



Superconducting Magnets

Part I

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Doctoral Studies Course “Accelerator technologies” (HEP700)

CERN – 9 May 2022



Overview

- Why superconductors ? A motivation
 - Superconducting magnet design
 - Magnetic field and field quality
 - Margins and stability
 - Quench protection
 - Forces and mechanics
-
- A brief introduction to superconductivity
 - Discoveries and principles
 - Technical superconductors
 - Superconducting HEP magnets
 - Other superconducting magnet systems

Part I

Part II

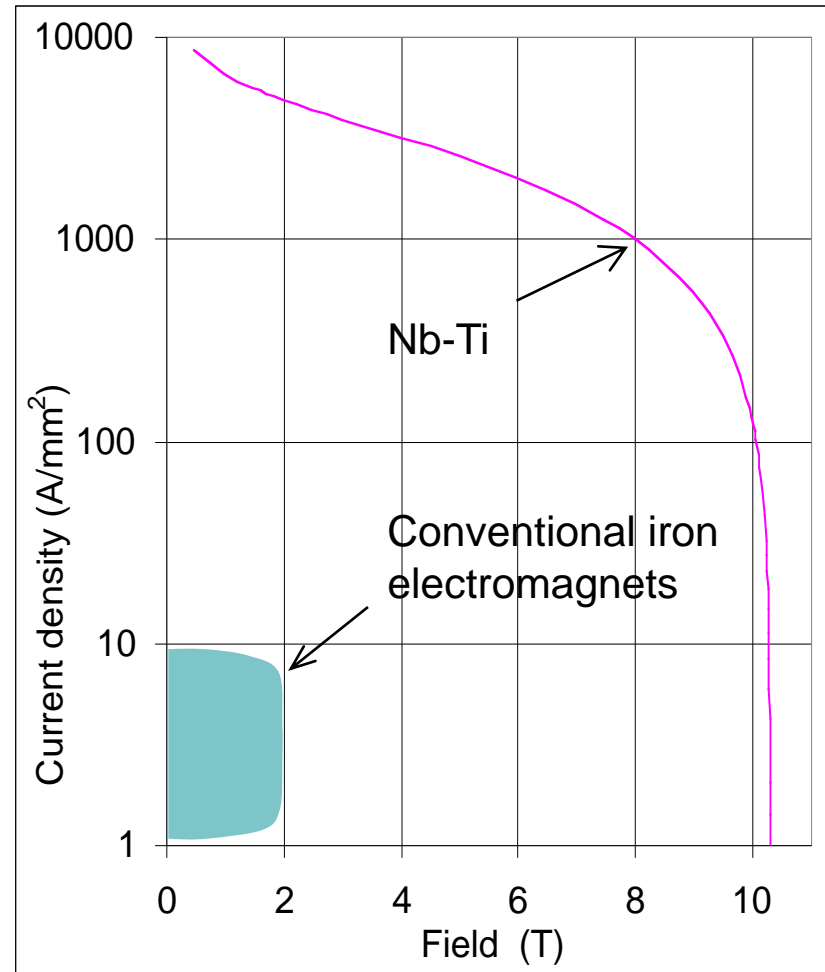


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- Superconducting magnet design
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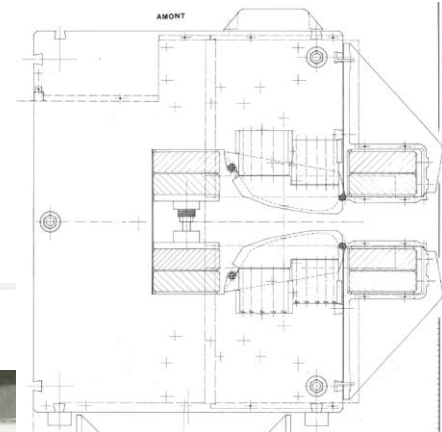
Why superconductivity anyhow ?

- **Abolish Ohm's law !**
 - no power consumption (although need refrigeration power)
 - high current density
 - ampere turns are cheap, so don't need iron (although often use it for shielding)
- **Consequences**
 - lower running cost \Rightarrow new commercial possibilities
 - energy savings
 - high current density \Rightarrow smaller, lighter, cheaper magnets \Rightarrow reduced capital cost
 - higher magnetic fields economically feasible \Rightarrow new research possibilities



NC vs. SC Magnets - 1/2

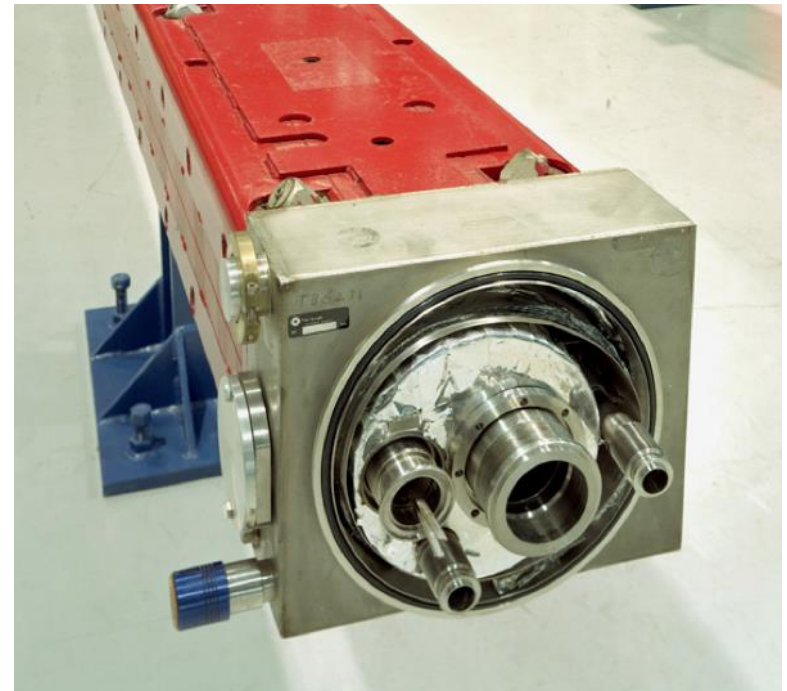
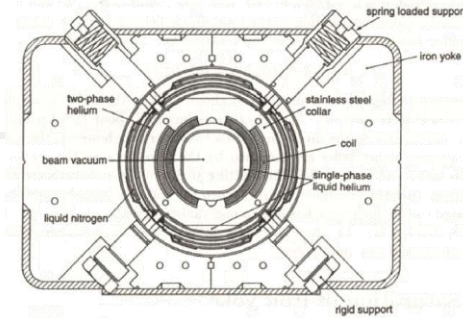
- *Normal conducting* accelerator magnets
 - Magnetization *ampere-turns* are *cheap*
 - Field is generated by the iron yoke (but limited by saturation, e.g. ≈ 2 T for iron)
 - Low current density in the coils to limit electric power and cooling needs
 - Bulky and heavy, large mass of iron (cost driver)



One of the dipole magnets of the PS, in operation at CERN since 1959

NC vs. SC Magnets - 2/2

- Superconducting accelerator magnets
 - Superconducting *ampere-turns* are *cheap*
 - Field generated by the coil current (but limited by critical current, e.g. ≈ 10 T for NbTi)
 - High current density, compact, low mass of high-tech SC material (cost driver)
 - Requires efficient and reliable cryogenics cooling for operation (availability driver)



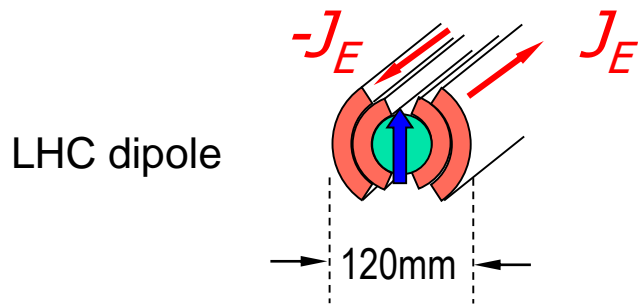
A superconducting dipole magnet of the Tevatron at FNAL, the first superconducting synchrotron, 1983

High current density - dipoles

- The field produced by an ideal dipole (see later) is:

$$B = \mu_0 f_{dip} J_e \frac{t}{2}$$

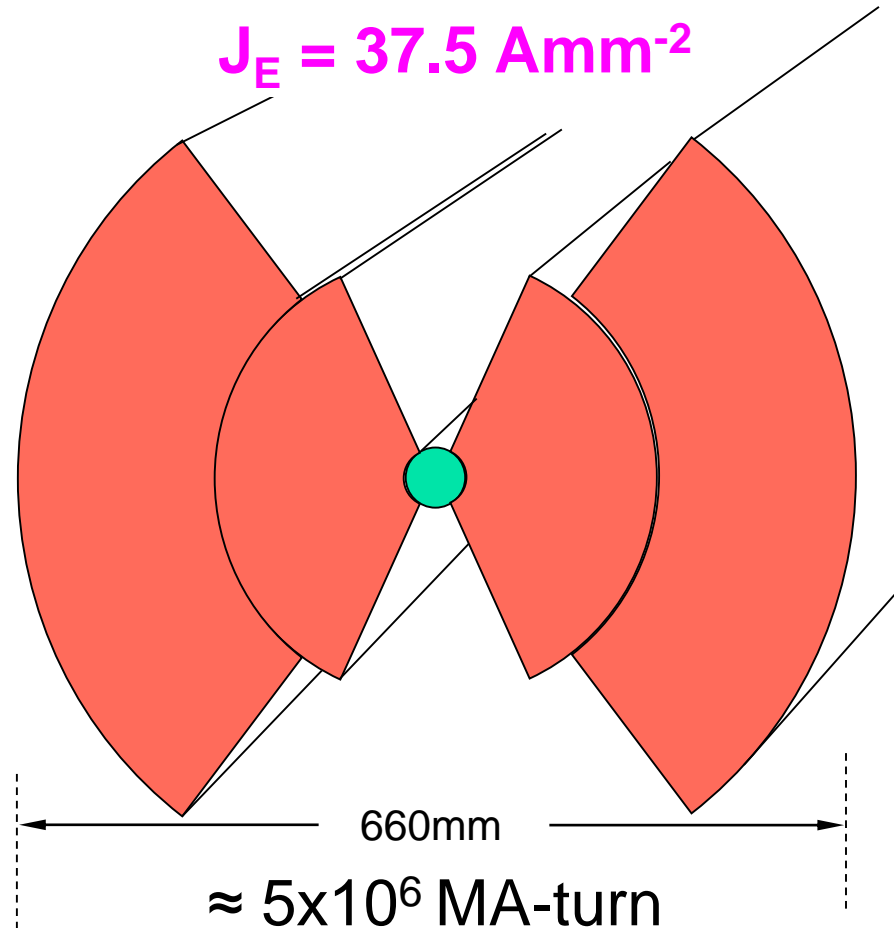
$J_E = 375 \text{ Amm}^{-2}$



$\approx 1 \times 10^6 \text{ MA-turn}$

all-SC dipole record field:
16.5 T (CERN, 2021)

$J_E = 37.5 \text{ Amm}^{-2}$



$\approx 5 \times 10^6 \text{ MA-turn}$

$\approx 6 \text{ MW/m}$

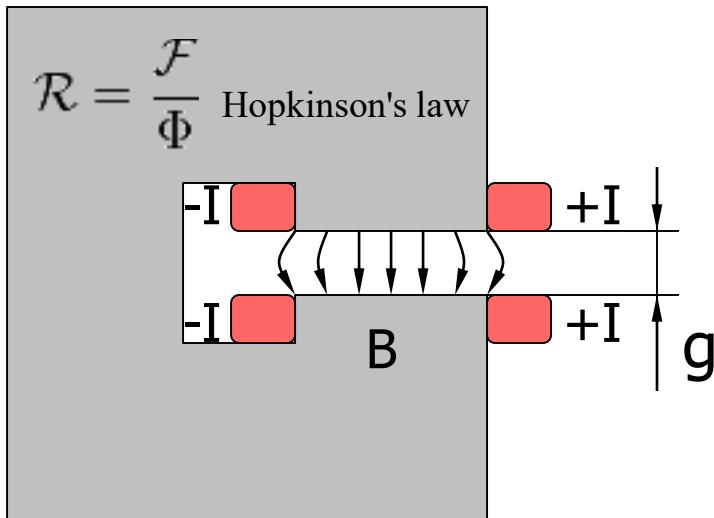


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- Why superconductors ? A motivation
- **Superconducting magnet design**
 - **Magnetic field and field quality**
 - Margins and stability
 - Quench protection
 - Forces and mechanics

Magnetic design - basics

- NC: magneto motive force, reluctance and pole shapes

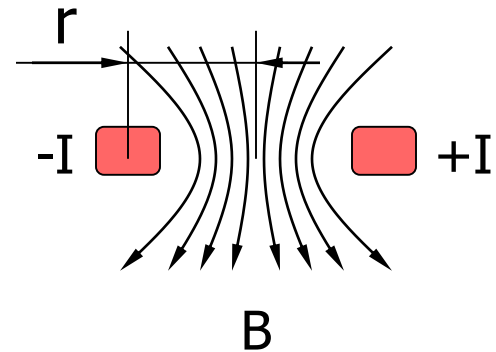


$$B \approx \mu_0 NI / g$$

g	=100 mm
NI	=100 kAturn
B	=1.25 T

- SC: Biot-Savart law and coil shapes

$$\mathbf{B} = \int \frac{\mu_0 I d\mathbf{l} \times \mathbf{r}}{4\pi |\mathbf{r}|^3} \quad \text{Biot-Savart law}$$



$$B \approx \mu_0 NI / \pi r$$

r	=45 mm
NI	=1 MAturn
B	=8.84 T

Definition of field and multipoles

- Accelerator magnets tend to be long and slender, to
 - Minimize the aperture (stored energy, material, cost)
 - Minimize lost space in interconnects (field integral)
- Example: the LHC bore has a ratio of length (16 m) to diameter (56 mm) larger than a *spaghetti*
- Field in accelerator magnets is 2-D in the magnet cross section (x,y), the third dimension can be ignored

$$B_y + iB_x = \sum_{n=1}^{\infty} (B_n + iA_n) z^{n-1}$$

normal and skew

Generalized gradients

Complex variable

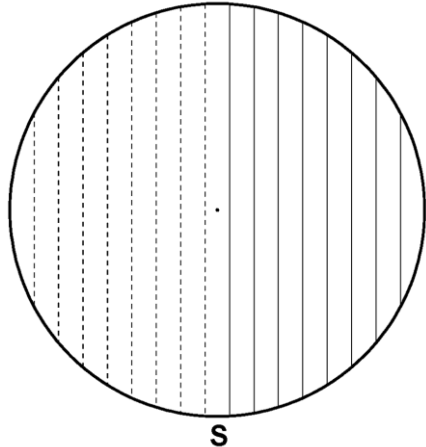
- *Multipole expansion* within the magnet aperture, based on a series of field *harmonics*

$$B_y + iB_x = \sum_{n=1}^{\infty} (B_n + iA_n) z^{n-1}$$

Multipoles and field gradients

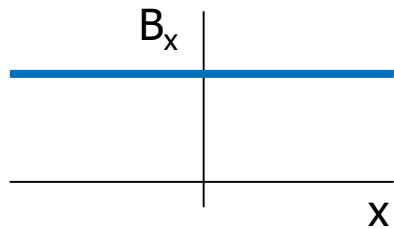
$B_1 \neq 0$

N

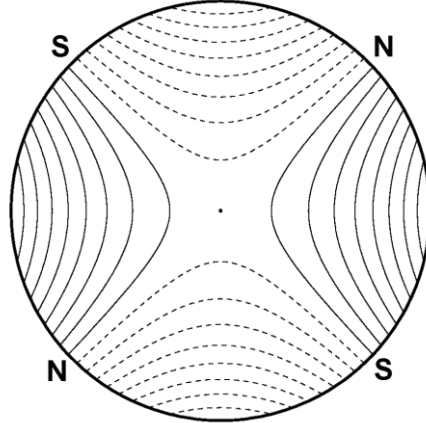


S

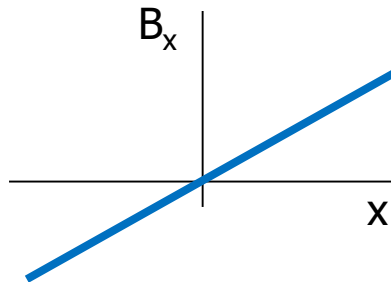
dipole



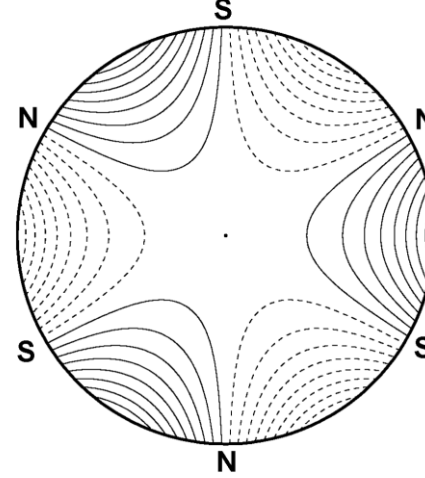
$B_2 \neq 0$



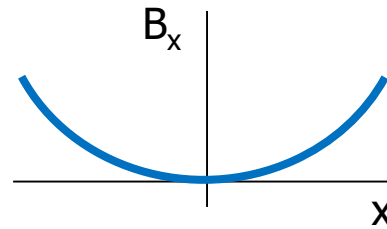
quadrupole



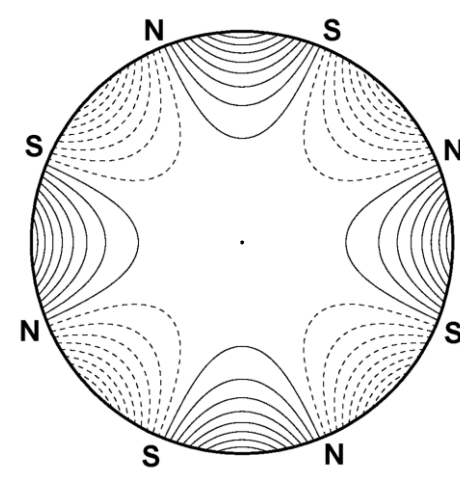
$B_3 \neq 0$



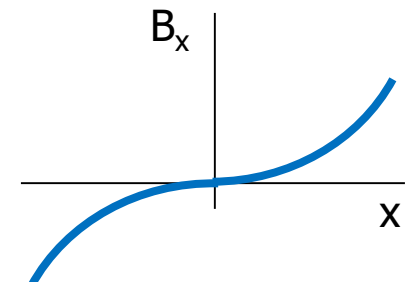
sextupole



$B_4 \neq 0$



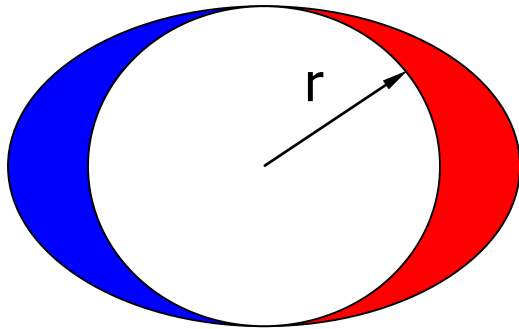
octupole



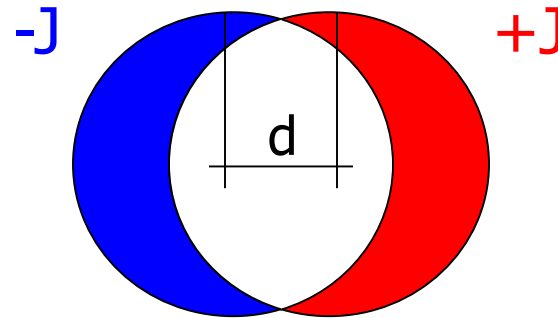
I will let you play with the other coefficients...

Design of an ideal dipole magnet

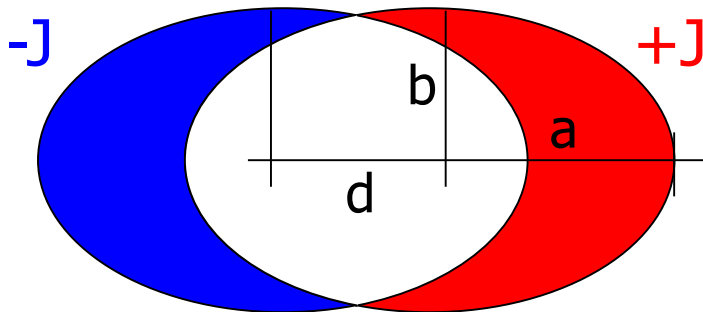
$$I = I_0 \cos(\theta) \Rightarrow B_I = -\mu_0 I_0 / 2 r$$



$$\text{Intersecting circles} \Rightarrow B_I = -\mu_0 J d / 2$$



$$\text{Intersecting ellipses} \Rightarrow B_I = -\mu_0 J d b / (a + b)$$

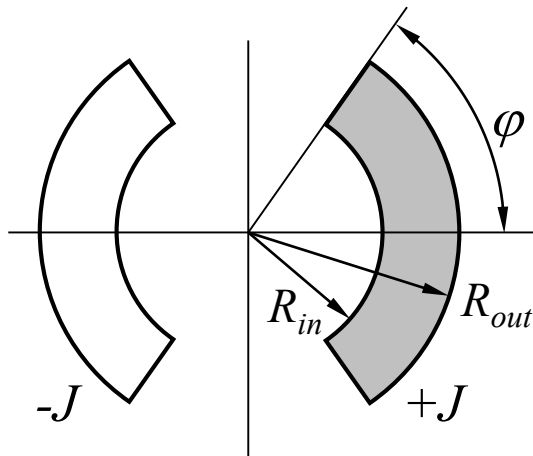


Several solutions are possible and can be extended to higher order multi-pole magnets

None of them is practical !

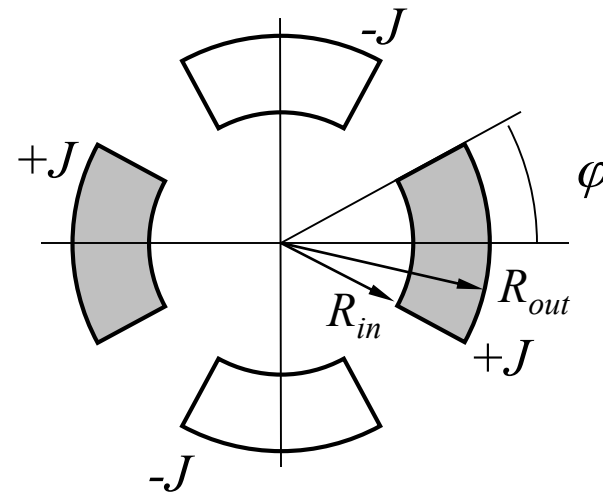
Magnetic design - sector coils

- Dipole coil



$$B_1 = -2\mu_0/\pi J (r_2 - r_1) \sin(\varphi)$$

- Quadrupole coil

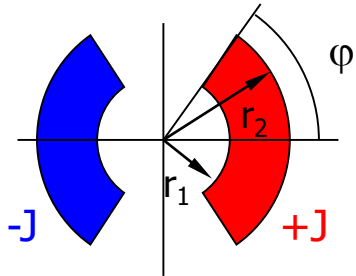


$$B_2 = -2\mu_0/\pi J \ln(r_2/r_1) \sin(2\varphi)$$

This is not an exact multipole magnet, but much more practical for the construction of a superconducting coil !

