CERN

Collective effects Part I: Introduction and space charge effect

Giovanni Rumolo

Outline

In this introductory part, we will first provide a qualitative description of **collective effects** and their **impact on particle beams**.

We will then introduce one first collective effect, known as **space charge** and will show **its possible effects**. This will be the first step towards other types of collective effects involving the parasitic interaction of the beam with its surrounding (next lectures).

- Part 1: Introduction + space charge
  - Introduction to collective effects
  - Examples of coherent and incoherent effects
  - Space charge
    - Basic concepts
    - Tune shift and tune spread

What are collective effects?

- We will study the dynamics of **charged particle beams** in a **particle accelerator environment**, taking into account the **beam self-induced electromagnetic fields**, i.e. not only the **impact of the machine onto the beam** but also the **impact of the beam onto the machine**.

What are collective effects?

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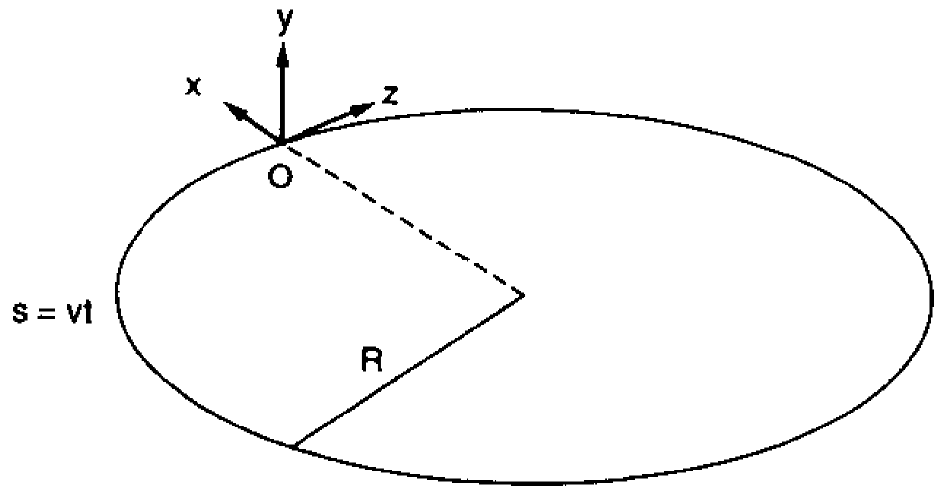
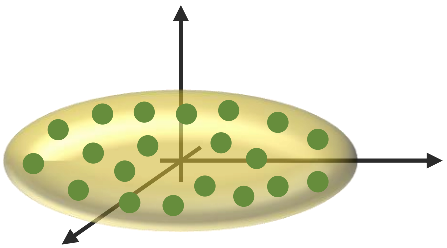
- First step → **Coordinates system** we will use throughout this set of lectures
  - The origin O is moving along with the "synchronous particle", i.e. a reference particle that has the design momentum and follows the design orbit
  - Transverse coordinates x (Horizontal) and y (Vertical) relative to reference particle (x,y << R), typically x is in the plane of the bending
  - Longitudinal coordinate z relative to reference particle
  - Position along accelerator is described by independent variable s = vt



What are collective effects?

- A charged particle beam is generally described as a **multiparticle system** via the **coordinates** and the **canonically conjugate momenta** of all of its particles – this makes up a distribution in the 6-dimensional phase space which can be described by a **particle distribution function**.
- Hence, we will study the **evolution of the phase space** occupied by this particle distribution (and described by its particle distribution function):



$$\frac{\partial}{\partial s} \psi \left(x, x', y, y', z, \delta, s\right)$$

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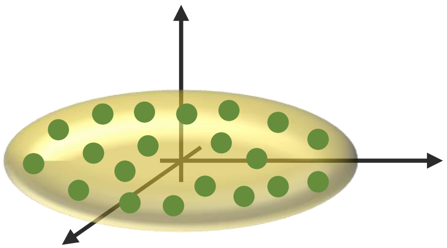
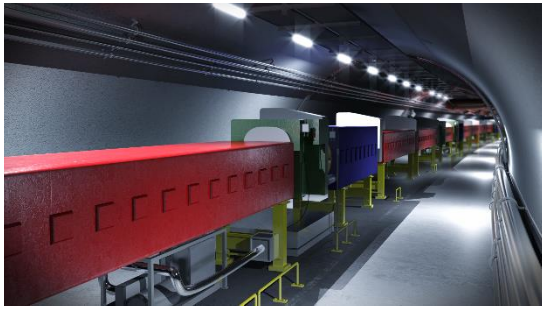
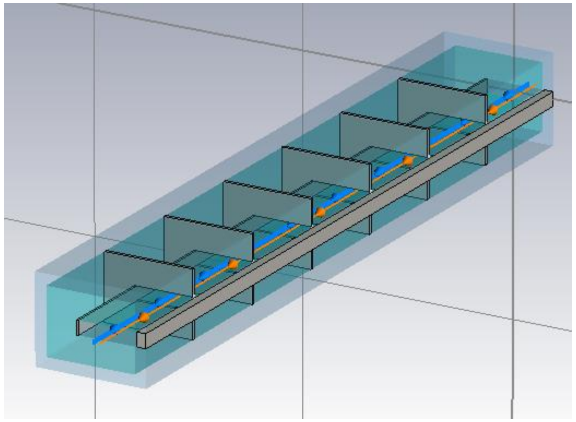
- Hence, we will study the **evolution of the phase space** occupied by this particle distribution (and described by its particle distribution function):
  - Optics defined by the machine lattice provides the **external force fields** (magnets, electrostatic fields, RF fields), e.g. for guidance and focusing
  - Collective effects add to this **distribution dependent force fields** (space charge, wake fields)







$$\frac{\partial}{\partial s}\psi\left(x,x',y,y',z,\delta,s\right)\propto f\left(F_{\text{extern}}+F_{\text{coll}}\left(\psi\right)\right)$$

What are collective effects?

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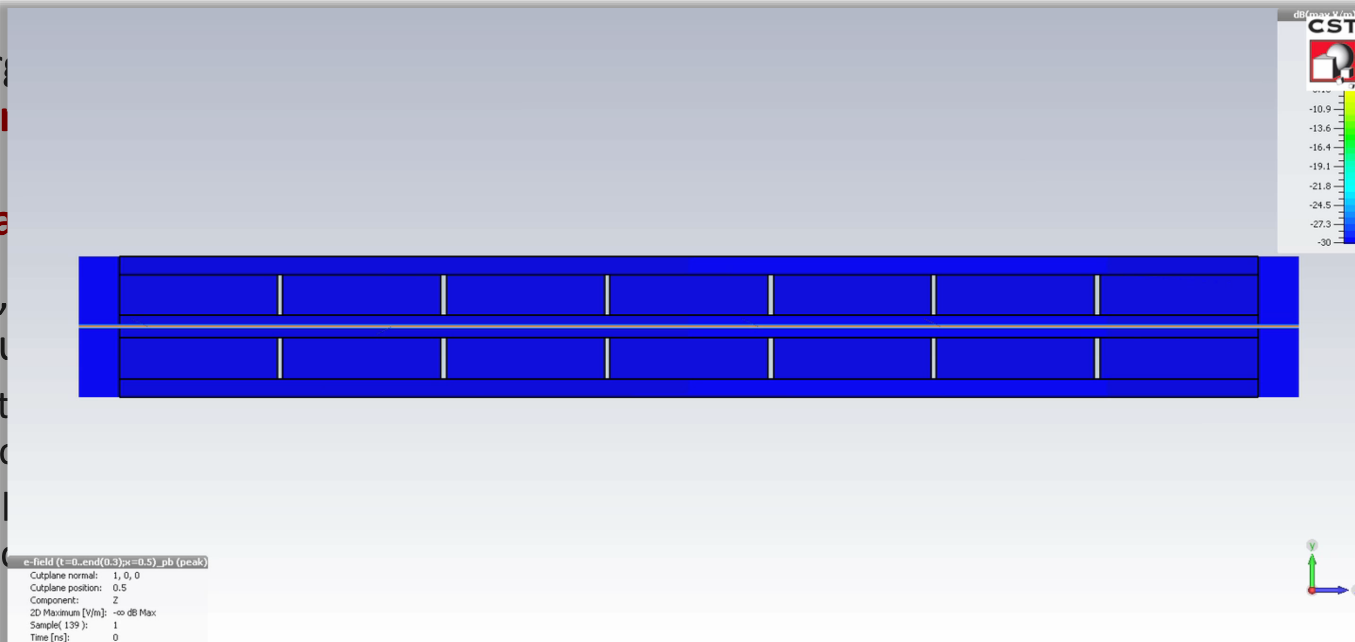
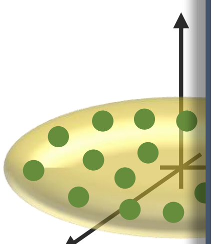
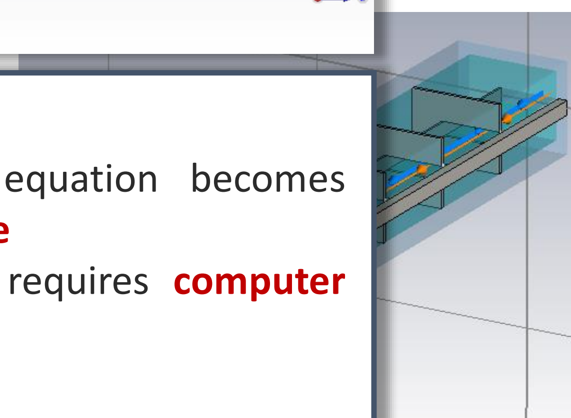
- Hence, [...]article distribu[…]
  - Opt[…] [...]agnets, elec[…]
  - Coll[…] [...], wake fiel[…]







- For a **multiparticle system** this self-consistent equation becomes arbitrarily complex and practically **impossible to solve**
- Obtaining the **multiparticle dynamics** very often requires **computer simulation codes**

$$\frac{\partial}{\partial s}\psi\left(x,x',y,y',z,\delta,s\right)\propto f\left(F_{\text{extern}}+F_{\text{coll}}\left(\psi\right)\right)$$

Formal description of collective beam motion

- Formally, instead of investigating the full set of equations for a multiparticle system, we typically instead describe the latter by a **particle distribution function**:

$$\psi = \psi(x, x', y, y', z, \delta, s)$$

where

$$dN(s) = \psi(x, x', y, y', z, \delta, s)\, dx dx' dy dy' dz d\delta$$

- The accelerator environment together with the multiparticle system forms a **Hamiltonian system** for which the **Hamilton equations of motion** hold:

$$\frac{\partial x}{\partial s} = \frac{\partial H}{\partial x'}, \qquad \frac{\partial x'}{\partial s} = -\frac{\partial H}{\partial x}$$

Formal description of collective beam motion

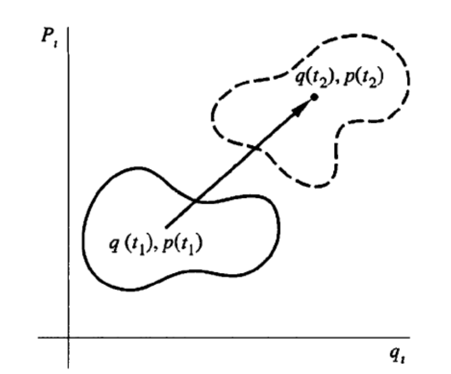
We can now derive the **Vlasov equation** which forms the **foundation of the theoretical treatment** of beam dynamics with collective effects:

- Consider an infinitesimal volume element dΩ containing a finite number of particles dN in phase space which evolve in time
  - dN is conserved as no particles can enter or leave the area (Picard-Lindelöf)
  - dΩ is conserved by means of the Hamilton equations of motion

- It follows that:

$$\frac{d}{ds}\psi = \frac{\partial\psi}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial\psi}{\partial x'}\frac{\partial x'}{\partial s} + \frac{\partial}{\partial s}\psi$$

$$= \underbrace{\frac{\partial\psi}{\partial x}\frac{\partial H}{\partial x'} - \frac{\partial\psi}{\partial x'}\frac{\partial H}{\partial x}}_{[\psi, H]:\ \text{Poisson bracket}} + \frac{\partial}{\partial s}\psi = 0$$



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Formal description of collective beam motion

- The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H}, \psi]$$

- With the Hamiltonian composed of **an external** and **a collective part**, and the particle distribution function decomposed into **an unperturbed part** and **a small perturbation** one can write

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H_0} + \boldsymbol{H_1}, \psi_0 + \psi_1]$$

- This becomes to **first order**

$$\boxed{\frac{\partial}{\partial s}\psi_1 =} \underbrace{[\boldsymbol{H_0}, \psi_1] + [\boldsymbol{H_1}(\psi_0 + \psi_1), \psi_0]}$$

$$\text{Linearization in } \psi_1: \quad \ldots \propto \boxed{\hat{\boldsymbol{\Lambda}}\psi_1}$$

Spatial component **Temporal component**

$$\Longrightarrow \quad \psi_1 = \sum_k \boxed{a_k\, \boldsymbol{v_k}}\, \boxed{\exp\left(\frac{i\Omega_k}{\beta c}s\right)}$$

We are looking for the EV of the evolution → **becomes an EV problem**!

Formal description of collective beam motion

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Linearization in ψ_1 ... $\propto \hat{\Lambda}\psi_1$

We call these distinct eigenvalues ψ_k **the coherent k-mode**.

The mode and thus for example also an instability is fully characterized by a single number:

the complex tune shift Ω_k

Spatial component | Temporal component

$$\Longrightarrow \psi_1 = \sum_k a_k \boldsymbol{v_k} \exp\left(\frac{i\Omega_k}{\beta c}s\right)$$

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Formal description of collective beam motion

- The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H}, \psi]$$

Remark:

- The stationary distribution ψ_0 is the distribution where

$$\frac{\partial}{\partial s}\psi_0 = [\boldsymbol{H}_0, \psi_0] = 0$$

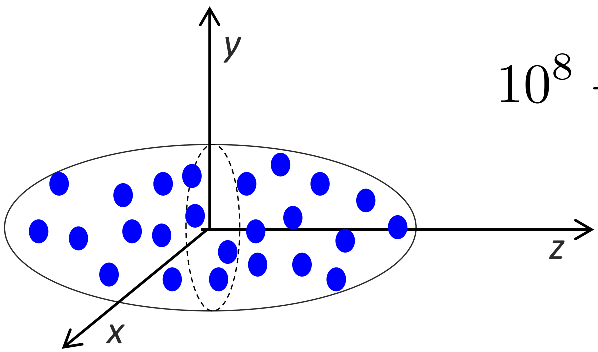
- In particular, a distribution is always stationary if

$$\psi_0 = \psi_0(\boldsymbol{H}_0)\,, \quad \text{as} \quad [\boldsymbol{H}_0, \psi_0(\boldsymbol{H}_0)] = 0$$

Solving for or finding the stationary solution for a given H_0 (which in fact represents the machine ‚potential‘) is referred to as **matching**.

Formal description of collective beam motion

- The evolution of a **multiparticle system** can also be studied via direct **macro-particle simulation**
  - Number of macro-particles needs to be chosen to have results statistically significant but in reasonable execution times within the available computing power
- Here we need to solve numerically a set of equations of motion corresponding to macro-particles representing the beam
  - The driving terms of these equations are the EM fields externally applied as well as the EM fields generated by the macro-particle distribution itself
  - Therefore, we typically need to couple with an EM solver



$$10^8 - 10^{11}\ \text{particles} \longrightarrow 10^4 - 10^6\ \text{macroparticles}$$

$$\frac{d\vec{p}_{\text{mp}}}{dt} = q\left(\vec{E} + \vec{v}_{\text{mp}} \times \vec{B}\right)$$

$$\begin{cases} \vec{E} = \vec{E}_{\text{ext}} + \vec{E}(\psi_{\text{mp}}) \\ \vec{B} = \vec{B}_{\text{ext}} + \vec{B}(\psi_{\text{mp}}) \end{cases}$$

Solutions of Maxwell's equations

Signpost

- We have seen the difference between **external forces** and **self-induced forces** and how these lead to **collective effects**.
- We have seen schematically how these collective effects can induce **coherent beam instabilities and some knobs to avoid them**.
- We have briefly sketched the **theoretical framework** within which the beam dynamics of collective effects is usually treated – we have encountered the Vlasov equation, bunch / beam eigenmodes and the complex tune shift.

- Part 1: Introduction + space charge
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Coherent effects

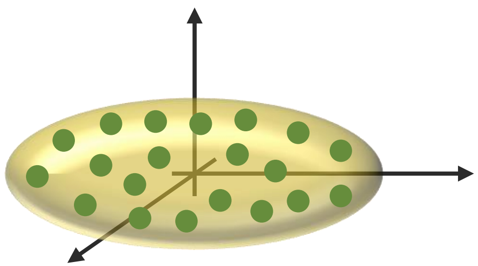
- Coherent effects, characterized by centroid motion, are mainly associated with beam instabilities

  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)

  - Understanding the type of instability limiting the performance, and its underlying mechanism, is essential because it:
    - Allows identifying the source and possible measures to mitigate/suppress the effect
    - Allows providing the specifications of an active feedback system to prevent the instability

What is a beam coherent instability?

- A beam becomes unstable when a **moment of its distribution** exhibits an **exponential growth** (e.g. mean positions, standard deviations, etc.), resulting into beam loss or emittance growth!



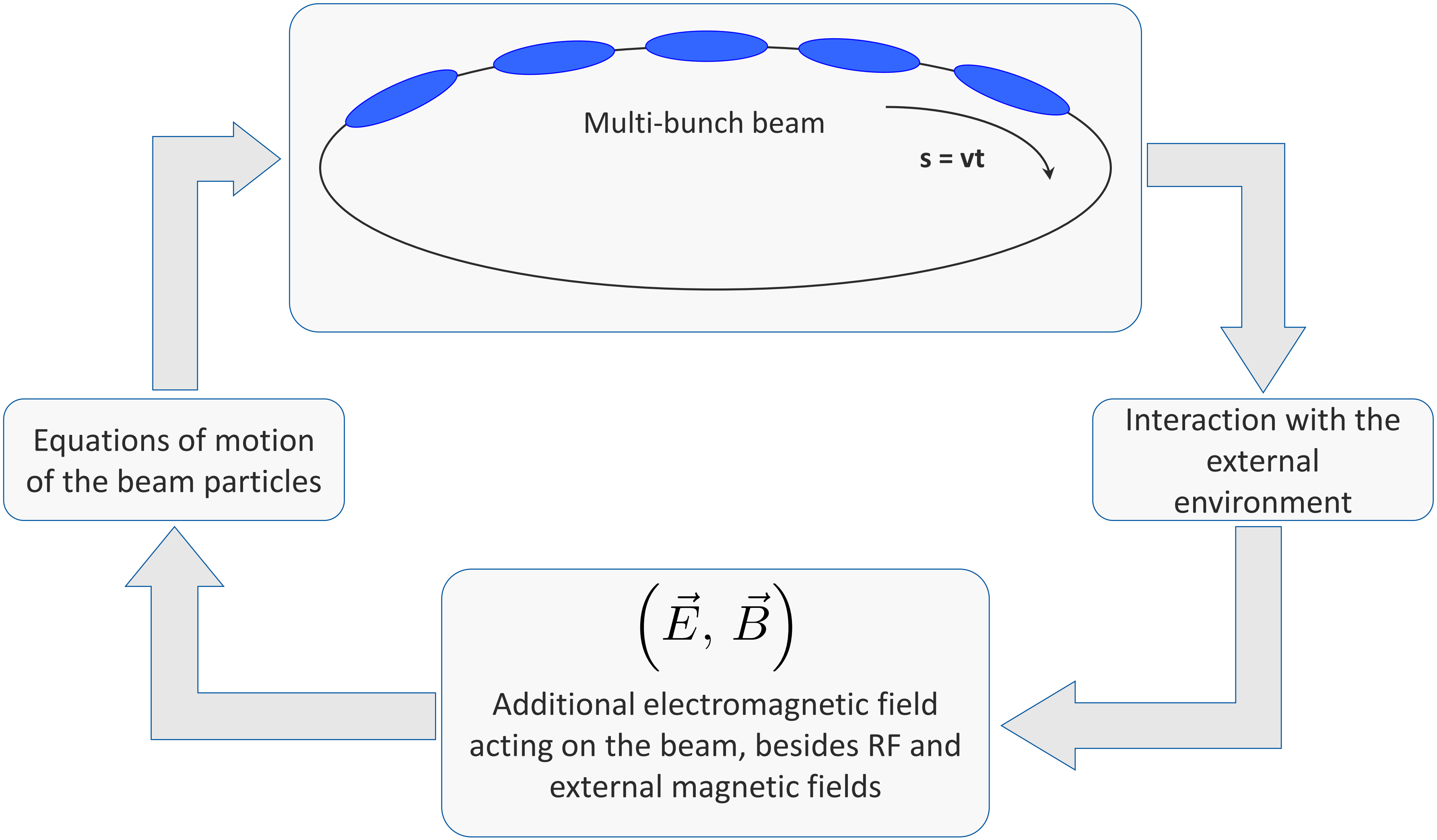
$$N = \int \psi\left(x, x', y, y', z, \delta\right)\, dx dx' dy dy' dz d\delta$$

$$\langle x \rangle = \frac{1}{N} \int x \cdot \psi\left(x, x', y, y', z, \delta\right)\, dx dx' dy dy' dz d\delta$$

