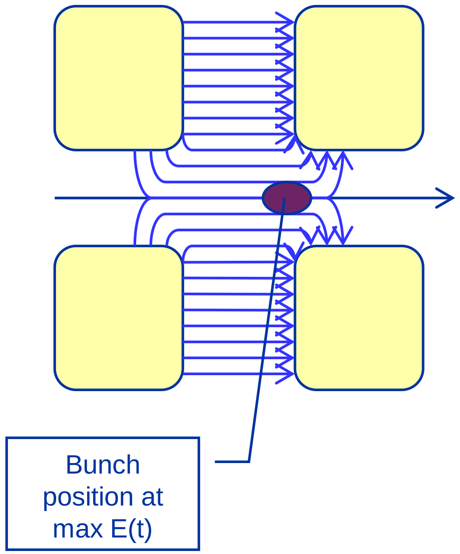


Force on a particle inside a long bunch with density n(r) traveling at velocity v:

$$E_r = \frac{e}{2\pi\varepsilon r}\int_0^r n(r)\,r\,dr \qquad B_\varphi = \frac{\mu}{2\pi}\frac{ev}{r}\int_0^r n(r)\,r\,dr$$

$$F = e(E_r - vB_\varphi) = eE_r\left(1-\frac{v^2}{c^2}\right) = eE_r(1-\beta^2) = \frac{eE_r}{\gamma^2}$$

Transverse dynamics - RF defocusing



→ RF defocusing experienced by particles crossing a gap on a longitudinally stable phase.

→ In the rest frame of the particle, only electrostatic forces → no stable points (maximum or minimum) → radial defocusing.

→ Lorentz transformation and calculation of radial momentum impulse per period (from electric and magnetic field contribution in the laboratory frame):

$$\Delta p_r = -\frac{\pi\, e\, E_0\, T\, L\, r \sin\varphi}{c\,\beta^2\,\gamma^2\,\lambda}$$

→ Transverse defocusing ~ $1/\gamma^2$ disappears at relativistic velocity (transverse magnetic force cancels the transverse RF electric force).

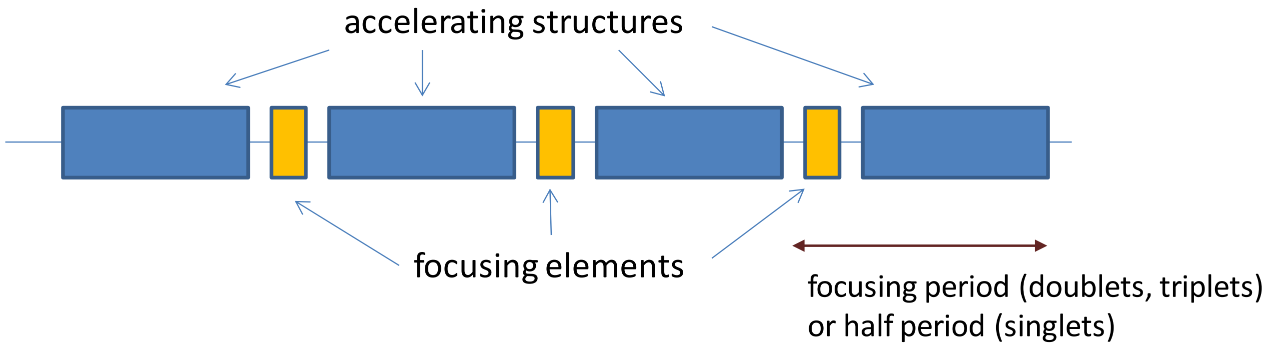
Focusing

Defocusing forces need to be compensated by focusing forces → alternating gradient focusing provided by quadrupoles along the beam line.

A linac alternates accelerating sections with focusing sections. Options are: one quadrupole (singlet focusing), two quadrupoles (doublet focusing) or three quadrupoles (triplet focusing).

Focusing period=length after which the structure is repeated (usually as $N\beta\lambda$).

The accelerating sections have to match the increasing beam velocity → the basic focusing period increases in length (but the beam travel time in a focusing period remains constant).
The maximum allowed distance between focusing elements depends on beam energy and current and change in the different linac sections (from only one gap in the DTL to one or more multi-cell cavities at high energies).



Transverse beam equilibrium in linacs

The equilibrium between external focusing force and internal defocusing forces defines the frequency of beam oscillations.
Oscillations are characterized in terms of *phase advance per focusing period* σ_t or *phase advance per unit length* k_t.

Ph. advance = Ext. quad focusing - RF defocusing - space charge

$$k_t^2 = \left(\frac{\sigma_t}{N\beta\lambda}\right)^2 = \left(\frac{qGl}{2mc\beta\gamma}\right)^2 - \frac{\pi q E_0 T \sin(-\varphi)}{mc^2 \lambda \beta^3\gamma^3} - \frac{3qI\lambda(1-f)}{8\pi\varepsilon_0 r_0^3 mc^3 \beta^2\gamma^3}$$

q=charge
G=quad gradient
l=length foc. element
f=bunch form factor
r_0=bunch radius
λ=wavelength
…

Approximate expression valid for:
FODO lattice, smooth focusing approximation, space charge of a uniform 3D ellipsoidal bunch.

A "low-energy" linac is dominated by space charge and RF defocusing forces !!

Phase advance per period must stay in reasonable limits (30-80 deg), phase advance per unit length must be continuous (smooth variations) $\rightarrow$ at low β, we need a strong focusing term to compensate for the defocusing, but the limited space limits the achievable G and l $\rightarrow$ needs to use short focusing periods N $\beta\lambda$.

Note that the RF defocusing term $\propto$f sets a higher limit to the basic linac frequency (whereas for shunt impedance considerations we should aim to the highest possible frequency, Z $\propto\sqrt{f}$) .

Electron linacs

Ph. advance = Ext. quad focusing - RF defocusing - space charge – Instabilities

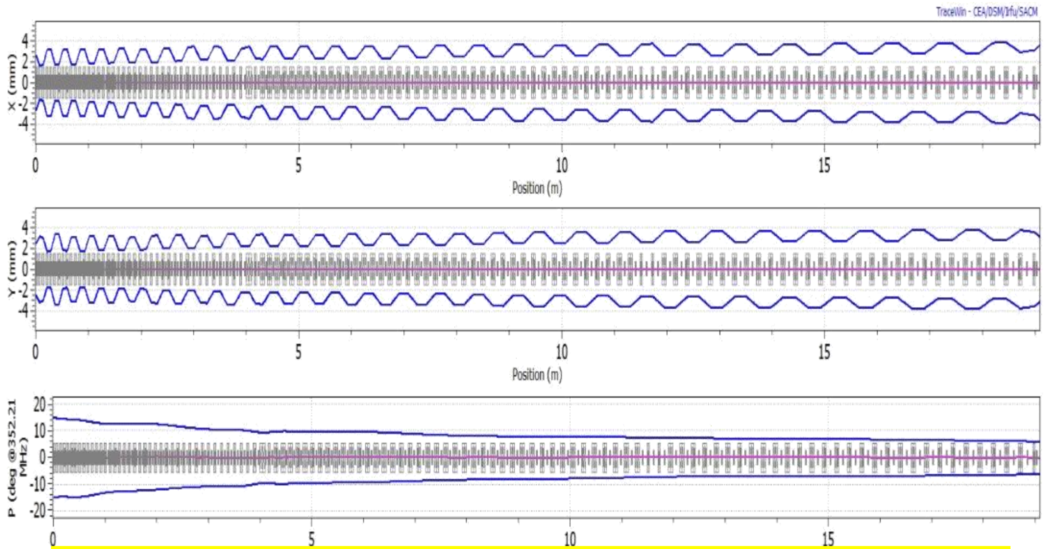
$$k_t^2 = \left(\frac{\sigma_t}{N\beta\lambda}\right)^2 = \left(\frac{q\,G\,l}{2\,mc\,\beta\gamma}\right)^2 - \frac{\pi\,q\,E_0 T \sin(-\varphi)}{mc^2\lambda\,\beta^3\gamma^3} - \frac{3q\,I\,\lambda(1-f)}{8\pi\varepsilon_0\,r_0^{\,3} mc^3\beta^2\gamma^3} - \ldots$$

Electron Linac:
Ph. advance = Ext. focusing + RF defocusing + space charge + Instabilities

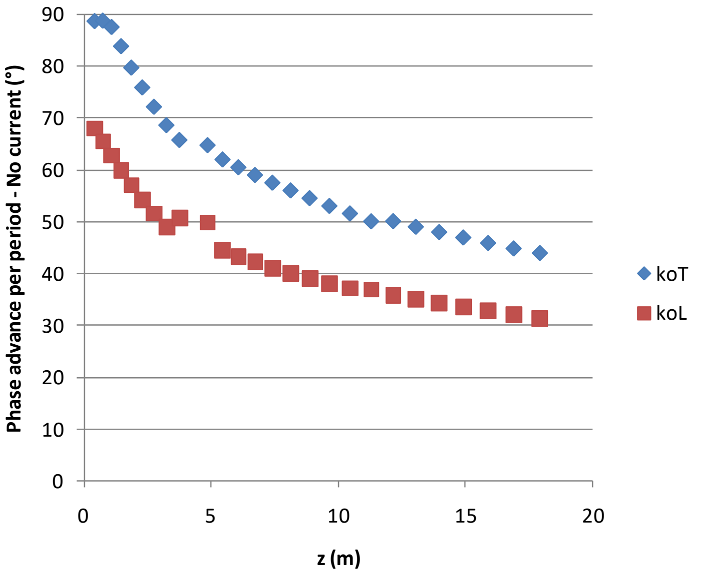
For $\gamma >> 1$ (electron linac): RF defocusing and space charge disappear, *phase advance →0*. External focusing is required only to control the emittance and to stabilize the beam against instabilities (as wakefields and beam breakup).

Phase advance – an example

Beam optics of the Linac4 Drift Tube Linac (DTL): 3 to 50 MeV, 19 m, 108 focusing quadrupoles (permanent magnets).



Oscillations of the beam envelope (coordinates of the outermost particle) along the DTL (x, y, phase)



Corresponding phase advance per period

Design prescriptions:

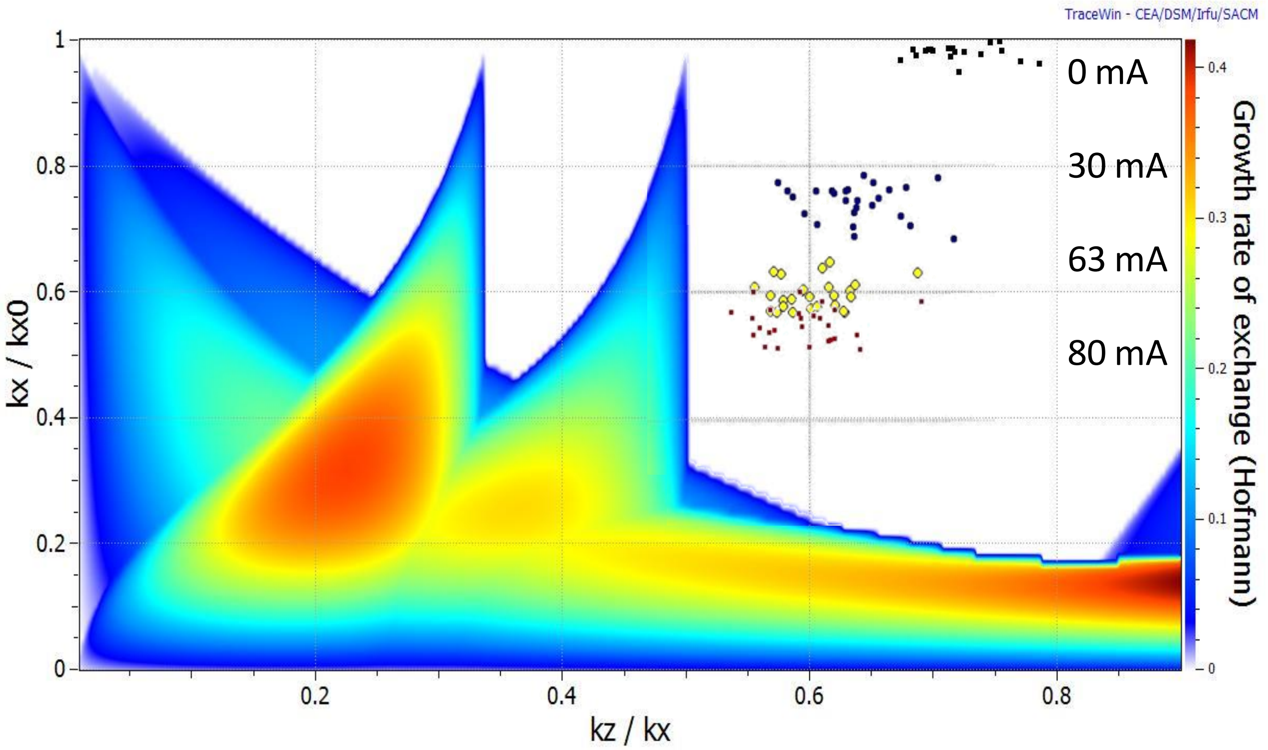
•Tranverse phase advance at zero current always less than 90°.

