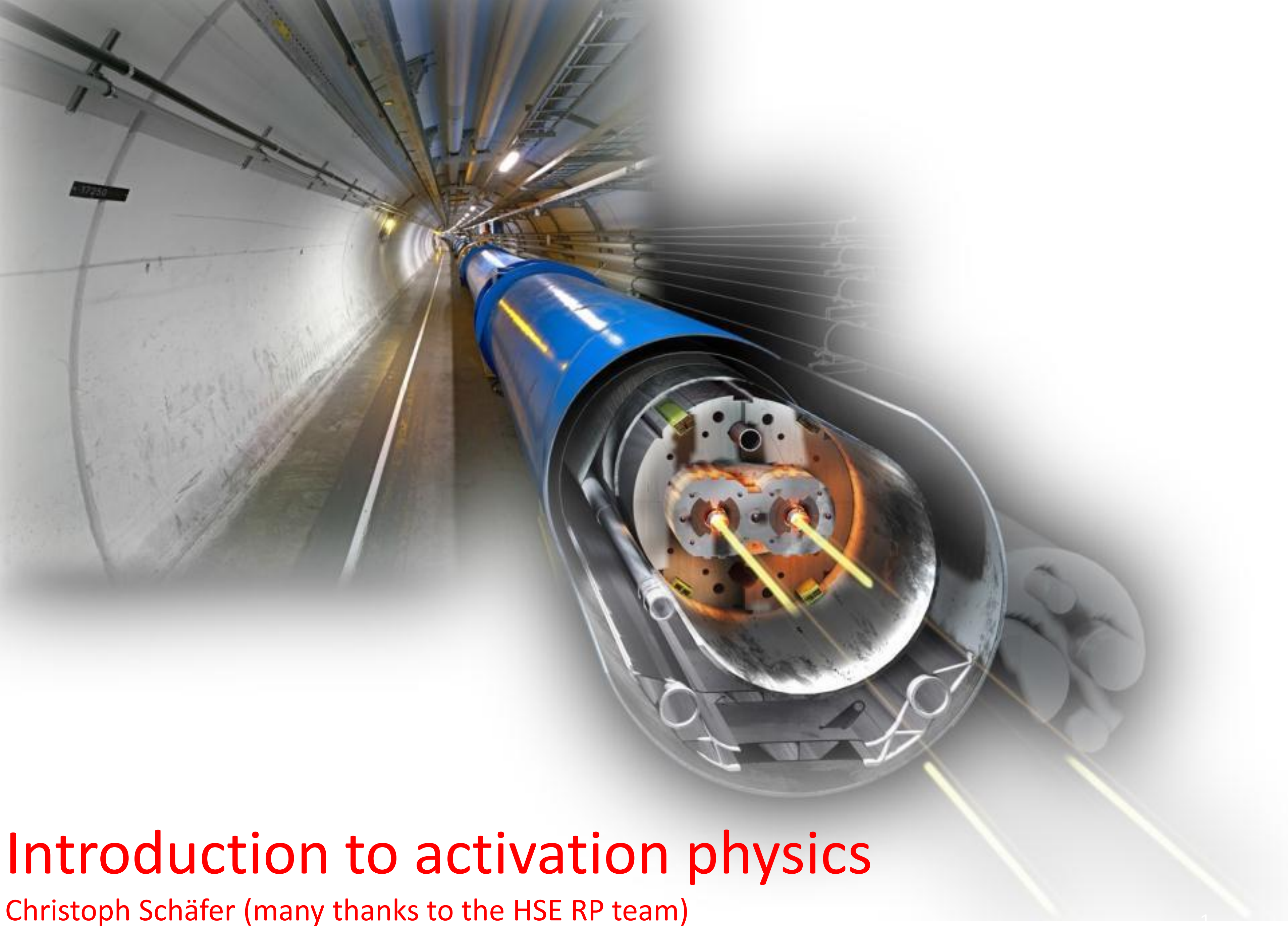


Introduction to activation physics

Christoph Schäfer (many thanks to the HSE RP team)

Contents

- Basics of activation physics
- Hands-on example

Basics of activation processes

Basics of activation physics

What is activation?

Activation can be described as the imposed change of nuclear composition of given isotopes resulting in the production of radioactivity

At which particle accelerators can activation occur?

In principle at all high-energy accelerators. However, the radionuclide production rate is much higher at hadron or ion accelerators than those experienced at electron/positron accelerators.

Where does activation occur at accelerators?

At locations with (high) particle losses like:

- Target area
- Dump area
- Beam cleaning sections like collimators and scraper areas
- High-energy physics experiments

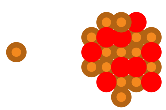
Activation processes

Which production mechanisms of activation occur at high-energy accelerators?

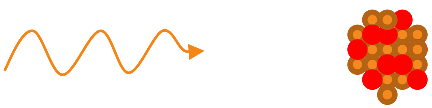
At high-energy accelerators primary particles interact with matter. The primary particle itself or secondary particles interacting with nuclei can produce radioactive isotopes. Main production channels of activation at high-energy accelerators are:

- Spallation and other inelastic hadronic interactions ((n,n'γ), (n,2n), (n,p), (n,α) …)

- Particle capture (mainly neutrons)