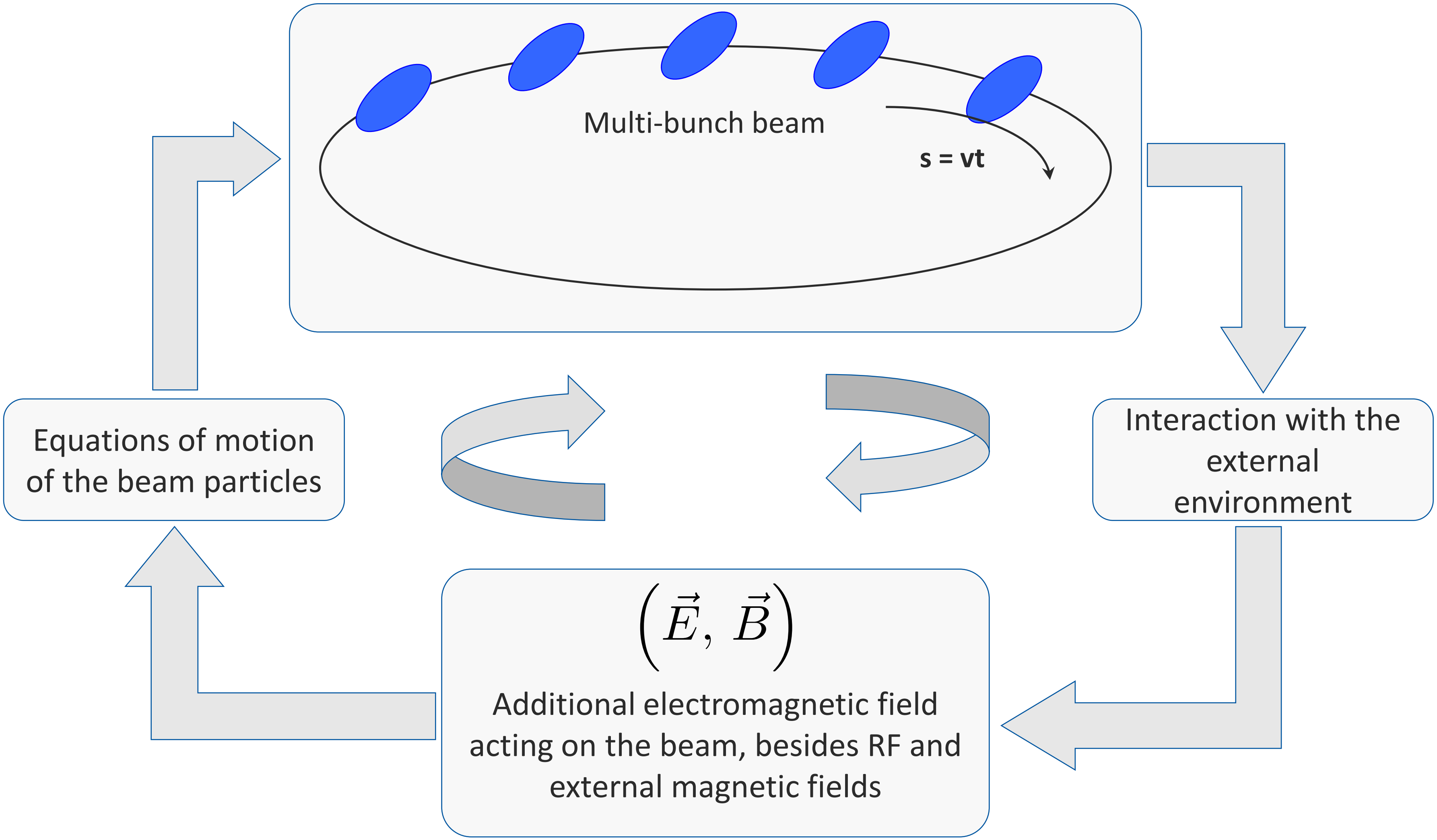
$$\sigma_x^2 = \frac{1}{N} \int \left(x - \langle x \rangle\right)^2 \cdot \psi\left(x, x', y, y', z, \delta\right)\, dx dx' dy dy' dz d\delta$$

and similar definitions for $\langle y \rangle, \sigma_y, \langle z \rangle, \sigma_z$

The instability loop

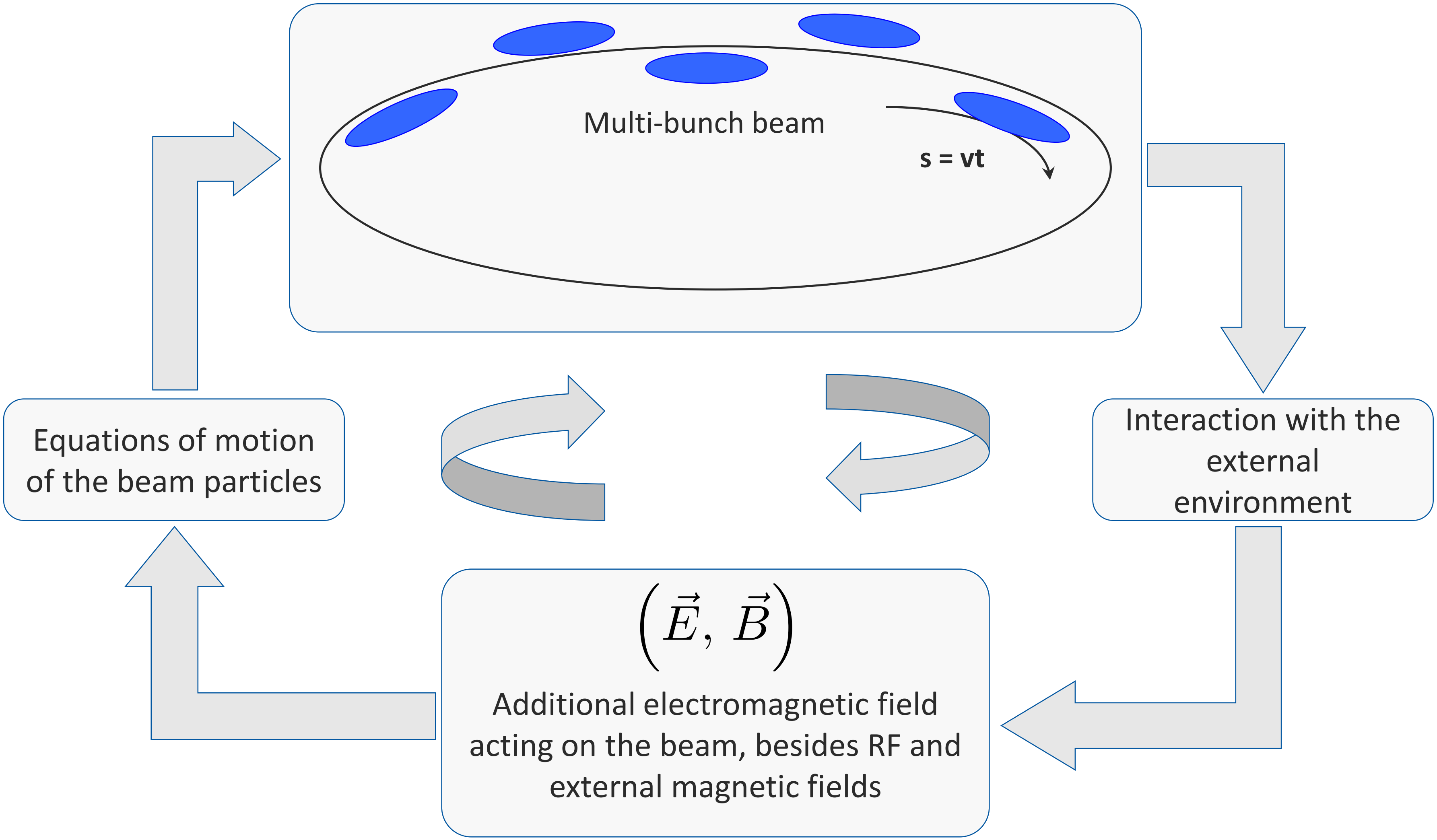


The instability loop



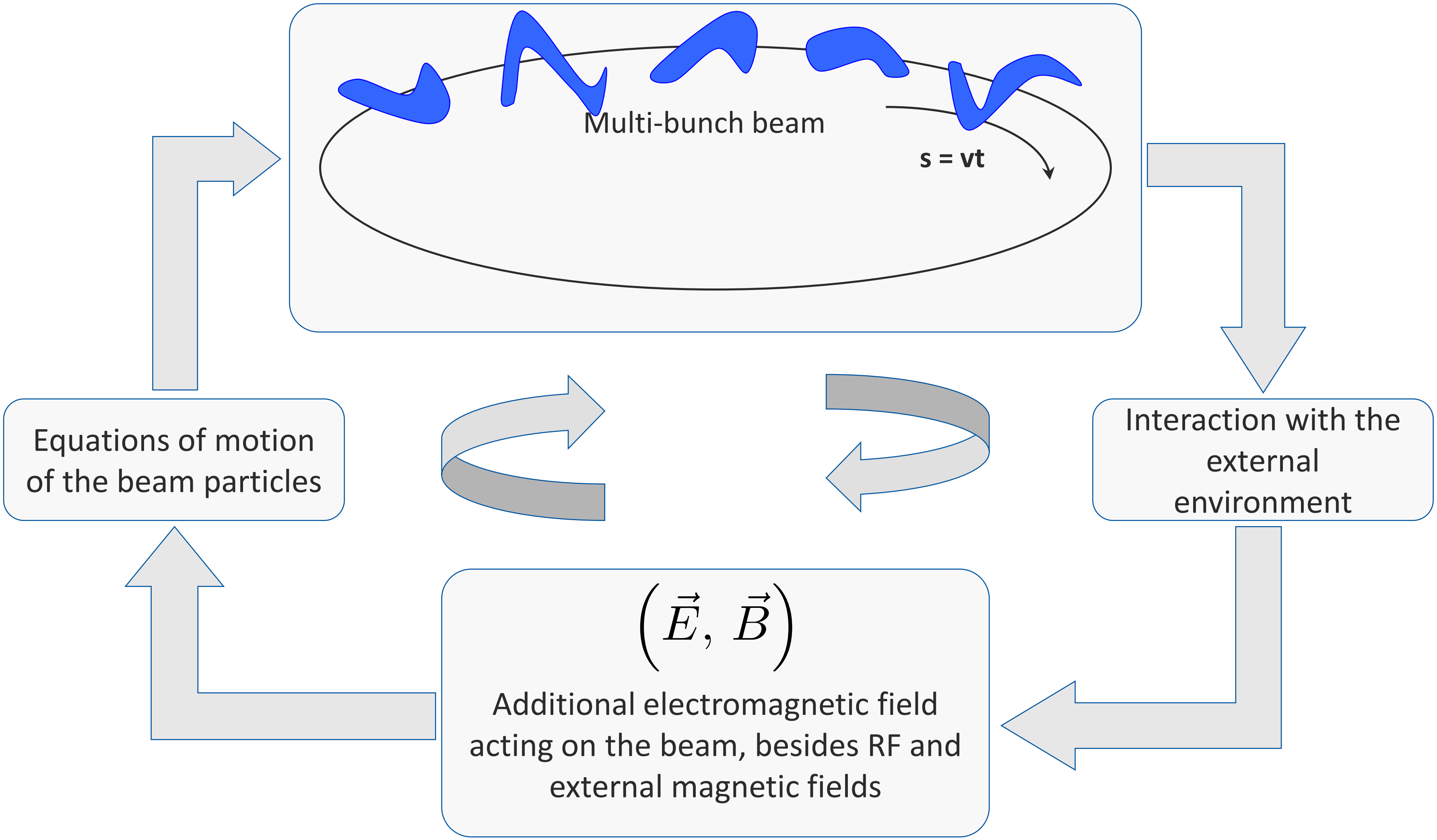
When the loop closes, either the beam will find a new stable equilibrium configuration …

The instability loop



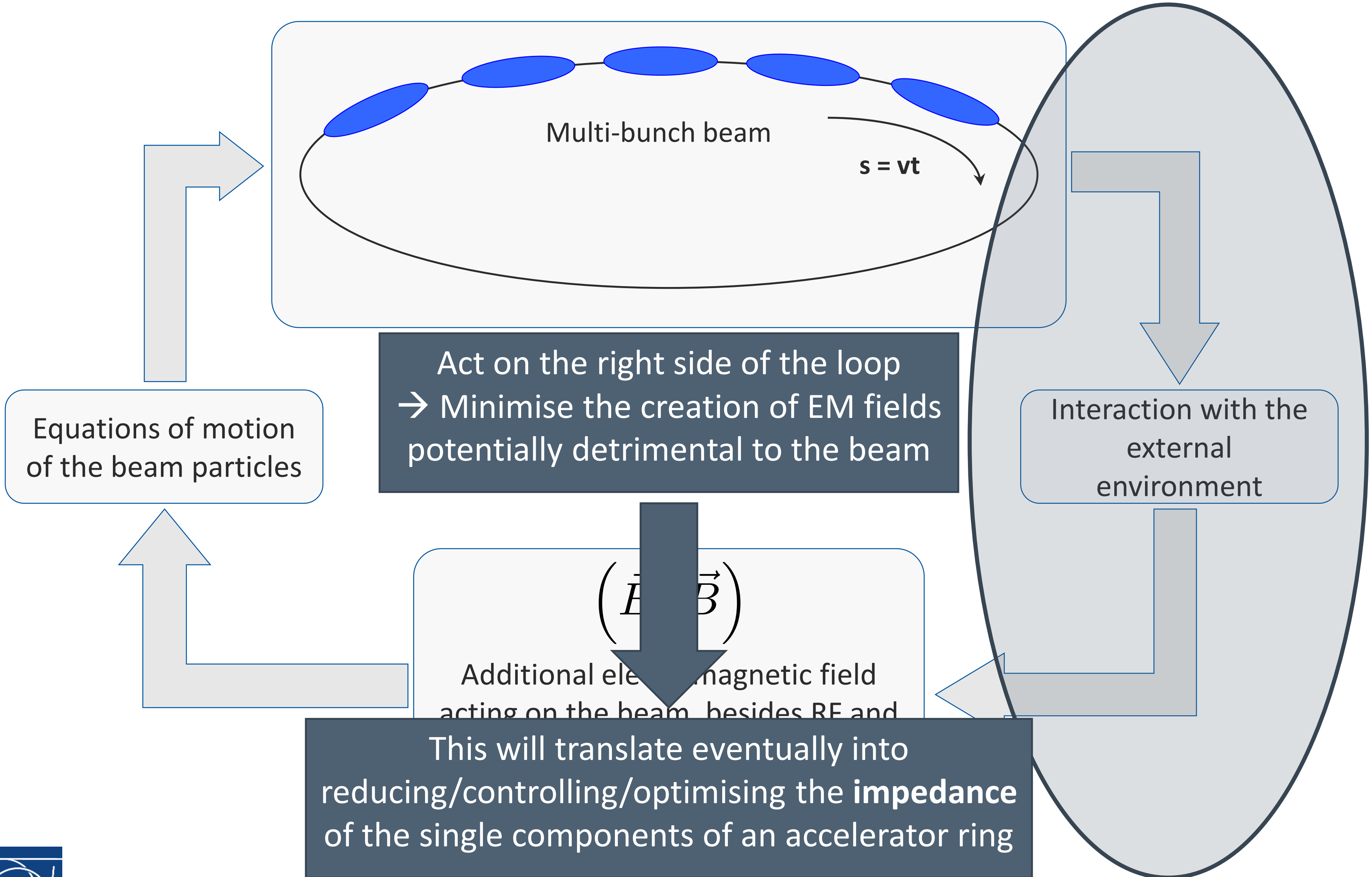
… or it might develop an instability along the bunch train …

The instability loop



… or also an instability affecting different bunches independently of each other

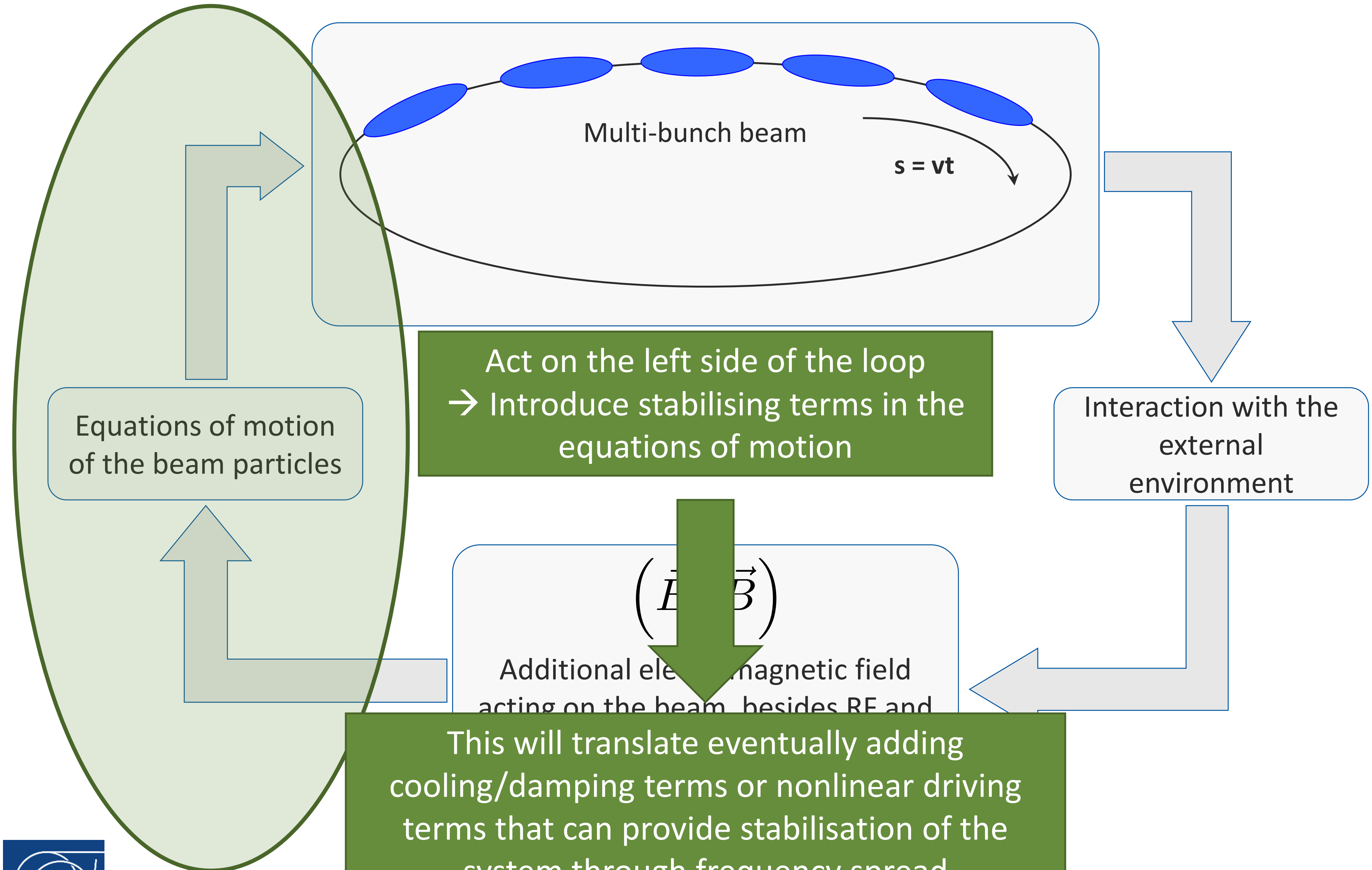
The instability loop



Act on the right side of the loop
→ Minimise the creation of EM fields potentially detrimental to the beam

This will translate eventually into reducing/controlling/optimising the **impedance** of the single components of an accelerator ring

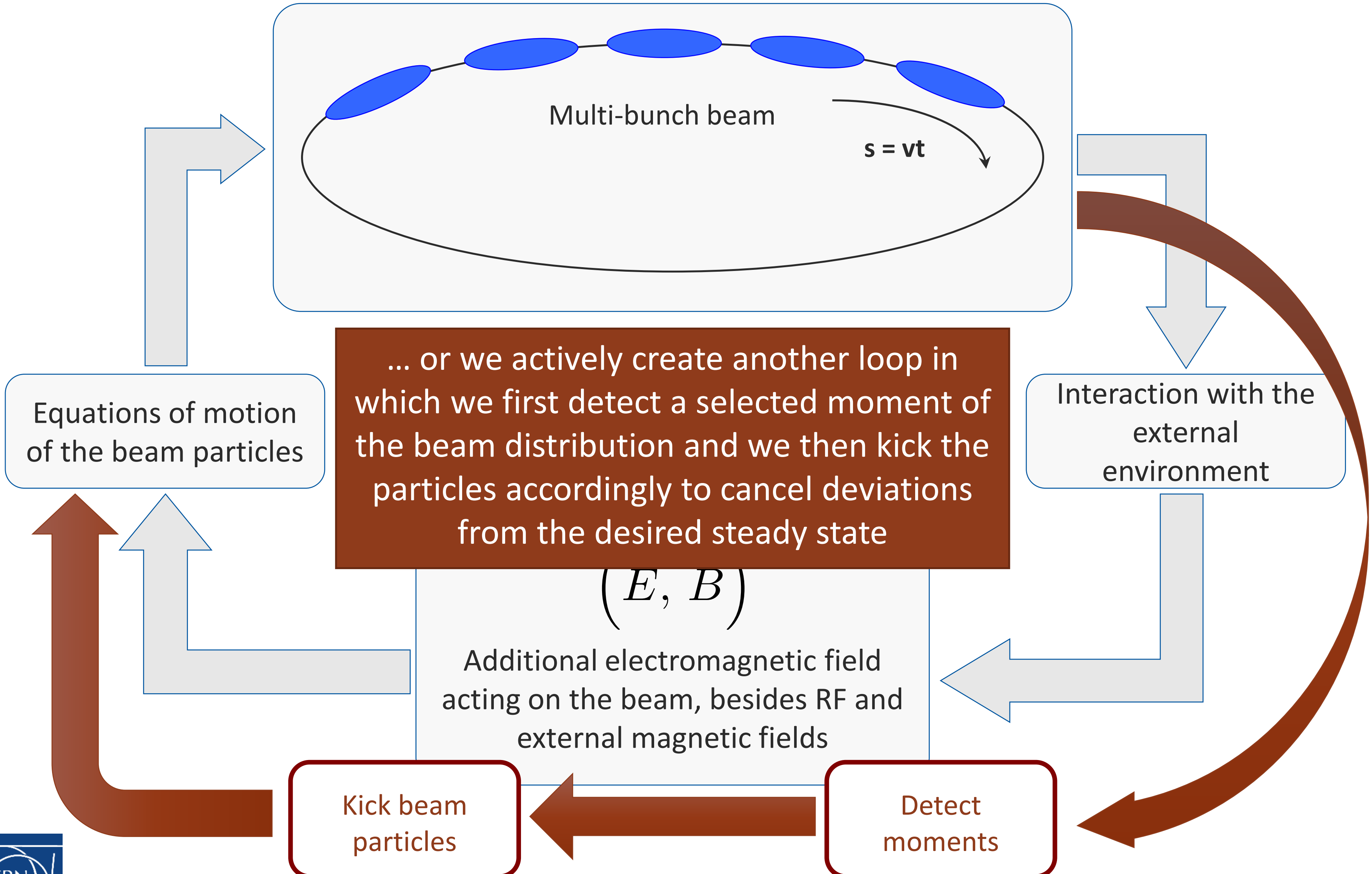
The instability loop



Act on the left side of the loop
→ Introduce stabilising terms in the equations of motion