• Smooth variation of the phase advance.

• Avoid resonances (see next slide).

Instabilities in linacs – the Hoffman plot

Ratio between transverse phase advance and zero current phase advance (a measure of space charge)



The "tune diagram" of a linac

Ratio between longitudinal and transverse phase advance

Linac4 DTL: the operating point(s) for all possible current levels are far from the resonances between transverse and longitudinal oscillations which are enhanced by space charge.

Effect of the resonances: emittance exchange, transverse emittance growth, migration of particles into the beam halo → particularly dangerous for high intensity machines (beam loss).

Focusing periods

Focusing usually provided by quadrupoles.

Need to keep the phase advance in the good range, with an approximately constant phase advance per unit length → The length of the focusing periods has to change along the linac, going gradually from short periods in the initial part (to compensate for high space charge and RF defocusing) to longer periods at high energy.

For Protons (high beam current and high space charge), distance between two quadrupoles (=1/2 of a FODO focusing period):
- $\beta\lambda$ in the DTL, from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV),
- can be increased to 4-10$\beta\lambda$ at higher energy (>40 MeV).
- longer focusing periods require special dynamics (example: the IH linac).

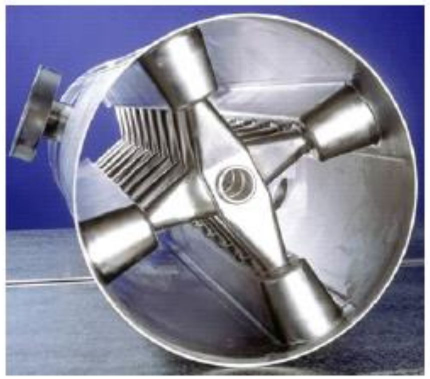
For Electrons (no space charge, no RF defocusing):
focusing periods up to several meters, depending on the required beam conditions. Focusing is mainly required to control the emittance.

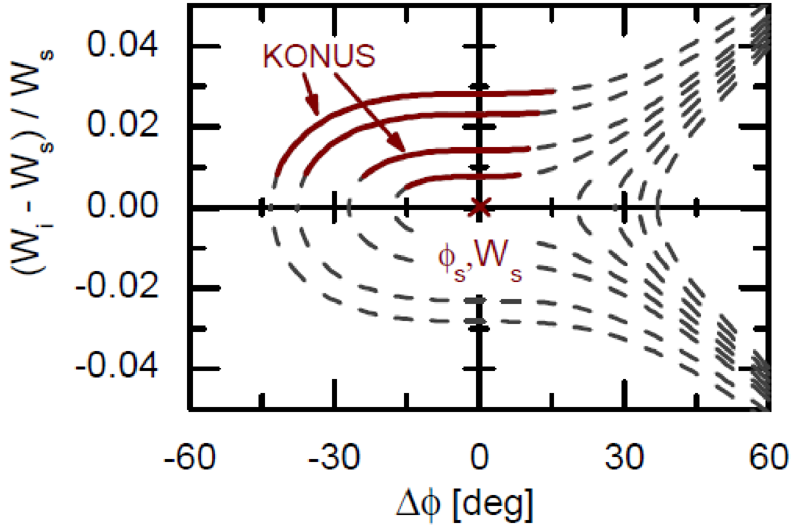
The IH and CH: a special case

The IH and CH structures used at GSI are based on a special beam dynamics called the KONUS dynamics.

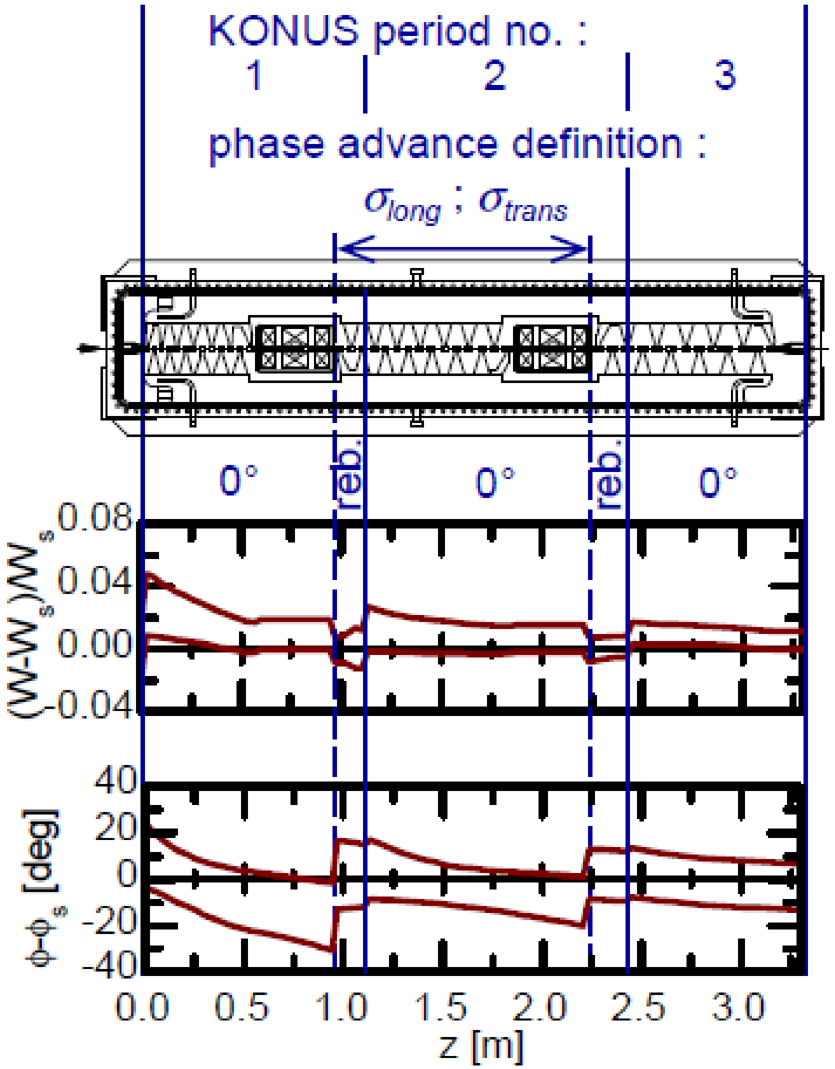
There is no stable particle. The beam from a relatively low energy goes through a series of accelerating gaps WITHOUT FOCUSING ELEMENTS.

The RF defocusing could completely destroy the beam: to avoid it, a gymnastics in the longitudinal plane is applied, where the beam rotates around a stable point corresponding to acceleration on the crest of the wave (phase=0).





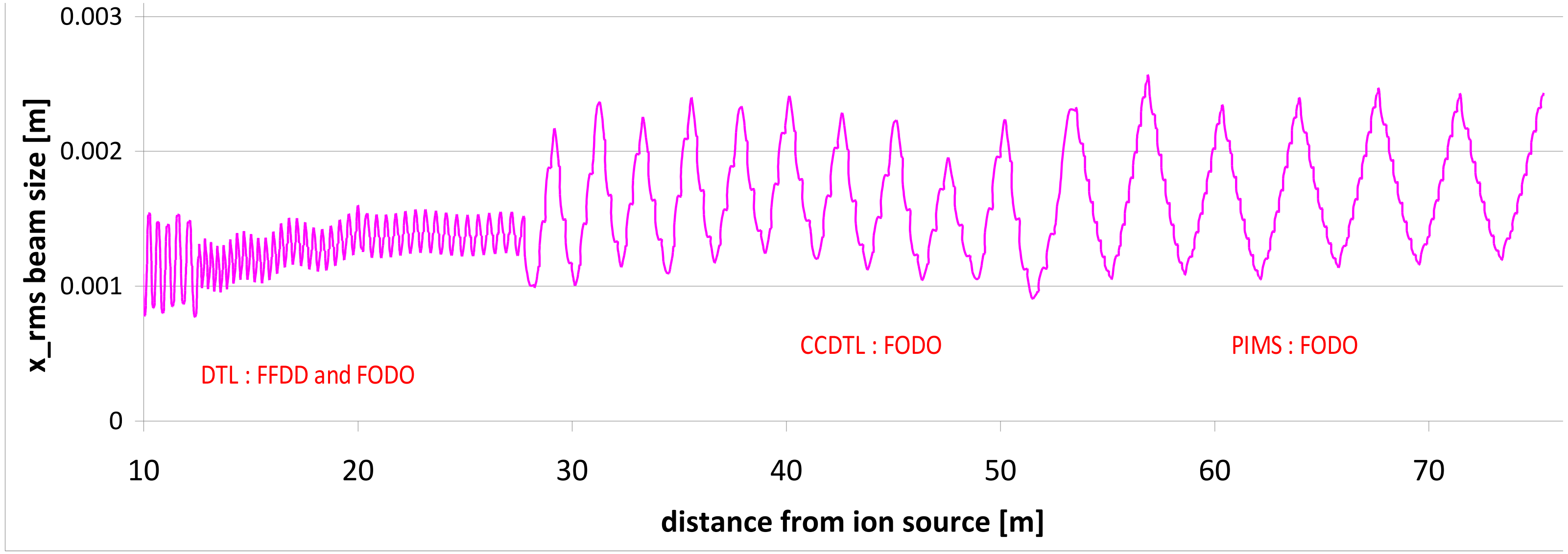
IH-CH rebunching



A rebunching section at the beginning of each period (few gaps)

High-intensity protons – the case of Linac4

Transverse (x) r.m.s. beam envelope along Linac4



Example: beam dynamics design for Linac4@CERN.

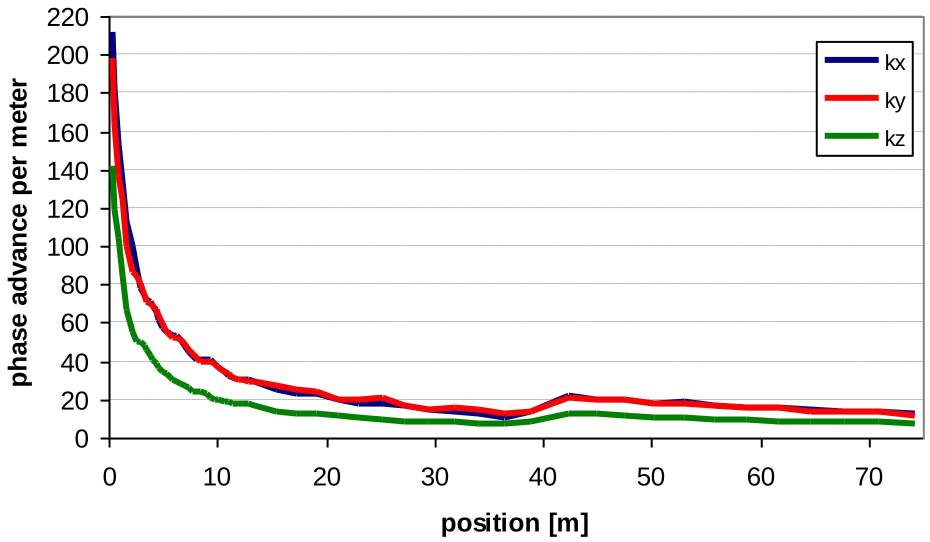
High intensity protons (60 mA bunch current, duty cycle could go up to 5%), 3 - 160 MeV

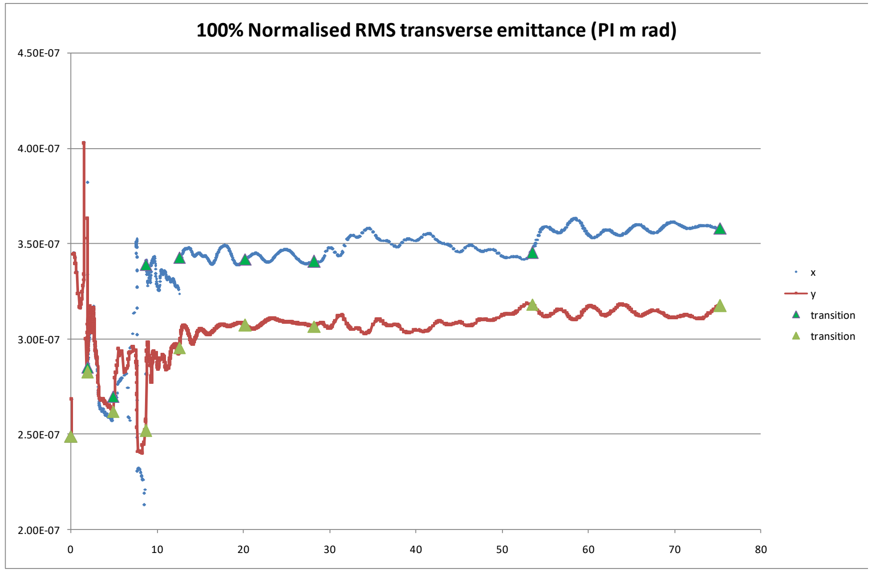
Beam dynamics design minimising emittance growth and halo development in order to:
1. avoid uncontrolled beam loss (activation of machine parts)
2. preserve small emittance (high luminosity in the following accelerators)