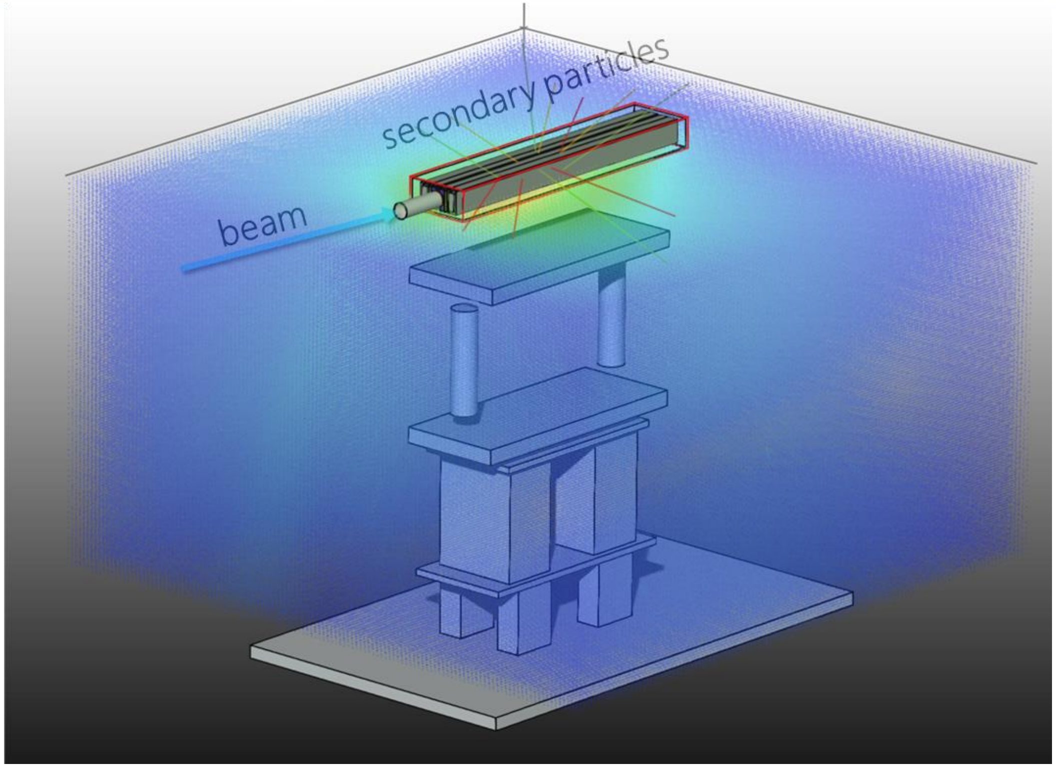


- (γ,n)-reactions (important for electron accelerators)



● proton ● neutron

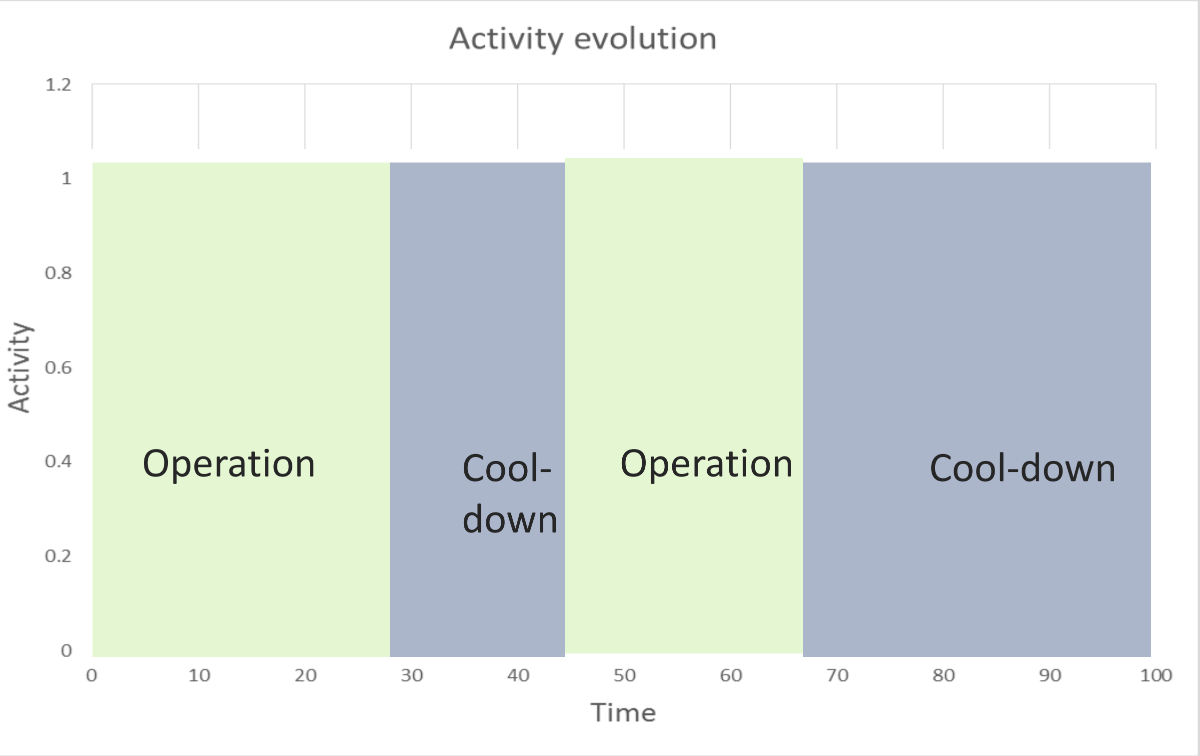
Isotope production

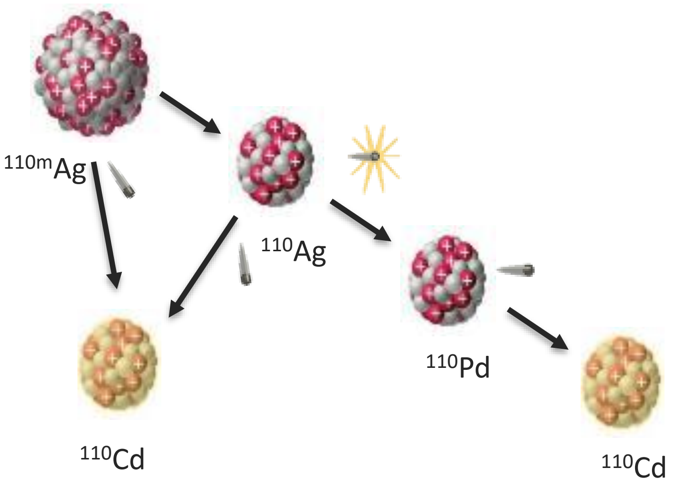


Activation depends on:

- chemical composition of target material
- incident particle type
- incident particle energy
- irradiation & cooling time