This will translate eventually adding cooling/damping terms or nonlinear driving terms that can provide stabilisation of the system through frequency spread

The instability loop



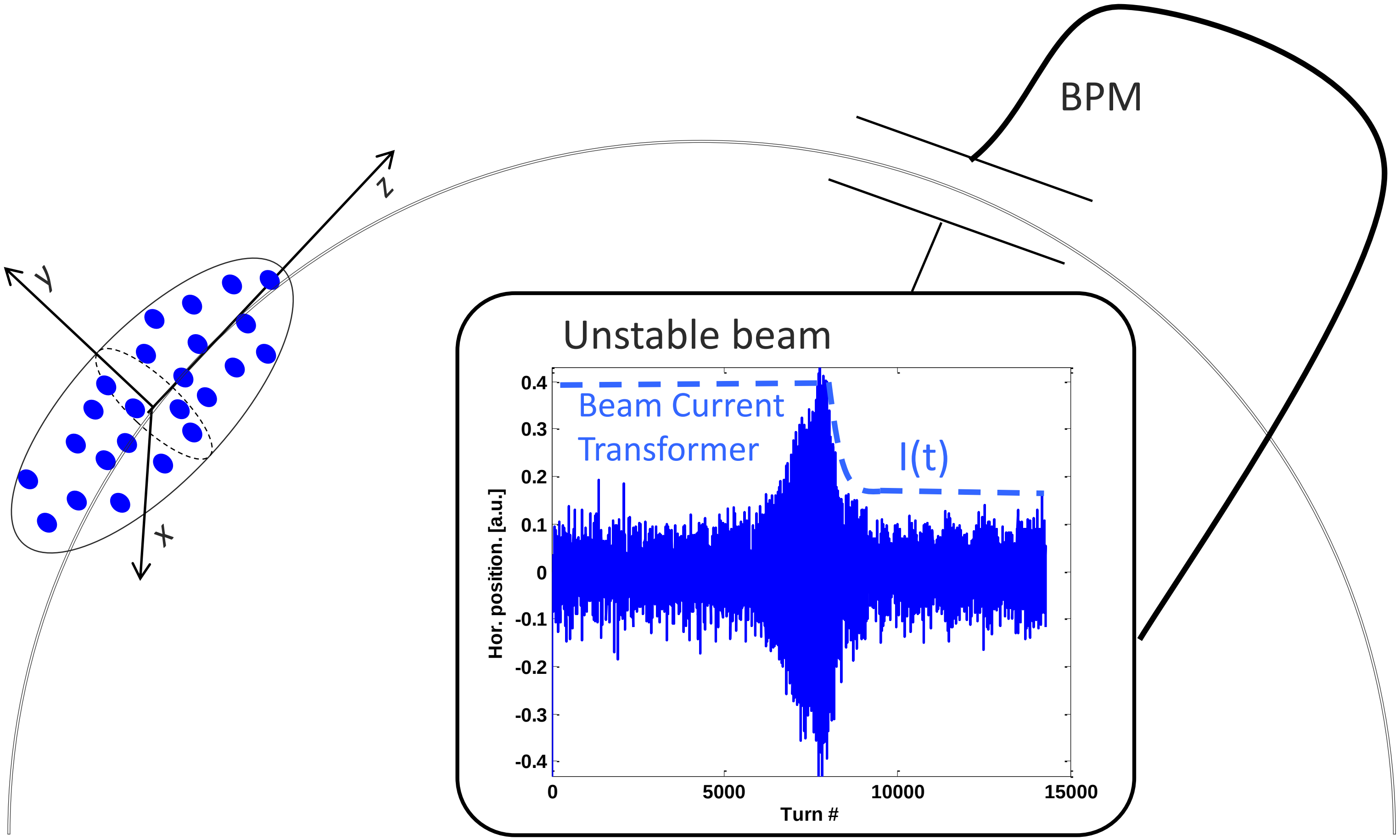
… or we actively create another loop in which we first detect a selected moment of the beam distribution and we then kick the particles accordingly to cancel deviations from the desired steady state

Incoherent effects

- Incoherent effects, in which typically no centroid motion is detectable, are mainly associated with emittance growth, halo and tail formation, slow losses
  - The strength of the excitation is not sufficient, or sufficiently resonant, to build up into a coherent centroid motion
  - They also result in performance limitations, as they can lead to intolerable loss of beam brightness due to emittance blow-up or intolerable level of losses in a machine leading to equipment activation or even damage

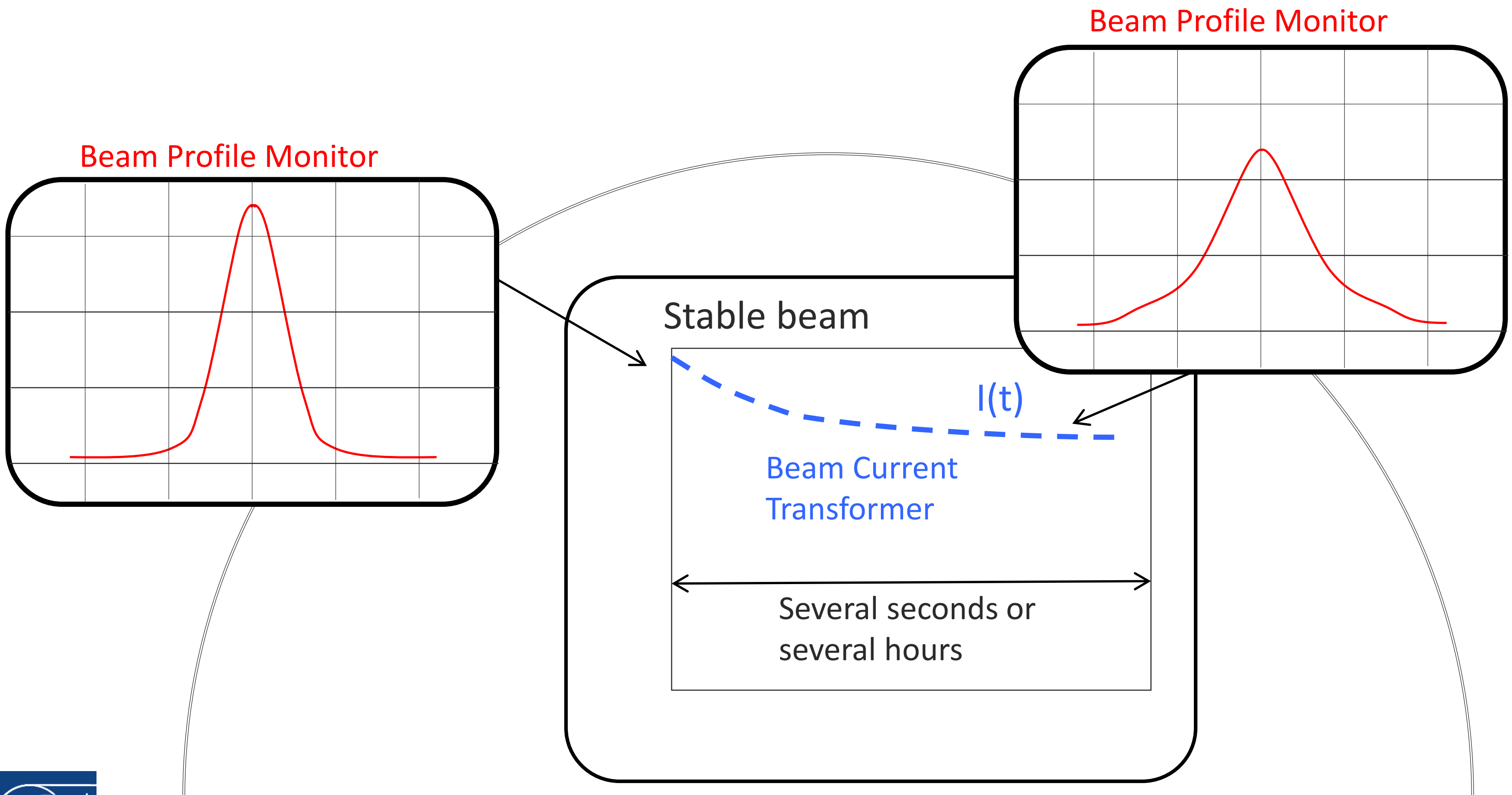
Coherent and incoherent effects

- Observation of a **transverse coherent instability**
  - The beam centroid, detected by a beam position monitor (BPM), exhibits an exponential growth on the scale of tens to thousands of turns (typically μs-ms)

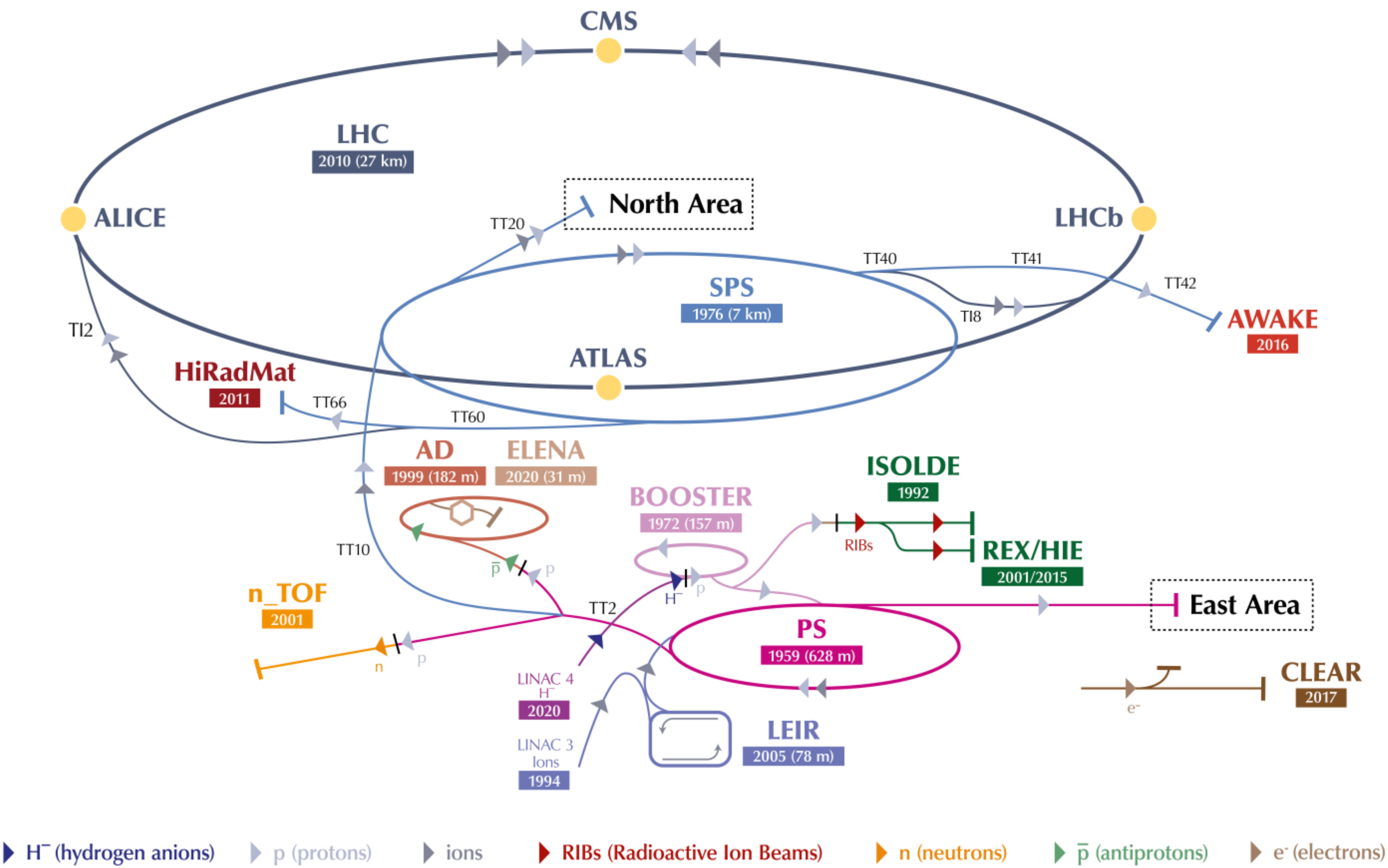


Coherent and incoherent effects

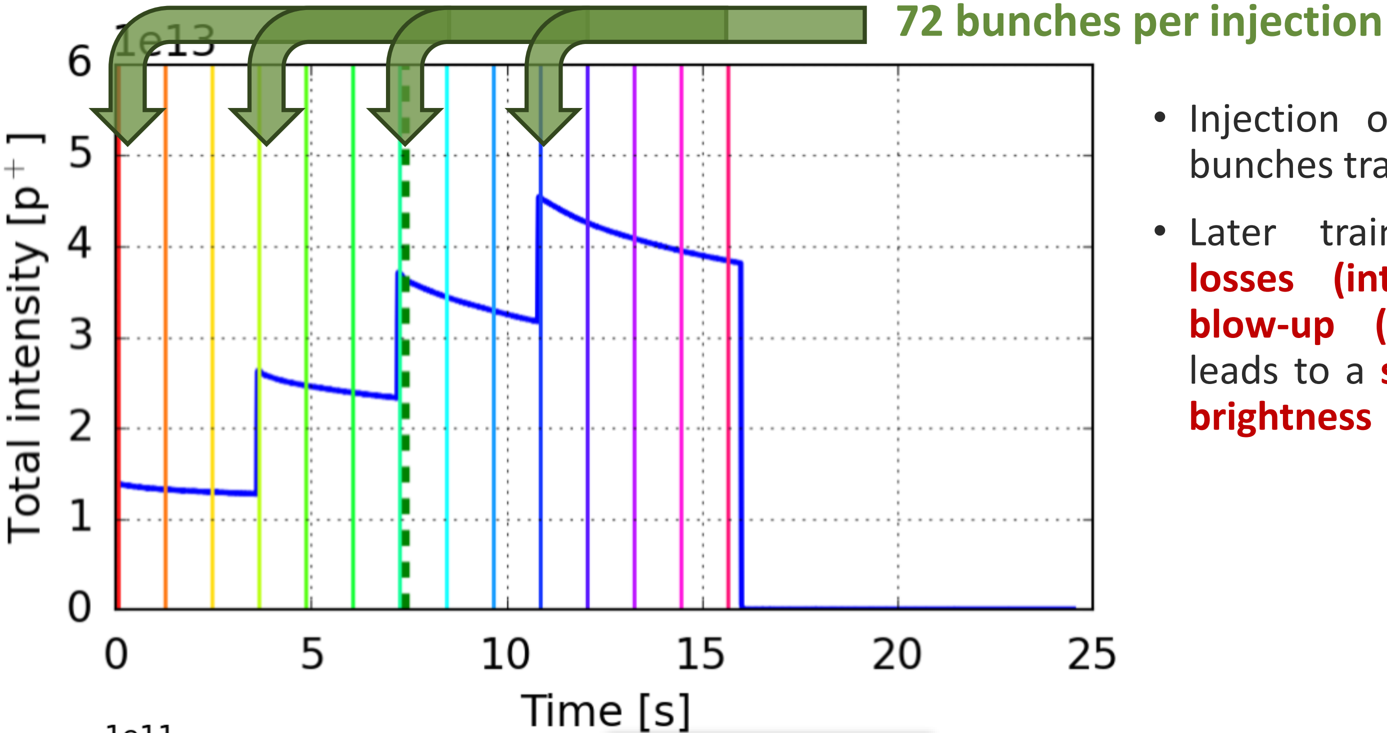
- Observation of a **transverse incoherent effect**
  - The beam exhibits slow losses (on the time scale of the cycle or store) and emittance growth visible on beam profile monitors



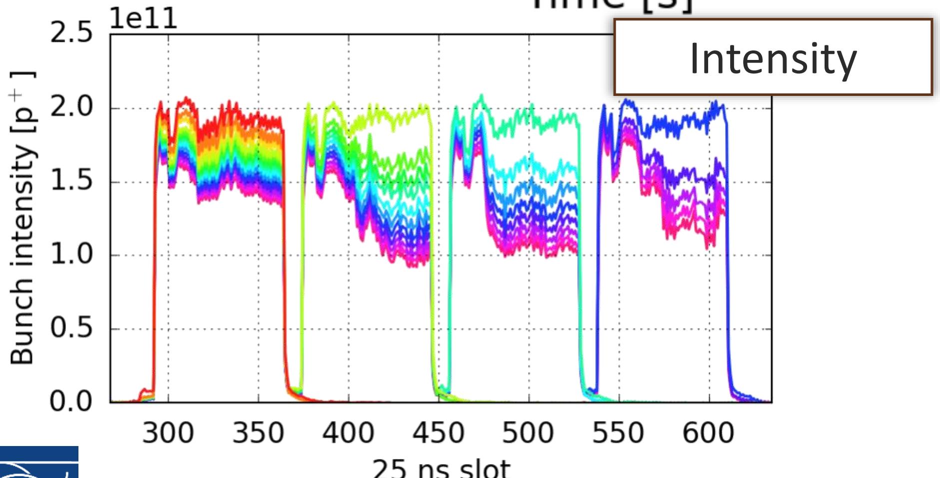
The CERN accelerator complex

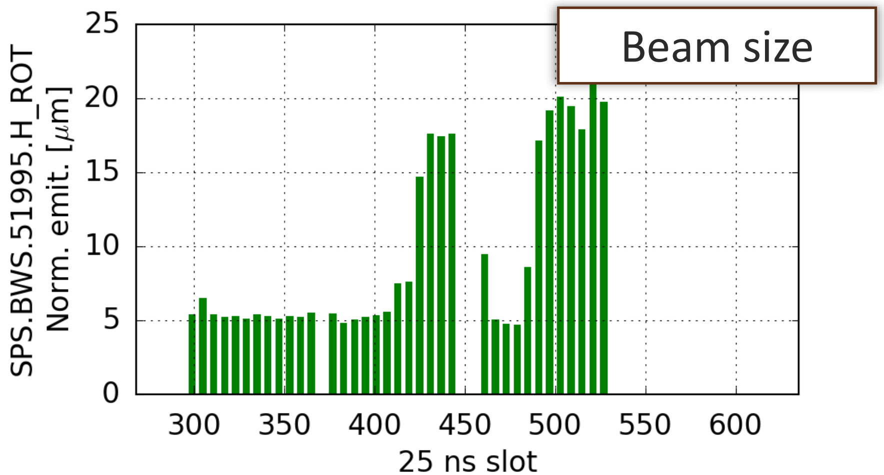


Coupled bunch instability in the SPS



- Injection of 4 batches of 72 bunches trains into the SPS
- Later trains feature **strong losses (intensity)** and **large blow-up (emittance)** – this leads to a **strong loss of beam brightness**