Field of a sector dipole coil

The field is proportional to the **current density J** and the coil width (**$R_{out} - R_{in}$**)



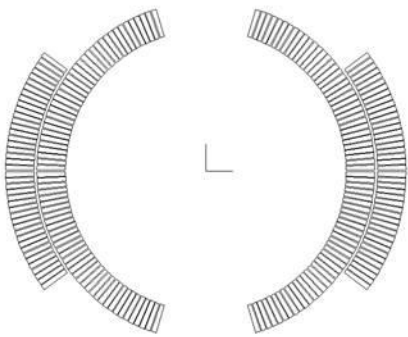
Main field	$B_1 = \frac{2\mu_0}{\pi} J (R_{out} - R_{in}) \sin(\varphi)$
Field errors $n = 3, 5, \dots, 2i - 1$	$B_n = \frac{2\mu_0}{\pi} J \frac{R_{out}^{2-n} - R_{in}^{2-n}}{n(2-n)} \sin(n\varphi)$ $A_n = 0$

Harmonics ***allowed*** by symmetry

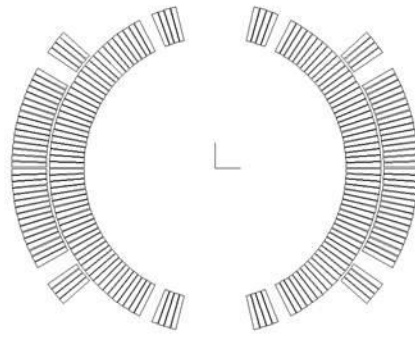
First allowed harmonic (B_3) can be made zero by taking $\phi = 60^\circ$

Evolution of coil cross sections

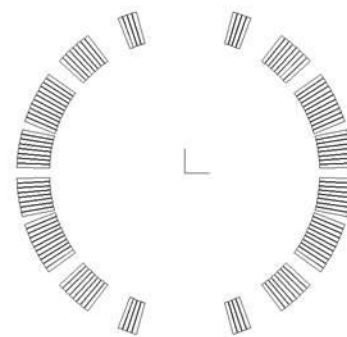
- Coil cross sections (to scale) of the four superconducting colliders



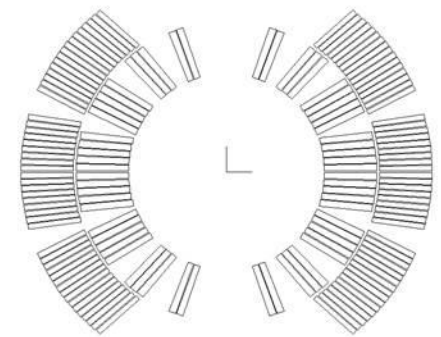
Tevatron



HERA



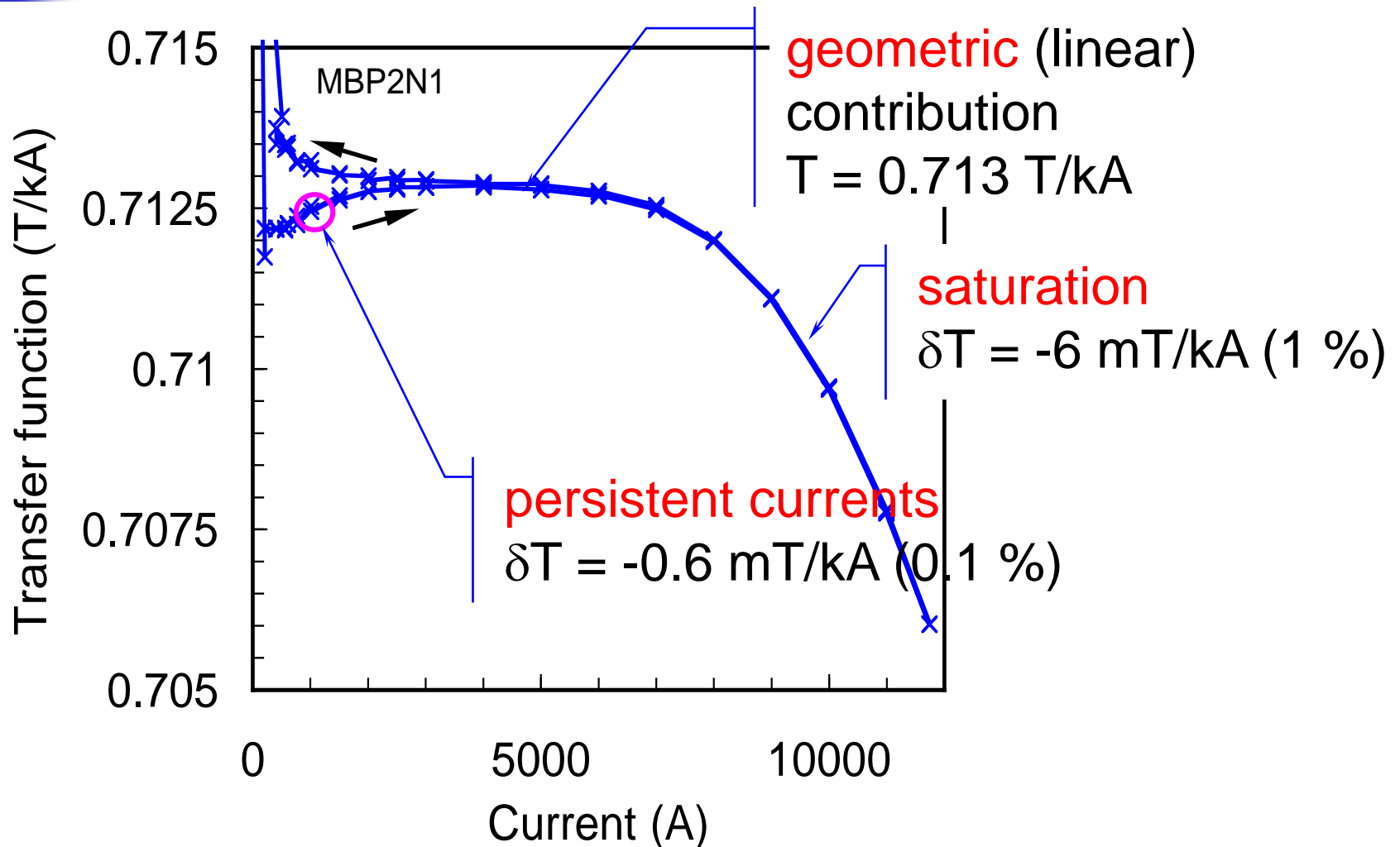
RHIC



LHC

- Increased coil complexity (nested layers, wedges and coil blocks) to achieve higher efficiency and improved field homogeneity

Field quality – “saturation”



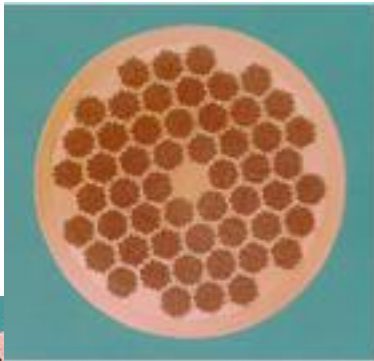


Overview

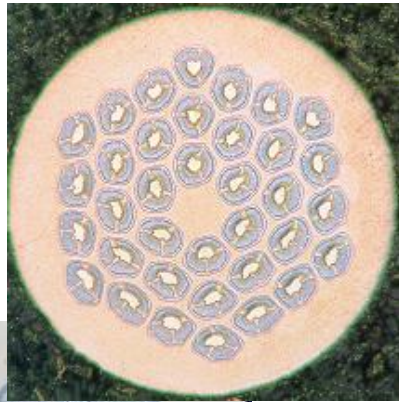
- Why superconductors ? A motivation
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Superconducting wires and tapes

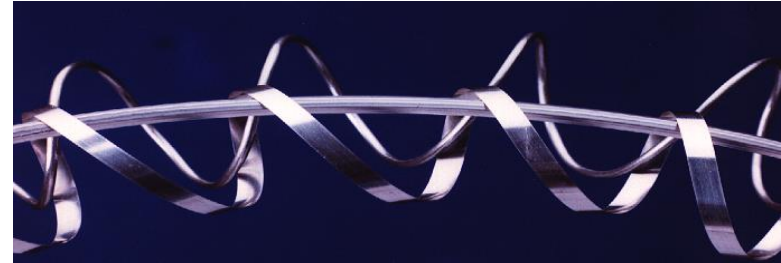
NbTi



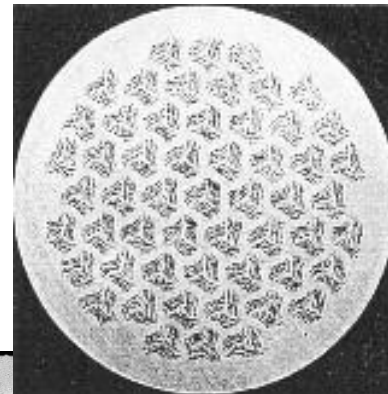
Nb₃Sn, Nb₃Al



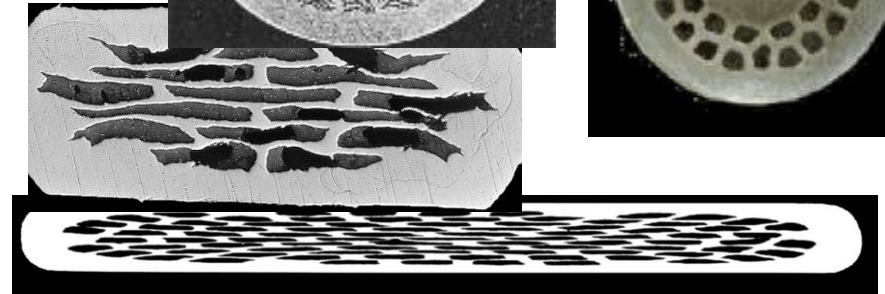
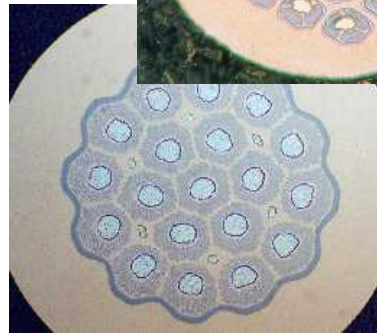
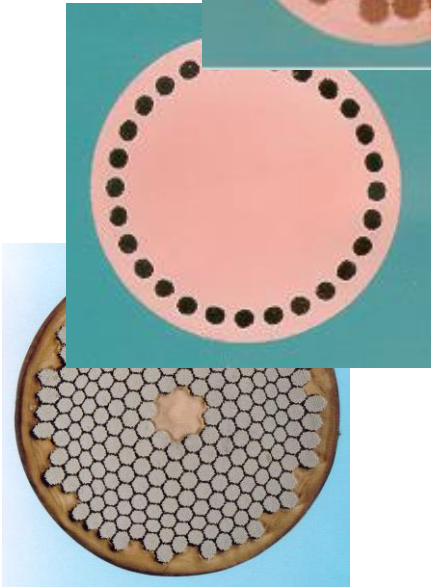
YBCO



BSCCO

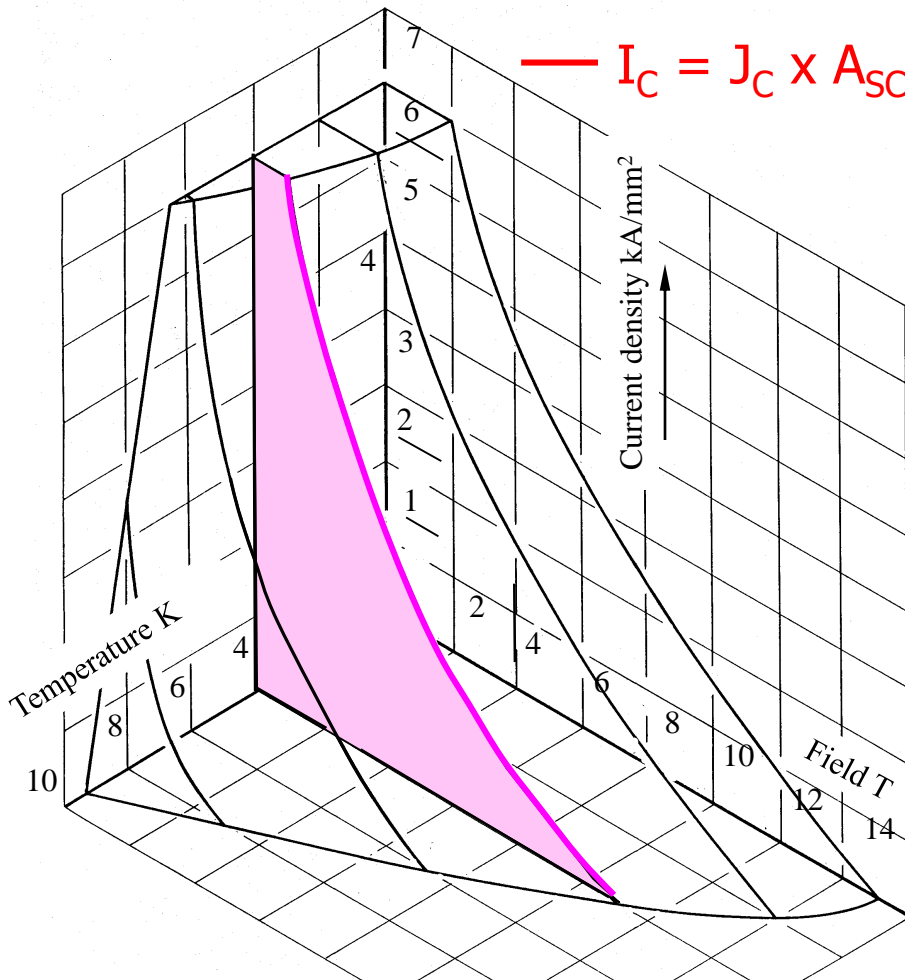


MgB₂

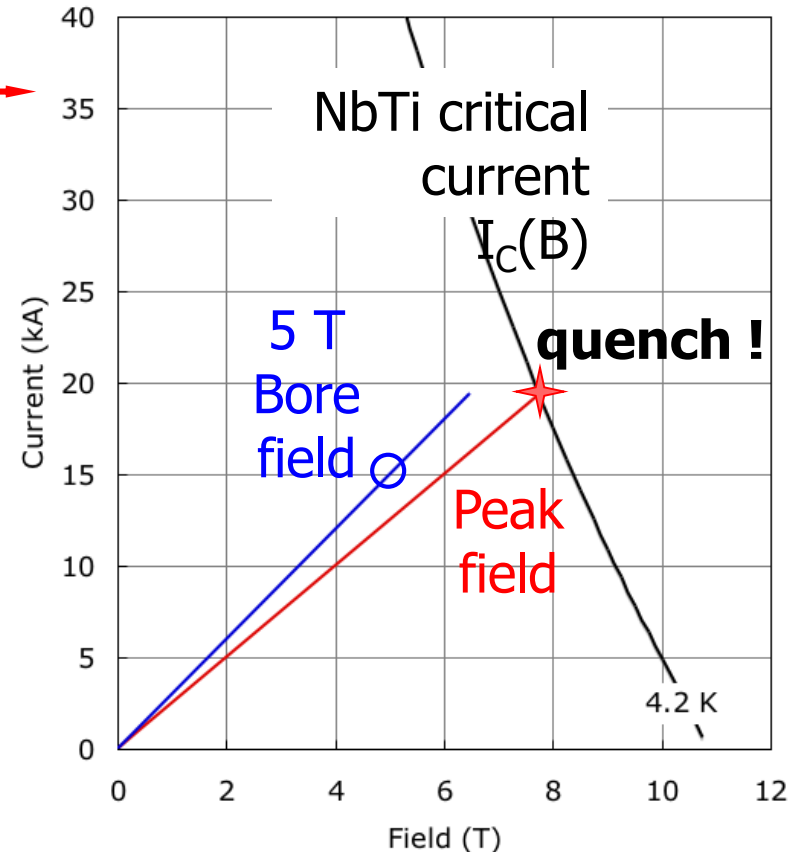


Critical line and magnet load lines

NbTi critical surface

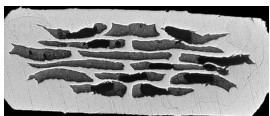
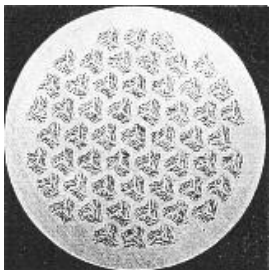
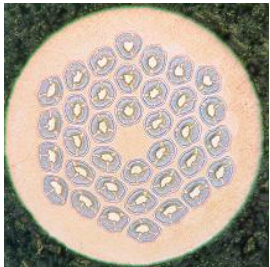
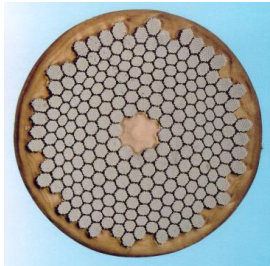


e.g. a 5 T magnet design



we expect the magnet to go resistive i.e. to **'quench'**, where the peak field load line crosses the critical current line

Engineering current density

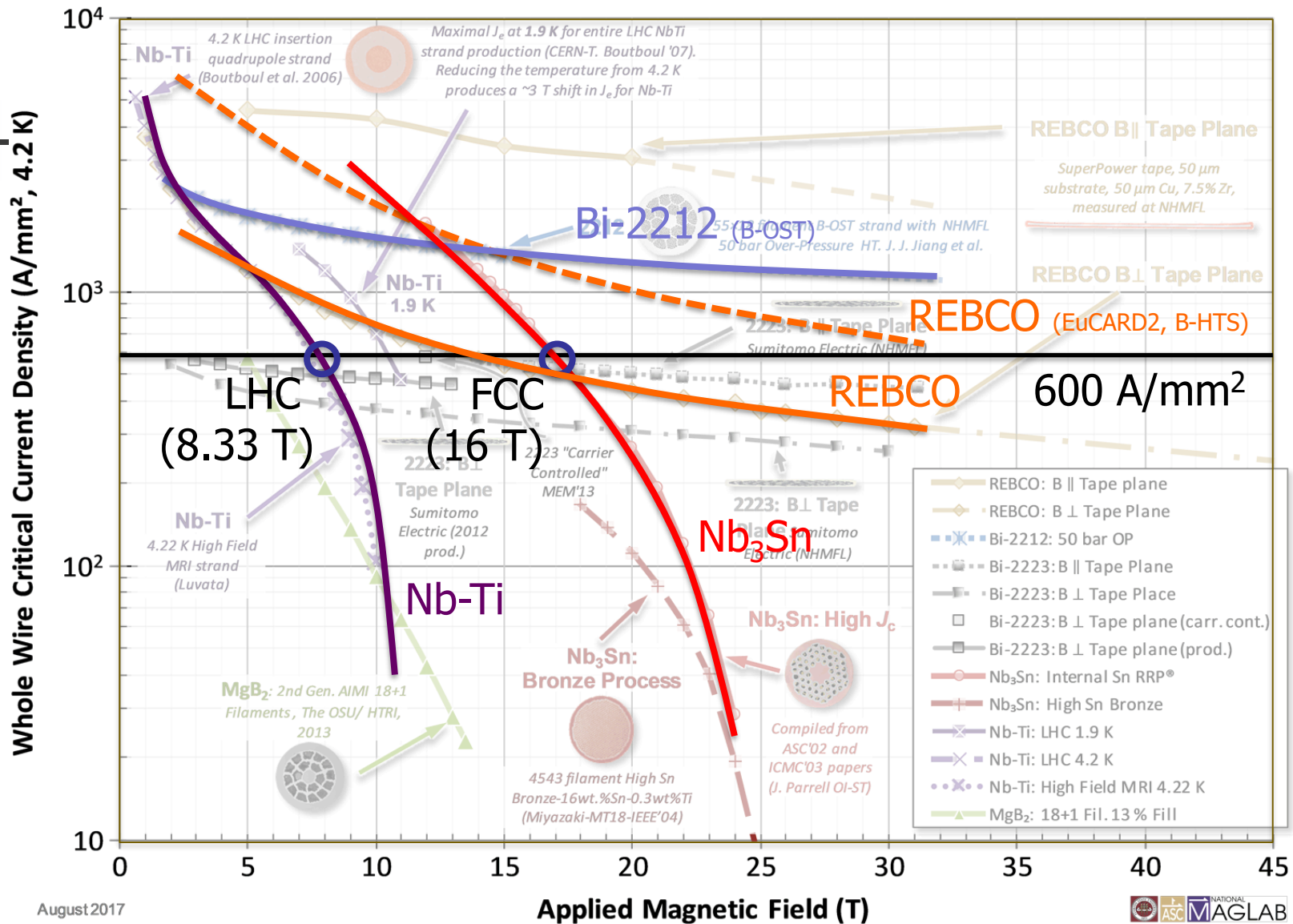


- All wires, tapes and cables contain additional components:
 - Low resistance matrices
 - Left-overs from the precursors of the SC formation
 - Barriers, texturing and buffering layers
- The *SC material fraction* is hence always only a part of the total cross section:

$$\lambda = A_{SC} / A_{total}$$

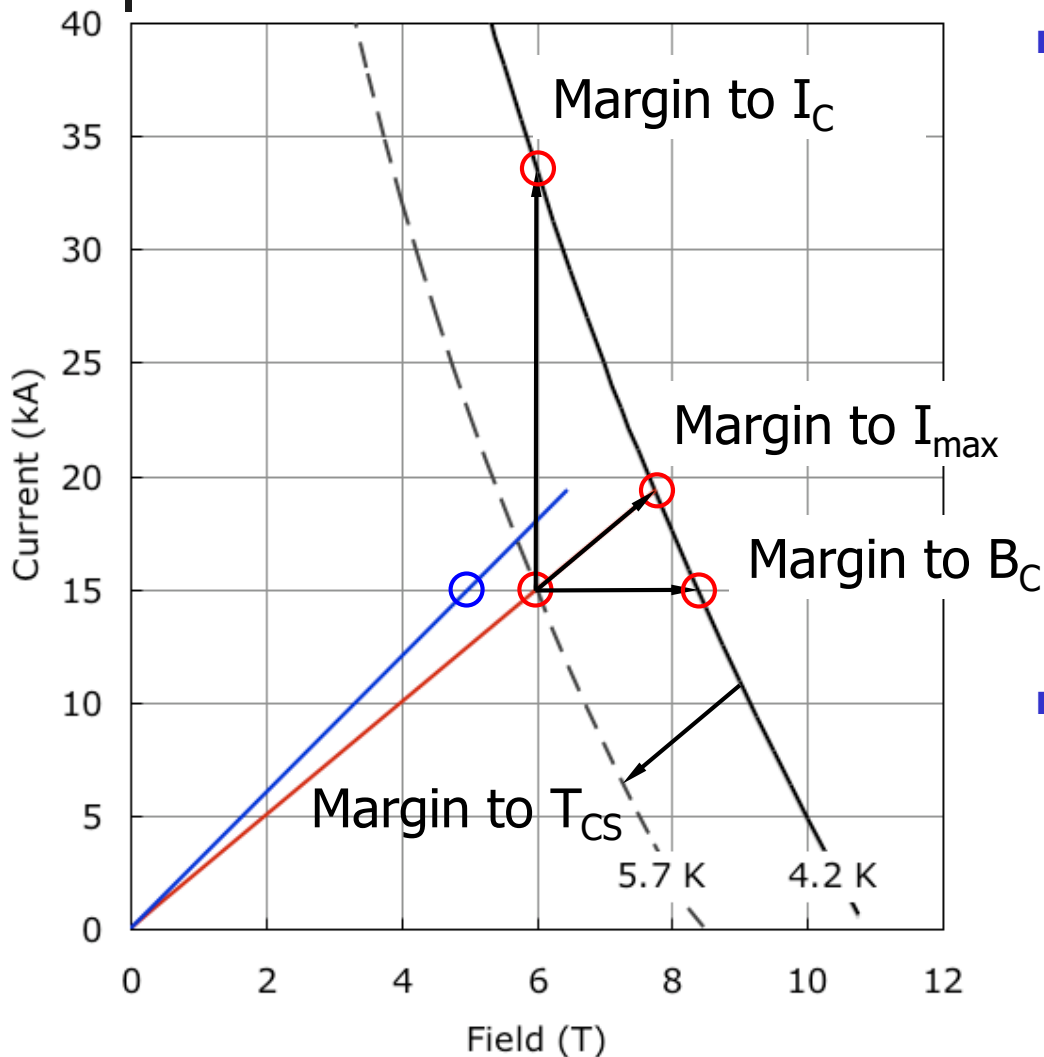
- To compare materials on the same basis, we use an *engineering current density*:

$$J_E = J_C \times \lambda$$



By courtesy of P. Lee, with stimulating inputs from D. Larbalestier and D. Abrahimov

Operating margins

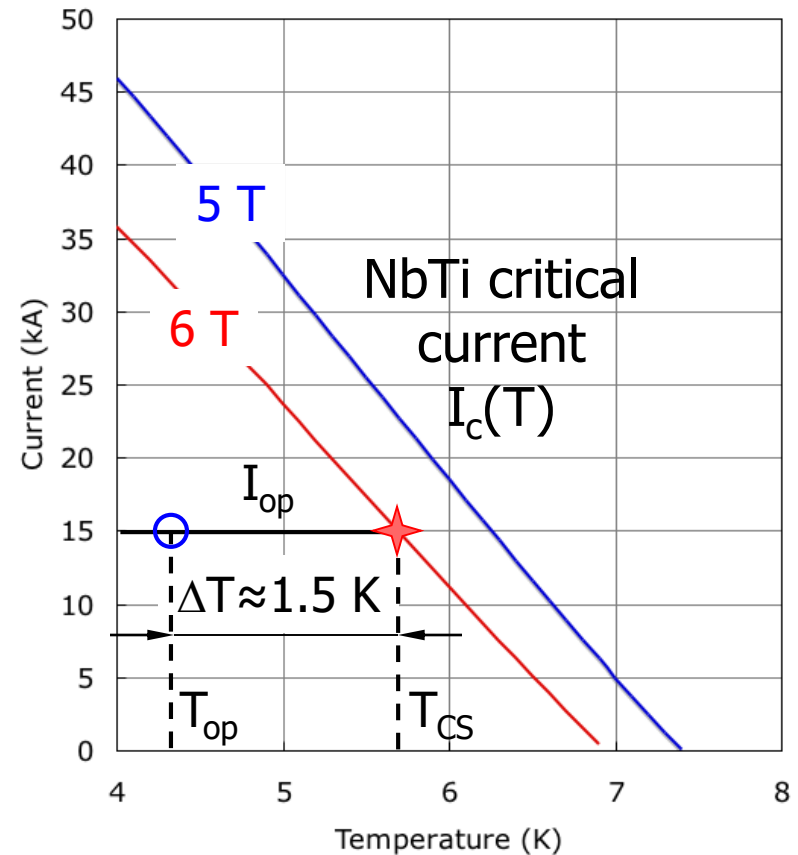


- Practical operation always requires margins:
 - Critical current margin:
 - $I_{op}/I_C \approx 50\%$
 - Critical field margin:
 - $B_{op}/B_C \approx 75\%$
 - Margin along the loadline:
 - $I_{op}/I_{max} \approx 85\%$
 - Temperature margin:
 - $T_{CS} - T_{op} \approx 1...2\text{ K}$
- The margin needed depends on the design and operating conditions

Temperature margin

- Temperature rise may be caused by
 - Sudden mechanical energy release
 - AC losses
 - Resistive heat at joints
 - Beams, neutrons, etc.
- We should allow *temperature headroom* for all foreseeable and unforeseeable events, i.e. a **temperature margin**:

$$\Delta T = T_{CS} - T_{op}$$

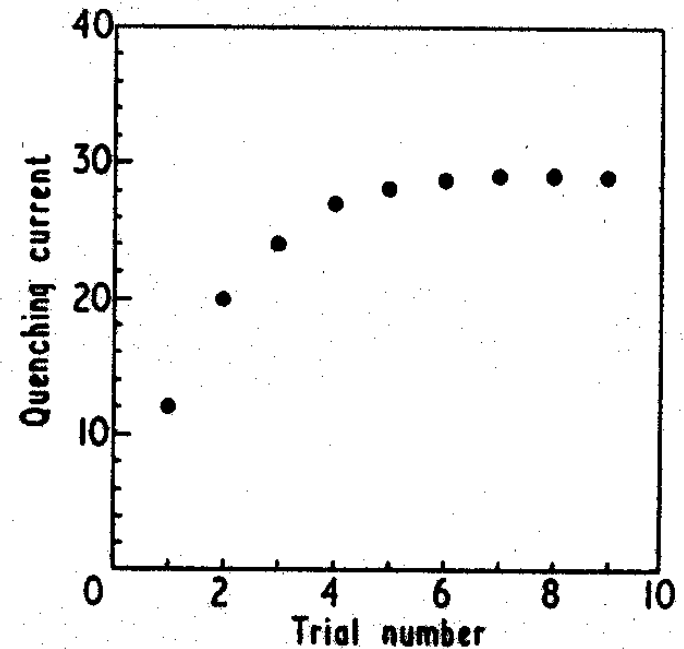


Training...

