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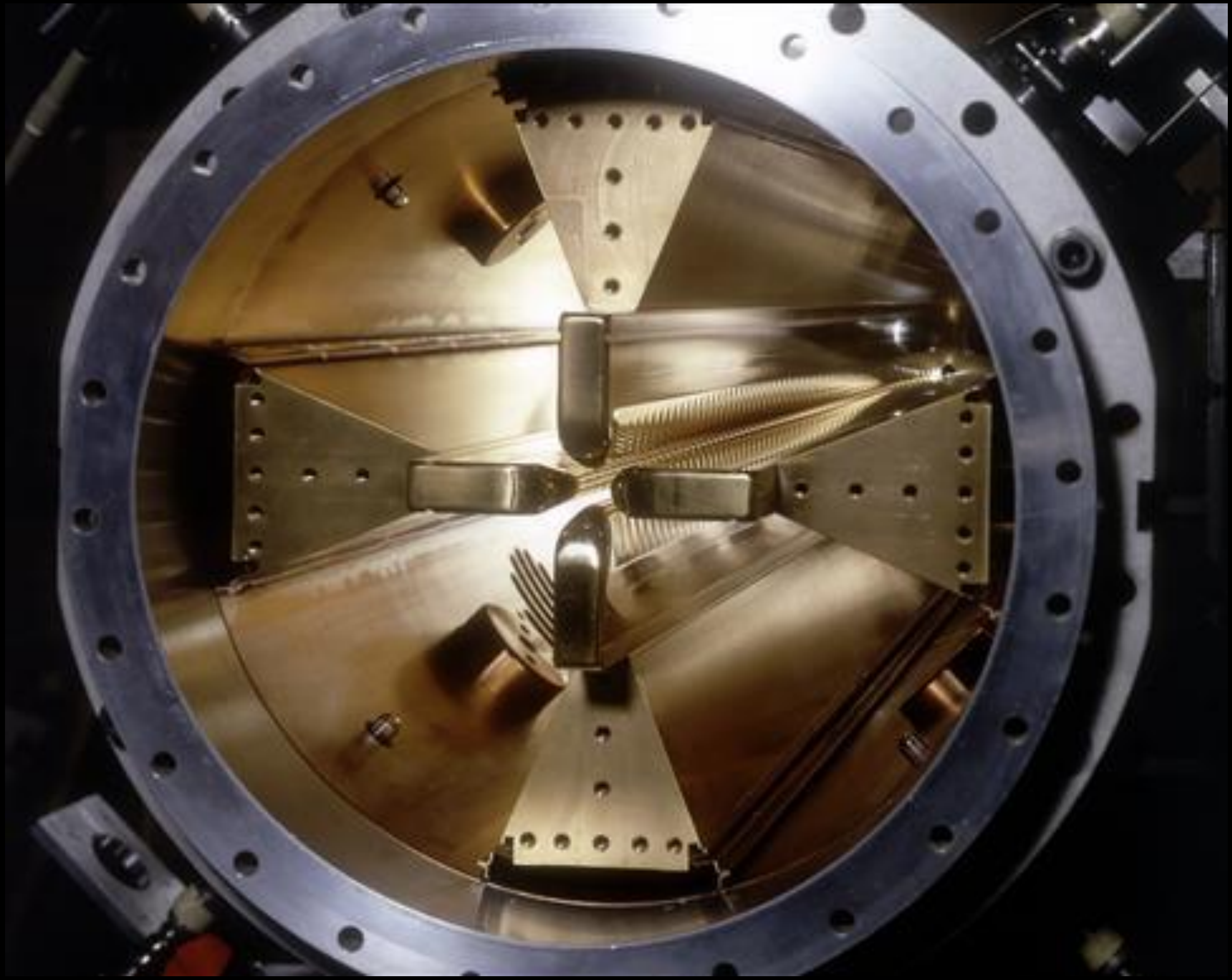
LECTURE 5

RTU April 2022

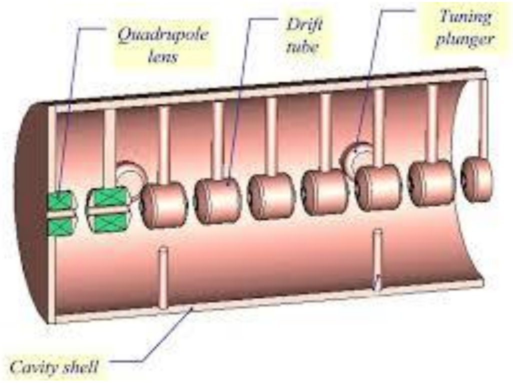


The Radio Frequency Quadrupole

Low-energy acceleration of protons and heavy ions

Low energy →
for protons, between ~ 50 keV (source extraction) and ~ 3 MeV (limit for an effective use of the DTL)
→ range $\beta = 0.01 – 0.10$

Why it is a problem?



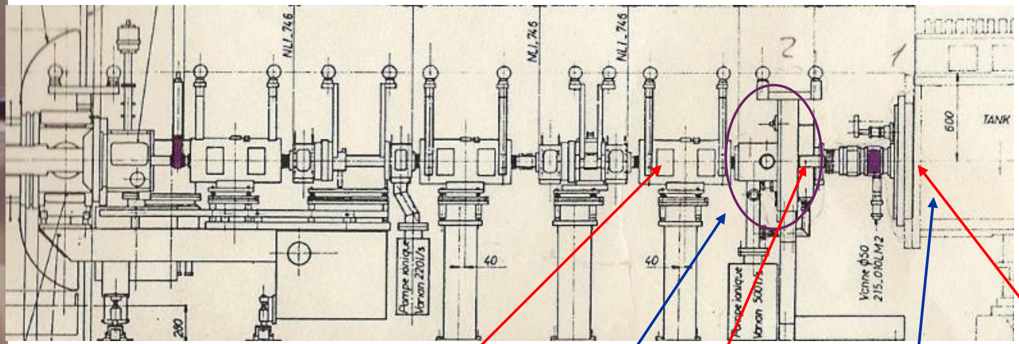
1. We have seen that at low energy we need strong focusing (strong space charge!), but the short cell length (~βλ) limits the length of quadrupoles, for ex. βλ(1MeV,352MHz) = 3.9cm
2. in this region the beam needs to be bunched → standard bunching systems are quite ineffective (~50% beam loss…).
3. At low energy, the usual accelerating structures have low efficiency (low shunt impedance).

The "old" solution:

1. Increase as much as possible the extraction voltage from the source → huge HV installations, up to the maximum of some 800 kV.
2. Add a bunching section (1 or 2 cavities) after the source extraction.
3. Start the first accelerating structure (usually a Drift Tube Linac) from the minimum possible energy.

The classical solution: HV column + LEBT + bunching

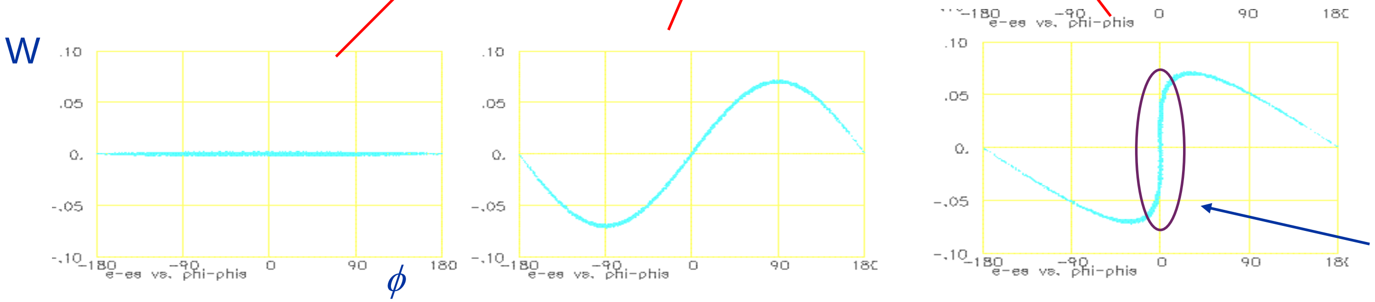
5.6m



Double harmonic buncher (200-400 MHz)

DTL

LEBT =
Low Energy Beam
Transport



Useful beam (inside DTL acceptance)

Principle of single-harmonic bunching

Drawbacks:
- -Large and expensive HV column
- -Reliability (800 kV…)
- -Bunching efficiency (~50%)
- -Long line with inefficient magnetic focusing ($\propto \beta$)
- -Difficult DTL at low energy (short tubes and quads)
- -Large emittances for high currents

RFQ compared to the old pre-injectors

The old pre-injector at CERN (1976):
Source+ Cockroft Walton
+line+bunching

The new RFQ2 pre-injector at CERN (1993):
Source+LEBT+RFQ

← 3.2m →

New ideas – history of the RFQ

The driving force for the development of something new for the low-energy section was the research in URSS and USA on high-current proton accelerators.
The goal is to break the limitation to current coming from: a) *space charge in the beam transport* and b) *bunching losses*.

- 1960's: Early works of I. Kapchinski at ITEP (Moscow): idea to *use at low energy an electric quadrupole focusing channel, excited at RF frequency, and modulated to add a longitudinal field component providing adiabatic bunching and acceleration*.
- 1969: an RF resonator is designed around Kapchinski's electrodes by V. Tepliakov (IHEP). First paper on the RFQ by Kapchinski and Teplyakov (in Russian). First experimental RFQ in Russia (1974).
- 1977: the idea arrives at Los Alamos (USA), introduced by a Czech refugee.

New ideas – history of the RFQ

- 1977-1980: the Los Alamos team is enthusiastic about this idea (for their Fusion Material Irradiation), makes some improvements to the original Kapchinski structure and develops a new resonator design. The first complete RFQ is built at Los Alamos and successfully operated (for a few hours…) in February 1980.
- 1980’s: the RFQ principle spreads around the world, more RFQs are built in the USA and in Europe (1st CERN RFQ: 1984). Long and difficult learning curve (RFQs are not simple devices…).
- 1985-1995 : reliable RFQ designs exist and progressively replace the old pre-injectors in most accelerator laboratories (CERN: 1993). Different design and applications are proposed all over the world.
- 1995-now : new RFQs are designed and built for extreme applications, like high intensity (CW, high current) or compact high-frequency.


atom

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contents

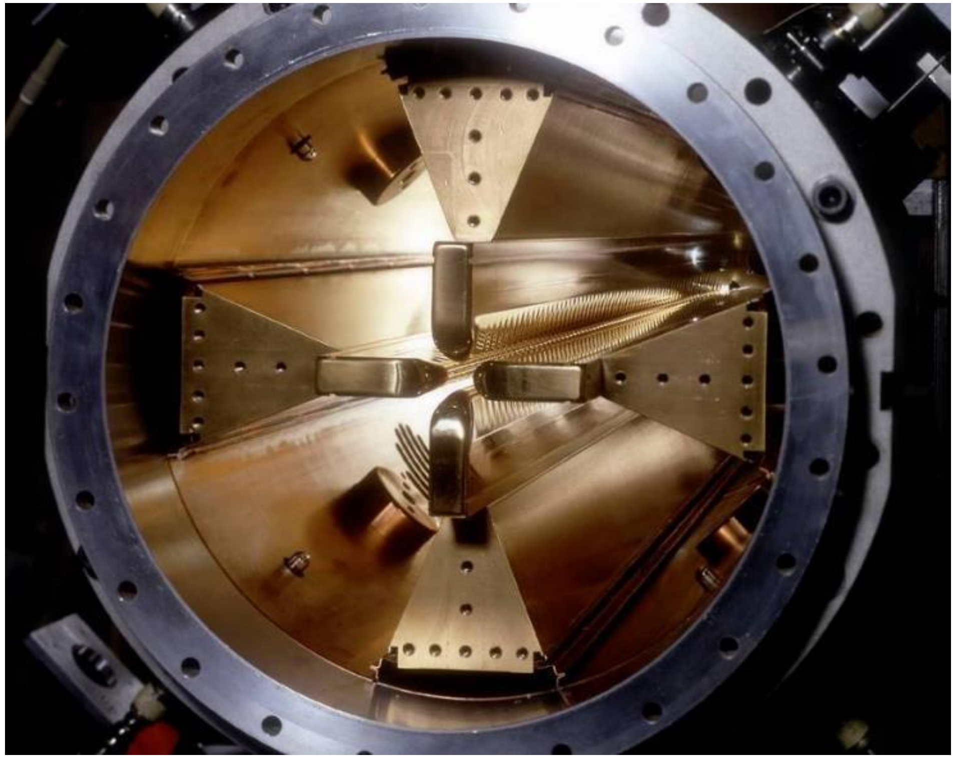
RFQ is Alive and Well 2

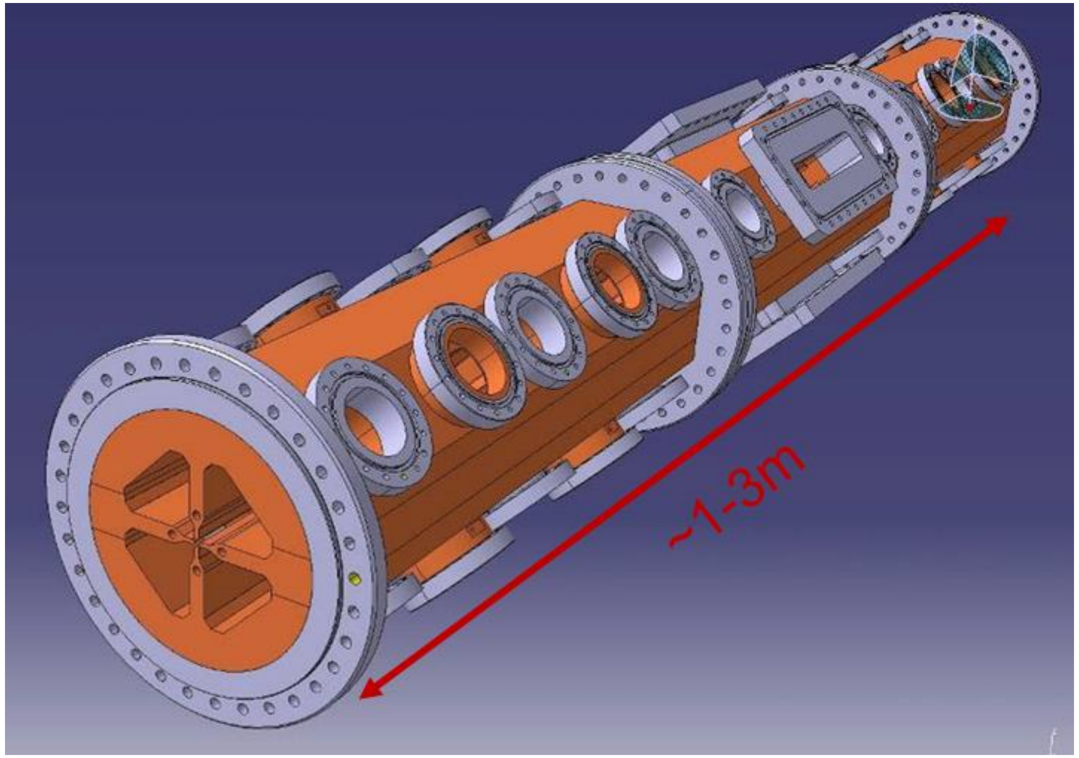
The missing link in linear accelerator development is progressing quickly from theory, to experimental device, to application.