Beam Optics Design Guidelines

Prescriptions to minimise emittance growth and halo formation:

1. Keep zero current phase advance always below 90°, to avoid resonances
2. Keep longitudinal to transverse phase advance ratio 0.5-0.8, to avoid emittance exchange
3. Keep a smooth variation of transverse and longitudinal phase advance per meter.
4. Keep sufficient safety margin between beam radius and aperture





Transverse r.m.s. emittance and phase advance along Linac4 (RFQ-DTL-CCDTL-PIMS)

Halo and beam loss

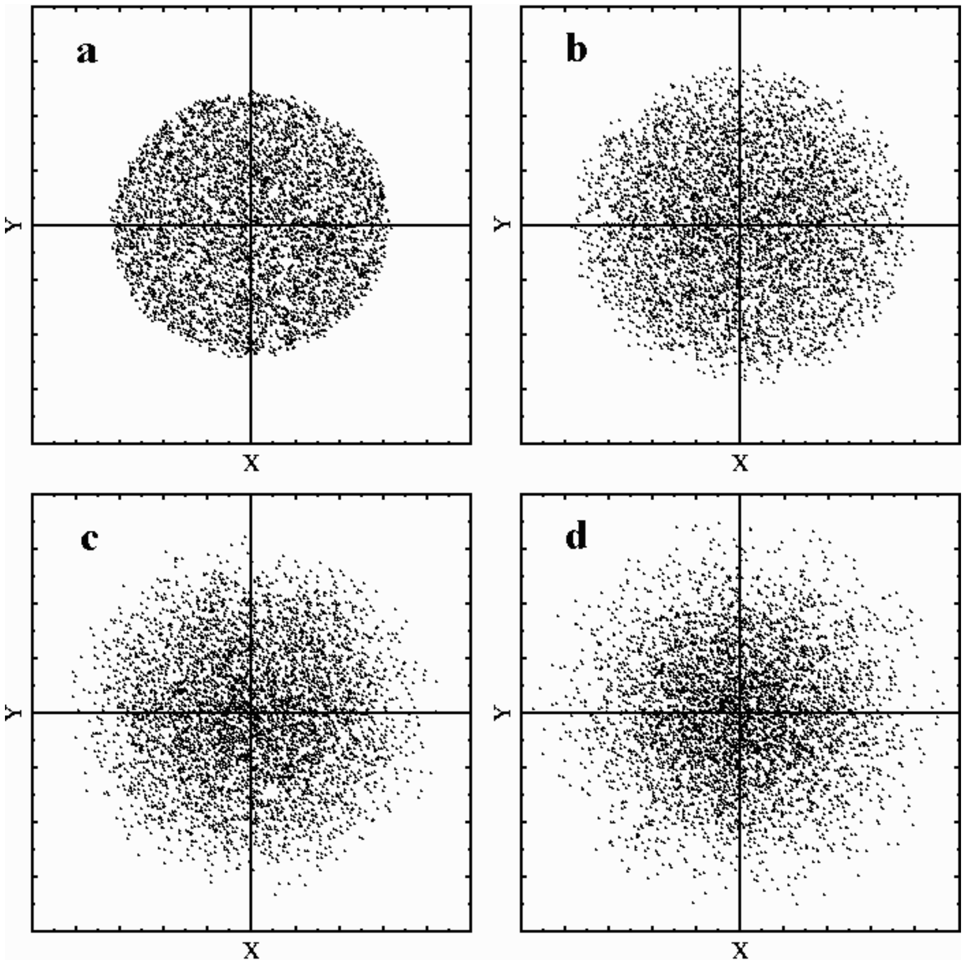
Additional challenge on beam dynamics:

Beam loss has to be avoided not because it reduces current, but because activation due to beam loss could come to levels preventing access to the machine.

Commonly accepted loss limit for hands-on maintenance is 1 W/m.

For example, in the case of the SPL design at CERN (5 GeV, 20 mA, 5% duty cycle) this corresponds to a maximum loss at 5 GeV of 0.2×10^{-6}/m, or 4 nA/m !!

In usual linacs, the beam distribution can be quite complicated, and presents a core surrounded by a “halo”. Halo formation has to be studied and controlled, but is at the limit of capability for modern computers

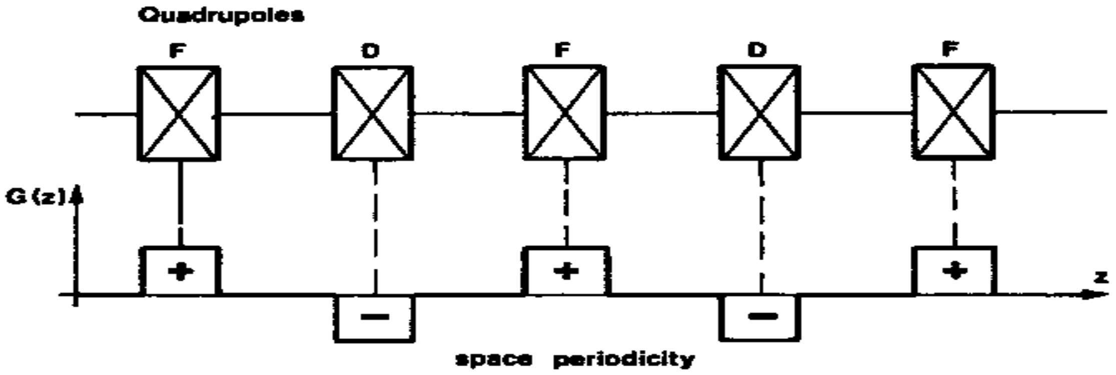


High brightness

Current and emittance are defined by the ion source: common ion sources have maximum currents ~ 80 mA for H-, ~ 200 mA for protons, in emittances of >0.02 π mm mrad.

In the linac, non-linearities in the focusing channel and in the space charge forces tend to increase the emittance and to decrease brightness.

Beam brightness: beam current/transverse emittance.
Corresponds to the “density” in phase space. Preserving brightness means minimizing emittance growth.

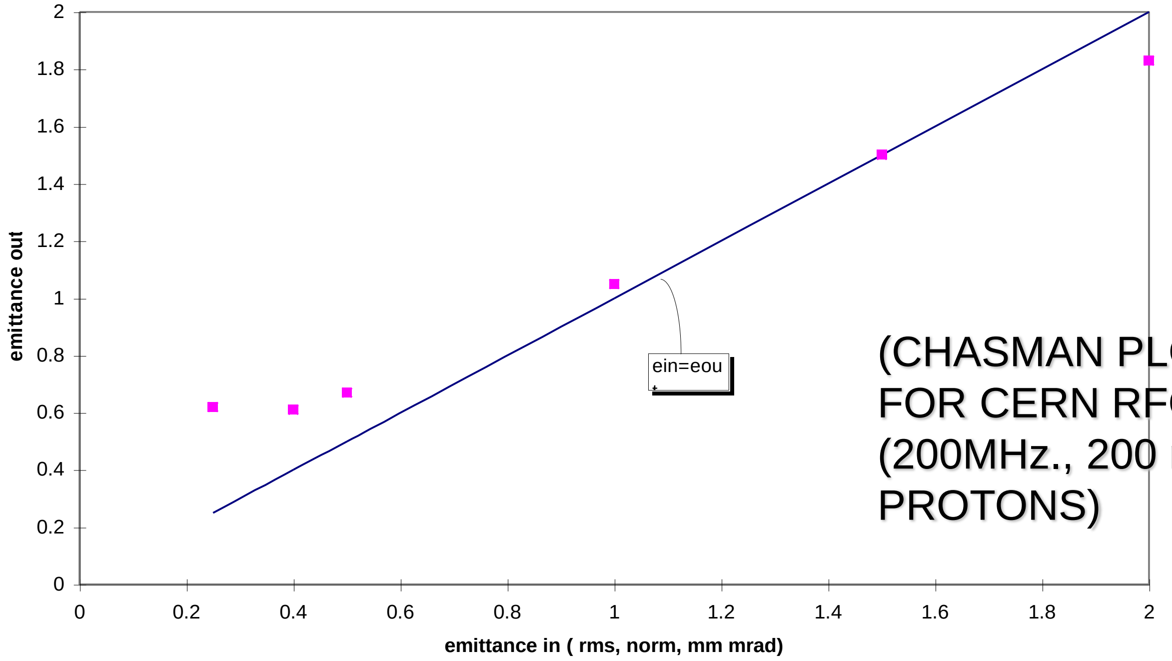


space charge – non linear effects

the more the beam is compressed in real space, the more the space charge effect is non linear

non linear space charge effect generates emittance growth

at low energy space charge is the limiting factor for the minimum emittance that can be produced out of an accelerator



(CHASMAN PLOT) FOR CERN RFQ2 (200MHz., 200 mA PROTONS)

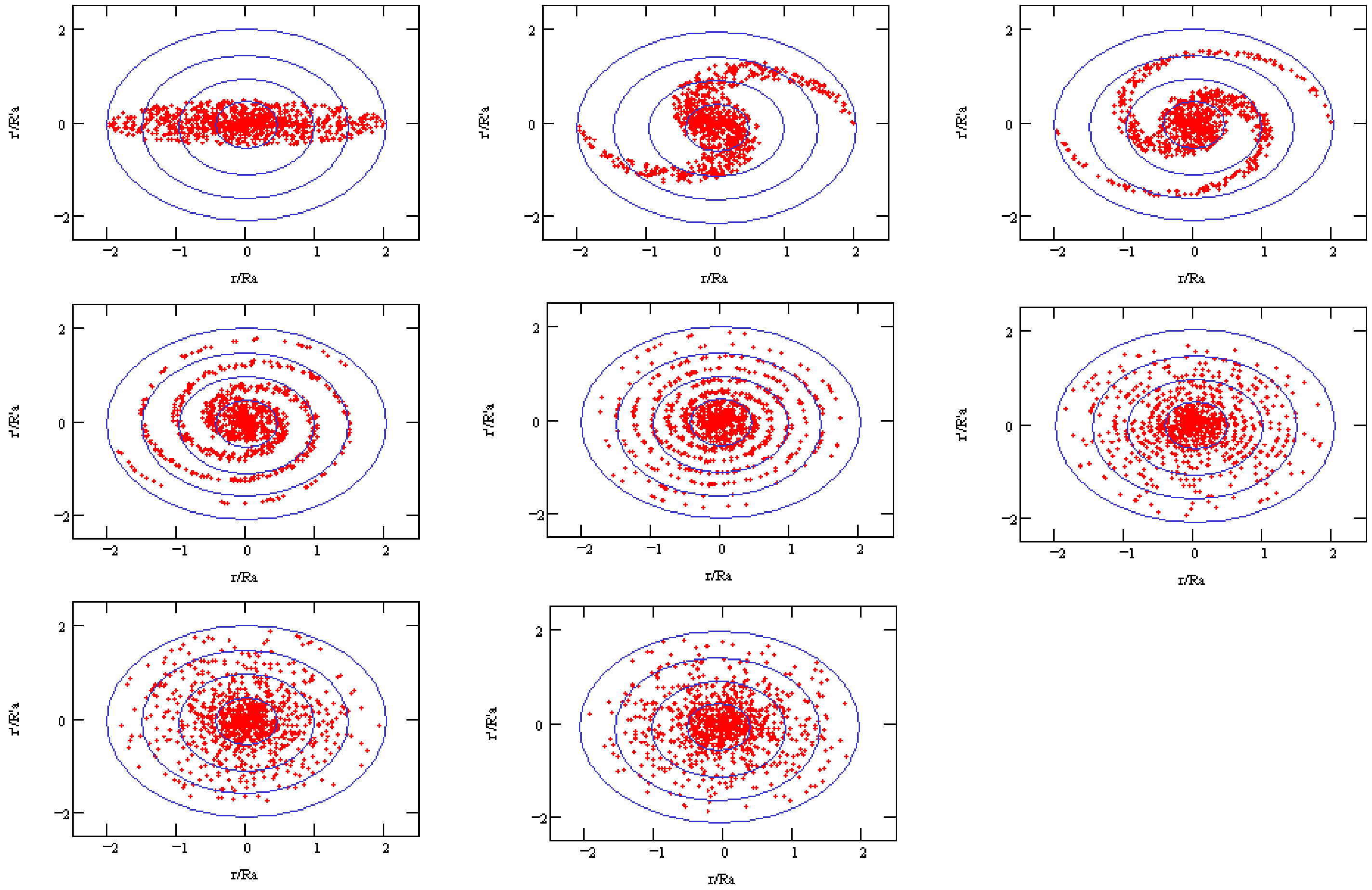
Minimum emittance from space charge

emittance growth due to filamentation

velocity of rotation in the transverse phase space with no space charge doesn't depend on the amplitude.