Creation of isotopes at an accelerator





decay-chains with branching

Operation: direct production + decay

Cool-down: indirect production via decay

Buildup & decay of isotopes

Decay of a mono-isotopic source described via an ordinary differential equation:

$$A = \frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda \cdot N$$

N	... Number of isotopes
λ	... Decay constant of isotope
N_0	... Number of isotopes at t = 0
$t_{1/2}$	... Half life
A	... Activity (Bq)
N_0	... Number of isotopes at t = 0

$$\lambda = \frac{\ln(2)}{t_{1/2}}$$

$$\int_{N_0}^{N} \frac{\mathrm{d}N}{N} = \int_0^t -\lambda \cdot dt$$

$$N(t) = N_0 e^{-\lambda t}$$

$$A(t) = N(t) \cdot \lambda$$

Buildup & decay of isotopes (simple case)

At an accelerator we also need to consider buildup → we need to solve a differential equations, considering beside the decay also a production term *P*.

$$\frac{\mathrm{d}N}{\mathrm{d}t} = P - \lambda \cdot N$$

Solution for this simple case:

!!! $$\boxed{N(t_{irr} + t_{cool}) = \frac{P}{\lambda}\left(1 - e^{-\lambda t_{irr}}\right) e^{-\lambda t_{cool}}}$$!!!

- the production rate *P* [nuclides/second] is constant
- decay chains are neglected
- consider only one irradiation & one cooling period

Buildup & decay of isotopes considering decay chains and multiple irradiation+cooling cycles

At accelerators the production & decay described via Bateman equations:

$$\frac{dN_1}{dt} = P_1 - \lambda_1 \cdot N_1$$

$$\frac{dN_2}{dt} = P_2 + (b_{1,2} \cdot \lambda_1 \cdot N_1) - \lambda_2 \cdot N_2$$

$$\vdots \qquad \vdots$$

$$\frac{dN_i}{dt} = P_i + (b_{i-1,i} \cdot \lambda_{i-1} \cdot N_{i-1}) - \lambda_i \cdot N_i$$

$$\vdots \qquad \vdots$$

$$\frac{dN_n}{dt} = \underbrace{P_n}_{\text{build-up}} + \underbrace{(b_{n-1,n} \cdot \lambda_{n-1} \cdot N_{n-1}) - \lambda_n \cdot N_n}_{\text{decay}}$$

N_n … Number of isotope n
P_n … Production rate of isotope n
l_n … Decay constant of isotope n
b_n … Branching ratio from isotope n-1 into n
k … index of irradiation/cooling cycle