Atom, July 1980

The Radio Frequency Quadrupole (RFQ)

<u>RFQ = Electric quadrupole focusing channel + bunching + acceleration</u>

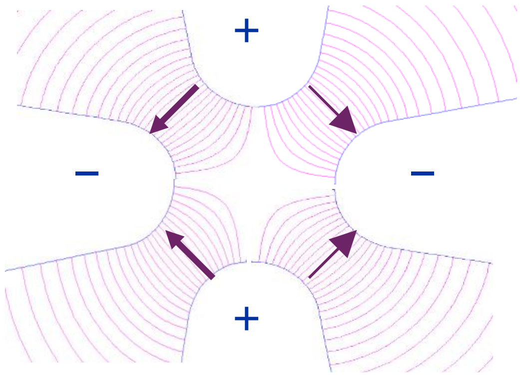




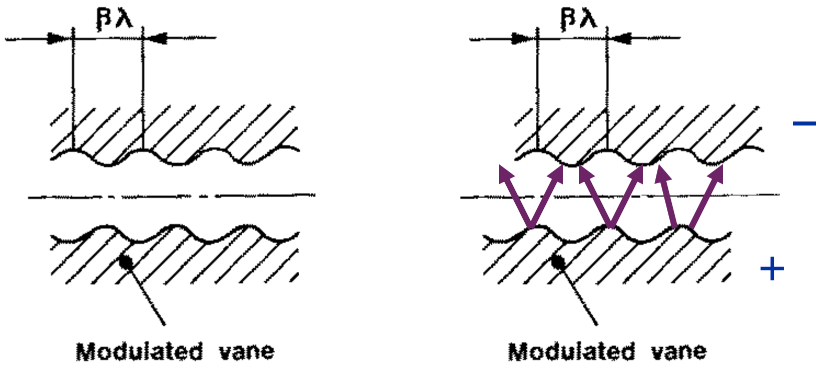
New and performing accelerator.
Compact and critical structure, where beam dynamics, RF and mechanical aspects are closely interconnected.

The basic RFQ principle

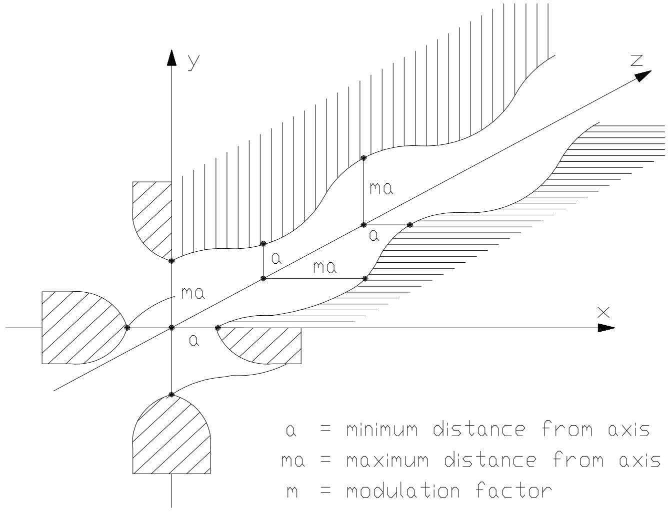
1. Four electrodes (called vanes) between which we excite an RF Quadrupole mode $\rightarrow$ Electric focusing channel, alternating gradient with the period of the RF. Note that electric focusing does not depend on the velocity (ideal at low β!)



2. The vanes have a longitudinal modulation with period = $\beta\lambda$ $\rightarrow$ this creates a longitudinal component of the electric field. The modulation corresponds exactly to a series of RF gaps and can provide acceleration.



Opposite vanes (180º) Adjacent vanes (90º)



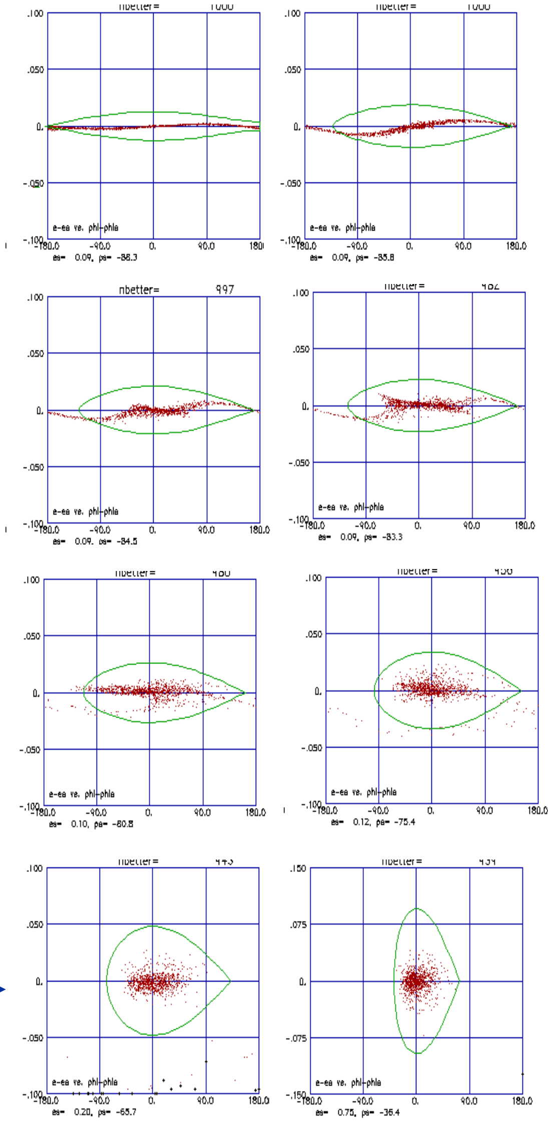
Bunching and acceleration

3. The modulation period (distance between maxima) can be slightly adjusted to change the phase of the beam inside the RFQ cells, and the amplitude of the modulation can be changed to change the accelerating gradient → we can start at -90° phase (linac) with some bunching cells, progressively bunch the beam (adiabatic bunching channel), and only in the last cells switch on the acceleration.

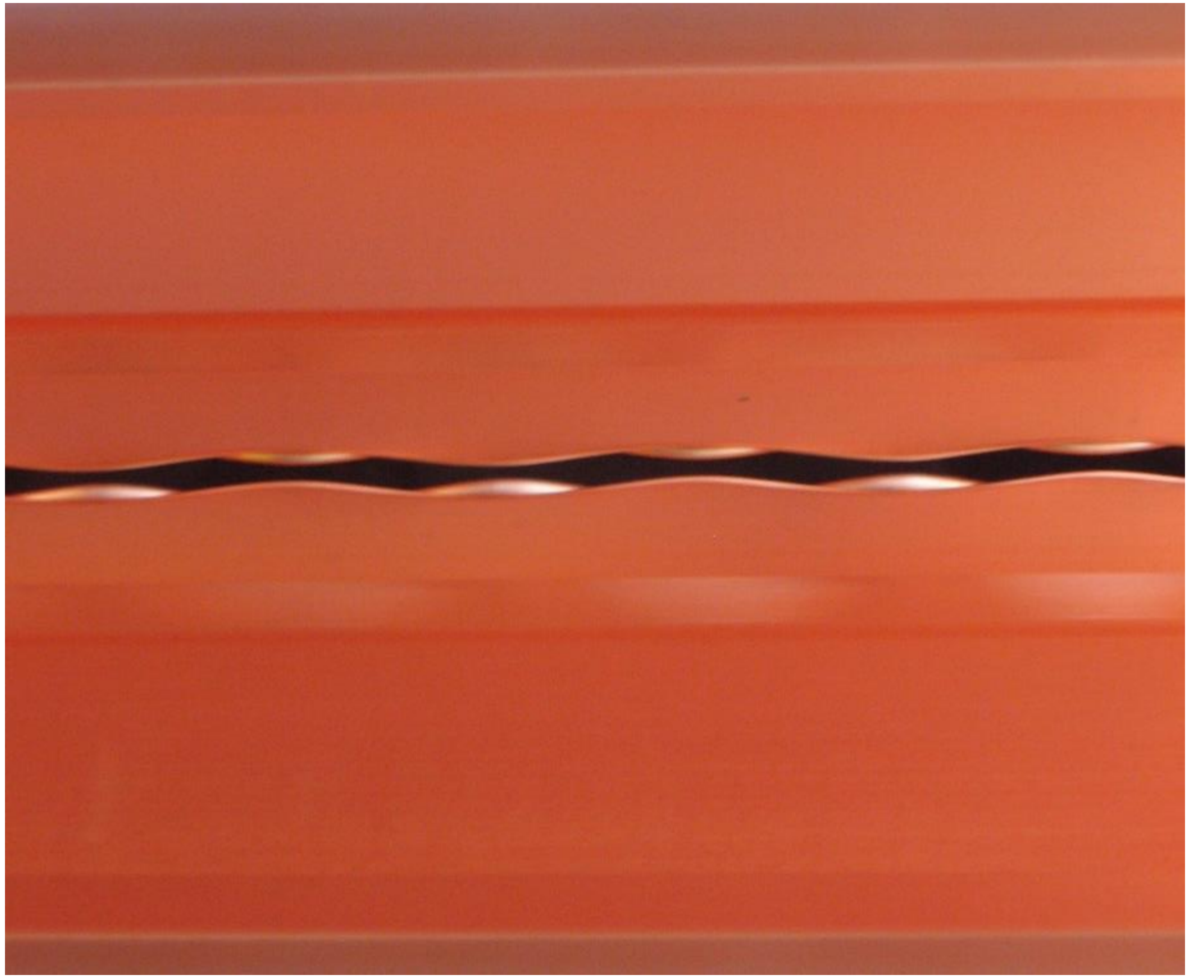
☞ An RFQ has 3 basic functions:

1. Adiabatically bunching of the beam.
2. Focusing, on electric quadrupole.
3. Accelerating.

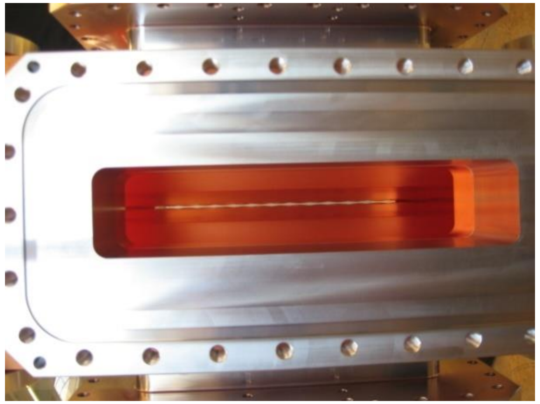
Longitudinal beam profile of a proton beam along the CERN RFQ2: from a continuous beam to a bunched accelerated beam in 300 cells.



Peeping into an RFQ…



Looking from the RF port into the new CERN RFQ (Linac4, 2011)

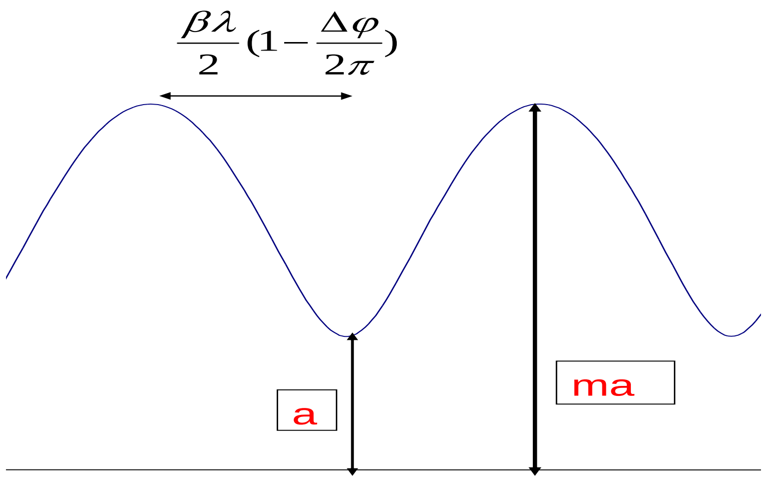


RFQ beam dynamics

An RFQ is made of a sequence of cells (length $\beta\lambda$ → in 1 m we can have > 100 cells) where the beam dynamics designer can vary 3 parameters for each cell:

1. Aperture a (defines the focusing strength)
2. Modulation factor m (defines the longitudinal component)
3. The beam phase ϕ, phase difference between bunch center and RF wave (defines the bunching and/or accelerating action).

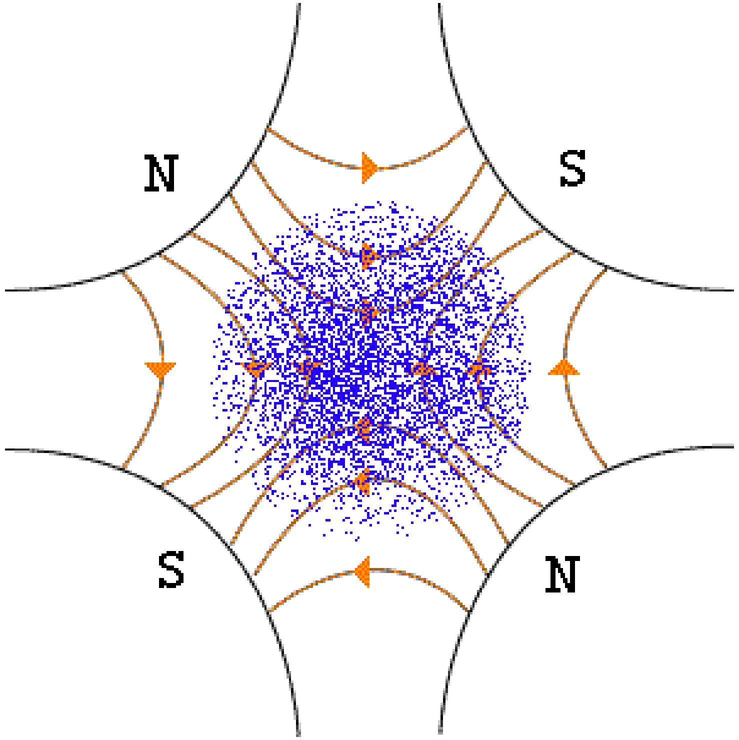
+ 1 more parameter that is common to all cells or can be changed only smoothly: the RF voltage V.

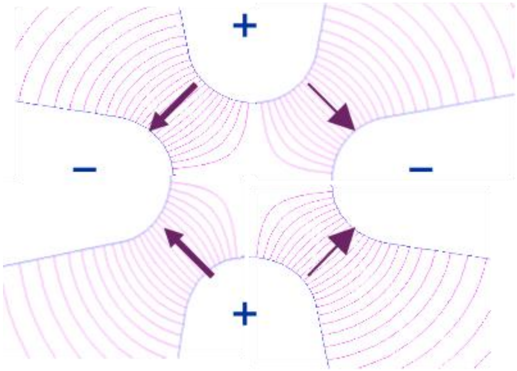


a = minimum aperture
m = modulation factor (ratio bw. max and min aperture)

cell length/$\beta\lambda$ = changing the length of the cell with respect to the optimum length for a given beta will change the RF phase seen by the beam.

Beam evolution through a RF quadrupole focusing channel





The electrodes change polarity with the RF period:
Strong focusing channel with alternating gradients in time and not in space!

The Kapchinski potential

In order to define the 3-dimensional shape of the RFQ electrodes, Kapchinski introduced an analytical expression for the fields in an RFQ channel :

- The region between the vanes is small w.r.t. the wavelength → static approximation, we can use the formulae for static fields.
- The potential in the intervane region is then a solution of the Laplace equation, which in cylindrical coordinates can be solved by a series of Bessel functions.
- Kapchinski's idea: of all the terms in the series, take only the 2 that are interesting for us (*the transverse quadrupole term + a longitudinal focusing and accelerating term*) and try to build some electrodes that give only those 2 terms.

$$V(r,\vartheta,z) = A_0 r^2 \cos 2\theta + A_{10} I_0(kr) \cos kz$$

k=2π/βλ

Transverse quadrupole term

"Longitudinal" term

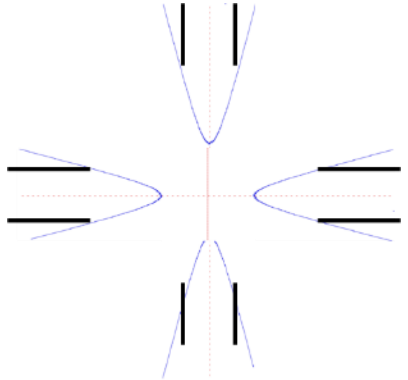
→ an RFQ cell is defined by the 2 parameters, A_0 and A_{10} (plus the phase)
→ the 3-dimensional profile of an RFQ electrode must correspond to an equipotential surface of V(r,theta,z)

RFQ beam dynamics - 2

$$V(r,\vartheta,z) = A_0 r^2 \cos 2\theta + A_{10} I_0(kr) \cos kz$$

→ The electrodes have to follow equipotential surfaces of this equation

These are hyperbolic surfaces with a longitudinal sinusoidal modulation.