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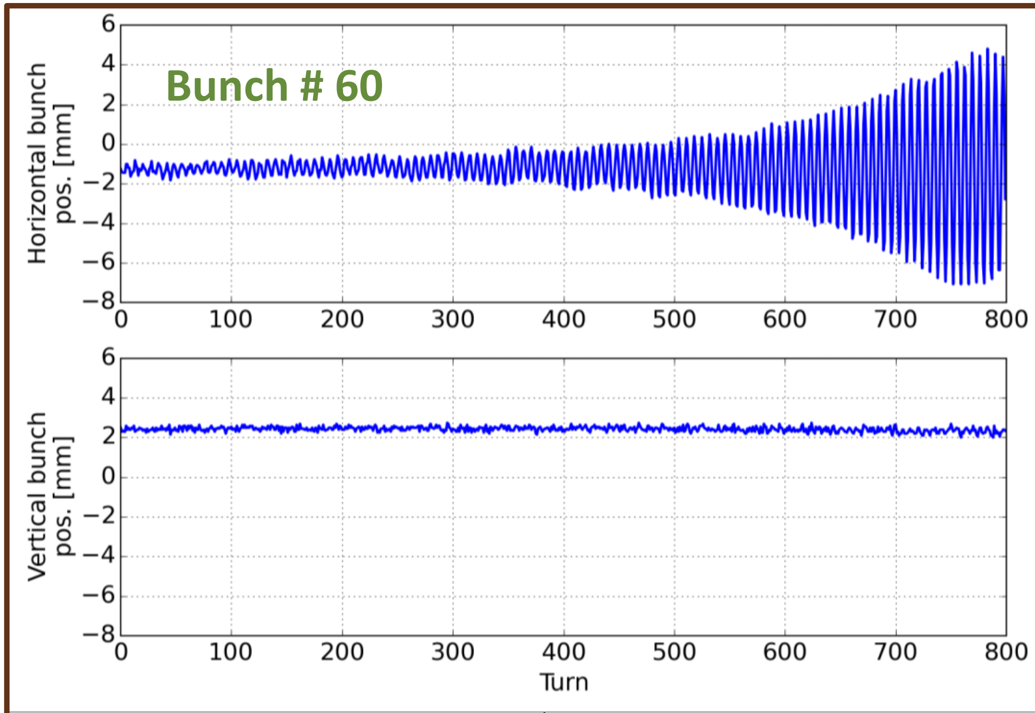
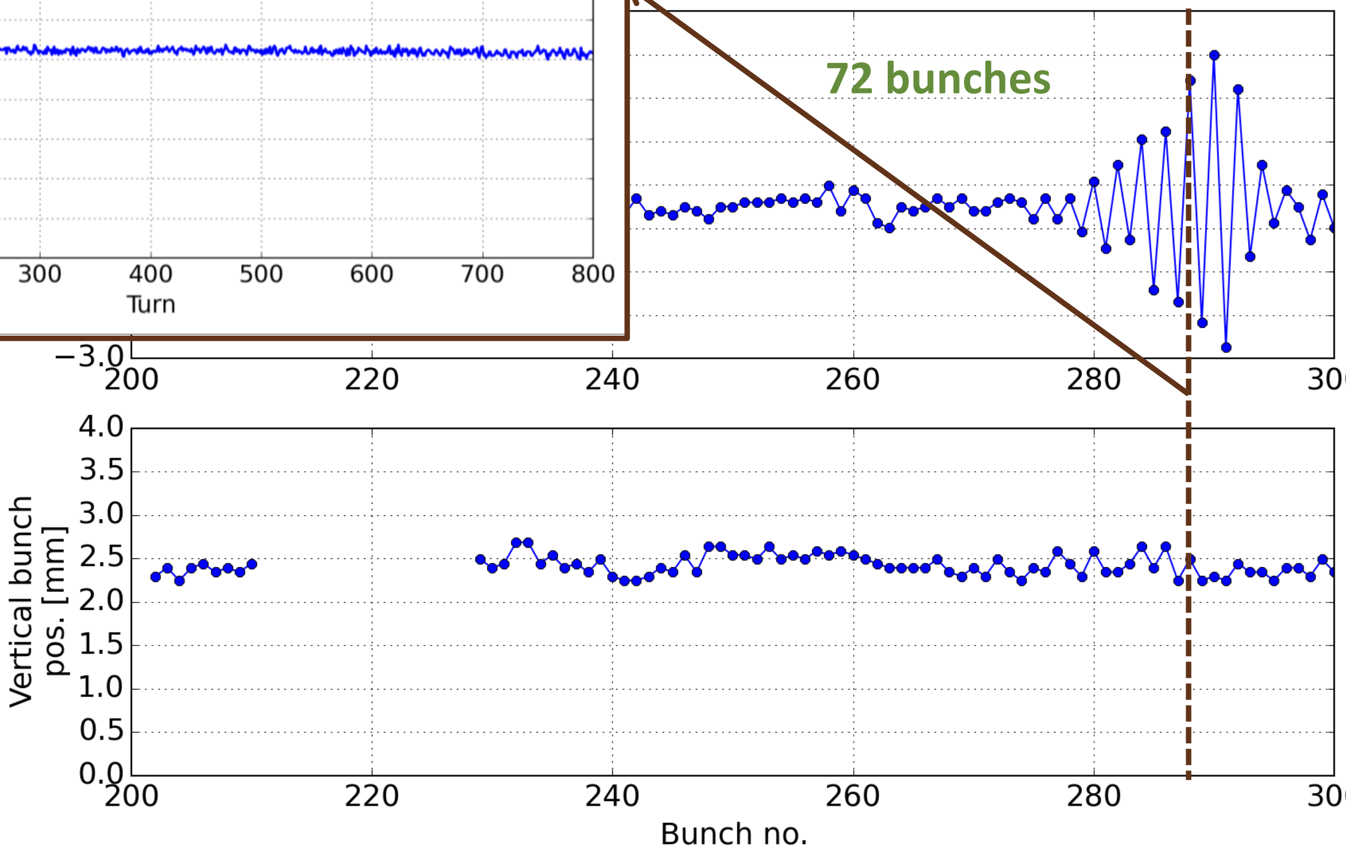


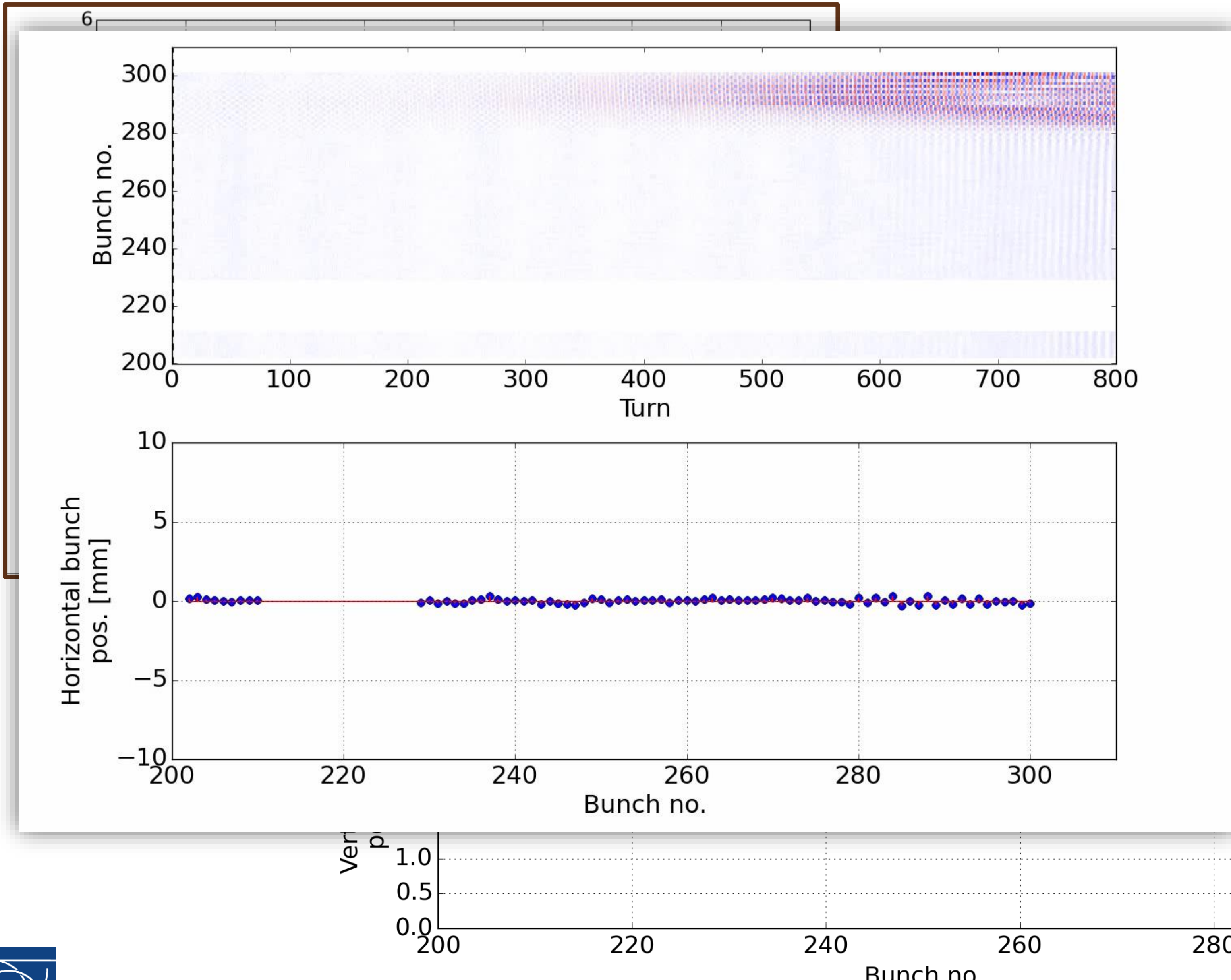
- Injection of 4 batches of 72 bunch trains into the SPS
- A closer look into one train exhibits **strong coherent coupled bunch oscillations**

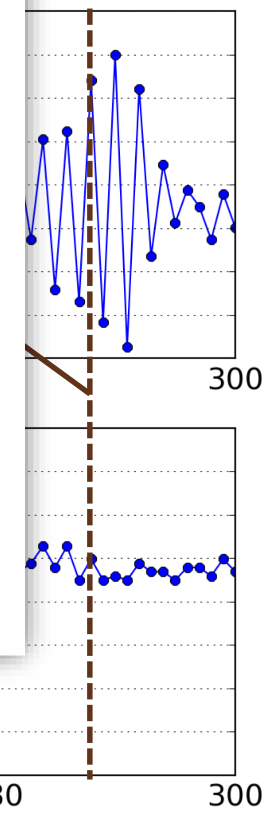


Coupled bunch instability in the SPS

...ches of 72
...e SPS

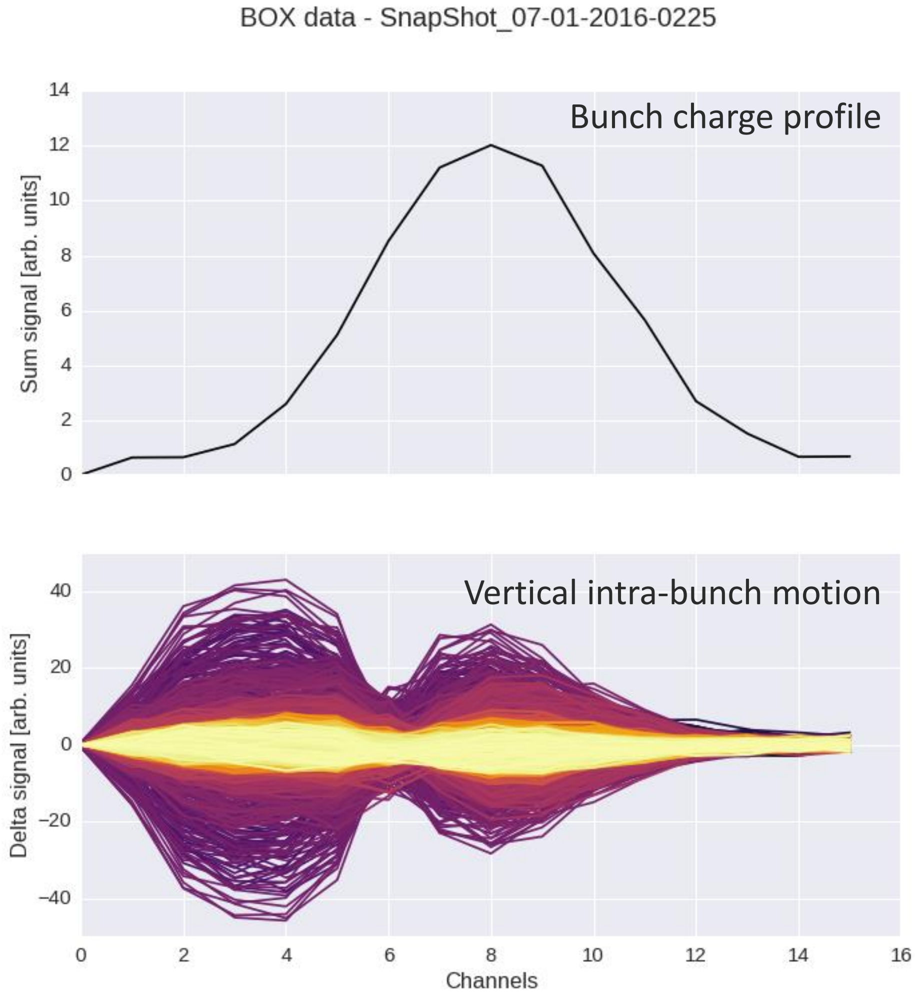
... one train
coherent
...lations

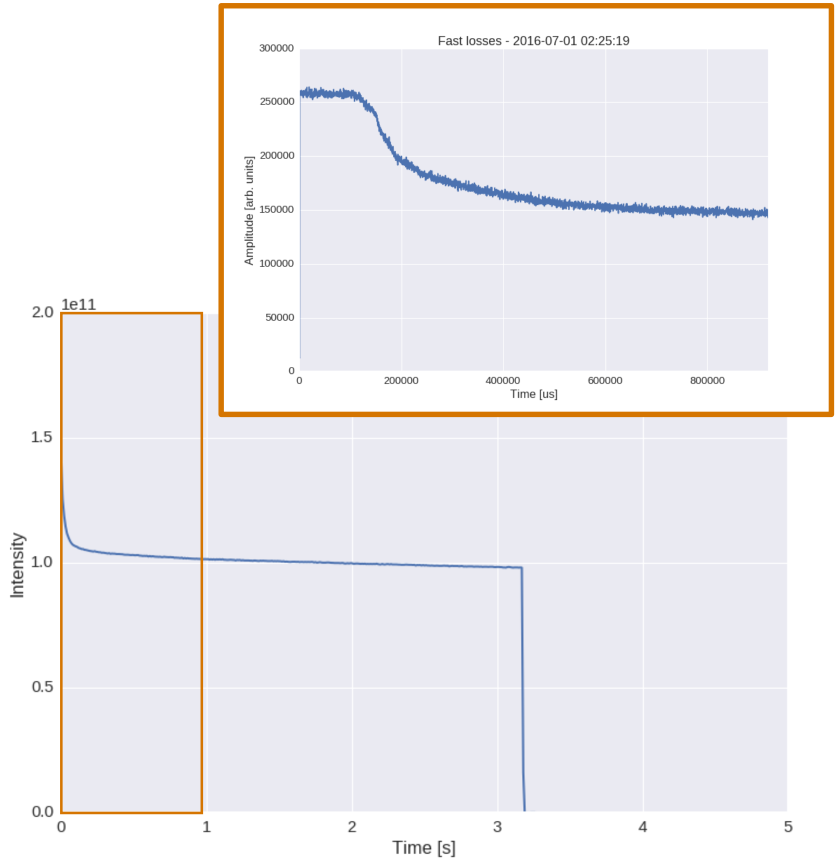




Single bunch instability in the SPS

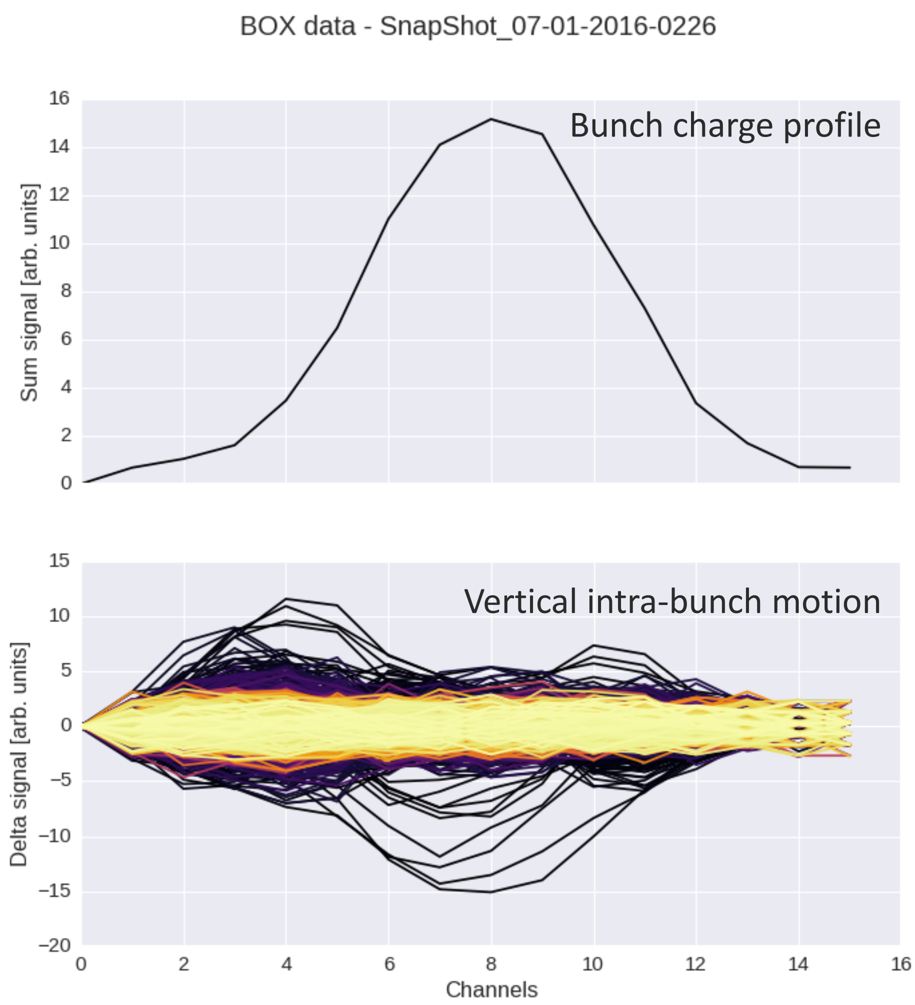
Open loop



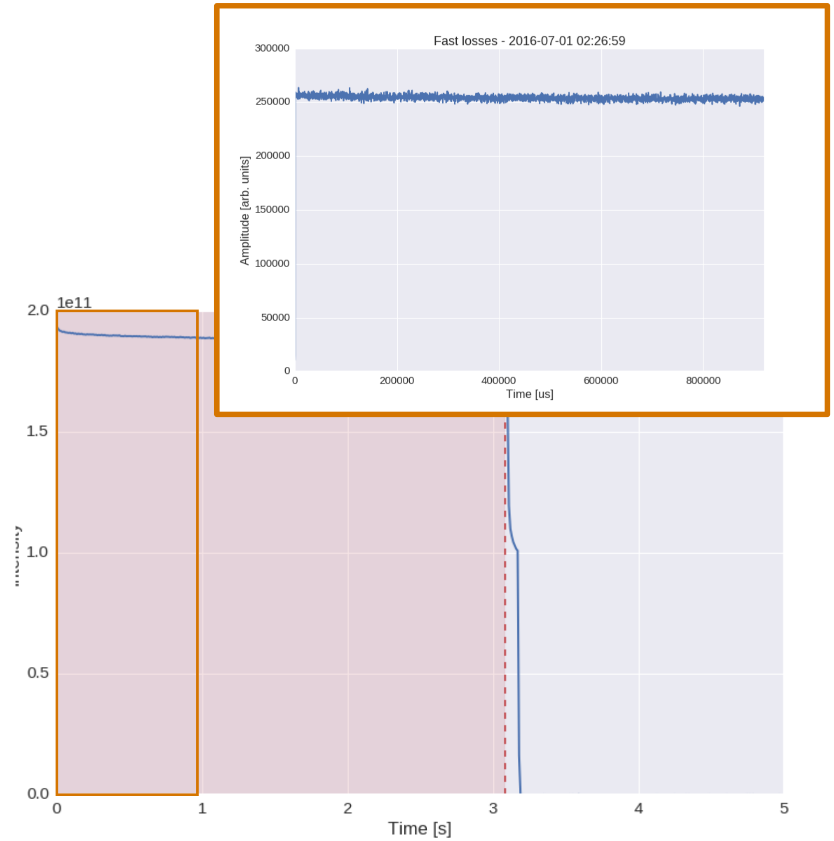


- Loss of more than 30% of the bunch intensity due to a **slow transverse mode coupling instability (TMCI)**.

