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Collective effects Part II: Wake fields – impact on machine elements and beam dynamics

Giovanni Rumolo

Signpost

- We had a general introduction on **collective effects** and focused on **direct space charge**, its effects and possible mitigations.
- We will learn the basic **concept of wake fields** and how these can be characterized as a **collective effect** in that they depend on the particle distribution.
- We will have a basic understanding of multiparticle systems and wakefields and are now ready to look at the **impact of these** in the longitudinal and transverse planes.

Part 2: Multiparticle dynamics with wake fields – impact on machine elements and beam dynamics

- General introduction to wake fields
- Longitudinal and transverse wake functions and impedance
- Energy loss – beam induced heating and stable phase shift
- Impedance models and effects in beam dynamics, including instabilities

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Part 2: Multiparticle dynamics with wake fields – impact on machine elements and beam dynamics

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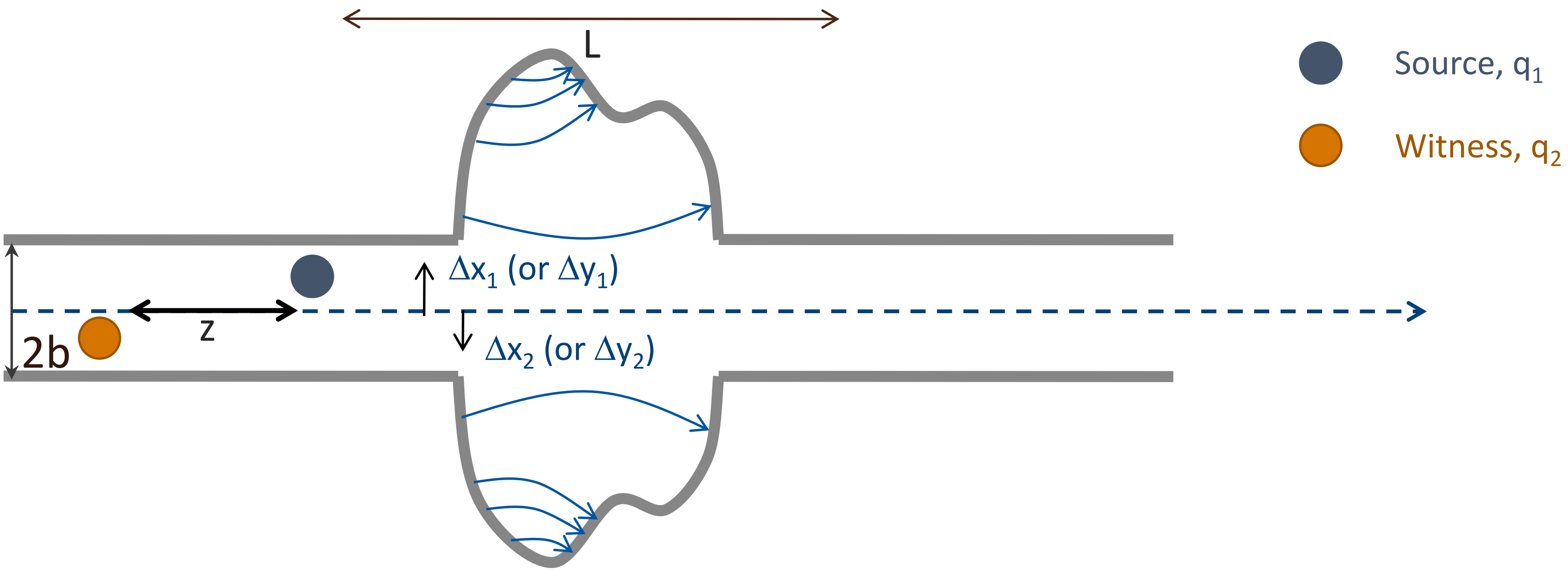
Wake functions: general definition



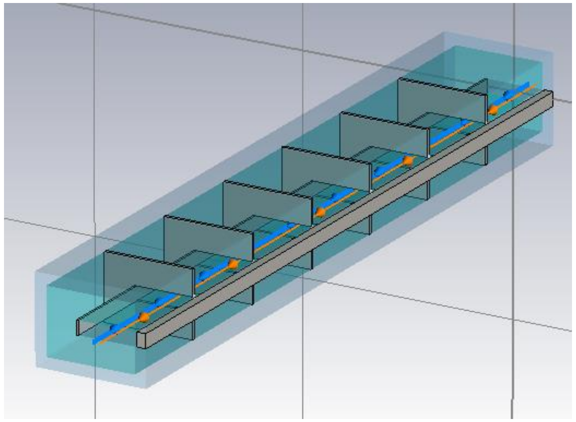
Wake function is the **integrated force** felt by a witness charge following a source charge, thus associated to an 'energy kick':

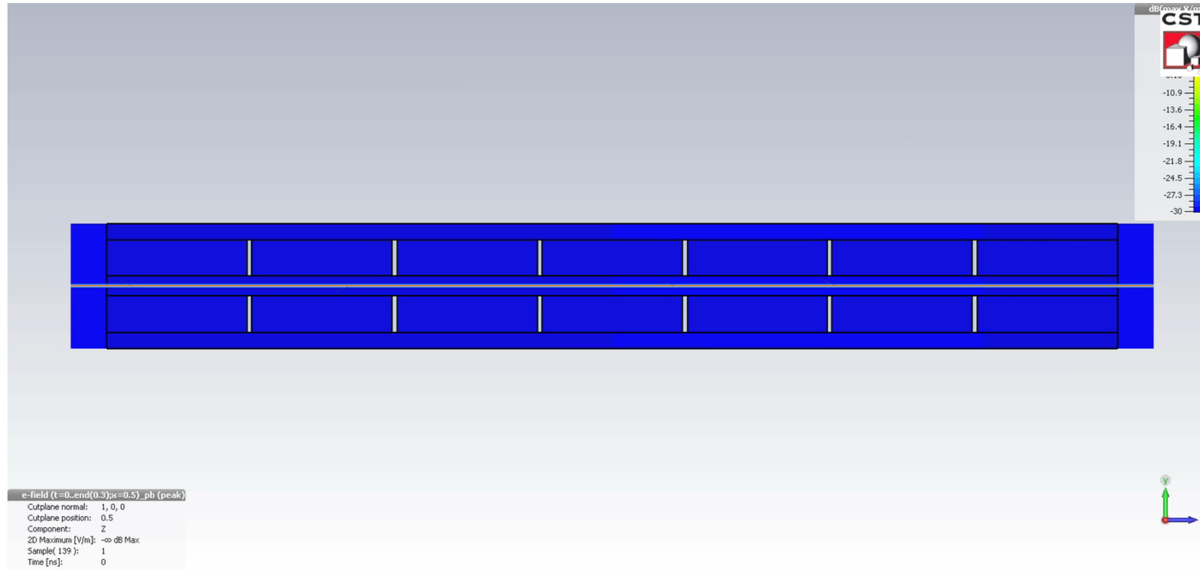
- In general, for two point-like particles, we have

$$\Delta E_2 = \int F(x_1, x_2, z, s)\, ds = -q_1 q_2\, \boldsymbol{w(x_1, x_2, z)}$$

w is typically expanded in the transverse offsets of source and witness particles. This yields the different types of wake fields (dipole, quadrupole, coupling wakes)

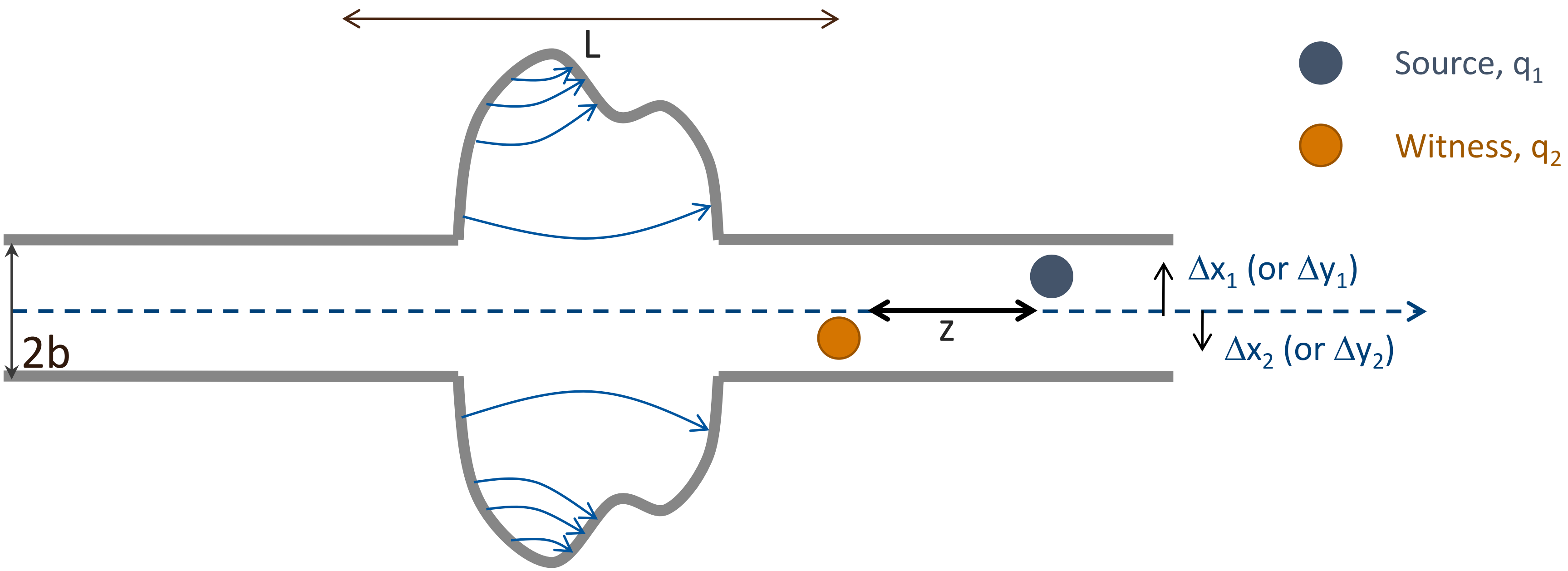
Wakefields as sources of collective effects





- The **wake function** is a type of **electromagnetic response** of a device to a charge pulse. It is an intrinsic property of this device and depends on
  - The device's **geometry** (transitions, cavities, etc.)
  - The **electromagnetic properties** of the materials exposed to the beam (e.g. PEC, finite conductivity, lossy materials, metamaterials, etc.)
- The wake function describes the **electromagnetic coupling between two point charges** as a function of the distance between them.

Longitudinal wake function



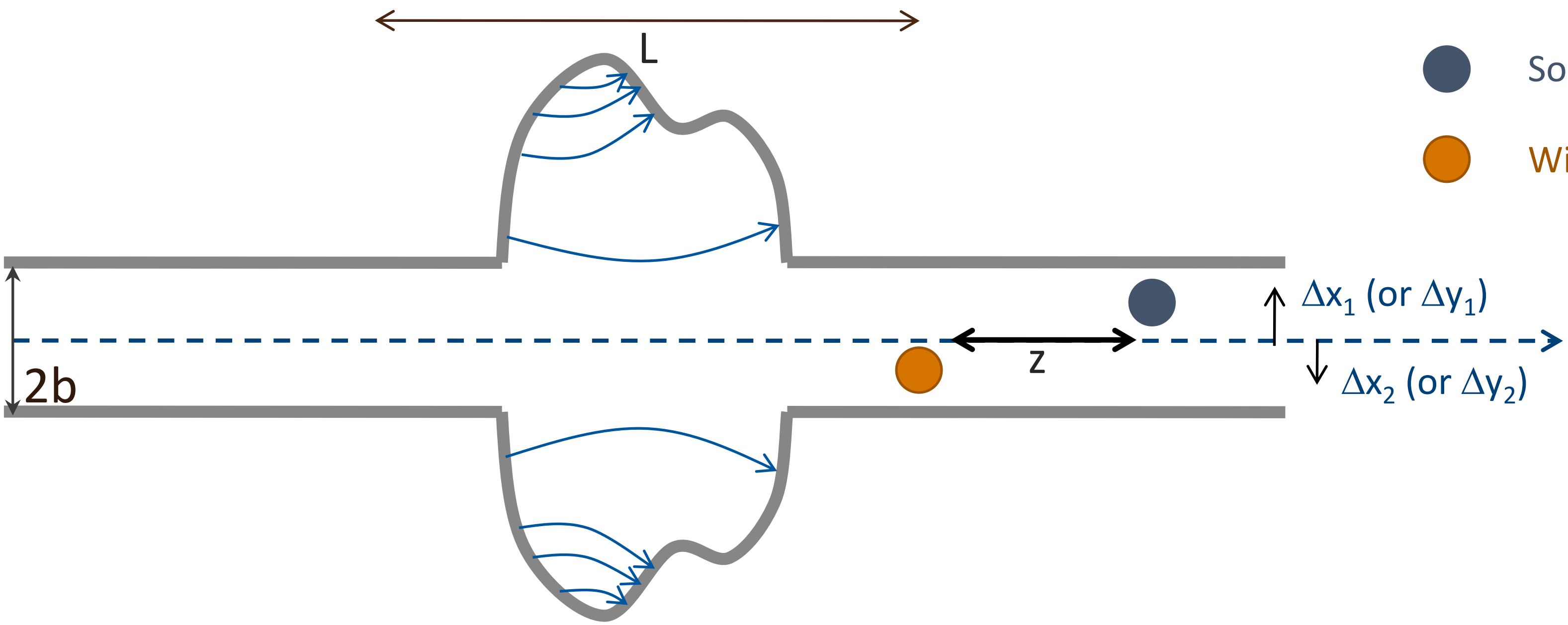
- Longitudinal wake fields

$$\int F_z(x_1, x_2, z, s)\, ds = -q_1 q_2 \left(W_{||}(z) + O(\Delta x_1) + O(\Delta x_2) \right)$$

Zeroth order with source and test centred usually dominant

Higher order terms
Usually negligible for small offsets

Longitudinal wake function



- Longitudinal wake fields

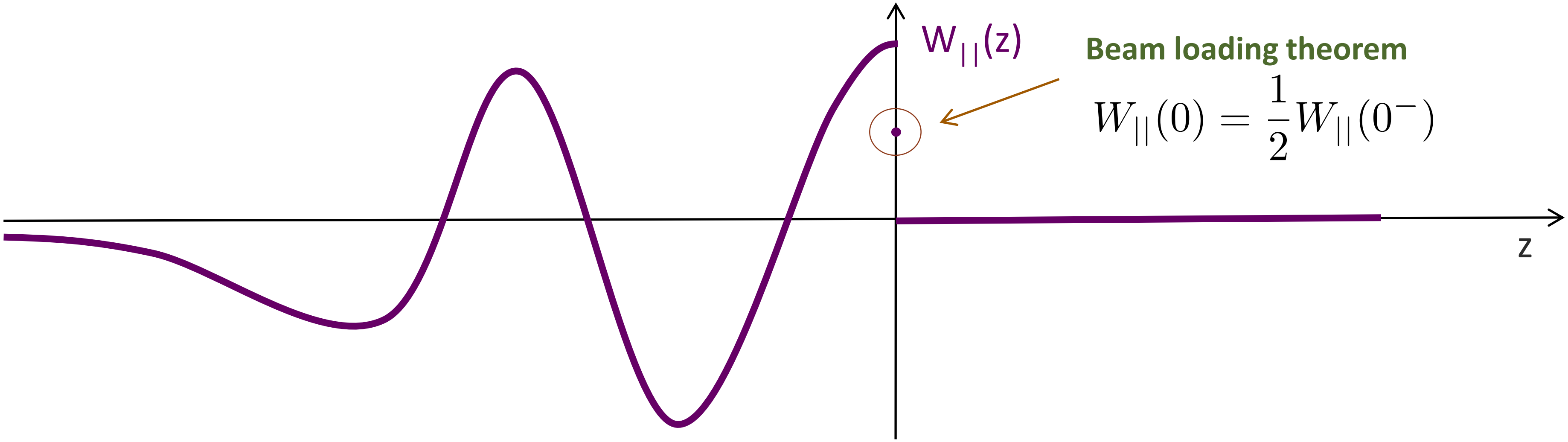
$$\Delta E_2 = \int F_z(z,s)\,ds = -q_1 q_2\,W_{||}(z)$$

Energy kick of the witness particle from longitudinal wakes

Longitudinal wake function

$$W_{||}(z) = -\frac{\Delta E_2}{q_1 q_2} \quad \xrightarrow[q_2 \to q_1]{z \to 0} \quad W_{||}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The value of the wake function in z=0 is related to the **energy lost by the source particle** in the creation of the wake
- $\boldsymbol{W_{||}(0)>0}$ since $\boldsymbol{\Delta E_1<0}$
- $\boldsymbol{W_{||}(z)}$ is discontinuous in z=0 and it vanishes for all z>0 because of the ultra-relativistic approximation



Longitudinal impedance

$$W_{||}(z) = -\frac{\Delta E_2}{q_1 q_2} \quad \xrightarrow[q_2 \to q_1]{z \to 0} \quad W_{||}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The **wake function** of an accelerator component is basically its **Green function in time domain** (i.e., its response to a pulse excitation)
  → Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!
- We can also describe it as a **transfer function in frequency domain**

→ This is the definition of **longitudinal beam coupling impedance** of the element under study

$$\boxed{Z_{||}(\omega)} = \int_{-\infty}^{\infty} \boxed{W_{||}(z)} \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$

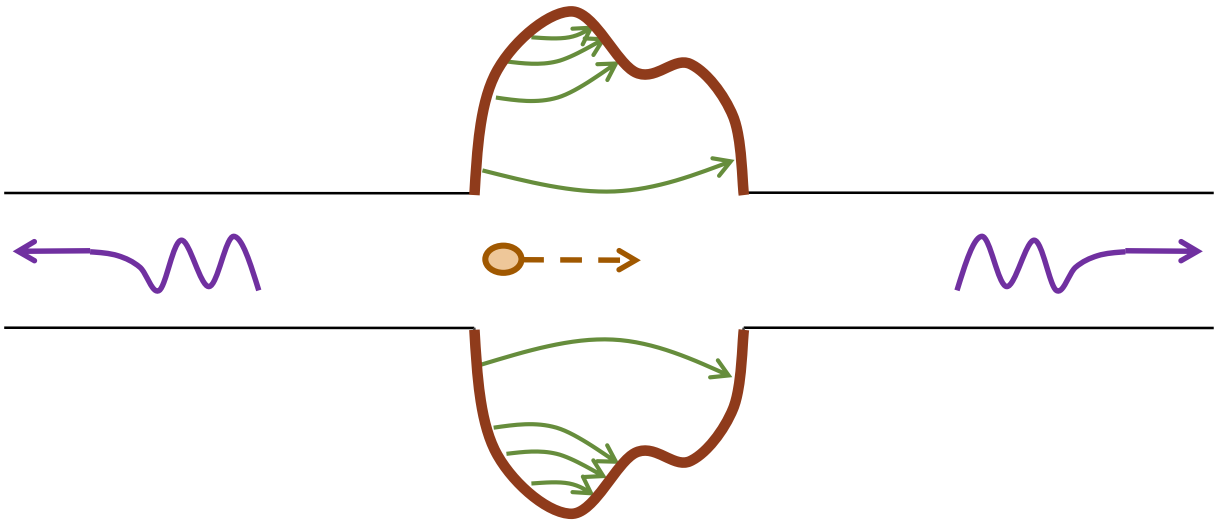
$[\Omega]$ $\qquad$ $[\Omega/s]$

The energy balance

$$W_{||}(0) = \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left(Z_{||}(\omega)\right)\, d\omega = -\frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

- In the global energy balance, the energy lost by the source splits into:
  - Electromagnetic energy of the **modes that remain trapped** in the object
    - → Partly dissipated on **lossy walls** or into purposely designed inserts or HOM absorbers
    - → Partly transferred to **following particles** (or the same particle over successive turns), possibly feeding into an instability!
  - Electromagnetic energy of **modes that propagate** down the beam chamber (above cut-off), eventually lost on surrounding lossy materials



The energy balance

$$W_{\parallel}(0) = \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left(Z_{\parallel}(\omega)\right)\, d\omega = -\frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

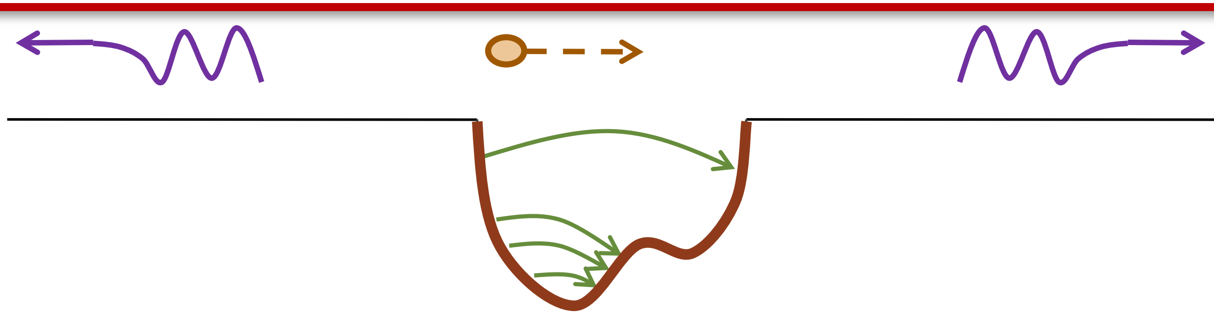
- In the global energy balance, the energy lost by the source splits into
  - ... ct ... absorbers ... ve turns),
  - ... chamber

The energy loss of a particle bunch

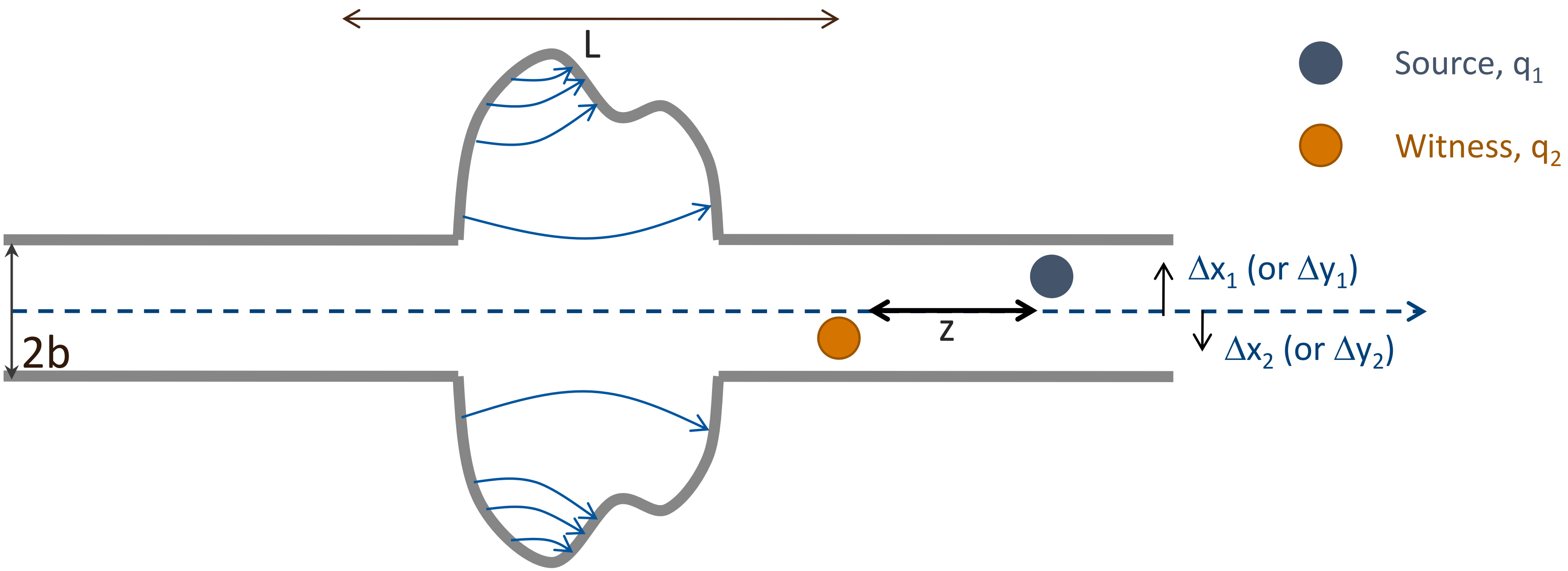
⇒ causes **beam induced heating** of the machine elements (damage, outgassing) or **sparking** due to high field

⇒ feeds into both **longitudinal and transverse instabilities** through the associated EM fields

⇒ is compensated by the RF system determining a **stable phase shift**



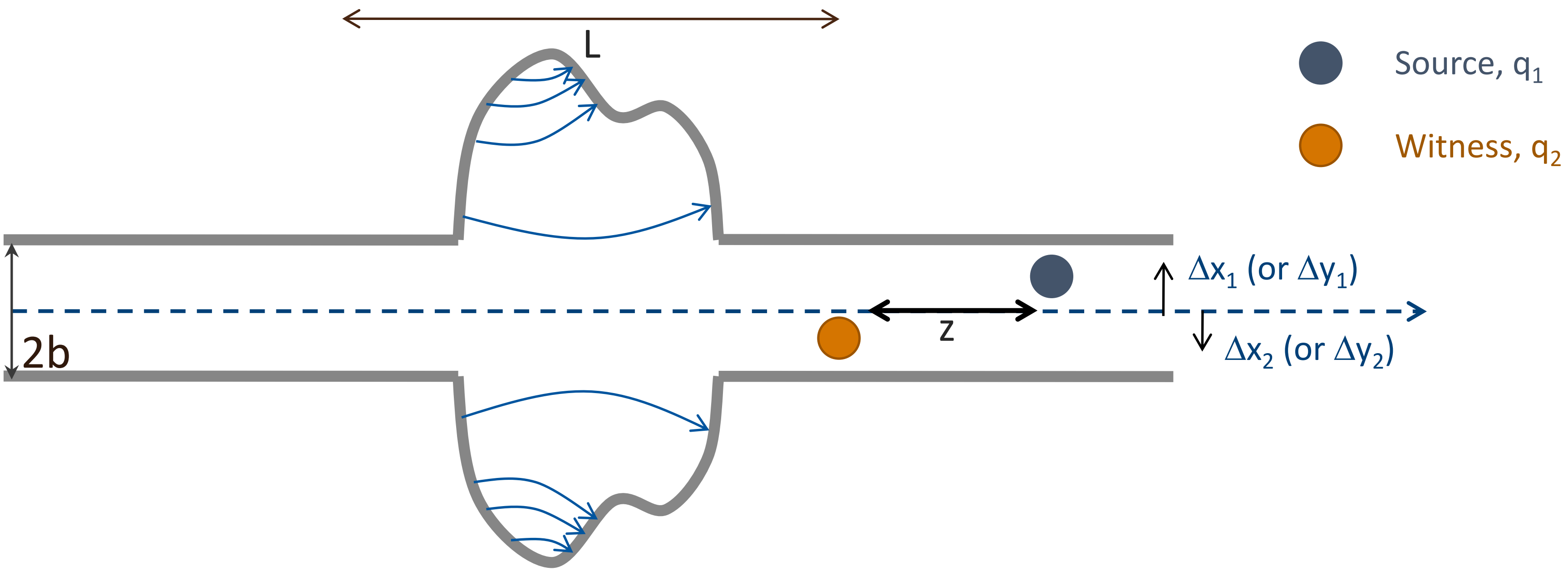
Transverse wake functions



- Transverse wake fields

$$\beta c\,\Delta p_{x2} = \int F_x(x_1, x_2, z, s)\, ds$$

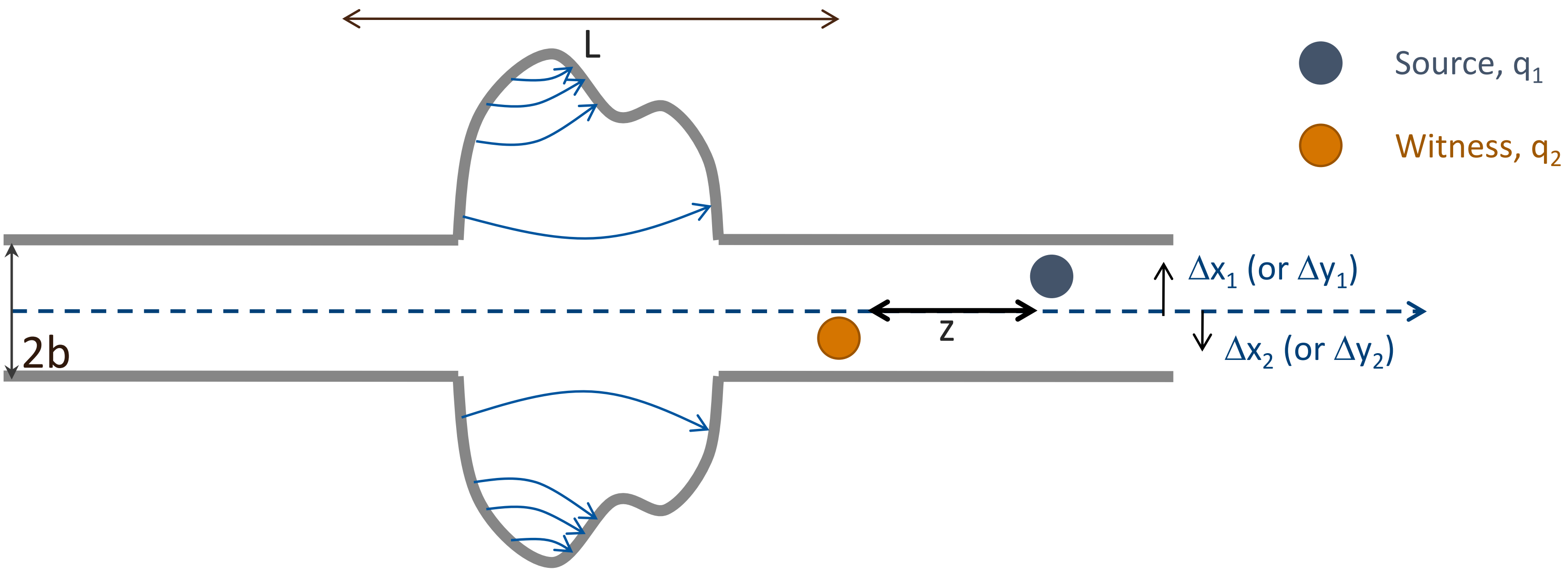
Transverse wake functions



- Transverse wake fields

$$\beta c\,\Delta p_{x2} = \int F_x(x_1, x_2, z, s)\,ds = -q_1 q_2 \left(W_{C_x}(z) + W_{Dx}(z)\,\Delta x_1 + W_{Q_x}(z)\,\Delta x_2\right)$$

Transverse wake functions



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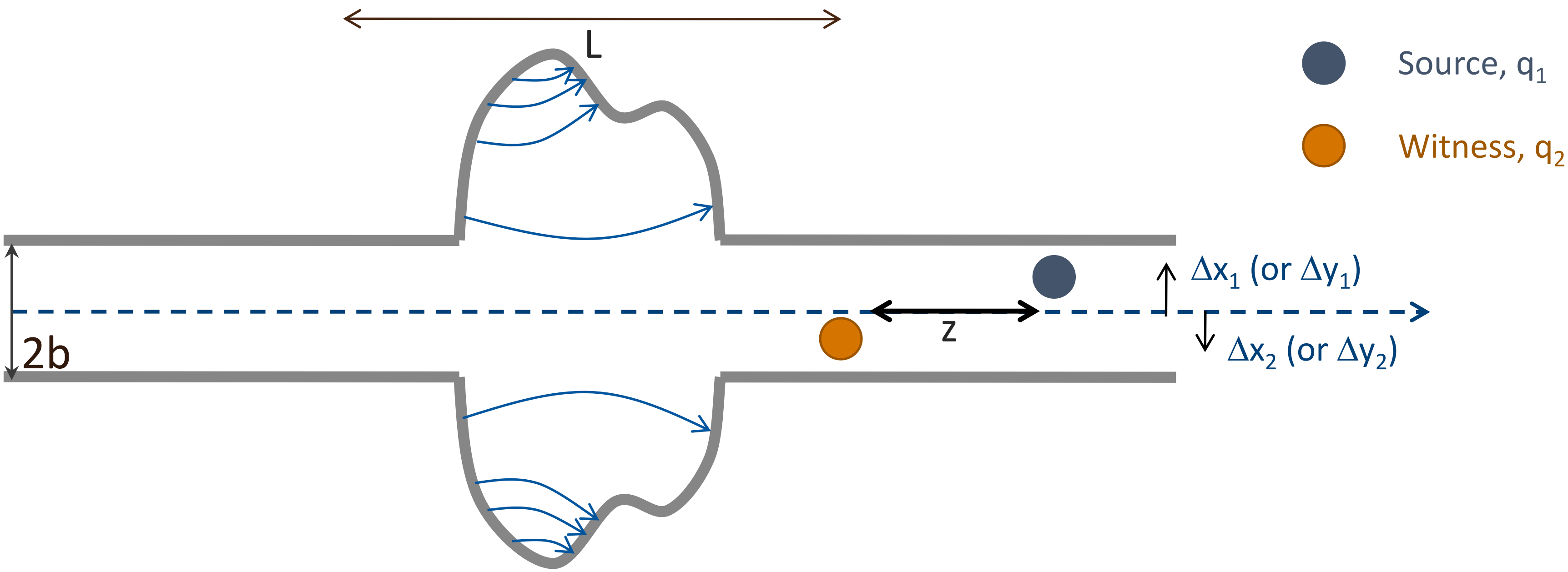
$$\longrightarrow \frac{\Delta p_{x2}}{p_0} = \Delta x_2'$$