Use

ActiWiz 3

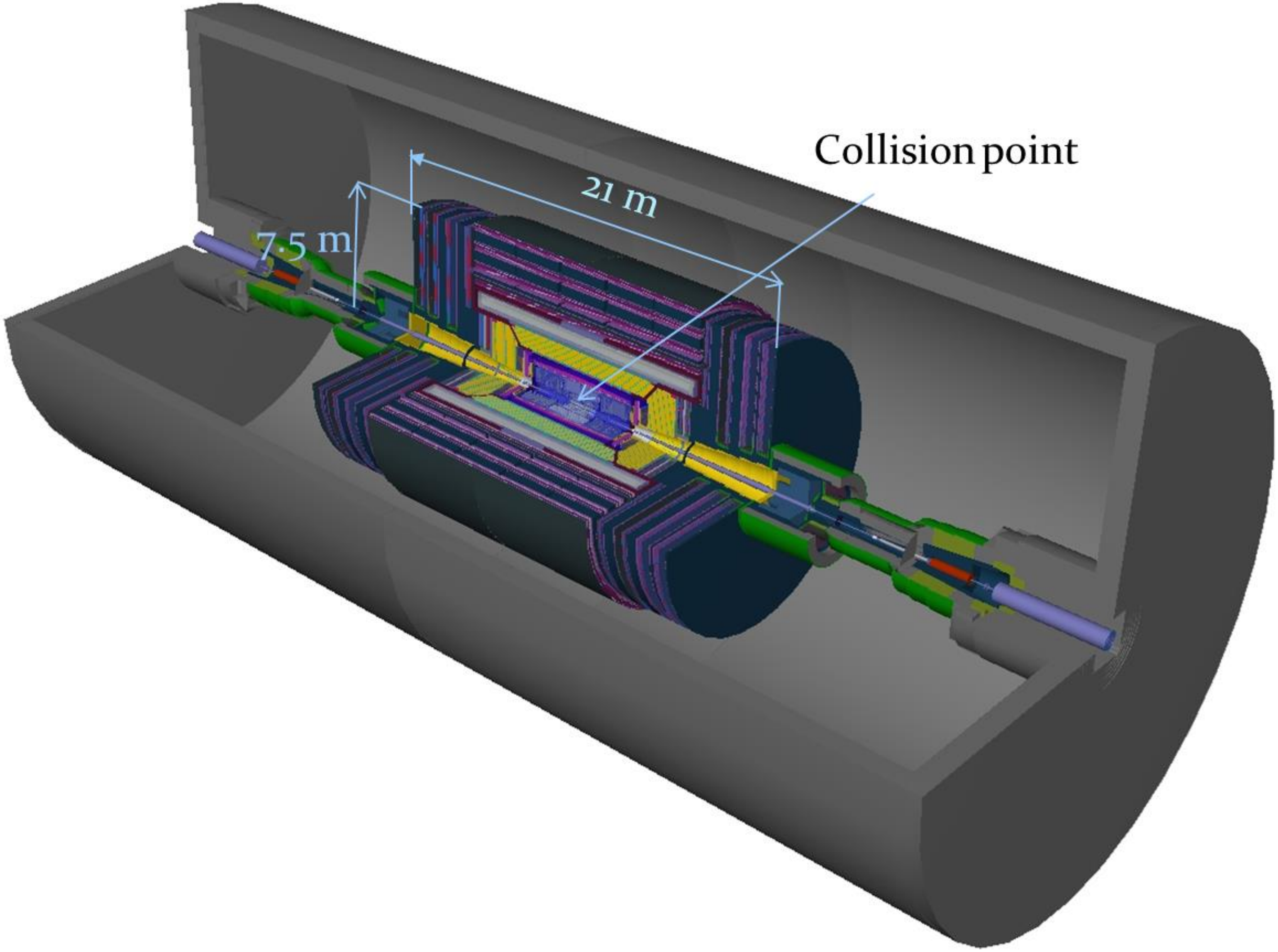
or FLUKA

To solve these problems

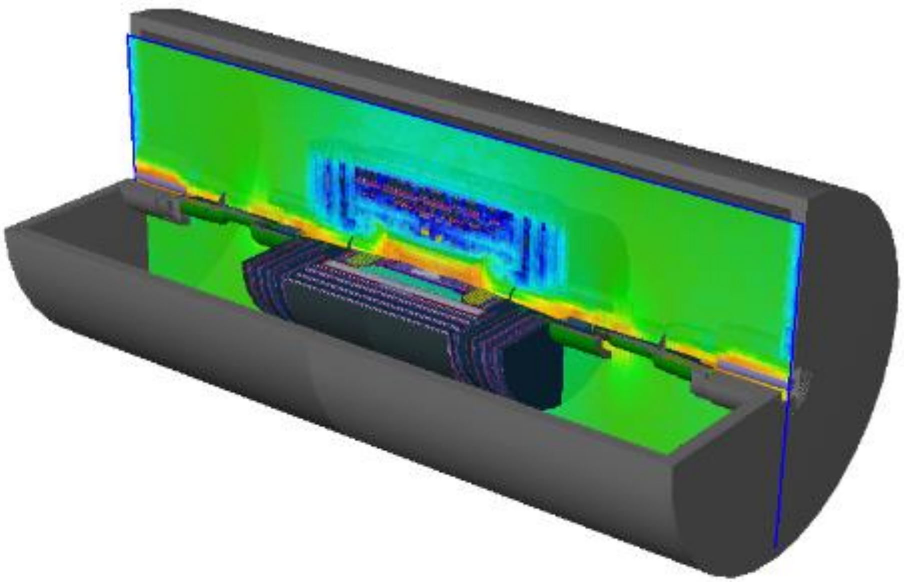
… Laplace transform + some algebra

$$N_n(t) = \sum_{k=1}^{m} \sum_{i=1}^{n} \left[\underbrace{\left(\prod_{j=i}^{n-1} b_{j,j+1} \right)}_{\text{branching}} \sum_{j=i}^{n} \left(\underbrace{\frac{N_i^k e^{-\lambda_j (t_{k,irr} + t_{k,cool})}}{\prod_{\substack{p=i \\ p\neq j}}^{n} (\lambda_p - \lambda_j)}}_{\text{cool-down} \rightarrow \text{decay}} + \underbrace{\frac{P_i^k (1 - e^{-\lambda_j t_{k,irr}}) e^{-\lambda_j t_{k,cool}}}{\lambda_j \prod_{\substack{p=i \\ p\neq j}}^{n} (\lambda_p - \lambda_j)}}_{\text{operation} \rightarrow \text{build-up \& decay}} \right) \right]$$