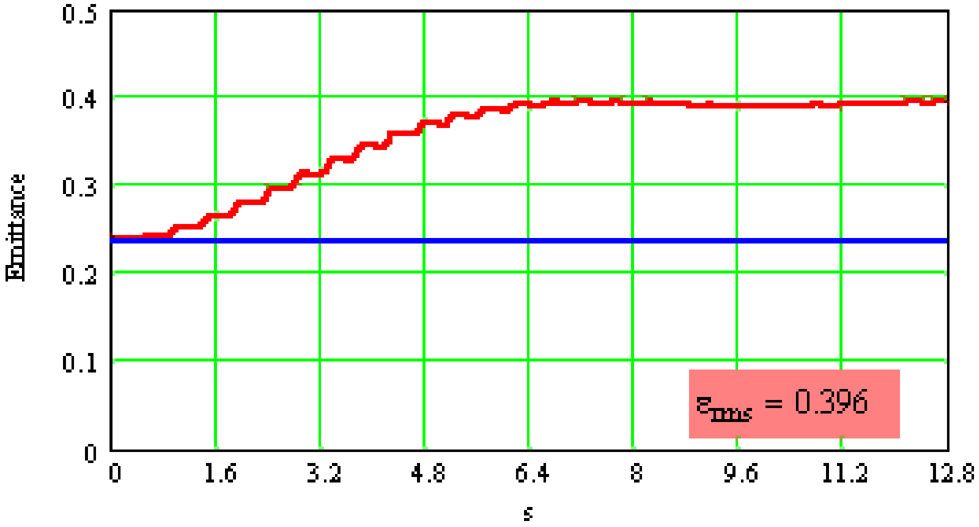
with linear space charge it is lowered but it still doesn't depend on the amplitude

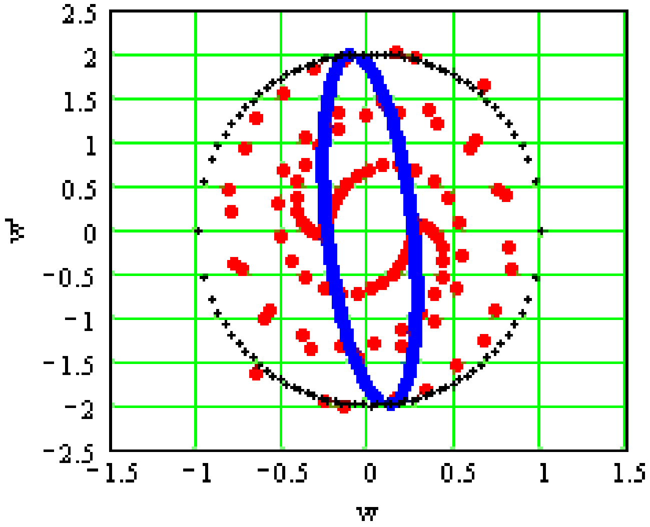
with non linear space charge it does depend on amplitude and therefore there are areas of the phase space move at different velocity. This generates emittance growth.



Filamentation: emittance increase

Evolution of the emittance along an accelerator under the influence of linear forces only (blue line) or non-linear forces (red line)





Linear force

Non linear force

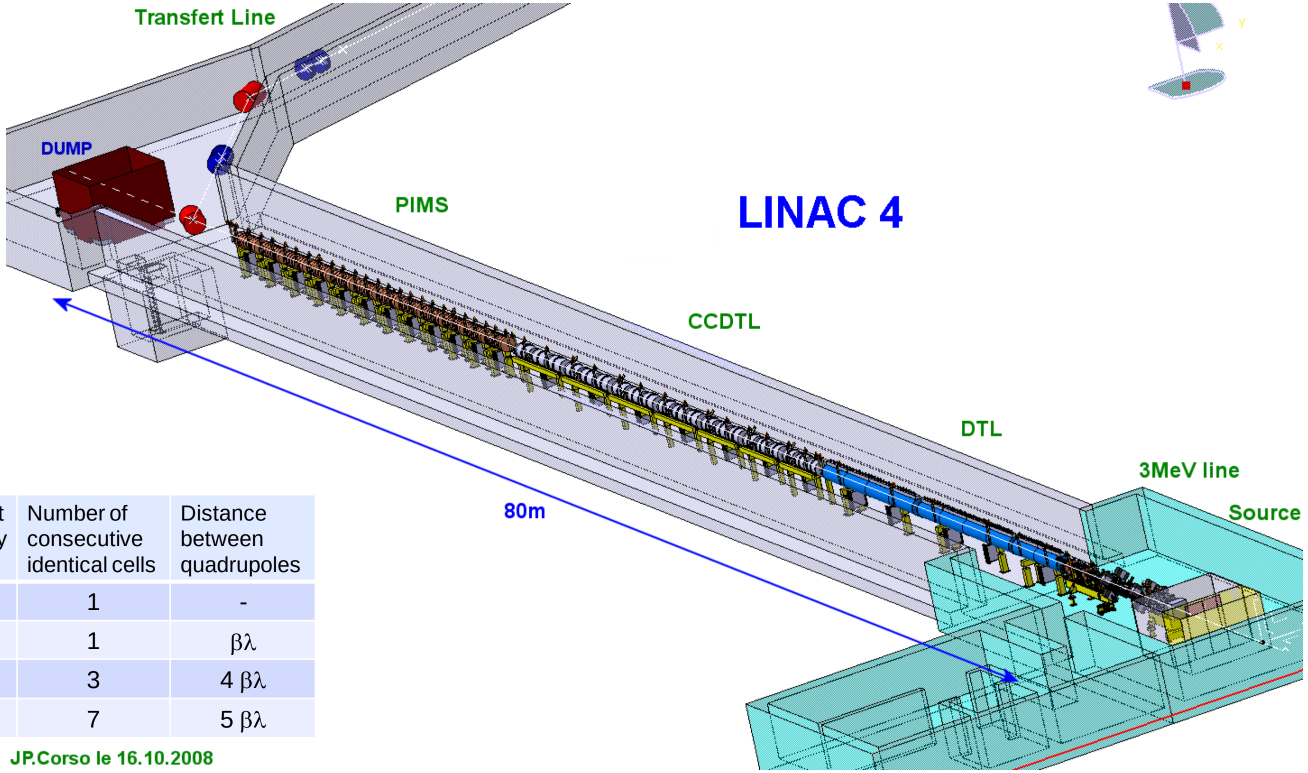
Module 4

Linac architecture (and limitations)

Proton linac architecture – cell length, focusing period

EXAMPLE: the Linac4 project at CERN. H-, 160 MeV energy, 352 MHz.
A 3 MeV injector + 22 multi-cell standing wave accelerating structures of 3 types

DTL: every cell is different, focusing quadrupoles in each drift tube
CCDTL: sequences of 2 identical cells, quadrupoles every 3 cells
PIMS: sequences of 7 identical cells, quadrupoles every 7 cells



Section	Output Energy [MeV]	Number of consecutive identical cells	Distance between quadrupoles
RFQ	3	1	-
DTL	50	1	$\beta\lambda$
CCDTL	100	3	4 $\beta\lambda$
PIMS	160	7	5 $\beta\lambda$

JP.Corso le 16.10.2008

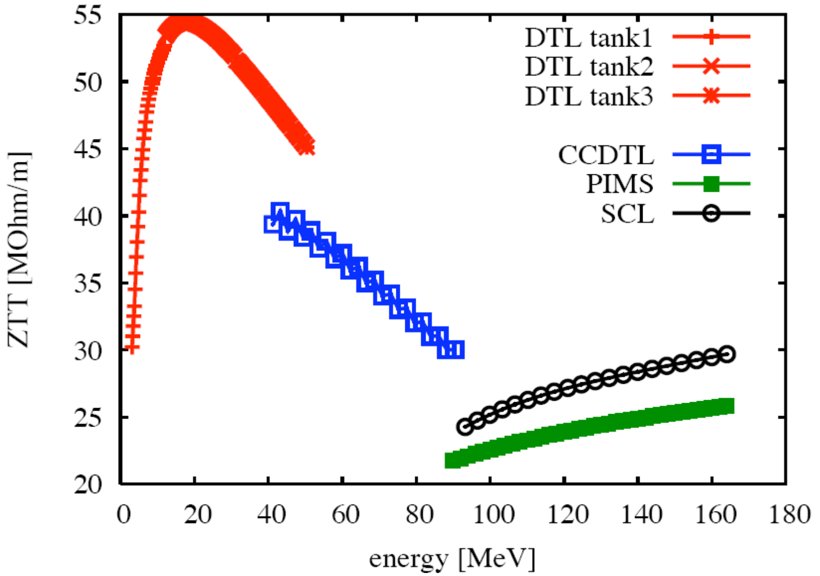
Two basic principles to remember:

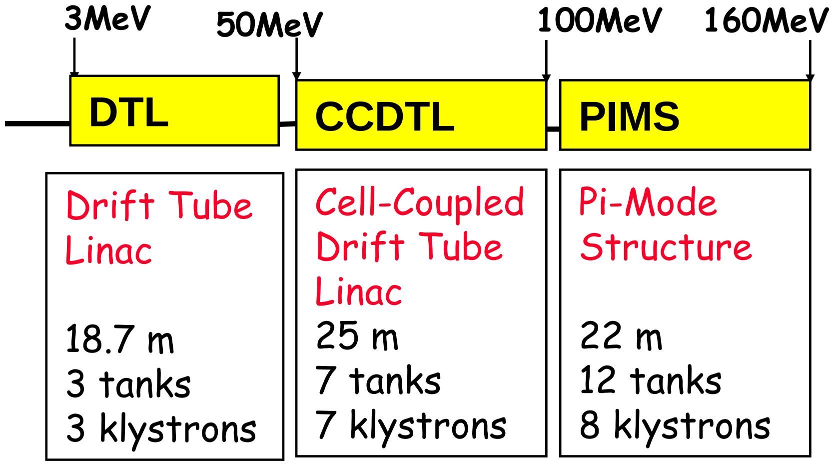
1. As beta increases, phase error between cells of identical length becomes small → we can have short sequences of identical cells (lower construction costs).

2. As beta increases, the distance between focusing elements can increase (more details in 2nd lecture!).

Proton linac architecture – Shunt impedance

A third basic principle:
Every proton linac structure has a characteristic curve of shunt impedance (=acceleration efficiency) as function of energy, which depends on the mode of operation.





The choice of the best accelerating structure for a certain energy range depends on shunt impedance, but also on beam dynamics and construction cost.

$$\Delta W = eE_0 T \cos\varphi$$

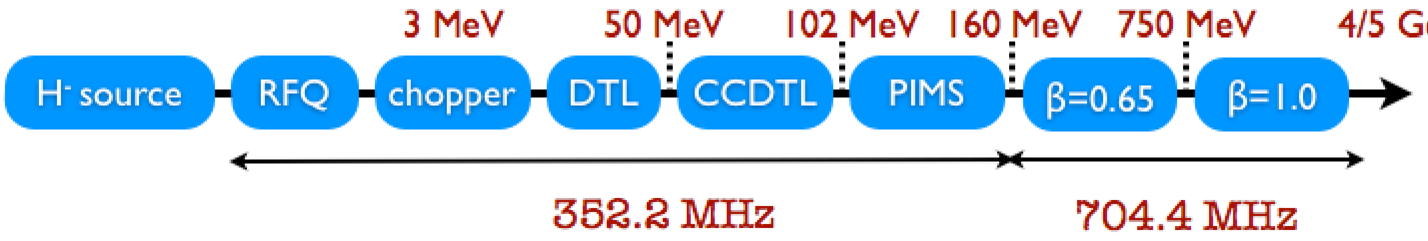
$$ZT^2 = \frac{V_{eff}^2}{P} = \frac{(E_0 T)^2}{P}$$

Effective shunt impedance ZT^2: ratio between voltage (squared) seen by the beam and RF power.