- Superconducting solenoids built from NbZr and Nb₃Sn in the early 60's quenched much below the rated current ...
- ... the quench current increased gradually quench after quench: **training**

M.A.R. LeBlanc, Phys. Rev., **124**, 1423, 1961.

NbZr solenoid
Chester, 1967

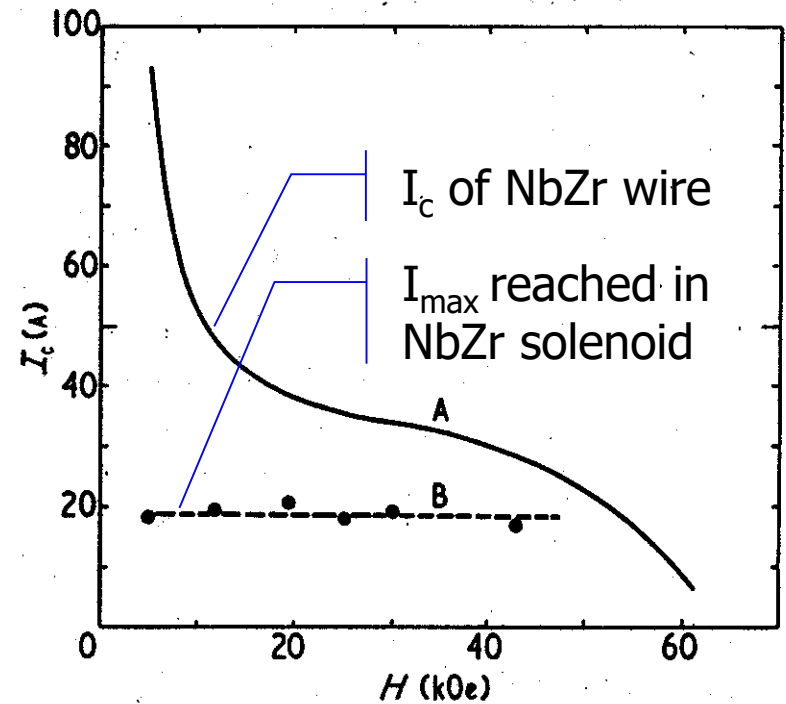


P.F. Chester, Rep. Prog. Phys., **XXX**, II, 561, 1967.

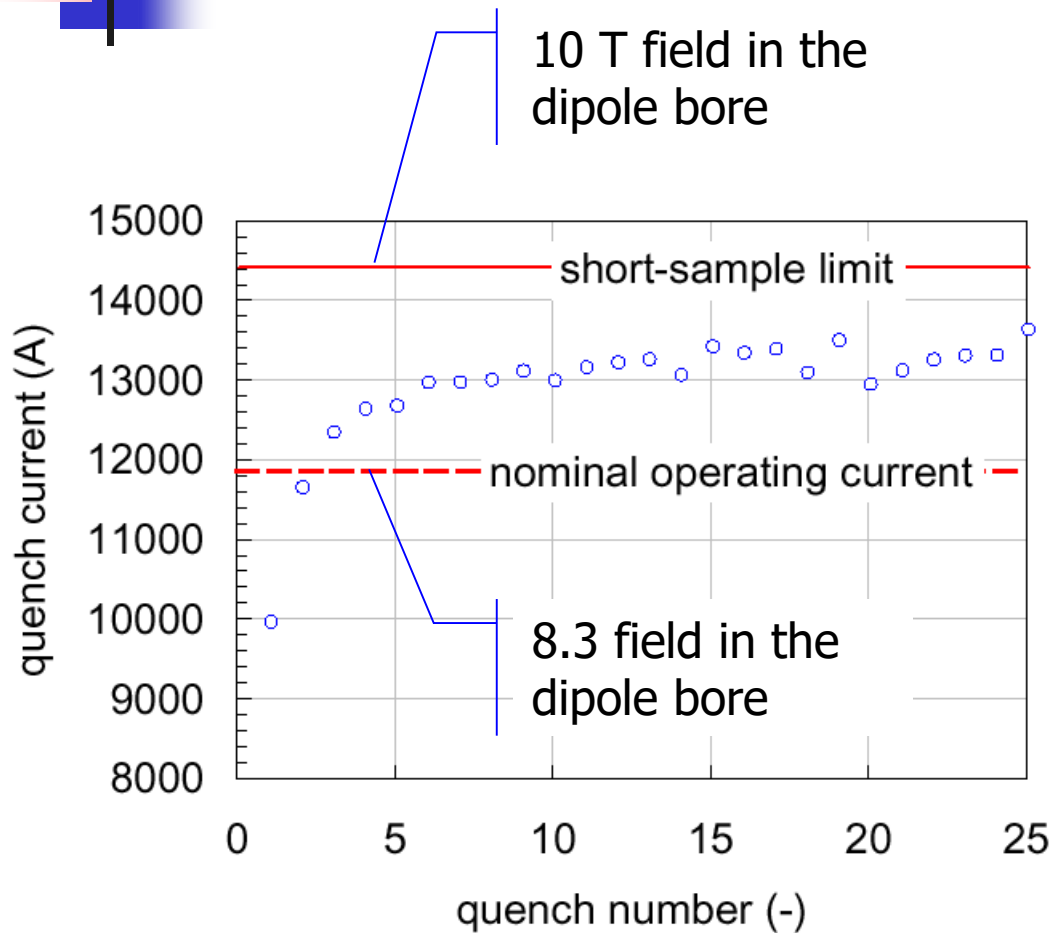
... and degradation

- ... but did not quite reach the expected maximum current for the superconducting wire !
- This was initially explained as a local damage of the wire: *degradation*, a very misleading name.
- All this had to do with *stability* !

NbZr solenoid vs. wire
Chester, 1967



Training today

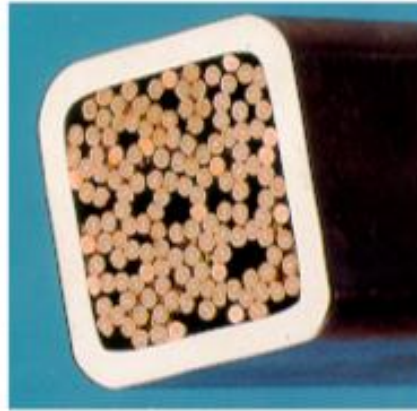
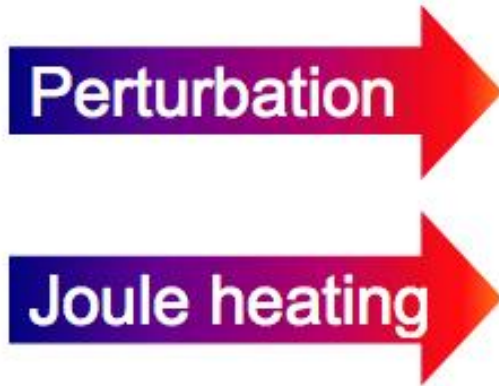


Courtesy of A. Siemko, CERN, 2002

- Training of an LHC short dipole model at superfluid helium
 - still (limited) training may be necessary to reach nominal operating current
 - short sample limit is not reached, even after a long training sequence

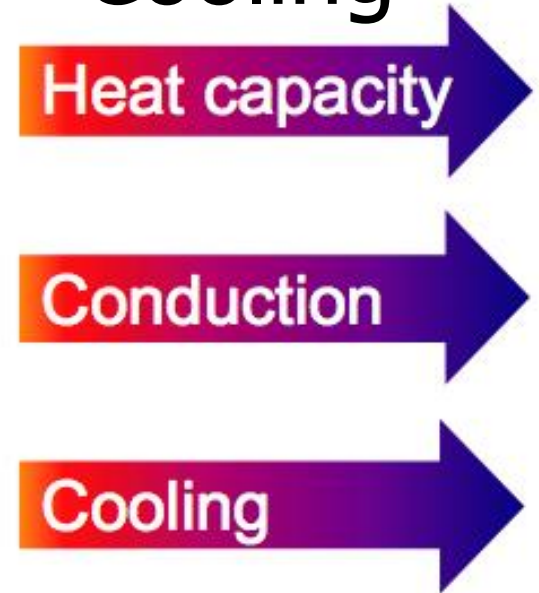
Stability as a heat balance

Heat generation



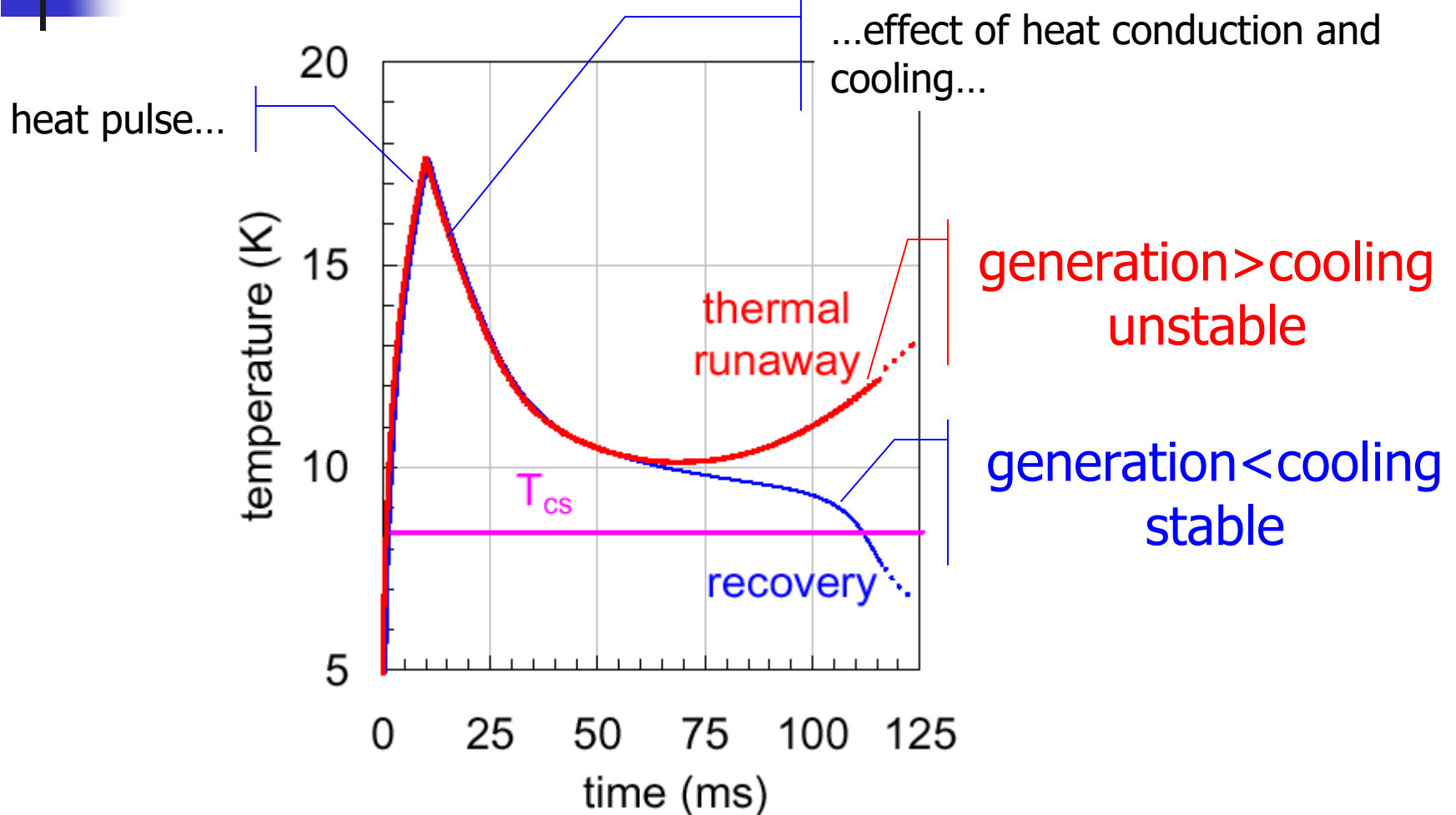
superconducting cable

Cooling

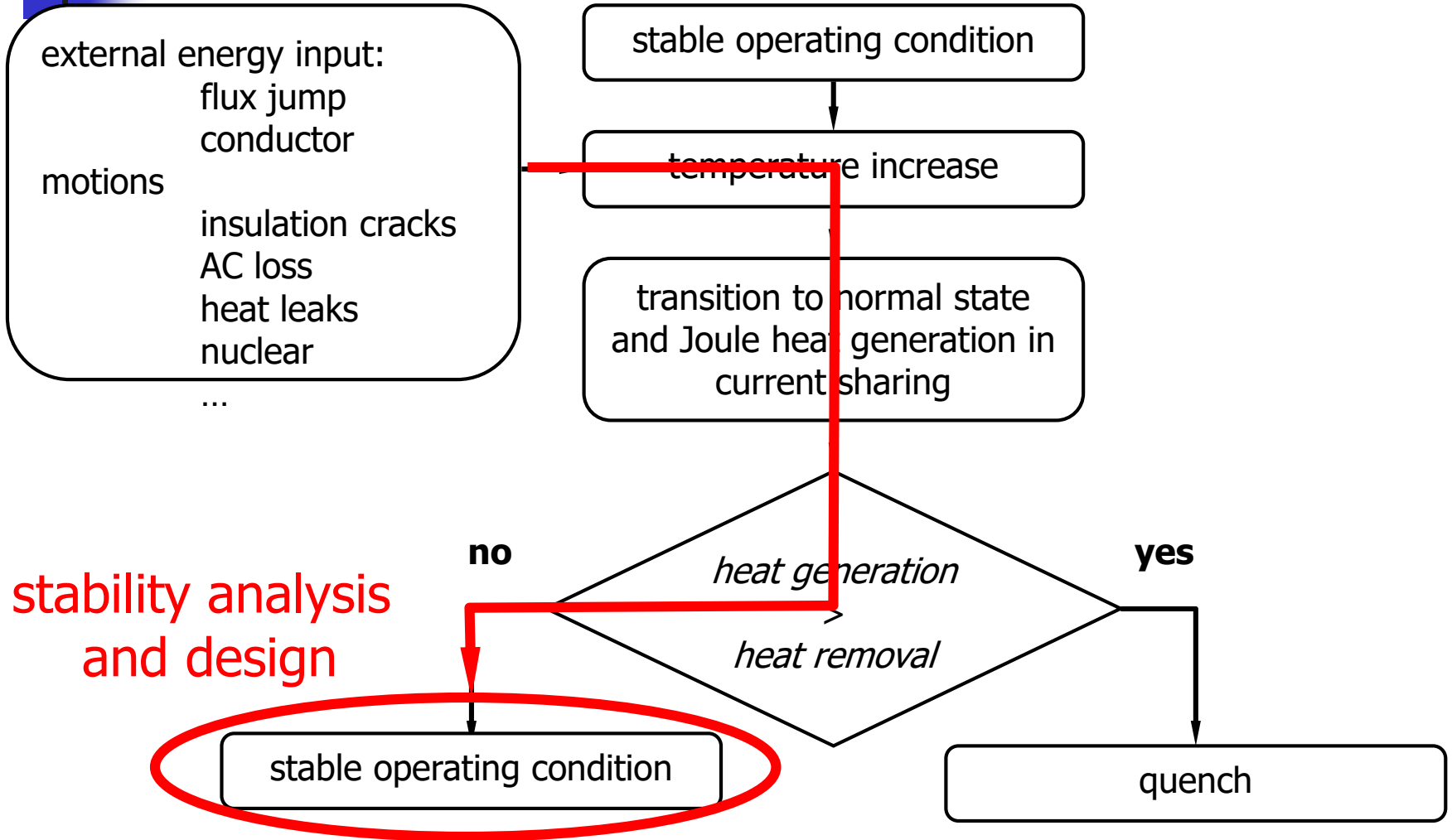


$$C \frac{dT}{dt} = q_{ext} + q_J + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) - \frac{wh}{A} (T - T_{he})$$

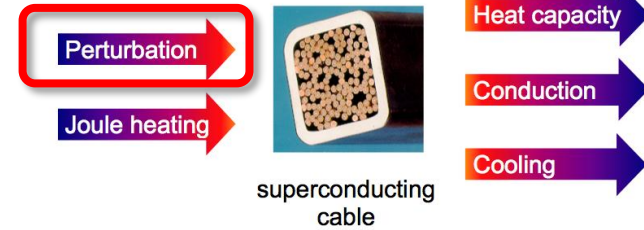
A prototype temperature transient



Stability analysis

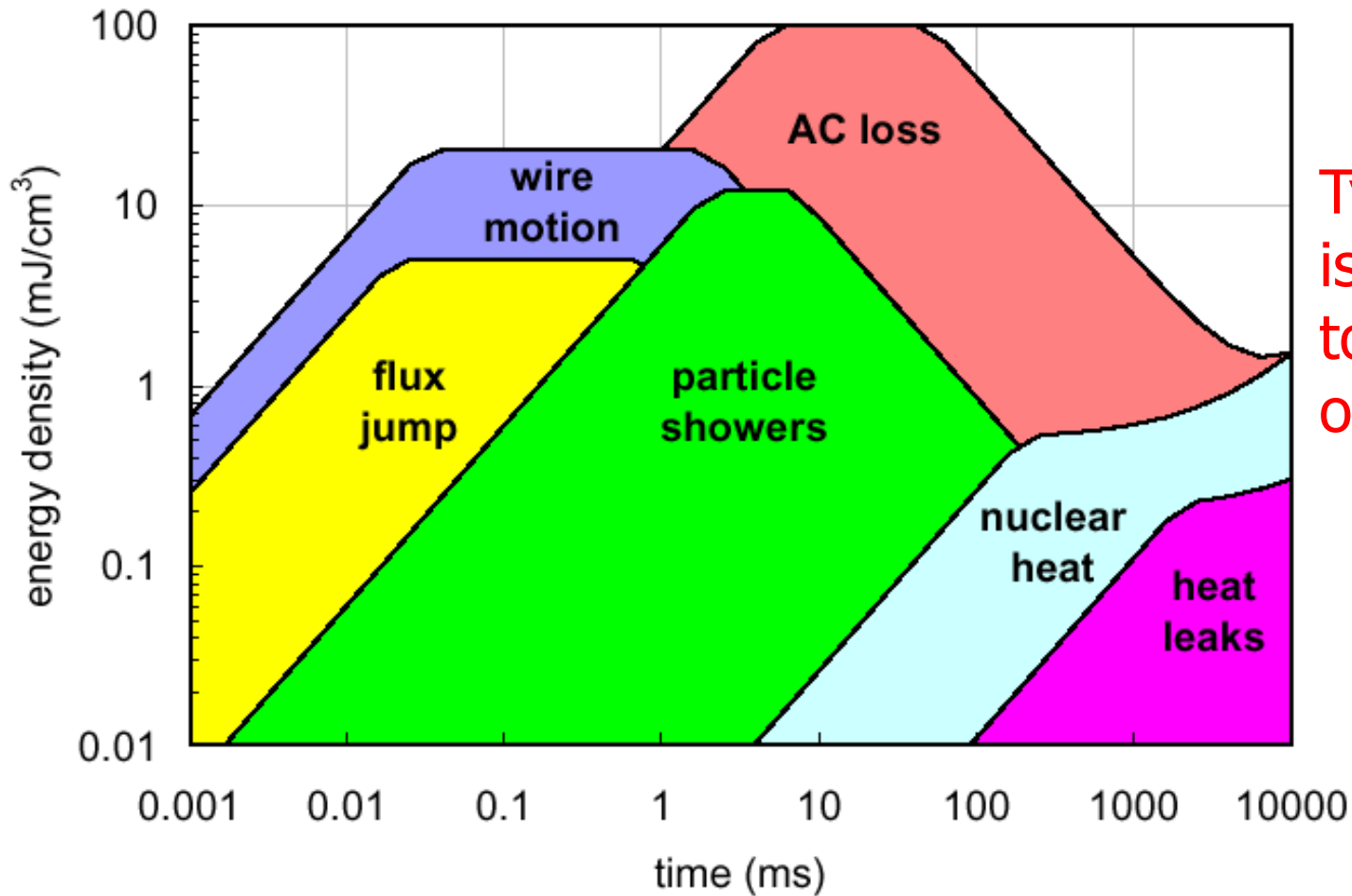


Perturbation spectrum



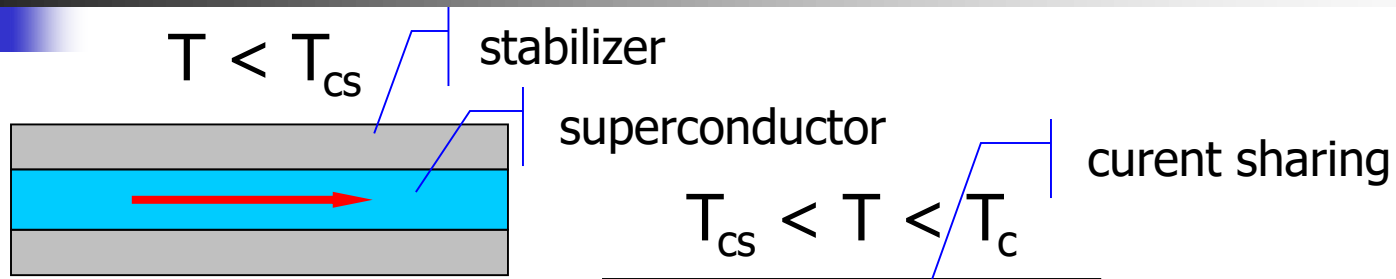
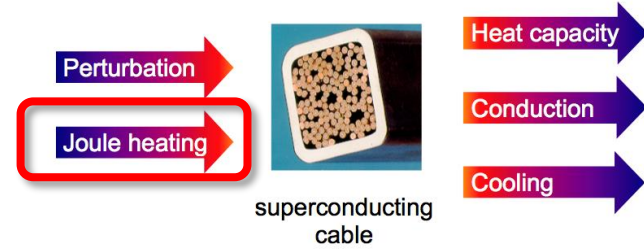
- mechanical *events*
 - wire motion under Lorentz force, micro-slips
 - winding deformations
 - failures (at insulation bonding, material yield)
- electromagnetic *events*
 - flux-jumps (important for large filaments, old story !)
 - AC loss (most magnet types)
 - current sharing in cables through distribution/redistribution
- thermal *events*
 - current leads, instrumentation wires
 - heat leaks through thermal insulation, degraded cooling
- nuclear *events*
 - particle showers in particle accelerator magnets
 - neutron flux in fusion experiments