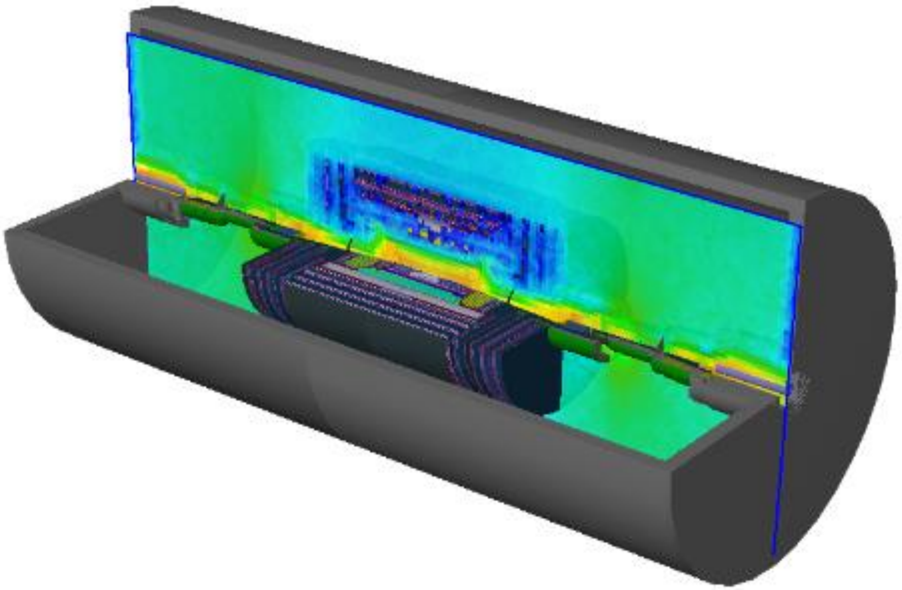
Example of residual activation @ CMS



Example of residual activation @ CMS



180 days of irradiation,
10^9 pp/s, 1h of cooling



180 days of irradiation,
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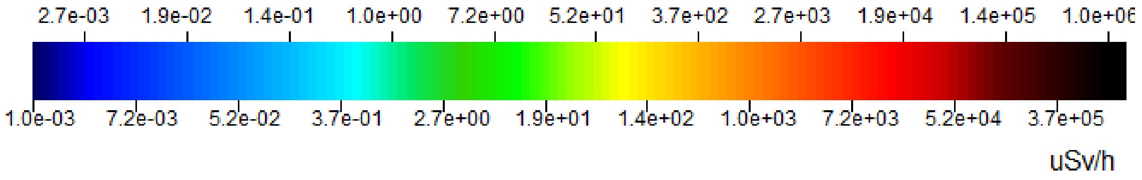
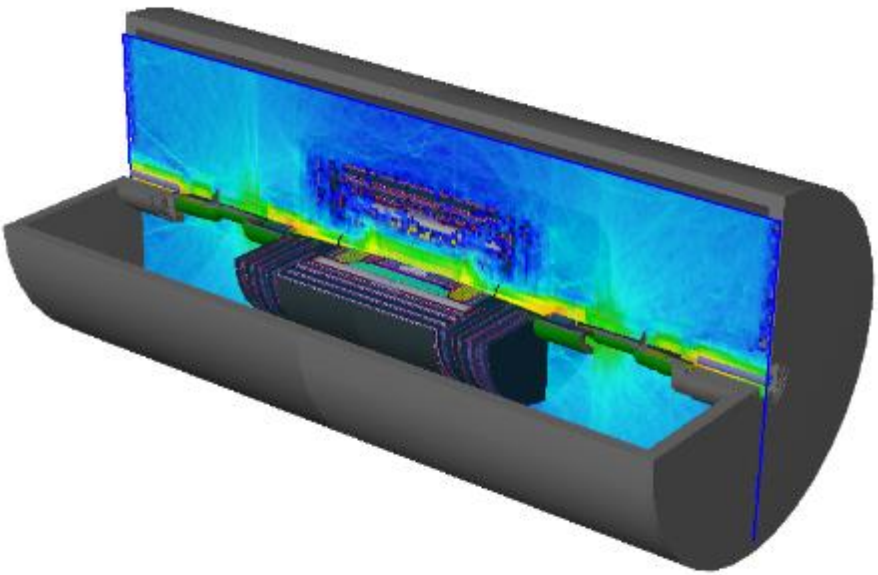
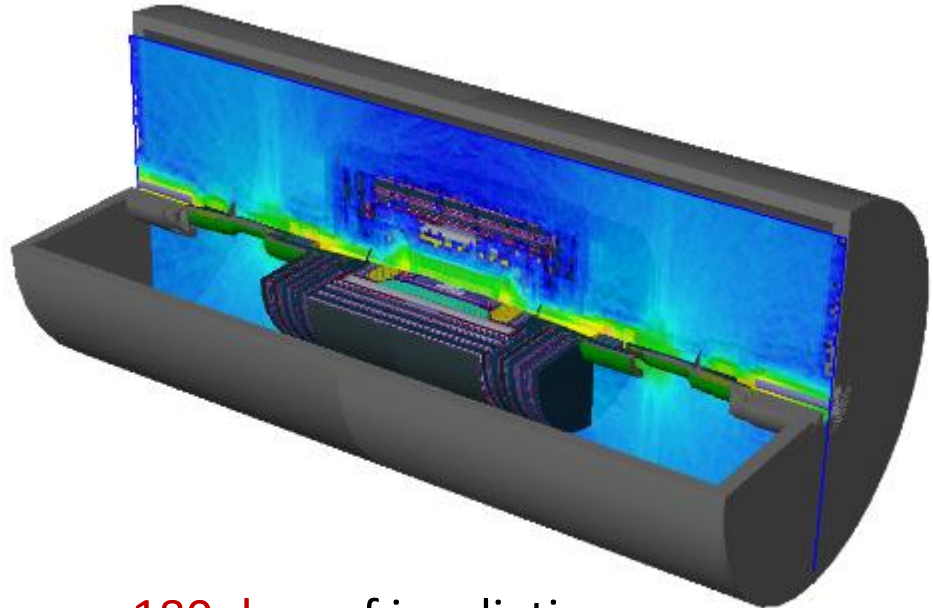


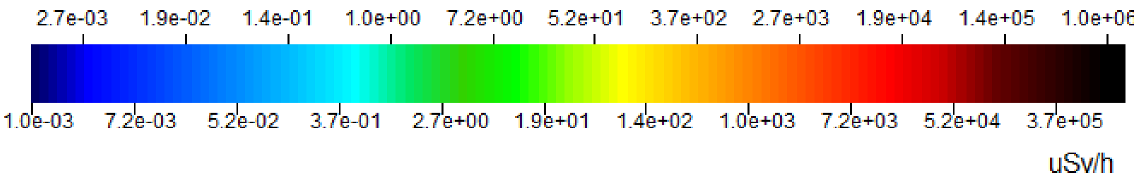
Example of residual activation @ CMS



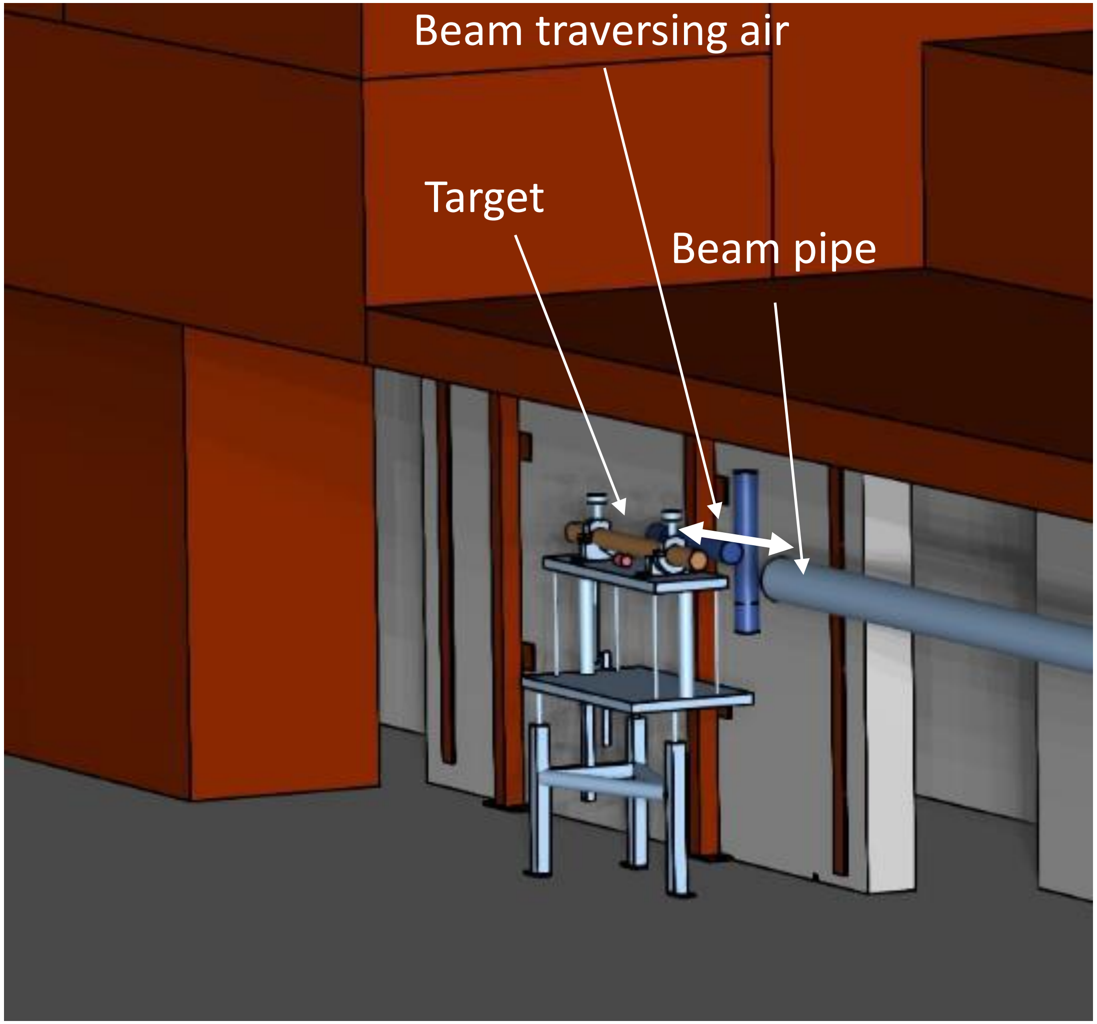
180 days of irradiation,
10^9 pp/s, 1w of cooling