It corresponds to the parallel resistance of the equivalent circuit (apart a factor 2)

High Energy Linacs

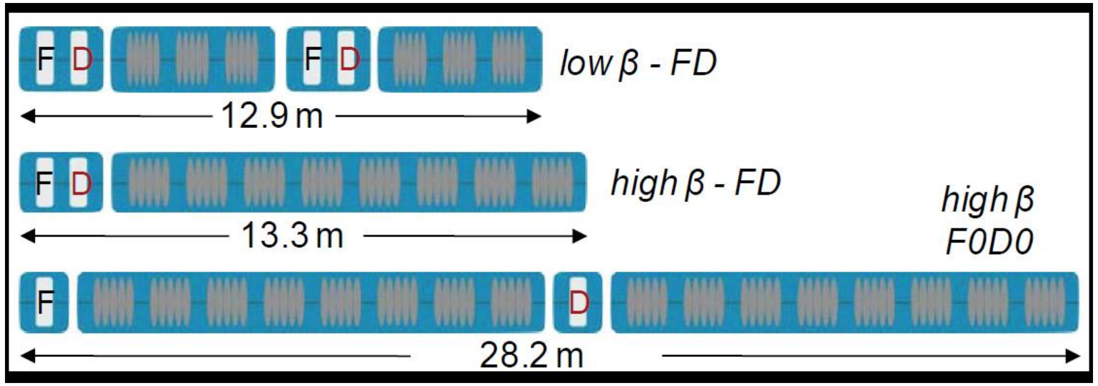
Superconducting Proton Linac a CERN project for extending Linac4 up to 5 GeV energy



low-beta: 20 cryo-modules with 3 cavities/cryo-module

high-beta -FD: 13 cryo-modules with 8 cavities/module

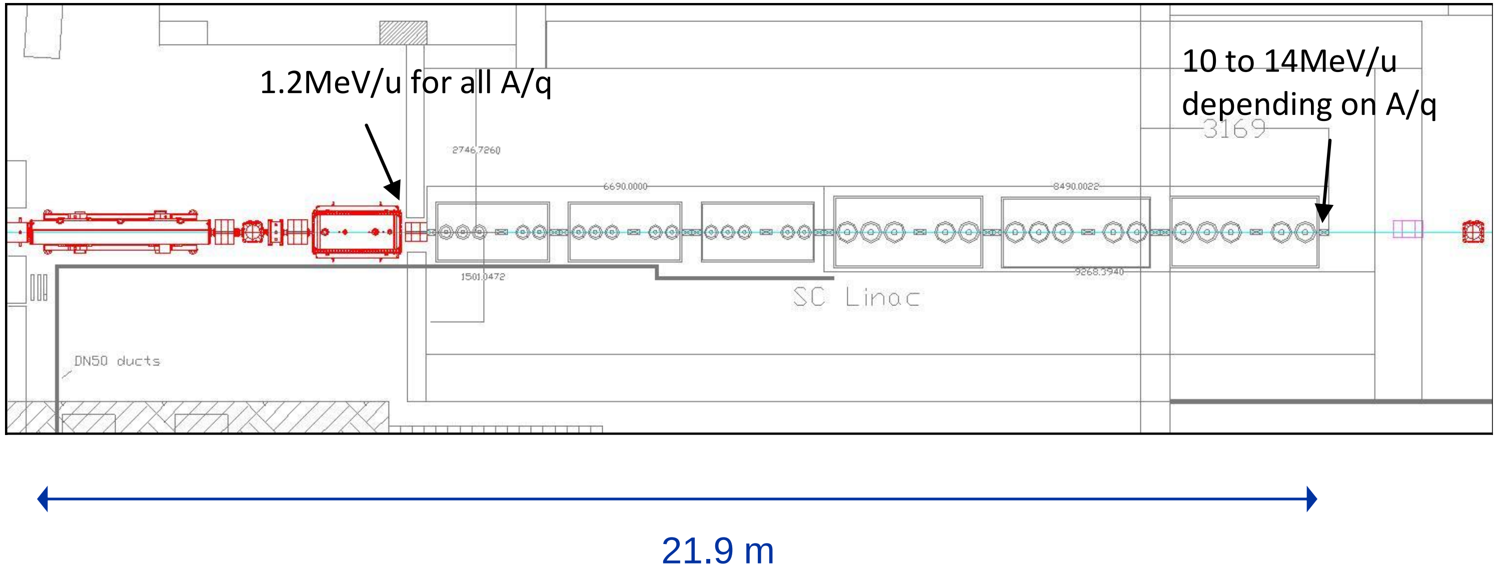
high-beta -FODO: 10 cryo-modules with 8 cavities/module



Section	Output Energy [MeV]	Lattice	Number of RF gaps between quadrupoles	Number of RF gaps per period	Number of consecutive identical cells
DTL	50	F0D0	1	2	1
CCDTL	102	F0D0	3	6	3
PIMS	160	F0D0	12	24	12
Low β	750	FD	15	15	300
High β / 1	2500	FD	35	35	920
High β / 2	5000	F0D0	35	70	

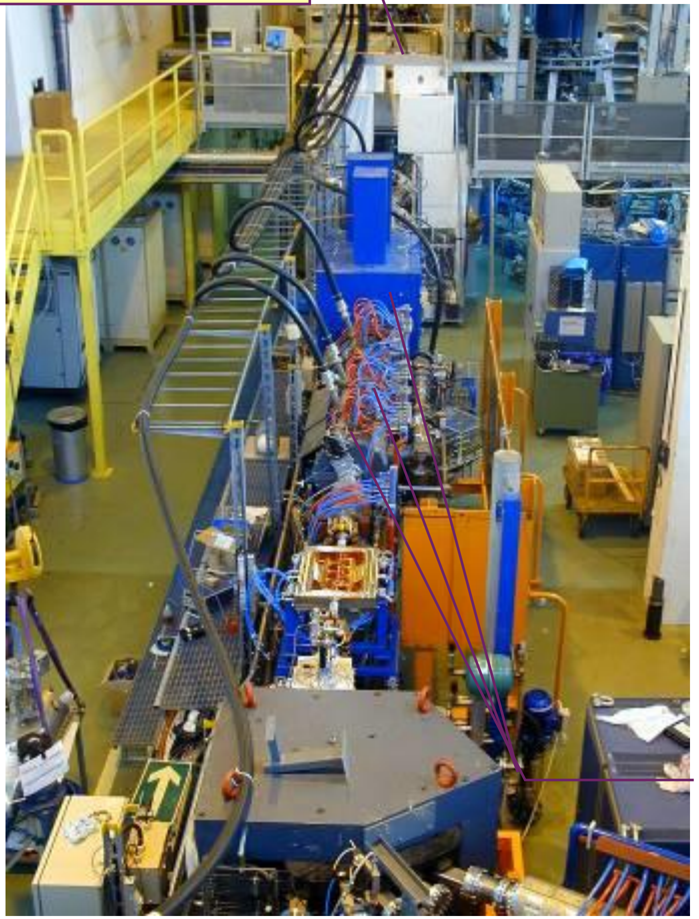
Heavy Ion Linac Architecture

EXAMPLE: the REX upgrade project at CERN-ISOLDE. Post-acceleration of radioactive ions with different A/q up to energy in the range 2-10 MeV.
An injector (source, charge breeder, RFQ) + a sequence of short (few gaps) standing wave accelerating structures at frequency 101-202 MHz, normal conducting at low energy (Interdigital, IH) and superconducting (Quarter Wave Resonators) at high energy → mix of NC-SC, different structures, different frequencies.

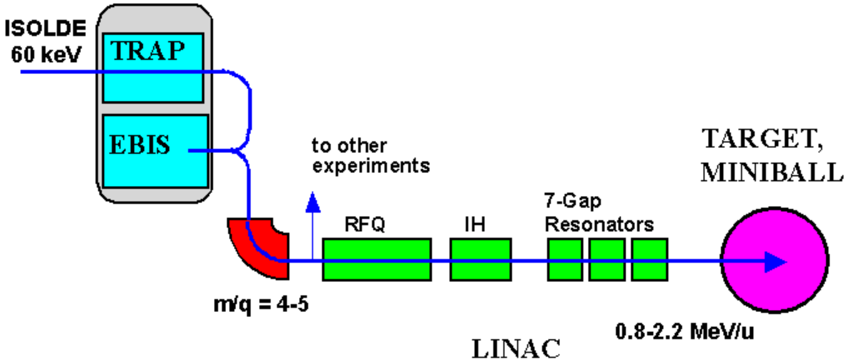


Examples: a heavy ion linac

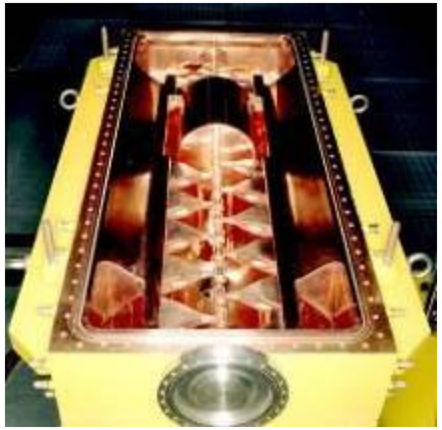
Particle source



Accelerating structures

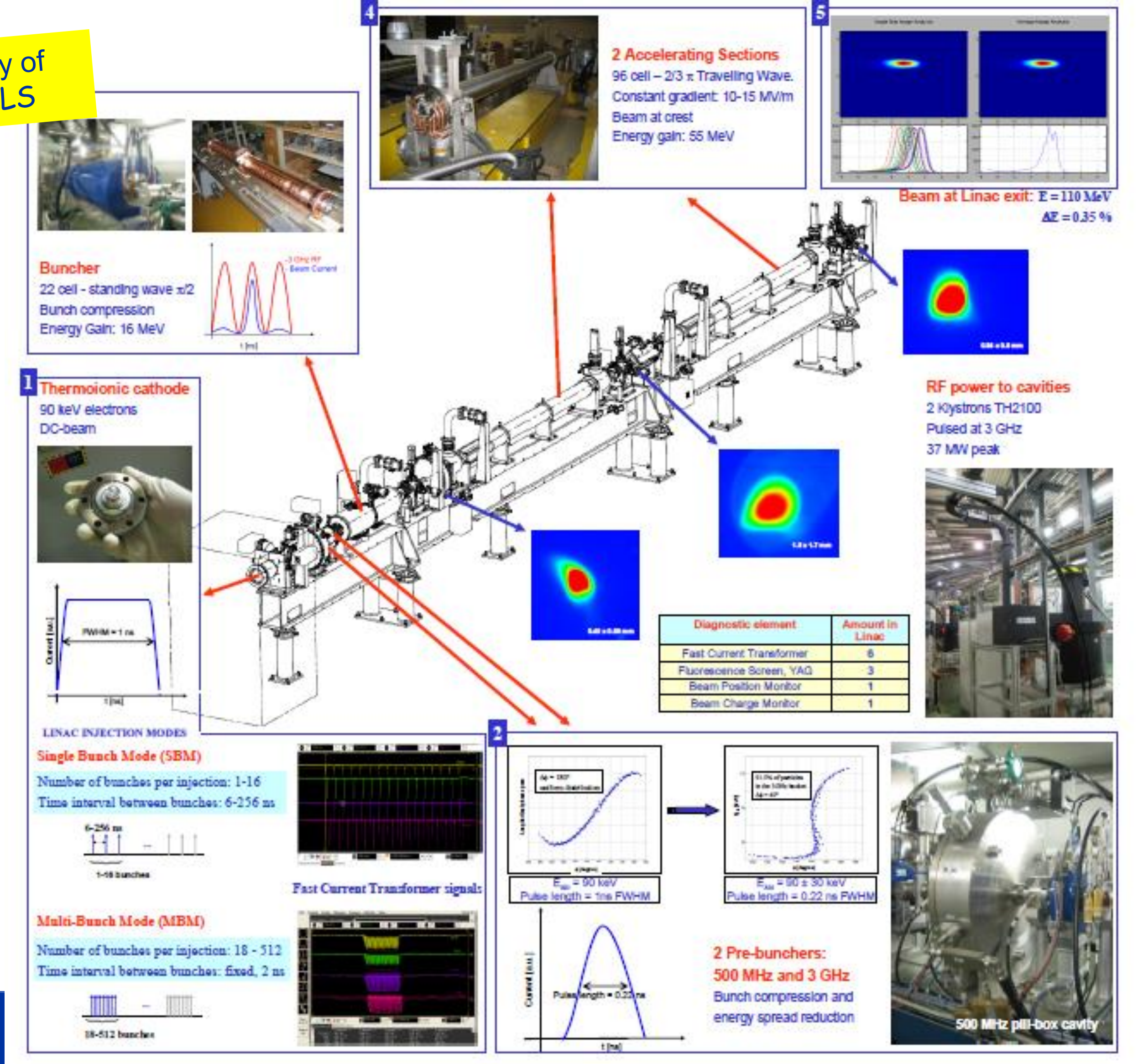


The REX heavy-ion post accelerators at CERN. It is made of 5 short standing wave accelerating structures at 100 MHz, spaced by focusing elements.