Single bunch instability in the SPS

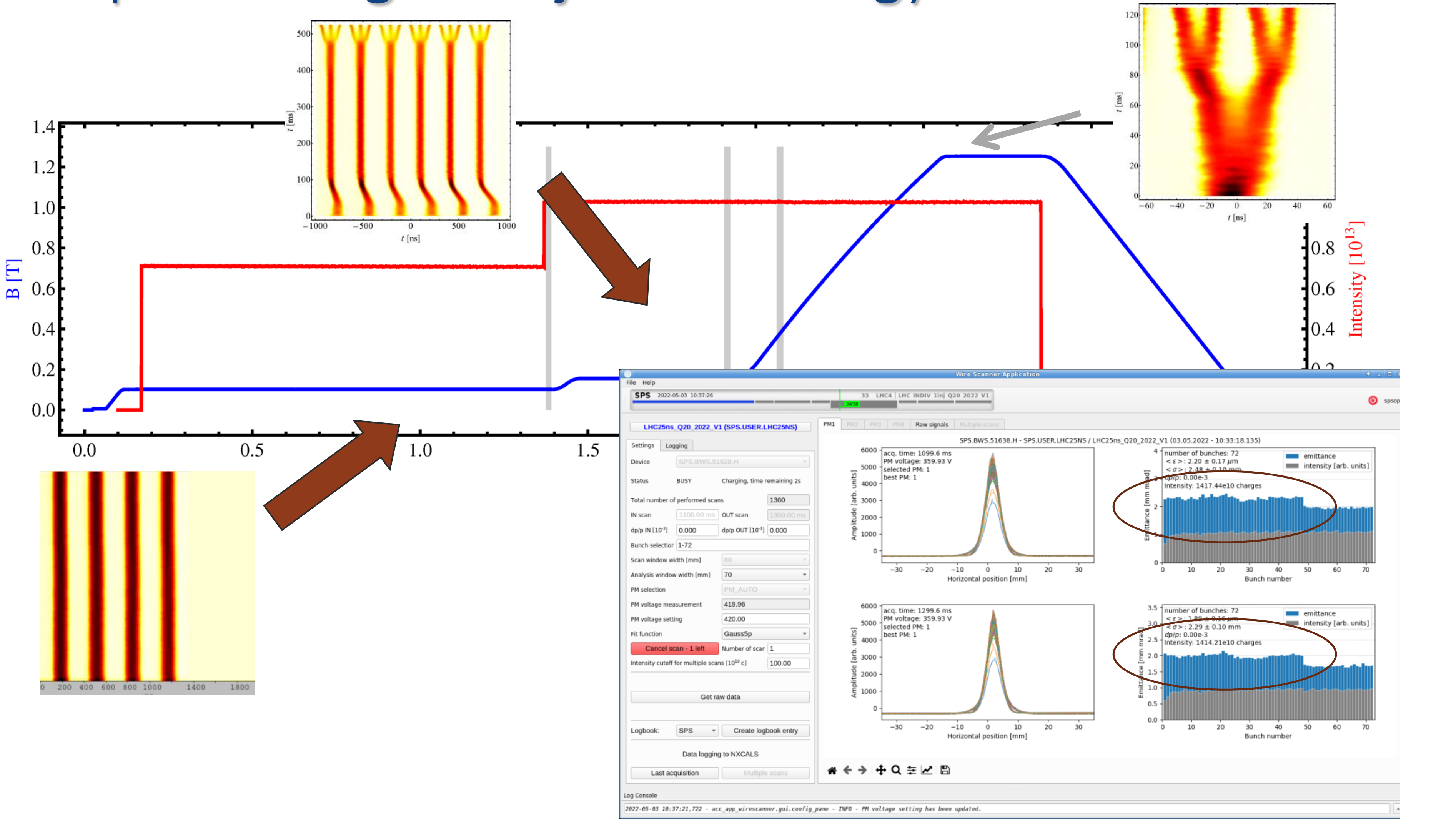


Closed loop



- Loss of more than 30% of the bunch intensity due to a **slow transverse mode coupling instability (TMCI)** → can be mitigated by a **wideband feedback system**.

Space charge at injection energy in the PS



- The bunches injected 1.2 s earlier from the PSB suffer of emittance growth visible in the bunch-by-bunch emittance profile measured at high energy

Signpost

- We now understand that collective effects can have a **huge potentially detrimental impact** on the machine performance and why, therefore, the study and the understanding of the underlying mechanism is important.
- We have encountered some **real world examples** of collective effects observed throughout the CERN accelerator chain.
- Next step is now having a look at the effect on the beam of the self-generated EM field, i.e. what we call **direct space charge**.

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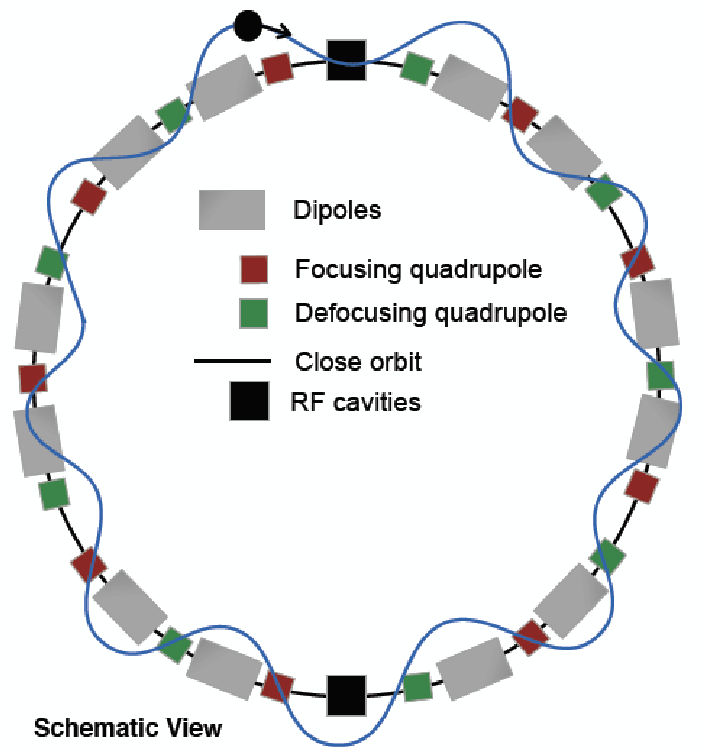
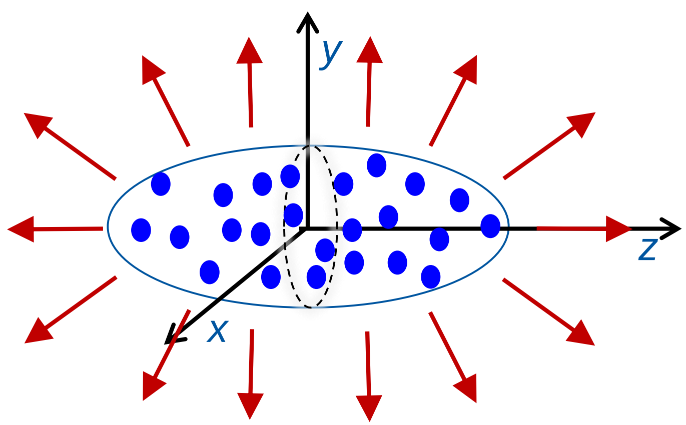
Space charge

- Particles within a bunch moving at any speed lower than speed of light generate a **repulsive force** acting on other particles

- Additional defocusing force on single particles with variable strength along the ring causing a decrease of their oscillation frequencies around the accelerator (tunes)
- Furthermore, particles feel different space charge defocusing forces according to their positions → Spread of tunes within the bunch

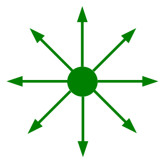




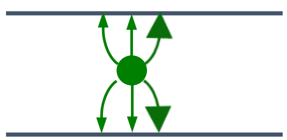
Space charge

- Particles within a bunch moving at any speed lower than speed of light generate a **repulsive force** acting on other particles

- Space charge includes typically two aspects



Direct space charge:
interaction of charged particles in free space

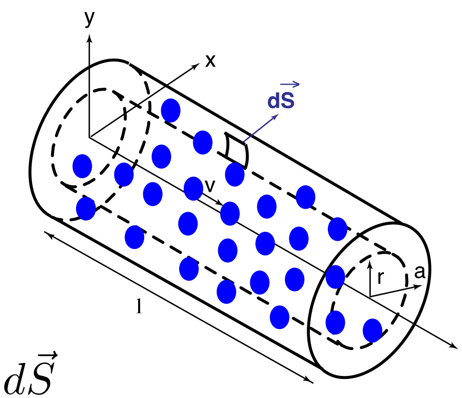


Indirect space charge:
interaction with image charges and currents induced by perfect conducting walls and ferromagnetic materials close to the beam pipe

- Expected dependence on
  - Beam current (intensity) and particle distribution
  - ~~Geometry and material of the surrounding vacuum chamber and machine elements~~

Space charge (direct)

- Coasting beam of circular cross section with radius a and uniform charge density $\eta=\lambda/\pi a^2$ [Cb/m^3] moving at constant velocity $v=\beta c$
  - Calculate the **electric field** using Gauss' law
  - Calculate the **magnetic field** using Stokes' law



Maxwell equation

$$\vec{\nabla} \cdot \vec{E} = \frac{\eta}{\varepsilon_0}$$

Gauss' law

$$\iiint \vec{\nabla} \cdot \vec{E}\, dV = \iint \vec{E}\, d\vec{S}$$

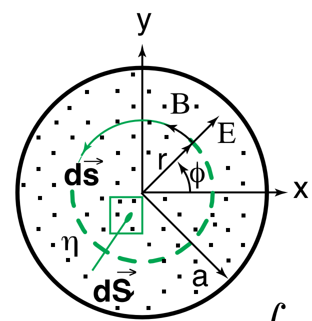
$$\Rightarrow \quad \pi l r^2 \frac{\eta}{\varepsilon_0} = 2\pi l r E_r$$

due to symmetry the electric field has only a radial component

with line density $\lambda = \pi a^2 \eta$

$$E_r = \frac{\lambda}{2\pi\epsilon_0} \frac{r}{a^2}$$

(for *r* < *a*)



Maxwell equation

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Stokes' law

$$\oint \vec{B}\, d\vec{s} = \iint \vec{\nabla} \times \vec{B}\, d\vec{S}$$

$$\Rightarrow \quad 2\pi r B_\phi = \mu_0 \pi r^2 J$$

with current density $J = \beta c \eta = \beta c \lambda / \pi a^2$

and $\mu_0 = 1/\varepsilon_0 c^2$

$$B_\phi = \frac{\lambda \beta}{2\pi\varepsilon_0 c} \frac{r}{a^2}$$

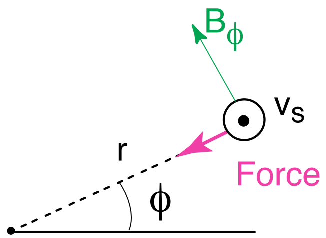
(for *r* < *a*)

Space charge (direct)

- Coasting beam of circular cross section with radius a and uniform charge density $\eta=\lambda/\pi a^2$ [Cb/m^3] moving at constant velocity $v=\beta c$
- Calculate the resulting force on a test particle with charge e

$$E_r = \frac{\lambda}{2\pi\epsilon_0}\frac{r}{a^2}$$

$$B_\phi = \frac{\lambda\beta}{2\pi\varepsilon_0 c}\frac{r}{a^2}$$



Lorentz force for the geometry studied

$$F_r = e\left(E_r - v_s B_\phi\right)$$

$$F_r = \frac{e\lambda}{2\pi\varepsilon_0}\left(1-\beta^2\right)\frac{r}{a^2} = \frac{e\lambda}{2\pi\varepsilon_0}\frac{1}{\gamma^2}\frac{r}{a^2}$$

Electric and magnetic components have opposite signs and scale between them with β^2 → there is perfect compensation when β=1

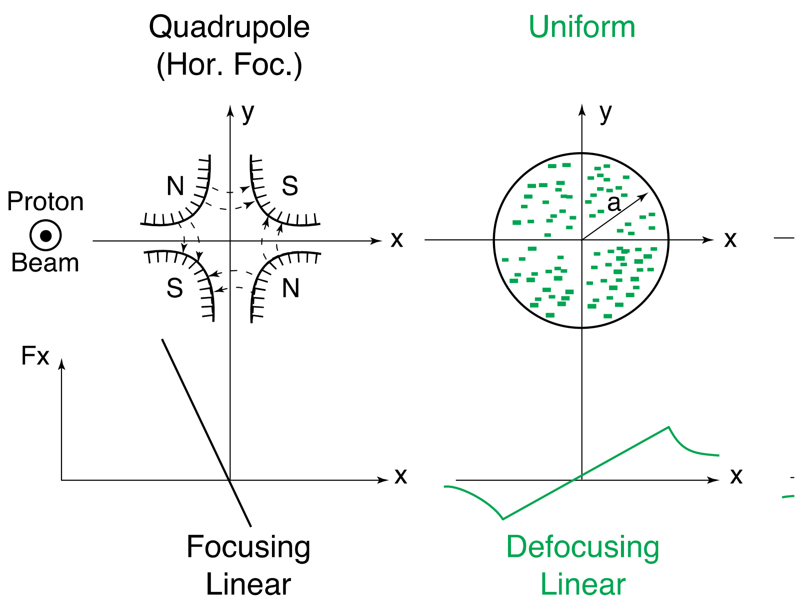
writing the force in x and y:

$$F_x = \frac{e\lambda}{2\pi\varepsilon_0\gamma^2 a^2}x$$

$$F_y = \frac{e\lambda}{2\pi\varepsilon_0\gamma^2 a^2}y$$

Space charge (direct)

- Coasting beam of circular cross section with radius a and uniform charge density $\eta=\lambda/\pi a^2$ [Cb/m^3] moving at constant velocity $v=\beta c$
- In this case the direct space charge force is linear in x and y



$$F_x = \frac{e\lambda}{2\pi\varepsilon_0\gamma^2a^2}x$$

$$F_y = \frac{e\lambda}{2\pi\varepsilon_0\gamma^2a^2}y$$

direct space charge is like a defocusing quadrupole…

however, direct space charge is always defocusing in both planes, while quadrupole is focusing in one and defocusing in the other plane

Space charge tune shift

- The direct space charge force for a beam with uniform charge distribution is linear in x and y → results in the direct space charge tune shift
  - We derive it here for the vertical plane …
  - Express y'' in terms of F_y using Newton's law

$$F_y = \frac{e\lambda}{2\pi\varepsilon_0\gamma^2 a^2}y$$

$$y'' = \frac{1}{\beta^2 c^2}\frac{d^2y}{dt^2} = \frac{1}{\beta^2 c^2}\frac{F_y}{m\gamma} = \frac{2r_0\lambda}{ea^2\beta^2\gamma^3}y$$

classical particle radius

$$r_0 = e^2/(4\pi\varepsilon_0 mc^2)$$

  - Generalize Hill's equation including defocusing space charge term

$$\left.\begin{aligned} y'' + K_y(s)\,y = 0 \\ K_y^{SC}(s) = -\frac{2r_0\lambda}{ea^2(s)\beta^2\gamma^3}\end{aligned}\right\} \Longrightarrow \quad y'' + \left(K_y(s) - \frac{2r_0\lambda}{ea^2\beta^2\gamma^3}\right)y = 0$$

Generalized Hill's equation

  - Calculate the tune shift treating space charge like a focusing error

$$\Delta Q_y = \frac{1}{4\pi}\oint K_y^{SC}(s)\beta_y(s)ds = -\frac{1}{4\pi}\oint \frac{2r_0\lambda\beta_y(s)}{ea^2(s)\beta^2\gamma^3}ds = -\frac{r_0 R\lambda}{e\beta^2\gamma^3}\left\langle\frac{\beta_y(s)}{a^2(s)}\right\rangle$$

Space charge tune shift

- After some reshuffling we notice that the direct space charge tune shift
  - is negative, because space charge transversely always defocuses
  - is proportional to the line density and thus to the number of particles in the beam
  - decreases with energy like $\beta^{-1}\gamma^{-2}$ (when expressed in terms of normalized emittance) and therefore vanishes in the ultrarelativistic limit
  - does not depend on the local beta functions or beam sizes but is inversely proportional to the normalized beam emittance

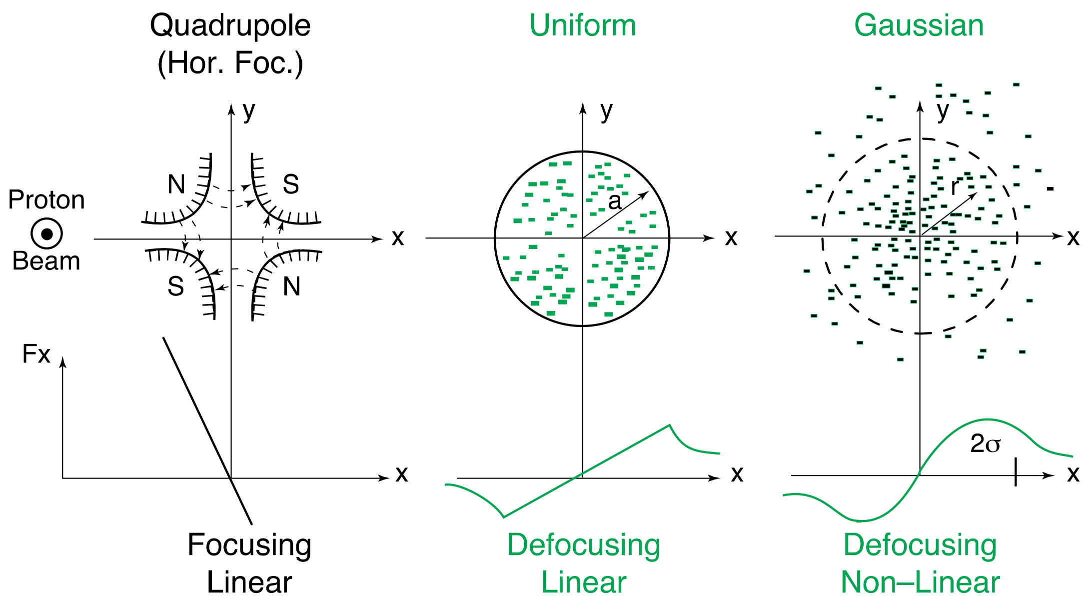
$$\left.\begin{aligned} \Delta Q_{x,y} &= -\frac{r_0 R \lambda}{e\beta^2\gamma^3}\left\langle \frac{\beta_{x,y}(s)}{a^2(s)} \right\rangle \\ a(s) &= \sqrt{\beta_{x,y}(s)\hat{\varepsilon}^n_{x,y}/\beta\gamma} \end{aligned}\right\}$$

$$\boxed{\Delta Q_{x,y} = -\frac{r_0 R \lambda}{e\beta\gamma^2 \hat{\varepsilon}^n_{x,y}}}$$

$$r_0 = e^2/(4\pi\varepsilon_0 mc^2) = \begin{cases} 1.54\cdot 10^{-18}\ \text{m (proton)} \\ 2.82\cdot 10^{-15}\ \text{m (electron)} \end{cases}$$

Space charge: Nonlinear

- For the (quite realistic) case of a beam with a transverse bi-Gaussian distribution the space charge force becomes nonlinear
  - Beam field over the beam cross section is nonlinear
  - Particles will see different tune shifts according to their betatron amplitudes. This causes a tune spread rather than a tune shift!
  - We can calculate the maximum tune shift, i.e. the tune shift in the beam center …



Space charge: Gaussian beam

- We consider first the case of a **circular beam** with rms beam size $\int$

$$\eta(r) = \frac{\lambda}{2\pi\sigma^2} e^{-r^2/2\sigma^2} \qquad \text{with rms beam size} \qquad \sigma = \sqrt{\beta_{x,y}\varepsilon^n_{x,y}/\beta\gamma}$$

  - The following fields satisfy Maxwell's equations
  - We obtain the radial Lorentz force and linearize it for small *r* to calculate the tune shift for particles around the beam center

$$E_r(r) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}\left(1 - e^{-r^2/2\sigma^2}\right)$$

$$B_\phi(r) = \frac{\lambda\beta}{2\pi\varepsilon_0 c}\frac{1}{r}\left(1 - e^{-r^2/2\sigma^2}\right)$$

$$F_r(r) = \frac{e\lambda}{2\pi\varepsilon_0\gamma^2}\frac{1}{r}\left(1 - e^{-r^2/2\sigma^2}\right)$$

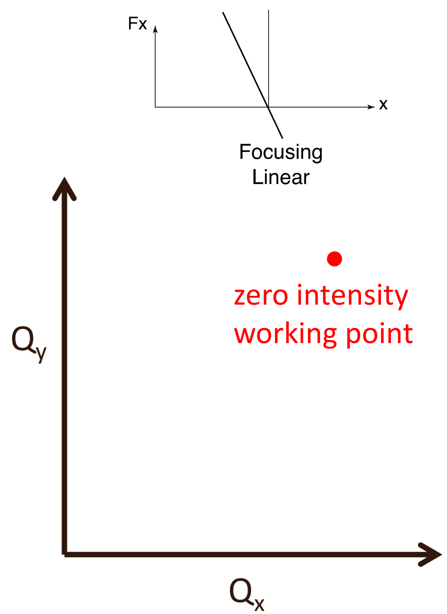
$$F_r(r) = \frac{e\lambda}{2\pi\varepsilon_0\gamma^2}\frac{1}{r}\left(1 - 1 + \frac{r^2}{2\sigma^2} - \dots\right) \approx \frac{e\lambda}{2\pi\varepsilon_0\gamma^2}\frac{r}{2\sigma^2}$$

maximum tune shift for a Gaussian beam distribution

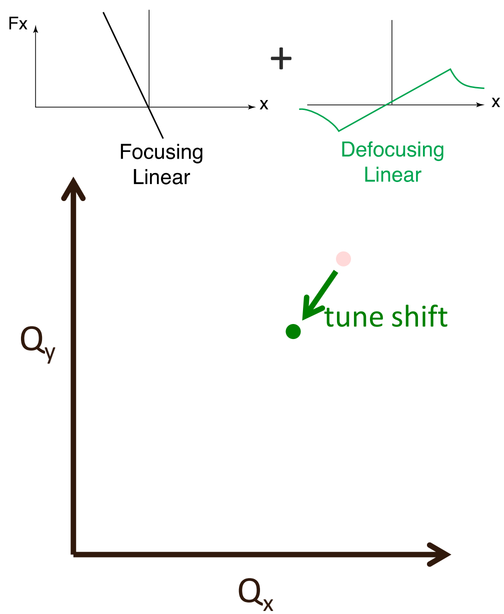
$$\Rightarrow \quad \Delta Q_{x,y} = -\frac{r_0\lambda}{2\pi e\beta^2\gamma^3}\oint\frac{\beta_{x,y}}{2\sigma^2}ds = -\frac{r_0 R\lambda}{e\beta\gamma^2}\frac{1}{2\varepsilon^n_{x,y}}$$