Perturbation overview

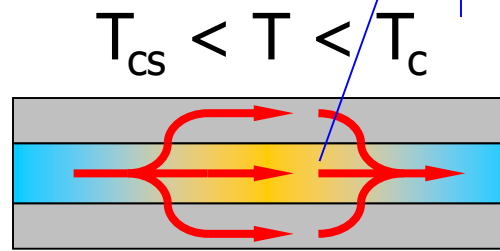


Typical range is from a few to a few tens of mJ/cm³

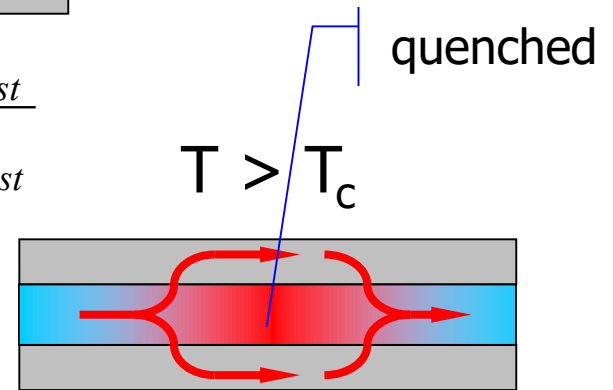
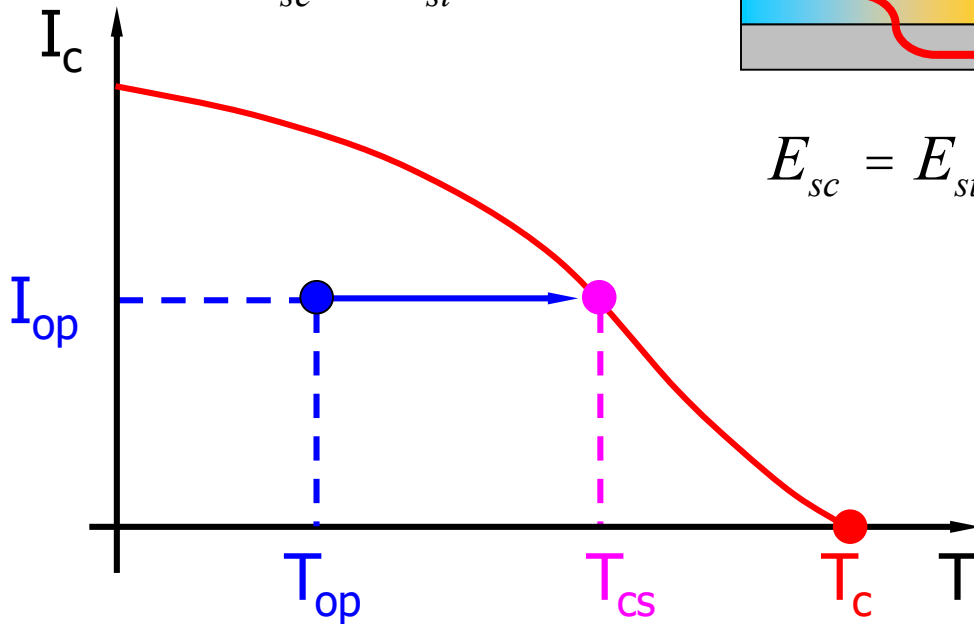
Current sharing



$$E_{sc} = E_{st} = 0$$

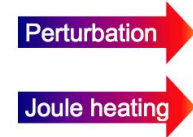


$$E_{sc} = E_{st} = I_{st} \frac{h_{st}}{A_{st}}$$

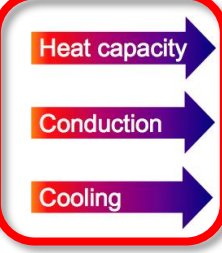


$$E_{sc} = E_{st} = I_{op} \frac{h_{st}}{A_{st}}$$

Adiabatic stability



superconducting cable



- adiabatic conditions:
 - no cooling (dry or impregnated windings)
 - energy perturbation over large volume (no conduction)

$$C \frac{dT}{dt} = q_{ext} + q_J + \frac{\rho \frac{dT}{dx}}{\rho \frac{dx}{dt}} - \frac{wh}{A} (T - T_{he})$$

(The terms $\frac{\rho \frac{dT}{dx}}{\rho \frac{dx}{dt}}$ and $(T - T_{he})$ are crossed out with red X's.)

- stable only if $q'''_{Joule} = 0$ ($T \leq T_{cs}$) ! Integrate:

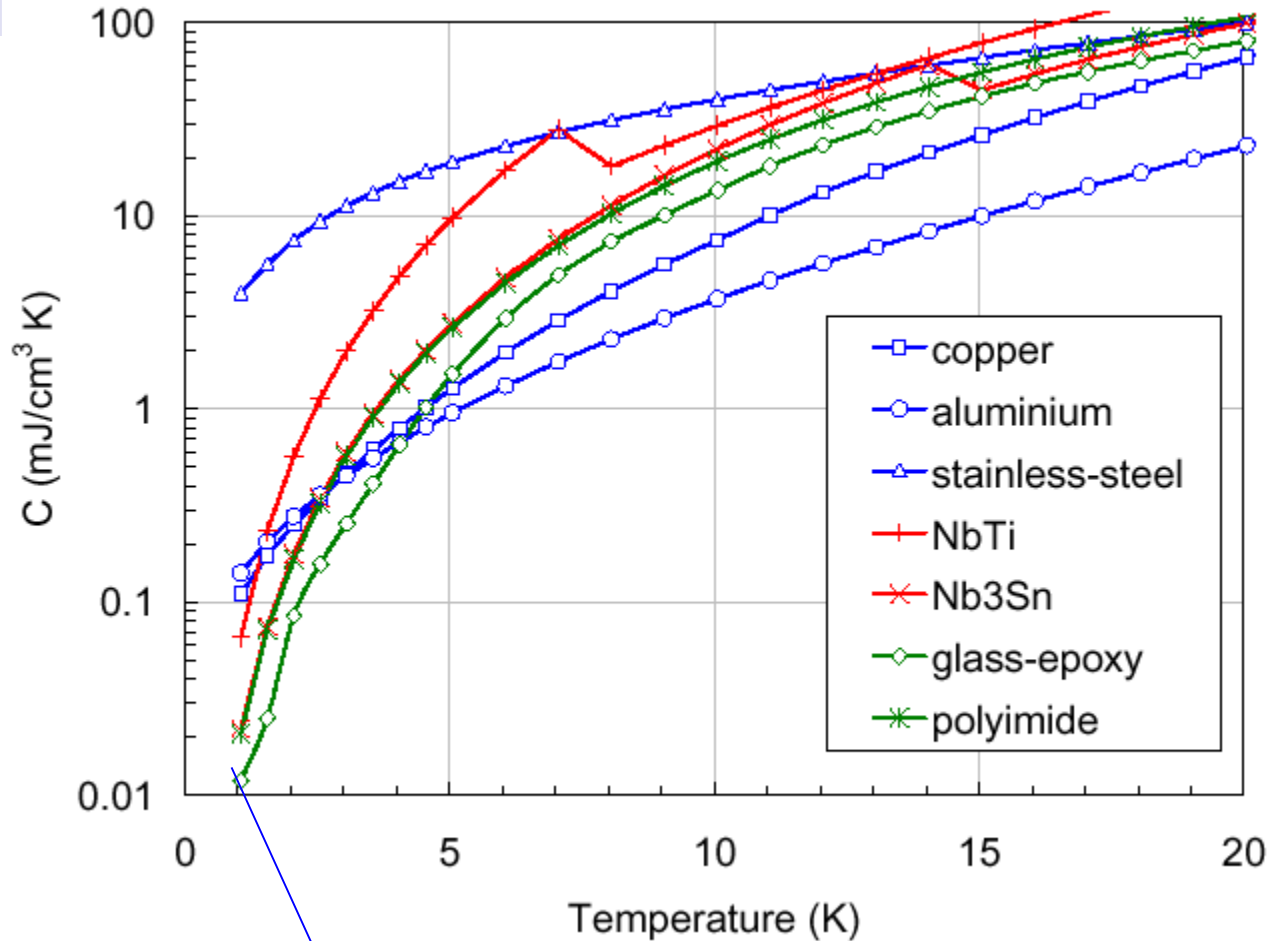
energy margin

$$\int_0^{T_{cs}} q'''_{ext} dt = \int_{T_{op}}^{T_{cs}} C dT \quad \longrightarrow \quad DQ''' = H(T_{cs}) - H(T_{op})$$

volumetric enthalpy

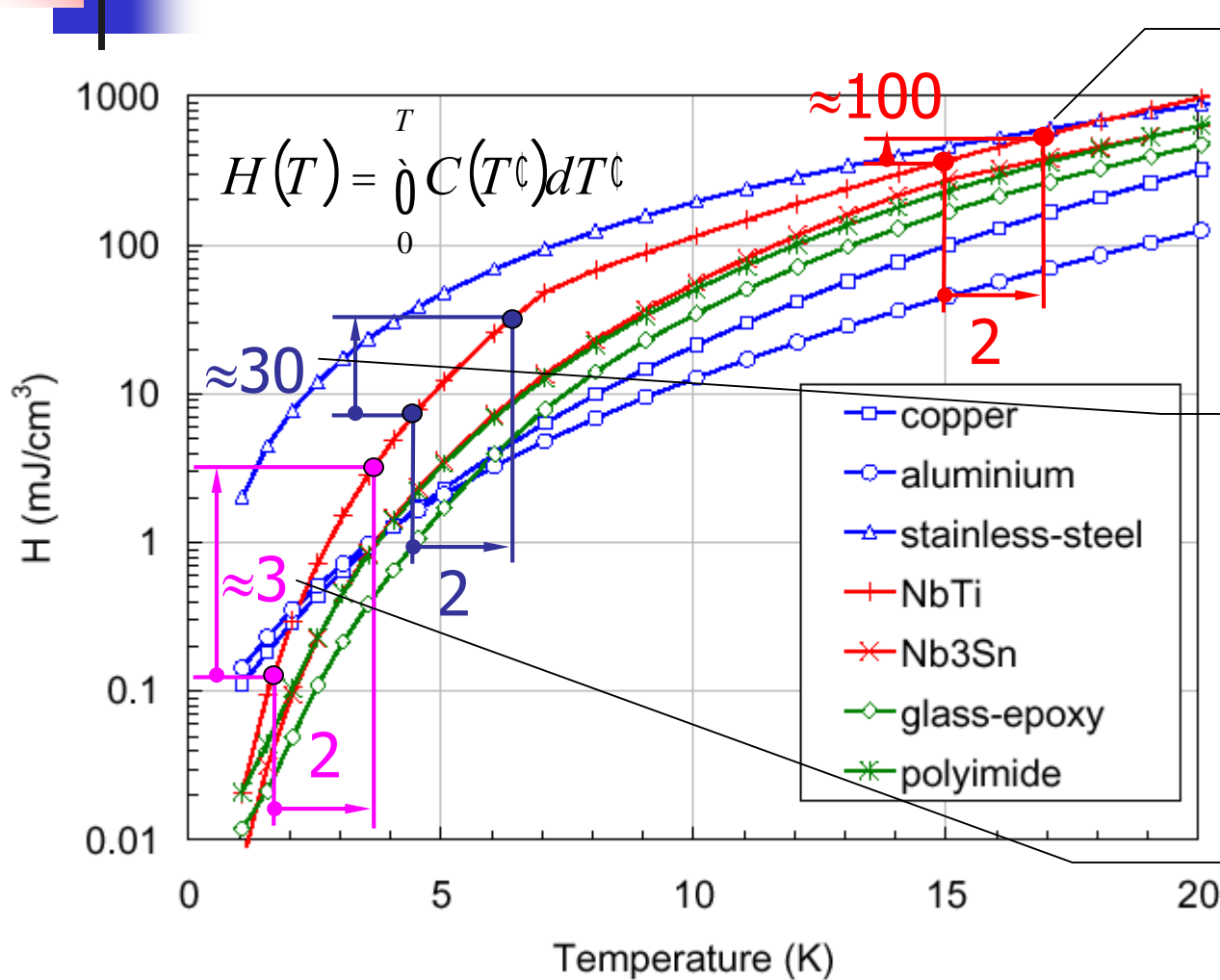
$$H(T) = \int_0^T C(T) dT$$

Low temperature heat capacity



Note that $C \Rightarrow 0$ for $T \Rightarrow 0$!

Enthalpy reserve



Enthalpy reserve increases massively at increasing T: **stability is not an issue for HTS materials**

Enthalpy reserve is of the order of the expected perturbation spectrum: **stability is an issue for LTS magnets**

do not sub-cool if you can only avoid it !



Stability - Re-cap

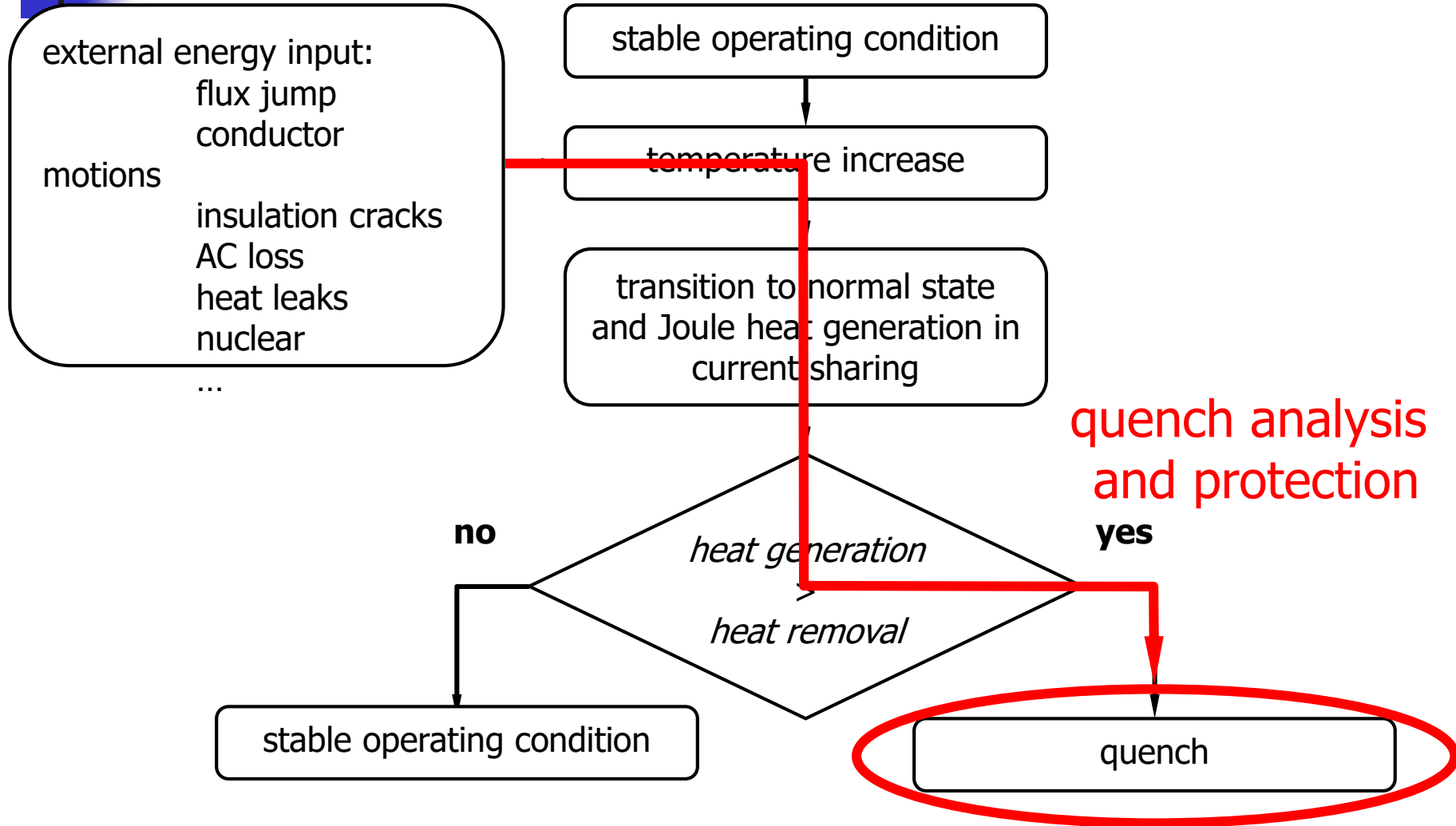
- A sound design is such that **the expected energy spectrum is smaller than the expected stability margin**
- To increase stability:
 - Increase **temperature margin**
 - Increase **heat removal** (e.g. conduction or heat transfer)
 - Decrease Joule heating by using a stabilizer with **low electrical conductance**
 - Make best use of **heat capacity**
 - Avoid sub-cooling (heat capacity increases with T , this is why stability is not an issue for HTS materials)
 - Access to helium for low operating temperatures



Overview

- Why superconductors ? A motivation
- **Superconducting magnet design**
 - Magnetic field and field quality
 - Margins and stability
 - **Quench protection**
 - Forces and mechanics

What is a quench ?



Why is it a problem ?

- the magnetic energy stored in the field:

$$E_m = \int_V \frac{B^2}{2\mu_0} dv = \frac{1}{2} LI^2$$

is converted to heat through Joule heating RI^2 .

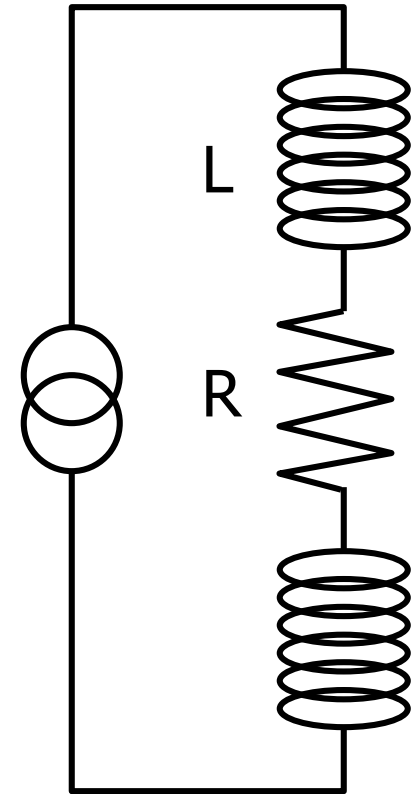
If this process happened uniformly in the winding pack:

- Cu melting temperature 1356 K
- corresponding $E_m = 5.2 \cdot 10^9 \text{ J/m}^3$

limit would be $B_{max} \leq 115 \text{ T}$: **NO PROBLEM !**

BUT

the process does not happen uniformly (as little as 1 % of mass can absorb total energy)

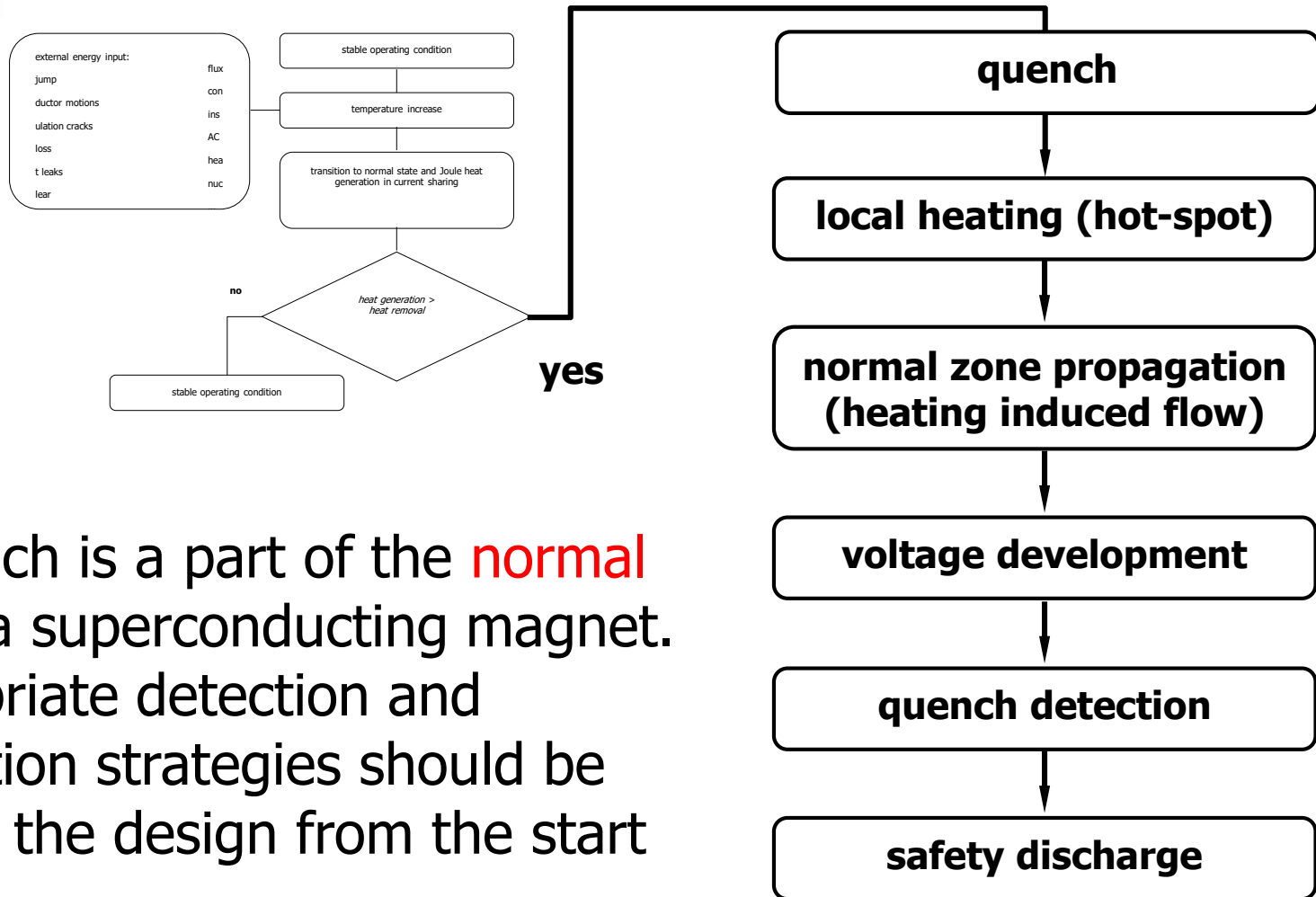


This is why it is important !