Electron linac architecture

Linac scheme courtesy of
R. Muñoz, ALBA-CELLS

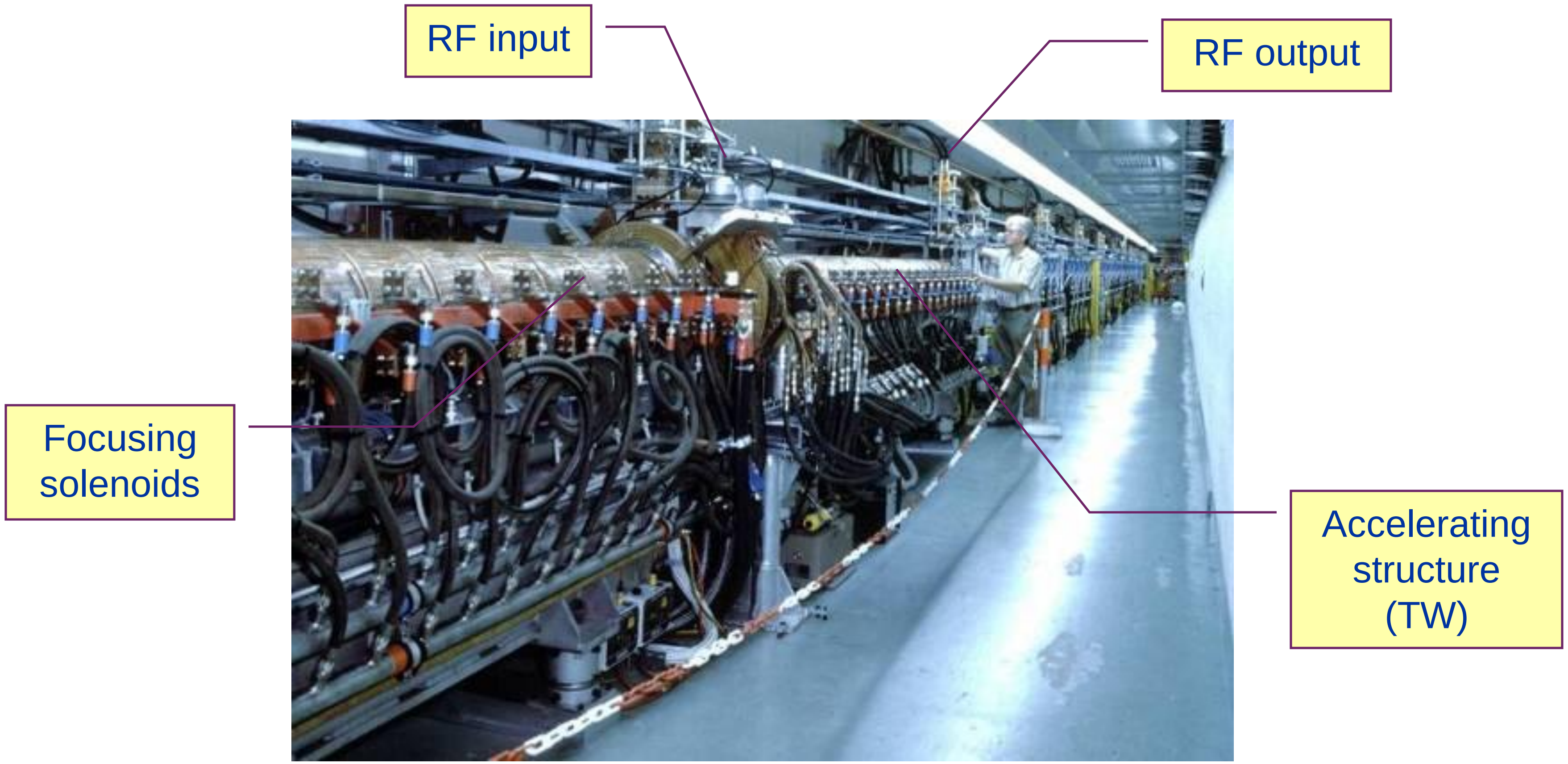


EXAMPLE:
njector linac of the ALBA Synchrotron
ight Facility (Barcelona):

00 MeV electron linac supplied by Thales in
008. Produces a beam up to 4 nC/bunch in either
ingle or multi-bunch mode at repetition rate up
o 5 Hz. Normalized beam emittance below 30 π
m mrad.

njector + sequence of identical multi-cell
raveling wave accelerating structures.

Examples: an electron linac



The old CERN LIL (LEP Injector Linac) accelerating structures (3 GHz). The TW structure is surrounded by focusing solenoids, required for the positrons.

Linac architecture: superconductivity

Advantages:

- - Much smaller RF system (only beam power) → prefer low current/high duty
- Large aperture (lower beam loss in the SC section).
- Lower operating costs (electricity consumption).

Disadvantages:

- Need cryogenic system (in pulsed machines, size dominated by static loss → prefer low repetition frequency or CW to minimize filling time/beam time).
- Need cold/warm transitions to accommodate quadrupoles → becomes more expensive at low energy (short focusing periods).
- Individual gradients difficult to predict (large spread) → need large safety margin in gradient at low energy.

Conclusions:

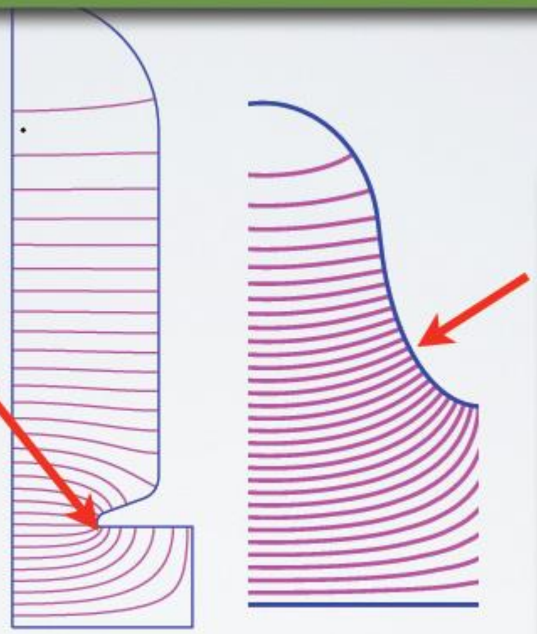
1. Superconductivity gives a large advantage in cost at high energy / high duty cycle.
2. At low energy / low duty cycle superconducting sections become expensive.

Comparing NC and SC

NC and SC half cells (typical shapes)

normal conducting:

- nose cones reduce the gap length & increase the transit time factor and eff. shunt impedance ZT^2,
- high peak fields,
- $P_{beam} \lesssim P_{diss}$
- **design goal:** maximise ZT^2 and keep Kilpatrick below a certain value (1.2 - 2.4)



superconducting:

- ZT^2 has no big importance ($P_{beam} \gg P_{diss}$),
- cryogenic losses (P_{diss}) can be optimised with the temperature (2 K/ 4.5 K),
- keep the ratio **$E_{peak,surface}/E_{peak,axis}$ as small as possible** (for $\beta=1 \Rightarrow P_s/P_a \approx 2$),

$$P_d = \frac{V_{acc}^2}{ZT^2 L}$$

$$P_d = \frac{V_{acc}^2}{(R/Q)Q_0}$$

When are SC cavities attractive?

Instead of Q values in the range of ~10^4, we can now reach 10^9 - 10^{10}, which drastically reduces the surface losses (basically down to ~0) ➜ high gradients with low surface losses

$$P_d = \frac{V_{acc}^2}{(R/Q)Q_0}$$

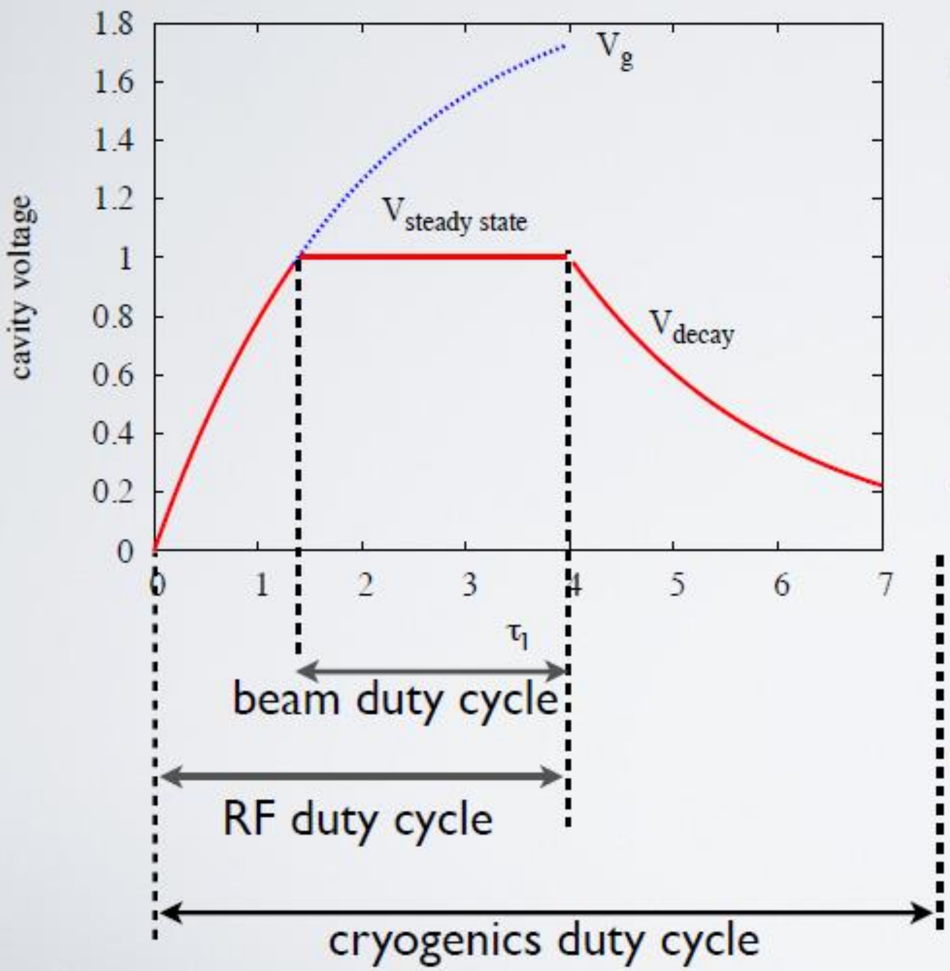
However, due to the large stored energy, also the filling time for the cavity increases (often into the range of the beam pulse length):

$$\tau_l = \frac{Q_l}{\omega_0} = \frac{Q_0}{\omega_0(1+\beta)} \approx \frac{Q_0}{\omega_0 \cdot P_b/P_d}$$

using: $\beta = 1 + \frac{P_b}{P_d} \approx \frac{P_b}{P_d}$

↑ only for SC cavities!

Pulsed operation for SC cavities



- **beam duty cycle:** covers only the beam-on time,
- **RF duty cycle:** RF system is on and needs power (modulators, klystrons)
- **cryo-duty cycle:** cryo-system needs to provide cooling (cryo-plant, cryo-modules, RF coupler, RF loads)
- RF and cryo-duty cycle have to be calculated as **integrals** of voltage over time.

Transition warm/cold

The RFQ must be normal conducting (construction problems / inherent beam loss).

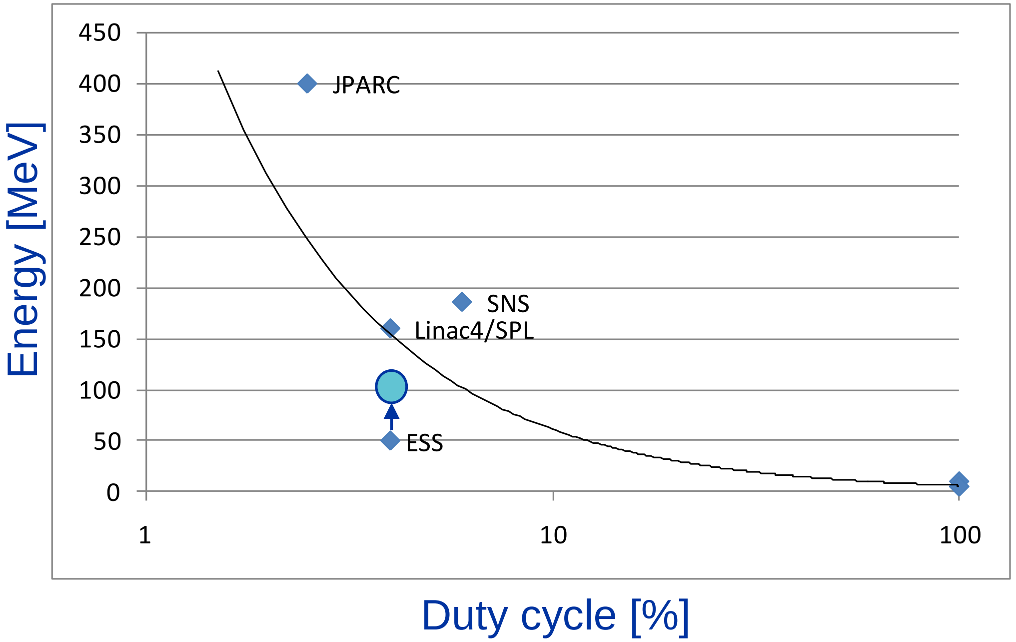
Modern high-energy (>200 MeV) sections should be superconducting.