→ The vanes in the 1st generation of RFQs were perfect truncated hyperbolae (the hyperbolae are truncated in straight lines far from the beam axis).

V=voltage applied between 2 adjacent vanes

The constants A_0, A_{10} depend on the geometry, and can be related to the modulation factors and to the intervane voltage V:

$$A_0 = \frac{V_0}{2a^2} \frac{I_0(ka) + I_0(kma)}{m^2 I_0(ka) + I_0(kma)} \qquad A_{10} = \frac{V_0}{2} \frac{m^2 - 1}{m^2 I_0(ka) + I_0(kma)}$$

But truncated hyperbolic surfaces are difficult to machine, while modern field calculation codes allow to use vane profiles that cannot be analyzed analytically.

→ after the first generation of RFQs, the designers are now using simplified vane profiles with constant curvature radius or simplified surfaces → introduction of multipoles, can be calculated and kept within acceptable limits.

Parameters of the RFQ

Transverse focusing coefficient

$$B=\left(\frac{q}{m_0}\right)\left(\frac{V}{a}\right)\left(\frac{1}{f^2}\right)\frac{1}{a}\left(\frac{I_o(ka)+I_o(mka)}{m^2 I_o(ka)+I_o(mka)}\right)$$

limited by sparking

Transverse field distortion due to modulation (=1 for un-modulated electrodes)

Longitudinal bunching and accelerating field

$$E_0T=\frac{m^2-1}{m^2 I_o(ka)+I_o(mka)}\cdot V\frac{2}{\beta\cdot\lambda}\frac{\pi}{4}$$

Accelerating efficiency : fraction of the field deviated in the longitudinal direction (=0 for un-modulated electrodes)

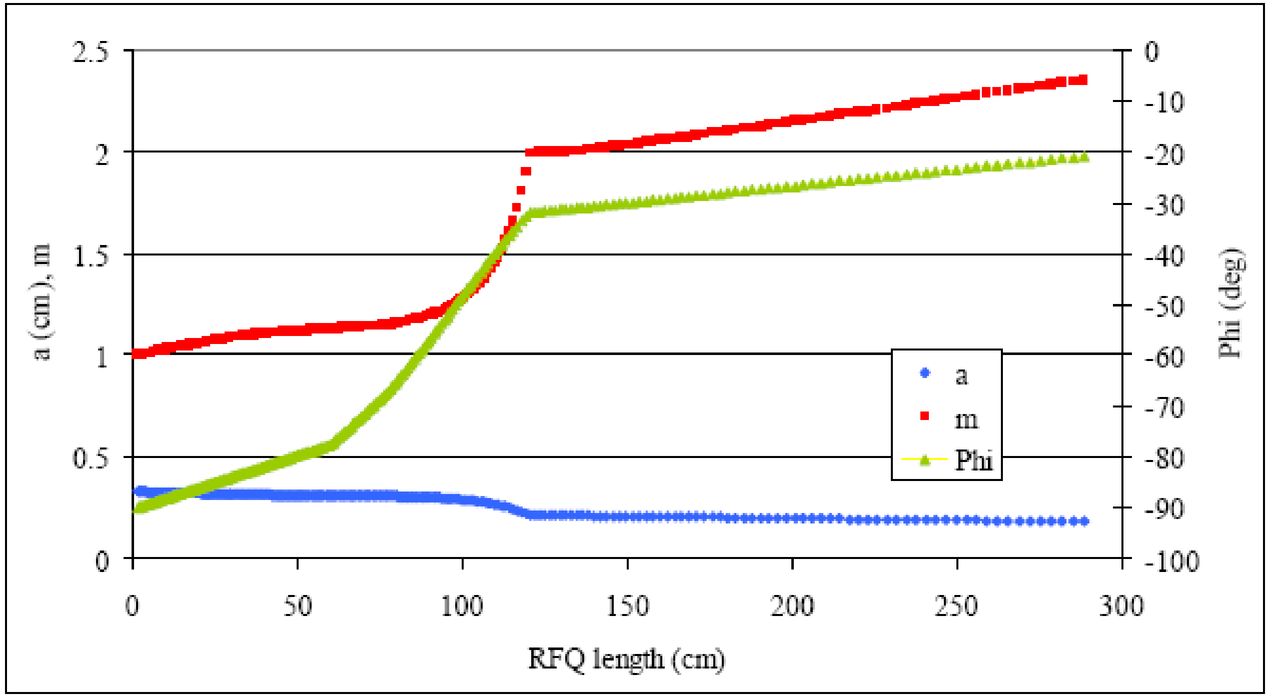
cell length

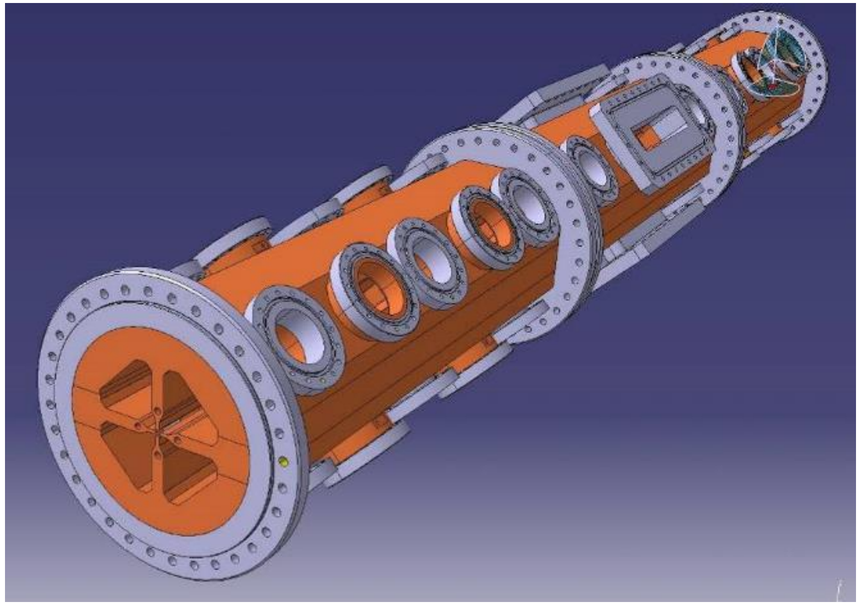
Transverse focusing B is the external focusing contribution to phase advance (see linac lecture)

$$\sigma=\sqrt{\frac{B^2}{8\pi^2}-\frac{\pi q E_0 T\sin(\varphi)\lambda}{mc^2\beta\gamma^3}-\frac{3Z_0 q I\lambda^3(1-f(p))}{8\pi mc^2\gamma^3 r^2 b}}$$

Adiabatic bunching – the RFQ Beam Dynamics design

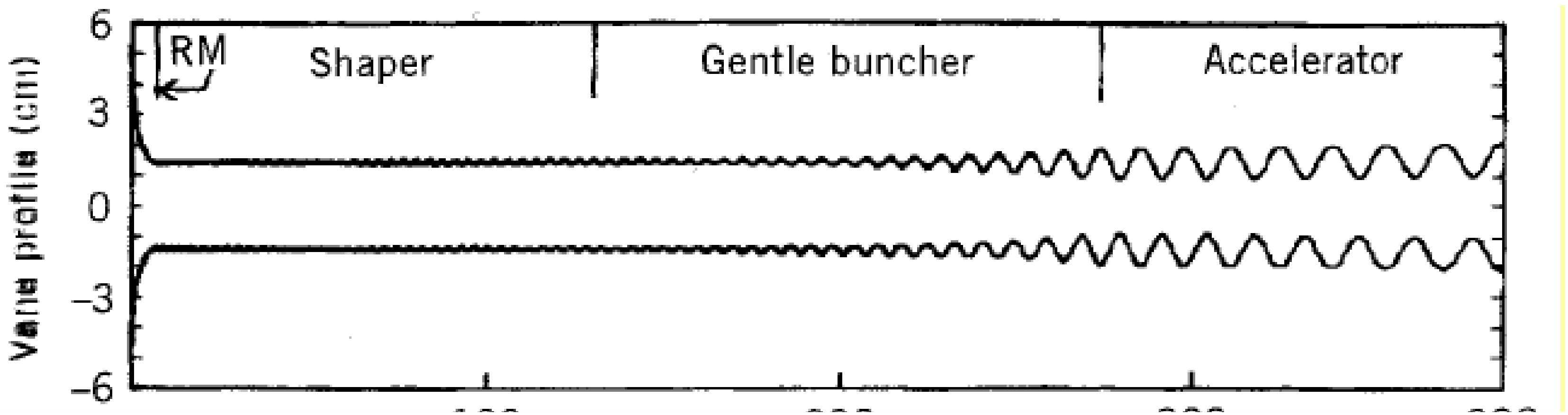
The new CERN Linac4 RFQ:
352 MHz, 45 keV to 3 MeV, 303 cells, 3 m length, 70 mA beam current
Beam transmission 95 % (calculated)





The first ~200 cells are used for adiabatic bunching of the beam: the synchronous phase is slowly increased from -90 to -20 deg
→ bunching with low beam loss!

RFQ sections

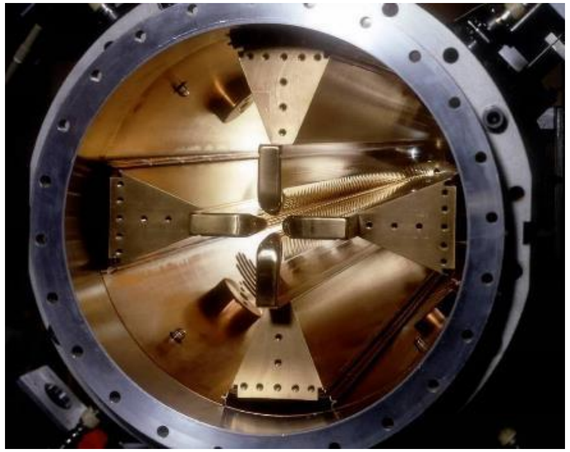


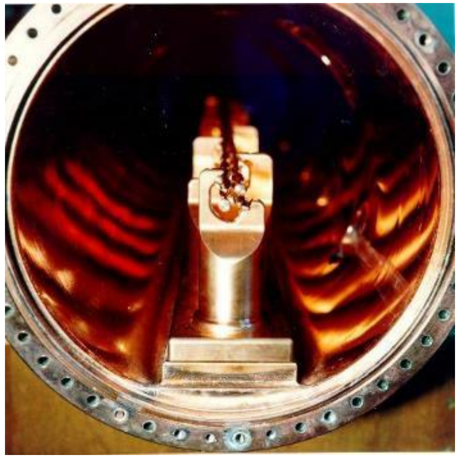
Radial matching to adapt the beam to a time-varying focusing system		
		aperture smoothly brought to the average value
shaping to give the beam a longitudinal structure		
Taper phase to –80,–60 deg	start modulation	aperture such that focusing is constant
bunching to bunch and begin acceleration		
Taper phase to –30,-20 deg	modulation to max	aperture such that focusing is constant
acceleration to bring the beam to the final energy.		
Constant phase	Constant modulation	Constant aperture
output matching to adapt the beam to the downstream user's need.		

The RFQ resonator

Problem:
How to produce on the electrodes the quadrupole RF field?

2 main families of resonators: 4-vane and 4-rod structures





plus some more exotic options
(split-ring, double-H, etc.)

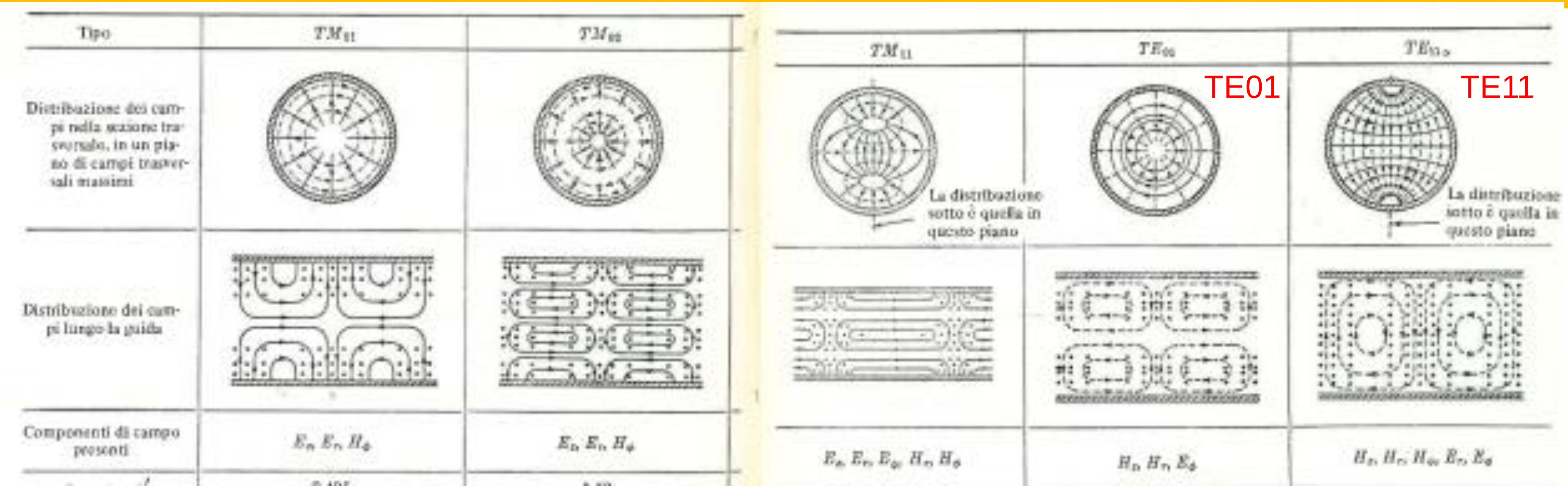
Remark:
what is the ideal frequency for an RFQ?

Cell length $\beta\lambda/2$ at injection should be mechanically achievable, of the order of few mm.

For heavy ions,
$\beta \sim 10^{-4} – 10^{-3}$
corresponding to
f~ 10 – 100 MHz

For protons,
$\beta \sim 10^{-2}$ makes higher frequencies possible, but beam dynamics (focusing $\sim f^{-2}$) and technology limit to
f ~ 200 – 400 MHz

The “4-vane” RFQ – TE modes in a cylindrical waveguide

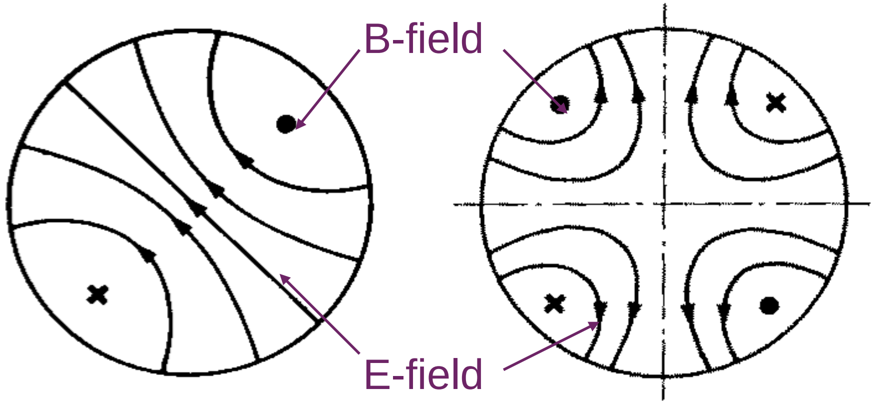


In the RFQ we want an “electric quadrupole” = a field distribution where the electric field is transverse to the beam axis and has a quadrupole symmetry.