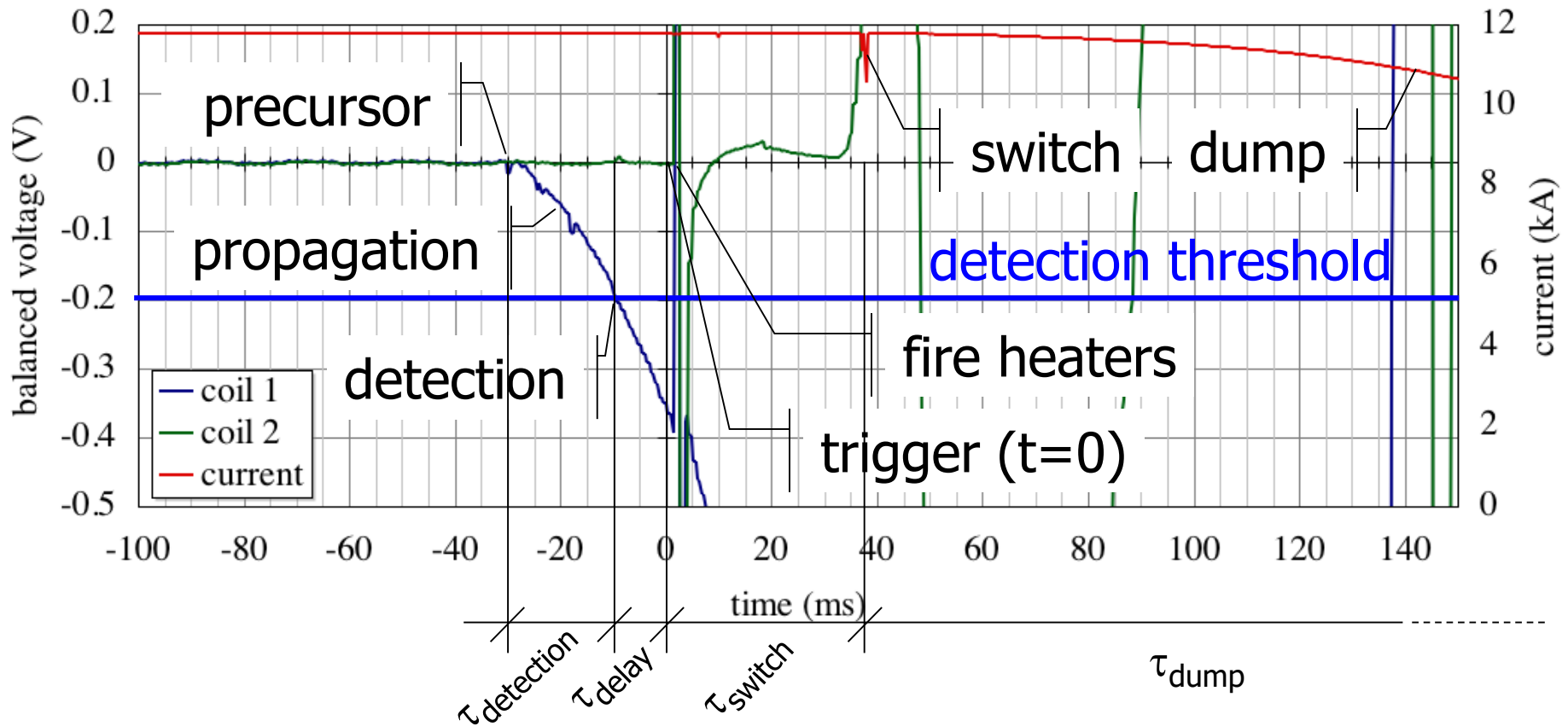
A large magnetic energy dissipated in a small volume

Quench sequence



A quench is a part of the **normal life** of a superconducting magnet. Appropriate detection and protection strategies should be built in the design from the start

Detection, switch and dump



$$\tau_{\text{discharge}} \approx \tau_{\text{detection}} + \tau_{\text{delay}} + \tau_{\text{switch}} + \tau_{\text{dump}}$$



Adiabatic propagation

for *constant* properties (η , k , C)

$$v_{adiabatic} = \frac{J_{op}}{C} \sqrt{\frac{h_{st} k_{st}}{(T_J - T_{op})}}$$

Example LTS:

$$J_{op} \approx 100 \times 10^6 \text{ (A/mm}^2\text{)}$$

$$C \approx 10^3 \text{ (J/m}^3 \text{ K)}$$

$$\eta \approx 10^{-9} \text{ (\Omega m)}$$

$$k \approx 100 \text{ (W/m K)}$$

$$T_J - T_{op} \approx 2 \text{ (K)}$$

$$v \approx 22 \text{ m/s}$$

Example HTS:

$$J_{op} \approx 100 \times 10^6 \text{ (A/mm}^2\text{)}$$

$$C \approx 10^6 \text{ (J/m}^3 \text{ K)}$$

$$\eta \approx 10^{-8} \text{ (\Omega m)}$$

$$k \approx 30 \text{ (W/m K)}$$

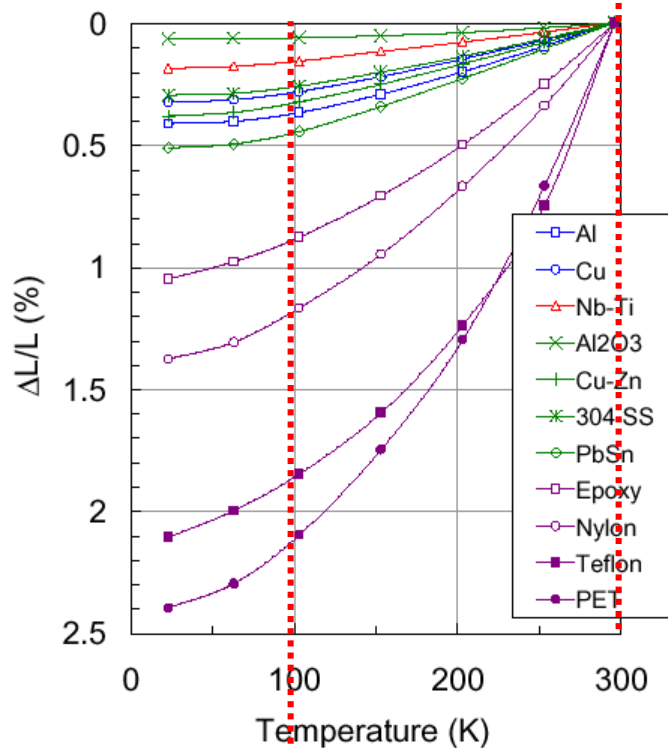
$$T_J - T_{op} \approx 20 \text{ (K)}$$

$$v \approx 1 \text{ cm/s}$$

- Constant quench propagation speed
- Scales linearly with the current density (and current)
- Practical estimate. HOWEVER, it can give largely inaccurate (over-estimated) values

Hot-spot limits

$T_{max} < 300$ K for highly supported coils (e.g. accelerator magnets)



$T_{max} < 100$ K for negligible effect

- the quench starts in a point and propagates with a *quench propagation velocity*
- the initial point will be the *hot spot* at temperature T_{max}
- T_{max} must be limited to:
 - limit thermal stresses (see graph)
 - avoid material damage (e.g. resins have typical $T_{glass} \approx 100$ ° C)

Adiabatic hot spot temperature

- adiabatic conditions at the hot spot :

$$C \frac{dT}{dt} = q_{st}$$

where:

$$q_{st} = \frac{h_{st}}{A_{st}} \frac{I^2}{A}$$

- can be integrated:

total volumetric heat capacity

stabilizer resistivity

stabilizer fraction

cable operating current density

$$\int_{T_{op}}^{T_{max}} \frac{C}{h_{st}} dT = \frac{1}{f_{st}} \int_0^{\infty} J^2 dt$$

$$Z(T_{max}) = \int_{T_{op}}^{T_{max}} \frac{C}{h_{st}} dT$$

$$\int_0^{\infty} J^2 dt \gg J_{op}^2 t_{decay}$$

The function $Z(T_{max})$ is a *cable property*

How to limit T_{max}

stabilizer material property

$$Z(T_{max}) \gg \frac{1}{f_{st}} J_{op}^2 \tau_{decay}$$

electrical operation of the coil (energy, voltage)

cable fractions design

implicit relation between T_{max} , f_{st} , J_{op} , τ_{decay}

■ to decrease T_{max}

- reduce operating current density ($J_{op} \downarrow \downarrow$)
- discharge quickly ($\tau_{decay} \downarrow \downarrow$)
- add stabilizer ($f_{st} \uparrow \uparrow$)
- choose a material with large $Z(T_{max}) \uparrow \uparrow$

May reduce quench propagation speed and cause long detection times ! (see later)



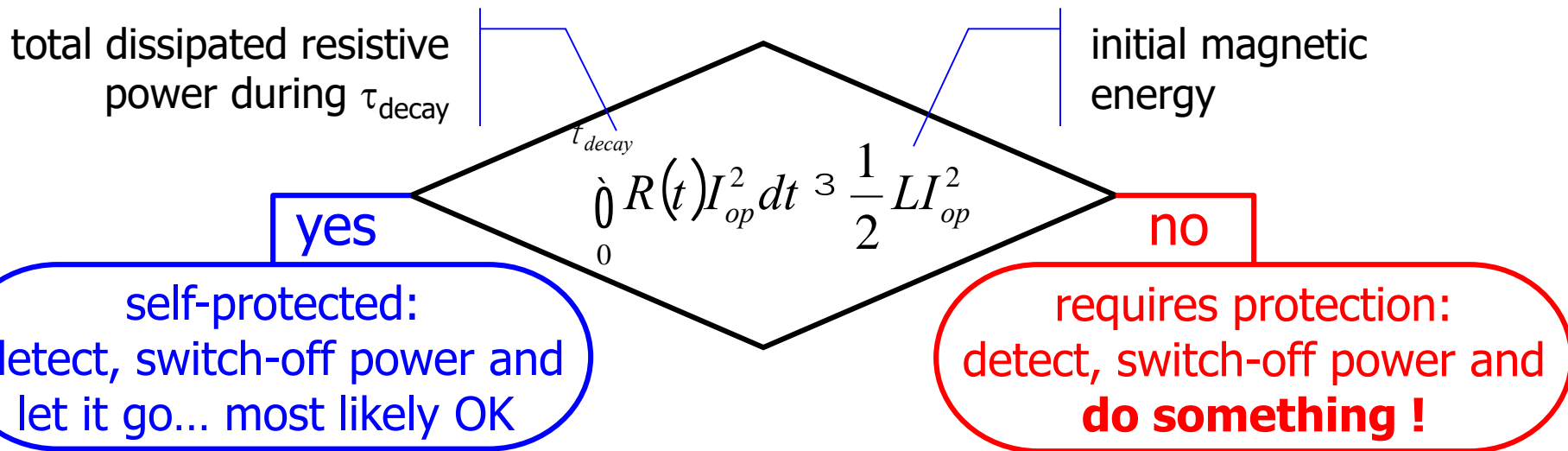
The quench dump

- the quench propagates in the coil at speed v_{quench} longitudinally ($v_{\text{longitudinal}}$) and transversely ($v_{\text{transverse}}$)...
- ...the total resistance of the normal zone $R_{\text{quench}}(t)$ grows in time following
 - the temperature increase, and
 - the normal zone evolution...
- ...a resistive voltage $V_{\text{quench}}(t)$ appears along the normal zone...
- ...that dissipates the magnetic energy stored in the field, thus leading to a discharge of the system in a time $\tau_{\text{discharge}}$.

the knowledge of $R_{\text{quench}}(t)$ is mandatory to verify the protection of the magnetic system !

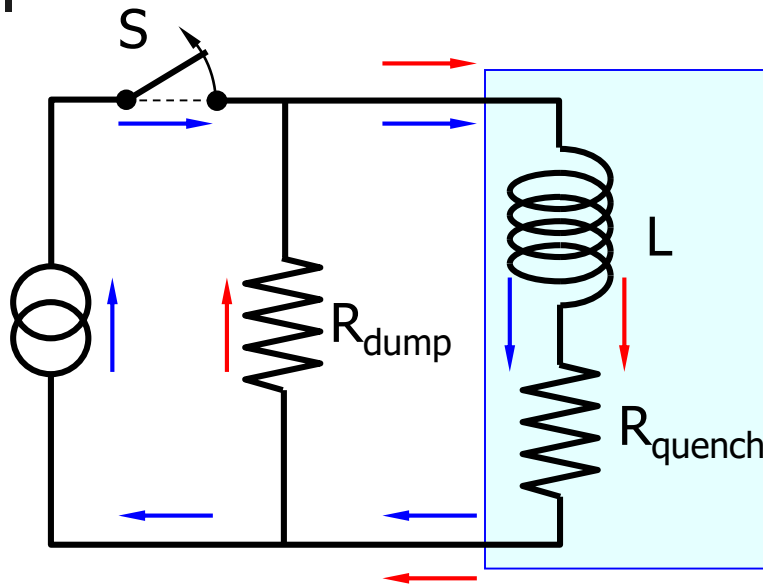
Quench protection concepts

- The magnet stores a magnetic energy $\frac{1}{2} L I^2$
- During a quench it dissipates a power $R I^2$ for a duration τ_{decay} characteristic of the powering circuit



WARNING: the reasoning here is qualitative, conclusions require in any case detailed checking

Strategy 1: energy dump



$$R_{dump} \gg R_{quench}$$

← normal operation

← quench

- the magnetic energy is extracted from the magnet and dissipated in an external resistor:

$$I = I_{op} e^{-\frac{(t-t_{detection})}{t_{dump}}} \quad t_{dump} = \frac{L}{R_{dump}}$$

- the integral of the current:

$$\int_0^{\infty} J^2 dt \gg J_{op}^2 \left(\frac{t_{detection}}{e} + \frac{t_{dump}}{2} \right)$$

- can be made small by:

- fast detection
- fast dump (large R_{dump})

Note: this is why we are limited to 500...700 A/mm2

Protection at high fields

$$E/l = \frac{\rho B^2 R_{in}^2}{m_0} + \frac{2}{3} \frac{w}{R_{in}} + \frac{1}{6} \frac{w}{R_{in}}$$

$$V/l \gg \frac{2E/l}{t I_{op}}$$

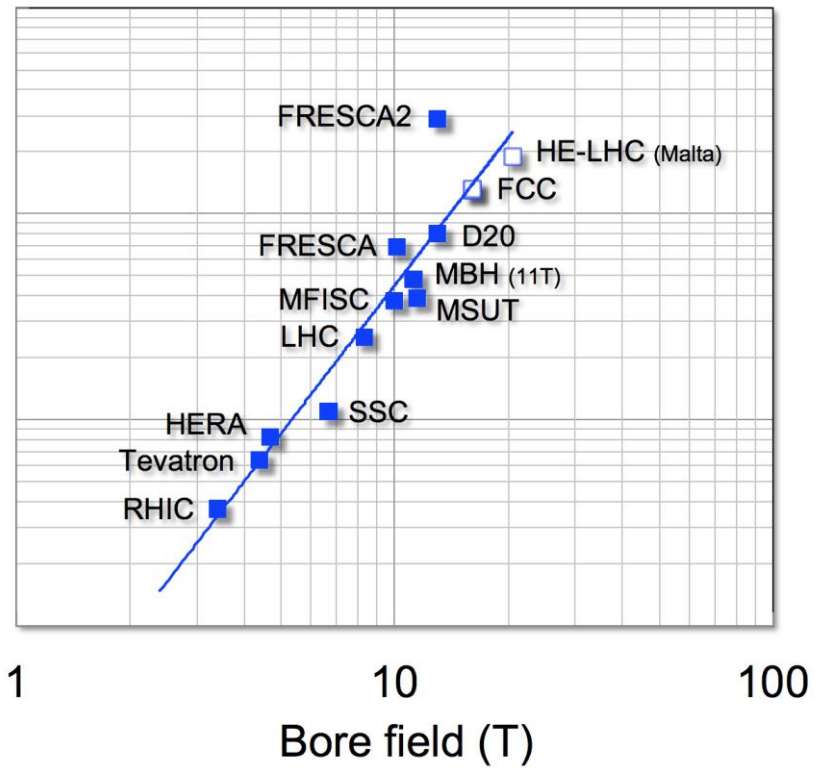
Voltage per unit length for an external dump with time constant τ

A simple exercise:

- $J_{Cu} \approx 1000...1250 \text{ (A/mm}^2\text{)}$
- $dT/dt \approx 1000...2000 \text{ (K/s)}$
- $\tau(300 \text{ K}) \approx 0.15...0.3 \text{ (s)}$
- $I_{op} \approx 15 \text{ (kA)}$
- $E/l \approx 1000 \text{ (kJ/m)}$
- $V/l \approx 500...1000 \text{ (V/m)}$

Energy per unit length in a sector coil of inner radius R_{in} , outer radius R_{out} , coil width $w = R_{out} - R_{in}$ producing a dipole field B

Stored energy (kJ/m)



It is not possible to protect accelerator magnet strings using an external dump

Strategy 2: coupled secondary

- the magnet is coupled inductively to a secondary that absorbs and dissipates a part of the magnetic energy

- advantages:

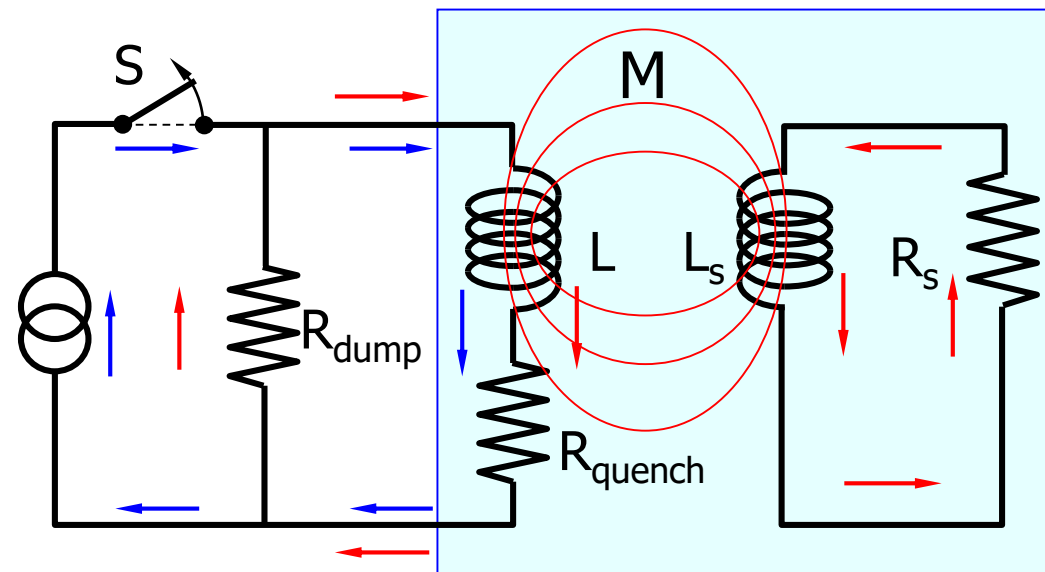
- magnetic energy partially dissipated in R_s (lower T_{\max})
- lower effective magnet inductance (lower voltage)
- heating of R_s can be used to speed-up quench propagation (quench-back)

- disadvantages:

- induced currents (and dissipation) during ramps

← normal operation

← quench



Strategy 3: subdivision

- the magnet is divided in sections, with each section shunted by an alternative path (resistance) for the current in case of quench

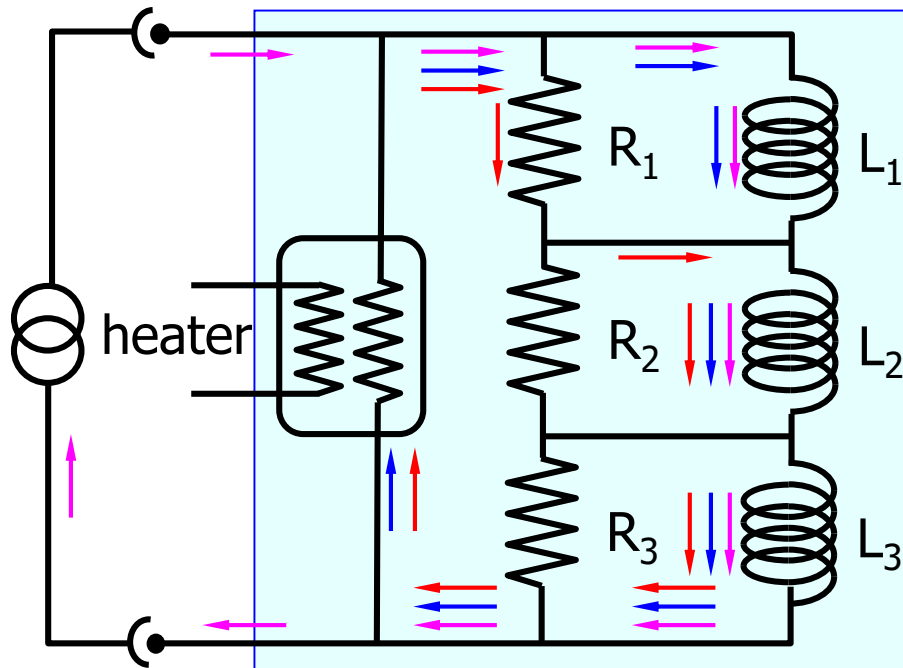
- advantages:

- passive
- only a fraction of the magnetic energy is dissipated in a module (lower T_{\max})
- transient current and dissipation can be used to speed-up quench propagation (quench-back)

- disadvantages:

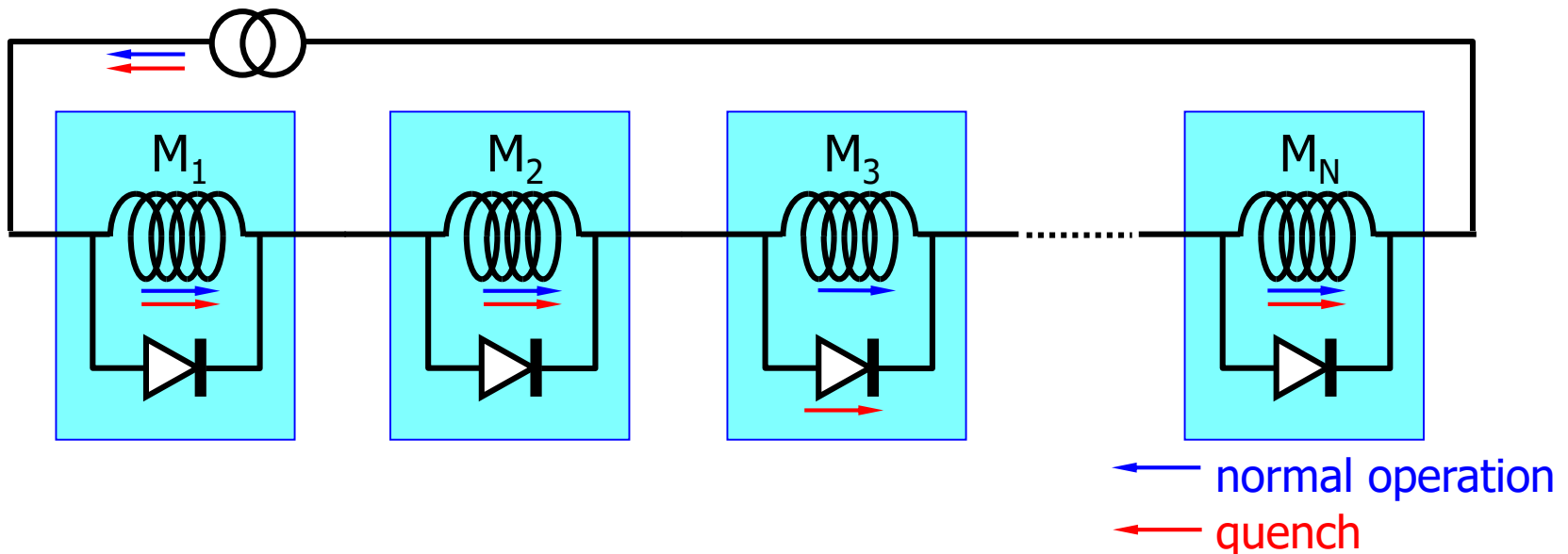
- induced currents (and dissipation) during ramps