But where is the optimum transition energy between normal and superconducting?

The answer is in the cost → the economics has to be worked out correctly !

Overview of warm/cold transition energies for linacs (operating and in design)

Project	Duty Cycle [%]	Transition Energy [MeV]
SNS	6	186
JPARC	2.5	400
Linac4/SPL	4	160
ESS	4	90
Project-X	100	10
EUROTRANS	100	5
EURISOL	100	5
IFMIF/EVEDA	100	5



The choice of the frequency

approximate scaling laws for linear accelerators:

➲ RF defocusing (ion linacs)	~ frequency
➲ Cell length ($=\beta\lambda/2$)	~ $(\text{frequency})^{-1}$
➲ Maximum surface electric field	~ $(\text{frequency})^{1/2}$
➲ Shunt impedance (power efficiency)	~ $(\text{frequency})^{1/2}$
➲ Accelerating structure dimensions	~ $(\text{frequency})^{-1}$
➲ Machining tolerances	~ $(\text{frequency})^{-1}$

- Higher frequencies are economically convenient (shorter, less RF power, higher gradients possible) but limitation comes from mechanical precision in construction (tight tolerances are expensive!) and beam dynamics for ion linacs at low energy.
- Electron linacs tend to use higher frequencies (0.5-12 GHz) than ion linacs. Standard frequency 3 GHz (10 cm wavelength). No limitations from beam dynamics, iris in TW structure requires less accurate machining than nose in SW structure.
- Proton linacs use lower frequencies (100-800 MHz), increasing with energy (ex.: 350 – 700 MHz): compromise between focusing, cost and size.
- Heavy ion linacs tend to use even lower frequencies (30-200 MHz), dominated by the low beta in the first sections (CERN lead ion RFQ at 100MHz, 25 keV/u: $\beta\lambda/2=3.5$mm !)

Linac architecture: optimum gradient (NC)

Note that the optimum design gradient (E_0T) in a normal-conducting linac is not necessarily the highest achievable (limited by sparking).

The cost of a linear accelerator is made of 2 terms:
- a "structure" cost proportional to linac length
- an "RF" cost proportional to total RF power

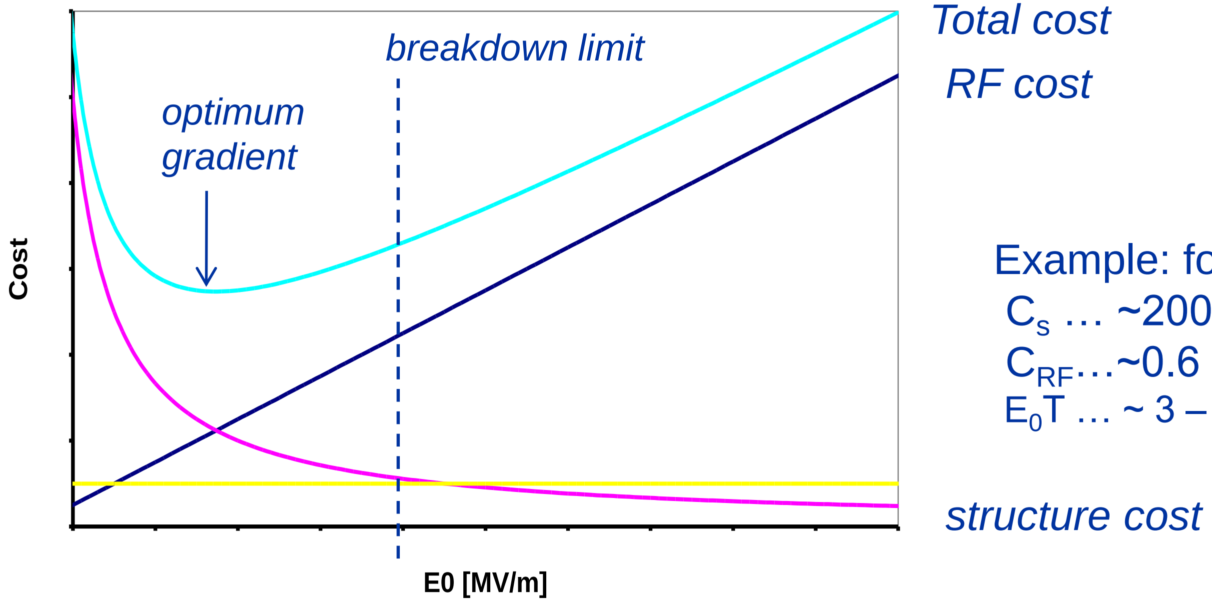
$$C = C_s l + C_{RF} P$$

$$l \propto 1/E_0T$$

$$P \propto (E_0T)^2 l \propto E_0T$$

$$C \propto C_s \frac{1}{E_0T} + C_{RF} E_0T$$

C_s, C_{RF} unit costs (€/m, €/W)



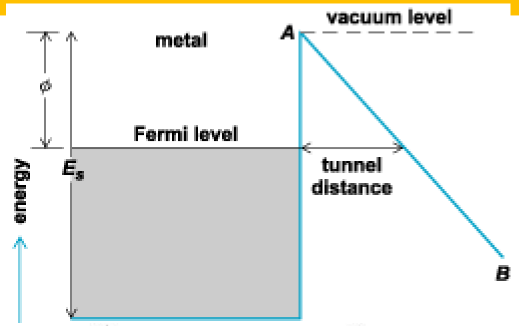
Overall cost is the sum of a structure term decreasing with the gradient and of an RF term increasing with the gradient → there is an optimum gradient minimizing cost.

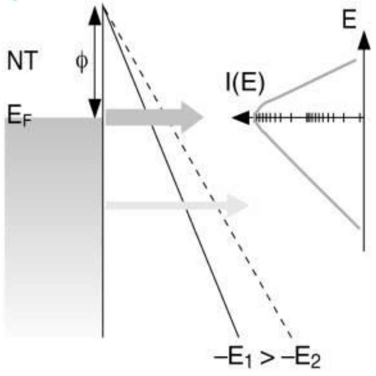
Example: for Linac4
C_s … ~200 kCHF/m
C_{RF}…~0.6 CHF/W (recuperating LEP equipment)
E_0T … ~ 3 – 4 MV/m

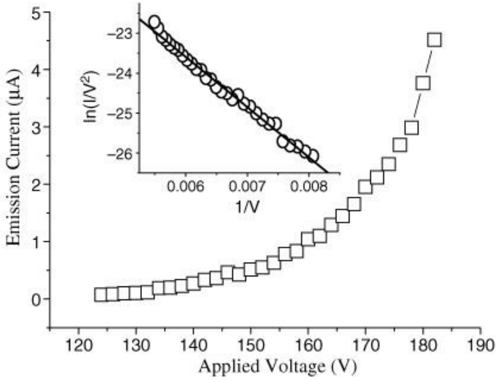
Appendix: breakdowns, conditioning, multipacting

This block of 6 slides can go either in the linac or in the RF lectures – depends on timing

On the breakdown limit…







The origin of breakdowns is FIELD EMISSION:
Tunneling of electrons through the potential barrier at the boundary metal/vacuum in presence of an applied voltage. Increasing the surface electric field reduces the barrier and increases the number of escaping electrons. The temperature enhances the emitted current (field assisted thermoionic emission).

Quantum mechanics fenomenon, can be calculated: Fowler-Nordheim relation:

$$J(E) = \frac{1.54 \cdot 10^{-10}}{\phi} \cdot E^2 \cdot \exp\left(-\frac{3.21 \cdot 10^{-9} \cdot \phi^{3/2}}{E}\right)$$

Current density (A/cm²) emitted by a metal of extraction potential Φ with an applied electric field E, in the limit case of T→0 (with some additional approximations).

The current goes up very quickly (exponential); a F.-N. behaviour is characterised by a linear ln(I/V)=f(1/V)

Breakdown and impurities

Looking at the numbers, for copper the F.E. current starts to become important only for fields in the region of few GV/m, well beyond normal operating fields in the order of 10-40 MV/m.

(E=1 GV/m → J=5e-17 A/m^2, E=5 GV/m → J=7e11 A/m^2 !!)

But: the theory is valid for smooth and perfect surfaces. A real electrode has marks coming from machining (finite surface roughness) and contains impurities incrusted on the surface (grains). Both these elements increase the surface field (edges for the roughness and dielectric constant for the impurities).

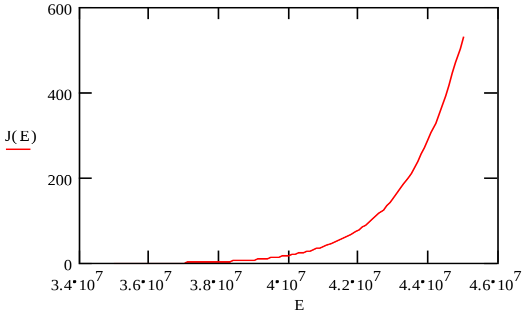
"Mountain and snow": The real field on the surface is βE, adding an "enhancement" factor β.

$$J(E) = 4.83 \cdot 10^{-11} \cdot (\beta \cdot E)^{2.5} \cdot \exp\left(-\frac{6.55 \cdot 10^{10}}{\beta \cdot E}\right)$$

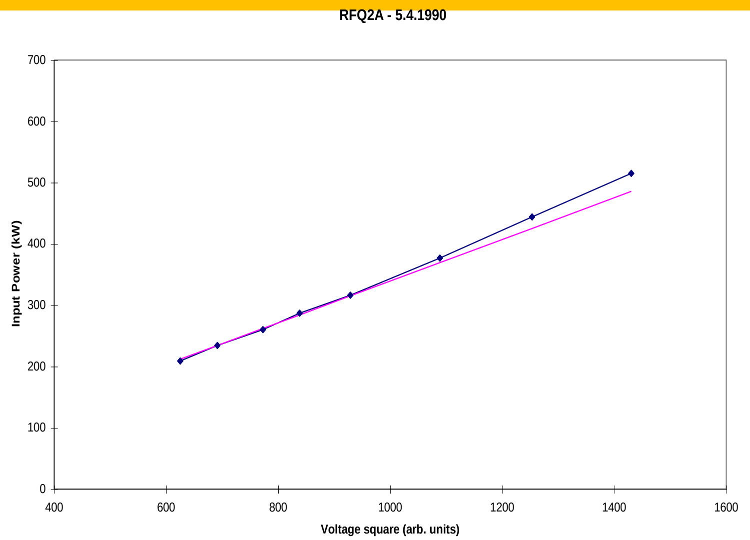
F.-N. formula for copper, with the enhancement factor

Field emission current is a pre-breakdown phenomenon. When the current at a certain spot goes beyond a certain limit a breakdown starts (the real physics of the phenomenon is still under discussion).

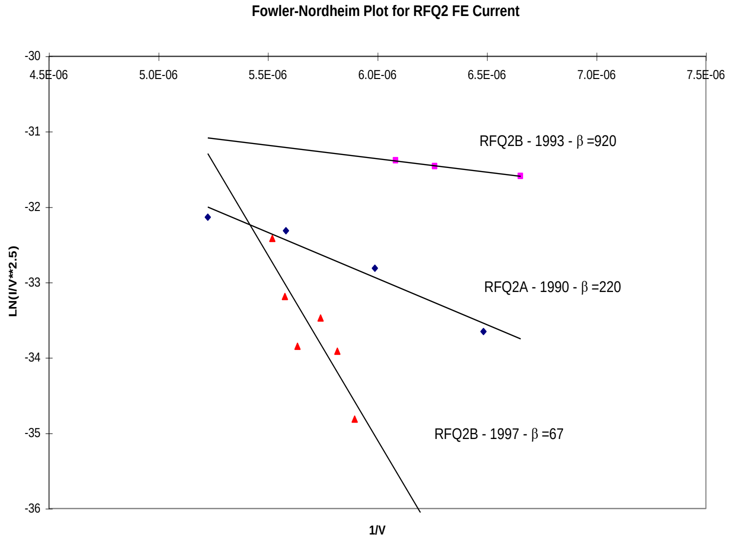
Field emission current is often called « dark current » and can be measured.