Transverse deflecting kick of the witness particle from transverse wakes

Transverse wake functions



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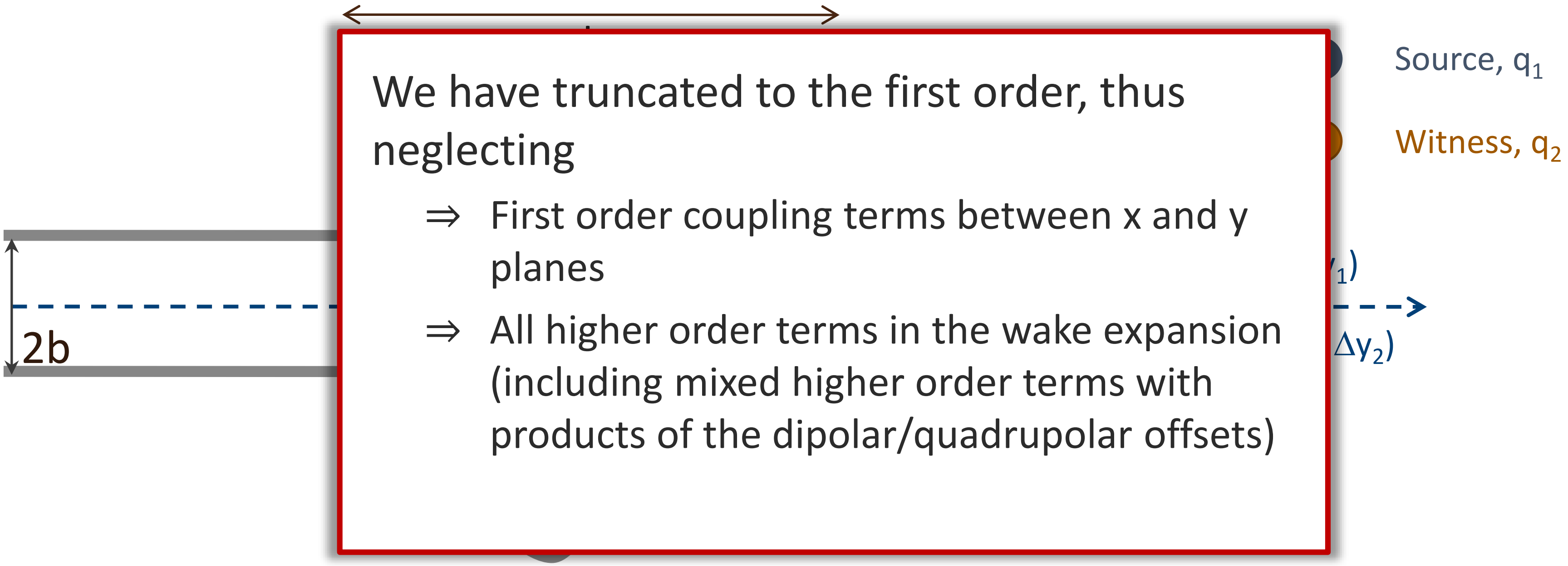
Zeroth order for asymmetric structures → Orbit offset

Dipole wakes – depends on **source particle**

Quadrupole wakes – depends on **witness particle**

Transverse wake functions



We have truncated to the first order, thus neglecting

- ⇒ First order coupling terms between x and y planes
- ⇒ All higher order terms in the wake expansion (including mixed higher order terms with products of the dipolar/quadrupolar offsets)

- Transverse wake fields

$$\beta c\,\Delta p_{x2} = \int F_x(x_1, x_2, z, s)\, ds = -q_1 q_2 \left(W_{C_x}(z) + W_{Dx}(z)\Delta x_1 + W_{Q_x}(z)\Delta x_2\right)$$

Zeroth order for asymmetric structures → Orbit offset

Dipole wakes – depends on **source particle**

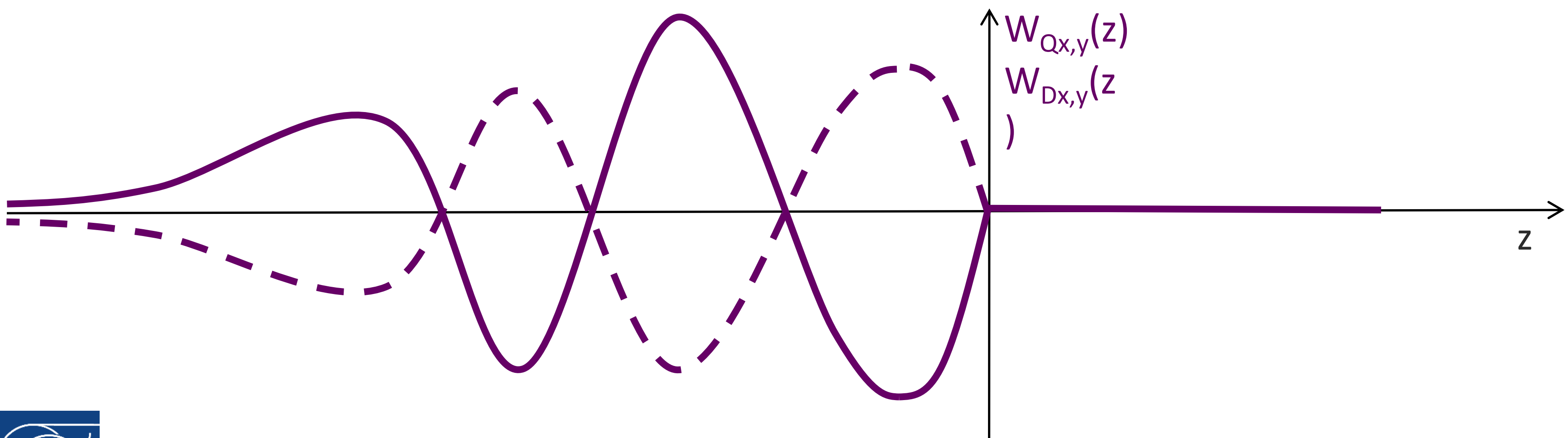
Quadrupole wakes – depends on **witness particle**

Transverse wake functions (detuning)

$$W_{D_x}(z) = -\frac{\beta^2 E_0}{q_1 q_2} \frac{\Delta x_2'}{\Delta x_1} \quad \xrightarrow{z\to 0} \quad W_{D_x=0}(0) = 0$$

$$W_{Q_x}(z) = -\frac{\beta^2 E_0}{q_1 q_2} \frac{\Delta x_2'}{\Delta x_2} \quad \xrightarrow{z\to 0} \quad W_{Q_x=0}(0) = 0$$

- The transverse wake functions (dip and quad) **vanish in z=0** because source and witness particles are traveling parallel and they can only – mutually – interact through space charge, which is not included in this framework



Transverse impedance

$$W_{D_x}(z) = -\frac{\beta^2 E_0}{q_1 q_2}\frac{\Delta x_2'}{\Delta x_1} \qquad W_{Q_x}(z) = -\frac{\beta^2 E_0}{q_1 q_2}\frac{\Delta x_2'}{\Delta x_2}$$

- The **wake function** of an accelerator component is basically its **Green function in time domain** (i.e., its response to a pulse excitation)
  → Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!
- We can also describe it as a **transfer function in frequency domain**
  → This is the definition of **transverse beam coupling impedance** of the element under study

Dipolar (or driving)

$$Z_{D_x}(\omega) = i\int_{-\infty}^{\infty} W_{D_x}(z)\exp\left(-\frac{i\omega z}{c}\right)\frac{dz}{c}$$

Quadrupolar (or detuning)

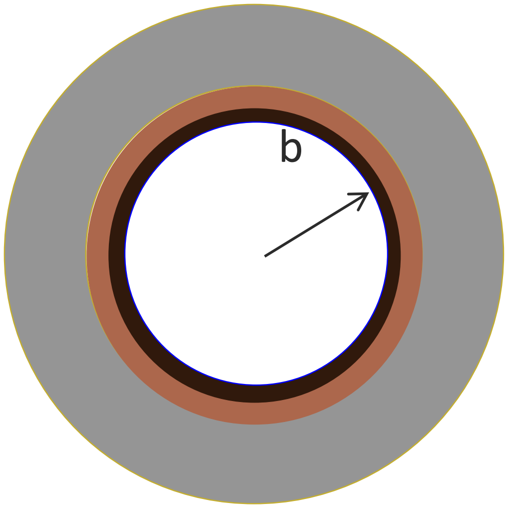
$$Z_{Q_x}(\omega) = i\int_{-\infty}^{\infty} W_{Q_x}(z)\exp\left(-\frac{i\omega z}{c}\right)\frac{dz}{c}$$

$[\Omega/\mathrm{m}]$

How are wakes and impedances computed?

- **Analytical or semi-analytical** approach, when geometry is simple (or simplified)
  - Solve Maxwell's equations with the correct source terms, geometries and boundary conditions up to an advanced stage
  - Find closed expressions or execute the last steps numerically to derive wakes and impedances

→ An example: **axisymmetric beam chamber** with several layers with different EM properties



$$\nabla \times \vec{E} = -i\omega \vec{B} \qquad \nabla \cdot \vec{E} = \frac{\tilde{\rho}}{\epsilon_0 \epsilon_1(\omega)}$$

$$\nabla \times \vec{B} = \mu_0 \mu_1(\omega) \vec{J} + i\omega \frac{\mu_1(\omega)\epsilon_1(\omega)}{c^2} \vec{E}$$

$$\nabla \cdot \vec{B} = 0$$

+ Boundary conditions

$$\tilde{\rho}(r, \theta, s, \omega) = \frac{q_1}{r_1 v} \delta(r - r_1) \delta_P(\theta) \exp\left(-\frac{i\omega s}{v}\right)$$

$$\vec{J}(r, \theta, s, \omega) = \tilde{\rho}(r, \theta, s, \omega) \vec{v}$$

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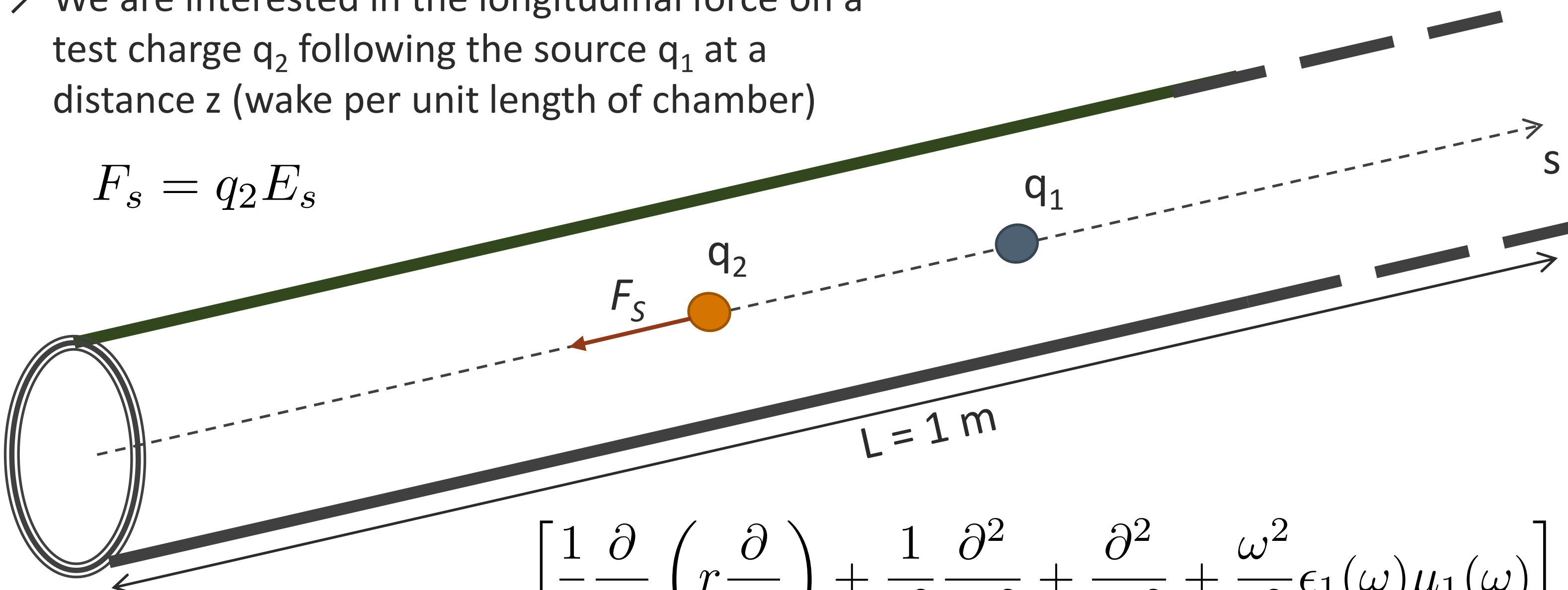
→ We are interested in the longitudinal force on a test charge q_2 following the source q_1 at a distance z (wake per unit length of chamber)

$$F_s = q_2 E_s$$



$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial s^2} + \frac{\omega^2}{c^2}\epsilon_1(\omega)\mu_1(\omega)\right] E_s =$$

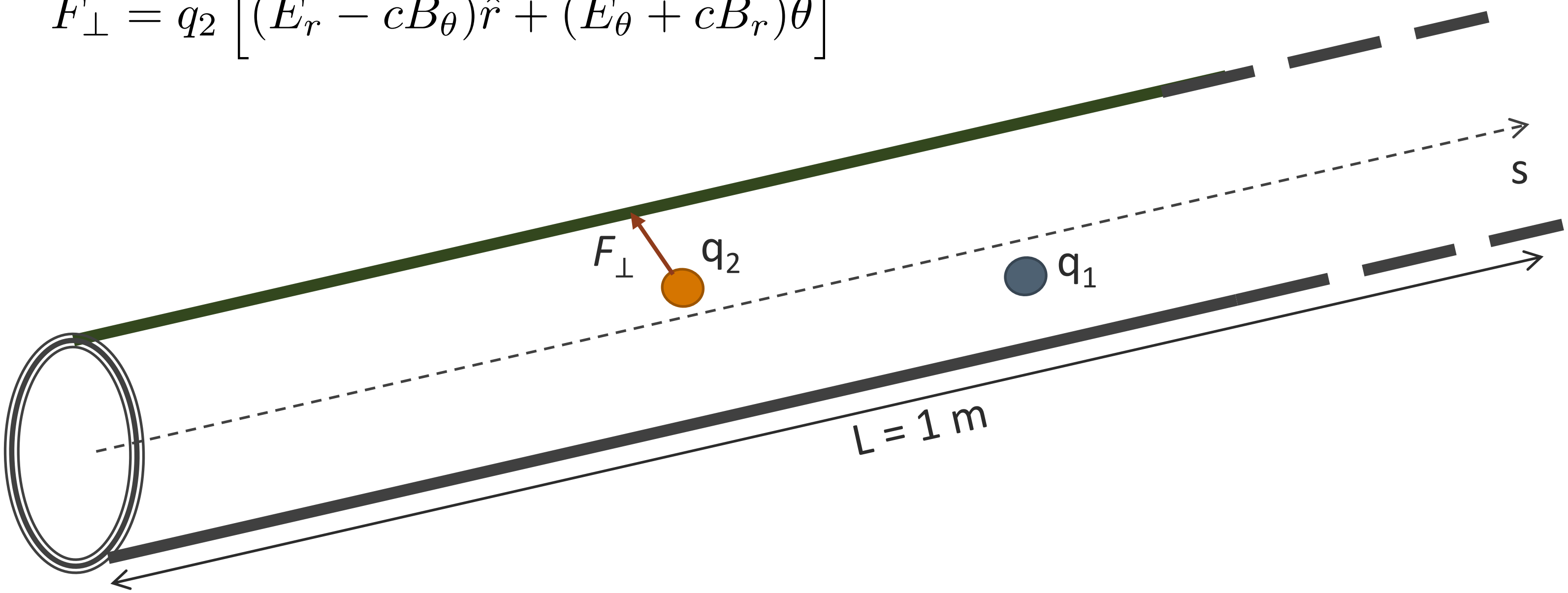
$$= \frac{1}{\epsilon_0\epsilon_1(\omega)}\frac{\partial\tilde{\rho}}{\partial s} + i\omega\mu_0\mu_1(\omega)\tilde{\rho}v$$

Examples of transverse wakes/impedances

- **Analytical or semi-analytical** approach, when geometry is simple (or simplified)
  - Same as in the longitudinal plane in terms of approach
  - But we have to calculate the transverse force from an (offset) source to an (offset) witness

→ We are interested in the transverse force on a test charge q_2 following the source q_1 at a distance z (wake per unit length of chamber)

$$F_{\perp} = q_2 \left[(E_r - cB_\theta)\hat{r} + (E_\theta + cB_r)\hat{\theta} \right]$$



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$$F_\perp = q_2 \left[\underbrace{(E_r - cB_\theta)}\hat{r} + \underbrace{(E_\theta + cB_r)}\hat{\theta} \right]$$

$$F_r = \frac{iq_2 v}{\omega} \frac{\partial E_s}{\partial r} \quad F_\theta = \frac{iq_2 v}{\omega r} \frac{\partial E_s}{\partial \theta}$$

Same as for the longitudinal plane

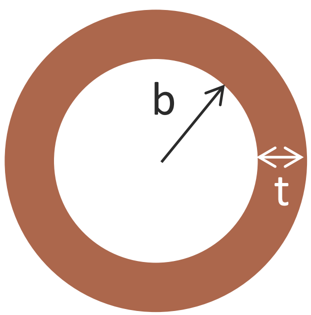
$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial s^2} + \frac{\omega^2}{c^2} \epsilon_1(\omega) \mu_1(\omega) \right] E_s =$$

$$= \frac{1}{\epsilon_0 \epsilon_1(\omega)} \frac{\partial \tilde{\rho}}{\partial s} + i\omega\mu_0\mu_1(\omega)\tilde{\rho}v$$

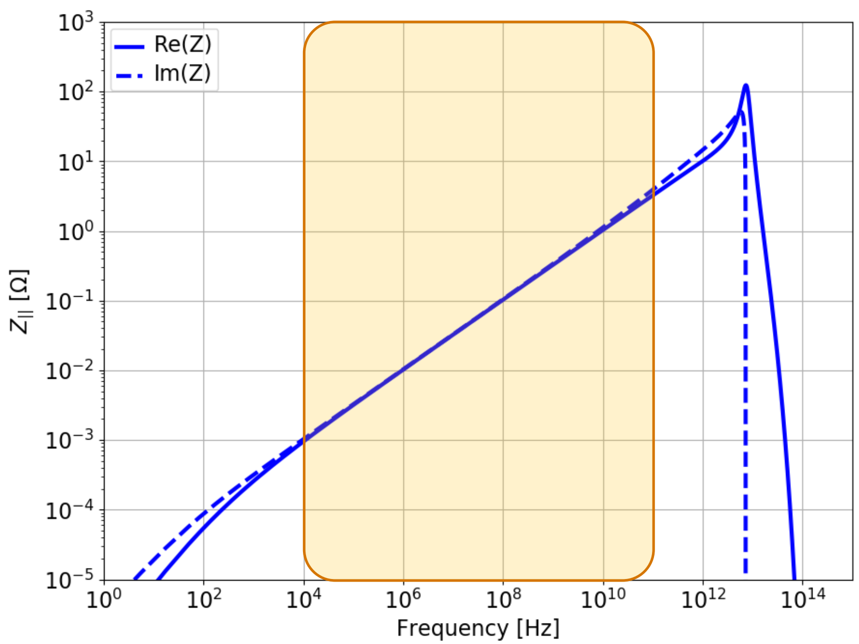
How are wakes and impedances computed?

- **Analytical or semi-analytical** approach, when geometry is simple (or simplified)
  - Solve Maxwell's equations with the correct source terms, geometries and boundary conditions up to an advanced stage
  - Find closed expressions or execute the last steps numerically to derive wakes and impedances

→ An example: a 1 m long Cu pipe with radius b=2 cm and thickness t = 4 mm in vacuum

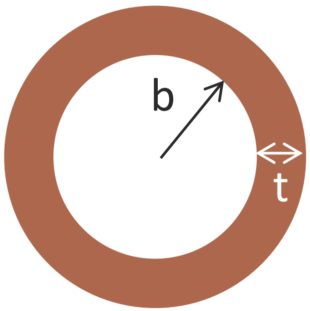


- Highlighted region shows the typical $\omega^{1/2}$ scaling
- Scaling is with respect to b:
  - Longitudinal impedance $\sim b^{-1}$

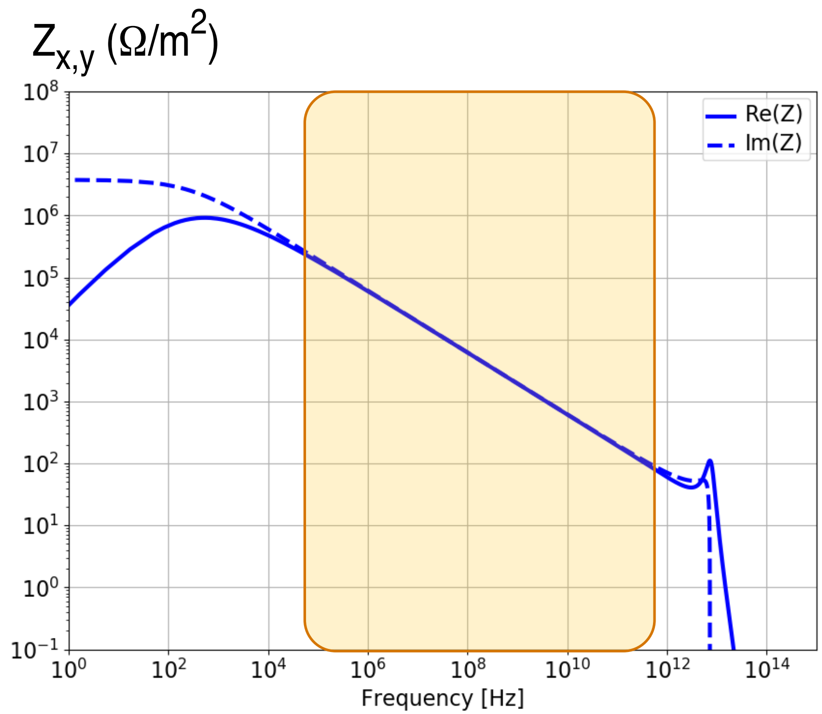


Examples of transverse wakes/impedances

- **Analytical or semi-analytical** approach, when geometry is simple (or simplified)
  - Same as in the longitudinal plane in terms of approach
  - But we have to calculate the transverse force from an (offset) source to an (offset) witness
  - We just need E_s also to characterize the transverse wake function

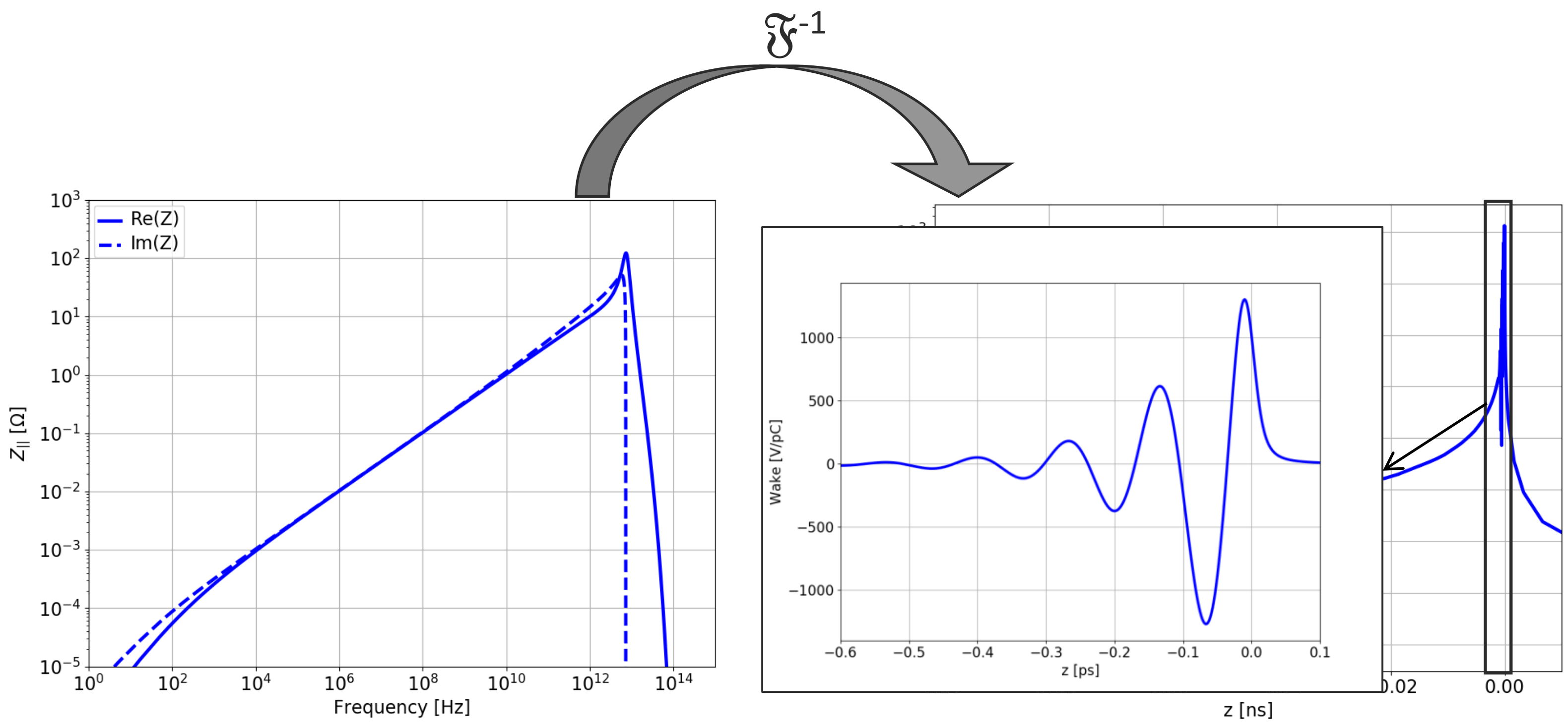


- Highlighted region shows the typical $\omega^{-1/2}$ scaling
- Scaling with respect to b:
  - Transverse impedance $\sim b^{-3}$



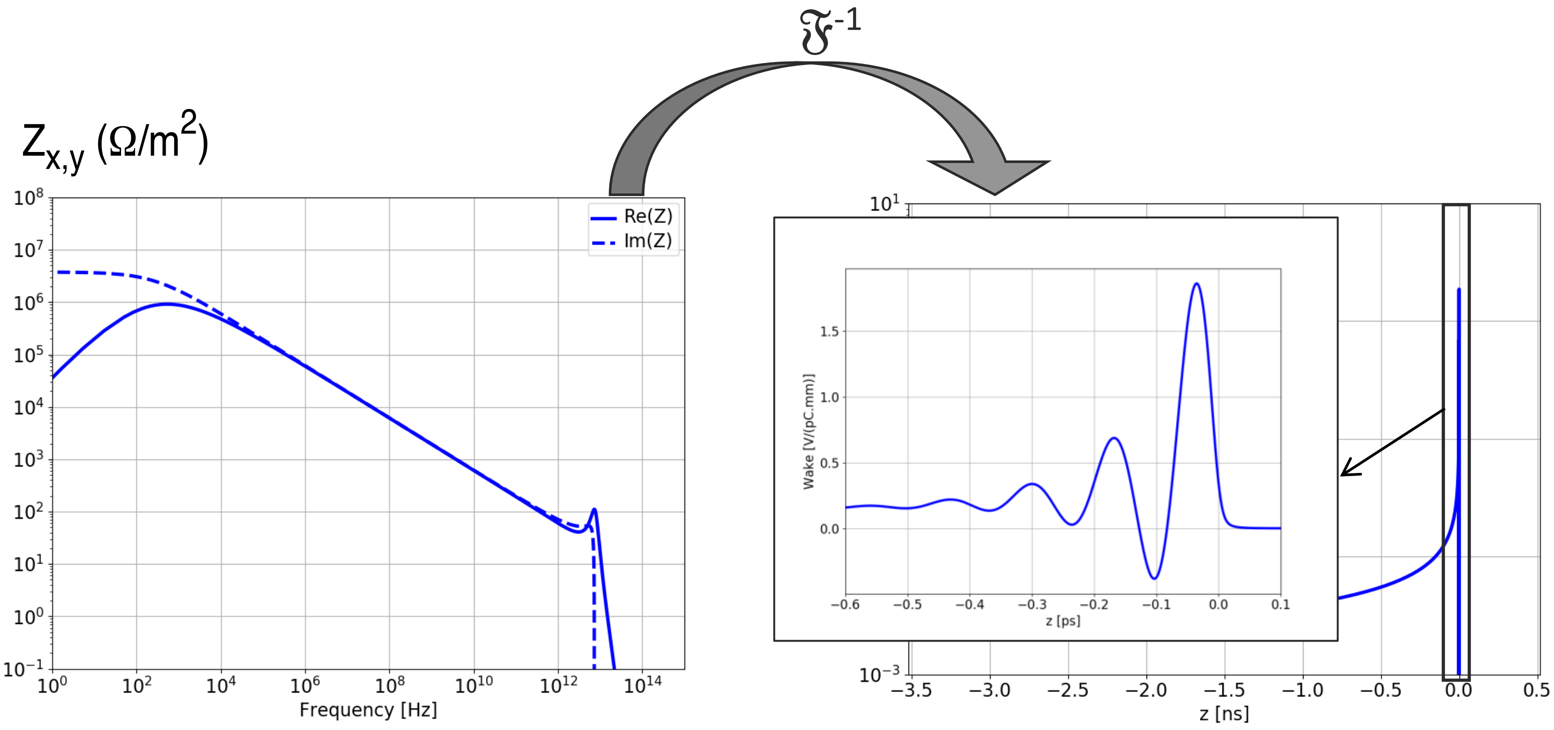
How are wakes and impedances computed?

- **From impedance to wakes – longitudinal**



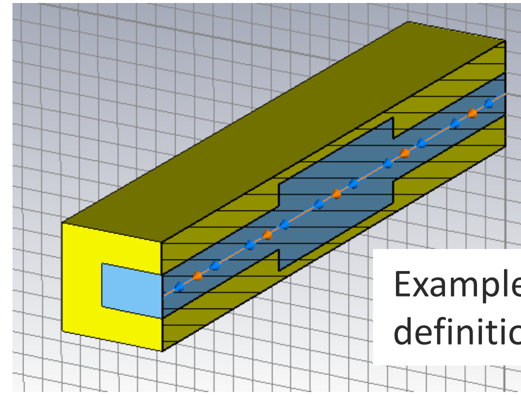
How are wakes and impedances computed?

- **From impedance to wakes – transverse**

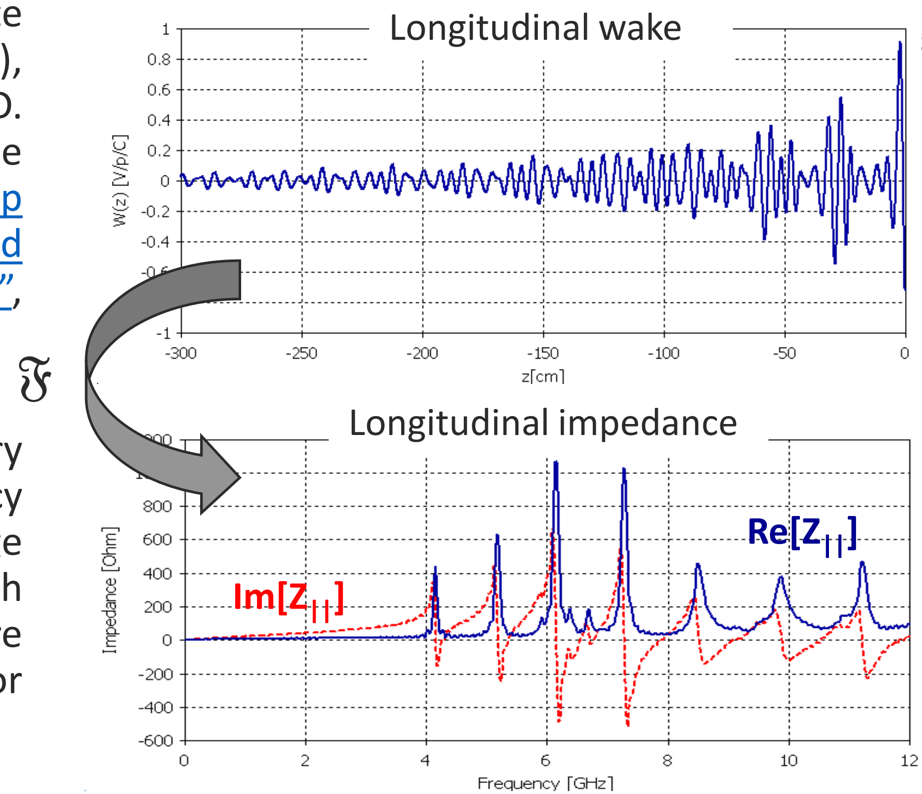


How are wakes and impedances computed?

- **Numerical approach**
  - Different codes have been developed over the years to solve numerically Maxwell's equations in arbitrarily complicated structures
  - Examples are CST Studio Suite (Particle Studio, Microwave Studio), ABCI, GdFidL, HFSS, ECHO2(3)D. Exhaustive list can be found from the program of the [ICFA mini-Workshop on "Electromagnetic wake fields and impedances in particle accelerators"](), Erice, Sicily, 23-28 April, 2014
  - Computations can become very **challenging** if high frequency resolution (long wake) or knowledge of impedance spectrum at high frequency (short excitation) are required, especially for large/complicated geometries

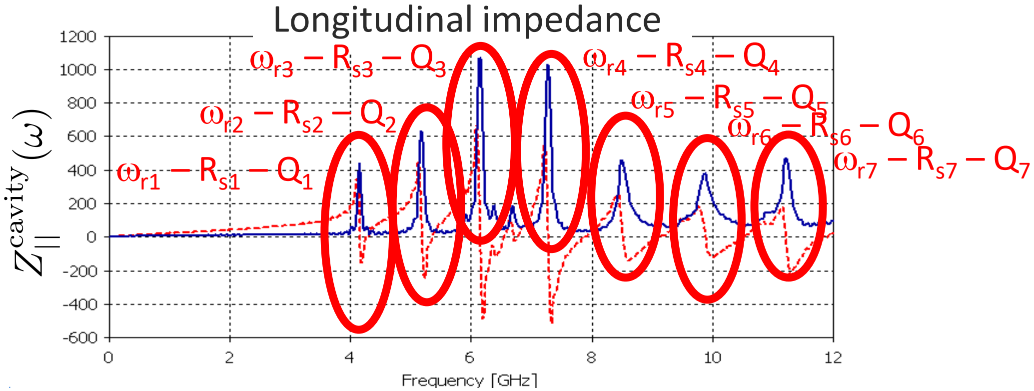


Example of a cavity definition in CST



How are wakes and impedances computed?

- **Numerical approach**
  - To limit numerical noise, in cases with many resonances, the resonances are first characterized through their frequencies (ω_{ri}), shunt impedances (R_{si}) and quality factors (Q_i)



  - Then analytical formulae for resonators are used in computations

$$Z_{||}^{\text{Res}}(\omega) = \frac{R_{s||}}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

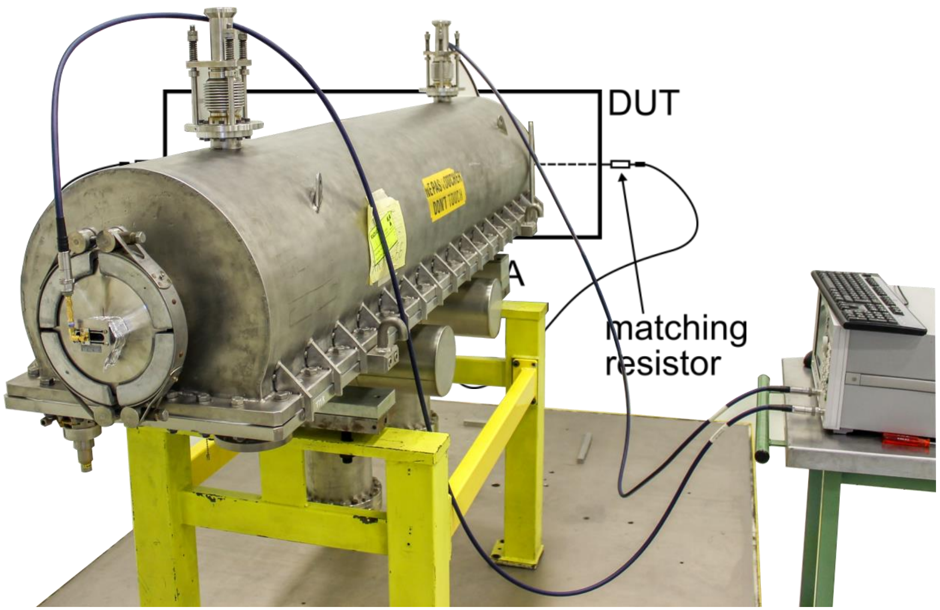
$$W_{||}^{\text{Res}}(z) = \begin{cases} 2\alpha_z R_{s||} \exp\left(\frac{\alpha_z z}{c}\right)\left[\cos\left(\frac{\bar{\omega} z}{c}\right) + \frac{\alpha_z}{\bar{\omega}} \sin\left(\frac{\bar{\omega} z}{c}\right)\right] & \text{if } \ z < 0 \\ \alpha_z R_{s||} \ \text{ if } \ z = 0 \\ 0 \ \text{ if } \ z < 0 \end{cases}$$

$$\alpha_z = \frac{\omega_r}{2Q} \qquad \bar{\omega} = \sqrt{\omega_r^2 - \alpha_z^2}$$

$$Z_{||}^{\text{cavity}}(\omega) = \sum Z_{||i}^{\text{Res}}(\omega)$$

How are wakes and impedances computed?