- charge
- normal operation
- quench



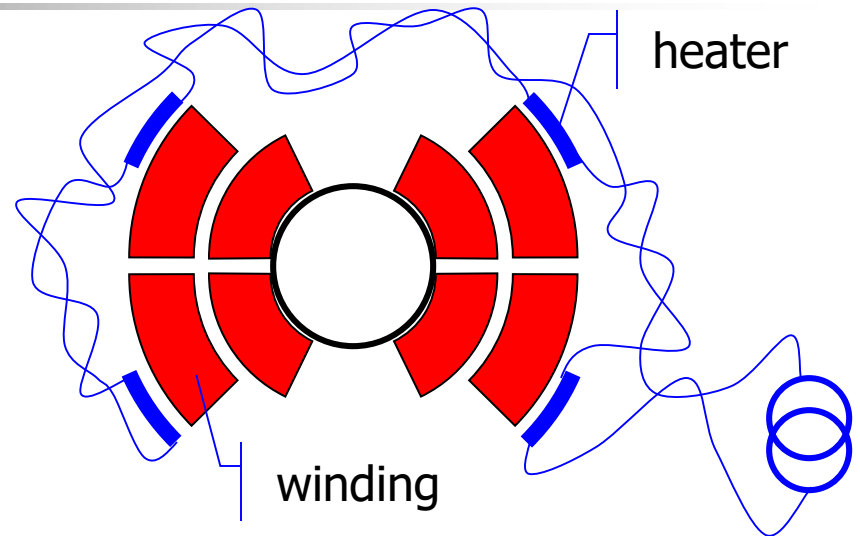
Magnet strings

- magnet strings (e.g. accelerator magnets, fusion magnetic systems) have exceedingly large stored energy (10' s of GJ):
 - energy dump takes very long time (10...100 s)
 - the magnet string is *subdivided* and each magnet is by-passed by a diode (or thyristor)



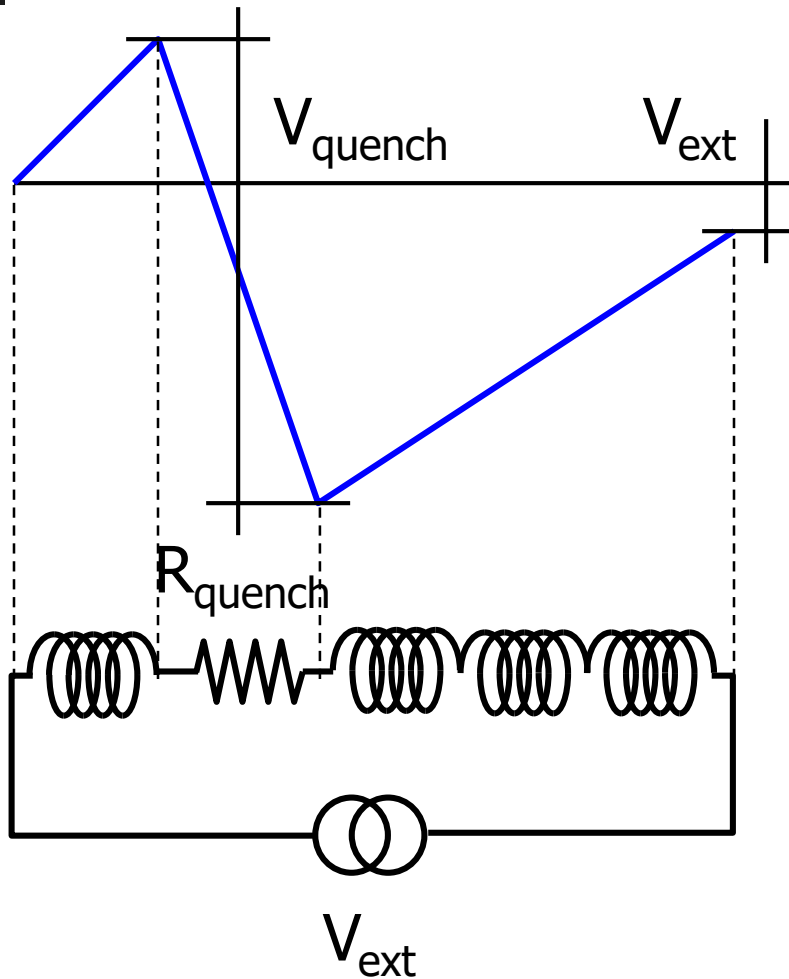
Strategy 4: heaters

- the quench is spread actively by firing heaters embedded in the winding pack, in close vicinity to the conductor
- heaters are mandatory in:
 - high performance, aggressive, cost-effective and highly optimized magnet designs...
 - ...when you are really desperate



- **advantages:**
 - homogeneous spread of the magnetic energy within the winding pack
- **disadvantages:**
 - active
 - high voltages at the heater

Quench voltage



- electrical stress can cause serious damage (arcing) to be avoided by proper design:
 - insulation material
 - insulation thickness
 - electric field concentration
- **REMEMBER: in a quenching coil the maximum voltage is not necessarily at the terminals**
- the situation in subdivided and inductively coupled systems is complex, may require extensive simulation



Quench and protection - Re-cap

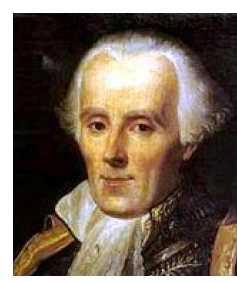
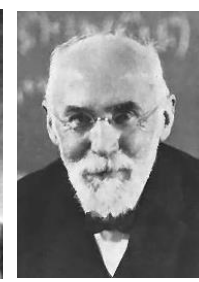
- A **good conducting material** (Ag, Al, Cu: large $Z(T_{\max})$) must be added in parallel to the superconductor to limit the maximum temperature during a quench
- The effect of a quench can be mitigated by
 - Adding stabilizer (\Leftrightarrow operating margin, stability)
 - Reducing operating current density (\Leftrightarrow economics of the system)
 - **Reducing the magnet inductance (large cable current) and increasing the discharge voltage** to discharge the magnet as quickly as practical



Overview

- Why superconductors ? A motivation
- **Superconducting magnet design**
 - Magnetic field and field quality
 - Margins and stability
 - **Forces and mechanics**
 - Quench protection

Electromagnetic force



(O. Heaviside) E.A. Lorentz, P.S. Laplace

- An electric charged particle q moving with a velocity v in a field B experiences a force F_L called electromagnetic (Lorentz) force (N):

$$\vec{F}_L = q\vec{v} \times \vec{B}$$

- A conductor carrying current density J (A/mm²) experiences a (Laplace) force density f_L (N/m³):

$$\vec{f}_L = \vec{J} \times \vec{B}$$

Magnetic pressure

- Ideal case of an infinite solenoid

- Vertical and uniform magnetic field

$$B_0 = \mu_0 J t$$

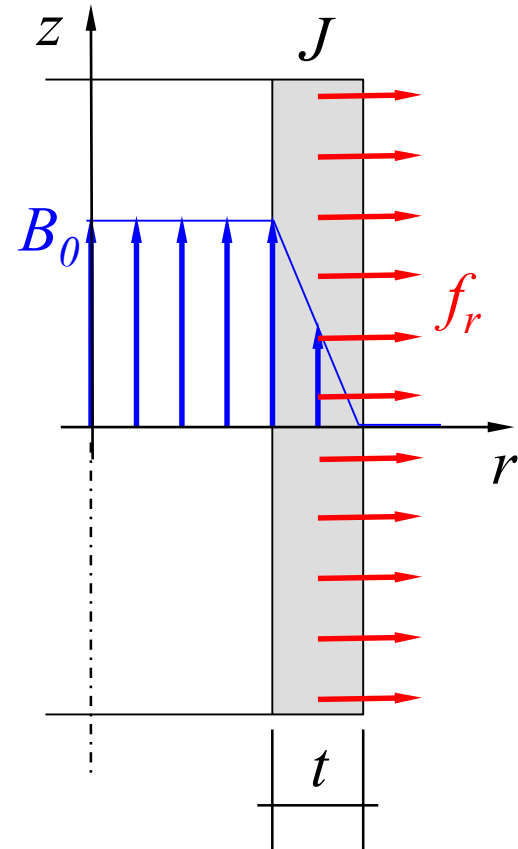
- Radial and uniform electromagnetic force

$$f_z = \int B(r) J dr = \int_0^t B_0 \left(1 - \frac{r}{t}\right) J dr = B_0 J \frac{t}{2} = \frac{B_0^2}{2\mu_0}$$

- Magnetic pressure

$$p = \frac{B_0^2}{2\mu_0}$$

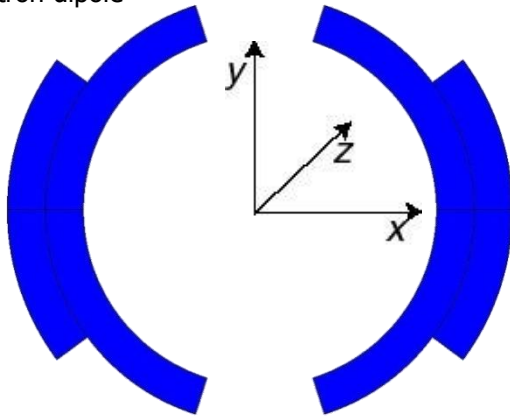
$$B_0 = 10 \text{ T} \rightarrow p = 400 \text{ bar}$$



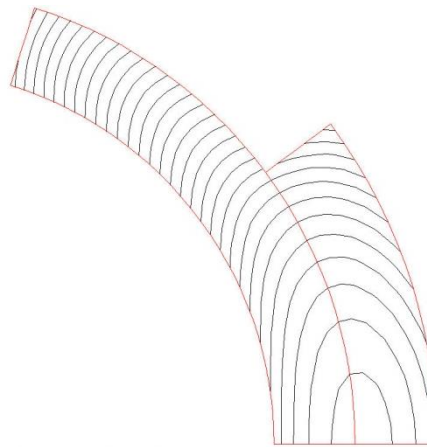
Electromagnetic forces - dipole

- The electromagnetic forces in a dipole magnet tend to push the coil:
 - Vertically, towards the mid plane ($F_y < 0$)
 - Horizontally, outwards ($F_x > 0$)

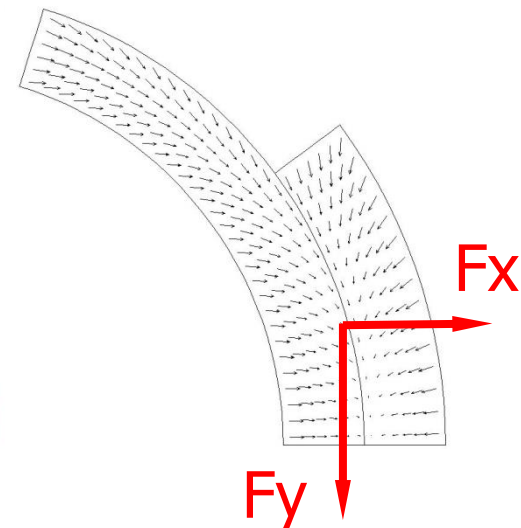
Tevatron dipole



Field

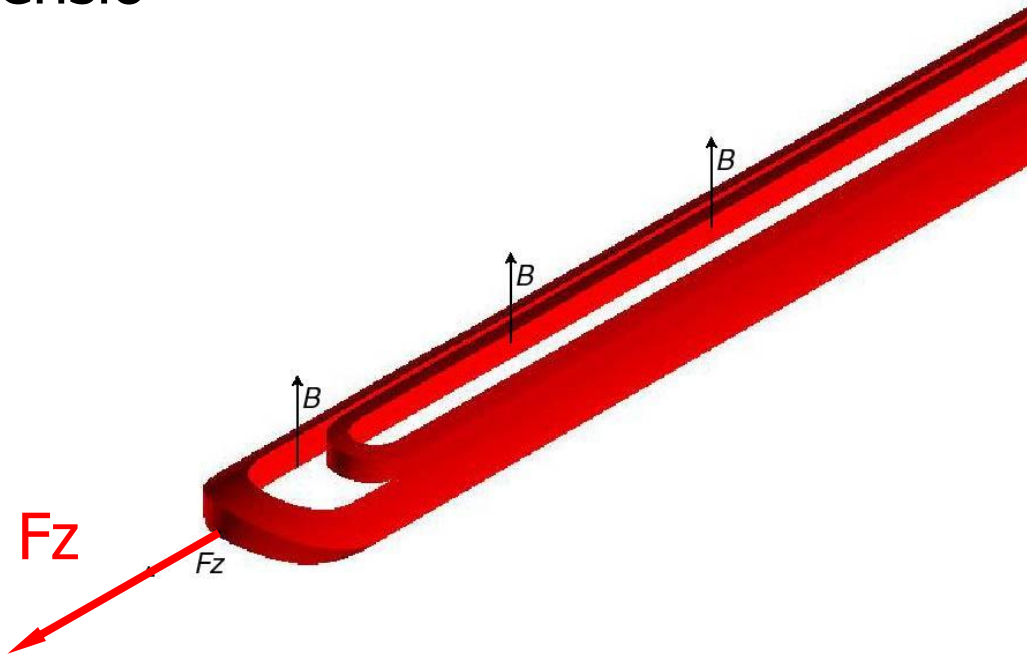


Force



Electromagnetic forces - ends

- In the coil ends the Lorentz forces tend to push the coil:
 - Outwards in the longitudinal direction ($F_z > 0$), and, similar to solenoids, the coil straight section is in tension



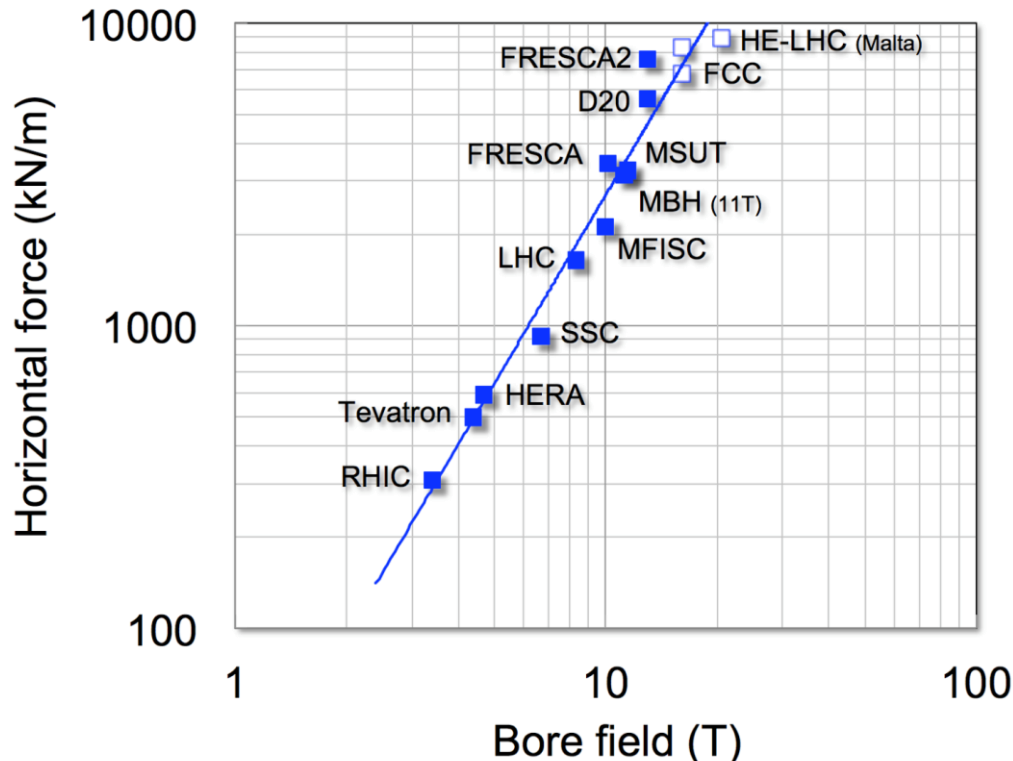
Note: this is why we are limited to 500...700 A/mm²

The real challenge of very high fields

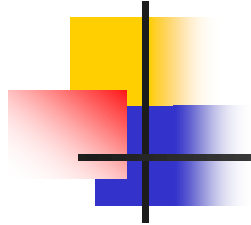
Lorentz forces in the plane of a thin coil of radius R_{in} generating a dipole field B (thin shell approximation), referred to a coil quarter

$$F_x = -F_y \gg \frac{4}{3} \frac{B^2}{2m_0} R_{in}$$

Force per coil quadrant in high-field dipoles built or designed for accelerators applications and R&D



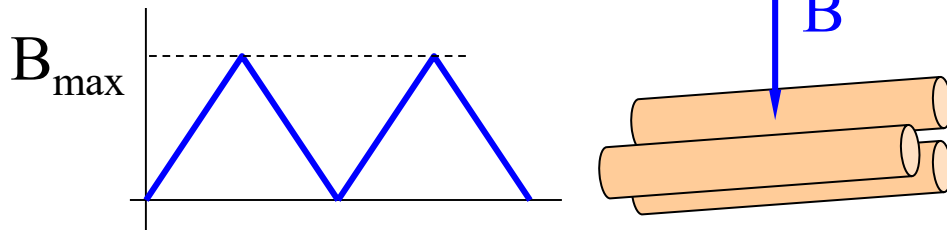
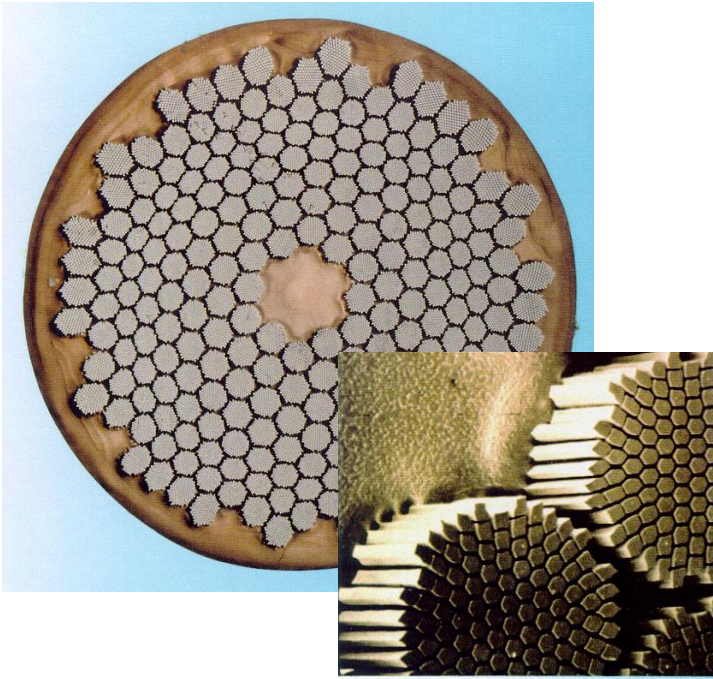
- Force increases **with the square of the bore field**
 - Requires massive structures (high-strength materials, volume, weight)
 - The stress limit is usually in the superconducting coil (superconductor and insulation, mitigated by $J_e \approx 1/B$)
- In practice the design of high field magnets is **limited by mechanics**



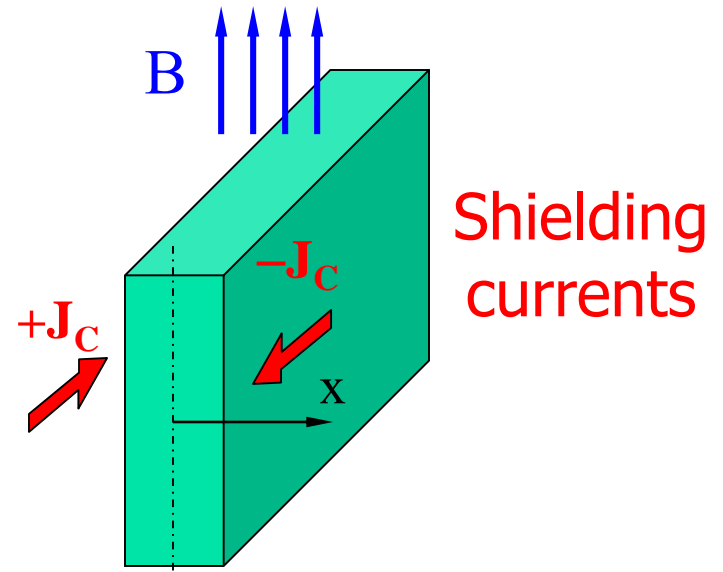
End of Part I

A superconductor in varying field

A simpler case: an infinite slab in a uniform, time-variable field



A filament in a time-variable field

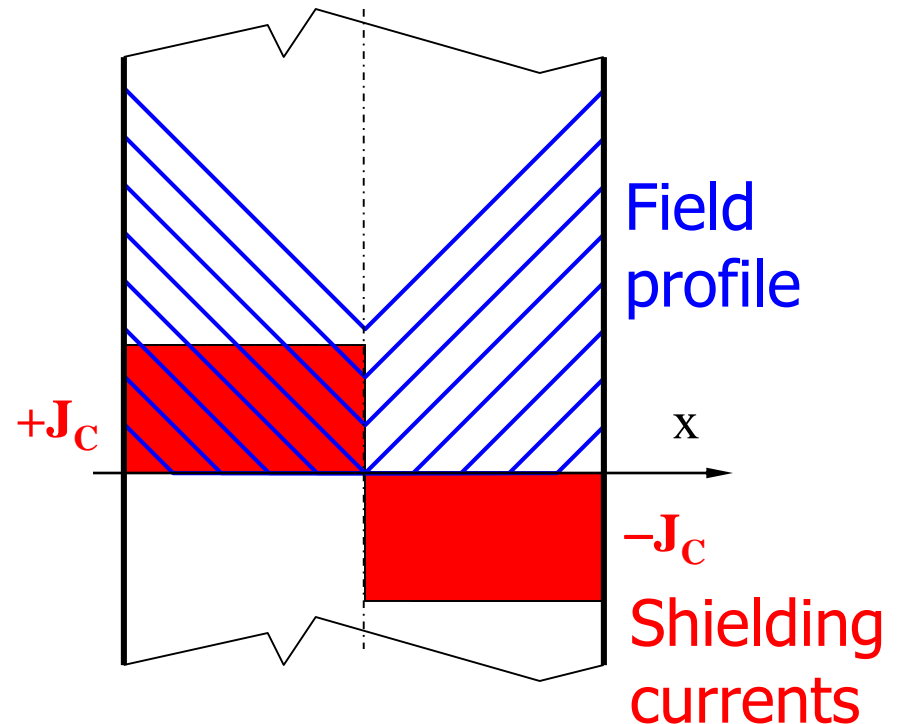
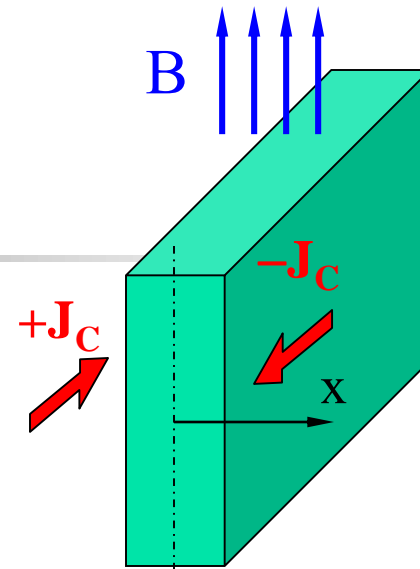


Quiz: how much is J ?