Space charge effect on the tune

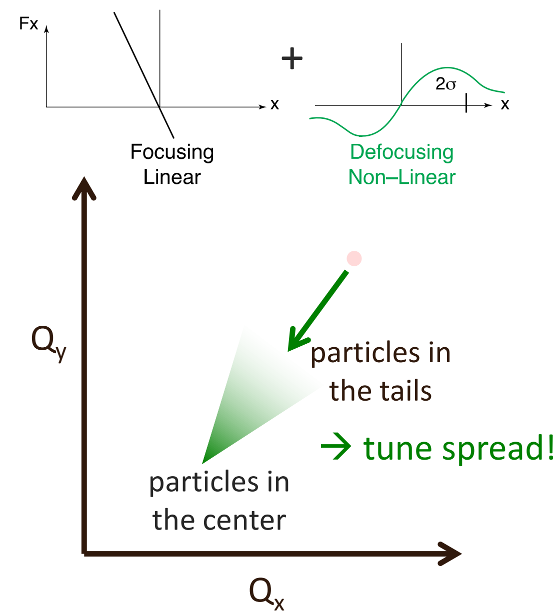
without space charge



with space charge
uniform distribution



with space charge
Gaussian distribution



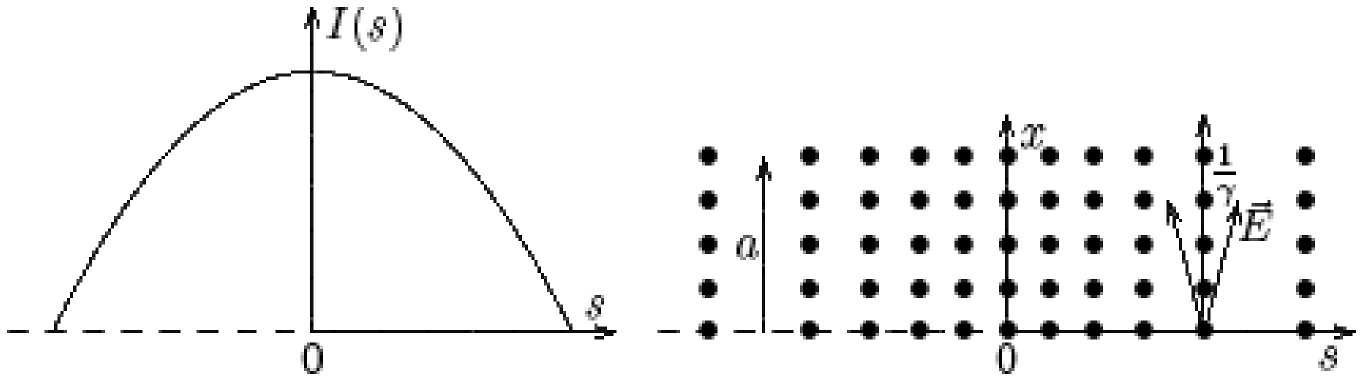
- All particles have the tunes Q_x and Q_y determined by the machine quadrupoles

- All particles have the tunes Q_x and Q_y determined by the machine quadrupoles and the linear defocusing from space charge

- Particles have a different tunes, since the space charge defocusing depends on the particles' amplitude
- The tune shift is largest for particles in the beam center

Space charge: Bunched beams

- If the beam is bunched and has an s-dependent line density
  - The relativistic field has an opening angle of $1/\gamma$
  - Bunched beam as locally continuous if the density $\lambda(s)$ changes smoothly (i.e. does not change much over $\Delta s=a/\gamma$ which means if $\gamma >> a/\Delta s$)
  - Can still use the formulas derived for coasting beams but use the s (or in fact z) dependent line density → the tune shift will also depend on the position of the particle along the bunch
  - This translates into tune modulation with twice the synchrotron period, because particles execute longitudinal oscillations and sample different line densities



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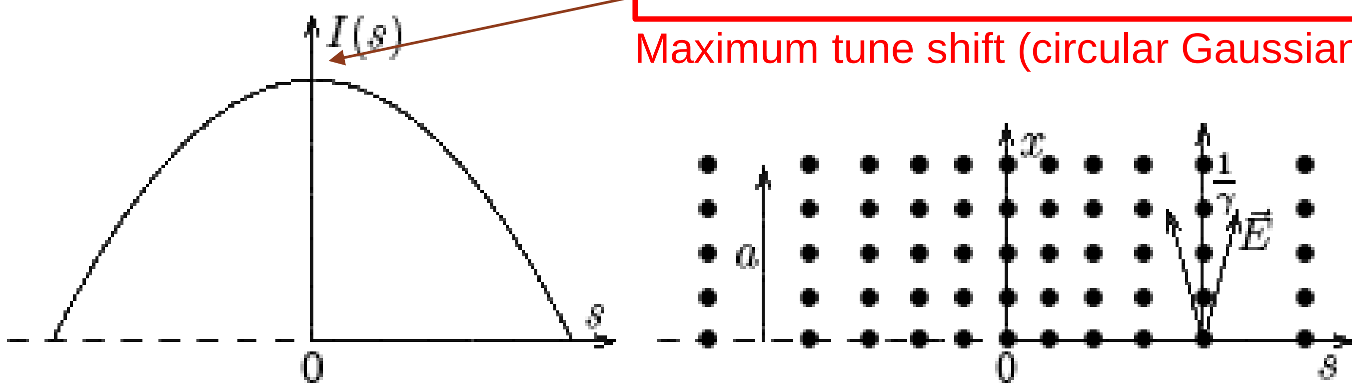
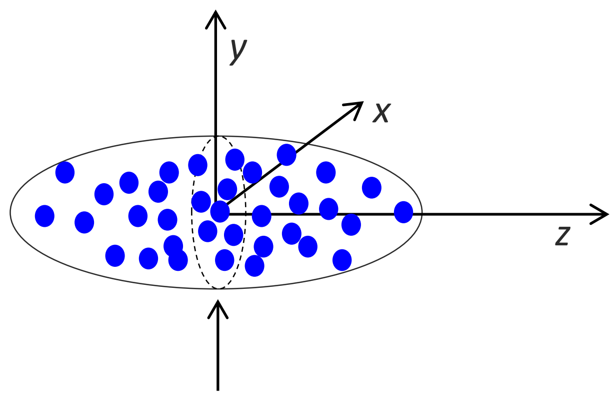
$$\Delta\hat{Q}_{x,y} = -\frac{r_0 C \hat{\lambda}}{2\pi e \beta \gamma^2} \frac{1}{2\varepsilon^n_{x,y}}$$

Maximum tune shift (circular Gaussian)

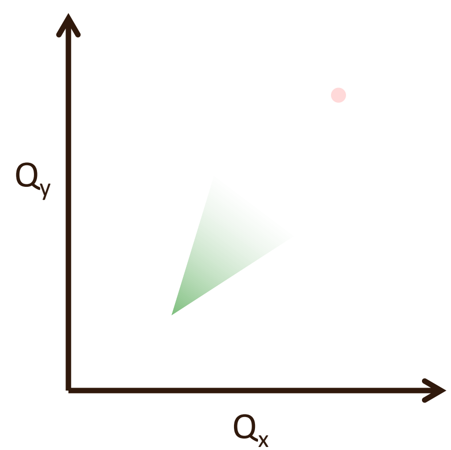


Space charge tune spread

- In case a Gaussian bunched beam, the maximum tune spread is associated to particles close to the peak line density
- Due to the Gaussian transverse distribution the tunes of these particles are also spread around the point of maximum shift

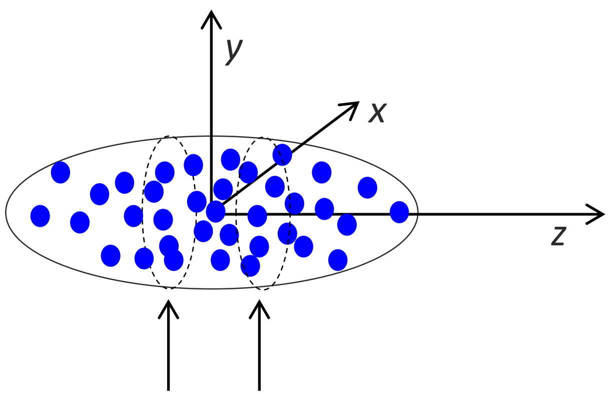


Particles close to the peak line density (often in the bunch center) will have the largest tune spread

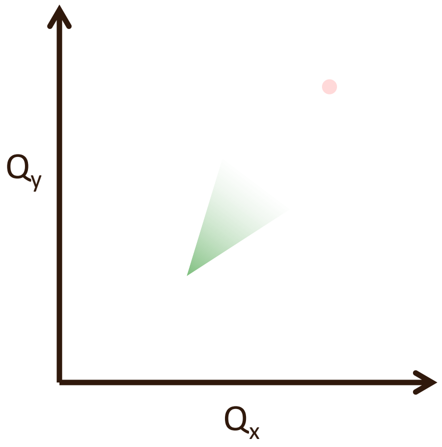


Space charge tune spread

- In case a Gaussian bunched beam, the maximum tune spread is associated to particles close to the peak line density
  - As we move away from the peak density the 'longitudinal' shift decreases
- Due to the Gaussian transverse distribution the tunes of these particles are also spread around the point of maximum shift



The tune spread is reduced for particles further away from the peak line density



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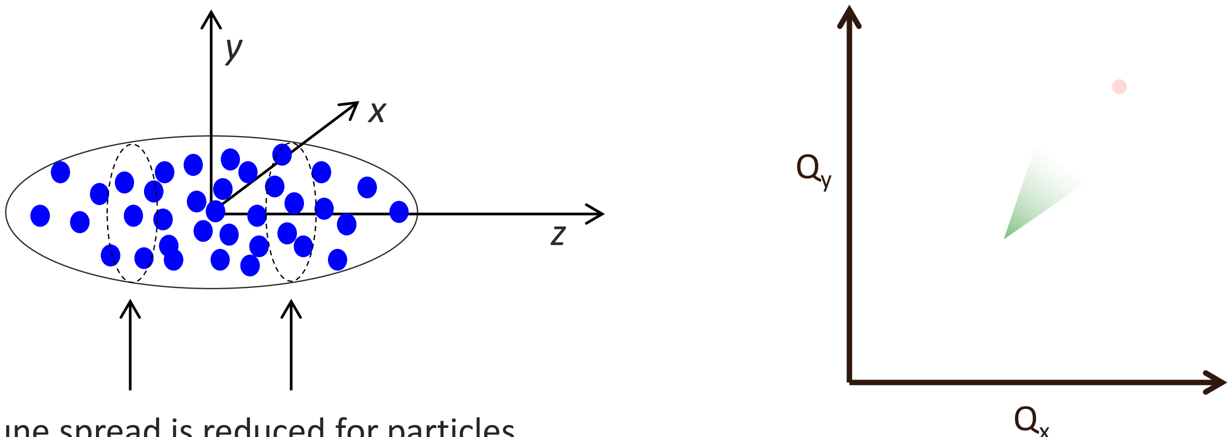

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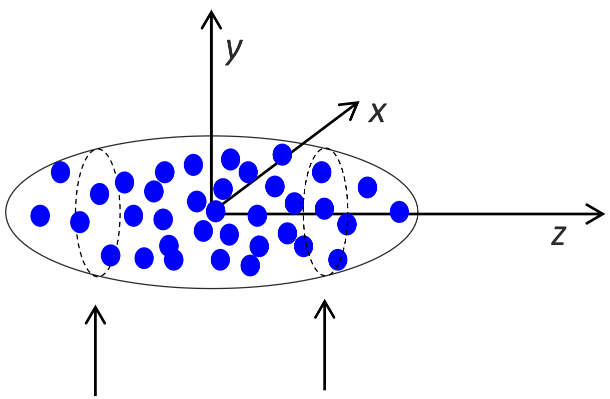

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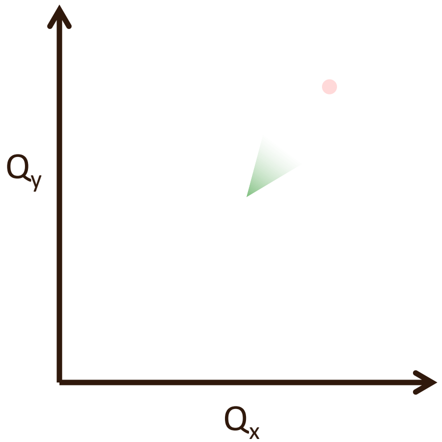

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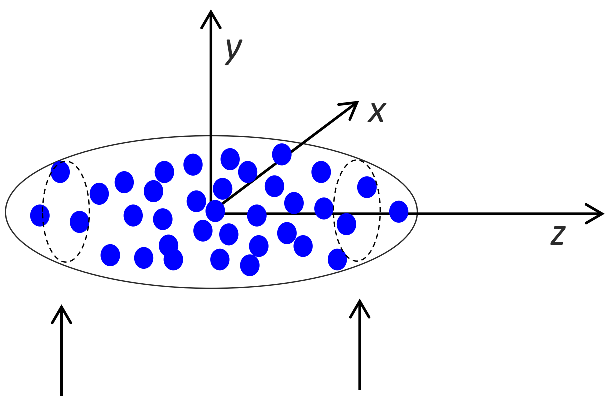

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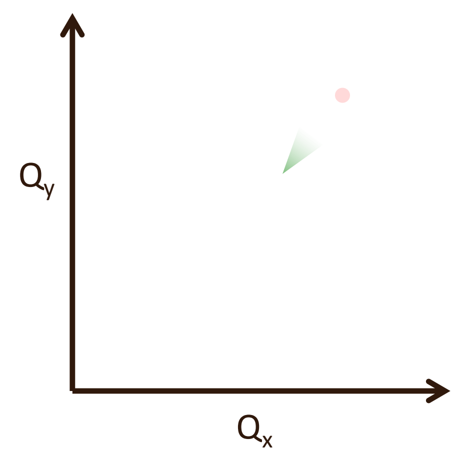

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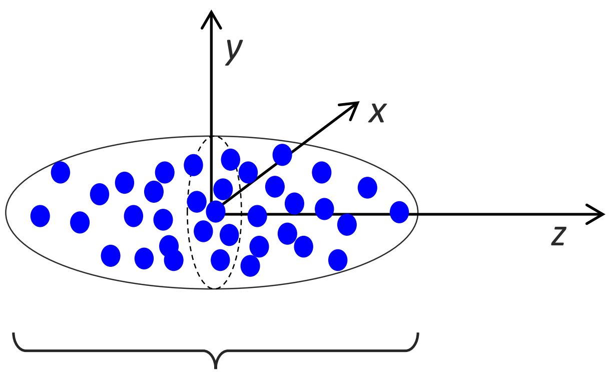
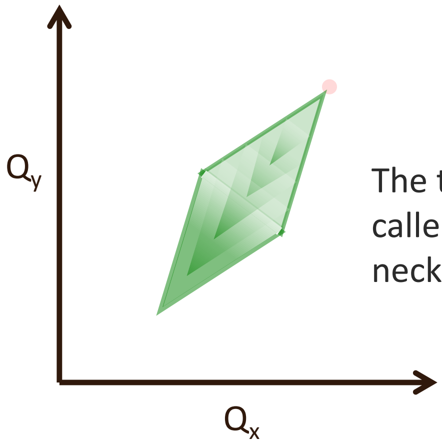
  - As we move away from the peak density the ‘longitudinal’ shift decreases
- Due to the Gaussian transverse distribution the tunes of these particles are also spread around the point of maximum shift
- Globally, the bunched beam will exhibit a tune spread occupying almost all the space between bare tune and maximum shifted tune



For bunched beams the tune spread is as big as the maximum tune shift!

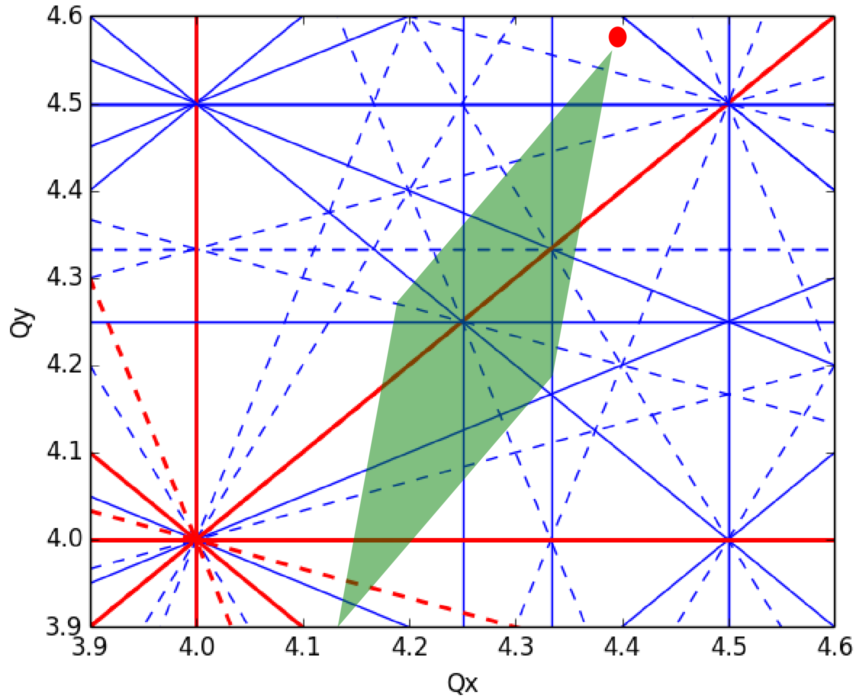


The tune footprint is also called the space charge necktie due to its shape

Brightness limitation from space charge

- A space charge tune spread beyond 0.5 cannot be tolerated without excessive emittance blow-up and/or particle loss due to resonances
  - Dipole errors in the machine excite the integer resonances (Q=n)
  - Quadrupole errors excite the half integer resonances (Q=n+1/2)
  - Higher order resonances can be excited due to sextupoles and multipole errors

- Imagine that a beam with a tune spread of beyond 0.5 is injected



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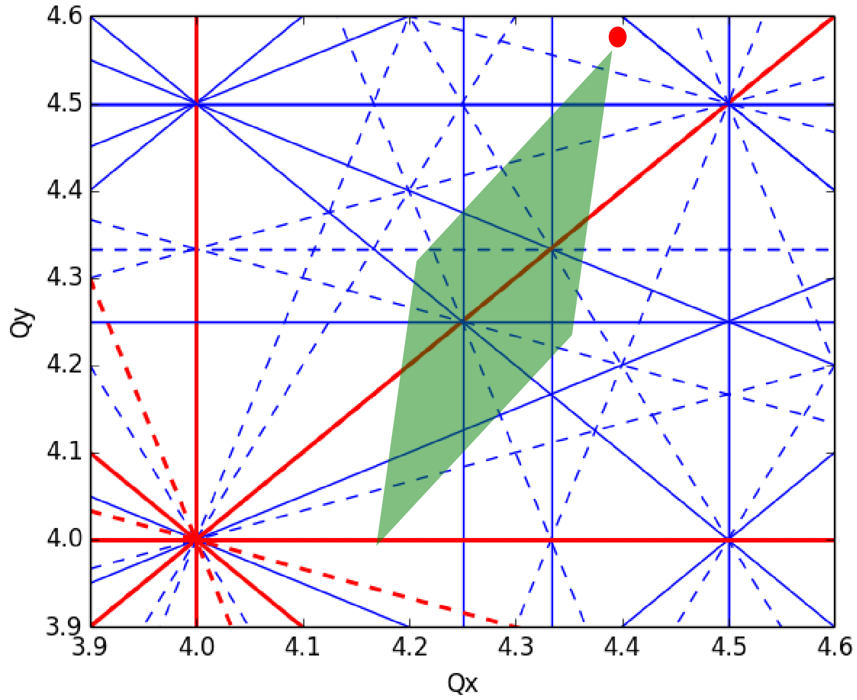
- Particles in the beam core will cross the integer resonance resulting in **emittance blow-up** and a reduction of the tune spread



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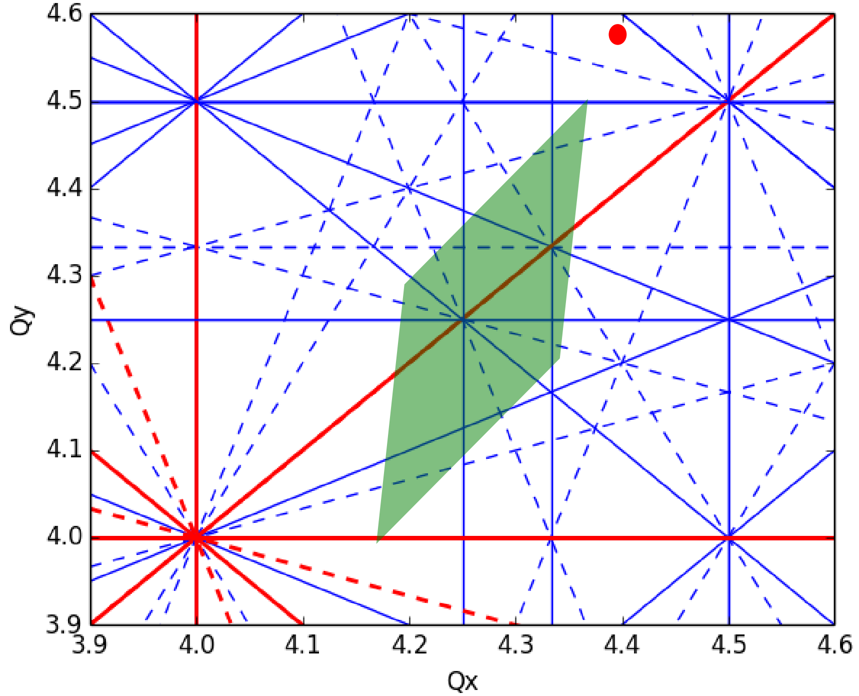
- Imagine that a beam with a tune spread of beyond 0.5 is injected
- Particles in the beam core will cross the integer resonance resulting in **emittance blow-up** and a reduction of the tune spread
- Particles in the beam tails can be pushed onto the half integer resonance resulting in **losses due to aperture restrictions**



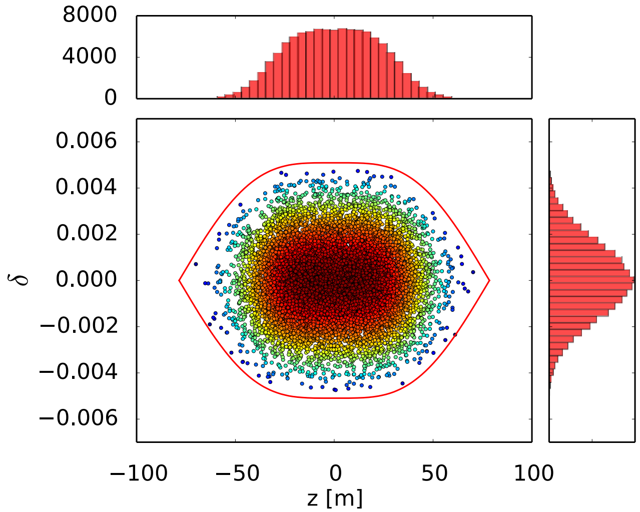
Brightness limitation from space charge

- To cope with space charge
  - Minimise tune charge tune spread

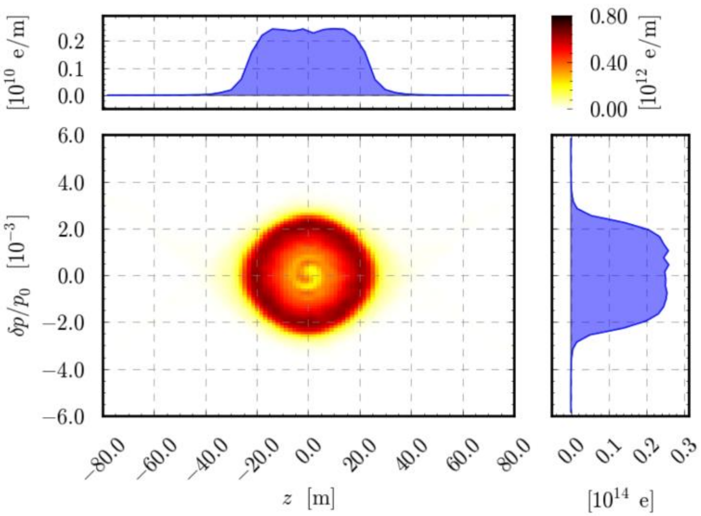
Maximum tune shift (circular Gaussian)

$$\Delta \hat{Q}_{x,y} = -\frac{r_0 C \hat{\lambda}}{2\pi e \beta \gamma^2} \frac{1}{2\varepsilon^n_{x,y}}$$

Using a double RF system ..