- **Bench measurements** based on transmission/reflection measurements with stretched wires
  - Seldom used independently to assess impedances due to the perturbation introduced by the measurement set up (flanging, presence of wire)
  - Usefulness mainly lies in that they can be used for validating 3D EM models for simulations



- A **wire** is stretched in the middle of the device to simulate the beam
- **Reflection and transmission coefficients** are measured via a VNA
- The impedance can be calculated by plugging the measured scattering parameters into the **LOG formula**

$$Z_{||} = 2Z_L \ln(S_{21})$$

Signpost

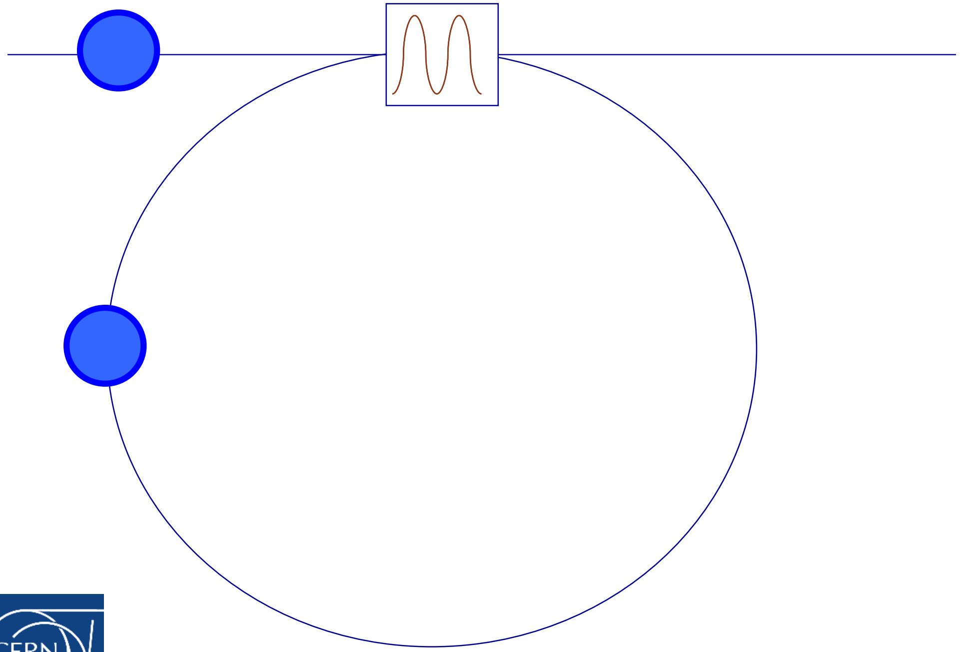
- We have learnt what are wake functions and impedances in both **longitudinal and transverse planes**.
- We have shown how wake functions and impedances **can be computed and given some examples of the different methods**

Part 2: Multiparticle dynamics with wake fields – impact on machine elements and beam dynamics

- General introduction to wake fields
- Longitudinal and transverse wake functions and impedance
- Energy loss – beam induced heating and stable phase shift
- Impedance models and effects in beam dynamics, including instabilities

Bunch energy loss per turn

- Single traversal of a bunch through an impedance source
  - We assume a single bunch of particles that goes only once through a known (characterized) wake/impedance source, representing both
    - Single passage (e.g. in a line)
    - Energy loss per turn if the bunch passes every turn but the wake fully decays between subsequent turns
  - Our goal is to calculate how much energy the bunch loses in this passage due to the electromagnetic interaction



Bunch energy loss per turn

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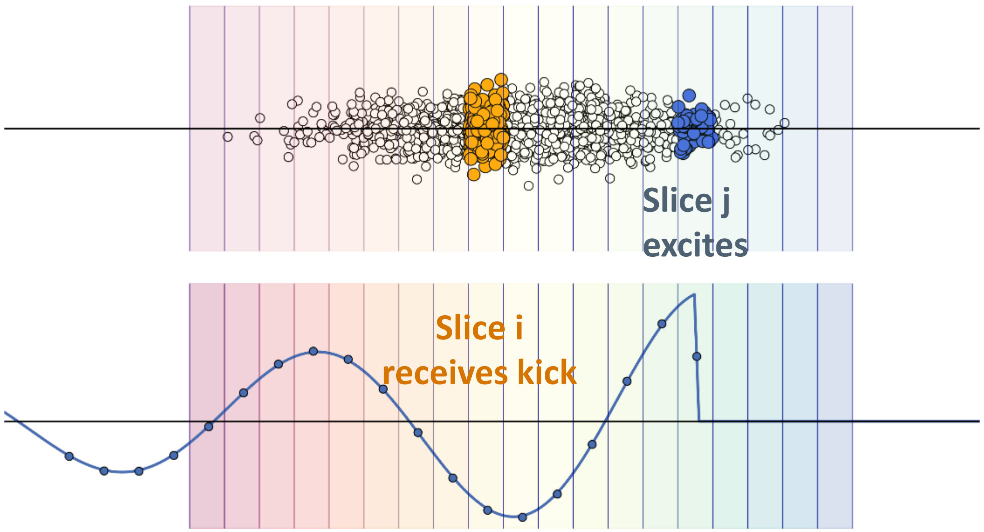


$$\Delta E_{ij} = -e^2 W_{||}(z_{ij})$$

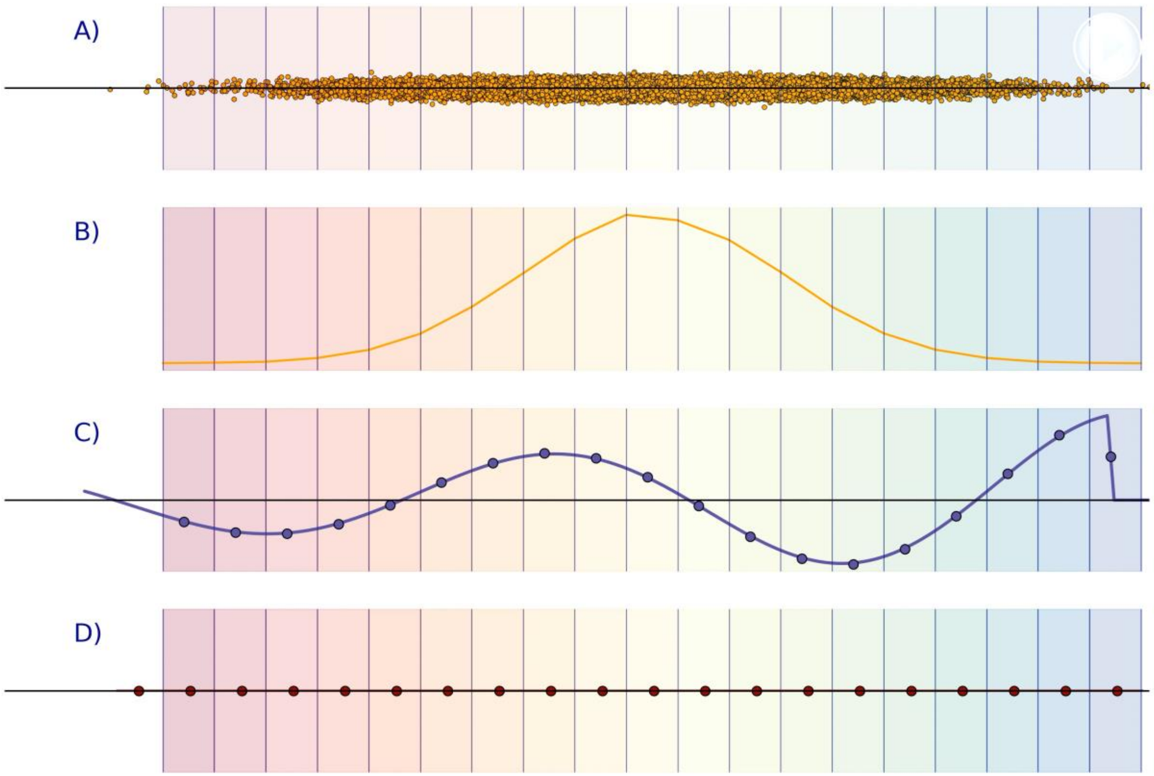
$$\Delta E_{bunch} = -e^2 \sum_{j=1}^{N_b} \sum_{i=1}^{N_b} W_{||}(z_{ij})$$

$$\Delta E_{ij} = -e^2 N[j] N[i] W_{||}[(i-j)\Delta z]$$

$$\Delta E_{i} = -e^2 N[i] \sum_{j=0}^{i} N[j] W_{||}[(i-j)\Delta z]$$

Bunch energy loss per turn

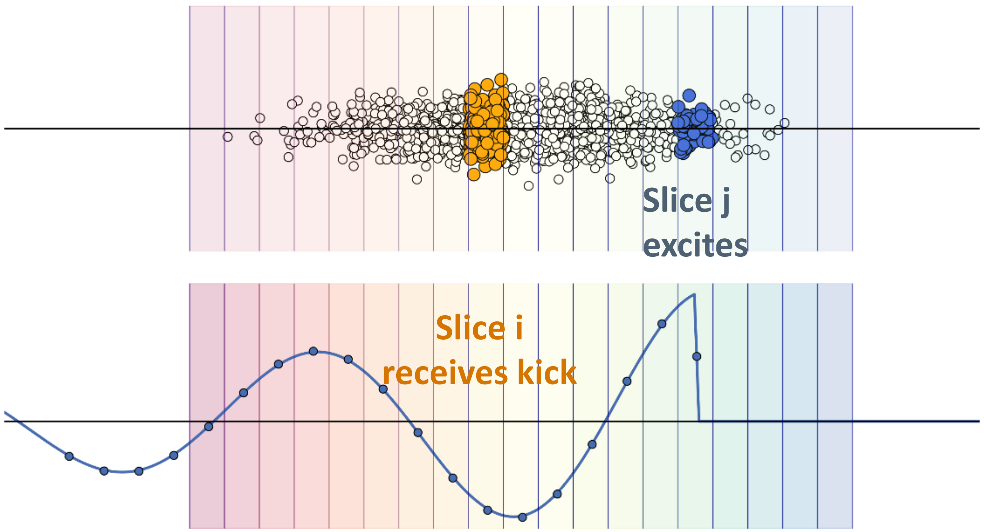
- Single traversal of a bunch through an impedance source



$$\Delta E_i = -e^2 N[i] \sum_{j=0}^{i} N[j] W_{||}[(i-j)\Delta z]$$

Bunch energy loss per turn

- Single traversal of a bunch through an impedance source



$$\Delta E_{ij} = -e^2 W_{||}(z_{ij})$$

$$\Delta E_{bunch} = -e^2 \sum_{j=1}^{N_b} \sum_{i=1}^{N_b} W_{||}(z_{ij})$$

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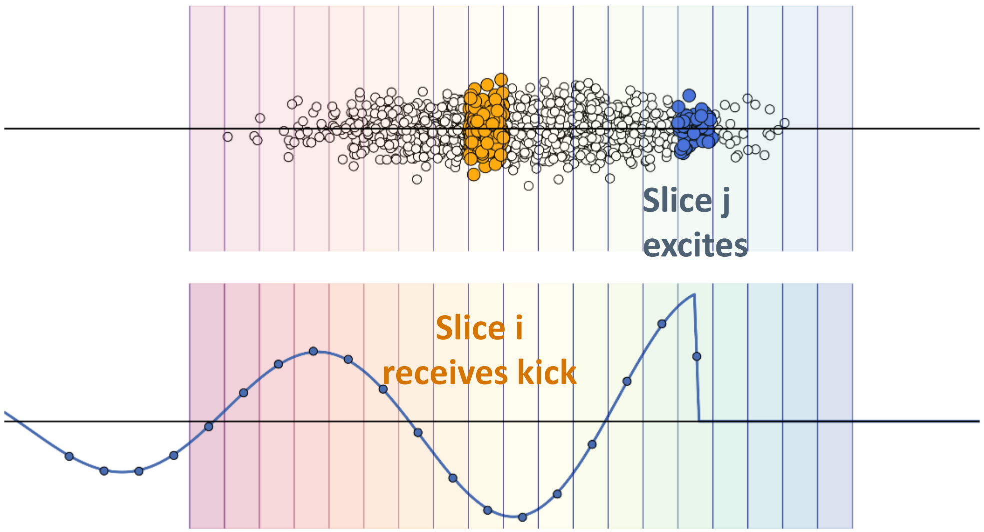
$$\Delta E_{bunch} = -e^2 \sum_{i=0}^{N_{\text{slices}}} N[i] \sum_{j=0}^{i} N[j] W_{||}[(i-j)\Delta z]$$

Bunch energy loss per turn

- Single traversal of a bunch through an impedance source



$\Delta z \to 0$ $\quad N[i] \to \lambda(z)dz$ $\quad N[j] \to \lambda(z')dz'$

$$\Delta E_{ij} = -e^2 W_{||}(z_{ij})$$

$$\Delta E_{bunch} = -e^2 \sum_{j=1}^{N_b} \sum_{i=1}^{N_b} W_{||}(z_{ij})$$

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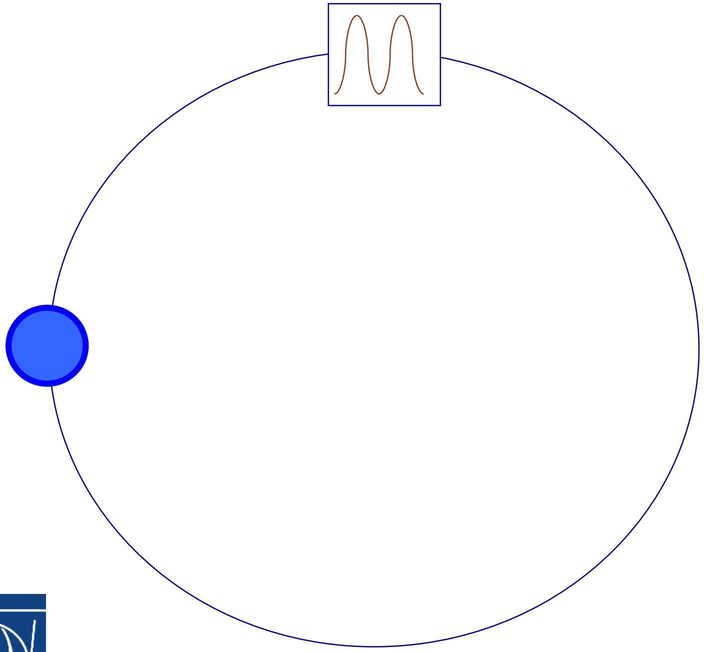
$$\Delta E_i = -e^2 N[i] \sum_{j=0}^{i} N[j] W_{||}[(i-j)\Delta z]$$

$$\Delta E_{bunch} = -e^2 \int \lambda(z)dz \int \lambda(z') W_{||}(z-z')dz'$$

$$\Delta E_{bunch} = -\frac{e^2}{2\pi} \int \left|\hat{\lambda}(\omega)\right|^2 \mathrm{Re}\left[Z_{||}(\omega)\right]$$

Bunch energy loss per turn

- Multiple traversal of a bunch through an impedance source
  - We assume a single bunch of particles that goes multiple times through a known (characterized) wake/impedance source, representing
    - Energy loss per turn if the bunch passes every turn and the wake fully keeps ringing between subsequent turns
  - Our goal is to calculate how much energy the bunch loses at each passage due to the electromagnetic interaction over several turns



Bunch energy loss per turn

$$\Delta E = -\frac{e^2}{2\pi}\int_{-\infty}^{\infty}\lambda(z)dz\int_{-\infty}^{\infty}\underbrace{dz'\lambda(z')}\sum_{k=-\infty}^{\infty}W_{||}(kC+z-z')dz'$$

$\lambda(z'+kC)=\lambda(z')$, i.e. assuming that the distribution doesn't change from turn to turn

$$\sum_{k=-\infty}^{\infty}W_{||}(kC+z-z') = \frac{c}{C}\sum_{p=-\infty}^{\infty}Z_{||}(p\omega_0)\exp\left[-\frac{ip\omega_0(z-z')}{c}\right]$$

$$\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}Z_{||}(p\omega_0)\underbrace{\int_{-\infty}^{\infty}\lambda(z)\exp\left(\frac{-ip\omega_0 z}{c}\right)dz}_{\hat{\lambda}(p\omega_0)}\underbrace{\int_{-\infty}^{\infty}\lambda(z')\exp\left(\frac{ip\omega_0 z'}{c}\right)dz'}_{\hat{\lambda}^*(p\omega_0)}$$

$$\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}\left|\hat{\lambda}(p\omega_0)\right|^2 \mathrm{Re}\left[Z_{||}(p\omega_0)\right]$$

Beam energy loss per turn

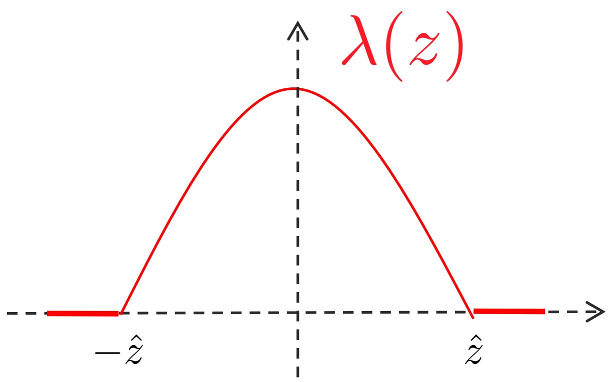
Replacing the **bunch spectrum with the beam spectrum**, we can calculate the energy loss from a beam

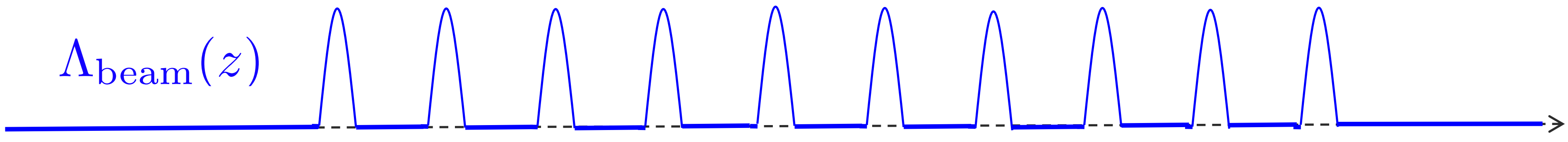
Bunch profile and spectrum

$\lambda(z)$

Beam profile and spectrum

$\Lambda_{\mathrm{beam}}(z)$





Beam energy loss per turn

Replacing the **bunch spectrum with the beam spectrum**, we can calculate the energy loss from a beam

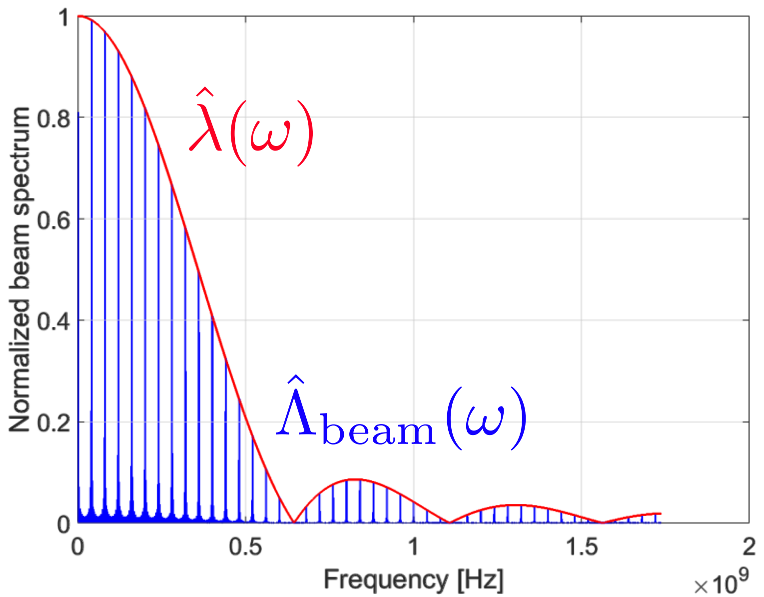
Bunch profile and spectrum

$$\lambda(z) \leftrightarrow \hat{\lambda}(\omega)$$

Beam profile and spectrum

$$\Lambda_{\mathrm{beam}}(z)$$

Ex. parabolic, as shown in the previous slide



$$\Delta E_{\mathrm{beam}} = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{+\infty}|\Lambda_{\mathrm{beam}}(p\omega_0)|^2\,\mathrm{Re}\left[Z_{||}(p\omega_0)\right]$$

Beam energy loss per turn

Replacing the **bunch spectrum with the beam spectrum**, we can calculate the energy loss from a beam

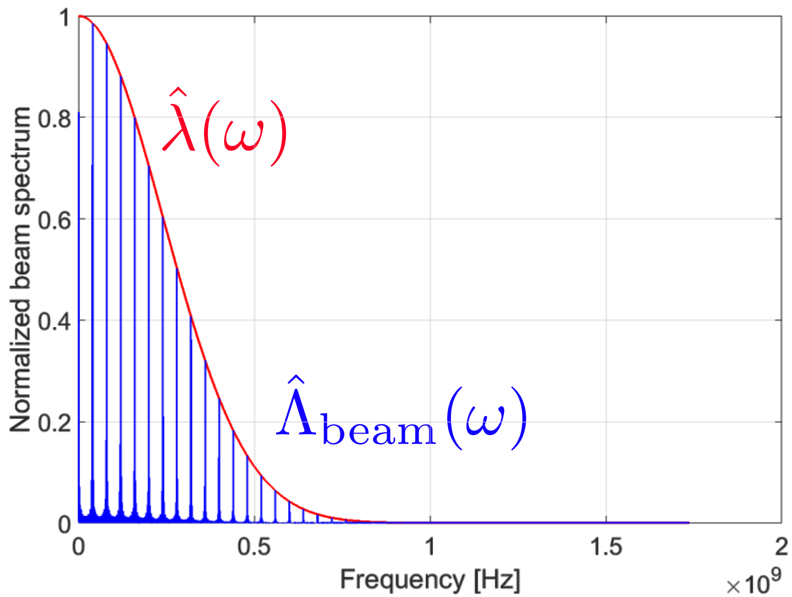
Bunch profile and spectrum

$$\lambda(z) \leftrightarrow \hat{\lambda}(\omega)$$

Beam profile and spectrum

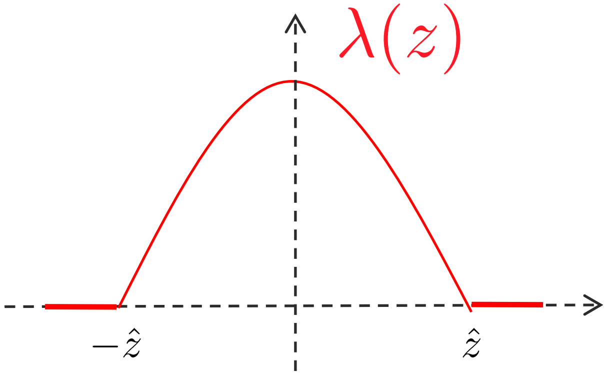
$$\Lambda_{\rm beam}(z) \leftrightarrow \hat{\Lambda}_{\rm beam}(\omega)$$

Or for a train of Gaussian bunches

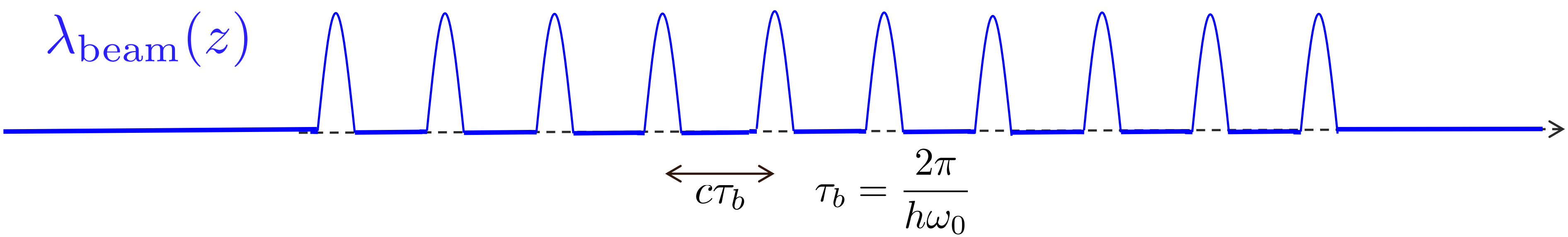


$$\Delta E_{\rm beam} = -\frac{e^2\omega_0}{2\pi} \sum_{p=-\infty}^{+\infty} \left|\Lambda_{\rm beam}(p\omega_0)\right|^2 {\rm Re}\left[Z_{||}(p\omega_0)\right]$$

Energy loss of a train of M identical bunches



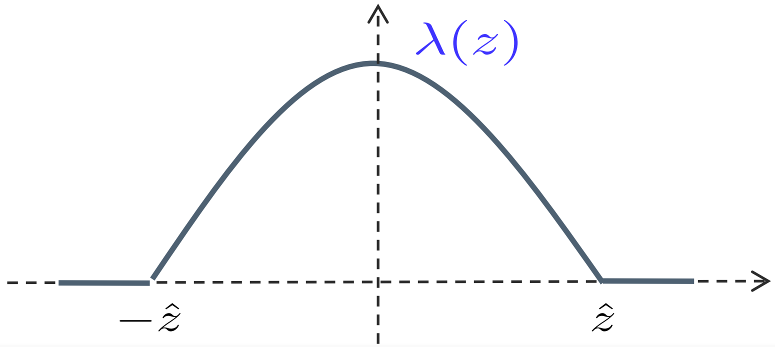
Exercise: Energy loss of a train of *M* identical equally spaced bunches circulating in a ring



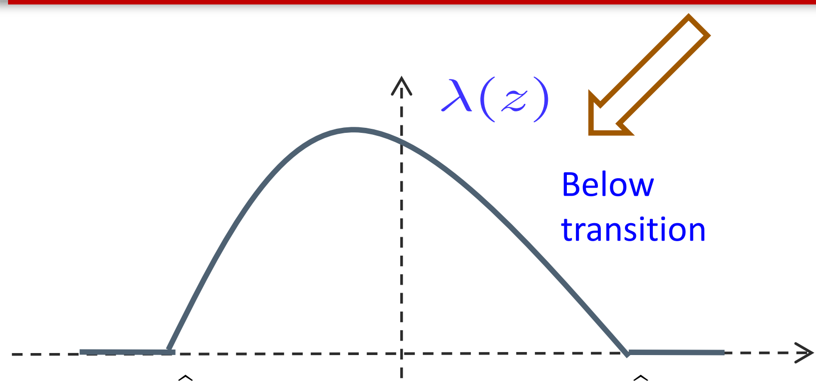
$$\lambda_{\text{beam}}(z) = \sum_{n=0}^{M-1} \lambda(z - nc\tau_b) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \Lambda_{\text{beam}}(\omega) = \hat{\lambda}(\omega) \sum_{n=0}^{M-1} \exp(-in\omega\tau_b)$$

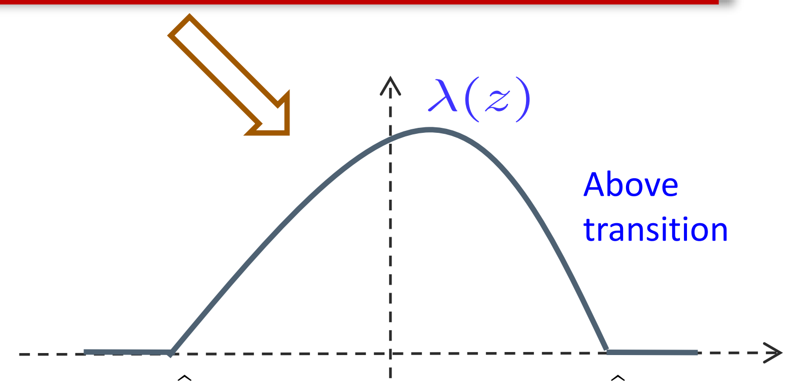
Bunch energy loss per turn and stable phase

- The **RF system compensates** for the energy loss by imparting a net acceleration to the bunch
- Therefore, the bunch readjusts to a **new equilibrium distribution** in the bucket and moves to an average synchronous angle $\Delta\Phi_s$



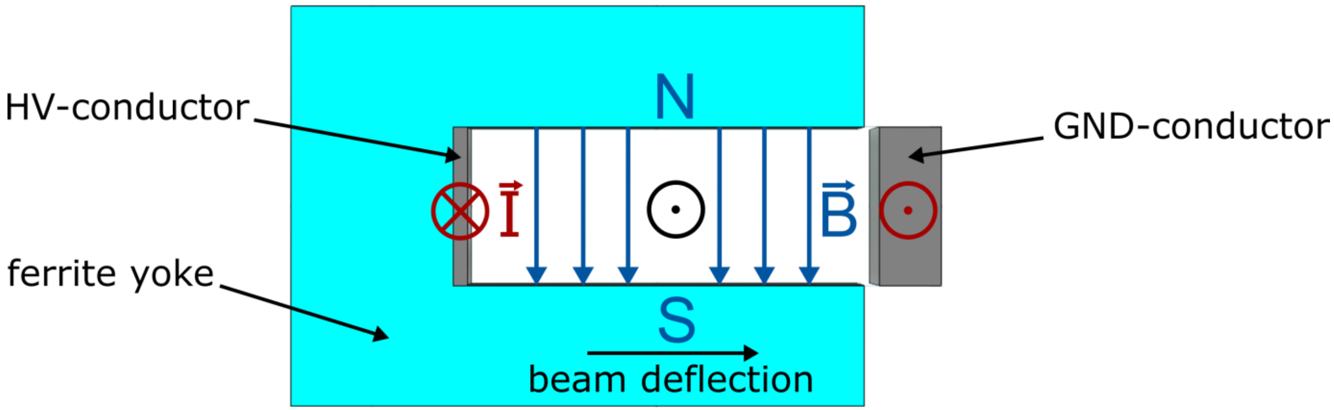
$$\sin\Delta\Phi_s = \frac{\Delta E_{\mathrm{turn}}}{NeV_m} = -\frac{e\omega_0}{2\pi N V_m}\sum_{p=-\infty}^{\infty}\left|\hat{\lambda}(p\omega_0)\right|^2 \mathrm{Re}\left(Z_{\parallel}(p\omega_0)\right)$$





Application to the SPS extraction kickers

- Problem with SPS extraction kickers (MKE)
  - Extraction elements through which the beam passes every turn
    - Based on a fast pulsed magnet capable of deflecting the whole beam over one turn
    - Active only on turn in which beam has to be extracted, otherwise passive but with all its elements (ferrite, conductors) exposed to the beam



Application to the SPS extraction kickers

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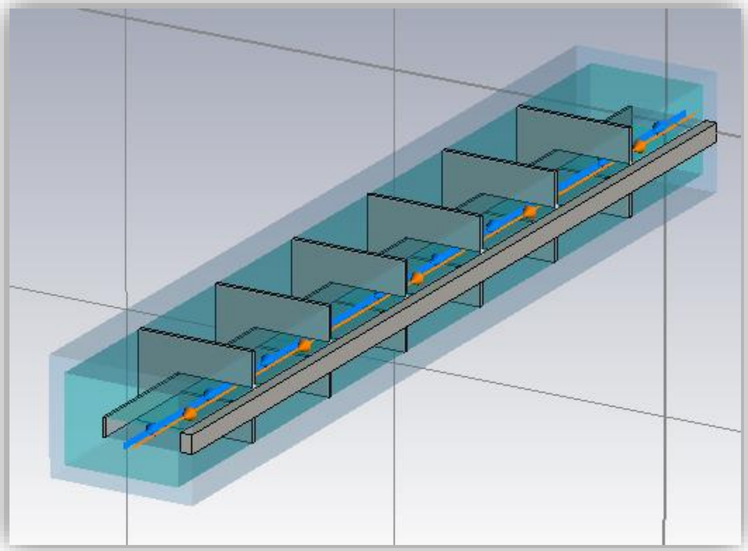
    - Based on a fast pulsed magnet capable of deflecting the whole beam over one turn
    - Active only on turn in which beam has to be extracted, otherwise passive but with all its elements (ferrite, conductors) exposed to the beam
  - Use of beam for LHC filling (4x 200-ns spaced trains of 72x 25-ns spaced bunches) led to inacceptable heating of these elements)
    - Heating above Curie temperature leads to ferrite degradation → Beam cannot be extracted anymore from the SPS
    - Heating causes outgassing and strong pressure rise in the kicker sector, with consequent beam interlocking due to poor vacuum