180 days of irradiation,
10^9 pp/s, 1m of cooling



Air activation



High-energy particles and neutrons are crossing the air which is surrounding beam impact points

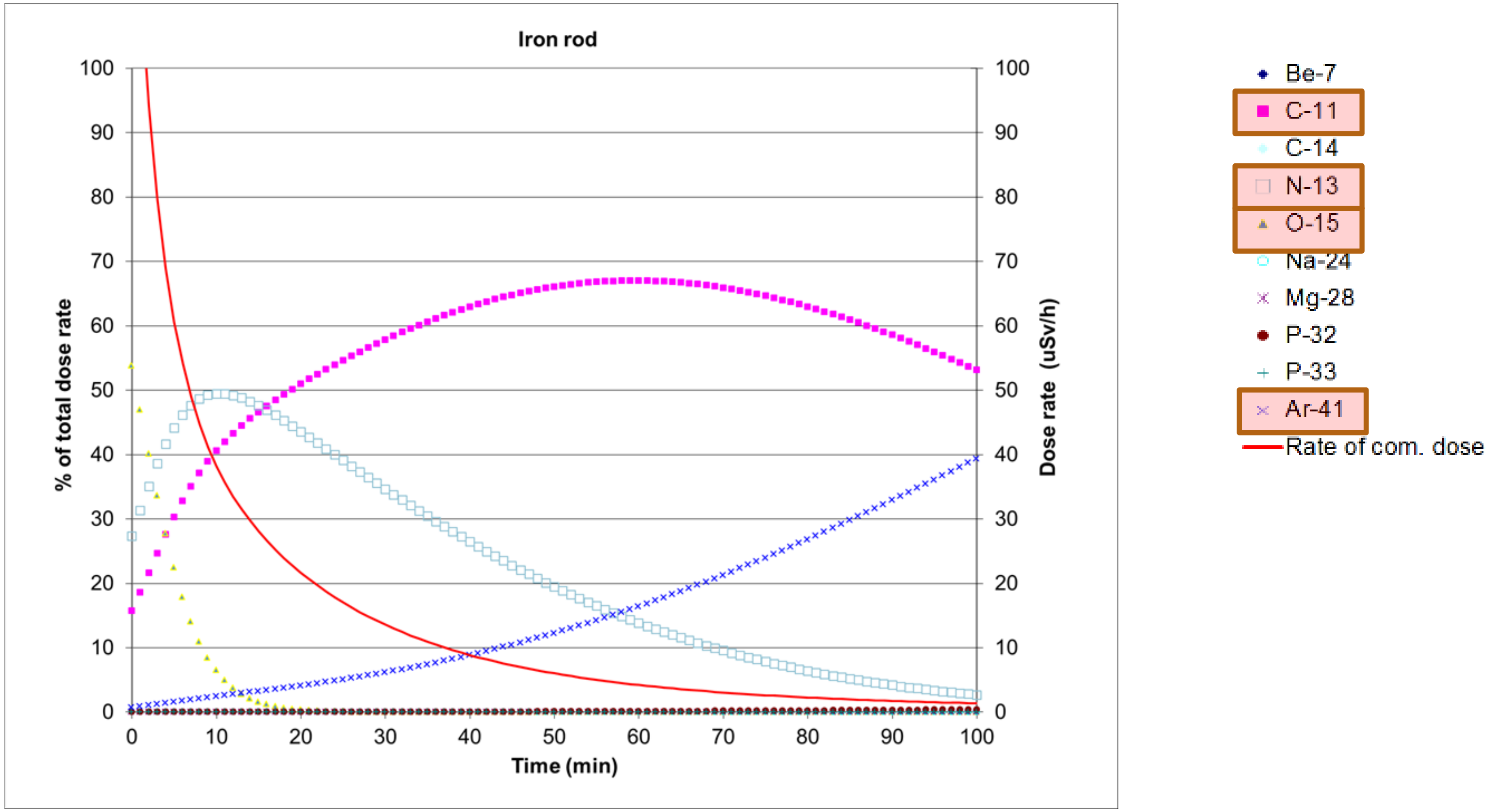
Production of radioactive air.