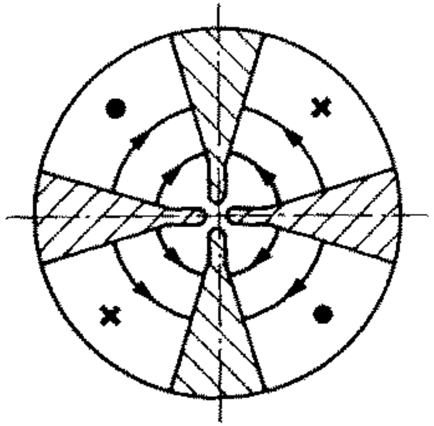
Start from the modes propagating into a cylindrical waveguide.
TE modes have transverse E-field. Do we have a transverse quadrupole?

Reminder:
the index represents the number of spatial half periods in ϕ, r, z

The “4-vane” RFQ – TE11 (dipole) and TE21 (quadrupole)



Empty cavity; mode TE_{11} Empty cavity; mode TE_{21}



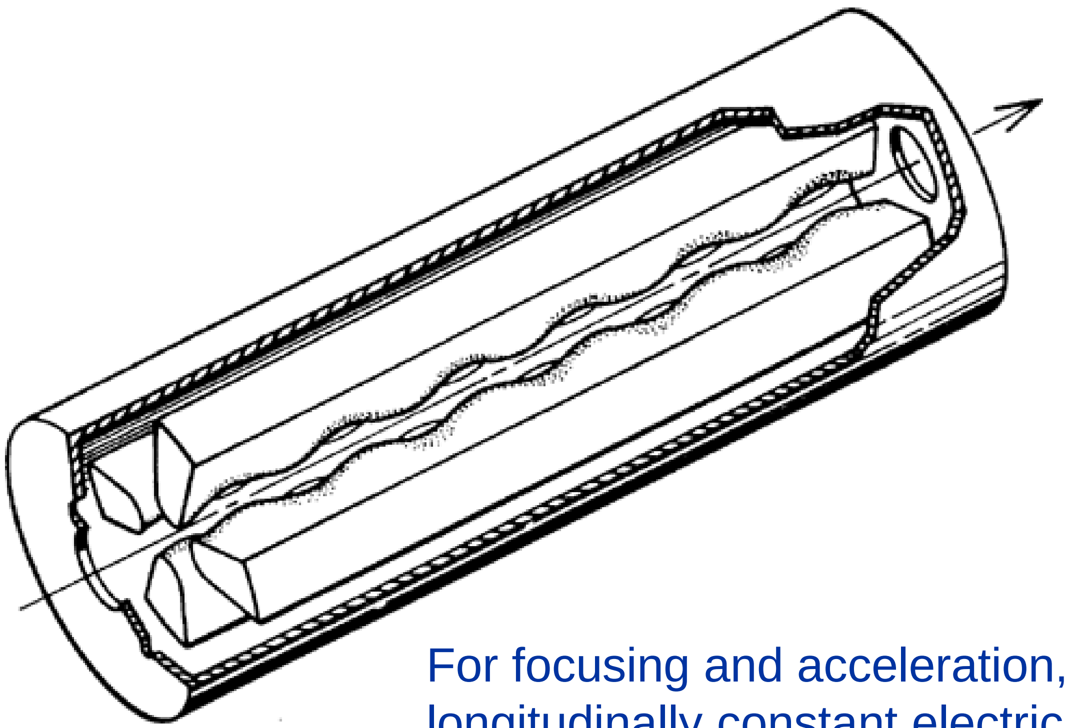
Cavity with vanes

1. In the empty cylindrical cavity, we have the TE21 (quadrupole) mode – very close in frequency to the TE11 (dipole) mode.

2. Introducing into the cavity 4 electrodes (the “**vanes”**) we can “load” the TE210 mode, with 2 effects:
   - Concentrate the electric field on the axis, increasing the focusing efficiency.
   - Lower the frequency of the TE210 mode, separating it from the other modes of the cylindrical waveguide.

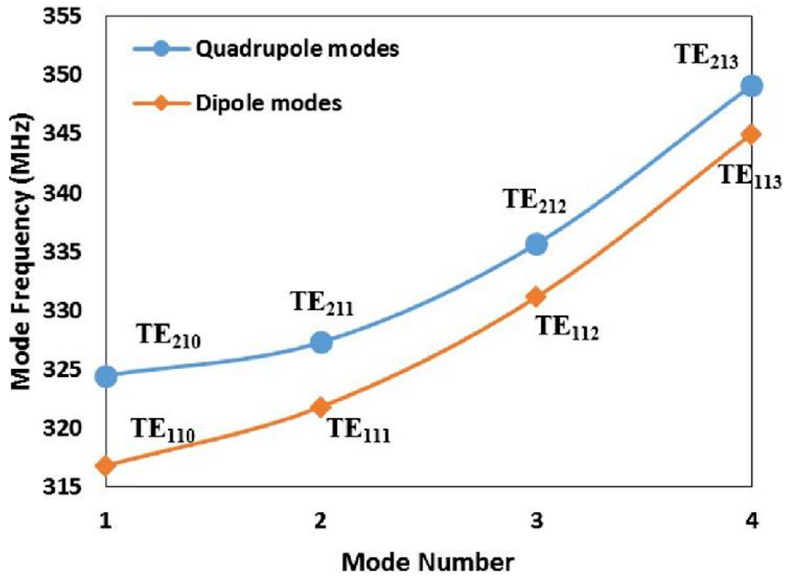
Unfortunately, the **dipole mode TE110** is lowered as well, and remains as a perturbing mode in this type of RFQs.

The 4-vane RFQ - longitudinally



The RFQ will result in cylinder containing the 4 vanes, which are connected to the cylinder along their length.

The loaded cylindrical waveguide sustaining the TE11 and TE21 mode bands will be closed at the 2 ends → on the dispersion curve are selected only the modes where the length contains an integer number of (half) longitudinal wavelengths.



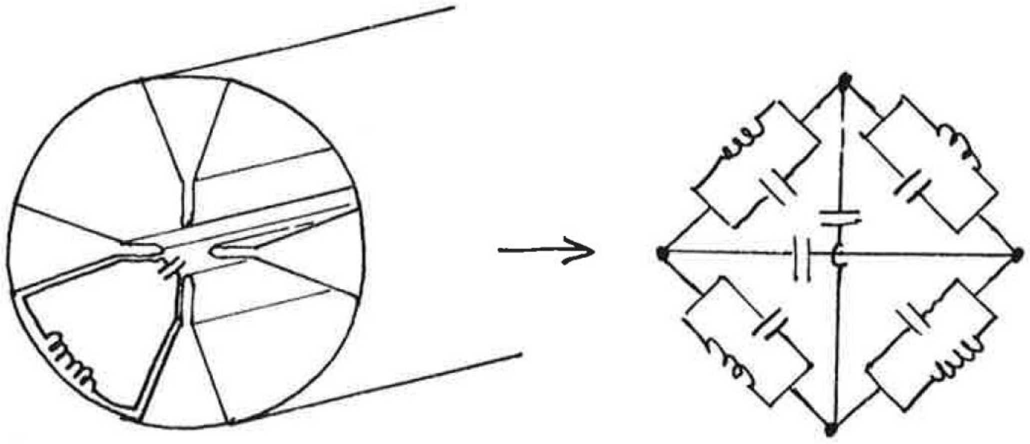
For focusing and acceleration, we need a mode with longitudinally constant electric field.

We need to operate in the **TE210** mode

But a TE mode with 0 longitudinal variation cannot exist in a waveguide short-circuited at the ends.

We need a "trick" to establish a TE210 mode!

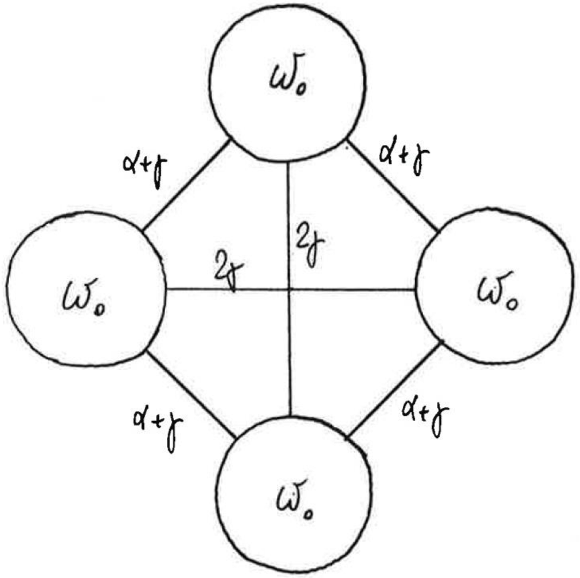
RFQ equivalent circuit, coupled resonator model

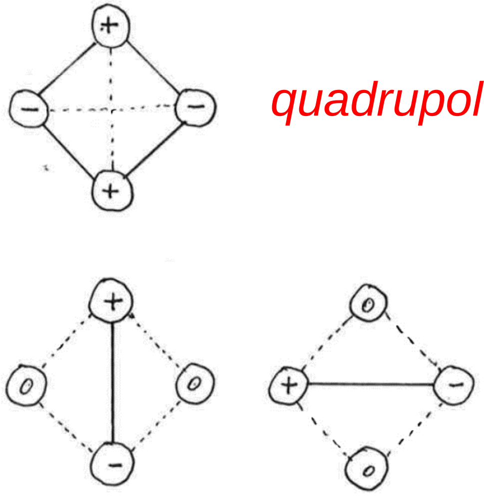


- Each quadrant of an RFQ can be seen as an RLC resonator.
- The 4 resonators are strongly coupled to the neighbouring resonators and weakly coupled to the opposite one via an intervane capacitance.

A system of 4 coupled resonators has 4 oscillation modes:

- +/+/+/+: a “zero” mode, not permitted by boundary conditions
- +/0/-/0 and 0/+/0/-, two “polarities” of a dipole mode.
- +/-/+/-, the quadrupole mode

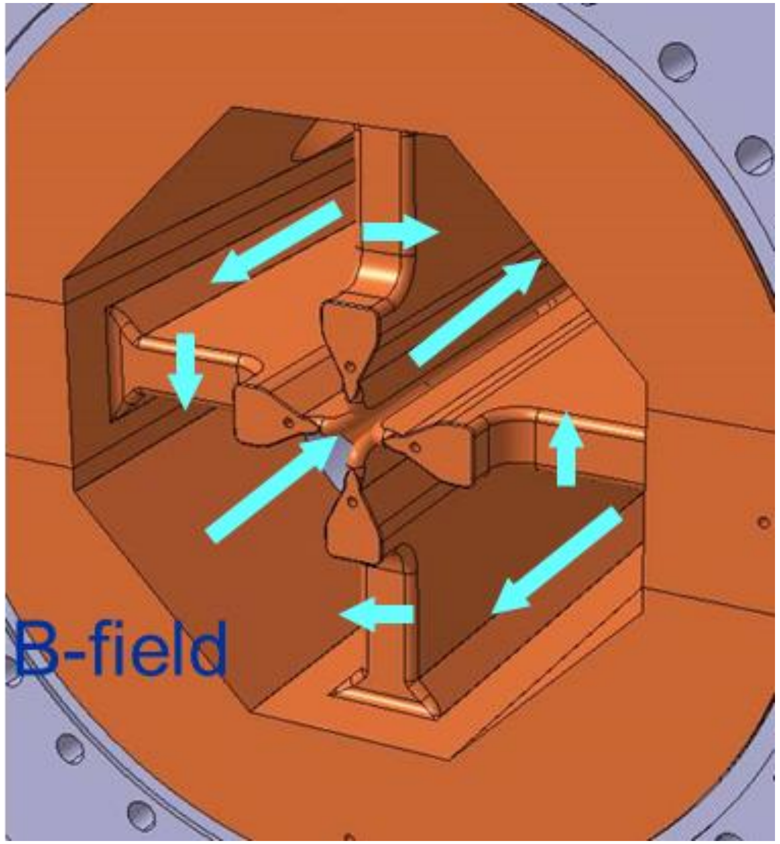




quadrupole

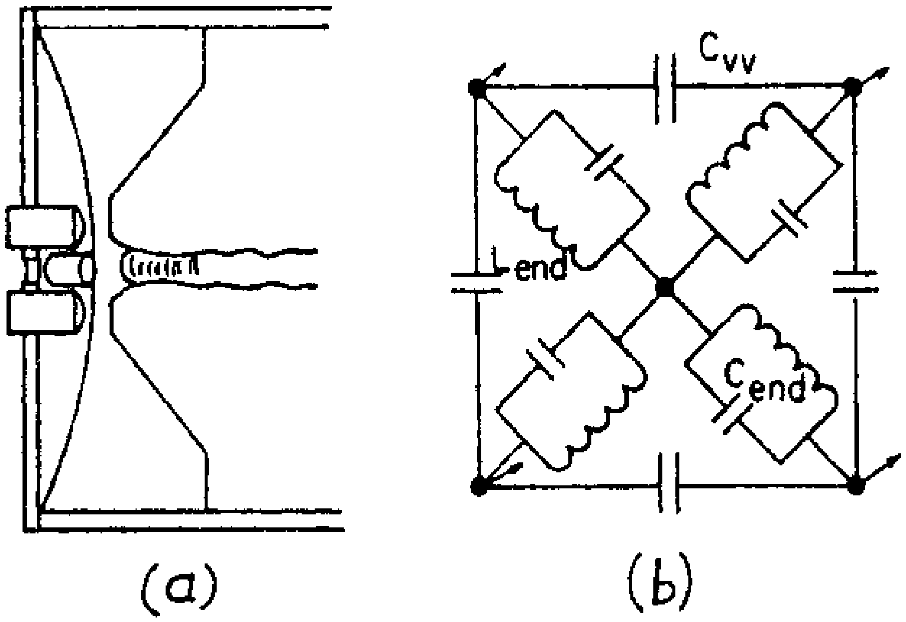
2 dipole modes

The end cells – simulating an infinite waveguide!



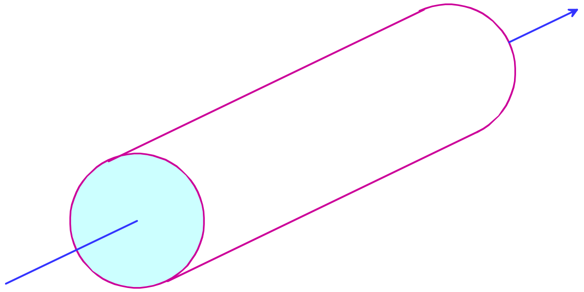
A critical feature are the end cells:

The magnetic field flowing longitudinally in the 4 “quadrants” has to close its path and pass from one quadrant to the next via some openings at the end of the vanes, tuned at the RFQ frequency!



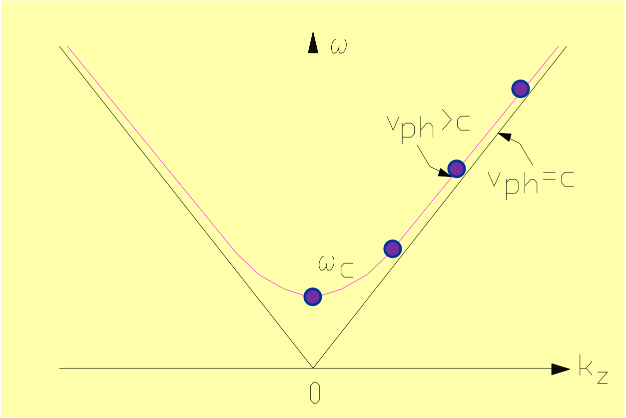
. 3 (a) The end region of a standard four-vane RFQ, showing an end

Length of an RFQ



Longitudinally, the RFQ behaves like a waveguide terminated at both ends by a metallic plate

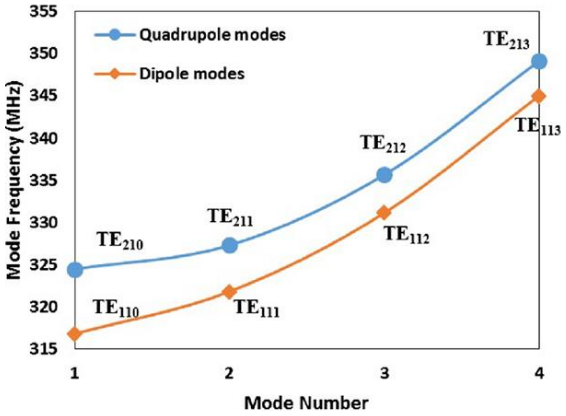
In the dispersion curve of a waveguide only some modes are allowed because of the terminations.