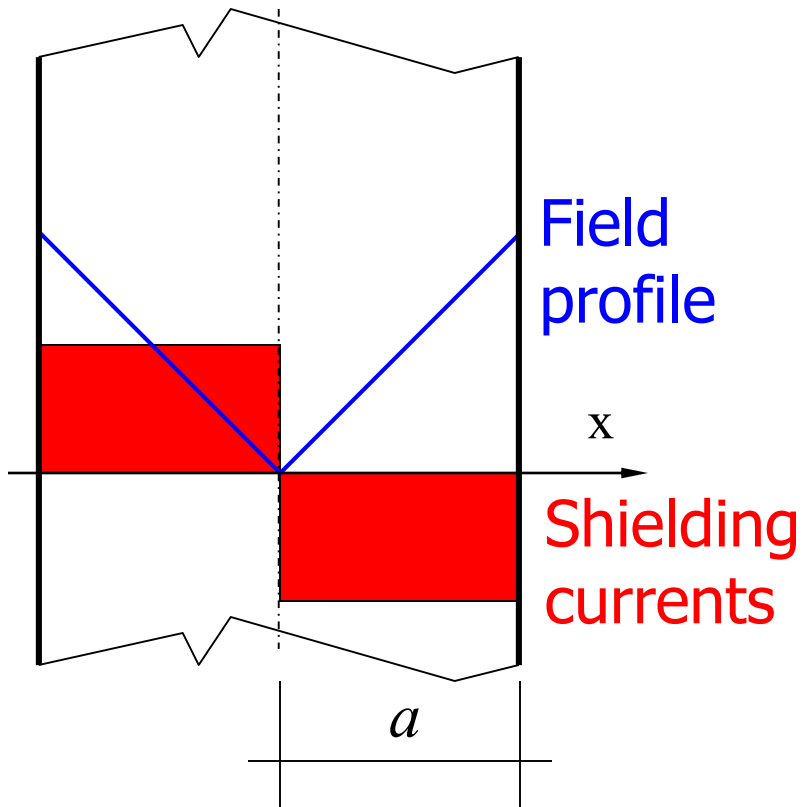
Persistent currents

- dB/dt produces an electric field E in the superconductor which drives it into the resistive state
- When the field sweep stops the electric field vanishes $E \Rightarrow 0$
- The superconductor goes back to J_C and then stays there
- This is the critical state (Bean) model: *within a superconductor, the current density is either $+J_C$, $-J_C$ or zero, there's nothing in between!*

$$J = \pm J_C$$



Magnetization



- Seen from outside the sample, the persistent currents produce a magnetic moment. We can define a *magnetization*:

$$M = \frac{1}{a} \int_0^a J_c x dx = \frac{J_c a}{2}$$

- The magnetization is proportional to the critical current density and to the size of the superconducting slab

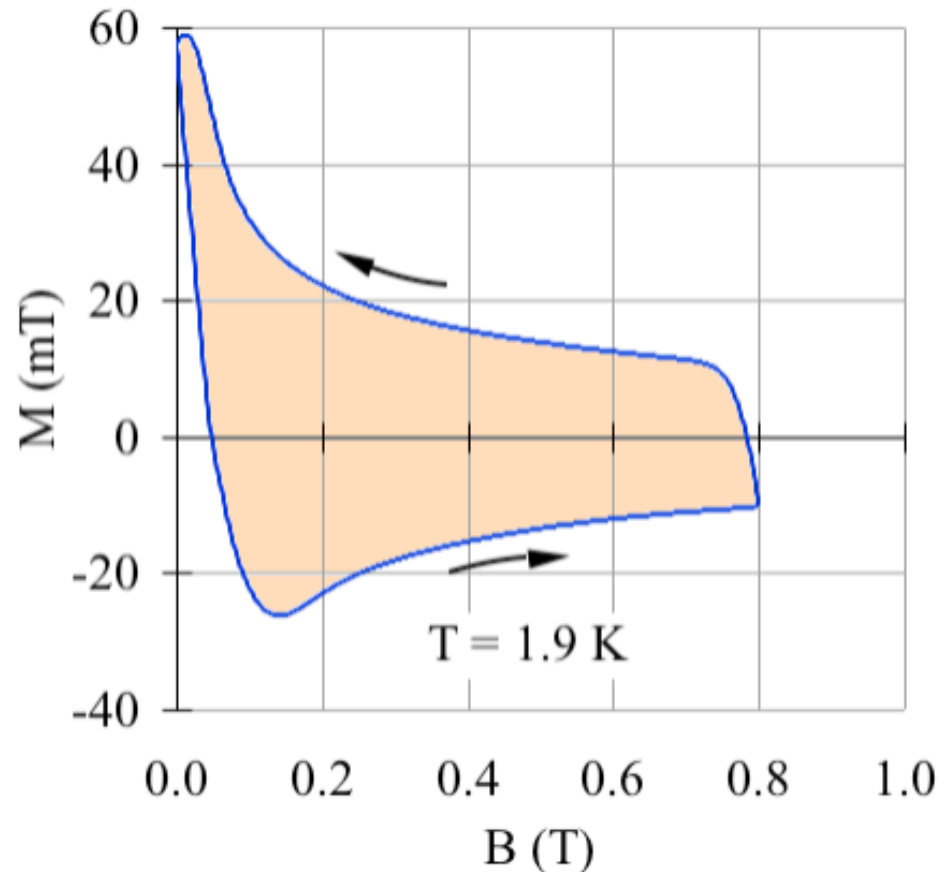
Hysteresis loss

- The response of a superconducting wire in a changing field is a field-dependent magnetization (remember $M \propto J_c(B)$)
- The work done by the external field is:

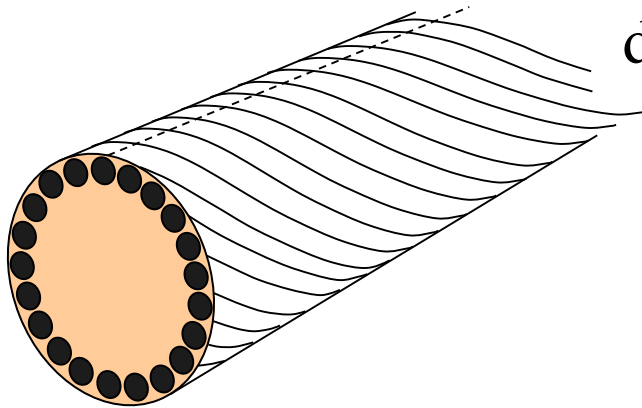
$$Q = \oint \mu_0 M dH = \oint \mu_0 H dM$$

i.e. the **area of the magnetization loop**

Remark: AC loss !?!



Filaments coupling



All superconducting wires are twisted to **decouple the filaments** and reduce the magnitude of eddy currents and associated loss

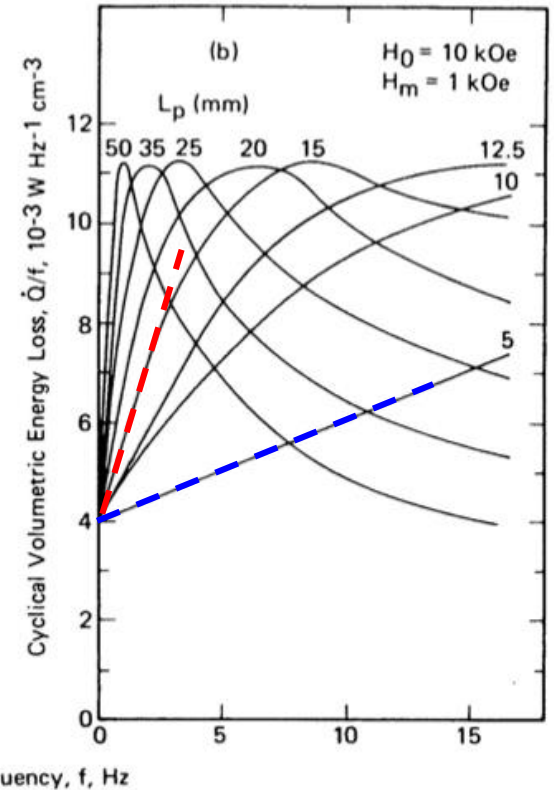
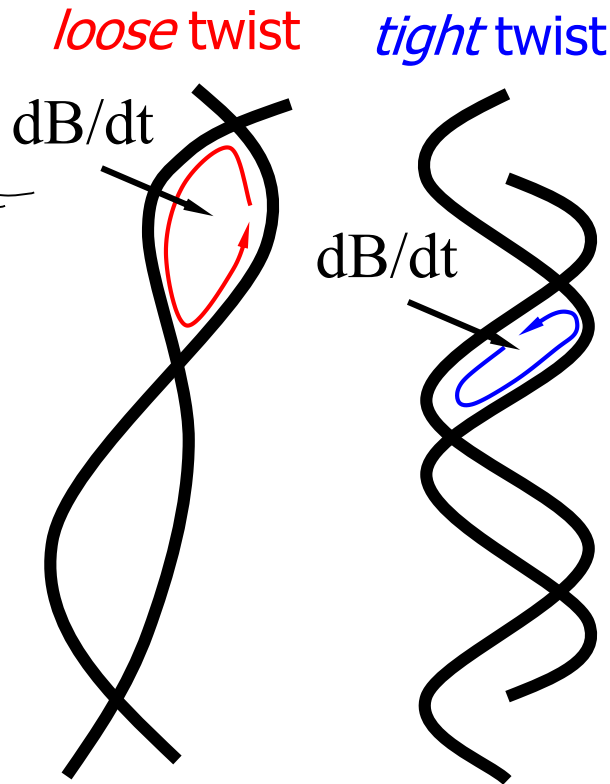
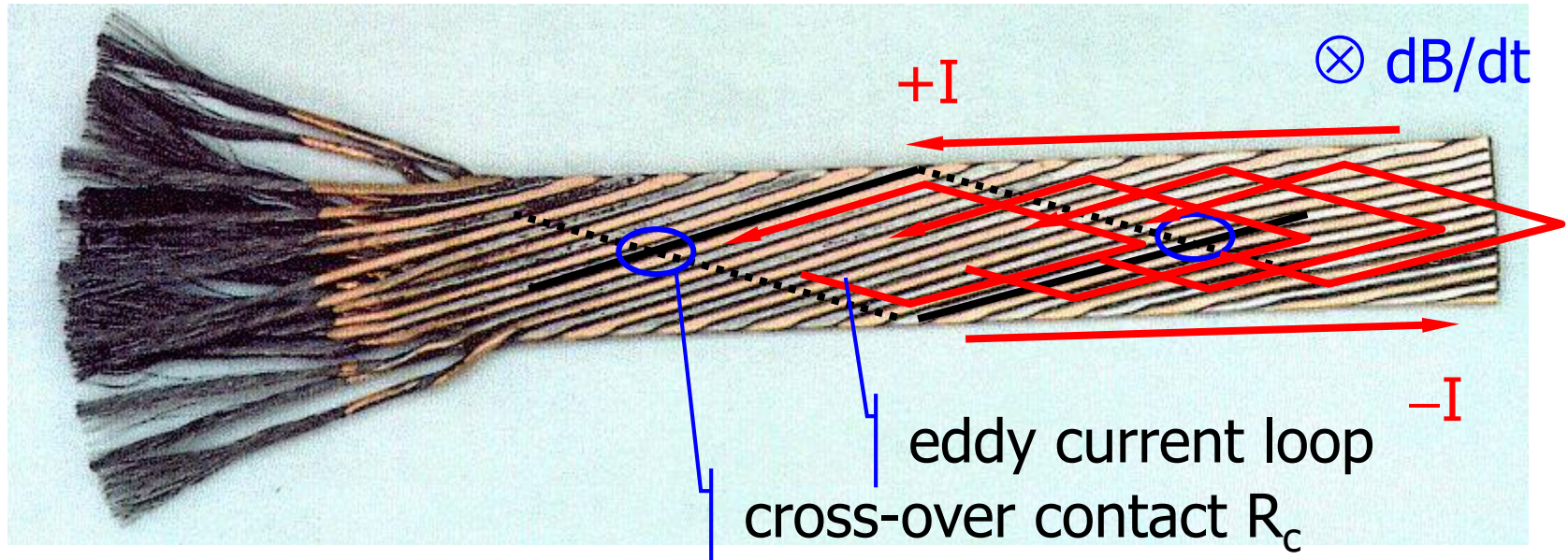


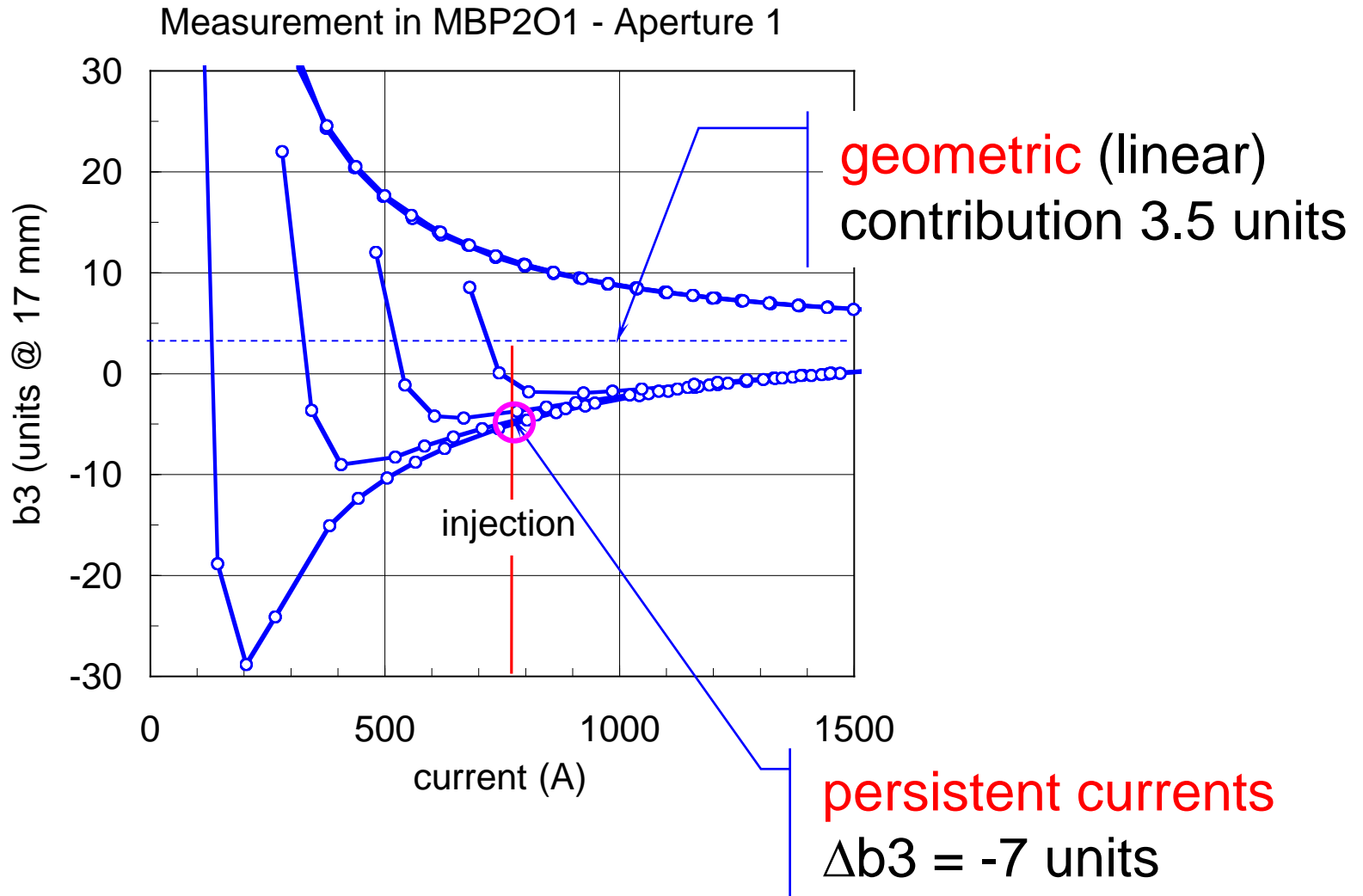
Figure 26-8. Energy loss per cycle ($\equiv \dot{Q}/f$) plotted versus frequency of the alternating component of an applied field $H_a(\omega) = H_0 + H_m \sin \omega t$. (a) The per-cycle coil loss is plotted for six values of H_m between 0.25 and 1.25 kOe at $H_0 = 10$ kOe; (b) the per-cycle volumetric loss of the composite is plotted for eight values of the twist pitch, L_p , at $H_0 = 10$ kOe, $H_m = 1$ kOe—after KWASNITZA and HORVATH [KWA74, KWA76].

Coupling in cables



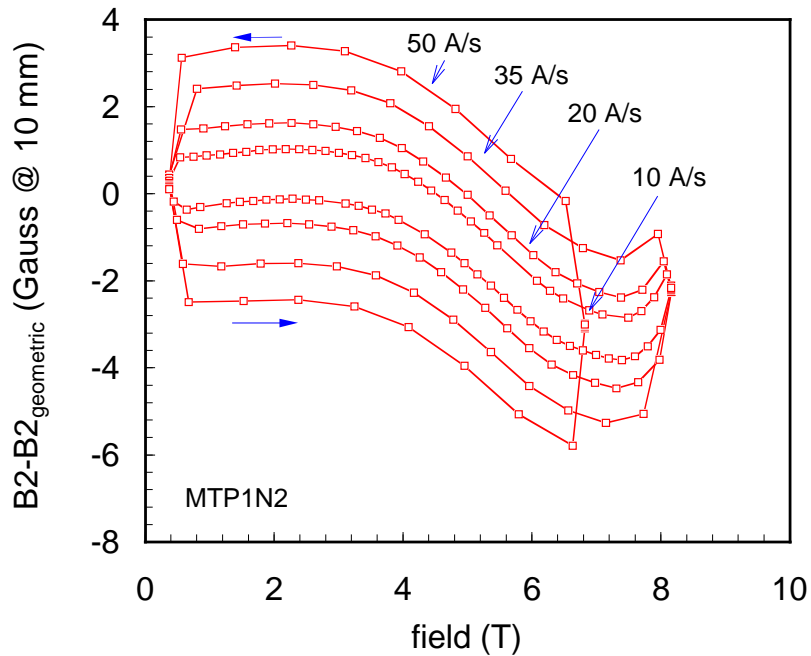
The strands in a cable are coupled (as the filaments in a strand). To decouple them we require to twist (**transpose**) the cable and to control the **contact resistances**

Field quality – “persistent”

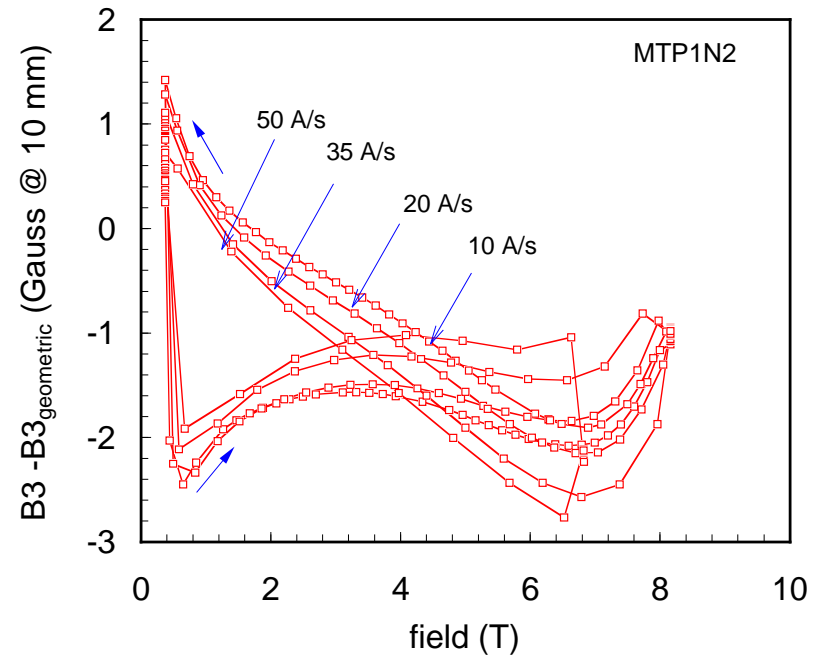


Field quality – “ramp”

Normal quadrupole during ramps

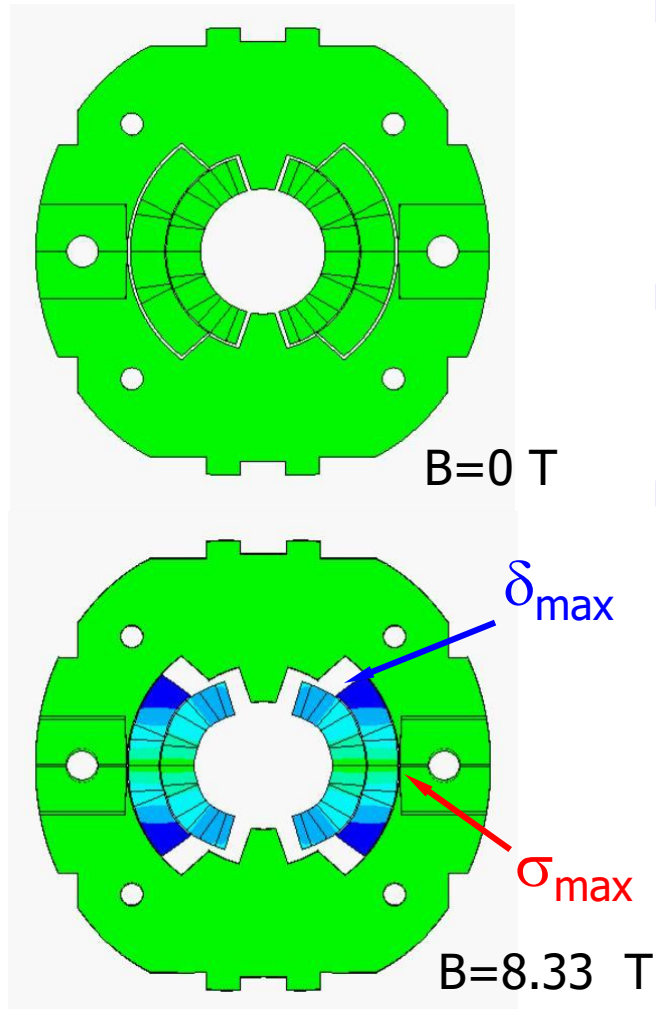


Normal sextupole during ramps



Stress and pre-stress - concepts

LHC dipole



- The peak stress is where the force accumulate, i.e. in the *mid-plane* for a $\cos(\theta)$ winding
- The *poles* of the coil tend to unload
- The coil needs **pre-loading** to avoid displacements
 - Mechanical energy release (cause quench and training)
 - Deformation of the coil geometry (affect field quality)

Effect of pre-load on training - *pro*

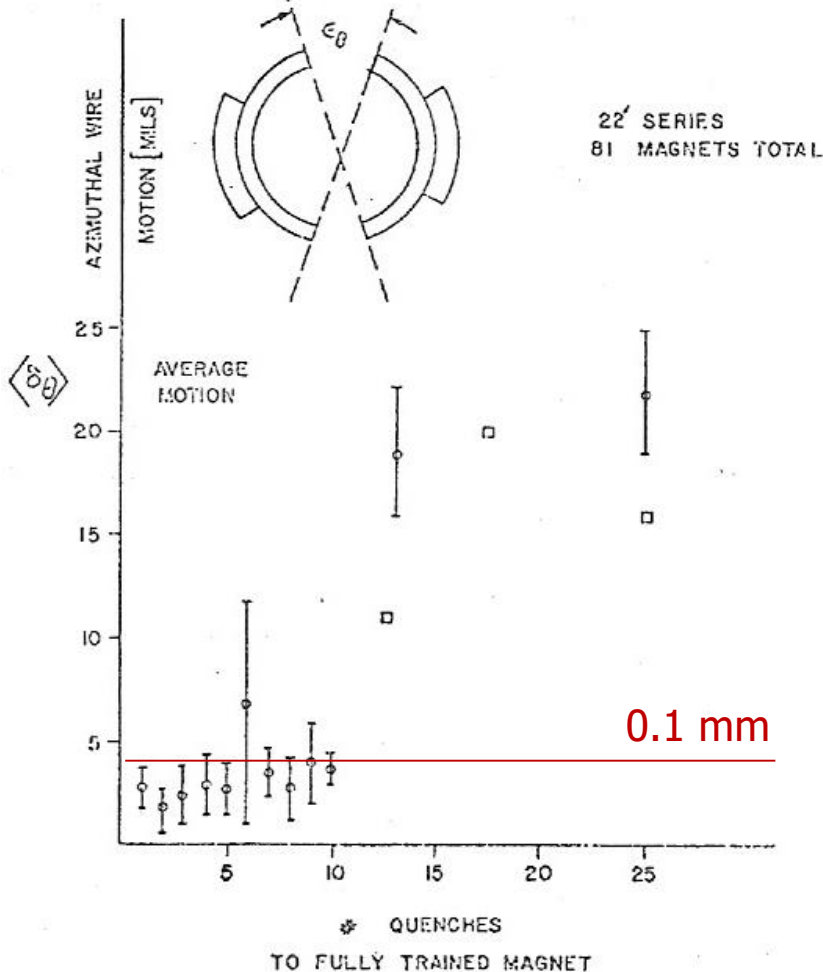
Training in Tevatron dipoles

elastic. It shows very little hysteresis. However, the azimuthal motion has given more trouble. Early in the program, magnets were built such that the elastic forces when cold were less than the magnetic forces, and the conductor at the key moved. Fig. 11 shows data from 81 magnets whose training took from one quench to over 25. Some magnets in this series had preload small enough so that there was motion of the wire at the col

Pre-load was not sufficient in the initial development of dipoles

error bars are just a square root of the number of magnets at each point. It is seen that this motion does not couple into the training until it is large enough so that the conductor is completely unclamped. Why it takes some magnets one quench and others 10 quenches to train when the conductor remains clamped is a mystery.