Problems to be considered:

1) People entering the area are exposed to and inhale the radioactive air
2) Release of airborne radioactivity to the environment

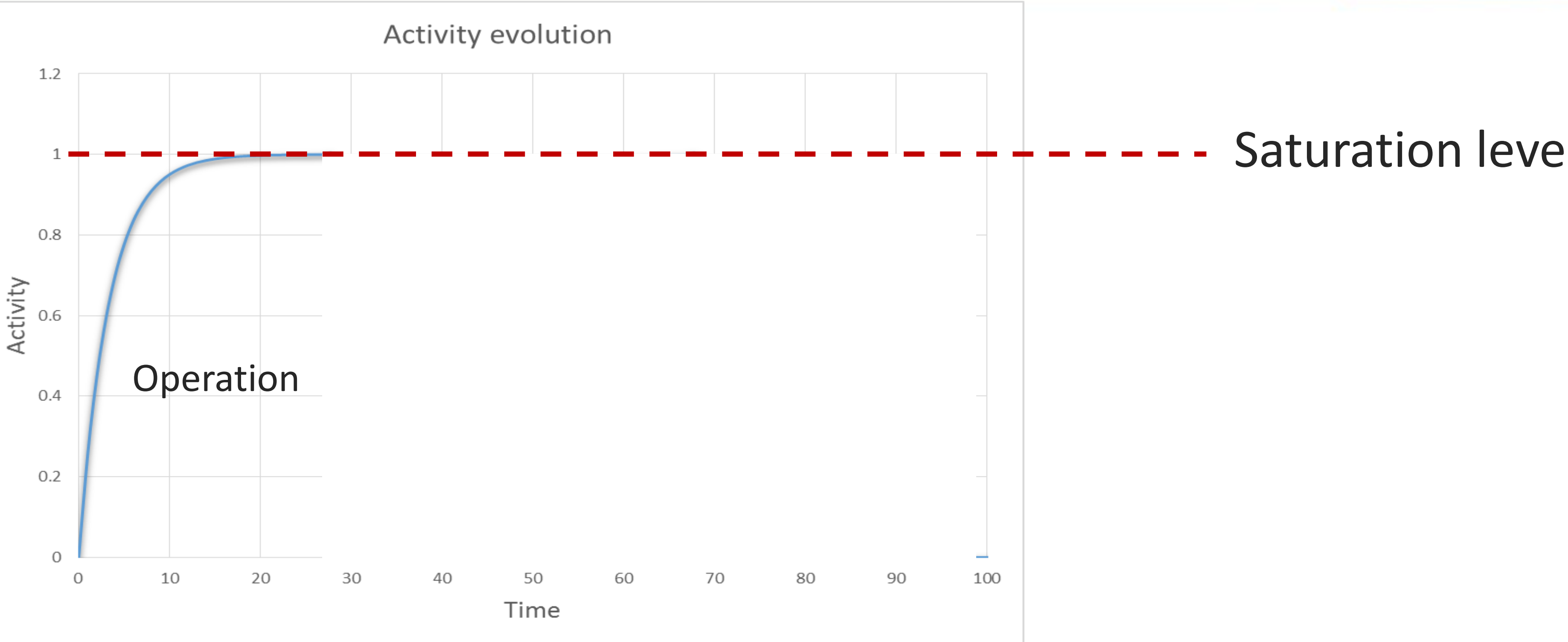
Air activation example

Example: Beam loss on an unshielded object in the SPS tunnel
(short term irradiation of air)



- Different main contributors to airborne radioactivity at different cooling times
- Long lived isotopes (e.g. Be-7) become important for confined spaces for longer irradiation and cooling times

Exercise



After which irradiation time (expressed in multiples of half-lives) do we reach 90% of the maximum saturation activity? (No cooling!)

Hints:

$$N(t_{irr} + t_{cool}) = \frac{P}{\lambda}(1 - e^{-\lambda t_{irr}})\, e^{-\lambda t_{cool}}$$

Solution of the excercise

$$N(t_{irr} + t_{cool}) = \frac{P}{\lambda}(1 - e^{-\lambda t_{irr}})\, e^{-\lambda t_{cool}}$$

- We talk about irradiation only, hence t_{cool} is 0.
- Full saturation occurrs after infinite time.
- Hence the full saturation level is given with:

$$N(t_{irr} = \infty, t_{cool} = 0) = \frac{P}{\lambda}(1 - 0.0) \text{ x } 1.0 = \frac{P}{\lambda}$$

We are looking now for 90% of the saturation level

$$0.9 \times \frac{P}{\lambda} = \frac{P}{\lambda}(1 - e^{-\lambda t_{irr}}) \Rightarrow 0.9 = 1 - e^{-\lambda t_{irr}}$$

$$\Rightarrow 0.1 = e^{-\lambda t_{irr}} \quad / \ln \quad \Rightarrow -2.3 = -\lambda t_{irr}$$

$$2.3/\lambda = t_{irr} \quad \text{with } \lambda = \frac{\ln(2)}{t_{1/2}} \Rightarrow t_{irr} = 2.3/\ln(2) \times t_{1/2}$$

$$\boldsymbol{t_{irr} = 3.3 \times t_{1/2}}$$

End of Part 1

Build-up and decay

Laplace transform of a function in the time domain f(t) into the Laplace domain F(s) for all real numbers t >= 0

$$F(s) = \int_{0}^{\infty} e^{-s \cdot t} f(t)\, dt$$

We also make use of the following identities:

Linearity $a \cdot f(t) + b \cdot g(t) \rightarrow a\, F(s) + b\, G(s)$

Differentiation $f'(t) \rightarrow sF(s) - f(0)$

Laplace transformation of system of differential equations ➔ system of linear algebraic equations

Buildup & decay of isotopes

At accelerators the production & decay described via Bateman equations:

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = P_1 - \lambda_1 \cdot N_1$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = P_2 + (b_{1,2} \cdot \lambda_1 \cdot N_1) - \lambda_2 \cdot N_2$$

$$\vdots$$

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = P_i + (b_{i-1,i} \cdot \lambda_{i-1} \cdot N_{i-1}) - \lambda_i \cdot N_i$$

$$\vdots$$

$$\frac{\mathrm{d}N_n}{\mathrm{d}t} = \underbrace{P_n}_{\text{build-up}} + \underbrace{(b_{n-1,n} \cdot \lambda_{n-1} \cdot N_{n-1}) - \lambda_n \cdot N_n}_{\text{decay}}$$