In the RFQ length *L* we must have an integer number of wavelengths; we can then take the fundamental waveguide relation:

$$n\frac{\lambda_g}{2} = L \qquad \frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2} = \frac{1}{\lambda_c^2} + \frac{n^2}{4L^2}$$

$$\Rightarrow f_N^2 = f_c^2 + \frac{c^2 n^2}{4L^2} \Rightarrow \frac{f_N^2 - f_c^2}{f_c^2} = \frac{n^2}{4}\left(\frac{\lambda_c}{L}\right)^2$$

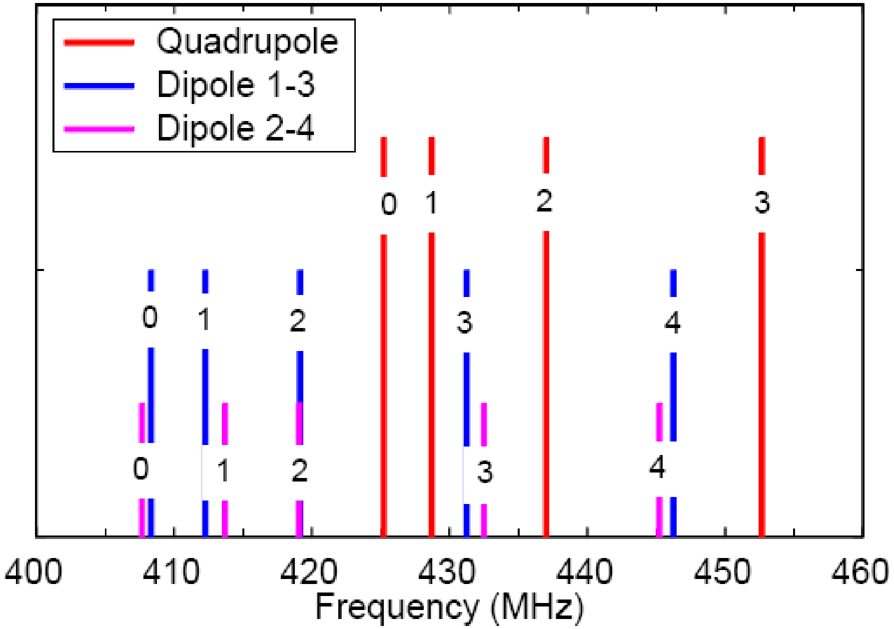


The length of an RFQ is limited by field errors: the higher $(L/\lambda)^2$, the higher will be the field error induced by a given perturbation (eg. error in the position of the vanes or in the tuning of the 4-rod cells) → the longer the RFQ, the closer the higher-order modes come to the operating mode and the more difficult becomes to keep the field flat.

Rule of thumb (4-vane): $L < 2\lambda$ → no problem, $2\lambda < L < 4\lambda$ → need some care

$L >$ ~4λ → difficult

Field symmetry and errors



Mode spectrum (after tuning) of a 425 MHz, 2.75m long RFQ (3.9 λ) – a limit case!

Why RFQs are so demanding in terms of tolerances?

☞ Beam dynamics wants a pure quadrupole mode (no dipole components, flat voltage along the RFQ).

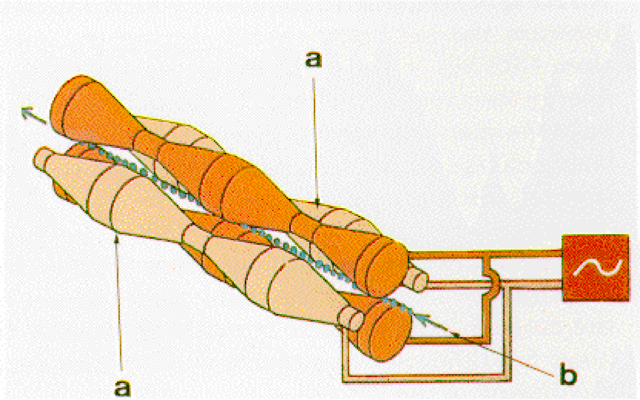
☞ But the TE210 quadrupole mode is not the only one: at frequencies close to the TE210 operating one there are all modes of the TE21 band (quadrupoles) and of the TE11 band (dipoles).

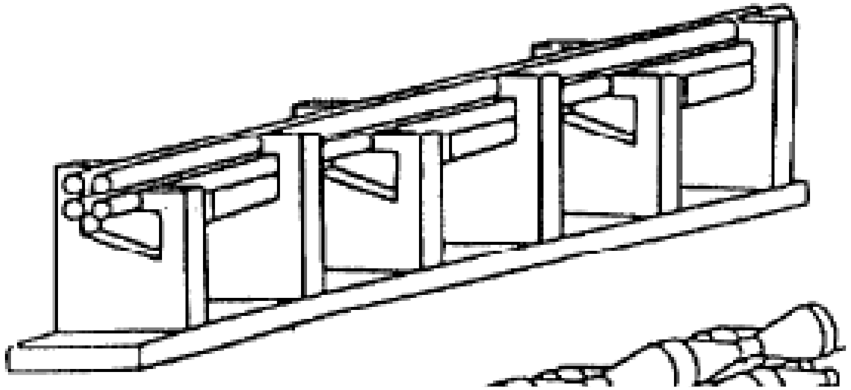
☞ Errors in the RFQ geometry (deviations from quadrupole symmetry) will induce errors in the electric field (dipole components, longitudinal errors) inversely proportional to the frequency separation between operating and perturbing modes.

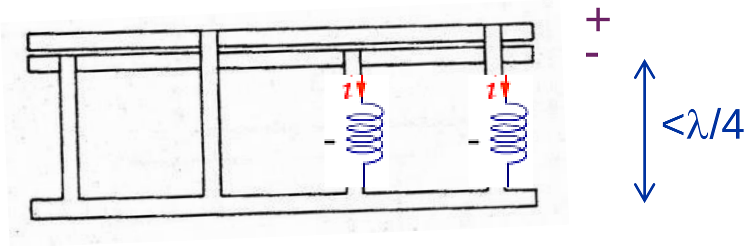
In order to keep the e.m. field error <~1% as required by beam dynamics are needed:

a. Careful design and in same cases stabilising schemes (to keep perturbing modes far from operating mode).
b. Tight mechanical tolerances in vane position (Linac4 RFQ: errors < 30 μm).
c. Correction schemes to flatten the voltage (local tuners, end cells, etc.).

The 4-rod RFQ







An alternative solution is to machine the modulation not on the tip of an electrode, but on a set of rods (machining on a lathe, old design) or on some small “vanelets”.

The 4 electrodes are then brought to the correct quadrupole potential by an arrangement of quarter-wavelength transmission lines. The set-up is then inserted into a cylindrical tank.

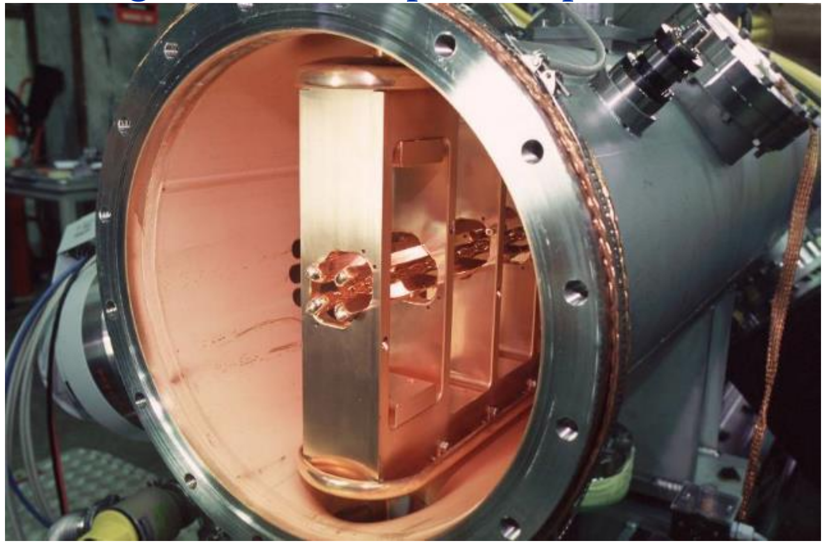
Cost-effective solution, becomes critical at high frequencies → dimensions become small and current densities go up.

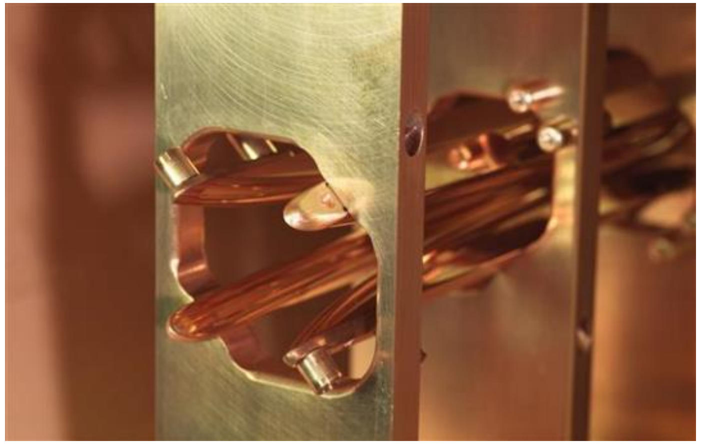
Power densities are higher than in the 4-vane → more problems for high power applications.

Commonly used for heavy ions and protons at low frequency – low duty cycle (f <200 MHz). Under development versions for high duty

Alternative "4-rod" geometries

The electrodes can also be "vane-like" in structures using doubled $\lambda/4$ parallel plate lines to create the correct fields.





Mechanical aspects – tolerances

Two main mechanical problems:

1. The need to achieve the tight tolerances in vane machining and positioning required by beam dynamics and RF.

Machining of a vane for the new CERN RFQ (linac4)

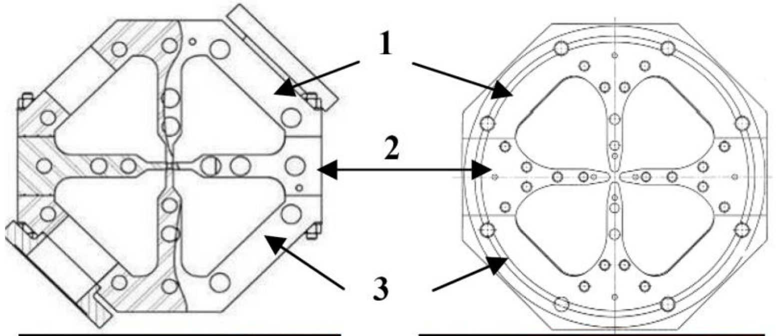
RF and beam dynamics both require tight tolerances in the position of the electrodes (Linac4 RFQ: <30 μm).
RF: presence of dipole and/or longitudinal components.
Beam dynamics: introduction of multipoles (Linac4 RFQ average aperture r_0 = 3.3 mm, 1% of aperture is ~30 μm). Minimum aperture a = 1.8 mm !!

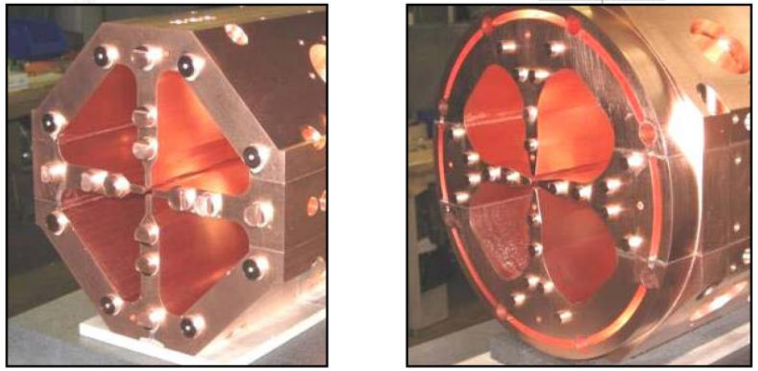
Linac4 RFQ Mechanical Tolerances	Value	Units
Machining error	±20	μm
Vane modulation error	±20	μm
Vane tilt over 1 m	±100	μm
Vane positioning error (displacement h+V)	±30	μm
Vane thickness error	±10	μm
Electrode gap (contiguous modules)	100±15	μm
Section tilt over 1 m	±30	μm
Electromagnetic field error	±1	%

Mechanical aspects – joining RFQ parts

2. The need to assemble a LEGO® of several components (tanks, vanes or rods, supports, etc.) that have to fit together keeping the tolerances and providing a good quality RF contact (large currents flowing transversally!).

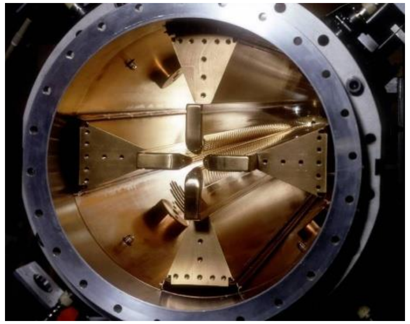
4-vane, high frequency: furnace brazing of 4 copper elements





TRASCO, LNL, Italy   IPHI, CEA-CNRS, France

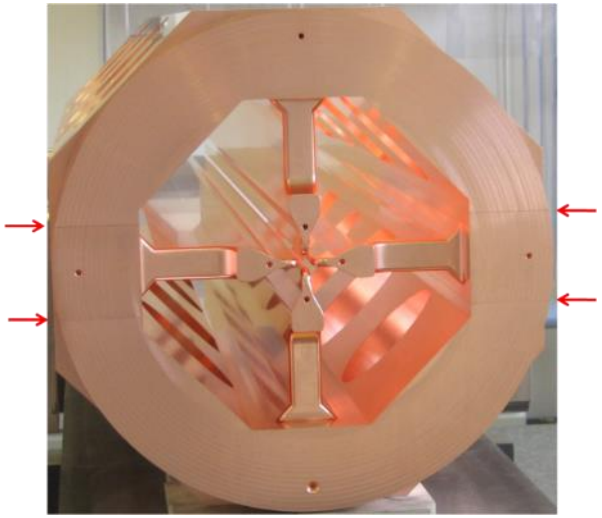
4-vane, low frequency: EB welding or bolting of copper or copper plated elements

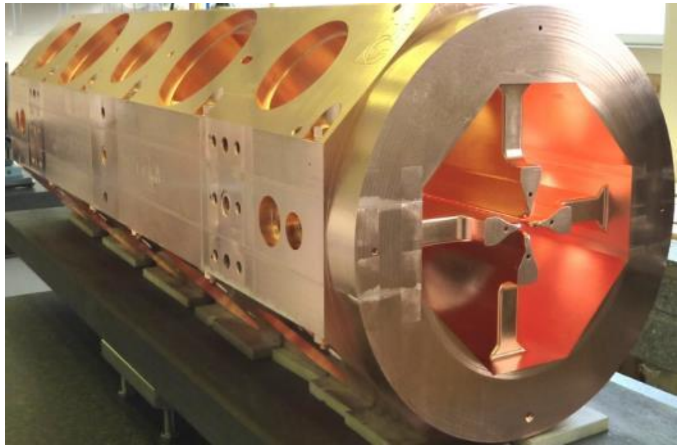


RFQ1 and RFQ2, CERN

SPIRAL2, CEA-CNRS, France

Errors before and after brazing



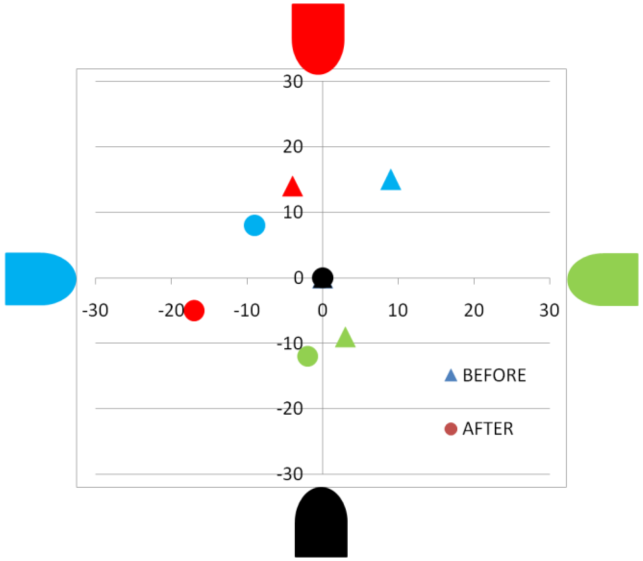


Linac4 RFQ:

☞ 3 segments of 1 m, each formed by 4 parts brazed together

☞ Required error in vane positions <30 μm

☞ Achieved by a) precise machining of the contact surfaces and b) appropriate thermal treatments to minimize vane deformation during brazing.