Application to the SPS extraction kickers

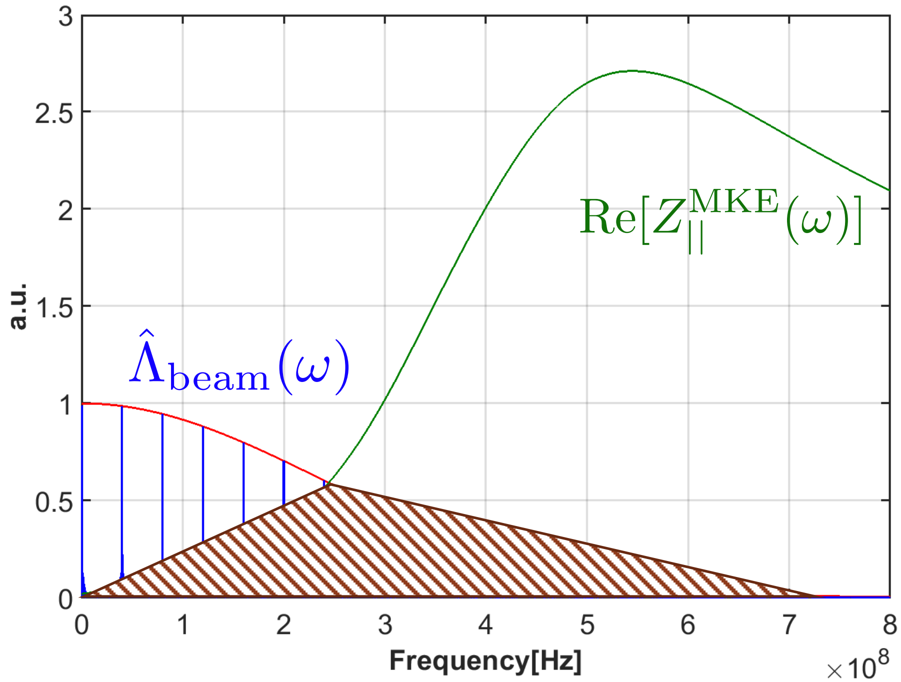
- We need to calculate the power loss in the kicker
  - Kicker impedance can be evaluated semi-analytically or via simulations
  - Then we apply the energy loss formula

$$\Delta E_{\rm beam} = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{+\infty}\left|\Lambda_{\rm beam}(p\omega_0)\right|^2 {\rm Re}\left[Z_{||}(p\omega_0)\right]$$

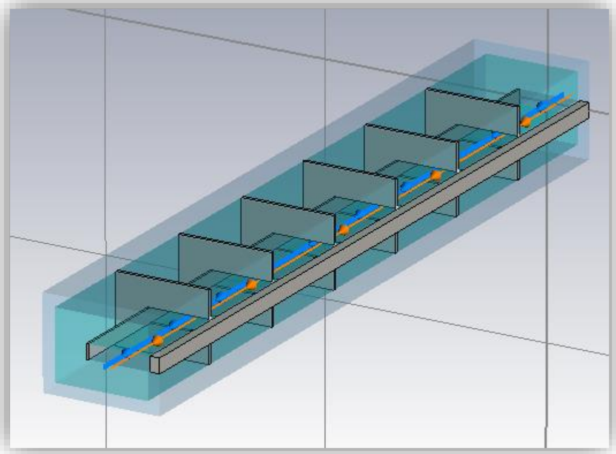
$$\Delta W = \frac{\Delta E_{\rm beam}}{T_0}$$

Application to the SPS extraction kickers

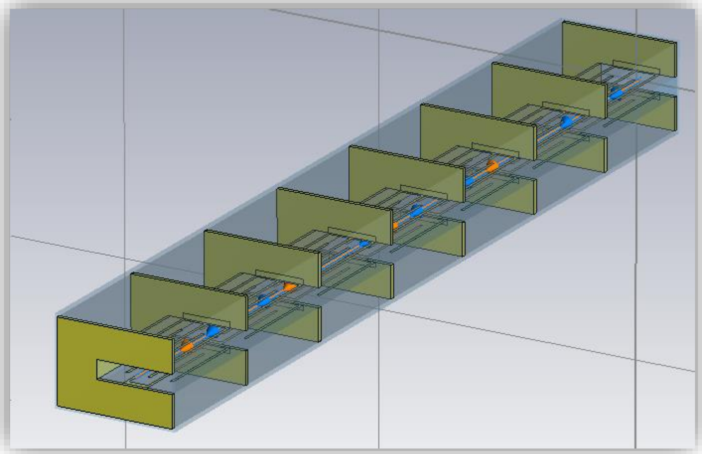
- We need to calculate the power loss in the kicker
  - Kicker impedance can be evaluated semi-analytically or via simulations
  - Then we apply the energy loss formula
- Kicker impedance already becomes significant at frequencies for which the beam spectrum has not fully decayed, causing the undesired heating
- We need to lower the kicker impedance → Impedance dominated by losses in ferrite → Ferrite shielding

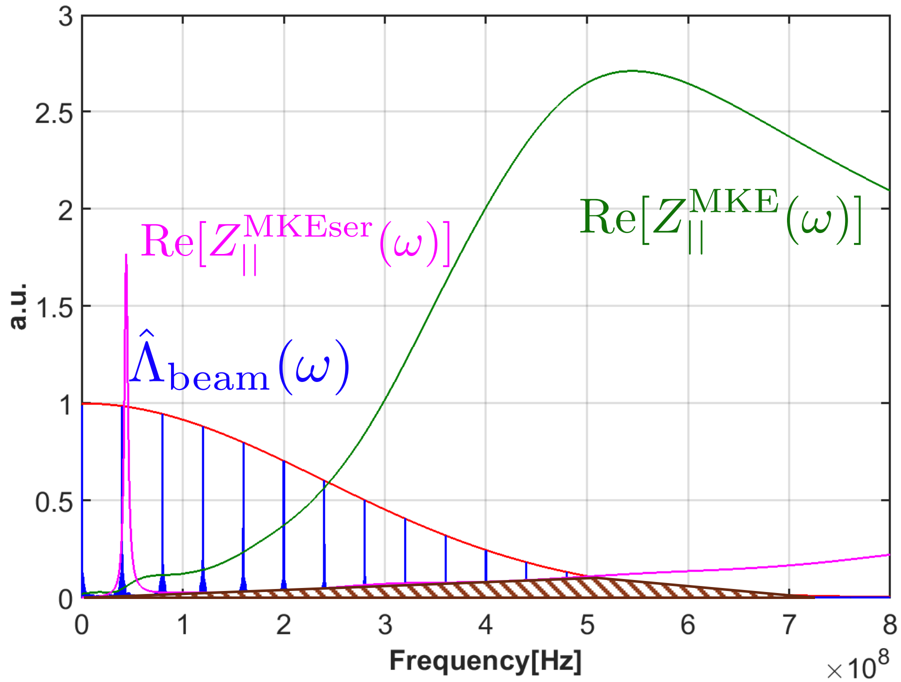


Application to the SPS extraction kickers



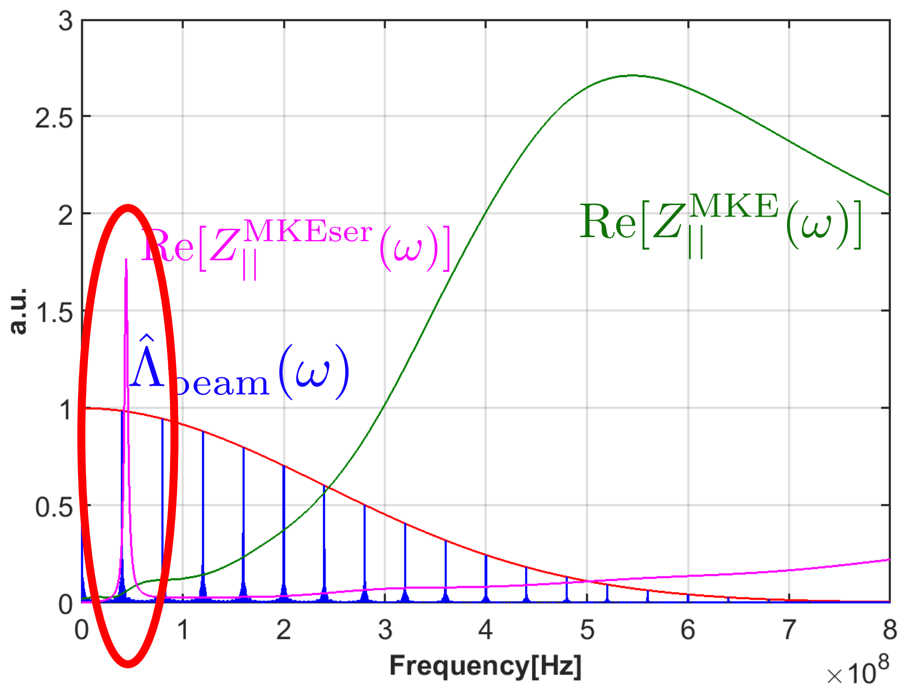
Print striped pattern of good conductor on ferrite (serigraphy)





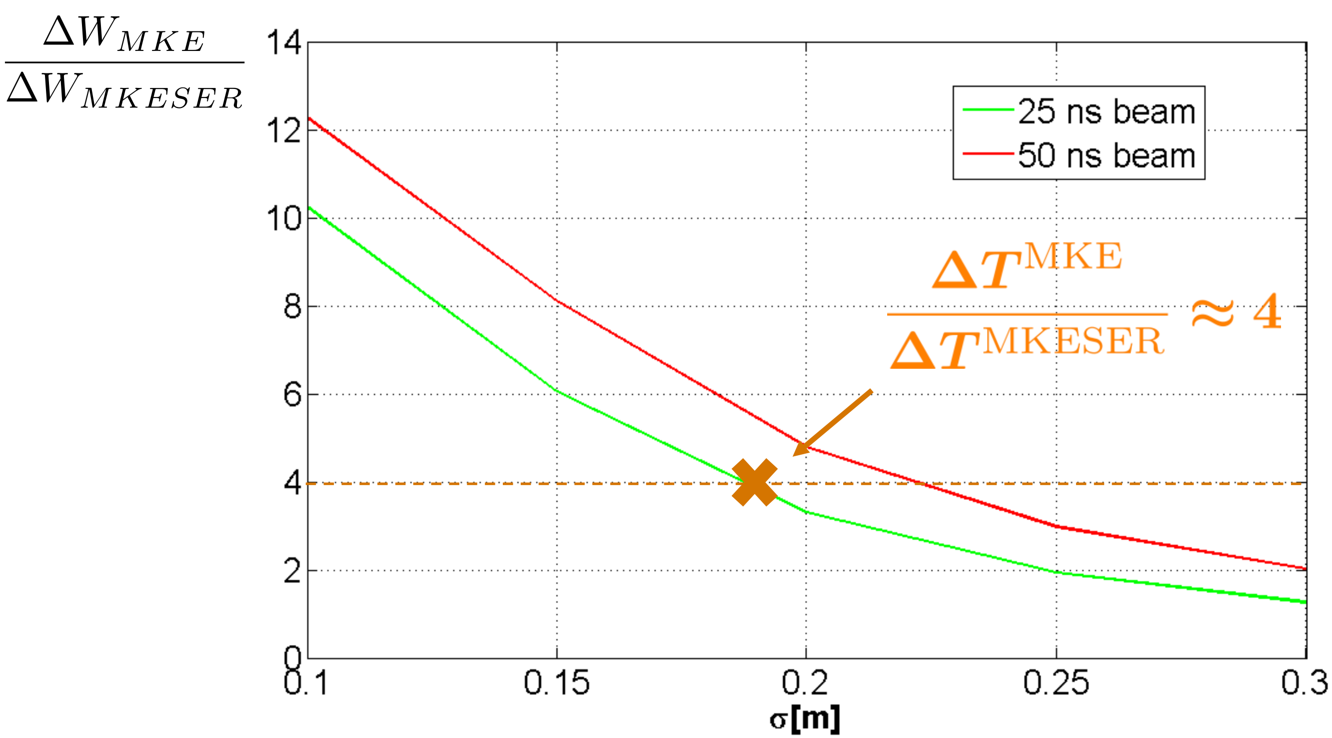
Application to the SPS extraction kickers

- This almost suppresses the impedance over the bunch spectrum
- It however introduces a low frequency peak, which needs to be kept far from beam spectral lines
  - Pay attention to do that for all needed bunch spacings



Application to the SPS extraction kickers

- This almost suppresses the impedance over the bunch spectrum
- It however introduces a low frequency peak, which needs to be kept far from beam spectral lines
- Expected up to 12-fold reduction of heat load, depending on bunch length and bunch spacing
  - Factor 4 for 25-ns LHC-type beam at 26 GeV



Application to the SPS extraction kickers

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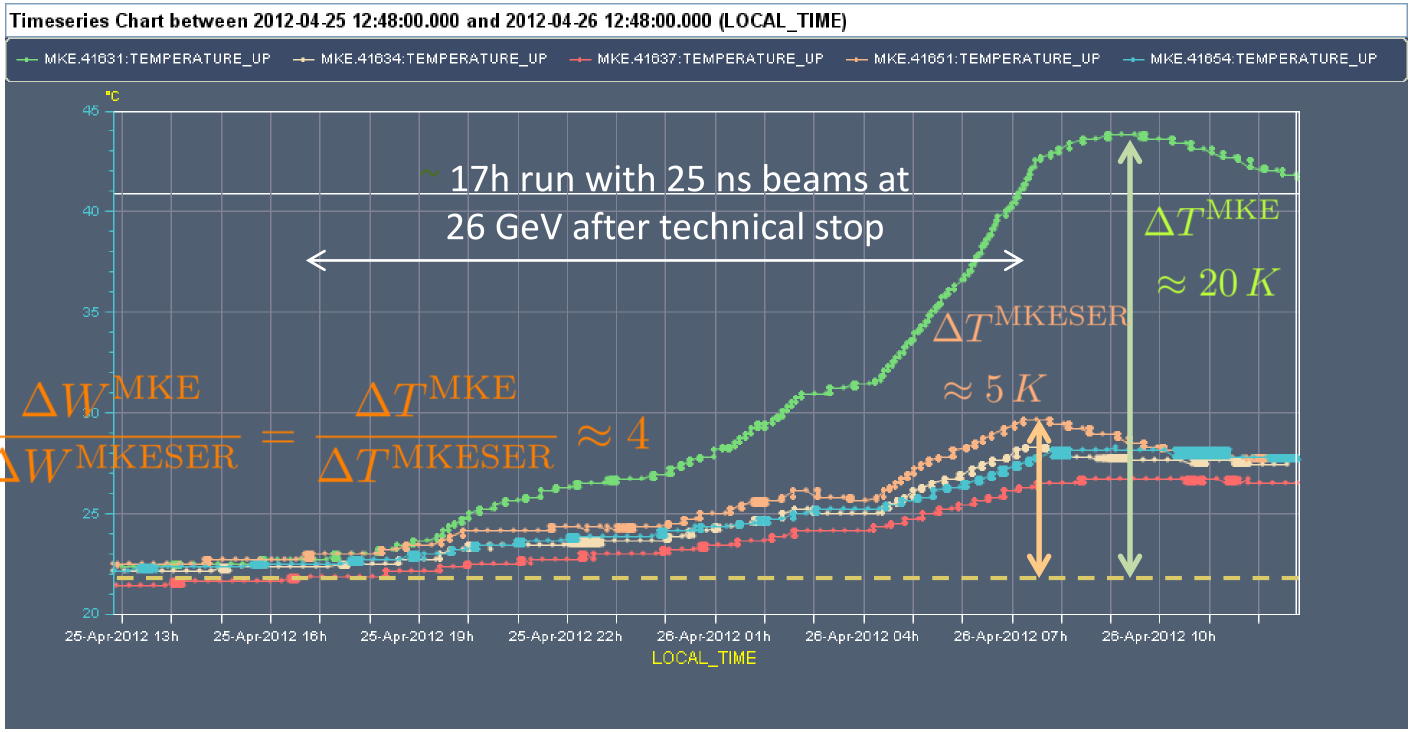
- Expected up to 12-fold reduction of heat load, depending on bunch length and bunch spacing
  - Factor 4 for 25-ns LHC-type beam at 26 GeV → Experimentally measured!



Signpost

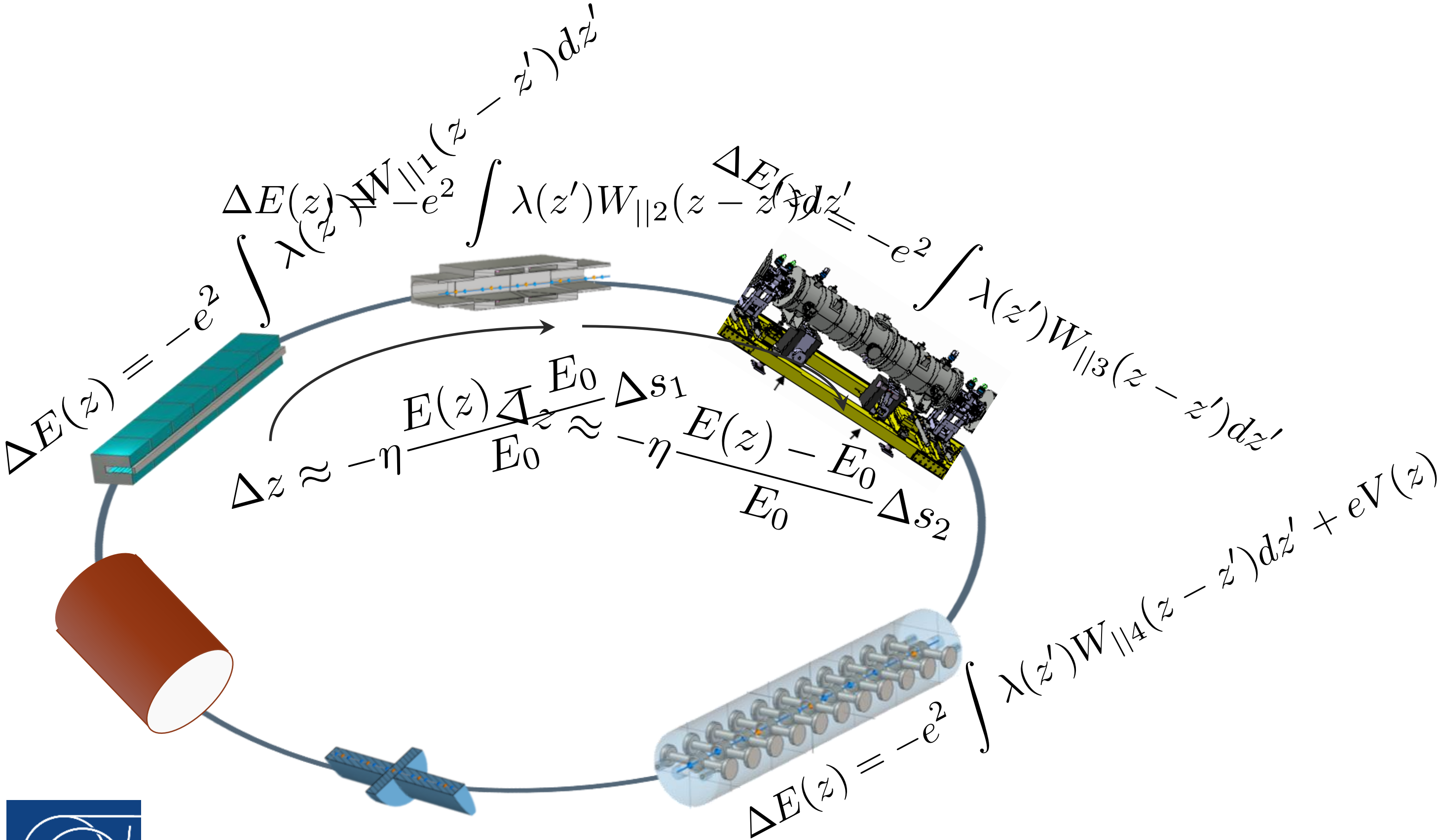
- We have further looked into the mechanism of energy loss and have seen the **impact of longitudinal impedances on machine elements** as these lead to **beam induced heating**.
- We have found that beam induced heating depends on the overlap of the **beam power spectrum** and the **impedance** of a given object.
- We have seen a **real world example** of the impact of an objects impedance on the beam induced heating.

Part 2: Multiparticle dynamics with wake fields – impact on machine elements and beam dynamics

- General introduction to wake fields
- Longitudinal and transverse wake functions and impedance
- Energy loss – beam induced heating and stable phase shift
- Impedance models and effects in beam dynamics, including instabilities

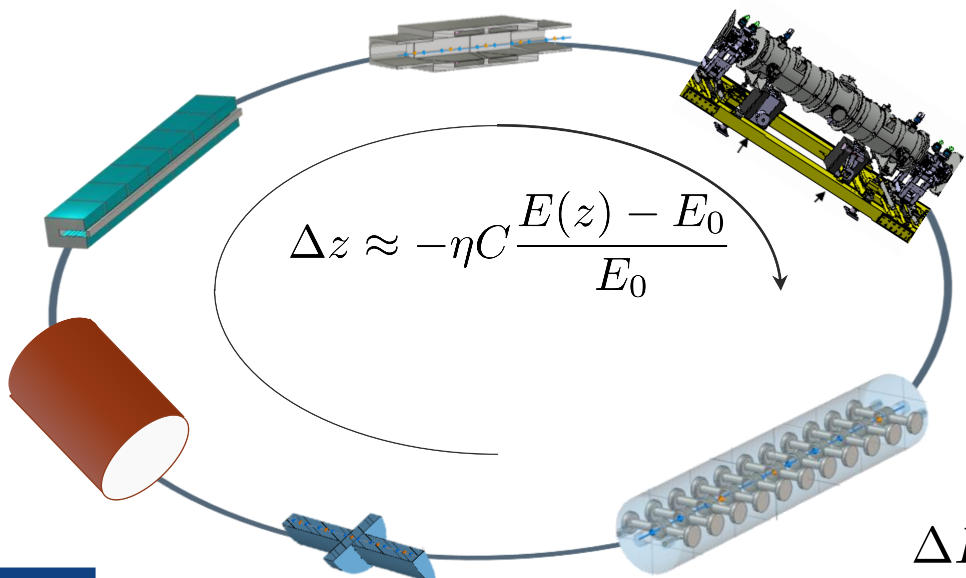
Longitudinal wakes in beam dynamics

- The effect of each localised wake/impedance on each particle in a beam can be described as an energy kick
- The accelerator is made of many components, each giving a small kick to the beam particles, which drift freely in between



Longitudinal wakes in beam dynamics

- The effect of each localised wake/impedance on each particle in a beam can be described as an energy kick
- The accelerator is made of many components, each giving a small kick to the beam particles, which drift freely in between
- For simulations, the impedance is lumped in one place and kicks to beam particles are applied once per turn, with free drift over one turn



$$\Delta z \approx -\eta C \frac{E(z) - E_0}{E_0}$$

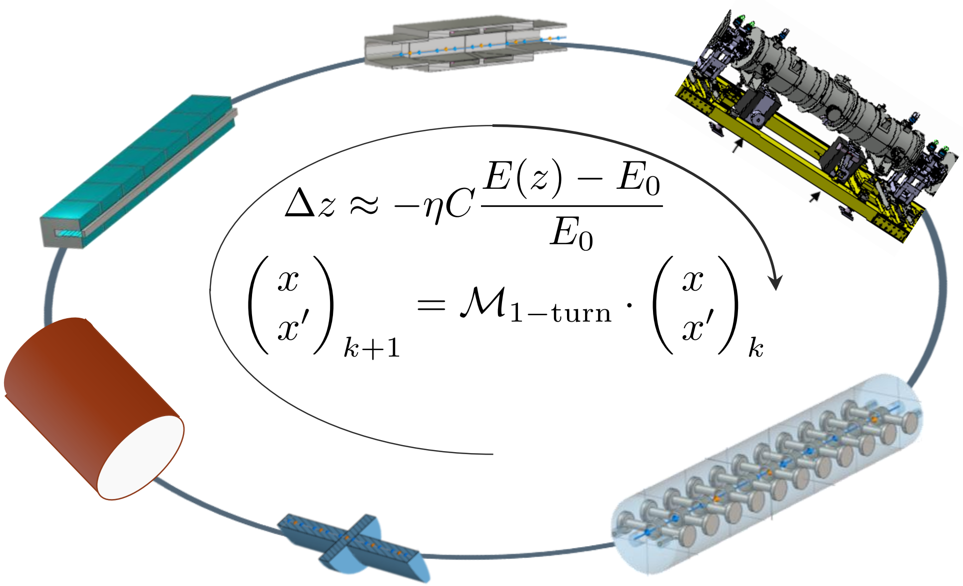
$$W_{||}^{\text{Ring}}(z) = \sum W_{||i}(z)$$

$$Z_{||}^{\text{Ring}}(\omega) = \sum Z_{||i}(\omega)$$

$$\Delta E(z) = -e^2 \int \lambda(z') W_{||}^{\text{Ring}}(z - z') dz'$$

Transverse wakes in beam dynamics

- Same approach as in the longitudinal plane to build the impedance model of a machine
- For simulations, the impedance is lumped in one place and kicks to beam particles are applied once per turn, with linear matrix transport between turns
  - One word of caution: The effect of the transverse impedance results in a combination of a dipole-type and quadrupole-type kick, therefore the beta functions at the real locations of the impedance source has to be taken into account when combining wakes/impedances



$$\Delta z \approx -\eta C \frac{E(z) - E_0}{E_0}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{k+1} = \mathcal{M}_{1-\text{turn}} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_k$$

$$W^{\text{Ring}}_{Cx,Dx,Qx}(z) = \sum_i \frac{\beta_{xi}}{\langle \beta_x \rangle} W^i_{Cx,Dx,Qx}(z)$$

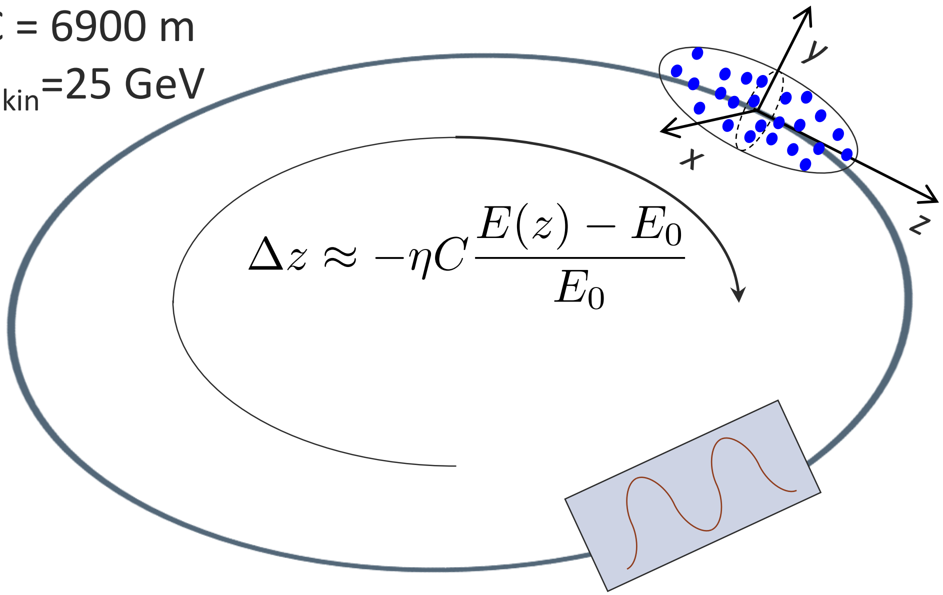
$$Z^{\text{Ring}}_{Cx,Dx,Qx}(\omega) = \sum_i \frac{\beta_{xi}}{\langle \beta_x \rangle} Z^i_{Cx,Dx,Qx}(\omega)$$

$$\Delta E = eV_{\text{rf}}(z)$$

$$\Delta x' = -\frac{e^2}{\beta^2 E_0} \int \lambda(z') \left[W^{\text{Ring}}_{Cx}(z - z') + \langle x \rangle (z') W^{\text{Ring}}_{Dx}(z - z') + x W^{\text{Ring}}_{Qx}(z - z') \right] dz'$$

Bunch lengthening and μW instability

SPS ring
C = 6900 m
E_{kin}=25 GeV



Single Gaussian bunch
σ_z = 0.2 m (0.67 ns)

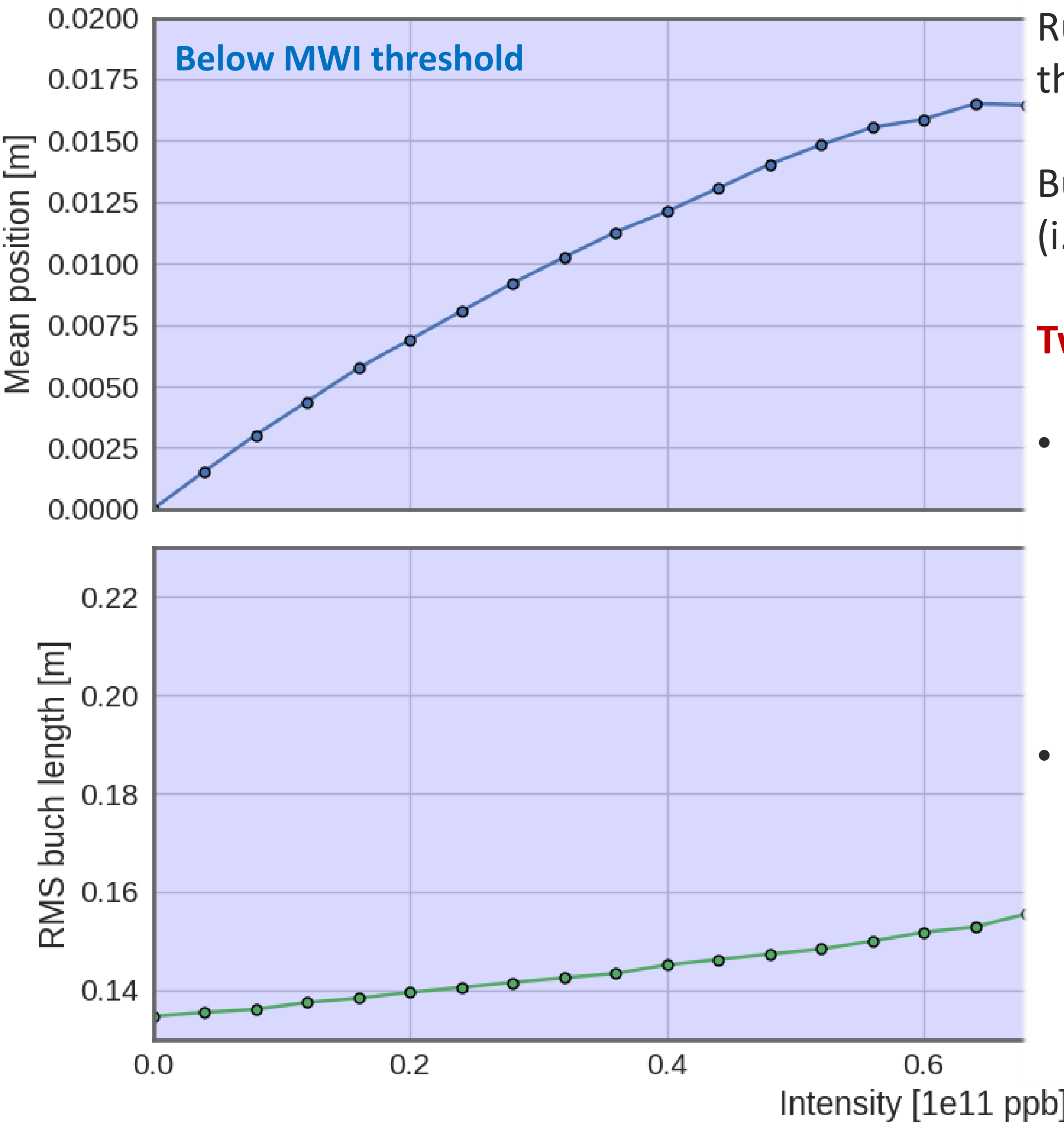
Ring impedance modeled as broad band resonator with
ω_r = 700 MHz
Q=1
R_s=

Single RF system
ω_{rf} = 200 MHz
V_{rf}^{max} = 3 MV

$$\Delta E(z) = -e^2 \int \lambda(z') W_{||}^{\mathrm{Res}}(z - z') dz' + eV_{\mathrm{rf}}(z)$$

$$Z_{||}^{\mathrm{Res}}(\omega) = \frac{R_{s||}}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

Bunch lengthening and μW instability



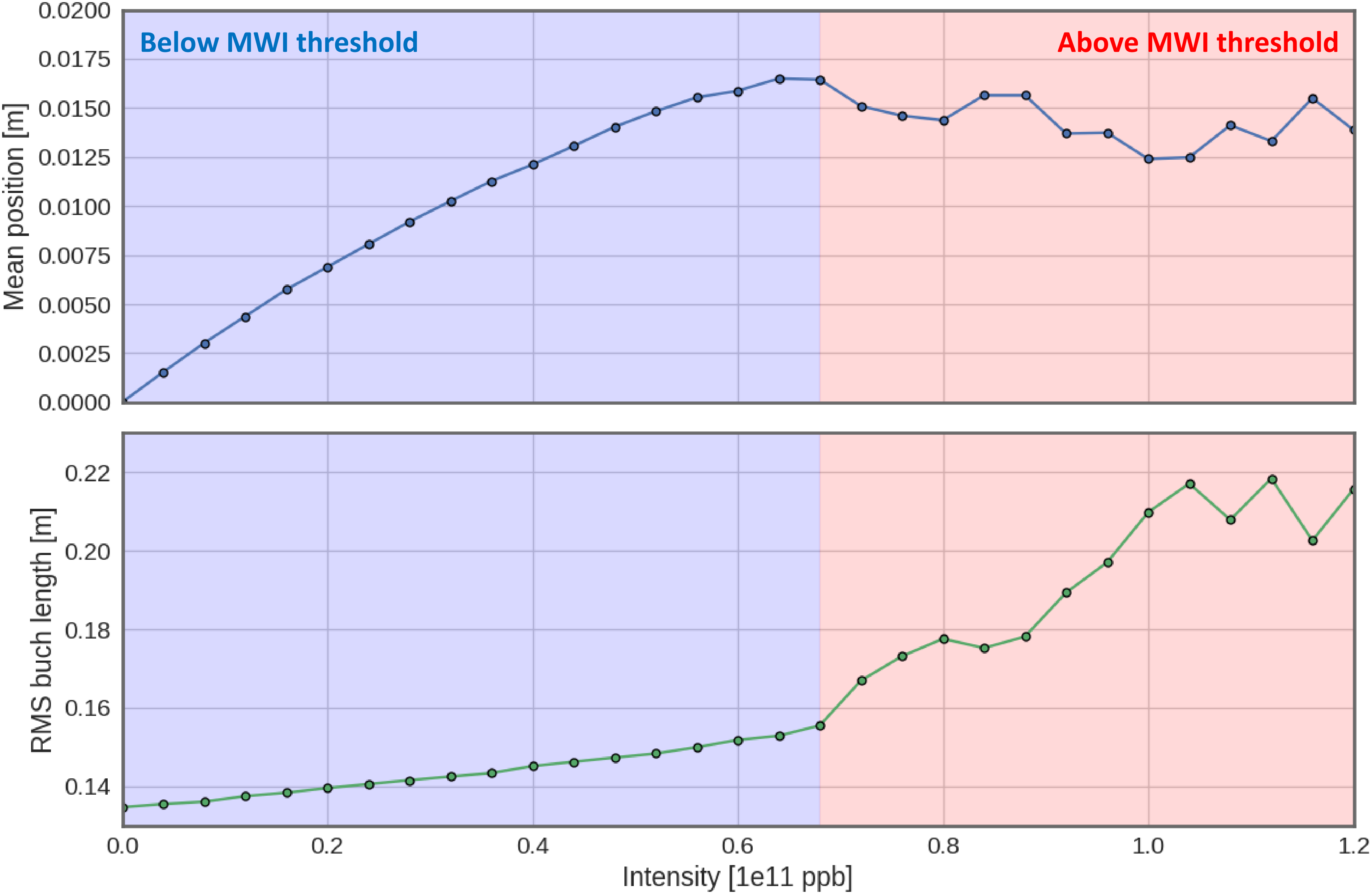
Running the numerical simulation for this case:

Bunch is matched at low intensity (i.e. without impedance)

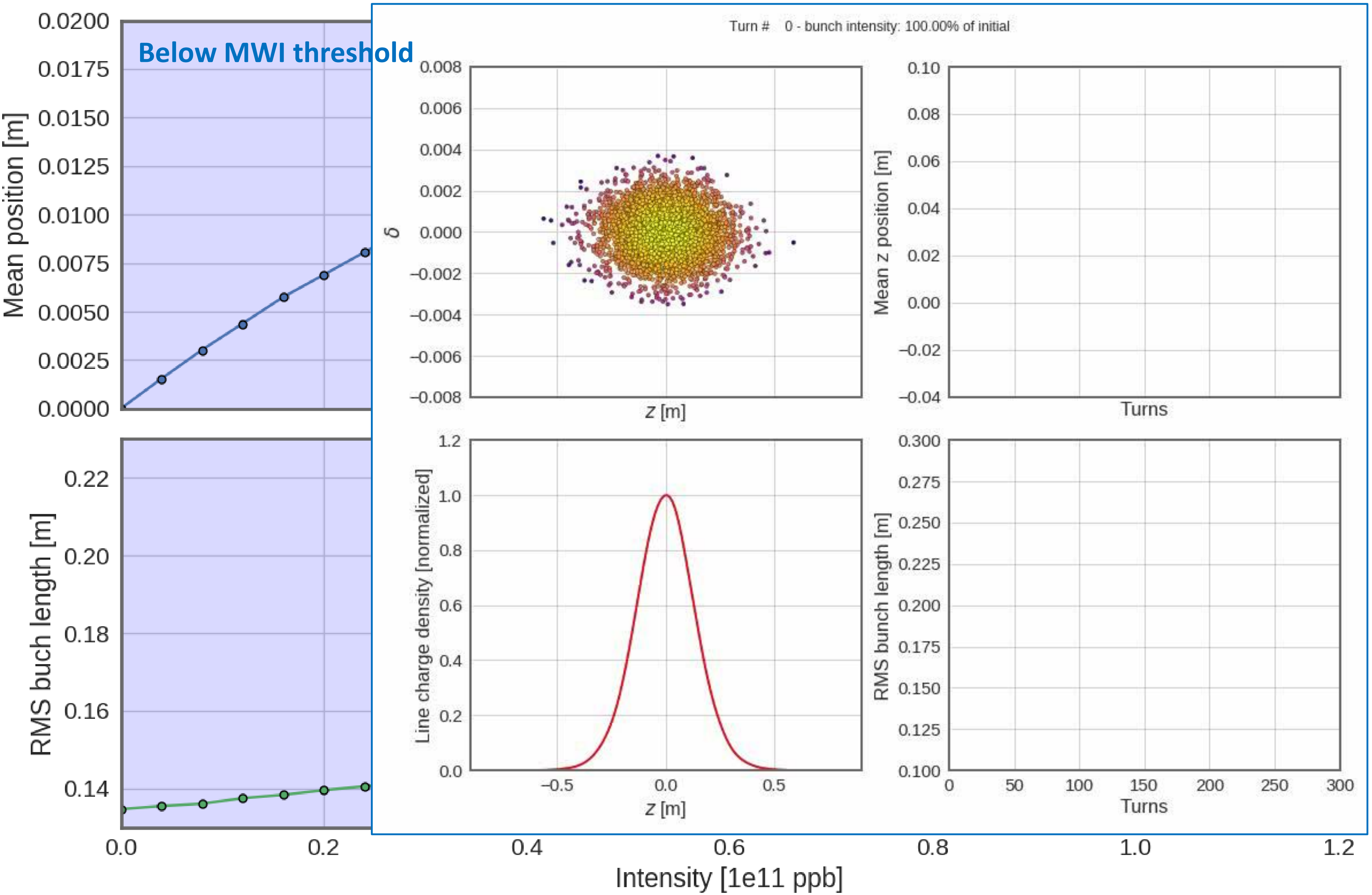
Two regimes are found:

- Bunch lengthening/emittance blow up regime with roughly linear increase of the **synchronous phase** and **bunch length** with intensity
- Unstable regime (**turbulent bunch lengthening**)

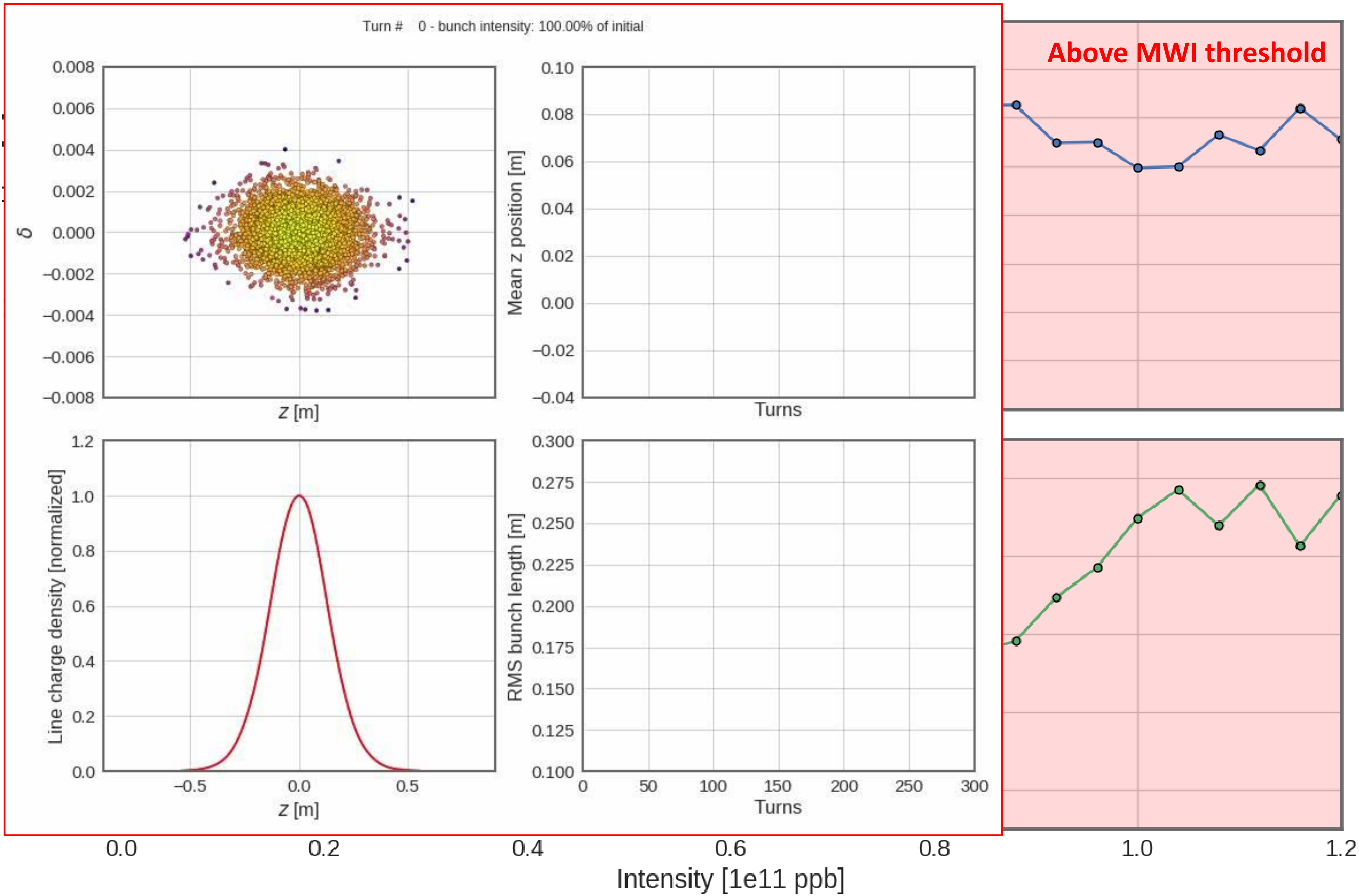
Bunch lengthening and μW instability



Bunch lengthening and μW instability

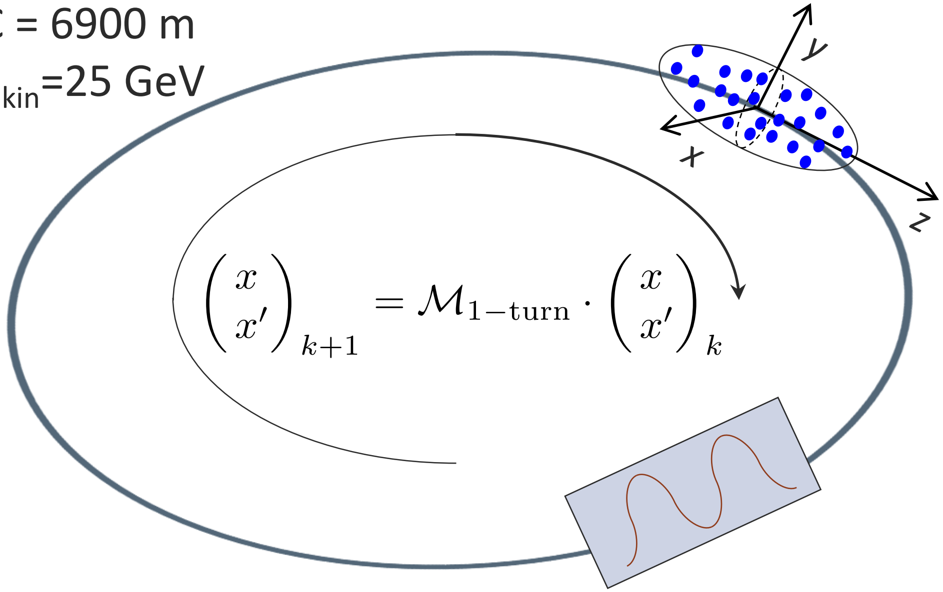


Bunch lengthening and μW instability



Effect of a transverse impedance on a bunch

SPS ring
C = 6900 m
E_{kin}=25 GeV



$$\begin{pmatrix} x \\ x' \end{pmatrix}_{k+1} = \mathcal{M}_{1-\mathrm{turn}} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_{k}$$

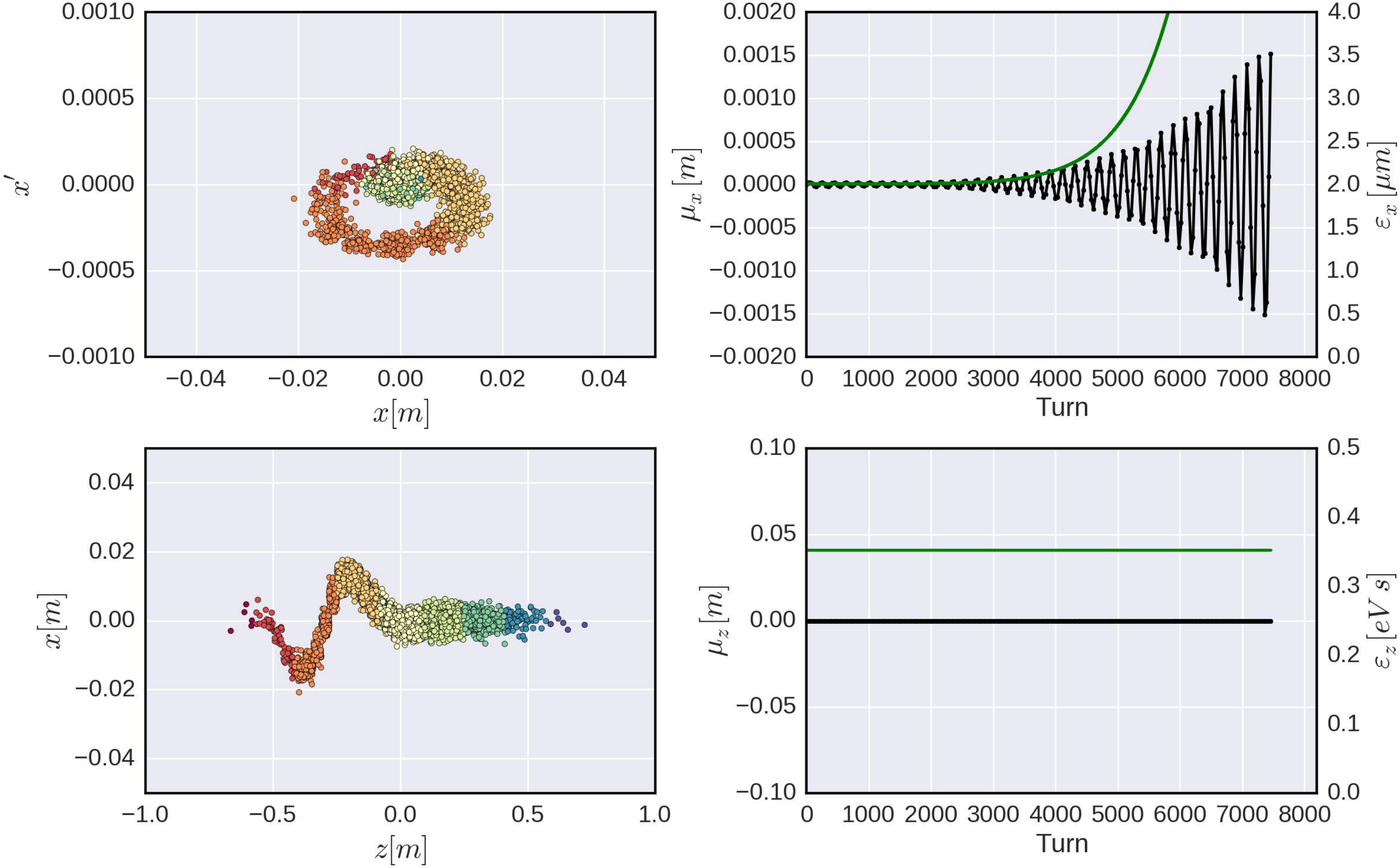
Single Gaussian bunch
σ_z = 0.2 m (0.67 ns)

Dipole horizontal wake in the form of broad-band resonator

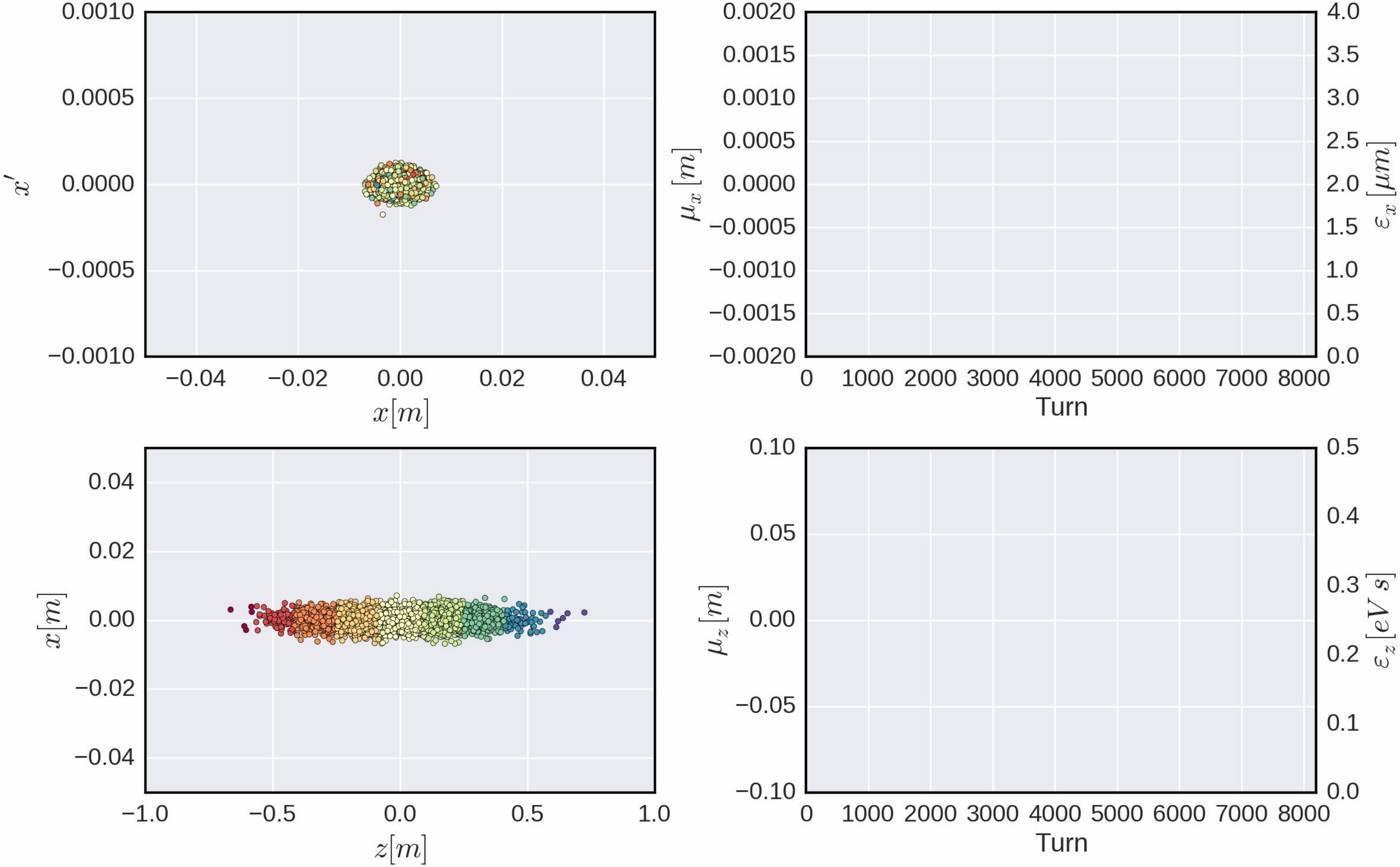
Frozen longitudinal motion or crossing transition ($\eta \approx 0$)

$$\Delta x' = -\frac{e^2}{\beta^2 E_0} \int \lambda(z') \langle x \rangle (z') W_{Dx}^{\mathrm{Ring}}(z - z') dz'$$

Dipole wakes – beam break-up

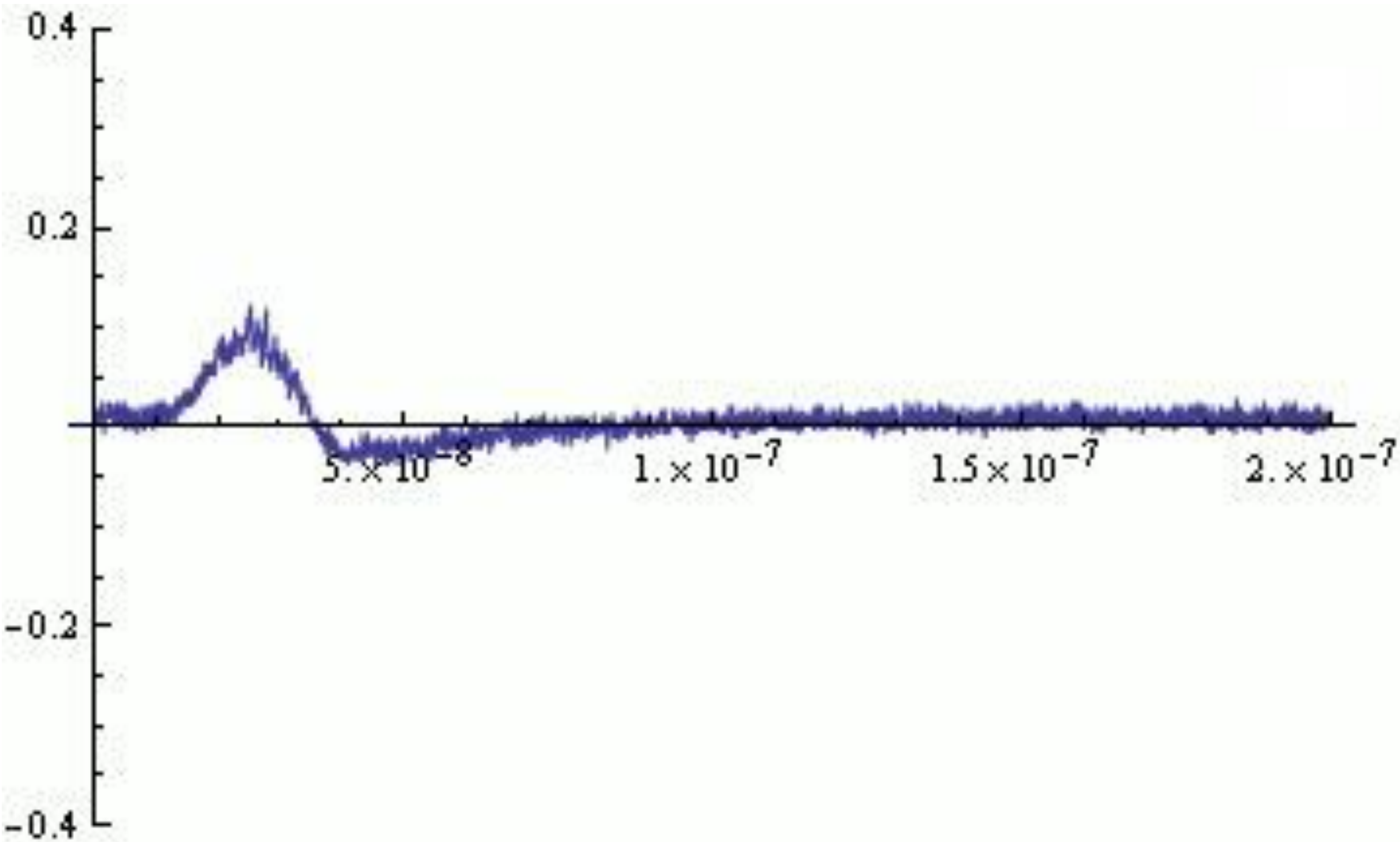


Dipole wakes – beam break-up



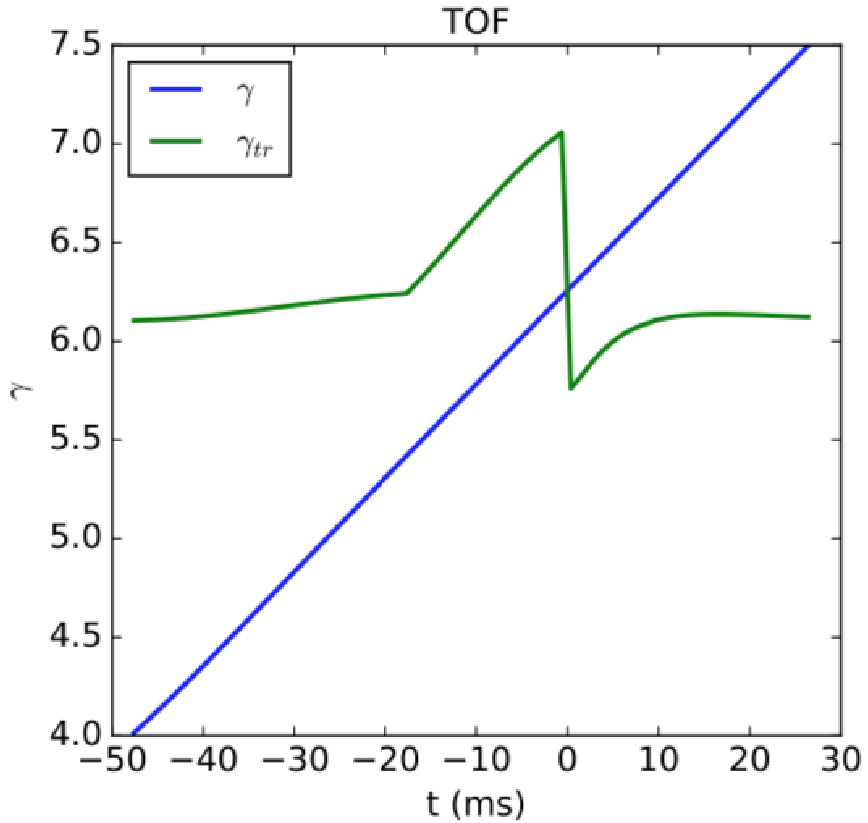
Measurement at CERN PS

- Beam break up type instabilities have been seen in the CERN PS when crossing transition with high intensity beams



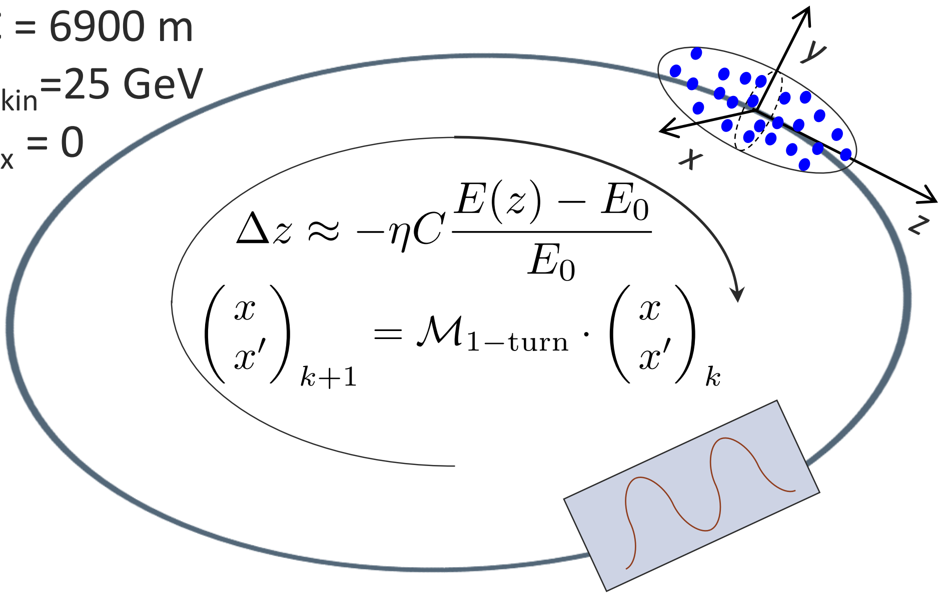
Measurement at CERN PS

- Beam break up type instabilities have been seen in the CERN PS when crossing transition with high intensity beams
- To increase the intensity reach, it is necessary to cross transition more quickly, gamma jump scheme implemented



Effect of a transverse impedance on a bunch

SPS ring
C = 6900 m
E_{kin}=25 GeV
ξ_x = 0



$$\Delta z \approx -\eta C \frac{E(z) - E_0}{E_0}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{k+1} = \mathcal{M}_{1-\text{turn}} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_k$$

Single Gaussian bunch
σ_z = 0.2 m (0.67 ns)

Dipole horizontal wake in the form of broad-band resonator

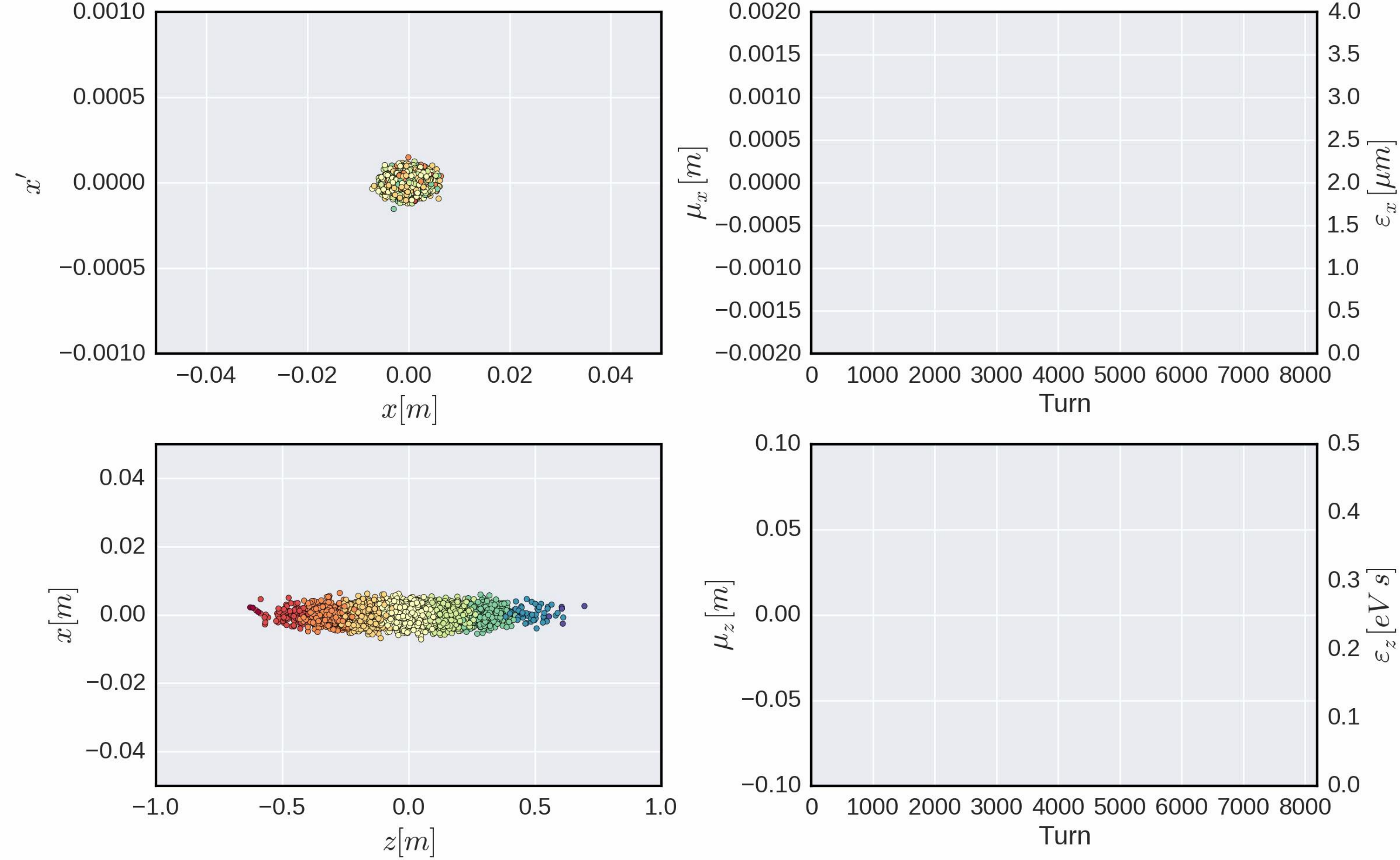
Single RF system
ω_{rf} = 200 MHz
V_{rf}^{max} = 3 MV

$$\Delta x' = -\frac{e^2}{\beta^2 E_0} \int \lambda(z') \langle x \rangle (z') W_{Dx}^{\text{Ring}}(z - z') dz'$$

$$\Delta E = e V_{\text{rf}}(z)$$

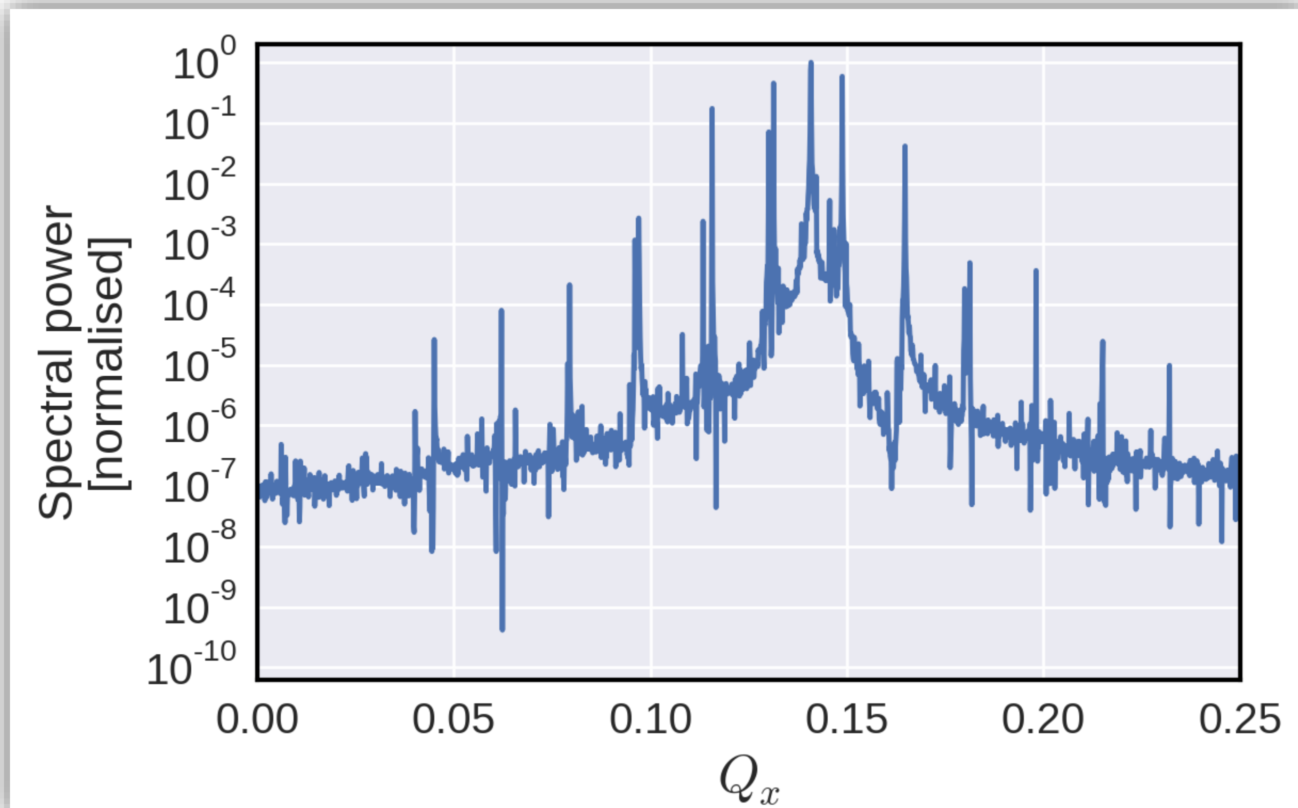
Dipole wakes – below instability threshold

- Bunch is stable up to a certain intensity ($N_b < N_{thr}$)