.. or creating a hollow bunch distribution



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- Increase the beam energy
- Examples: Increase of injection energy into PSB (50 → 160 MeV) and into PS (1.4 → 2 GeV) during Long Shutdown 2
  - The scope is to eventually increase the brightness of the LHC beams!

Brightness limitation from space charge

- To cope with space charge
  - Minimise tune charge tune spread
  - Compensate resonances
    - Some can be efficiently compensated by means of magnetic multipoles – and this frees up regions of the tune space in which larger tunes spreads can be accommodated without significant emittance growth and/or losses
    - Some may be however excited by space charge itself!
- Space charge compensation as well as longitudinal shape tailoring are essential beam dynamics knobs (sometimes requiring additional hardware, e.g., multipoles or 2nd harmonic RF system), especially in low energy machines, to preserve the high brightness needed for ex. in colliders

Signpost

- Space charge is an additional defocusing force that leads to, e.g., **negative tune shift** and **tune spread depending on transverse and longitudinal amplitudes**.
- This tune spread can intercept to **resonance lines** in the **tune diagram**.
- Space charge can therefore lead to are **emittance growth** and/or **particle loss – in either case loss of beam quality**.

- Part 1: Introduction + space charge
  - Introduction to collective effects
  - Examples of coherent and incoherent effects
  - Space charge
    - Basic concepts
    - Tune shift and tune spread

Signpost

- We have learned about some of the peculiarities of **collective effects**. We have also introduced **multi-particle systems** and have seen how these can be described and treated theoretically/numerically.
- We have seen some **real-world example of collective effects** manifesting themselves as coherent beam instabilities.
- We have looked at some specific **features of space charge** and how it can influence beam dynamics and lead to beam quality degradation.

- Part 1: Introduction + space charge
  - Introduction to collective effects
  - Examples of coherent and incoherent effects
  - Space charge
    - Basic concepts
    - Tune shift and tune spread

End part 1

Backup

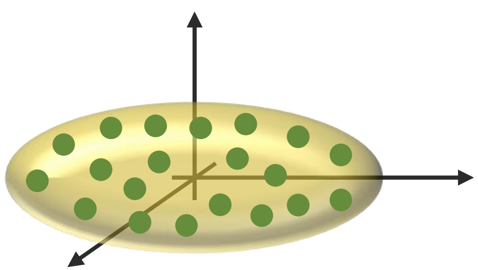
Signpost

- We have learned about the **particle description** of a beam.
- We have seen **macroparticles** and **macroparticle models**.
- We have seen how **macroparticle models** are **mapped and represented in a computational environment**.

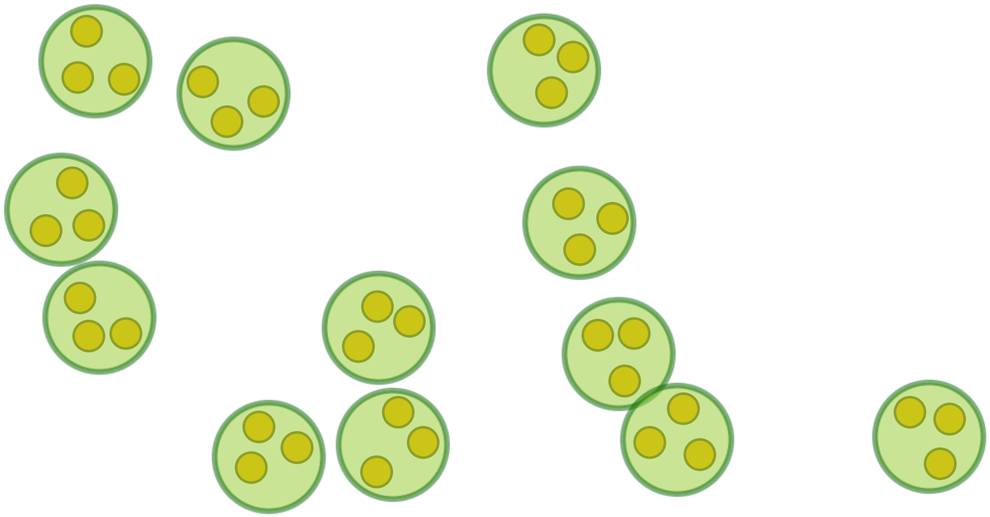
- Part 1: Introduction – multiparticle systems, macroparticle models and wake functions
  - Introduction to beam instabilities
  - Basic concepts
    - Particles and macroparticles – macroparticle distributions
    - Beam matching
    - Multiparticle effects – filamentation and decoherence
    - Wakefields as sources of collective effects

The particle description

- As seen earlier, and especially for the analytical treatment, we can represent a charged particle beam via a **particle distribution function**.
- In computer simulations, a charged particle beam is still represented as a multiparticle system. However, to be **compatible with computational resources**, we need to rely on **macroparticle models**.
- A **macroparticle** is a numerical **representation** of a **cluster of neighbouring physical particles**.
- Thus, instead of solving the system for the **N (~10^{11})** physical particles one can significantly **reduce the number of degrees of freedom** to **N_{MP} (~10^{6})**. At the same time one must be aware that this **increases of the granularity** of the system which gives rise to numerical noise.



$$\Psi\,(x, x', y, y', z, \delta)$$



Macroparticle representation of the beam

- Macroparticle models permit a **seamless mapping** of realistic systems into a **computational environment** – they are fairly easy to implement

Beam:

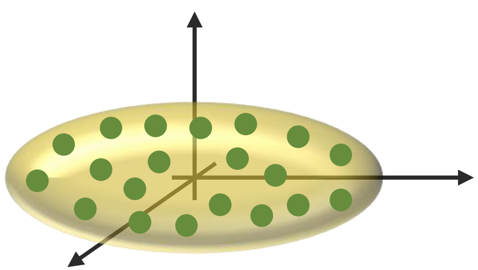
$$\begin{pmatrix} x_i \\ x'_i \end{pmatrix} \quad \begin{pmatrix} q_i \\ m_i \end{pmatrix}, \quad i = 1, \ldots, N$$

Macroparticlenumber

$$\begin{pmatrix} y_i \\ y'_i \end{pmatrix}$$

$$\begin{pmatrix} z_i \\ \delta_i \end{pmatrix}$$

Canonically conjugate coordinates and momenta



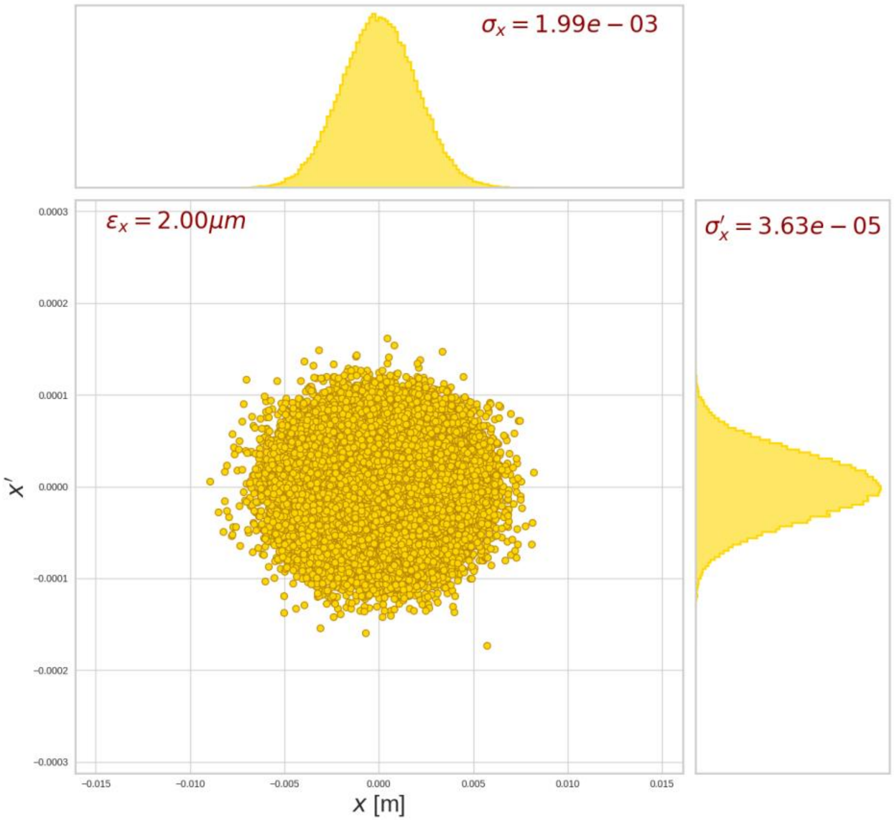
$$\Psi\left(x, x', y, y', z, \delta\right)$$

```
In [6]: df = pd.DataFrame(bunch.get_coords_n_momenta_dict())
        df
```

Out[6]:

	dp	x	xp	y	yp	z
0	0.001590	0.000566	-2.285393e-05	-0.001980	4.283152e-06	0.353427
1	0.001978	0.000370	1.954404e-05	-0.000359	5.543904e-05	0.159670
2	0.003492	-0.000829	-2.773707e-05	0.000291	6.627340e-05	-0.251489
3	0.002195	-0.001668	-2.317633e-05	0.001878	-1.870926e-05	-0.038597
4	0.000572	0.000990	5.493907e-05	0.000152	-1.951051e-05	0.492968
5	-0.000418	0.001088	4.778027e-05	0.003320	-7.716856e-06	0.415582
6	-0.000114	-0.000194	1.065400e-05	0.001798	-4.984276e-07	-0.349064
7	0.001100	-0.001257	-6.873217e-05	-0.002374	5.657645e-06	-0.023157
8	0.002706	0.005351	-1.867898e-07	-0.000765	3.012523e-05	-0.291095
9	0.003508	0.000499	1.865768e-05	-0.001032	-5.363820e-05	0.211726
10	-0.001711	-0.003168	4.372560e-05	-0.001933	-2.151020e-05	-0.145358
11	-0.002150	-0.000565	-1.853825e-05	-0.003895	-6.192450e-06	0.072499
12	0.002059	0.003453	-3.808703e-05	0.000118	3.179588e-05	-0.001816
13	0.002709	0.000241	-3.457535e-05	0.000474	5.057865e-05	-0.005464
14	-0.001593	0.000711	-1.667091e-05	-0.002523	-3.804168e-05	[illegible]
15	-0.000830	-0.000393	-7.473946e-05	[illegible]		
16	-0.001743	[illegible]				

Macroparticle representation of the beam




```
In [6]: df = pd.DataFrame(bunch.get_coords_n_momenta_dict())
        df
```

Out[6]:

	dp	x	xp	y	yp
0	0.001590	0.000566	-2.285393e-05	-0.001980	4.283152e-0
1	0.001978	0.000370	1.954404e-05	-0.000359	5.54390
2	0.003492	-0.000829	-2.773707e-05	0.000291	6.62
3	0.002195	-0.001668	-2.317633e-05	0.001878	
4					

- Initial conditions of the beam/particles

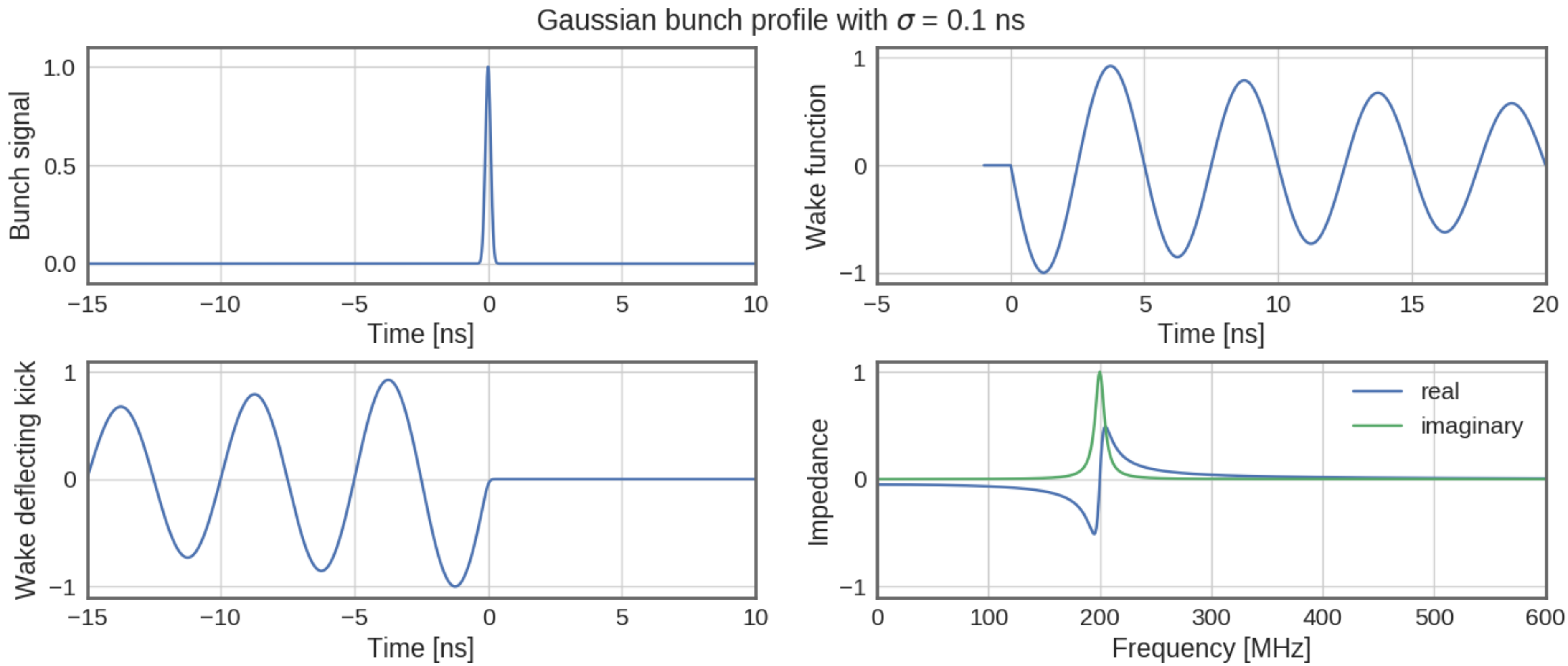
Profile	Size	Matching
Gaussian	Emittance	Optics
Parabolic		
Flat		
…		

- We use **random number generators** to obtain **random distributions of coordinates and momenta**
- Example transverse Gaussian beam in the SPS with normalized emittance of 2 um (0.35 eVs longitudinal)

$$\varepsilon_\perp = \beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$
$$= \beta\gamma\sigma_x\sigma_{x'}$$
$$\varepsilon_\parallel = 4\pi\sigma_z\sigma_\delta\frac{p_0}{e}$$

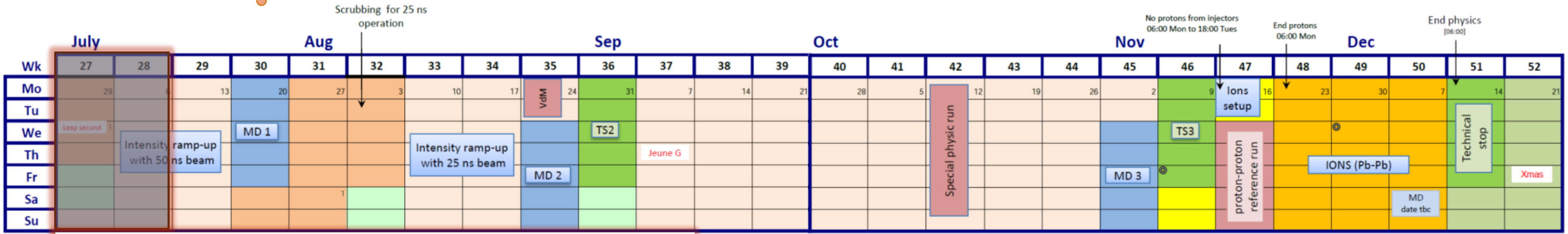
Wake fields illustrative examples

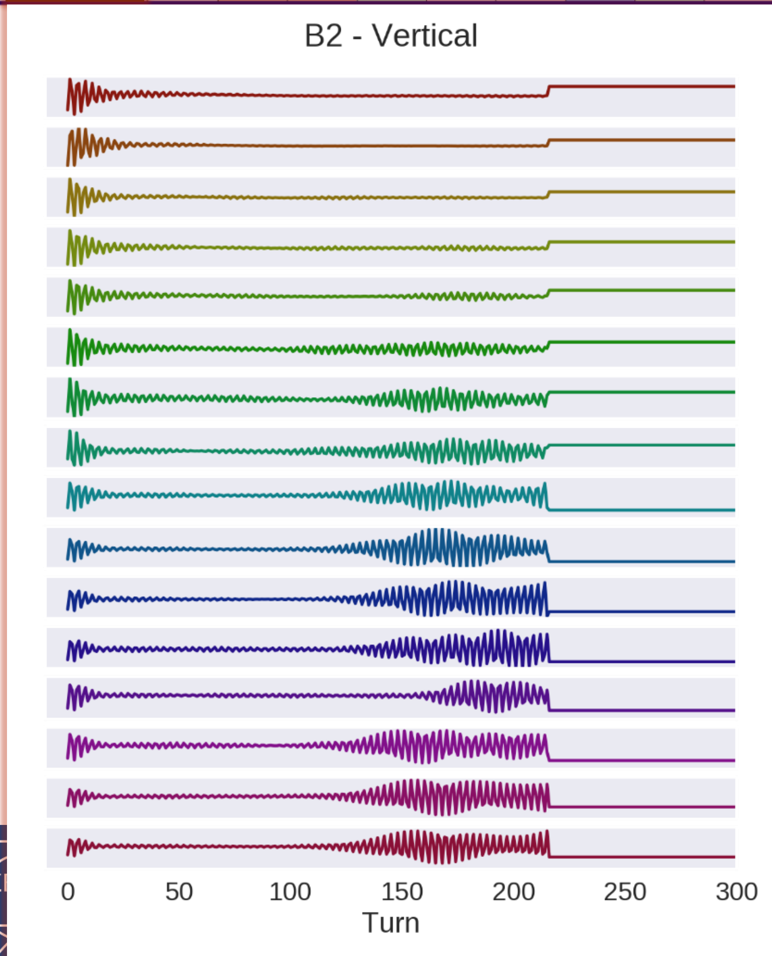
- Resonator wake: fr = 200 MHz, Q = 20 – Gaussian bunch charge profile
- The plots show how the bunch moments and the wake function **convolve into an integrated deflecting kick** at the different positions along the bunch