RFQ – thermal aspects

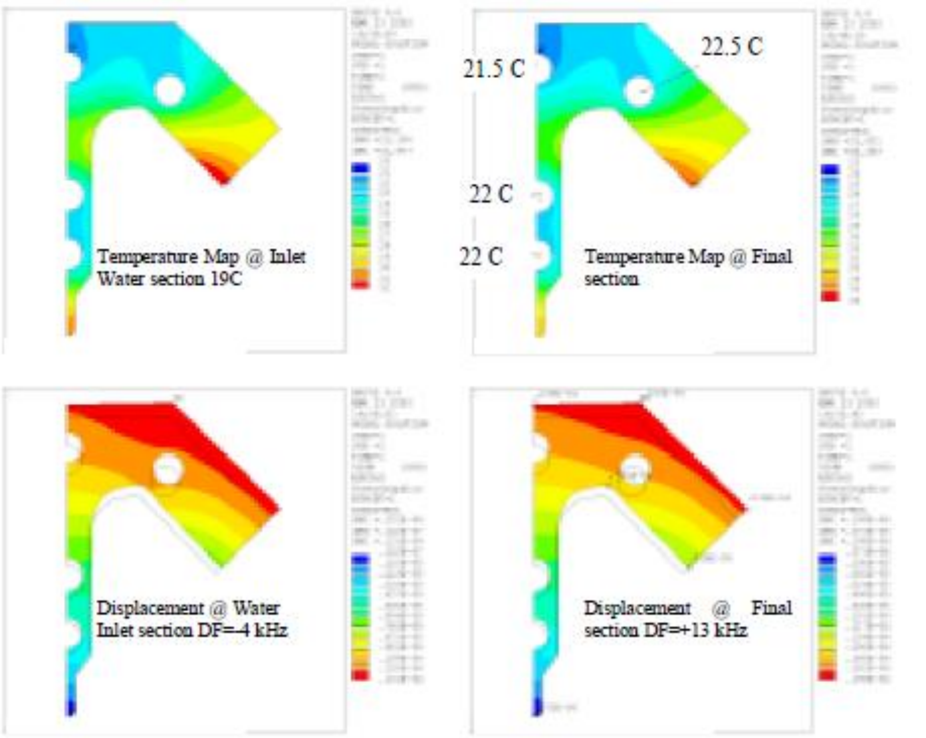


Fig. 6: Top: temperature maps at begin (left) and at the end (right) of one RFQ section. Bottom: deformation maps and frequency shifts.

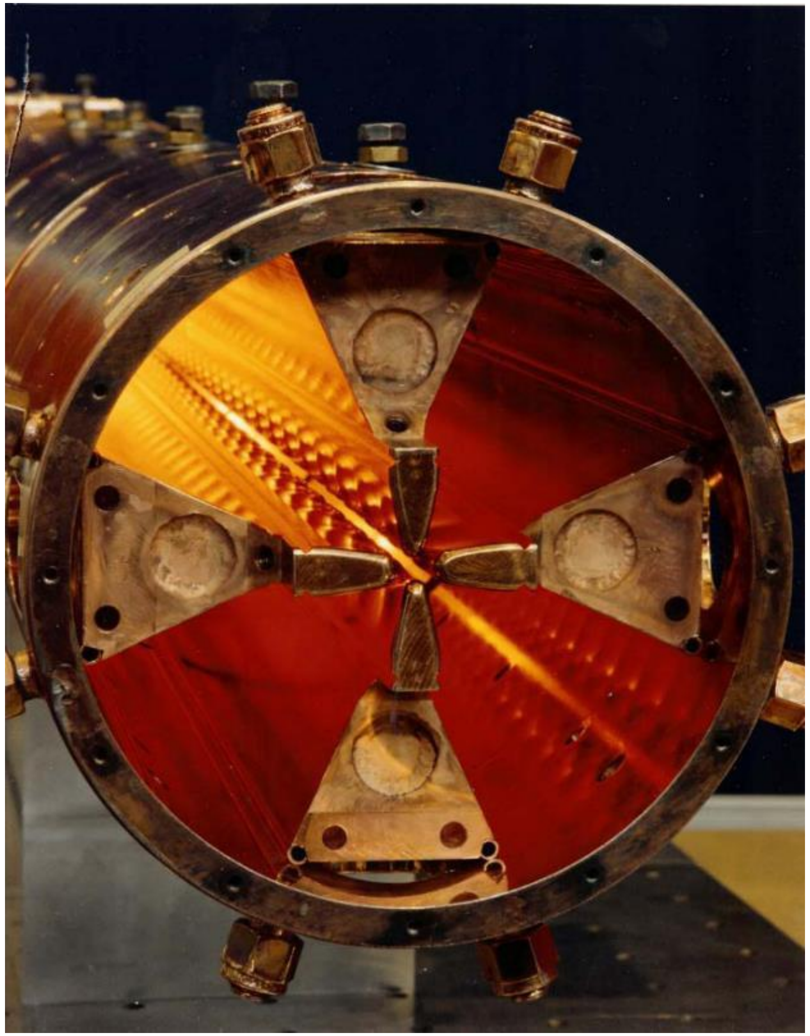
Example: thermal study of the TRASCO RFQ (CW, 352 MHz, 1 kW/cm) – courtesy of LNL, Legnaro

1. High (beam) power RFQs need to dissipate large amounts of RF power in small volumes (vanes are “thin” to maximize shunt impedance).
2. Thermal deformations can lead to large voltage variations and to beam loss.

Need to carefully design and dimension the cooling channels to keep High (beam) power RFQs need to dissipate large amounts of RF power in small volumes (vanes are “thin” to maximize shunt impedance).

Thermal deformations can lead to large voltage variations and to beam loss.

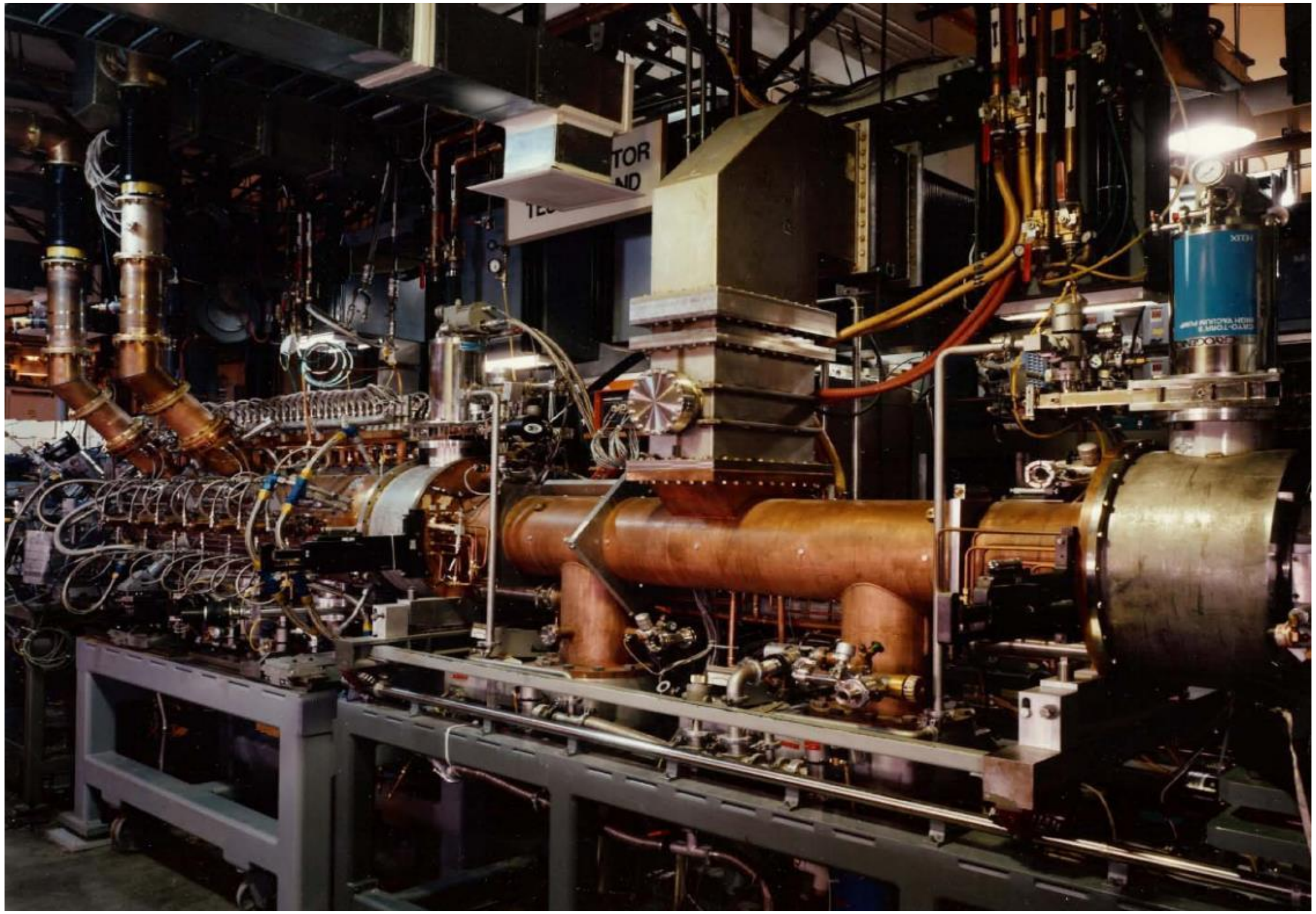
The 1st 4-vane RFQ



Proof of Principle (POP) RFQ, Los Alamos
1980 – the 1st vane-type RFQ
100 KeV - 650 KeV, 30 mA , 425 MHz

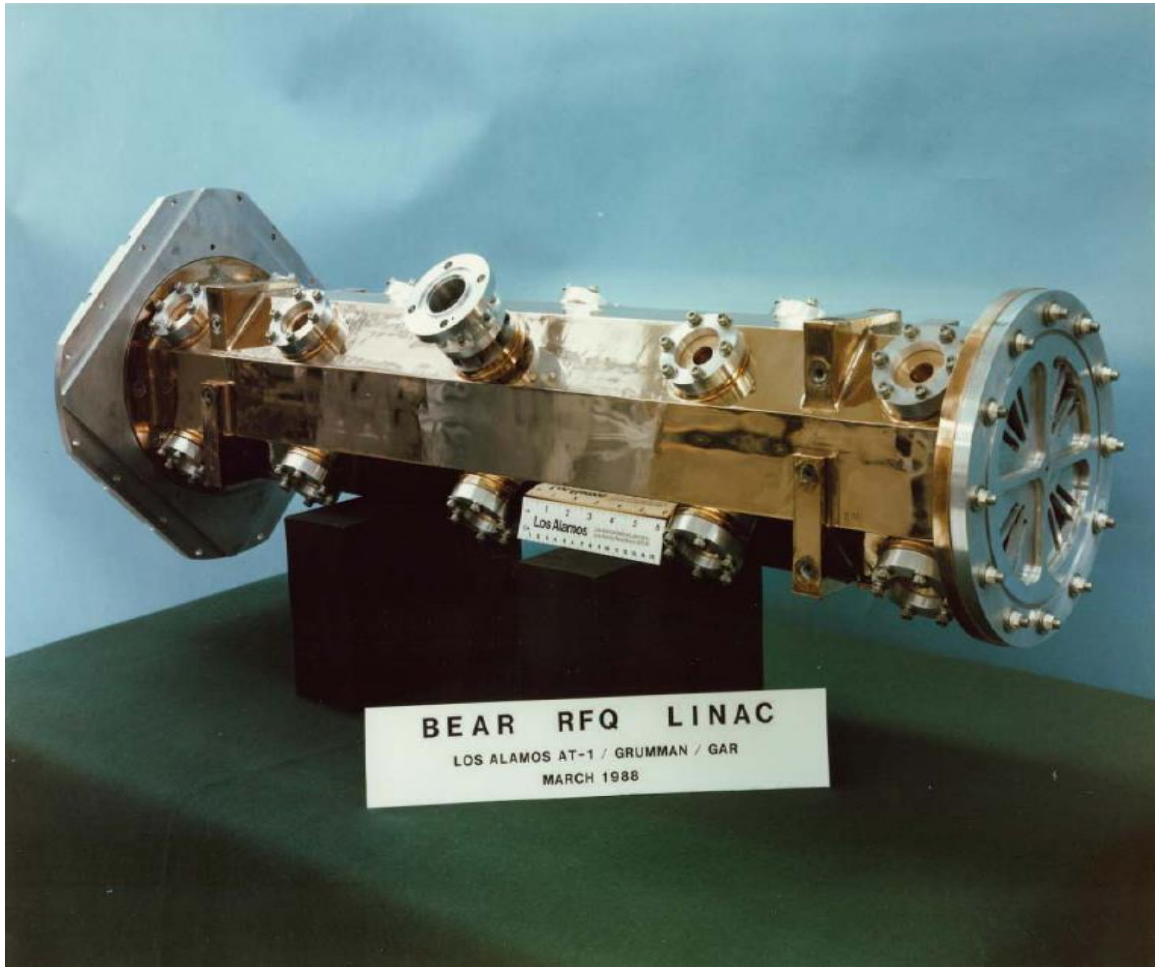
Examples of RFQs – 1

"Star Wars" RFQ (now de-classified), 1983, LANL
2 MeV, 100 mA, ~5% duty, H-minus, 425 MHz
Cu plated carbon steel vanes and cavity, manifold coupled
Demonstrated very small emittance H-minus beams