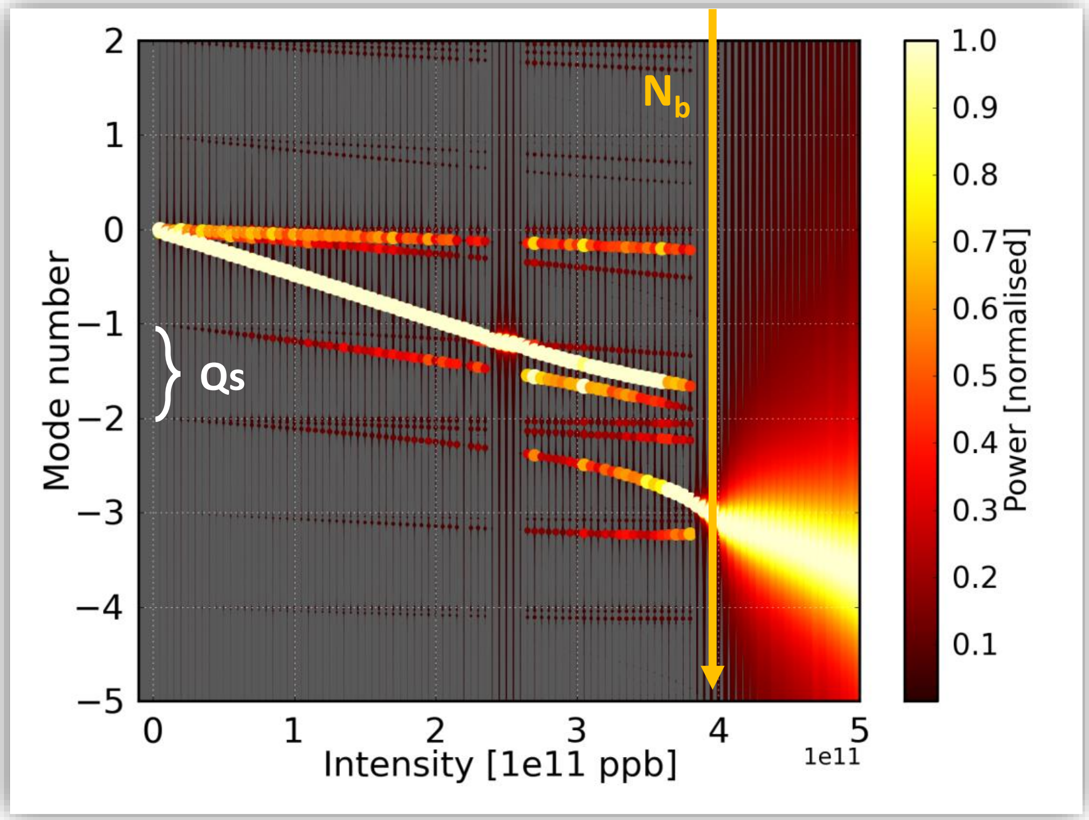
Coherent modes of the bunch

- Bunch is stable up to a certain intensity ($N_b < N_{thr}$)
- Fourier analysis of bunch centroid reveals the existence of many modes



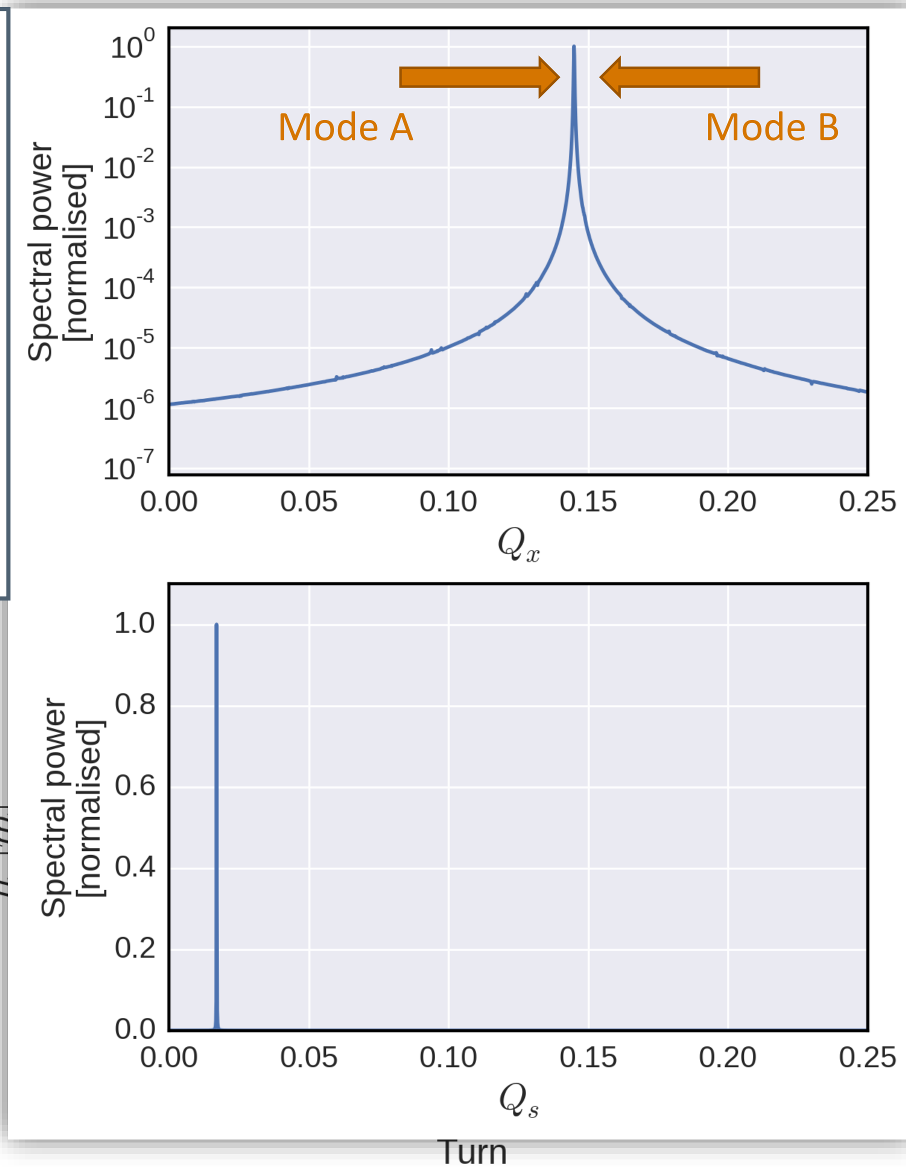
Coherent modes of the bunch

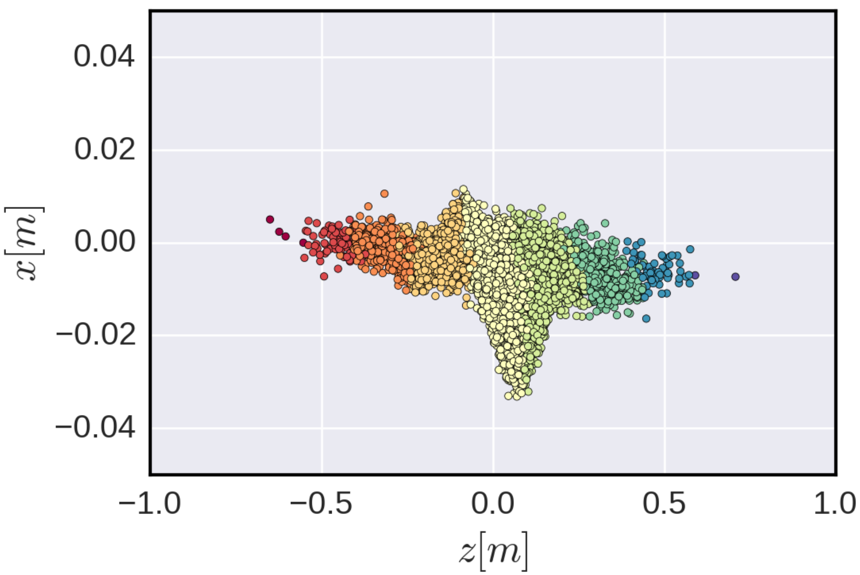
- Bunch is stable up to a certain intensity ($N_b < N_{thr}$)
- Fourier analysis of bunch centroid reveals the existence of many modes
  - Separated by ω_s at very low intensity
  - Shifting closer to each other for increasing intensity and eventually merging



Dipole wakes – above instability threshold

When the two modes merge a fast coherent instability arises – the transverse mode coupling instability (TMCI) which often is a hard intensity limit in many machines





Signpost

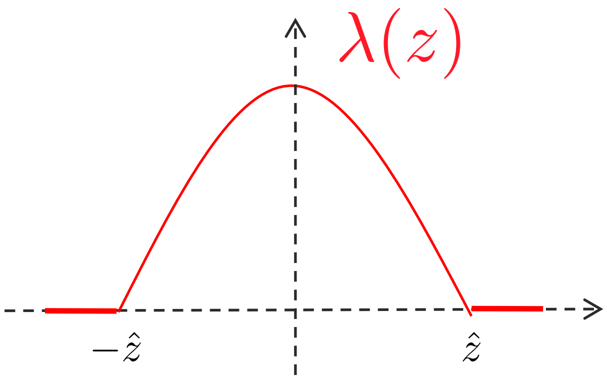
- We have **discussed longitudinal and transverse wake fields** and impedances and examples of their impact on both the machine as well as the beam.
- We have learned about **beam induced heating** and how it is related to the beam power spectrum and the machine impedance.
- We have seen some example of **longitudinal and transverse instabilities**

Next Part 3

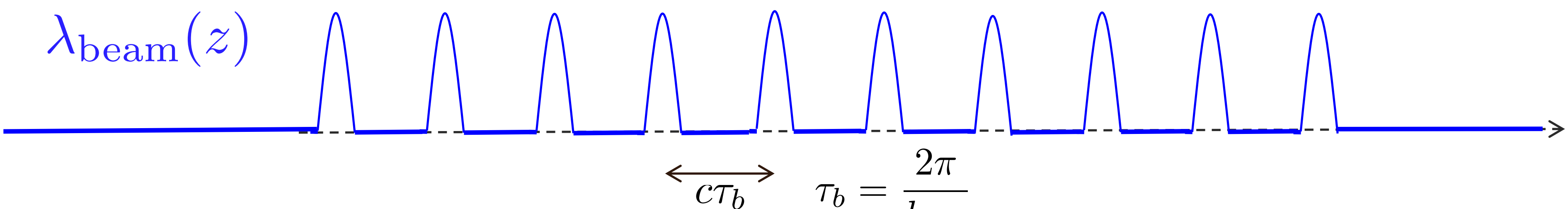
→ Electron cloud build up and effects on beam dynamics

End part 2

Energy loss of a train of M identical bunches



A train of *M* identical equally spaced bunches circulating in a ring



$$\tau_b = \frac{2\pi}{h\omega_0}$$

$$\Delta E_{\rm beam} = \frac{e^2\omega_0}{2\pi} \sum_{p=-\infty}^{+\infty} \left|\hat{\lambda}(p\omega_0)\right|^2 \mathrm{Re}\left[Z_{||}(p\omega_0)\right] \cdot \left[\frac{1-\cos\left(\frac{2\pi M p}{h}\right)}{1-\cos\left(\frac{2\pi p}{h}\right)}\right]$$

Energy loss of a train of M identical bunches

$$\Delta E_{\text{beam}} = \frac{e^2\omega_0}{2\pi} \sum_{p=-\infty}^{+\infty} \left|\hat{\lambda}(p\omega_0)\right|^2 \text{Re}\left[Z_{||}(p\omega_0)\right] \cdot \left[\frac{1-\cos\left(\frac{2\pi Mp}{h}\right)}{1-\cos\left(\frac{2\pi p}{h}\right)}\right]$$

- The potential leading terms in the summation are those with $p = k \cdot h$, as the ratio in brackets tends to M^2.
- Narrow-band impedances peaked around multiples of the harmonic number of the accelerator are **the most efficient to drain energy** from the beam → beam induced heating, instabilities.
- This type of impedances, usually **associated to the RF systems** and their higher order modes (HOMs), **need mitigation** in the accelerator design (e.g. detuners, HOM absorbers).

Longitudinal wakes in beam dynamics

- The effect of each localised wake/impedance on each particle in a beam can be described as an energy kick
- The accelerator is made of many components, each giving a small kick to the beam particles, which drift freely in between
- For simulations, the impedance is lumped in one place and kicks to beam particles are applied once per turn, with free drift over one turn
- For analytical calculations, both global impedance and RF are smeared over the ring

$$\begin{cases} \dfrac{dz}{ds} = -\eta\delta \\ \dfrac{d\delta}{ds} = \dfrac{e}{m_0\gamma c C}\left[V_{\rm rf}(z) - e\sum_k \int \lambda(z'+kC) W_{||}^{\rm Ring}(z-z'-kC)dz'\right] \end{cases}$$

$$H = -\frac{1}{2}\eta\delta^2 + \frac{e}{\beta^2 EC}U_{\rm rf}(z) +$$
$$+\frac{e^2}{\beta^2 EC}\int_{-\infty}^{z} dz'' \sum_k \int \lambda(z'+kC) W_{||}^{\rm Ring}(z''-z'-kC)dz'$$

Longitudinal wakes in beam dynamics

- For a bunch under the effect of longitudinal wake fields, two different regimes can be found:
  - Regime of **potential well distortion**, i.e. due to the impedance a new equilibrium distribution can be found for the bunch
    - Stable phase shift
    - Synchrotron frequency shift
    - Different matching (→ bunch lengthening for lepton machines)
  - Regime of **longitudinal instability**, i.e. no equilibrium distribution can be found under the effect of the impedance, a perturbation grows exponentially
    - Dipole mode instabilities
    - Coupled bunch instabilities
    - Microwave instability (longitudinal mode coupling)

Potential well distortion and Haissinki equation

- The **equilibrium distribution** in the presence of a longitudinal wake field can be found analytically. The (linearized) **longitudinal Hamiltonian** with longitudinal wake fields is given as:

$$H = -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{e^2}{\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)W_{\|}(z''-z'-kC)$$

A simple Taylor expansion in z already qualitatively reveals some of the effects of the longitudinal wake fields onto the beam:

1. First order:
   shift in the mean position (**stable phase shift**)
2. Second order:
   change in bunch length accompanied by an (incoherent) **synchrotron tune shift**

- The equilibrium (matched) line charge density is then given by the self-consistency equation (**Haissinski equation**):

$$\lambda(z) = A\,\exp\left(-\frac{1}{2}\left(\frac{\omega_s z}{\eta\sigma_\delta \beta c}\right)^2 + \frac{e^2}{\eta\sigma_\delta^2\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)W_{\|}(z''-z'-kC)\right)$$

Backup - wakefields

Signpost

- We have learned about the **impact of the longitudinal impedance on the beam**.
- We found the **Haissinki equation** and discussed the **potential well distortion** along with the **stable phase shift** and **synchrotron tune shift**.
- We looked at some generic wake fields and the **two regimes** of potential well distortion with **bunch lenthening** and its transition to the **microwave instability**.
- We will now look specifically at multi-turn wake fields and the phenomenon of **Robinson instability** and damping.

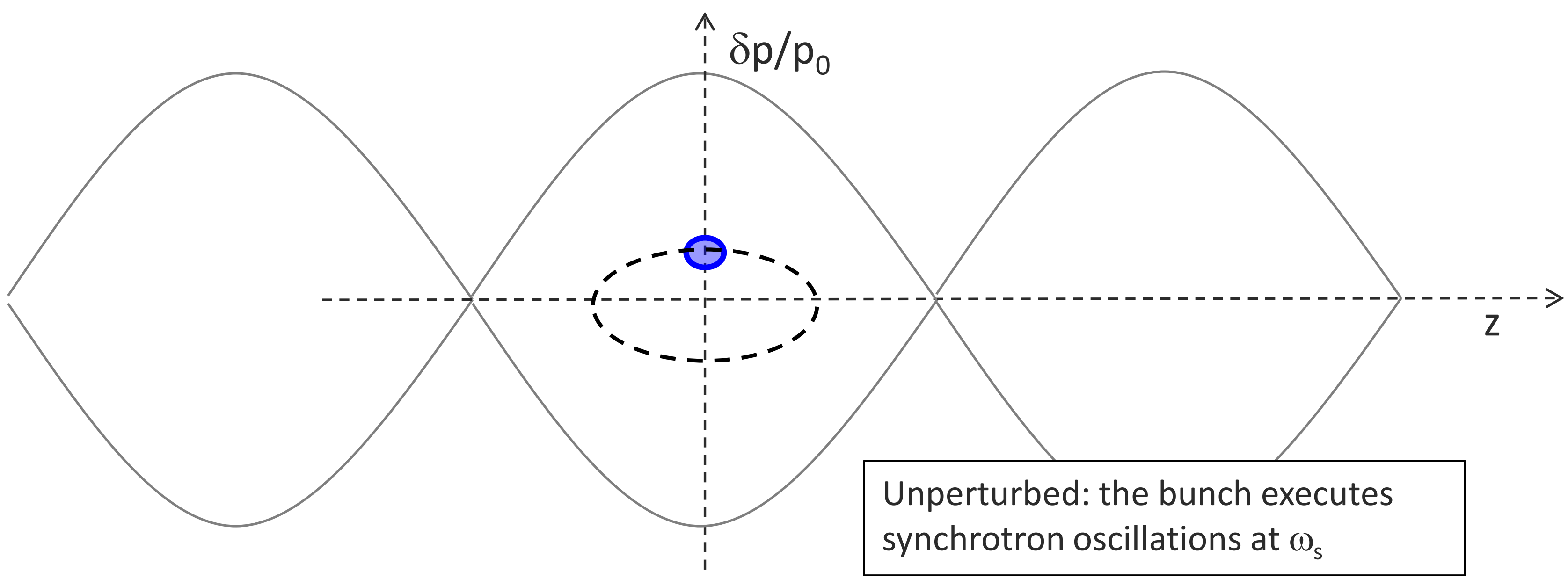
Part 2: Longitudinal wakefields –

impact on machine elements and beam dynamics

- Longitudinal wake fields and the longitudinal wake function
- Energy loss – beam induced heating and stable phase shift
- Potential well distortion, bunch lengthning and microwave instability
- Robinson instability

The Robinson instability

- To illustrate the Robinson instability we will use some simplifications:
  - The bunch is **point-like** and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  - The bunch additionally feels the effect of a **multi-turn wake**



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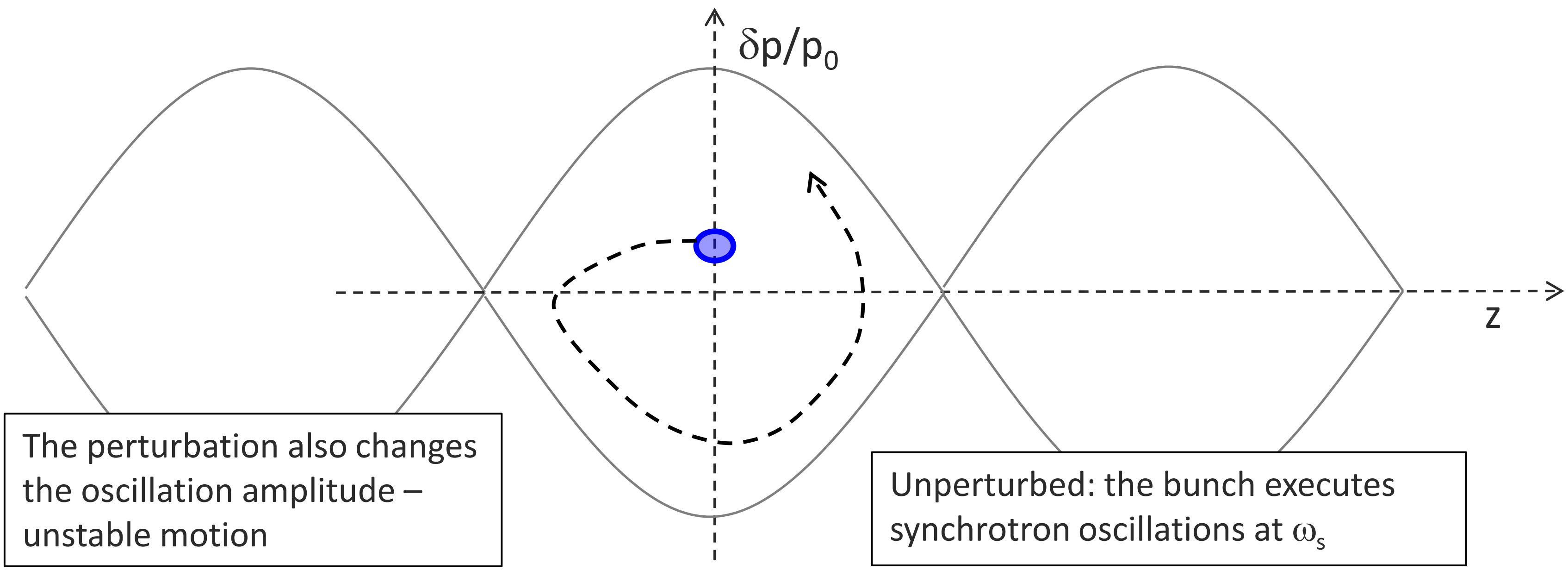

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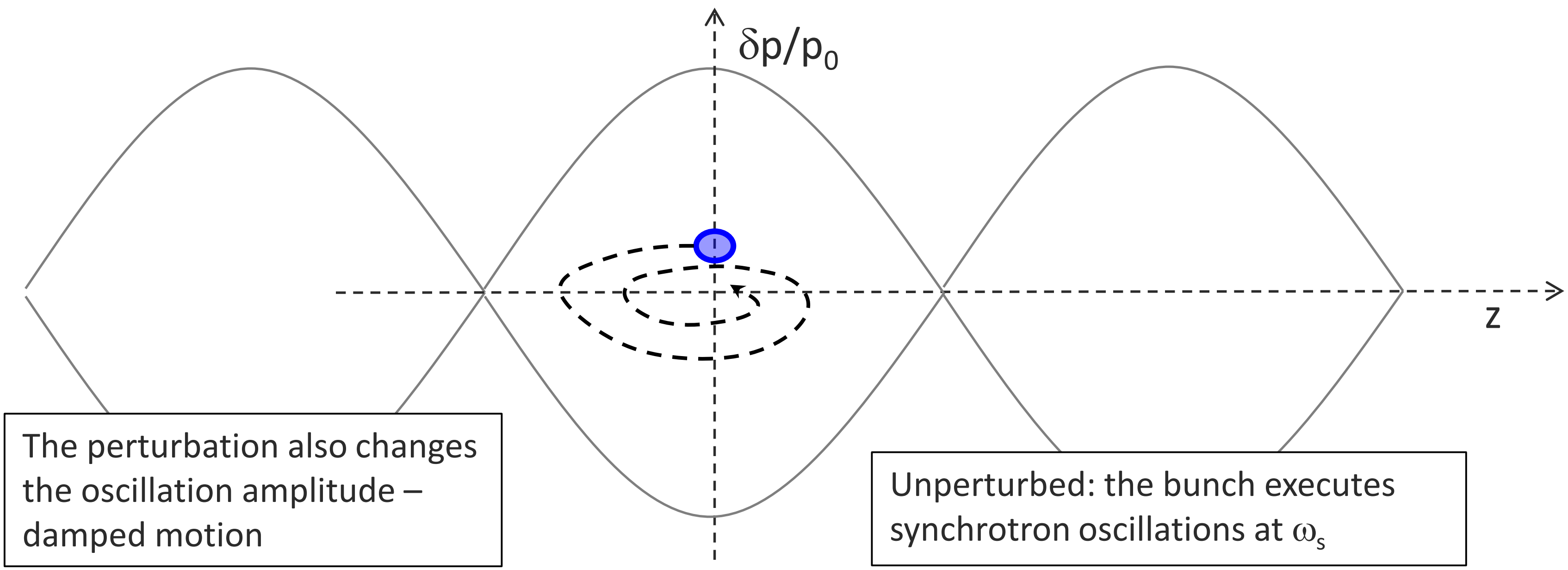

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  - The bunch is **point-like** and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  - The bunch additionally feels the effect of a **multi-turn wake**

- Longitudinal Hamiltonian

$$H = -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{e^2}{\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)\,W_{||}(z''-z'-kC)$$

$$= -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{Ne^2}{\beta^2 EC}\sum_k \int_0^z dz''\,W_{||}\Big(z(t) - z(t-kT_0) - kC\Big)$$

- Expansion of wake field (we assume that the wake can be linearized on the scale of a synchrotron oscillation)

$$W_{||}(z(t) - z(t-kT_0) - kC) \approx W_{||}(kC) + W'_{||}(kC)\Big(z(t) - z(t-kT_0)\Big)$$

$$\approx W_{||}(kC) + W'_{||}(kC)\,kT_0\,\frac{dz(t)}{dt}$$

The Robinson instability

- The **first term** only contributes as a constant term in the solution of the equation of motion, i.e. the synchrotron oscillation will be executed around a certain z0 and not around 0. This term represents the **stable phase shift** that compensates for the energy loss
- The **second term** is a dynamic term introduced as a **“friction” term** in the equation of the oscillator, which can **lead to instability**!

- Equations of motion

$$\frac{d^2 z}{dt^2} + \omega_s^2 \, z^2 = \frac{N e^2 \eta}{C m_0 \gamma} \sum_{k=-\infty}^{\infty} W_{\parallel}(kC) + W_{\parallel}'(kC)\, kT_0 \, \frac{dz}{dt}$$

The Robinson instability

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- Equations of motion

$$\frac{d^2 z}{dt^2} + \omega_s^2 z^2 = \frac{Ne^2\eta}{Cm_0\gamma} \sum_{k=-\infty}^{\infty} W_{||}(kC) + W'_{||}(kC)\, kT_0 \frac{dz}{dt}$$

- Ansatz

$$z(t) \propto \exp(-i\Omega t)$$

- Solution

$$(\Omega^2 - \omega_s^2) = -\frac{Ne^2\eta}{Cm_0\gamma} \sum_{k=-\infty}^{\infty} \left(1 - \exp(-ik\Omega T_0)\right) W'_{||}(kC)$$

Expressed in terms of impedance

$$\frac{i}{C} \sum_{p=-\infty}^{\infty} \left(p\omega_0 Z_{||}(p\omega_0) - (p\omega_0 + \Omega)\, Z_{||}(p\omega_0 + \Omega)\right)$$

The Robinson instability

- We assume a small deviation from the synchrotron tune:
  - Re(Ω - ω_s) → **Synchrotron tune shift**
  - Im(Ω - ω_s) → **Growth/damping rate**, only depends on the dynamic term, if it is positive there is an instability!

- **Solution:**

$$\left(\Omega^2 - \omega_s^2\right) = -\frac{iNe^2\eta}{C^2 m_0 \gamma} \sum_{p=-\infty}^{\infty} \left(p\omega_0 \, Z_{||}\left(p\omega_0\right) - \left(p\omega_0 + \Omega\right) \, Z_{||}\left(p\omega_0 + \Omega\right) \right)$$

$$\approx 2\omega_s \, \left(\Omega - \omega_s\right)$$

- **Tune shift:**

$$\Delta\omega_s = \mathrm{Re}\left(\Omega - \omega_s\right) = \frac{e^2}{m_0 c^2} \frac{N\eta}{2\omega_s \gamma T_0^2}$$

$$\sum_{p=-\infty}^{\infty} \left(p\omega_0 \, \mathrm{Im}\left[Z_{||}\right]\left(p\omega_0\right) - \left(p\omega_0 + \omega_s\right) \, \mathrm{Im}\left[Z_{||}\right]\left(p\omega_0 + \omega_s\right) \right)$$

- **Growth rate:**

$$\tau^{-1} = \mathrm{Im}\left[\Omega - \omega_s\right] = \frac{e^2}{m_0 c^2} \frac{N\eta}{2\omega_s \gamma T_0^2} \sum_{p=-\infty}^{\infty} \left(\left(p\omega_0 + \omega_s\right) \, \mathrm{Re}\left[Z_{||}\right]\left(p\omega_0 + \omega_s\right) \right)$$

The Robinson instability

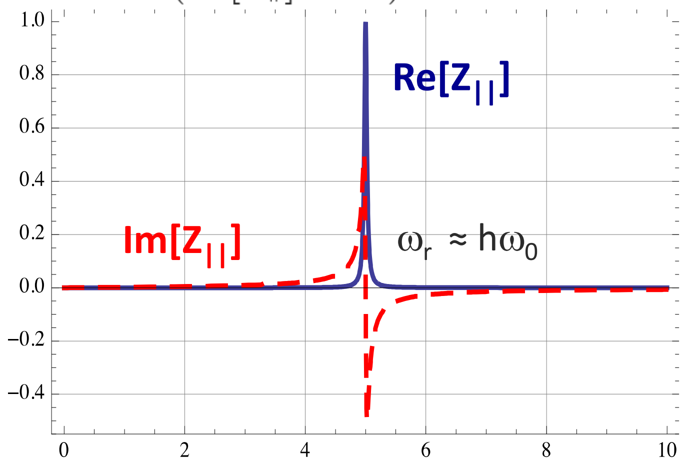
- We assume the impedance to be peaked at a frequency ω_r close to $h\omega_0 \gg \omega_s$ (e.g. RF cavity fundamental mode or HOM)
- Only two dominant terms are left in the summation at the RHS of the equation for the growth rate
- Stability requires that η and $\Delta\,\mathrm{Re}\left[Z_{\parallel}\right](p\omega_0)$ have different signs
- **Solution:**

$$\tau^{-1} = \mathrm{Im}\left(\Omega - \omega_s\right) = \frac{e^2}{m_0 c^2}\frac{N\eta}{2\omega_s \gamma T_0^2}\sum_{p=-\infty}^{\infty}\Big(\left(p\omega_0+\omega_s\right)\,\mathrm{Re}(Z)_{\parallel}(p\omega_0+\omega_s)\Big)$$

$$= \frac{e^2}{m_0 c^2}\frac{N\eta h\omega_0}{2\omega_s \gamma T_0^2}\underbrace{\Big(\mathrm{Re}\left[Z_{\parallel}\right](h\omega_0+\omega_s) - \mathrm{Re}\left[Z_{\parallel}\right](h\omega_0-\omega_s)\Big)}_{\Delta\left(\mathrm{Re}\left[Z_{\parallel}\right](h\omega_0)\right)}$$

- **Stability criterion:**

$$\eta\cdot\Delta\Big(\mathrm{Re}\left[Z_{\parallel}\right](h\omega_0)\Big) < 0$$



The Robinson instability

- **Stability criterion:** $\eta \cdot \Delta\Big(\mathrm{Re}\left[Z_{\parallel}\right](h\omega_0)\Big) < 0$

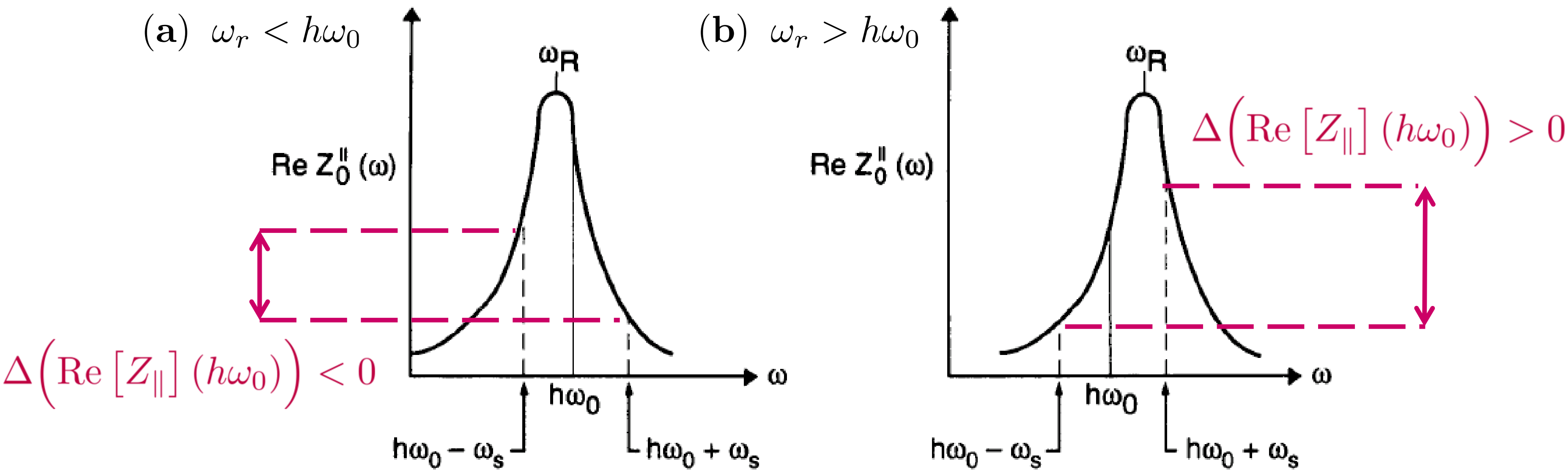
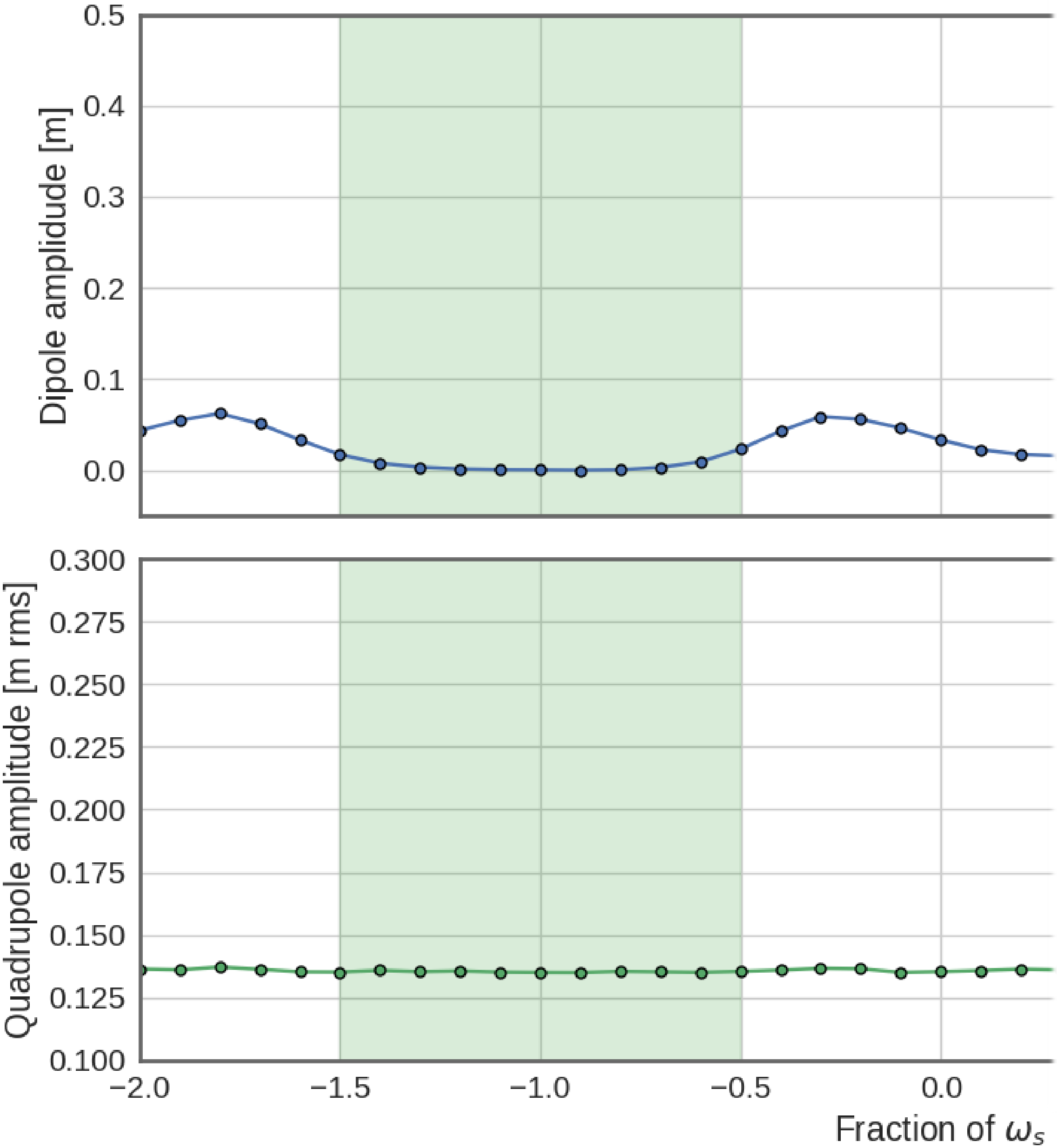


Figure 4.4. Illustration of the Robinson stability criterion. The rf fundamental mode is detuned so that ω_R is (a) slightly below $h\omega_0$ and (b) slightly above $h\omega_0$. (a) is Robinson damped above transition and antidamped below transition. (b) is antidamped above transition and damped below transition.