Measurement of dark current



Plotting RF power measured at the input of the cavity as function of cavity voltage square (arbitrary units, measures on a pick-up loop at the cavity) we see above a certain power the appearance of dark current accelerated on the gap and absorbing power.



Considering that the dark current power is proportional to gap voltage and to current intensity, we can plot ln(I/V)=f(1/V); the slope of the curve is the enhancement factor.

CERN RFQ2:

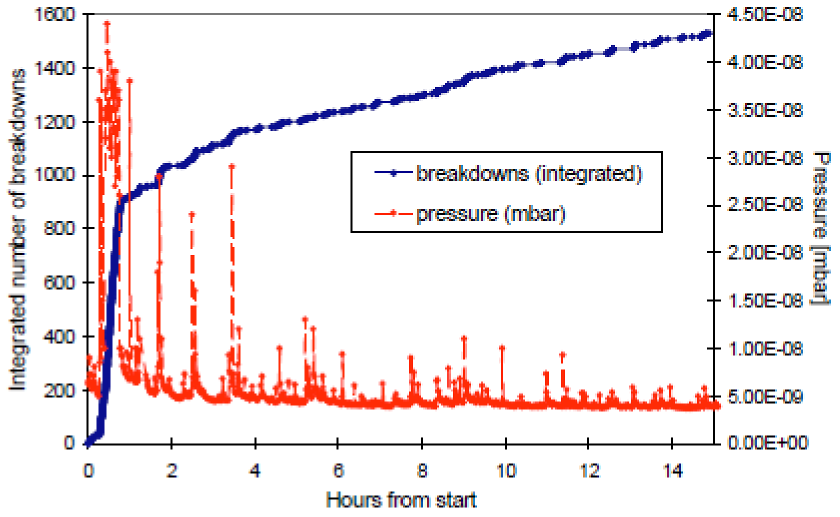
β=220 electrodes as out of the workshop

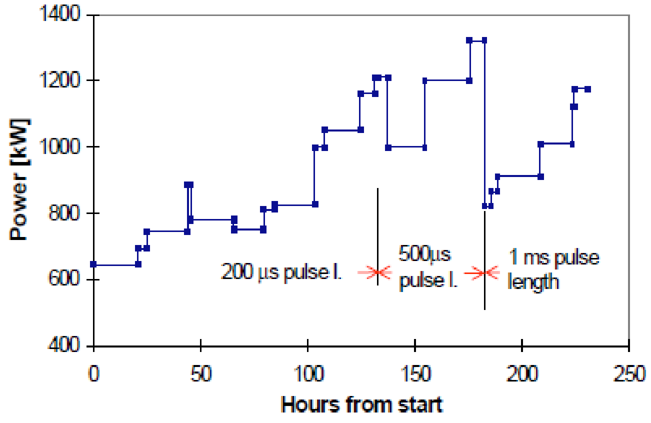
β=920 after a heavy pollution from hydrocarbons from the vacuum system

β=67 after long conditioning

Breakdowns and conditioning

Increasing the voltage, in one or few points the F.-E. current will go above the threshold and start a breakdown. But the breakdown will reconfigure the surface and there is a certain probability that the new surface will be better (melted spikes, degazed impurities) → we can continue and condition the cavity.
At some point, the number of emitting spots will be so high that we will always find a sparking point → limit of conditioning.
Note that breakdown is a statistic phenomenon. We cannot define a breakdown threshold, instead we can speak of sparking rate (number of breakdown per unit time) as funtion of voltage. The rate is decreased by conditioning; it will decrease asymptotically towards a limiting value.





The Kilpatrick field

W. Kilpatrick in 1956 fitted some experimental data on breakdown "levels" in RF regime with a F.-N. type formula, assuming that the breakdown is ignited by the impact on the surface of an ion accelerated on the gap, with energy W:

$$WE^2 \exp(-17/E) = 1.8$$

In the late 60's at Los Alamos they introduced a calculation of the ion velocity based on gap and frequency (the ion has a transit time factor!) → frequency dependant version of the Kilpatrick criterion:

$$f = 1.64E^2 \exp(-8.5/E)$$

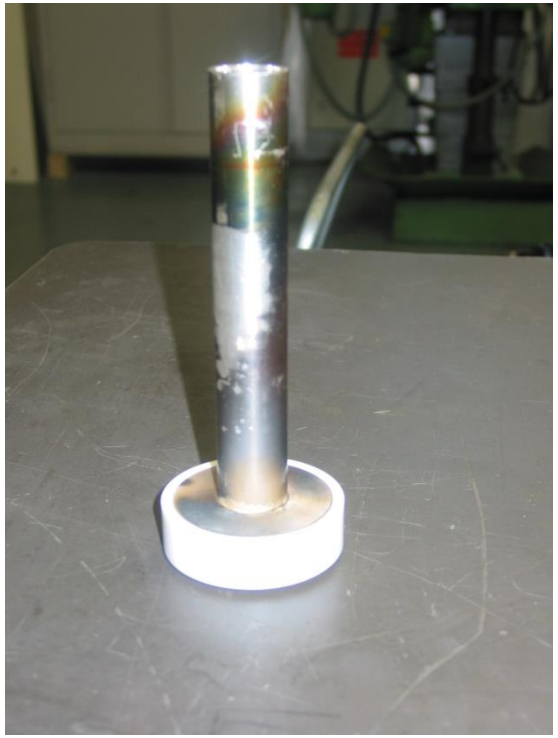
This formula is still used as a reference in the design of linear accelerators! For a given frequency, allows to calculate the "Kilpatrick field".

Very useful and a good reference, with some caveats:

1. It gives the famous result that maximum field goes as √f ; although demonstrated in several cases, now it is not considered as a fixed law and in particular does not hold >10 GHz, where other phenomena take place.
2. It is not valid for small gaps and low frequencies, where we approach instead the limits for DC field;
3. It gives the wrong message that there is a breakdown threshold;
4. The experimental data were taken in the 50's with bad vacuums, nowadays a cavity can operate at a few times the Kilpatrick limit.

Usual maximum fields range from about 1 Kilpatrick (CW systems high reliability) to 2.5 Kilpatrick (RFQ2)

Multipactoring



The inner conductor of a coaxial coupler that was (probably by mistake) silver plated and then installed in a CERN linac cavity in 1978. Multipactoring went on for 25 years during normal operation sputtering silver on the window, until it eventually short circuited a Friday night in 2003 stopping all CERN accelerators…

Electron resonance due to secondary emitted electrons from the surfaces. Occurs at low voltages and can completely stop normal operation of a cavity.

Some causes of multipactoring:

- Dirty surfaces (impurities emitting electrons)
- Air pockets (providing ions and electrons)
- Parallel plate and coaxial geometries (well-defined electron path)
- High pulsing rates (remaining electrons)
- Presence of silver (secondary emission >1)
- Vicinity of a ceramic insulator (high sec. emission)
- Bad luck…

Some cures:

- Conditioning: long times in the mp. region, possibly at higher repetition frequency, heating the surfaces to oxidize them and thus reduce the secondary emission coefficient below 1. Adding a frequency modulation increases the conditioned surface.
- Some paints (aquadag) were used in the past but are not very good for the vacuum.

Additional slides on coupling

Coupling between two cavities

In the PIMS, cells are coupled via a slot in the walls. But what is the meaning of coupling, and how can we achieve a given coupling?

Simplest case: 2 resonators coupled via a slot

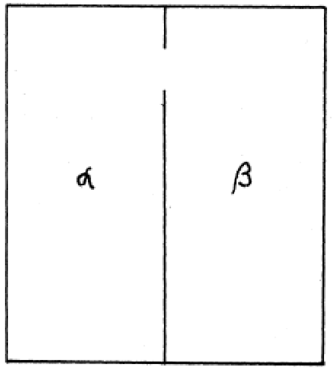


Fig. 1. Two cavities, α and β, coupled by a small hole.

Described by a system of 2 equations:

$$\begin{cases} X_1\left(1-\dfrac{\omega_1^2}{\omega^2}\right)+kX_2=0 \\ kX_1+X_2\left(1-\dfrac{\omega_2^2}{\omega^2}\right)=0 \end{cases} \quad \text{or} \quad \begin{vmatrix} 1-\dfrac{\omega_1^2}{\omega^2} & k \\ k & 1-\dfrac{\omega_2^2}{\omega^2} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = 0$$

E-field mode 1
E-field mode 2

If $\omega_1=\omega_2=\omega_0$, usual 2 solutions (mode 0 and mode π):

$$\omega_{c,1}=\frac{\omega_0}{\sqrt{1+k}} \quad \text{with} \quad \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \qquad \text{and} \qquad \omega_{c,2}=\frac{\omega_0}{\sqrt{1-k}} \quad \text{with} \quad \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} 1 \\ -1 \end{vmatrix}$$

Mode + + (field in phase in the 2 resonators) and mode + - (field with opposite phase)

Taking the difference between the 2 solutions (squared), approximated for k<<1

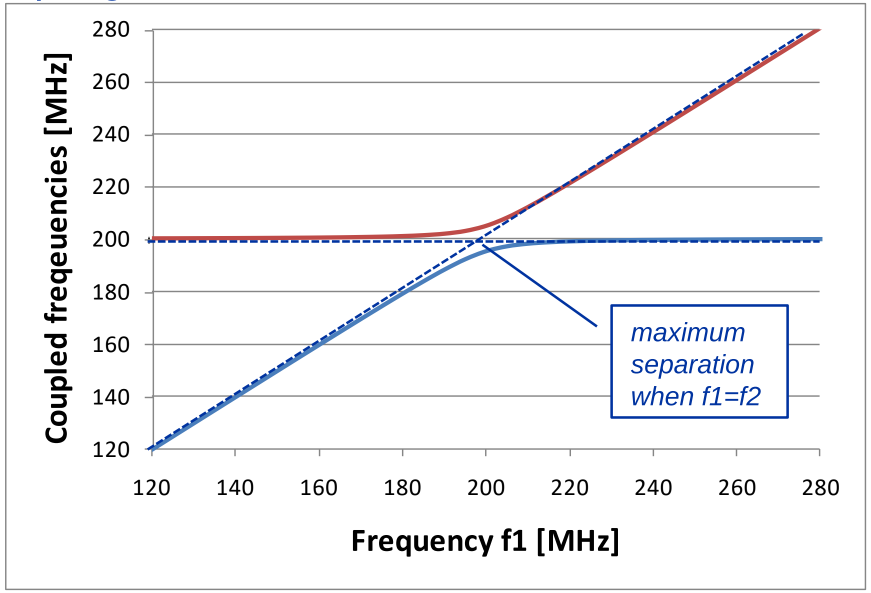
$$\frac{\omega_{c,2}^2-\omega_{c,1}^2}{\omega_0^2}=\frac{1}{1-k}-\frac{1}{1+k}\approx 2k \quad \text{or} \quad \boxed{\frac{\omega_{c,2}-\omega_{c,1}}{\omega_0}\approx k}$$

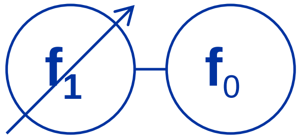
The coupling k is equal to the difference between highest and lowest frequencies.

→ k is the bandwidth of the coupled system.

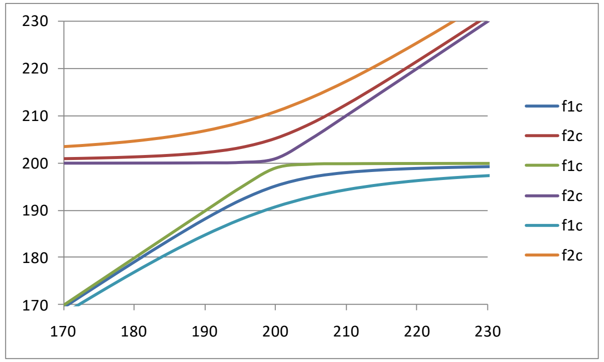
More on coupling

Solving the previous equations allowing a different frequency for each cell, we can plot the frequencies of the coupled system as a function of the frequency of the first resonator, keeping the frequency of the second constant, for different values of the coupling k.





- “Coupling” only when the 2 resonators are close in frequency.
- For $f_1=f_2$, maximum spacing between the 2 frequencies ($=kf_0$)



case of 3 different coupling factors (0.1%, 5%, 10%)

For an elliptical coupling slot:

$$k \approx F\, l^3 \left(\frac{H_1}{\sqrt{U_1}} \right) \left(\frac{H_2}{\sqrt{U_2}} \right)$$

F = slot form factor
l = slot length (in the direction of H)
H = magnetic field at slot position
U = stored energy

The coupling k is:
- Proportional to the 3rd power of slot length.
- Inv. proportional to the stored energies.

End of Lectures 3 & 4

Elwood Smith