Examples of RFQs - 2

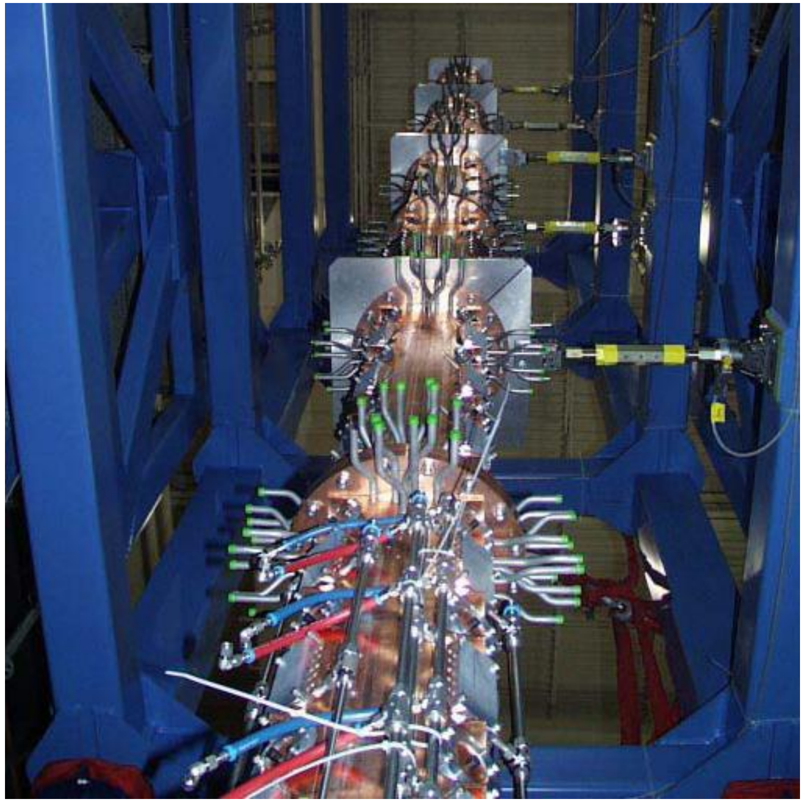
“BEAR” RFQ (beam experiment aboard a rocket)(partly classified) 1989
30 KeV – 1 MeV, 20 mA, <1% duty H-minus
425 MHz, solid-state RF system
Cu plated Al quadrants, joined by electroforming, 55 kg
Operated in sub-orbital flight with a “neutral” beam, LANL

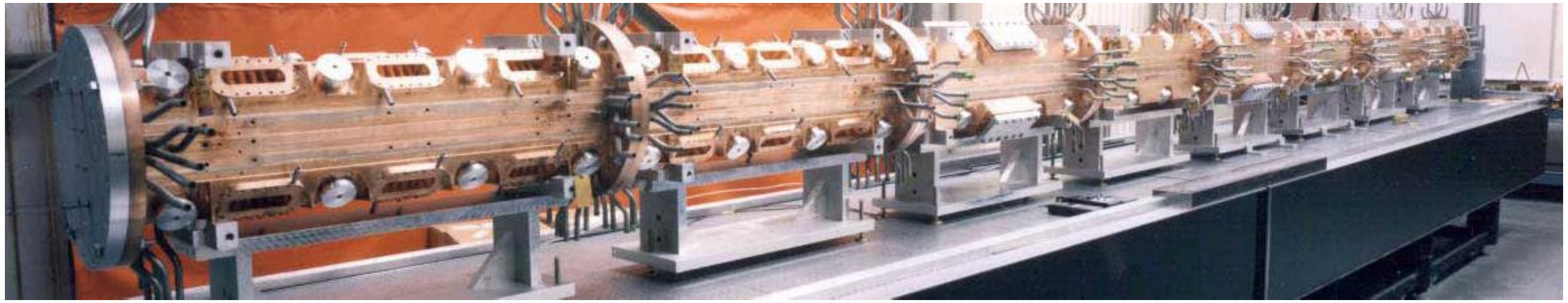


Examples of RFQs - 3

The first high-beam-power RFQ

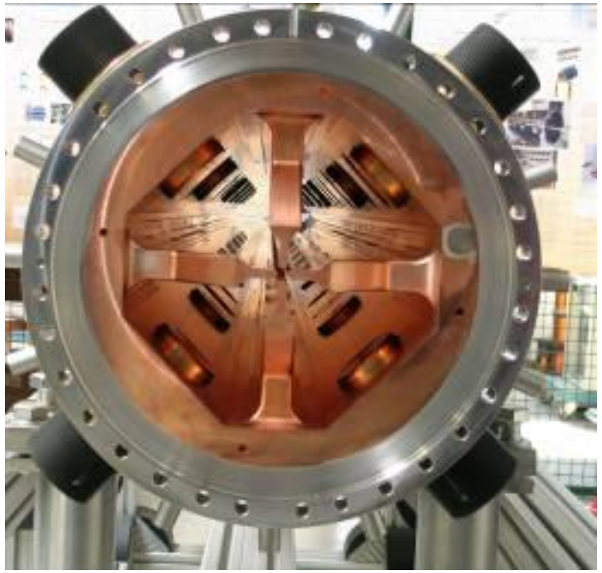
LEDA RFQ (low energy demonstration accelerator)
1999 - 2000
75 keV-6.7 MeV, 100 mA cw protons
350 MHz
Brazed OFE Cu quadrants
Resonantly coupled, 8 m long, LANL



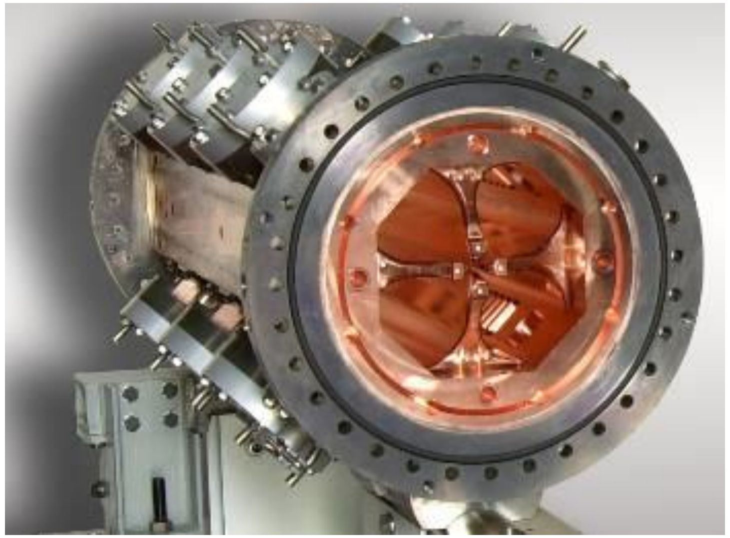


Examples of RFQs - 4

High frequency (352 MHz), high duty cycle (CW) for ADS studies and other applications.

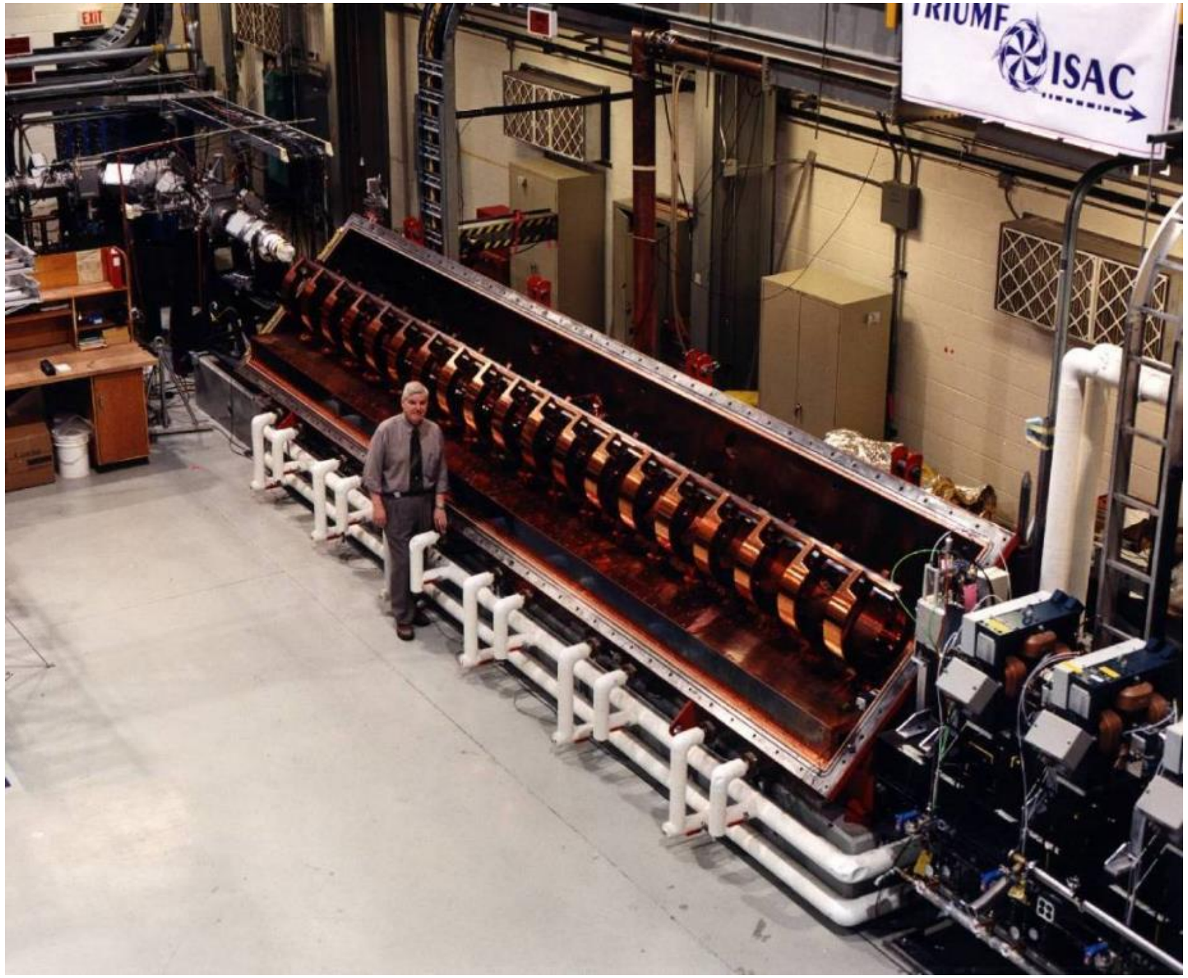


TRASCO@LegnaroINFN



IPHI@Saclay.CEA

Examples of RFQs - 5



Low frequency (35 MHz), high duty cycle (CW) for post-acceleration of radioactive ions.

The ISAC-II RFQ at TRIUMF (Canada)

Examples of RFQ - 6

Al prototype and the final installation of the superconducting RFQ at LNL, Italy

Superconducting RFQs:

Only one operating Superconducting RFQs built so far in the world (INFN Legnaro, Italy).

The modulation is extremely difficult to realise in Nb $\rightarrow$ a superconducting RFQ is limited to few cells at low frequency $\rightarrow$ heavy ions.

LNL superconducting RFQ: 2 separate structures, 1.4 m and 0.8 m, 41 and 13 cells

On proton RFQs with high intensity, the unavoidable beam loss during the bunching process would be very dangerous for a superconducting structure.

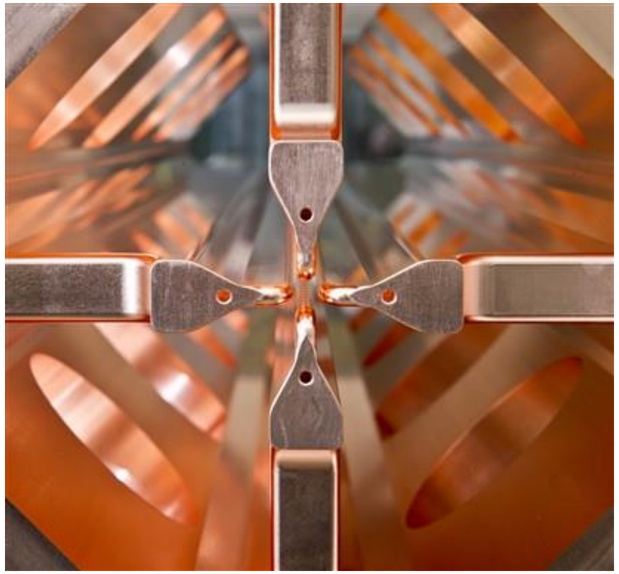
Examples of RFQ - 7

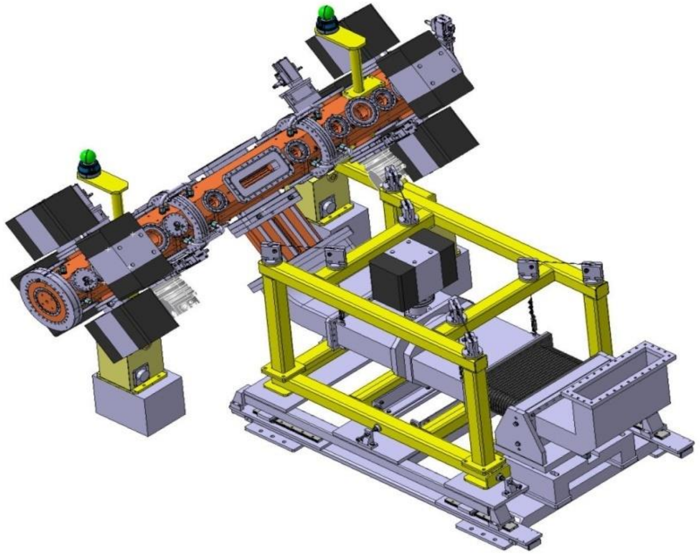
Medium frequency (176 MHz), high duty cycle (CW),
4-rod design
for high-intensity deuteron and proton acceleration.

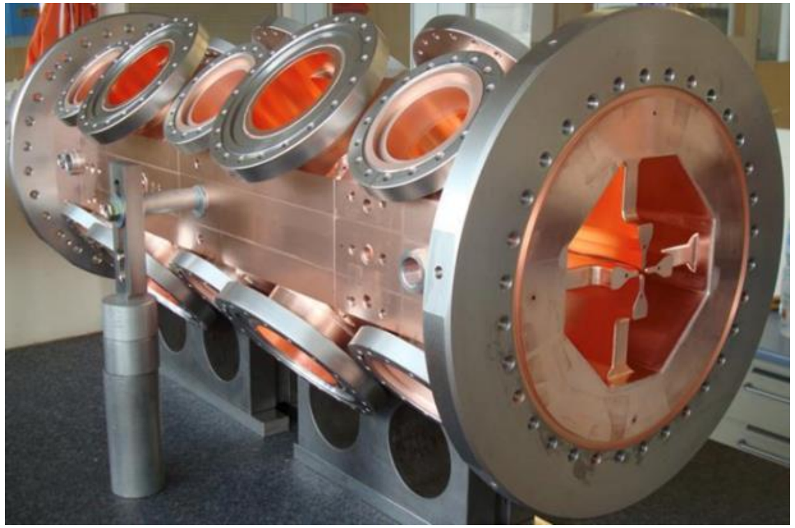
The SARAF RFQ, built by NTG for the Soreq Nuclear Research Center in Israel.