E-cloud instabilities in the LHC

Scrubbing run in 2015 – early stage





Head of batch

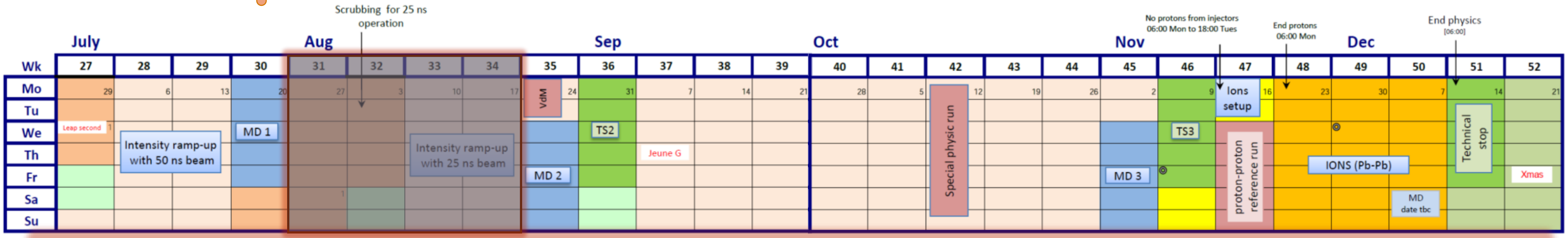
every 4th bunch just after injection

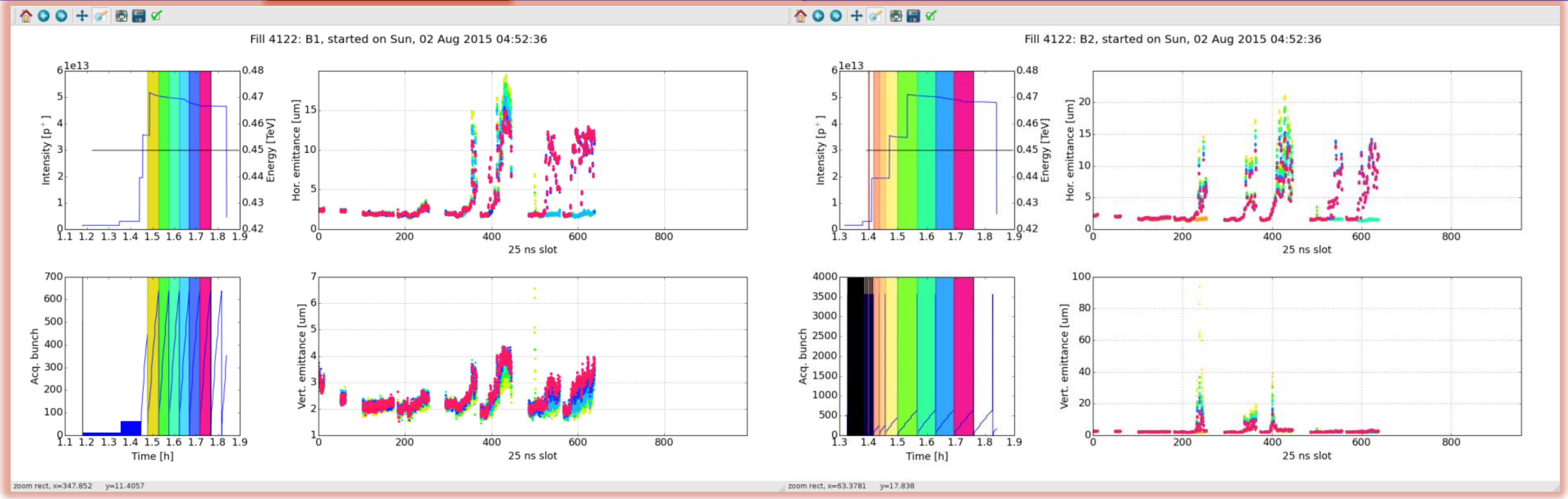
Tail of batch

- Injection of multiple bunch batches from the SPS into the LHC.
- Violent **instabilities during initial stages of scrubbing** – clear e-cloud signature
- Very hard to control in the beginning – **slow and staged ramp-up of intensity** (24 → 36 → 48 → 60 → 72 → 144 bpi)

E-cloud instabilities in the LHC

Scrubbing run in 2015 – second stage





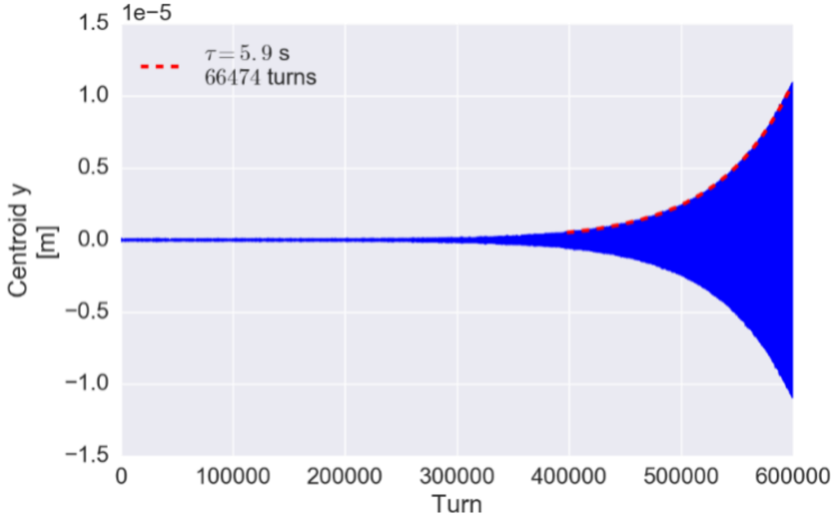
- At later stages dumps under control but still **emittance blow-up and serious beam quality degradation**.
- Beam and e-cloud induced h**eating of kickers and collimators**.

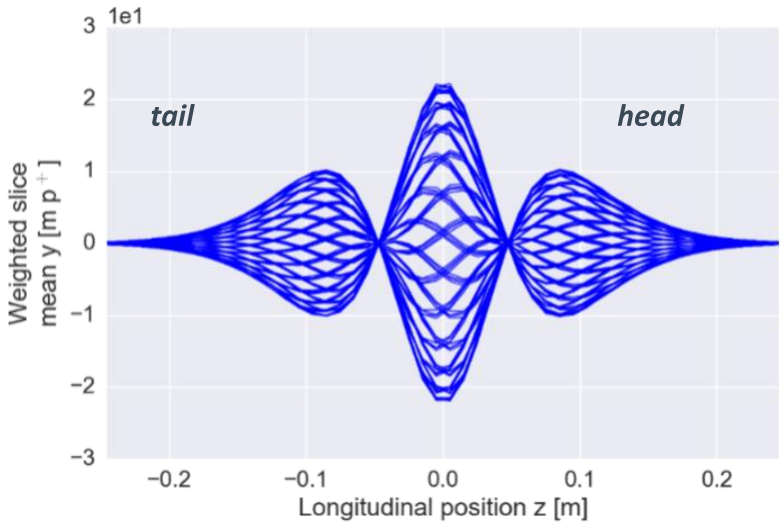
Headtail instabilities in the LHC

- The **impedance in the LHC** can give rise to coupled and single bunch instabilities which, when left untreated, can lead to **beam degradation and beam loss**.

- As an example, **headtail instabilities** are predicted from **macroparticle simulations** using the LHC impedance model.

- These simulations help to understand and to predict unstable modes which are observed in the real machine.

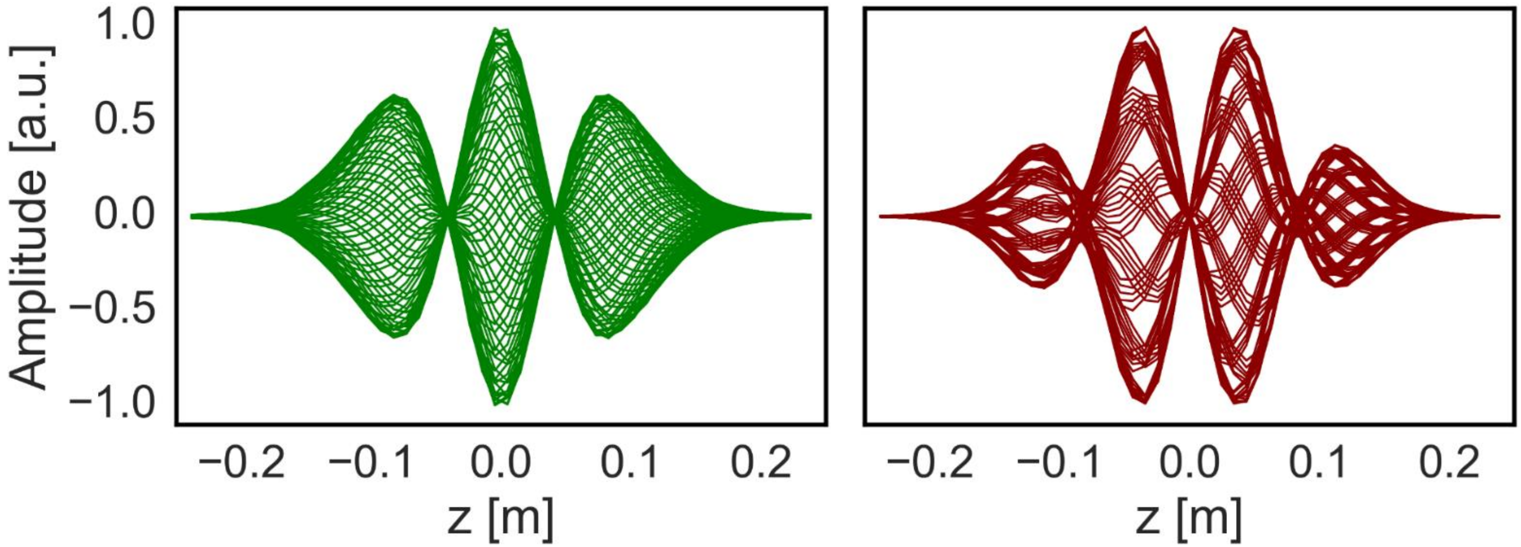




Headtail instabilities in the LHC

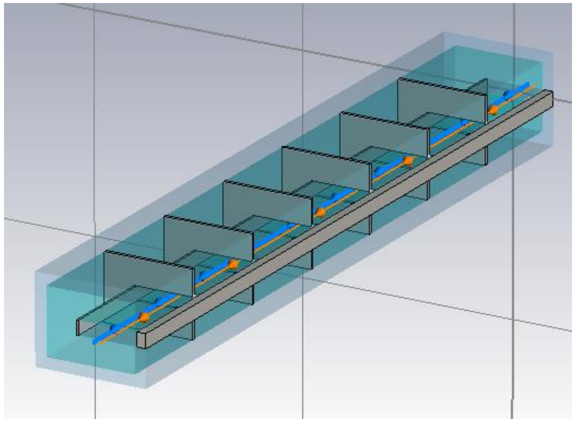
m = 0 m = -1

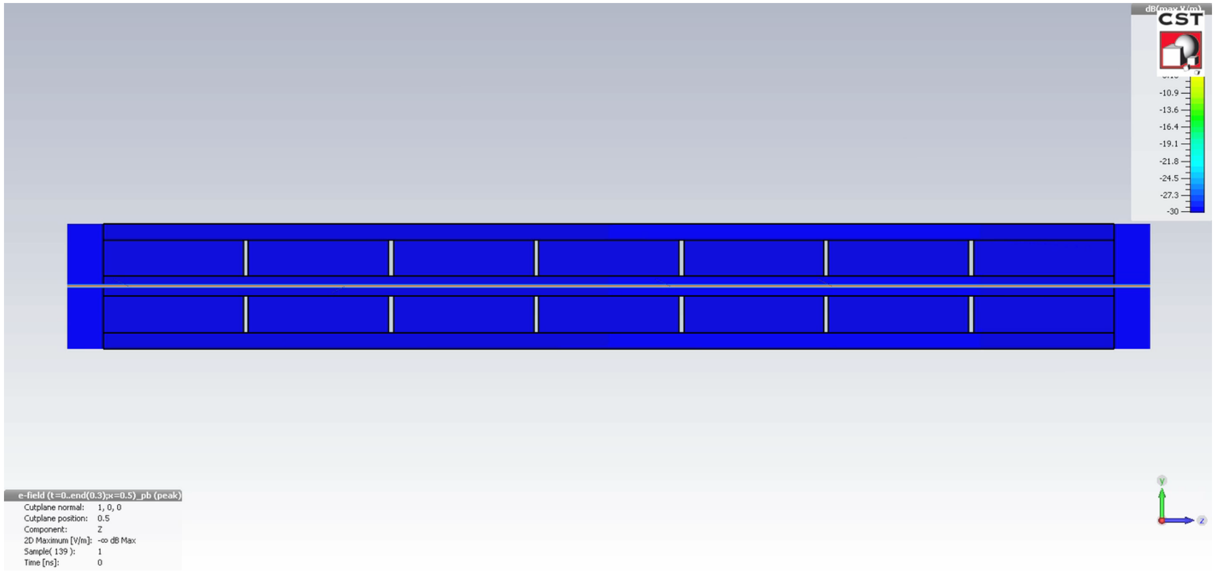
Macroparticle simulations (PyHEADTAIL)



- These simulations help to understand and to **predict instabilities** which are **observed in the real machine**.

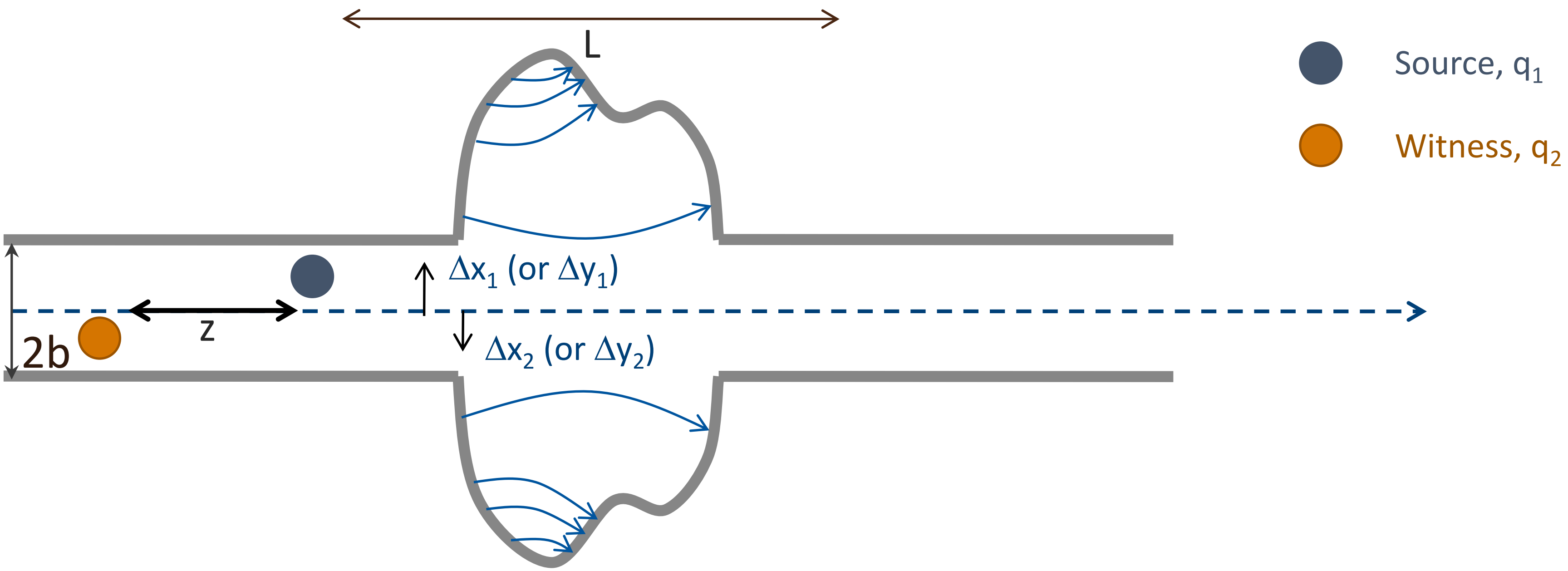
Wakefields as sources of collective effects





- The **wake function** is the **electromagnetic response** of an object to a charge pulse. It is an intrinsic property of any such object.
- The wake function **couples two charge distributions** as a function of the distance between them.
- The response depends on the boundary conditions and can occur e.g. due to **finite conductivity** (resistive wall) or more or less sudden **changes in the geometry** (e.g. resonator) of a structure.

Wake functions in general



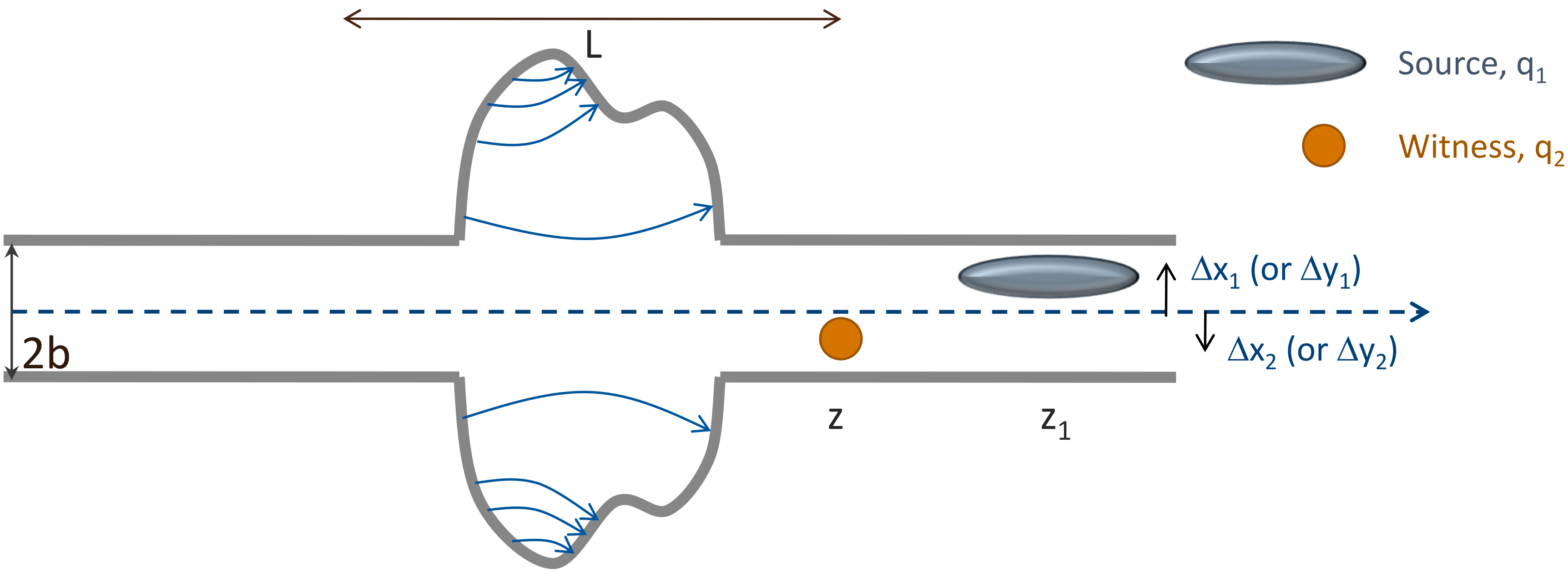
Definition as the **integrated force** associated to a change in energy:

- In general, for two point-like particles, we have

$$\Delta E_2 = \int F(x_1, x_2, z, s)\, ds = -q_1 q_2\, \boldsymbol{w(x_1, x_2, z)}$$

w is typically expanded in the transverse offsets of source and witness particles. This yields the different types of wake fields (dipole, quadrupole, coupling wakes)

Wake potential for a distribution of particles



Definition as the **integrated force** associated to a change in energy:

- For an extended particle distribution this becomes

$$\Delta E_2(z) \propto \int \boxed{\lambda_1(x_1, z_1)}\, \boldsymbol{w(x_1, x_2, z - z_1)}\, dx_1 dz_1$$

Forces become dependent on the **particle distribution function**

Wake fields – impact on the equations of motion

$$\Delta E_2(z) \propto \int \lambda_1(x_1, z_1)\, \boldsymbol{w(x_1, x_2, z - z_1)}\, dx_1 dz_1$$

- We include the impact of wake field into the standard Hamiltonian for linear betatron (or synchrotron motion):

$$H = \frac{1}{2} x' + \frac{1}{2}\left(\frac{Q_x}{R}\right)^2 x^2 + \frac{e^2}{\beta^2 EC} \int \lambda_1(x_1, z_1)\, w(x_1, x, z - z_1)\, dx_1 dz_1 dx$$

- The equations of motion become:

$$x'' + \left(\frac{Q_x}{R}\right)^2 x + \frac{e^2}{\beta^2 EC} \int \boxed{\lambda_1(x_1, z_1)\, w(x_1, x, z - z_1)}\, dx_1 dz_1 = 0$$

The presence of wake fields adds an **additional excitation** which depends on

1. The **moments of the beam distribution**
2. The **shape and the order** of the wake function

How are wakes and impedances computed?

- **Analytical or semi-analytical** approach, when geometry is simple (or simplified)
  - Solve Maxwell's equations with the correct source terms, geometries and boundary conditions up to an advanced stage (e.g. resistive wall for axisymmetric chambers)
  - Find closed expressions or execute the last steps numerically to derive wakes and impedances

- **Numerical approach**
  - Different codes have been developed over the years to solve numerically Maxwell's equations in arbitrarily complicated structures
  - Examples are CST Studio Suite (Particle Studio, Microwave Studio), ABCI, GdFidL, HFSS, ECHO2(3)D. Exhaustive list can be found from the program of the ICFA mini-Workshop on "Electromagnetic wake fields and impedances in particle accelerators", Erice, Sicily, 23-28 April, 2014

- **Bench measurements** based on transmission/reflection measurements with stretched wires
  - Seldom used independently to assess impedances, usefulness mainly lies in that they can be used for validating 3D EM models for simulations