	$\omega_r < h\omega_0$	$\omega_r > h\omega_0$
Above transition ($\eta > 0$)	stable	unstable
Below transition ($\eta < 0$)	unstable	stable

Robinson damping and instability



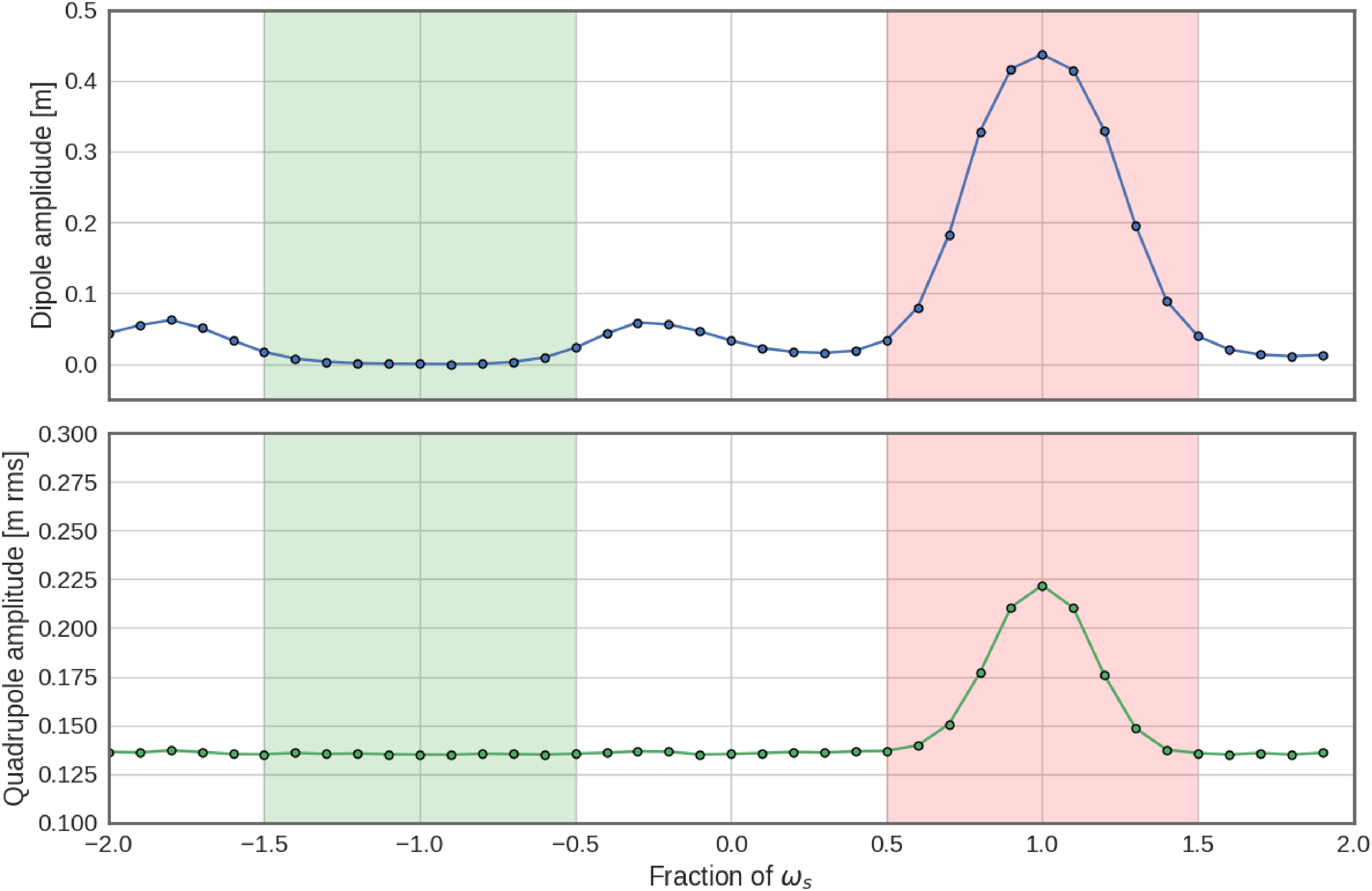
Examples of numerical simulations – SPS bunch with **single narrow-band resonator** wake:

Initializing an otherwise matched bunch with a slight momentum error, **two regimes are found**:

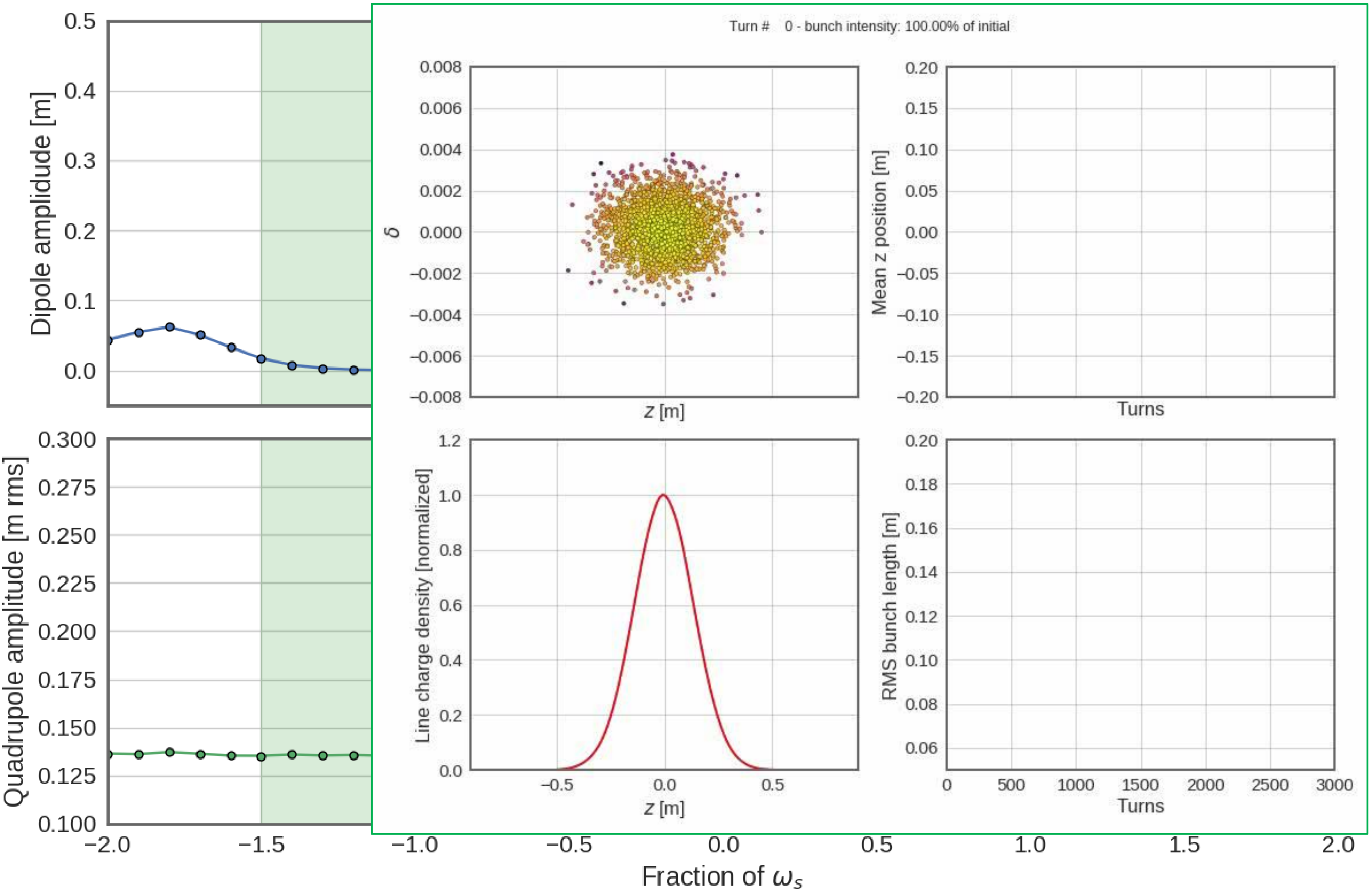
- Regime of **Robinson damping** when the resonator is **detuned to $h\omega_0 - \omega_s$**. Initial dipole oscillations are damped.

- Regime of **Robinson instability** when the resonator is **detuned to $h\omega_0 + \omega_s$**. Initial dipole oscillations start to are grow exponentially.

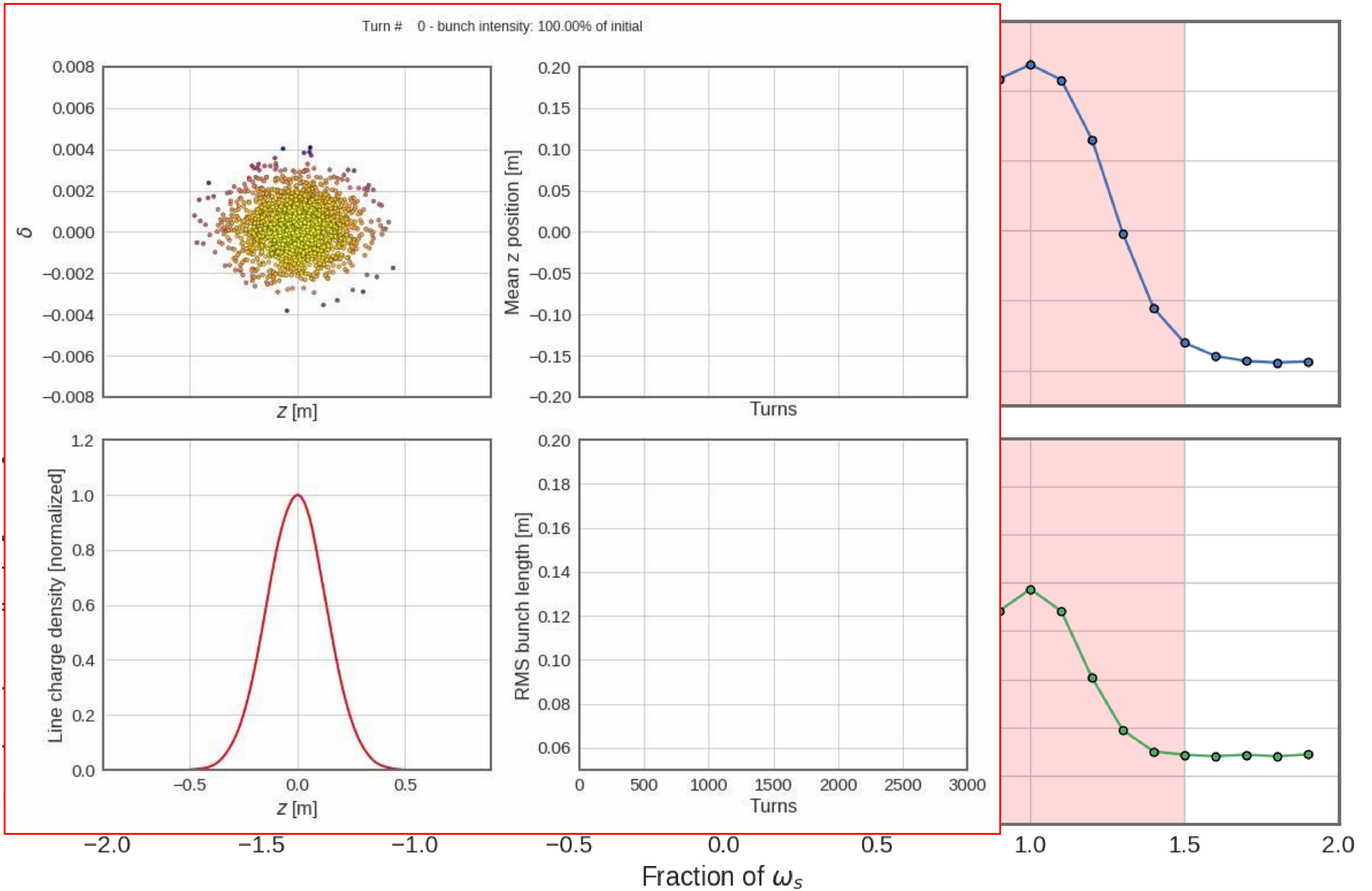
Robinson damping and instability



Robinson damping and instability



Robinson damping and instability



Other longitudinal instabilities

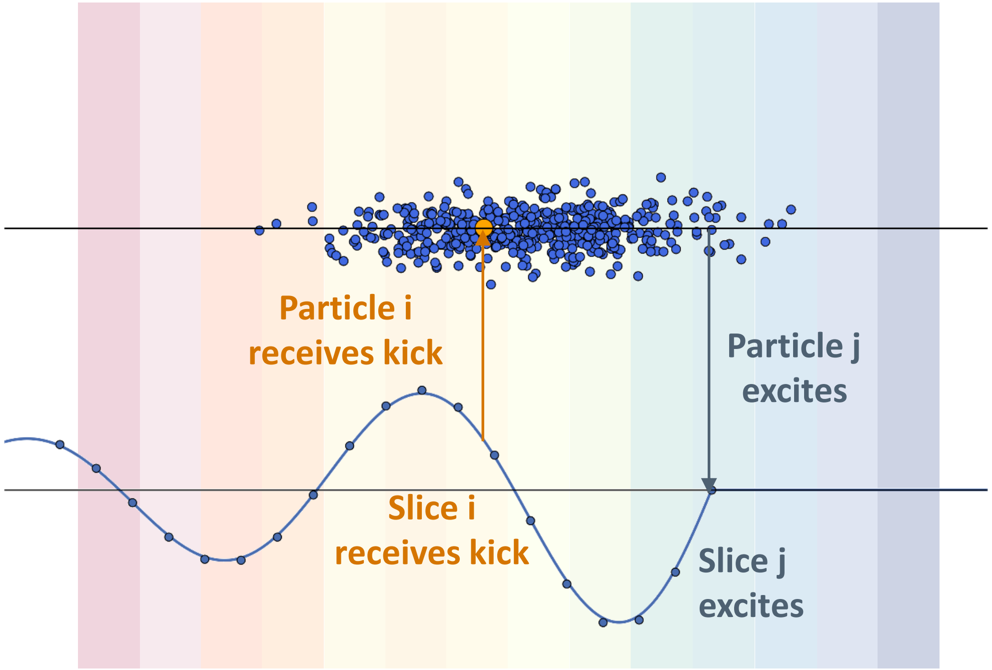
- The **Robinson instability** occurs for a single bunch under the action of a **multi-turn wake field**
  - It contains a term of coherent synchrotron tune shift which depends only on the imaginary part of the longitudinal impedance
  - It results into an unstable rigid bunch dipole oscillation where the growth rate depends on the real part of the longitudinal impedance

- Other **important collective effects** can affect a bunch in a beam – some of them of which we have also seen
  - Potential well distortion (resulting in synchronous phase shift, bunch lengthening or shortening, synchrotron tune shift/spread)
  - High intensity single bunch instabilities (e.g. microwave instability)
  - Coasting beam instabilities (e.g. negative mass instability)
  - Coupled bunch instabilities

- To be able to study these effects we would need to resort to a **more detailed description** of the bunch(es)
  - Vlasov equation (kinetic model)
  - Macroparticle simulations

Bunch energy loss per turn

- Single traversal of a bunch through an impedance source



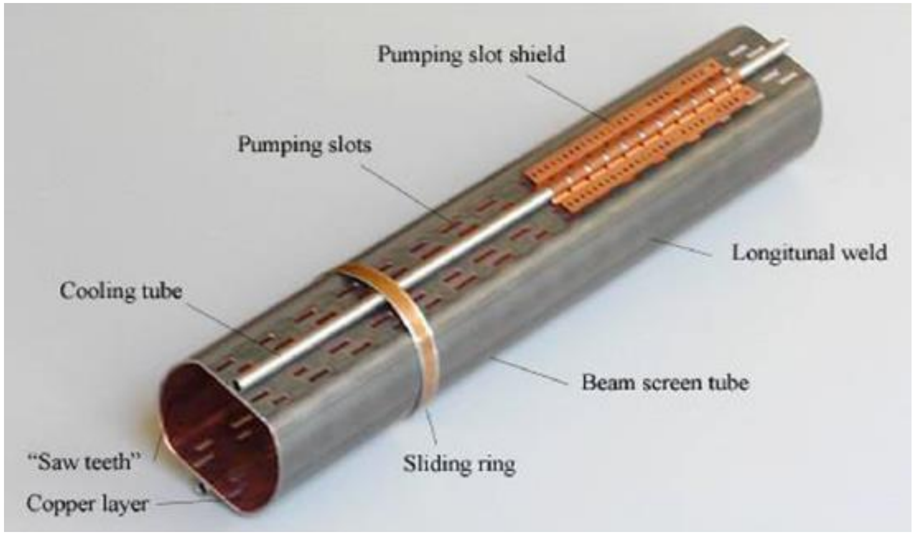
$$\Delta E_{ij} = -e^2 W_{||}(z_{ij})$$

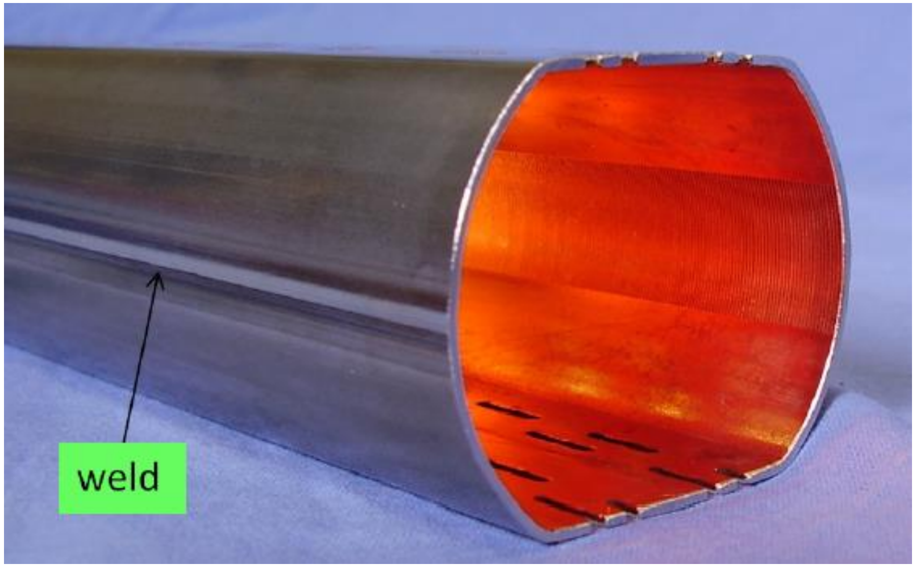
$$\Delta E_{bunch} = -e^2 \sum_{j=1}^{N_b} \sum_{i=1}^{N_b} W_{||}(z_{ij})$$

$$\Delta E_{ij} = -e^2 N[j] N[i] W_{||}[(i-j)\Delta z]$$

$$\Delta E_i = -e^2 N[i] \sum_{j=0}^{i} N[j] W_{||}[(i-j)\Delta z]$$

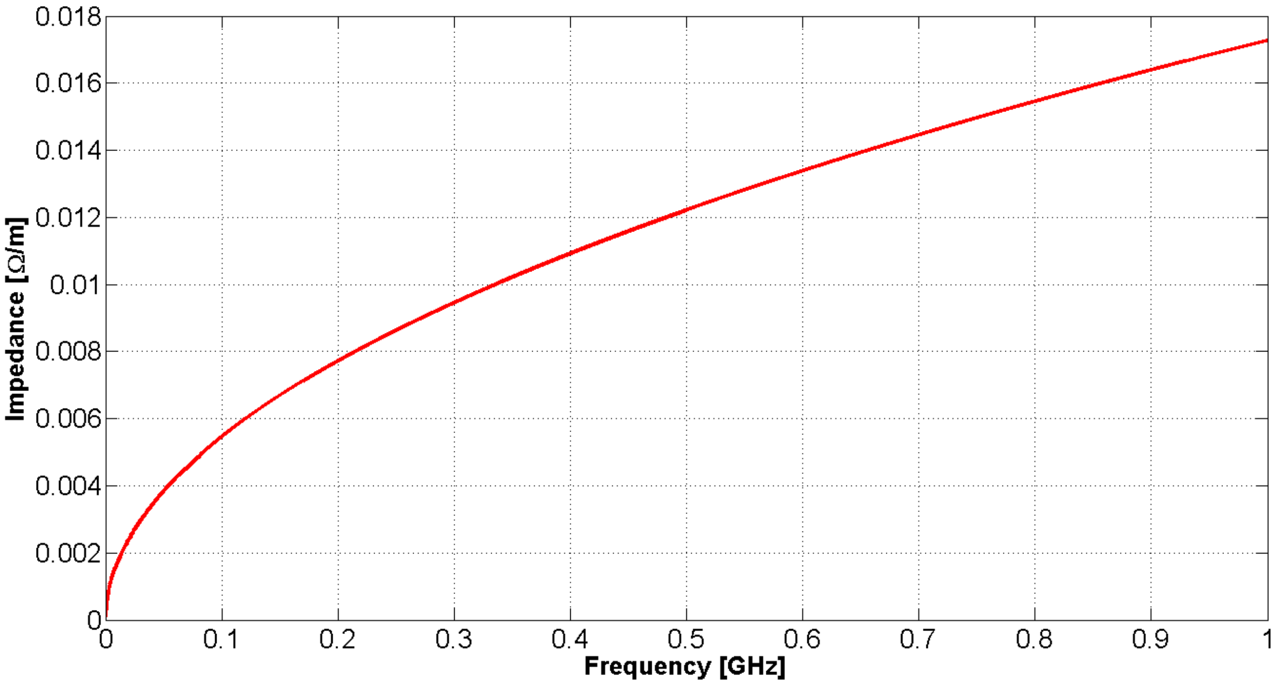
Application to the LHC beam screen





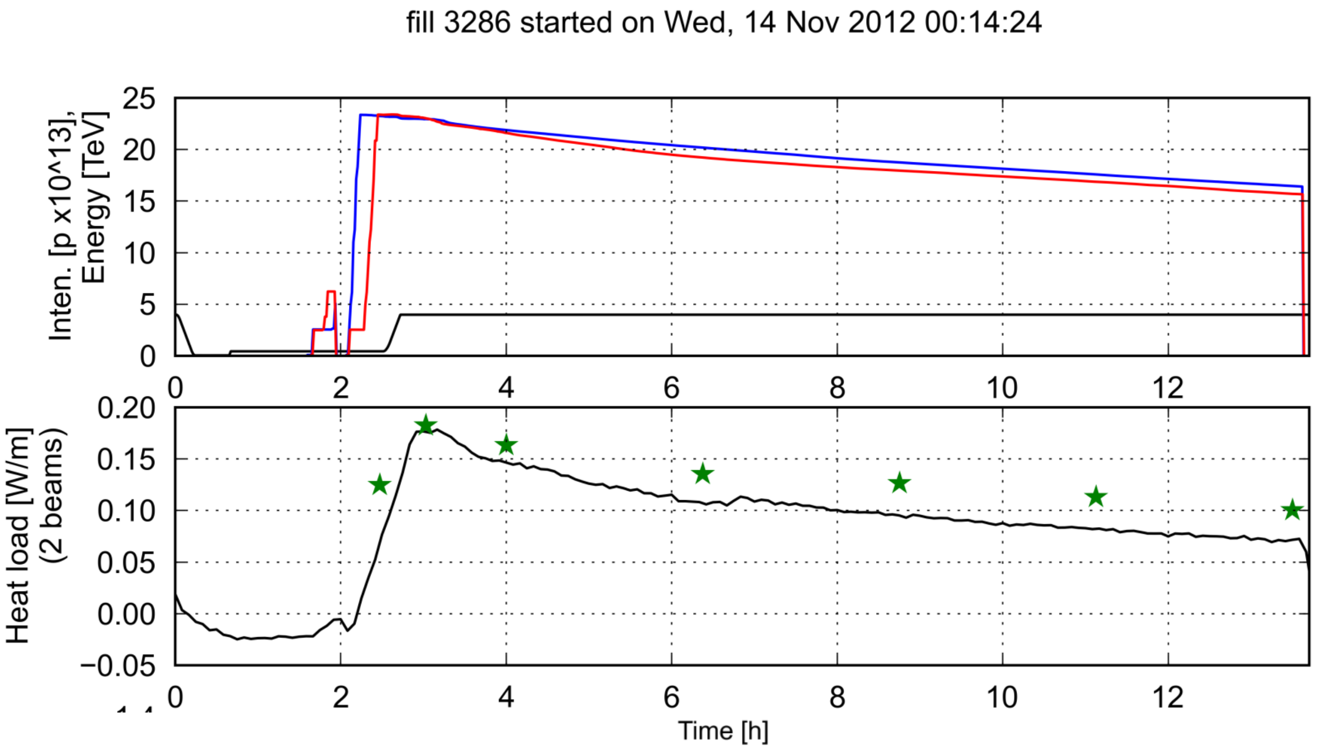
- All along the arcs and in other cold regions of the LHC, a beam screen is interposed between the beam and the cold bore
- The LHC beam screen is made of stainless steel with a layer of **few mm of co-laminated copper**
- Due to the production procedure, there is **a stainless steel weld** on one side of the beam screen that remains exposed to the beam.
- The screen has **holes for pumping** on top and bottom

Application to the LHC beam screen



- The impedance model includes the **weld on one side of the beam screen**, which means a small longitudinal stripe of exposed StSt, as well as **the pumping holes**

Application to the LHC beam screen



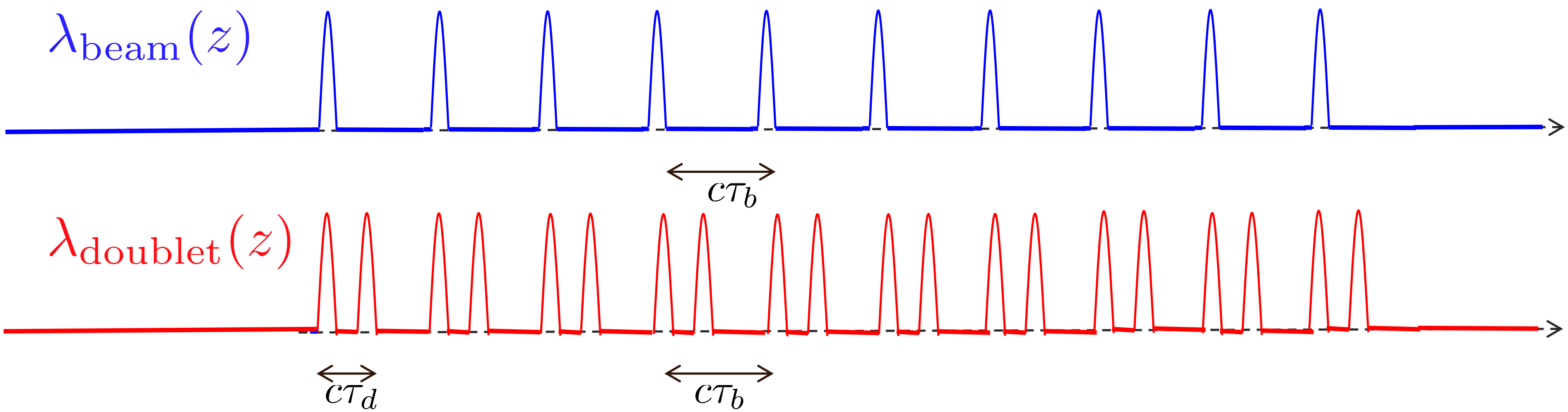
★ Estimation (impedance + synchrotron rad.)

Heat load measurement from cryogenics

$$\Delta E_{\text{beam}} = \frac{e^2\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 \text{Re}\left[Z_{||}(p\omega_0)\right] \cdot \left[\frac{1-\cos(\frac{2\pi M p}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$

- The heat dissipated on the beam screen **can be calculated for a beam made of bunches spaced by 50 ns** and compared to the measurement from cryogenics

Beam energy loss: a doublet beam



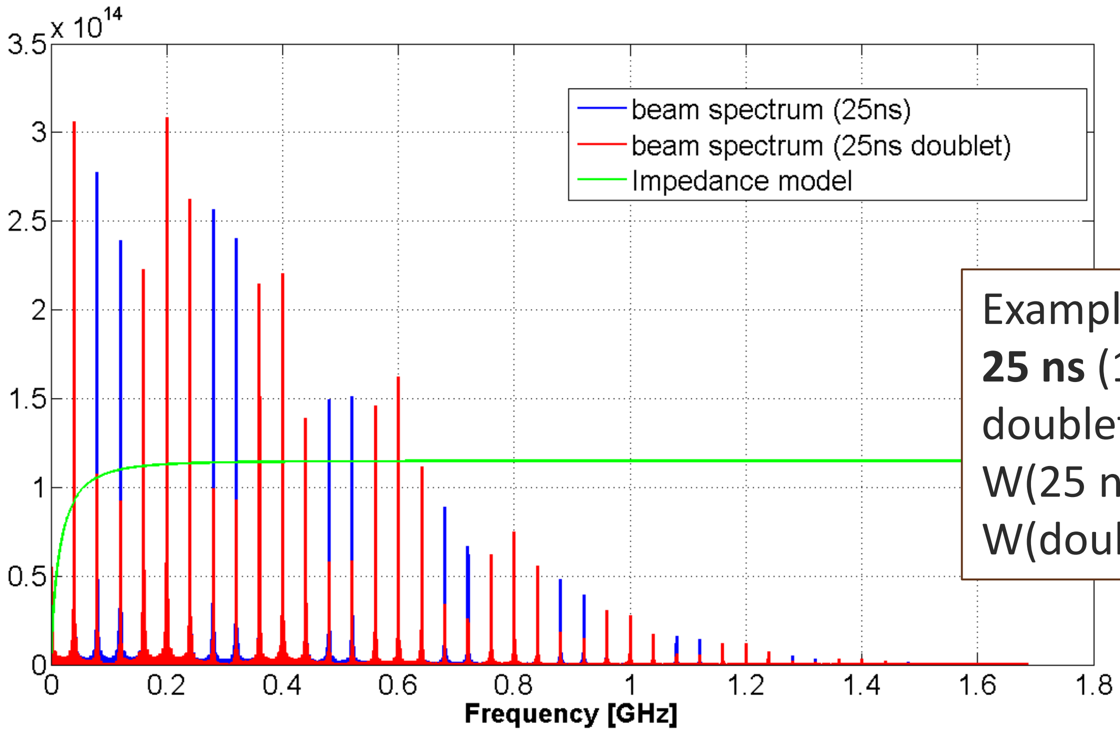
$$\Lambda_{\rm beam}(\omega) \rightarrow \Lambda_{\rm doublet}(\omega) = \Lambda_{\rm beam}(\omega)\left[1+\exp(-i\omega\tau_d)\right]$$

$$\Delta E_{\rm doublet} = \frac{2e^2\omega_0}{\pi}\sum_{p=-\infty}^{\infty} |\Lambda_{\rm beam}(p\omega_0)|^2 \cos^2\left(\frac{p\omega_0\tau_d}{2}\right) {\rm Re}\left[Z_{||}(p\omega_0)\right]$$

N.B. in this example the doublet has double total intensity than single beam

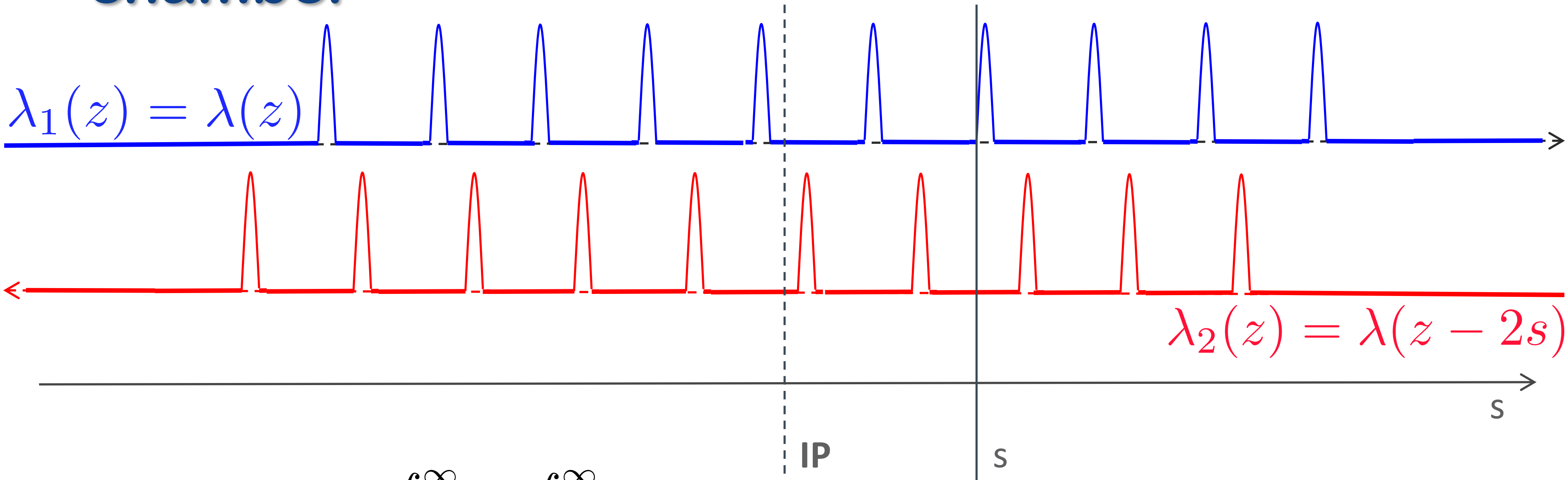
Beam energy loss: a doublet beam

- No additional impedance energy loss is expected with the doublet beam with respect to nominal beam for same total intensity
  - Beam power spectrum is modulated with $\cos^2$ function and lines are weakened by this modulation
  - For higher doublet intensity, global effect depends on the impedance spectrum
  - Example → LHC injection beam stopper (TDI)