N_n ... Number of isotope n
P_n ... Production rate of isotope n
l_n ... Decay constant of isotope n
b_n ... Branching ratio from isotope n-1 into n
k ... index of irradiation/cooling cycle

... Laplace transform + some algebra

$$N_n(t) = \sum_{k=1}^{m} \sum_{i=1}^{n} \left[\underbrace{\left(\prod_{j=i}^{n-1} b_{j,j+1} \right)}_{\text{branching}} \sum_{j=i}^{n} \left(\underbrace{\frac{N_i^k e^{-\lambda_j (t_{k,irr} + t_{k,cool})}}{\prod_{\substack{p=i \\ p \neq j}}^{n} (\lambda_p - \lambda_j)}}_{\text{cool-down} \rightarrow \text{decay}} + \underbrace{\frac{P_i^k (1 - e^{-\lambda_j t_{k,irr}}) e^{-\lambda_j t_{k,cool}}}{\lambda_j \prod_{\substack{p=i \\ p \neq j}}^{n} \left(\lambda_p - \lambda_j \right)}}_{\text{operation} \rightarrow \text{build-up \& decay}} \right) \right]$$

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = P_1 - \lambda_1 \cdot N_1$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = P_2 + (b_{1,2} \cdot \lambda_1 \cdot N_1) - \lambda_2 \cdot N_2$$

$$\vdots \qquad\qquad \vdots$$

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = P_i + (b_{i-1,i} \cdot \lambda_{i-1} \cdot N_{i-1}) - \lambda_i \cdot N_i$$

$$\vdots \qquad\qquad \vdots$$

$$\frac{\mathrm{d}N_n}{\mathrm{d}t} = P_n + (b_{n-1,n} \cdot \lambda_{n-1} \cdot N_{n-1}) - \lambda_n \cdot N_n$$

N_n … Number of isotope n
P_n … Production rate of isotope n
l_n … Decay constant of isotope n
b_n … Branching ratio from isotope n-1 into n
k … index of irradiation/cooling cycle

Transform system of ODE into Laplace domain using the identities from the previous slide

$$F(s) = \int_0^\infty e^{-s\cdot t} f(t)\, dt$$

Other approaches:

- algebraic mapping via matrix exponential → Eigenvalue problem

→ numerically tricky because short-lived isotopes introduce arbitrarily large eigenvalues

Build-up and decay

Laplace transformed equations = system of linear equations, to be solved in the Laplace domain as a function of *s.*

$$s \cdot F_1(s) - N_1(t=0) = \frac{P_1}{s} - \lambda_1 \cdot F_1(s)$$

$$\vdots \qquad \vdots$$

$$s \cdot F_n(s) - N_n(t=0) = \frac{P_n}{s} + b_{n-1,n} \cdot \lambda_{n-1} \cdot F_{n-1}(s) - \lambda_n \cdot F_n(s)$$

$$\vdots \qquad \vdots$$

$$s \cdot F_m(s) - N_m(t=0) = \frac{P_m}{s} + b_{m-1,m} \cdot \lambda_{m-1} \cdot F_{m-1}(s) - \lambda_m \cdot F_m(s)$$

N_n … number of isotope n
P_n … production rate of isotope n
λ_n … decay constant of isotope n
b_n … branching ratio from isotope n-1 into n
F_n (s) … Laplace transformed of N_n(t)

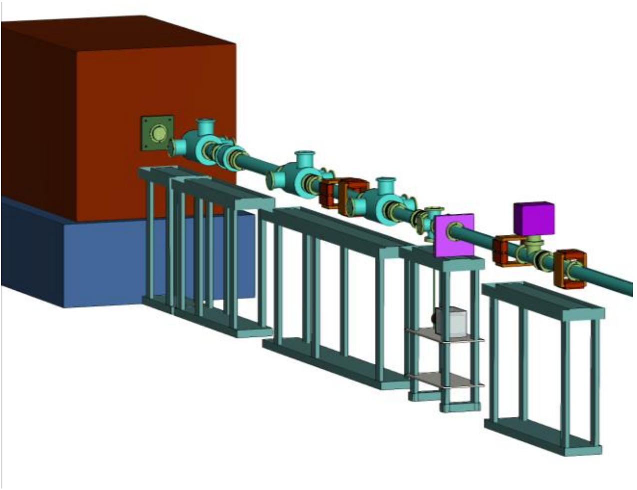
Inverse Laplace transformation of $F_n(s)$ $\quad \mathcal{L}^{-1}(F_n(s)) \Rightarrow N_n(t)$

$$\Rightarrow N_n(t) = \sum_{i=1}^{n}\left[\left(\prod_{j=i}^{n-1}\lambda_j b_{j,j+1}\right)\sum_{j=i}^{n}\left(\frac{N_i^0 e^{-\lambda_j t}}{\prod_{\substack{p=i \\ p\neq j}}^{n}\lambda_p - \lambda_j} + \frac{P_i(1-e^{-\lambda_j t})}{\lambda_j \prod_{\substack{p=i \\ p\neq j}}^{n}\lambda_p - \lambda_j}\right)\right]$$

To obtain the final result for a given isotope the contributions of the various chains have to be summed up.

C. Theis, H. Vincke, "Implementation of an activation build-up and decay engine in ActiWiz"
EDMS 1312453 & 1363771

Reduction of radioactive waste



- Optimization already crucial during the design phase
- Beside other aspects also the radiological consequences of the implementation of a material have to be considered
- Level of activation depends on the type of the material

Safety benefit	Operational benefit	End of life-cycle benefit
• Lower dose rates and committed doses	• Reduced downtime due to faster access • Less restrictions for manipulation & access	• Smaller amount and less critical radioactive waste • Smaller financial burden

S. Myers initiated the project concerning the radiological classification of materials