Examples of RFQs - 8

The CERN Linac4 RFQ
45 keV – 3 MeV, 3 m
80 mA H-, max. 10% duty cycle
Commissioned in 2013

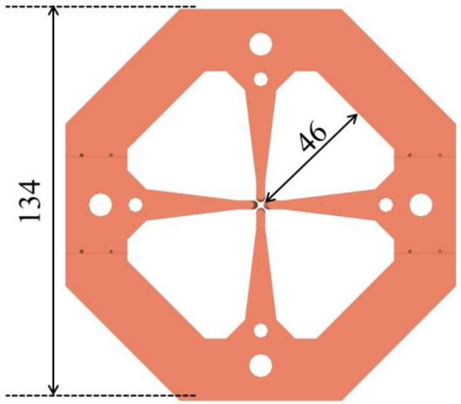


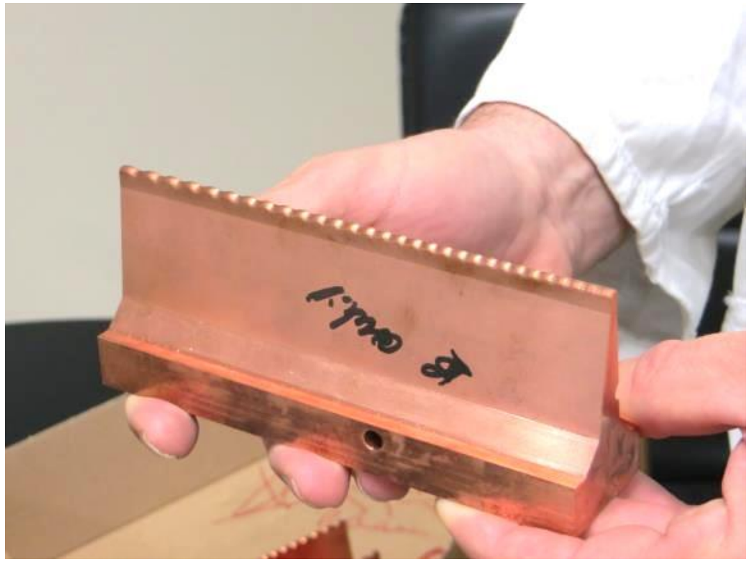


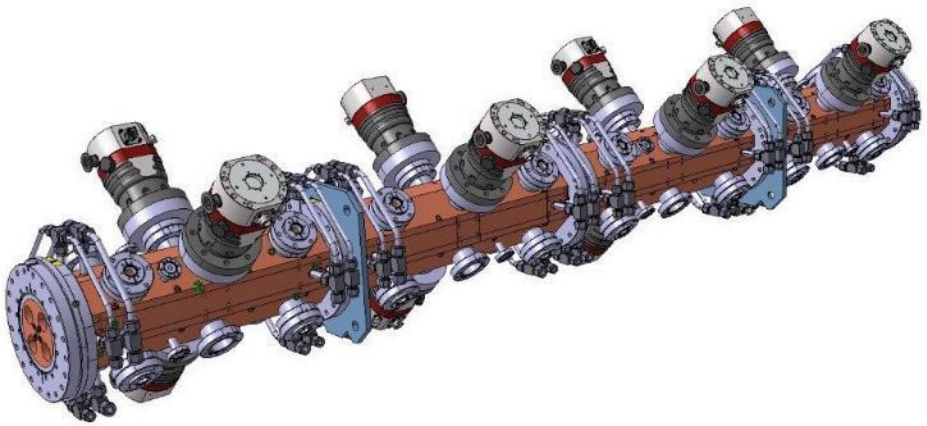


Examples of RFQ - 9

The compact high-frequency RFQ built at CERN
40 keV – 5 MeV, in only 2 m
750 MHz, max. 5% duty cycle
For medical and industrial applications
Highest frequency in the world







Modulation test machining

The 750 MHz RFQ

http://cds.cern.ch/journal/CERNBulletin

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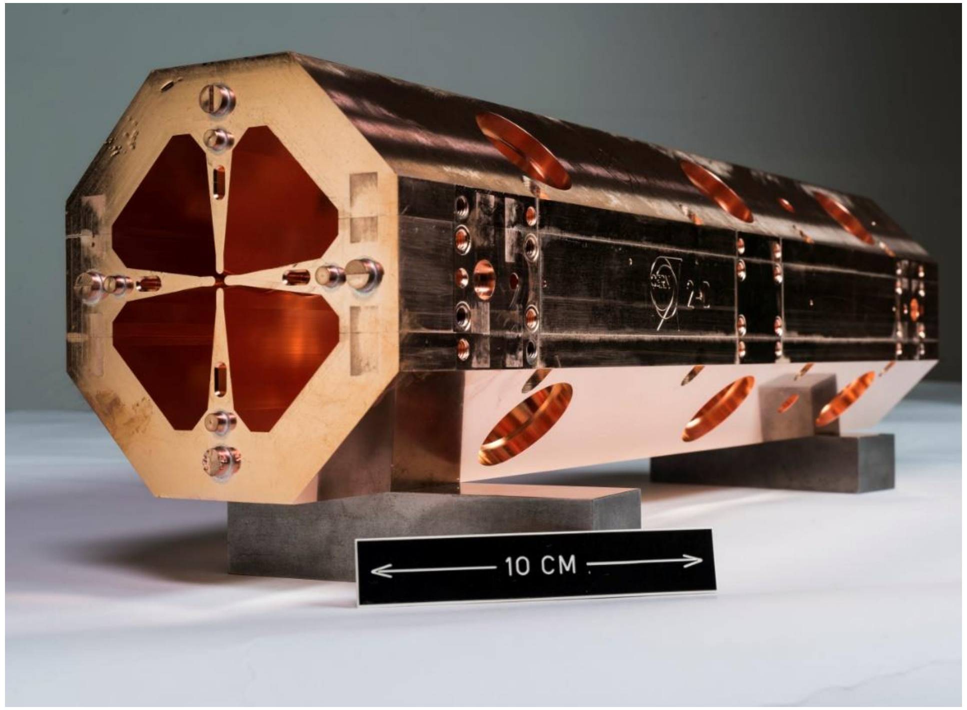
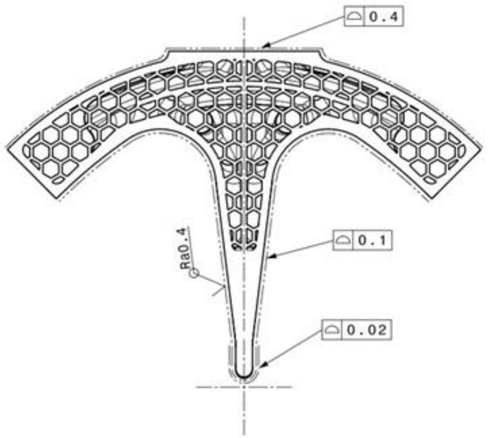
THE MINIATURE ACCELERATOR

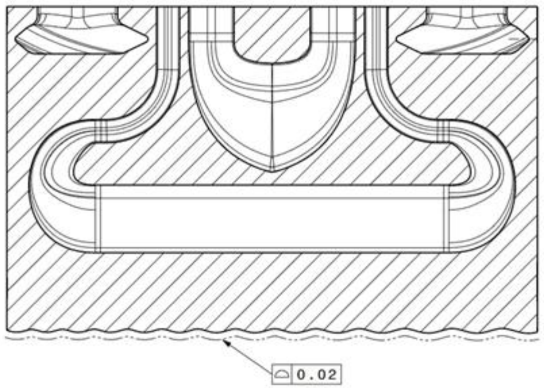
The image that most people have of CERN is of its enormous accelerators and their capacity to accelerate particles to extremely high energies. But thanks to some cutting-edge studies on beam dynamics and radiofrequency technology, along with innovative construction techniques, teams at CERN have now created the first module of a brand-new accelerator, which will be just 2 metres long. The potential uses of this miniature accelerator will include deployment in hospitals for the production of medical isotopes and the treatment of cancer. It's a real David-and-Goliath story. >>

Click to add notes



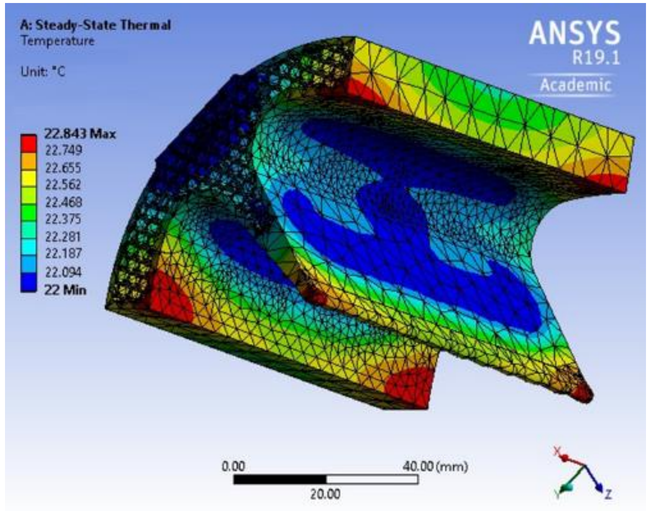
The first Additive Manufactured RFQ sample





Measured roughness 12-15 μm, required 0.4 μm.

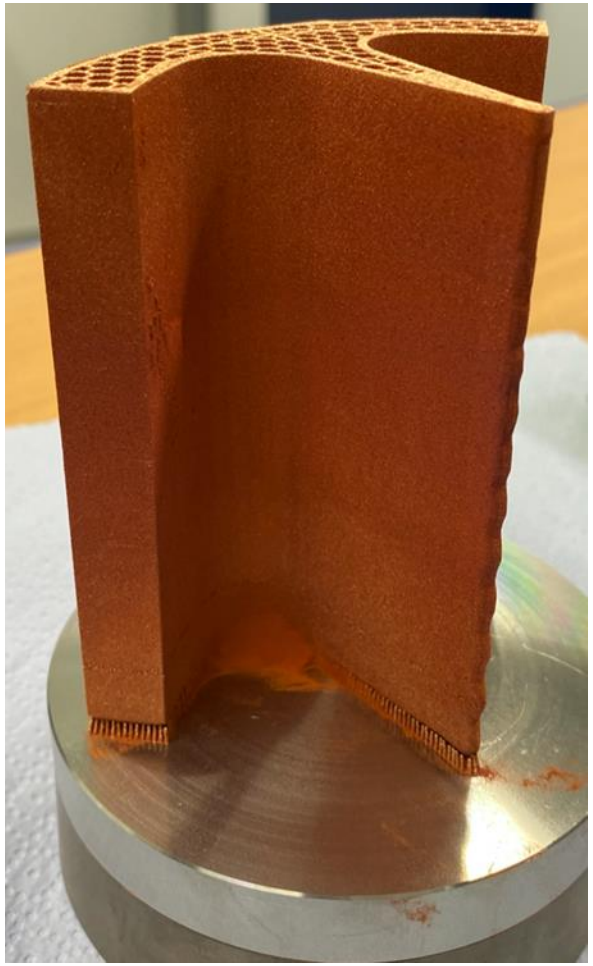
Measured machining errors slightly higher than required accuracy of 20 μm on vane tips, and 100 μm on walls.



FIRST PROOF-OF-CONCEPT PROTOTYPE OF AN ADDITIVE-MANUFACTURED RADIO FREQUENCY QUADRUPOLE

T. Torims[1,2], G. Pikurs[1,2], S. Gruber[3], M. Vretenar[2], A. Ratkus[1,2], M. Vedani[4], E. Lopez[3], F. Bruckner[3,5]

[1] Riga Technical University, Riga, Latvia
[2] CERN, Geneva, Switzerland
[3] Fraunhofer Institute for Material and Beam Technology, Dresden, Germany
[4] Politecnico di Milano, Milan, Italy
5 Luleå University of Technology, Sweden



End of Lecture 5

Elwood Smith

Additional slides on coupling

Coupling between two cavities

In the PIMS, cells are coupled via a slot in the walls. But what is the meaning of coupling, and how can we achieve a given coupling?

Simplest case: 2 resonators coupled via a slot

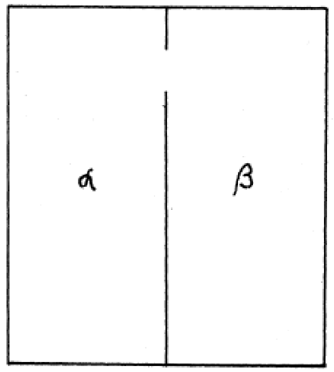


FIG. 1. Two cavities, α and β, coupled by a small hole.

E-field mode 1
E-field mode 2

Described by a system of 2 equations:

$$\begin{cases} X_1\left(1-\frac{\omega_1^2}{\omega^2}\right)+kX_2=0 \\ kX_1+X_2\left(1-\frac{\omega_2^2}{\omega^2}\right)=0 \end{cases} \quad \text{or} \quad \begin{vmatrix} 1-\frac{\omega_1^2}{\omega^2} & k \\ k & 1-\frac{\omega_2^2}{\omega^2} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = 0$$

If $\omega_1=\omega_2=\omega_0$, usual 2 solutions (mode 0 and mode π):

$$\omega_{c,1}=\frac{\omega_0}{\sqrt{1+k}} \quad \text{with} \quad \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \qquad \text{and} \qquad \omega_{c,2}=\frac{\omega_0}{\sqrt{1-k}} \quad \text{with} \quad \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} 1 \\ -1 \end{vmatrix}$$

Mode + + (field in phase in the 2 resonators) and mode + - (field with opposite phase)

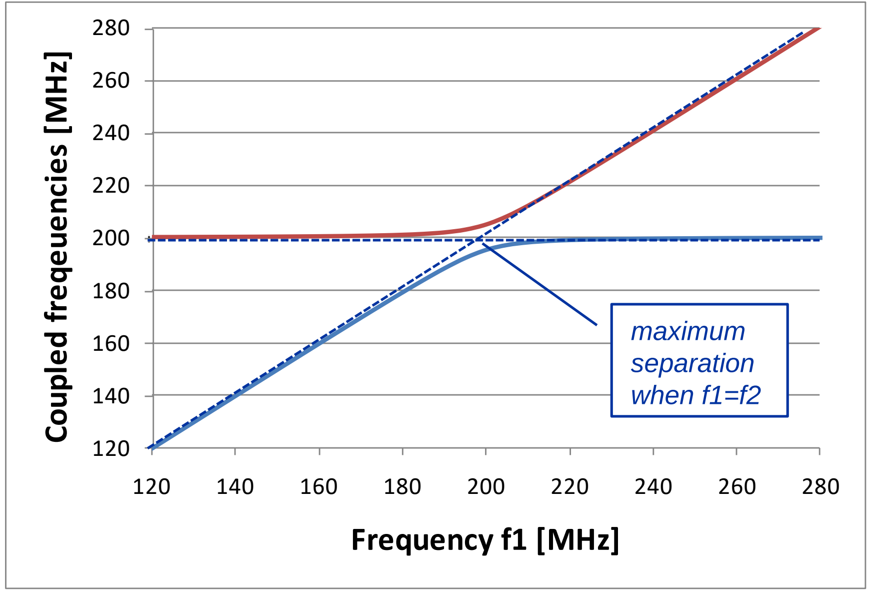
Taking the difference between the 2 solutions (squared), approximated for k<<1

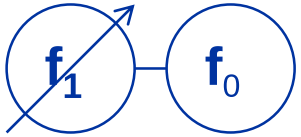
$$\frac{\omega_{c,2}^2-\omega_{c,1}^2}{\omega_0^2}=\frac{1}{1-k}-\frac{1}{1+k}\approx 2k \quad \text{or} \quad \boxed{\frac{\omega_{c,2}-\omega_{c,1}}{\omega_0}\approx k}$$

The coupling k is equal to the difference between highest and lowest frequencies.
→ k is the bandwidth of the coupled system.

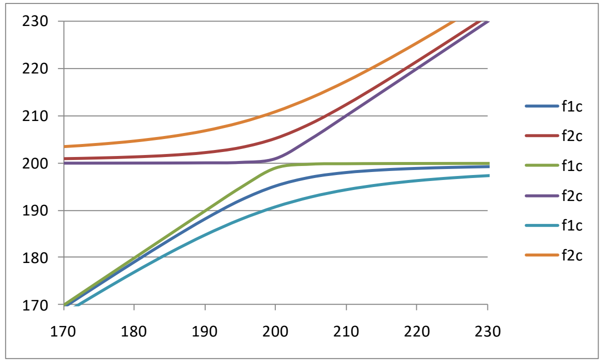
More on coupling

Solving the previous equations allowing a different frequency for each cell, we can plot the frequencies of the coupled system as a function of the frequency of the first resonator, keeping the frequency of the second constant, for different values of the coupling k.





- "Coupling" only when the 2 resonators are close in frequency.
- For $f_1=f_2$, maximum spacing between the 2 frequencies ($=kf_0$)



case of 3 different coupling factors (0.1%, 5%, 10%)

For an elliptical coupling slot:

$$k \approx F\, l^3 \left(\frac{H_1}{\sqrt{U_1}}\right)\left(\frac{H_2}{\sqrt{U_2}}\right)$$

F = slot form factor
l = slot length (in the direction of H)
H = magnetic field at slot position
U = stored energy

The coupling k is:
- Proportional to the 3rd power of slot length.
- Inv. proportional to the stored energies.