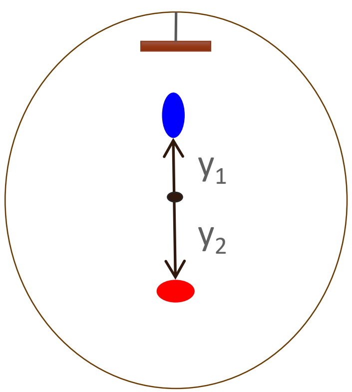


Example for LHC beam in TDI:
25 ns (1.2×10^{11} p/b) vs. **20+5 ns** doublet (1.5×10^{11} p/doublet)
W(25 ns) = 456 W
W(doublet) = 338 W

Beam energy loss: collider's common chamber



$$\Delta E_{\text{beam1}}(s) = e^2 \int_{-\infty}^{\infty} \lambda(z) \int_{-\infty}^{\infty} \left[\lambda(z') W_{||b1}(z-z') - \lambda(z'-2s) W_{||b2}(z-z') \right] dz' dz$$

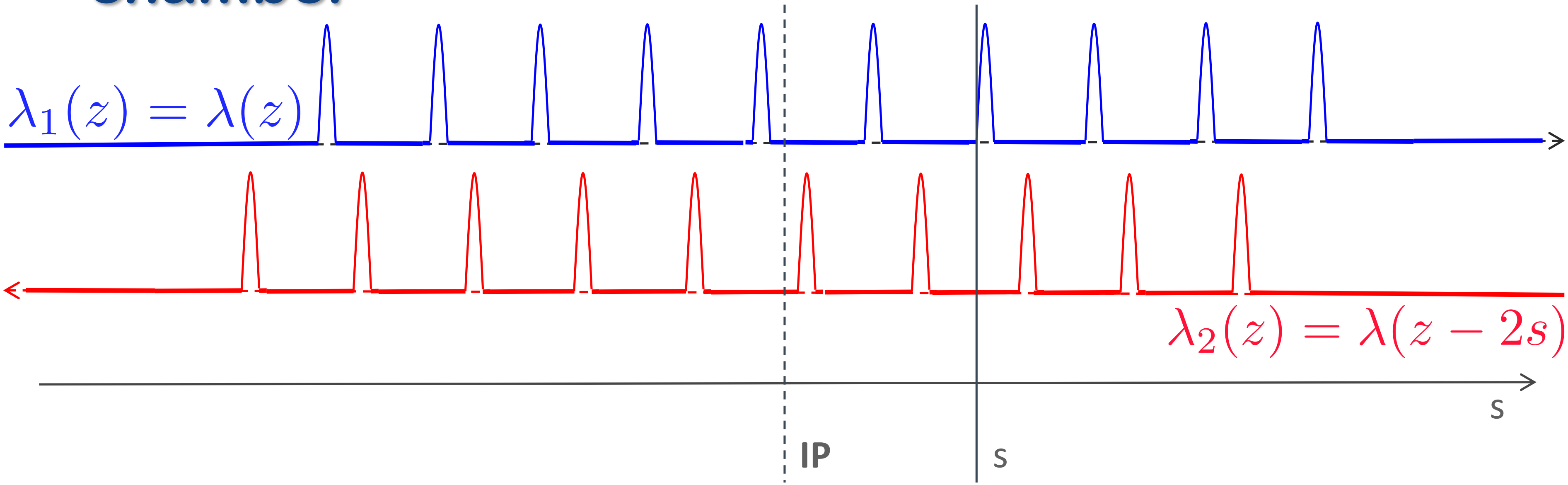


$$W_{||b1}(z) = W_{||}^{(0)}(z) + W_{||}^{(1d)}(z) y_1 + W_{||}^{(1q)}(z) y_1$$

$$W_{||b2}(z) = W_{||}^{(0)}(z) + W_{||}^{(1d)}(z) y_2 + W_{||}^{(1q)}(z) y_1$$

$$\text{with} \quad W_{||}^{1d}(z) = W_{||}^{1q}(z)$$

Beam energy loss: collider's common chamber



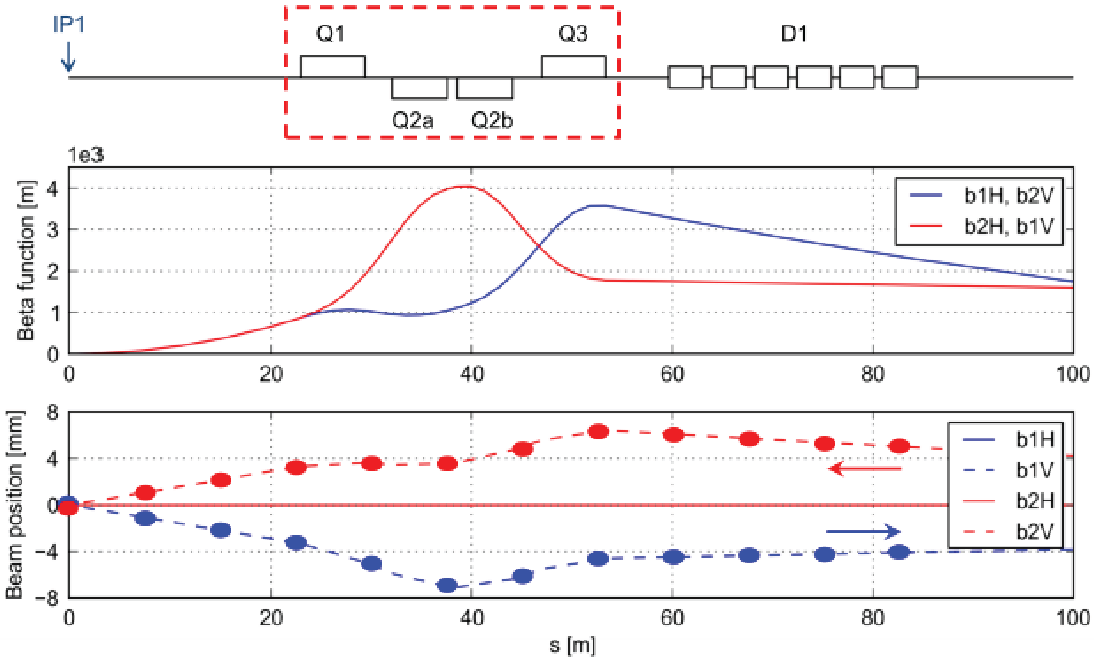
$$W_{||}^{1d}(z),\, W_{||}^{1q}(z) \overset{\mathcal{F}}{\Longleftrightarrow} Z_{||}^{1}(\omega) \qquad W_{||}^{0}(z) \overset{\mathcal{F}}{\Longleftrightarrow} Z_{||}^{0}(\omega)$$

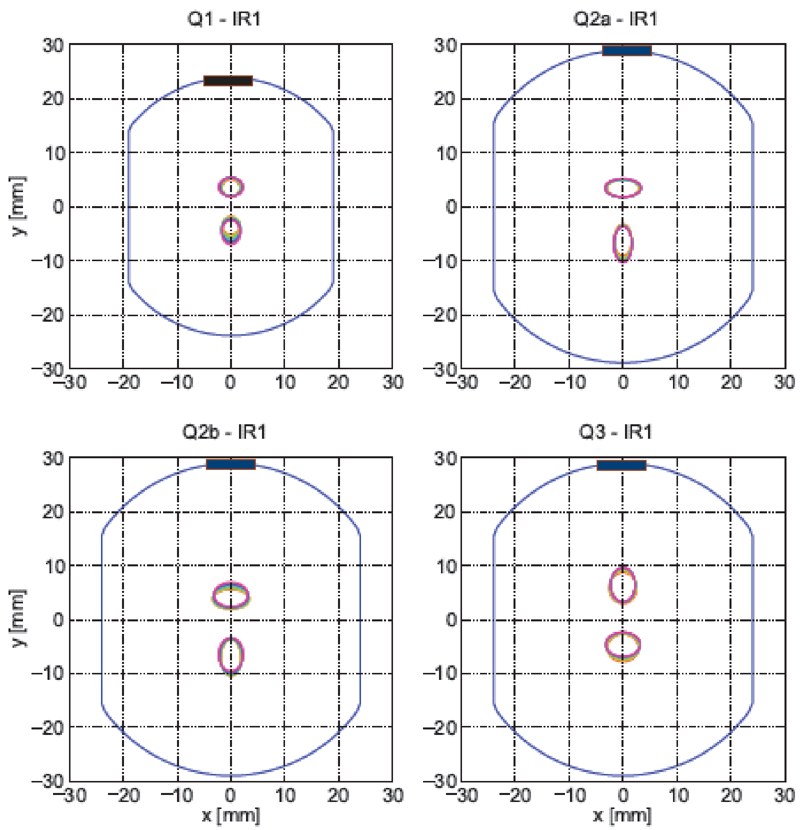
$$\Delta E_{\text{beam1}}(s) + \Delta E_{\text{beam2}}(s) =$$

$$\frac{4e^2\omega_0}{\pi}\sum_{p=0}^{\infty}|\Lambda(p\omega_0)|^2\left\{\text{Re}\left[Z_{||}^{0}(p\omega_0)\right] + [y_1(s)+y_2(s)]\,\text{Re}\left[Z_{||}^{1}(p\omega_0)\right]\right\}\cdot\sin^2\left(\frac{p\omega_0 s}{c}\right)$$

$$\Delta W_{CC} = \frac{\omega_0}{2\pi}\int_{-s_0}^{s_0}\left[\Delta E_{\text{beam1}}(s) + \Delta E_{\text{beam2}}(s)\right]ds$$

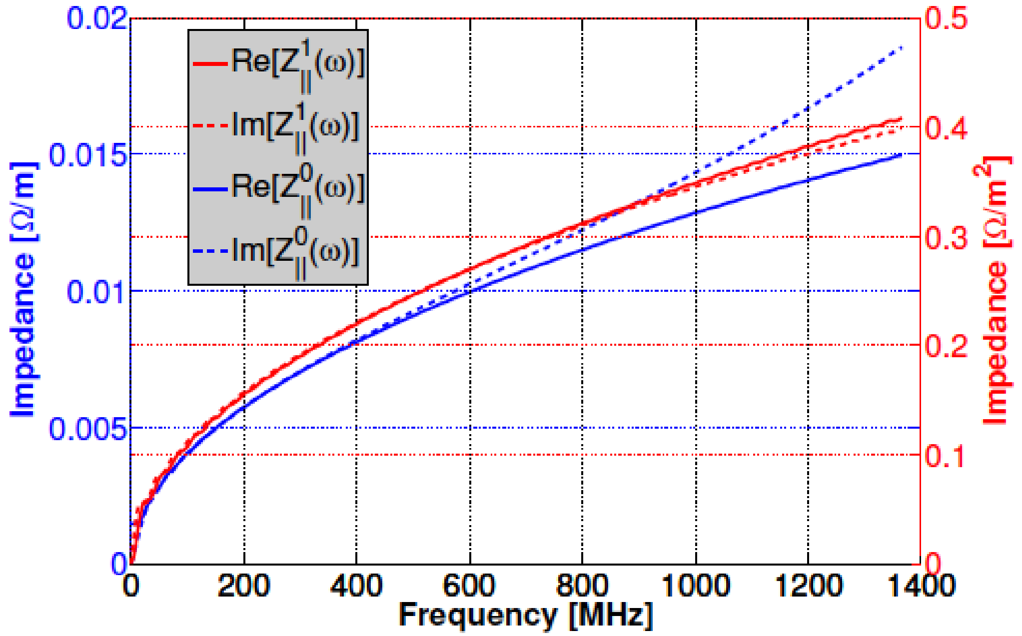
Beam energy loss in the LHC triplets





- Application to the LHC inner triplets
  - Beams are separated vertically (IP1) or horizontally (IP5)
  - Strongly off-axis for ~30m, all relative delays between beams swept
  - Asymmetric chamber in the direction of separation because of the weld

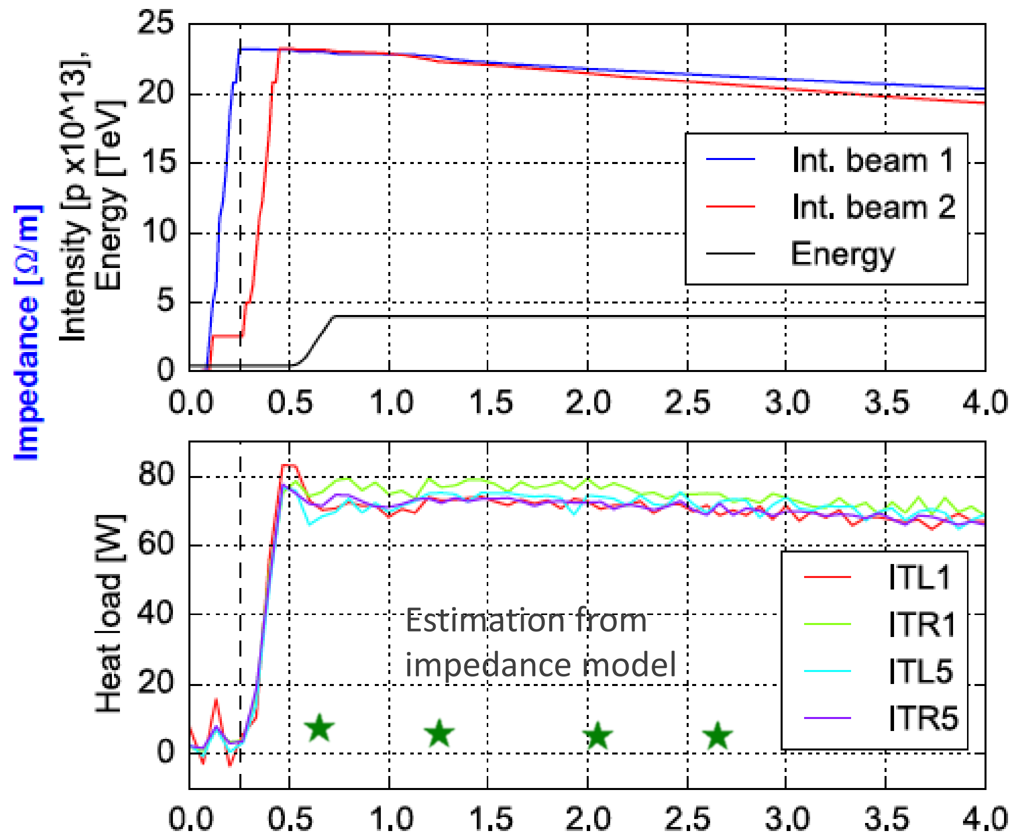
Beam energy loss in the LHC triplets



$$\Delta W_{IT} = 4\,\mathrm{W}$$

for a typical 50 ns fill of the LHC

Beam energy loss in the LHC triplets



- Comparison with measured data (L. Tavian)
  - Estimated heat load more than a factor 10 below measurement
  - Indication of a dominant contribution from electron cloud, also enhanced by the two-beam effect

$$\Delta W_{IT} = 4\,W$$

for a typical 50 ns fill of the LHC

Panofsky-Wenzel Theorem

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Source terms (displaced point charge traveling along s with speed v) in Cartesian coordinates:

$$\rho(x, y, s, t) = q_1 \delta(x - x_1)\delta(y - y_1)\delta(s - vt)$$

$$\vec{j}(x, y, s, t) = \rho(x, y, s, t)\vec{v}$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Source terms (displaced point charge traveling along s with speed v) in cylindrical coordinates:

$$\rho(r, \theta, s, t) = \frac{q_1}{r_1} \delta(r - r_1) \delta_P(\theta) \delta(s - vt) =$$

$$= \frac{q_1}{r_1} \delta(r - r_1) \delta(s - vt) \sum_{m=0}^{\infty} \frac{\cos m\theta}{\pi(1 + \delta_{m0})}$$

$$\vec{j}(r, \theta, s, t) = \rho(r, \theta, s, t) \vec{v}$$

$$v = \beta c \;\; \text{with} \;\; \beta \approx 1$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_s}{\partial s} = \frac{\rho}{\epsilon_0}$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_s}{\partial s} = 0$$

$$\frac{\partial B_s}{\partial y} - \frac{\partial B_y}{\partial s} - \frac{1}{c^2}\frac{\partial E_x}{\partial t} = 0$$

$$\frac{\partial B_x}{\partial s} - \frac{\partial B_s}{\partial x} - \frac{1}{c^2}\frac{\partial E_y}{\partial t} = 0$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \frac{1}{c^2}\frac{\partial E_s}{\partial t} = \mu_0 \rho c$$

$$\frac{\partial E_s}{\partial y} - \frac{\partial E_y}{\partial s} + \frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial E_x}{\partial s} - \frac{\partial E_s}{\partial x} + \frac{\partial B_y}{\partial t} = 0$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_s}{\partial t} = 0$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

We want to find relations between the forces on the witness charge:

$$\vec{F}_\perp = q_2[(E_x - cB_y)\hat{x} + (E_y + cB_x)\hat{y}]$$

$$F_s = q_2 E_s$$

with

$$s - ct = z$$

$$\frac{\partial}{\partial s} = \frac{\partial}{\partial z} = -\frac{1}{c}\frac{\partial}{\partial t}$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_s}{\partial s} = \frac{\rho}{\epsilon_0} \qquad\qquad \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_s}{\partial s} = 0$$

We first use this set of equations

$$\frac{\partial B_s}{\partial y} - \frac{\partial B_y}{\partial s} - \frac{1}{c^2}\frac{\partial E_x}{\partial t} = 0 \qquad\qquad \frac{\partial E_s}{\partial y} - \frac{\partial E_y}{\partial s} + \frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial B_x}{\partial s} - \frac{\partial B_s}{\partial x} - \frac{1}{c^2}\frac{\partial E_y}{\partial t} = 0 \qquad\qquad \frac{\partial E_x}{\partial s} - \frac{\partial E_s}{\partial x} + \frac{\partial B_y}{\partial t} = 0$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \frac{1}{c^2}\frac{\partial E_s}{\partial t} = \mu_0 \rho c \qquad\qquad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_s}{\partial t} = 0$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial F_x}{\partial z} - \frac{\partial F_s}{\partial x} = 0$$

$$\frac{\partial F_s}{\partial y} - \frac{\partial F_y}{\partial z} = 0$$

$$\frac{\partial \vec{F}_\perp}{\partial z} = \nabla_\perp F_s$$

$$\frac{\partial \int_0^L \vec{F}_\perp ds}{\partial z} = \nabla_\perp \int_0^L F_s ds$$

Result known as Panofsky-Wenzel theorem

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial \int_0^L \vec{F}_\perp ds}{\partial z} = \nabla_\perp \int_0^L F_s ds$$

$$\int_0^L F_x ds = W_x(z)\Delta x_1 + W_{Qx}(z) x$$

$$W'_x(z) = W_{||}^{(dq)}(z) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \frac{\omega}{c} Z_x(\omega) = Z_{||}^{(dq)}(\omega)$$

$$W'_{Qx}(z) = 2W_{||}^{(2q)}(z) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \frac{\omega}{c} Z_{Qx}(\omega) = 2Z_{||}^{(2q)}(\omega)$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial \int_0^L \vec{F}_\perp ds}{\partial z} = \nabla_\perp \int_0^L F_s ds$$

$$\int_0^L F_x ds = W_x(z)\Delta x_1 + W_{Qx}(z)x$$

The longitudinal and transverse wake functions are not independent, although in general no relation can be established between $W_{||}(z)$ and $W_{x,y}(z)$, which are the main wakes in the longitudinal and transverse planes, respectively.

$$W'_x(z) = W \ldots \quad \ldots {}^{(dq)}_{||}(\omega)$$

$$W'_{Qx}(z) = 2W_{||}^{(2q)}(z) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \frac{\omega}{c} Z_{Qx}(\omega) = 2Z_{||}^{(2q)}(\omega)$$

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$$\frac{\partial E_s}{\partial y} - \frac{\partial E_y}{\partial s} + \frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial E_x}{\partial s} - \frac{\partial E_s}{\partial x} + \frac{\partial B_y}{\partial t} = 0$$

We can now use also these two sets of equations to find additional properties of the wakes

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \frac{1}{c^2}\frac{\partial E_s}{\partial t} = \mu_0 \rho c$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_s}{\partial t} = 0$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial F_x}{\partial x} = -\frac{\partial F_y}{\partial y}$$

$$\frac{\partial \int_0^L F_x ds}{\partial x} = -\frac{\partial \int_0^L F_y ds}{\partial y}$$

$$W_{Qx}(z) = -W_{Qy}(z)$$

This is an interesting result!
The quadrupolar wakes in x and y must be equal with opposite signs

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

$$\frac{\partial \int_0^L F_x ds}{\partial y} = \frac{\partial \int_0^L F_y ds}{\partial x}$$

This relation means that the cross-wakes between x and y must be equal.
We have so far ignored these terms in our derivations.

Instabilities

Synchrotron tune shift

- The **equilibrium distribution** in the presence of a longitudinal wake field can be found analytically. The (linearized) **longitudinal Hamiltonian** with longitudinal wake fields is given as:

$$H = -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{e^2}{\beta^2 EC}\sum_k \int dz'' \int dz'\,\lambda(z'+kC)W_0'(z''-z'-kC)$$

- Remember the example of the harmonic oscillator:

$$H = \frac{1}{2}p^2 \qquad + \frac{1}{2}\boxed{W}\,q^2$$

Coefficient determines frequency/tune

Synchrotron tune shift

- The **equilibrium distribution** in the presence of a longitudinal wake field can be found analytically. The (linearized) **longitudinal Hamiltonian** with longitudinal wake fields is given as:

$$H = -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \underbrace{\frac{e^2}{\beta^2 EC}\sum_k \int dz'' \int dz'\,\lambda(z'+kC)W_0'(z''-z'-kC)}_{\text{we make an expansion in } z \text{ – factor out } \frac{1}{2\eta\beta^2c^2}}$$

- Remember the example of the harmonic oscillator:

$$H = \frac{1}{2}p^2 + V\,q + \frac{1}{2}W\,q^2$$

Coefficient determines frequency/tune (W)

Term determines center position/orbit ($+V\,q$)

Synchrotron tune shift

- The **equilibrium distribution** in the presence of a longitudinal wake field can be found analytically. The (linearized) **longitudinal Hamiltonian** with longitudinal wake fields is given as:

$$H = -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \underbrace{\frac{e^2}{\beta^2 EC}\sum_k \int dz'' \int dz'\,\lambda(z'+kC)W_0'(z''-z'-kC)}_{\text{expansion in } z \text{ – factor } \frac{1}{2\eta\beta^2c^2}}$$

- It follows then quite easily that:

$$\Delta\omega_s \approx -\frac{1}{2\omega_s}\frac{e^2\,\eta c^2}{EC}\int dz'\,\lambda(z')W''0(z-z')$$
$$= -\frac{i}{4\pi}\frac{e^2\,\eta c^2}{\omega_s\,EC}\int d\omega\,\hat{\lambda}(\omega)Z_0(\omega)\frac{\omega}{c}$$

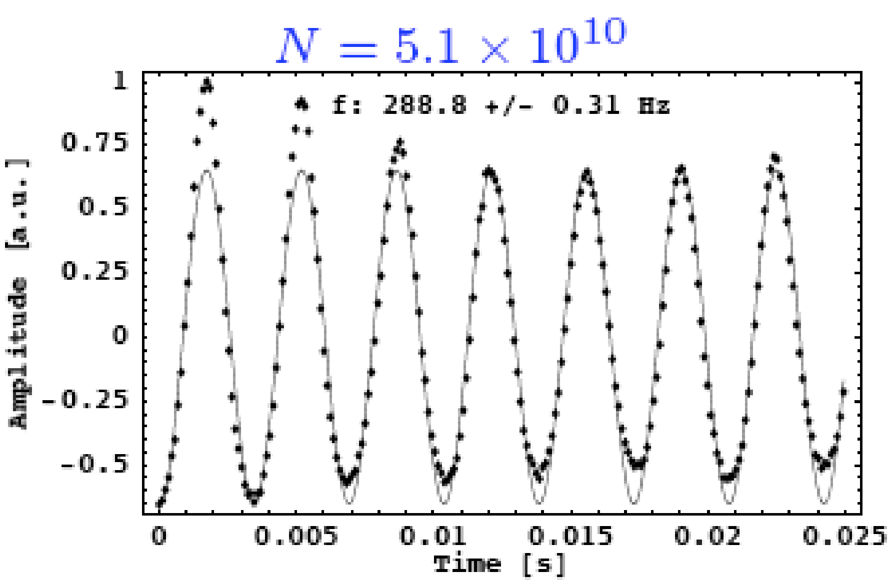
Remember, we make use of:
$\Omega^2 - \omega_s^2 \approx 2\omega_s\,\Delta\omega_s$

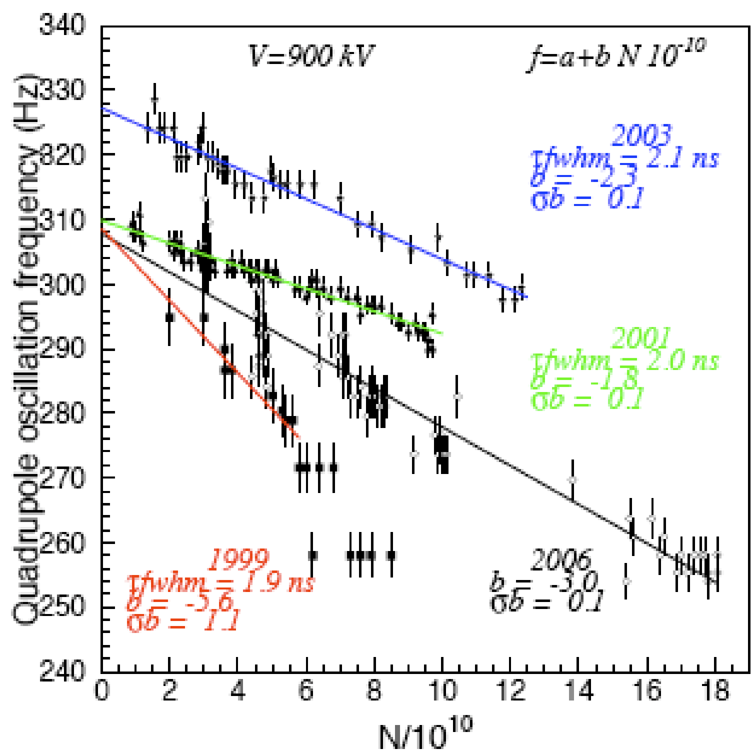
- The synchrotron tune shift from an impedance is, hence, given as:

$$\Delta Q_s = -\frac{1}{4\omega_s}\frac{e^2\,\eta}{(2\pi^2)E}\int d\omega\,\omega\hat{\lambda}(\omega)\,\mathrm{Im}[Z_0(\omega)]$$

Measurements of synchrotron tune shift at SPS

- The slope of the **incoherent synchrotron tune shift with intensity**, measured in reproducible conditions over the years, shows the evolution of the **imaginary part of the machine impedance** (E. Shaposhnikova, T. Bohl, J. Tuckmantel)
  - The technique uses the quadrupole oscillations of a bunch injected with a mismatch
  - Qs can be extrapolated from bunch length or peak amplitude measurements



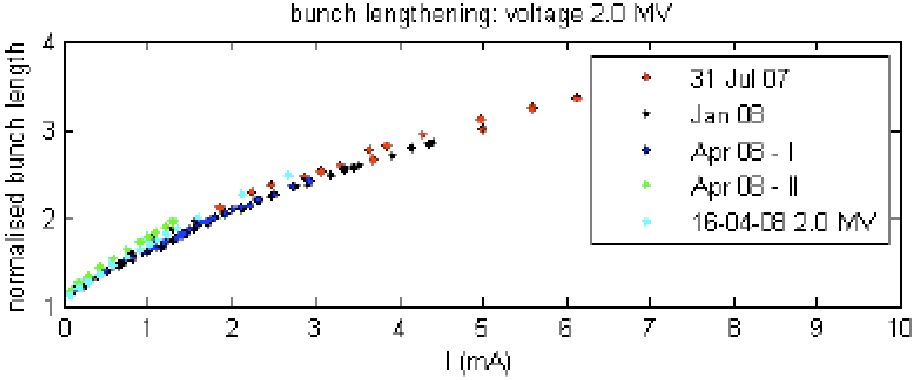


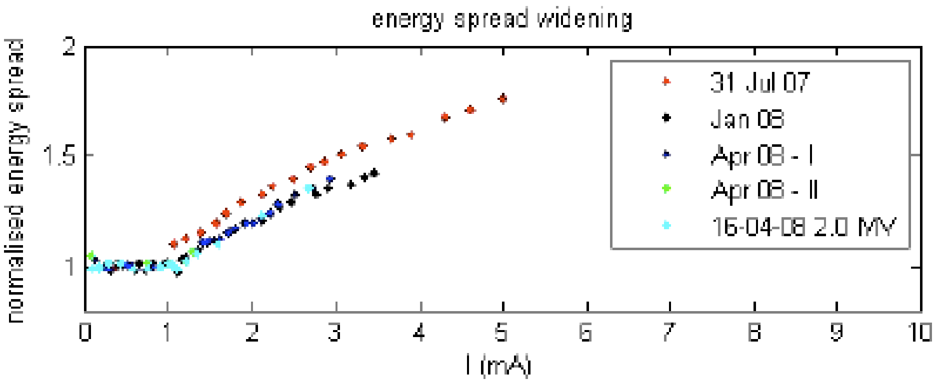
Measurements of potential well distortion
Stable phase and bunch lengthening

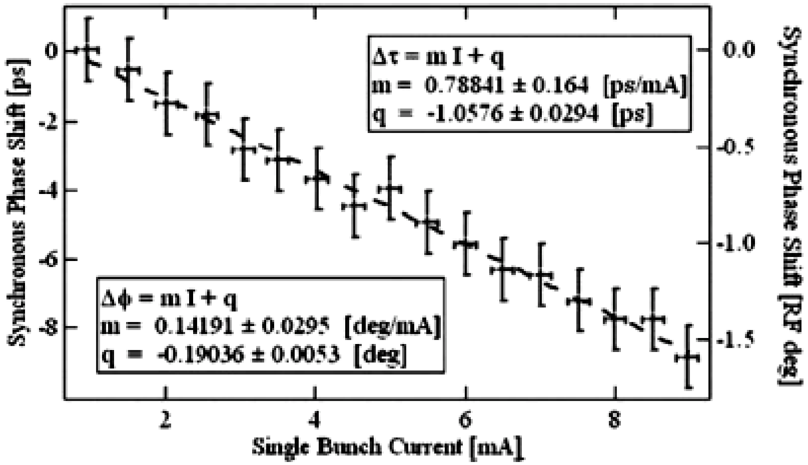
Measurements at light sources

⇒ Bunch lengthening @DIAMOND (left, R. Bartolini)

⇒ Energy loss measured through the synchronous phase shift @Australian light source (right, R. Dowd, M. Boland, G. LeBlanc, M. Spencer, Y. Tan, PAC07)







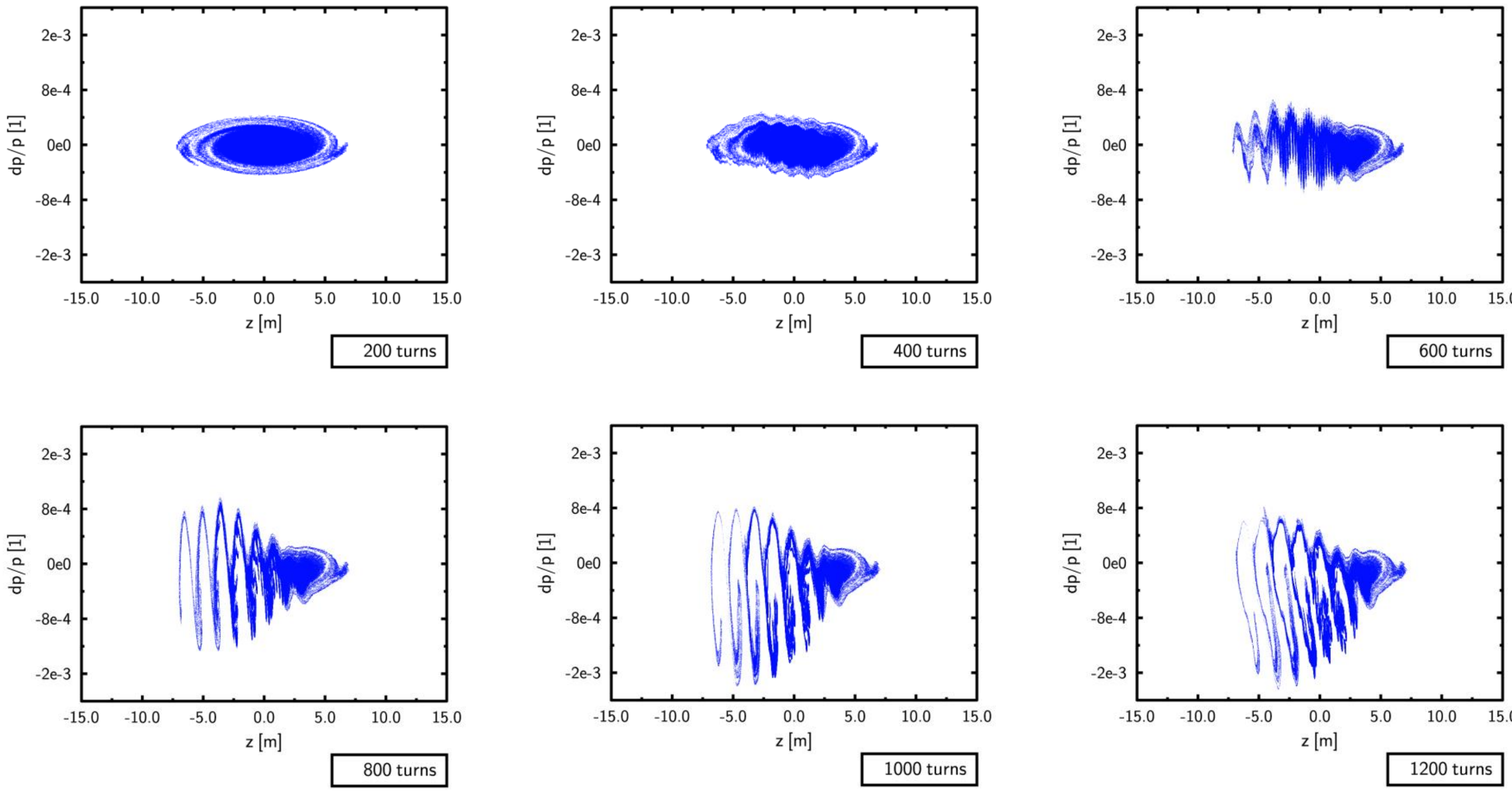
Synchronous phase shift measured with a streak camera in the Australian Synchrotron.

Examples of numerical simulations debunching bunch with SPS impedance model

Microwave instability on a debunching bunch is used at SPS for probing the machine impedance (E. Shaposhnikova, T. Bohl, H. Timkó, et al.)

⇒ Long bunch injected into SPS with RF off slowly debunches due to low momentum spread

⇒ Spectrum of bunch profile reveals important components for the impedance



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⇒ Long bunch injected into SPS with RF off slowly debunches due to low momentum spread

⇒ Spectrum of bunch profile reveals important components for the impedance

⇒ Simulations with impedance model are used to match measured profile