Large conductor movements were associated to long training

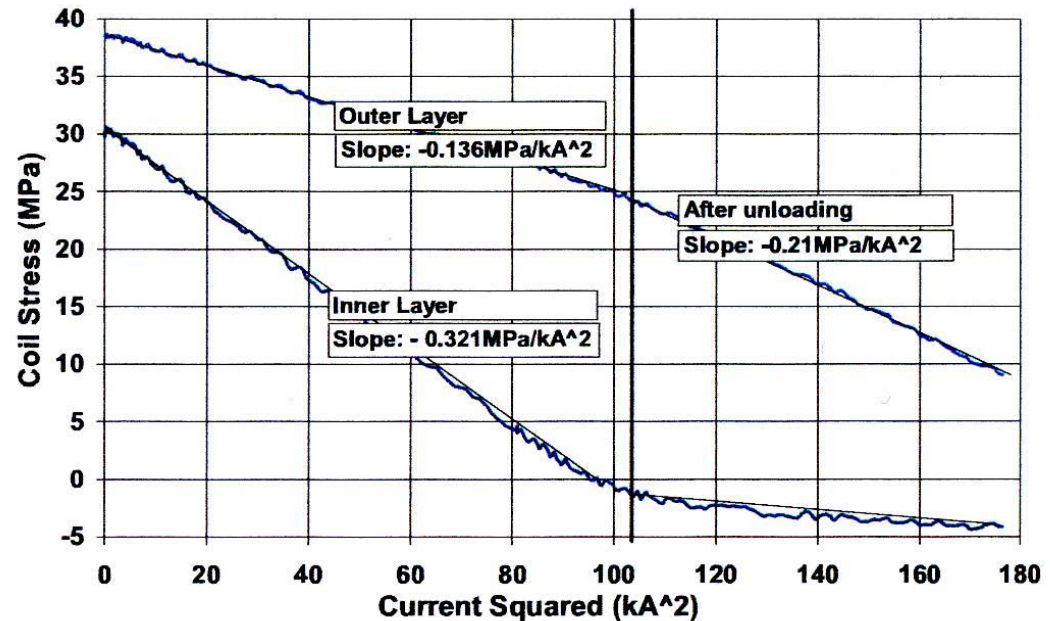


Effect of pre-load on training - *contra*

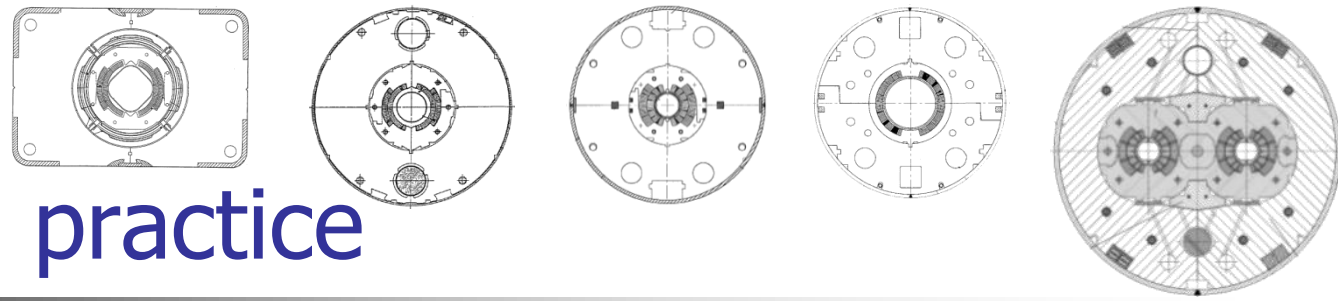
Evidence of pole unloading at 75 % of nominal current

It is worth pointing out that, in spite of the complete unloading of the inner layer at low currents, both low pre-stress magnets showed correct performance and quenched only at much higher fields

Pole force in a LHC model dipole

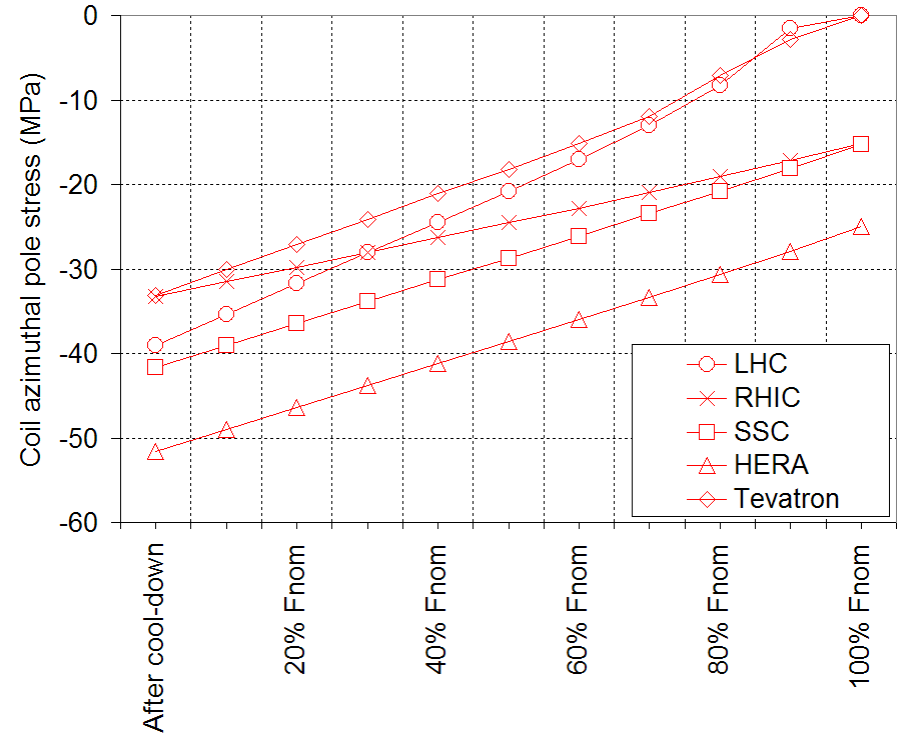


Large conductor movements do not seem to be associated to long training and degraded performance



Pre-load practice

- All SC accelerator magnets to date have been designed so that the coil retains the contact with the pole at nominal field
- In some cases an additional margin is taken, e.g. to deal with variations during manufacturing



- Whether and how much the pre-load affects the magnet performance is still a topic of (very) active research and development



Energy margin

- $\Delta Q'''$, energy margin
 - minimum energy density that leads to a quench
 - maximum energy density that can be tolerated by a superconductor, still resulting in recovery
 - simple and experimentally measurable quantity (...)
 - measured in [mJ/cm³] for convenience (values $\approx 1...1000$)
 - also called *stability margin*
 - compared to the energy spectrum to achieve stable design
- ΔQ , quench energy
 - better adapted for disturbances of limited space extension
 - measured in [μ J] to [mJ]

Flux-jumps energy

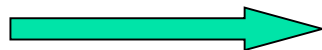
- During a complete flux-jump the field profile in a superconducting filament becomes flat:
 - e.g.: field profile in a fully penetrated superconducting slab

$$dB = m_0 J_c x$$

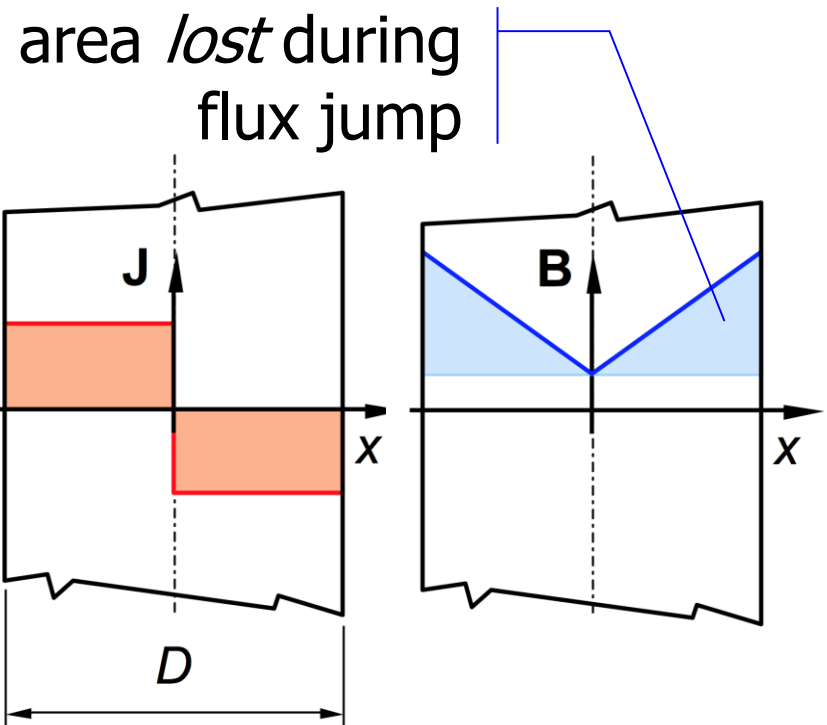
- energy stored in the magnetic field profile:

$$Q''' = \frac{2}{D} \int_0^{D/2} \frac{dB^2}{2m_0} dx = \frac{m_0 J_c^2 D^2}{24}$$

$$D = 50 \mu\text{m}, J_c = 10000 \text{ A/mm}^2$$



$$Q''' \approx 6 \text{ mJ/cm}^3$$



NOTE: to decrease Q''' , one can decrease D

Mechanical events

- a strand carrying a current I_{op} in a field B_{op} is subjected to a force F
- force per unit length acting on the strand F' :

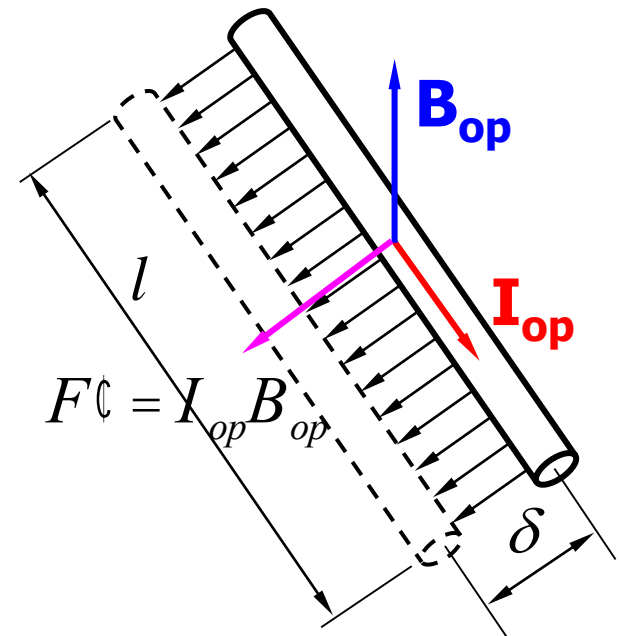
$$F \zeta = I_{op} B_{op}$$

$$J_{op} = 400 \text{ A/mm}^2, B_{op} = 10 \text{ T} \Rightarrow f = 4 \text{ GN/m}^3$$

- a displacement δ of a length l requires a work W :

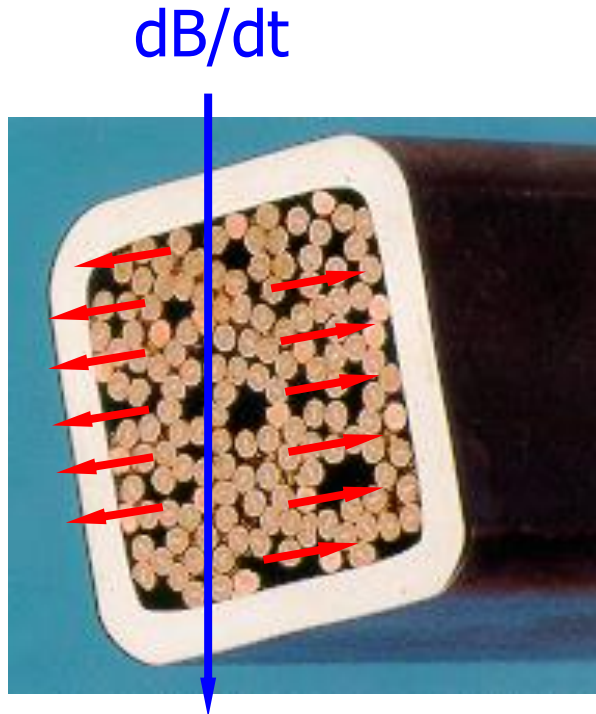
$$W = F \zeta d l$$

$$\delta = 10 \text{ } \mu\text{m}, l = 1 \text{ mm} \Rightarrow W'''' \approx 40 \text{ mJ/cm}^3$$



$$\longrightarrow Q'''' \approx 1...10 \text{ mJ/cm}^3$$

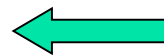
AC loss



- a changing magnetic field causes persistent and coupling currents in a superconducting cable
- these currents cause *hysteresis* or *coupling AC loss*
- e.g. coupling current loss due to a field ramp

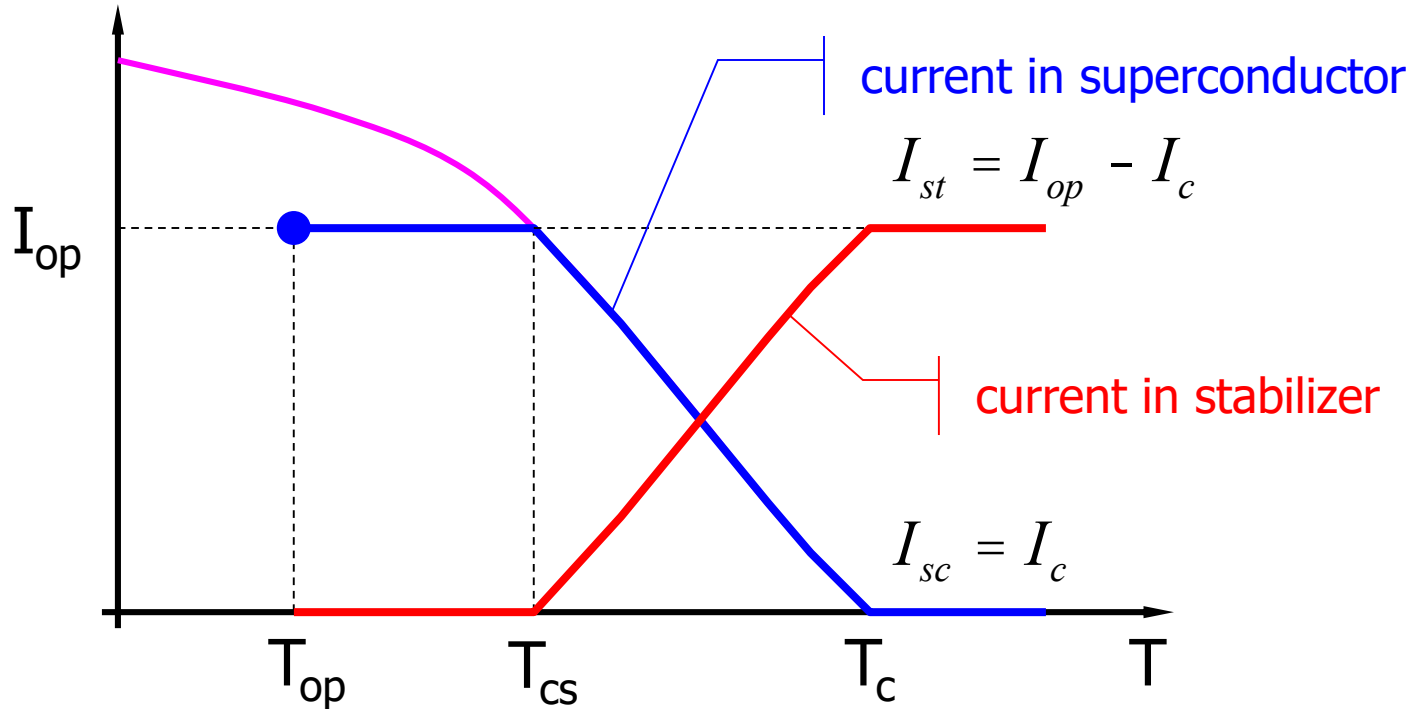
$$Q''' = \frac{n t}{m_0} \frac{dB}{dt} DB$$

$$Q''' \approx 80 \text{ mJ/cm}^3$$



$$n\tau = 100 \text{ ms}, \text{ dB/dt} = 1 \text{ T/s}, \Delta B = 1 \text{ T}$$

Joule heating



$$q_{\mathcal{J}} = \frac{EI_{st} + EI_{sc}}{A} = \frac{EI_{op}}{A} = \frac{h_{st}}{A_{st}} \frac{I_{op} (I_{op} - I_c)}{A}$$

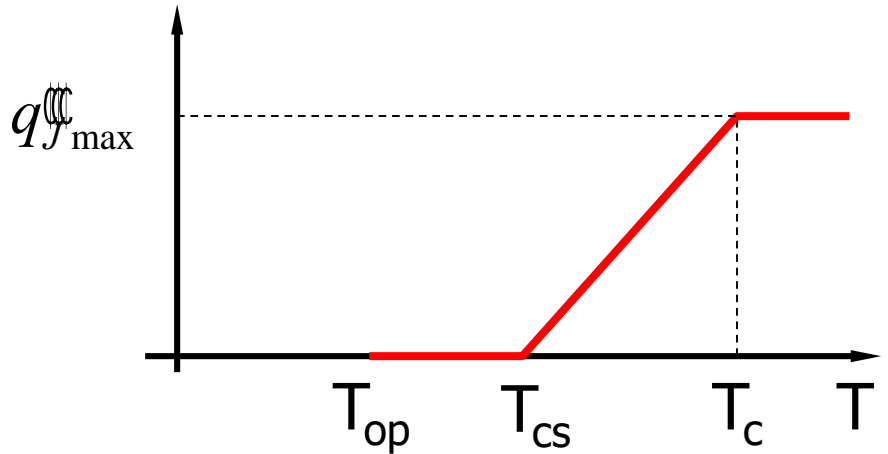
$$q_{\mathcal{J} \max} = \frac{h_{st}}{A_{st}} \frac{I_{op}^2}{A}$$

Joule heating (cont' d)

- linear approximation for $J_c(T)$

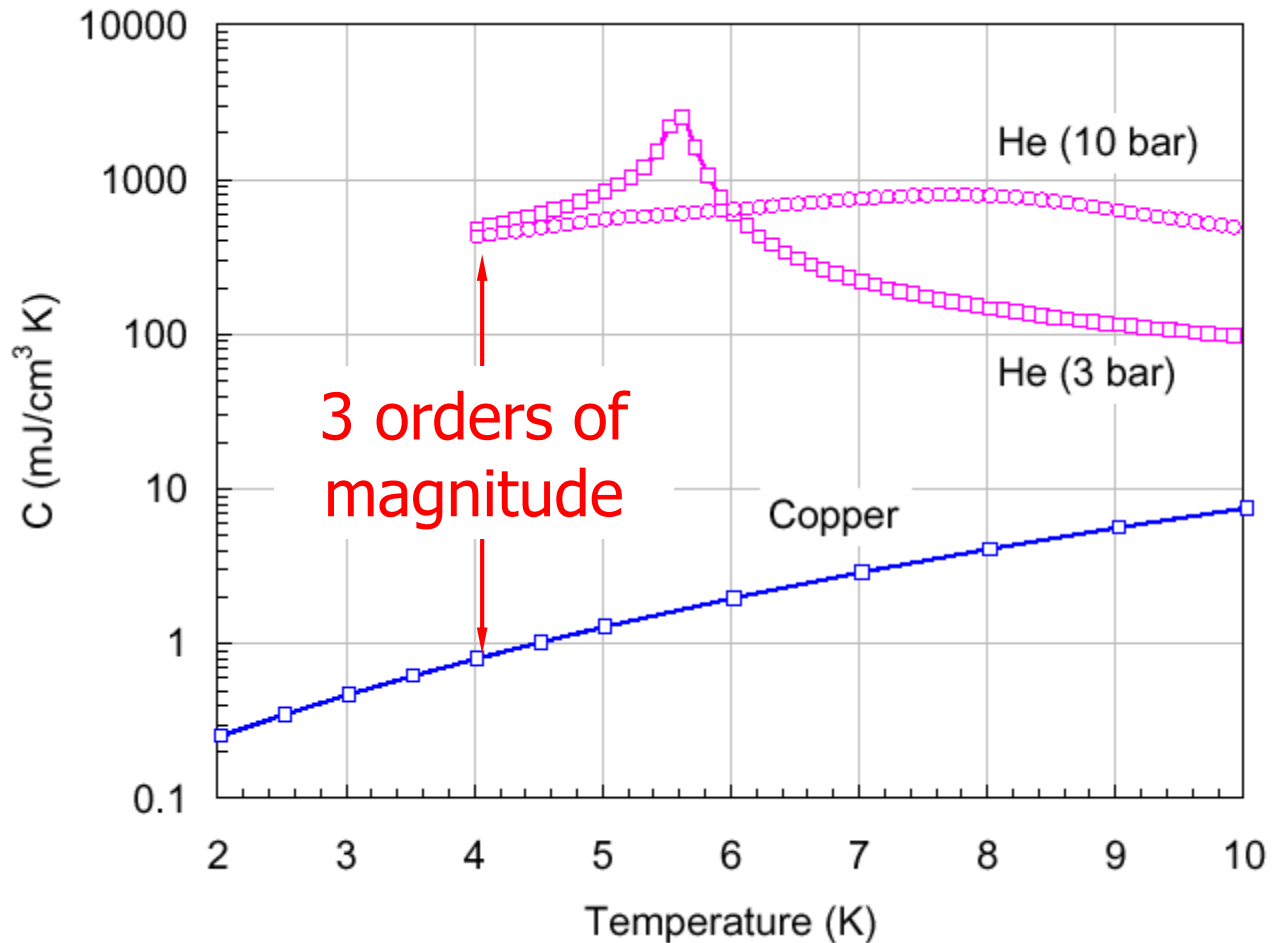
$$I_c \gg I_{op} \frac{T_c - T}{T_c - T_{cs}} \quad I_{op}$$

- Joule heating

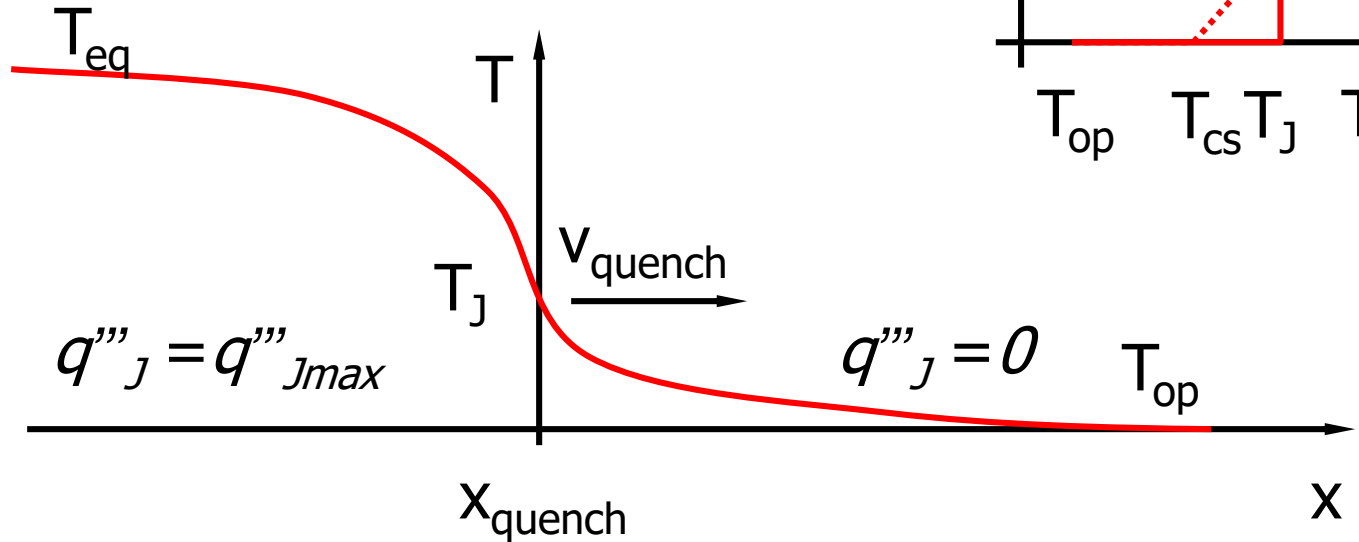


$$q_J = \begin{cases} 0 & \text{for } T < T_{cs} \\ q_{J_{max}} \frac{T - T_{cs}}{T_c - T_{op}} & \text{for } T_{cs} < T < T_c \\ q_{J_{max}} & \text{for } T > T_c \end{cases} \quad q_{J_{max}} = \frac{h_{st}}{A_{st}} \frac{I_{op}^2}{A}$$

Helium is a great heat sink !



Adiabatic propagation



$$C \frac{\partial T}{\partial t} = q''' + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right)$$

fixed reference frame

$$X = x - x_{quench} = x - v_{quench} t$$

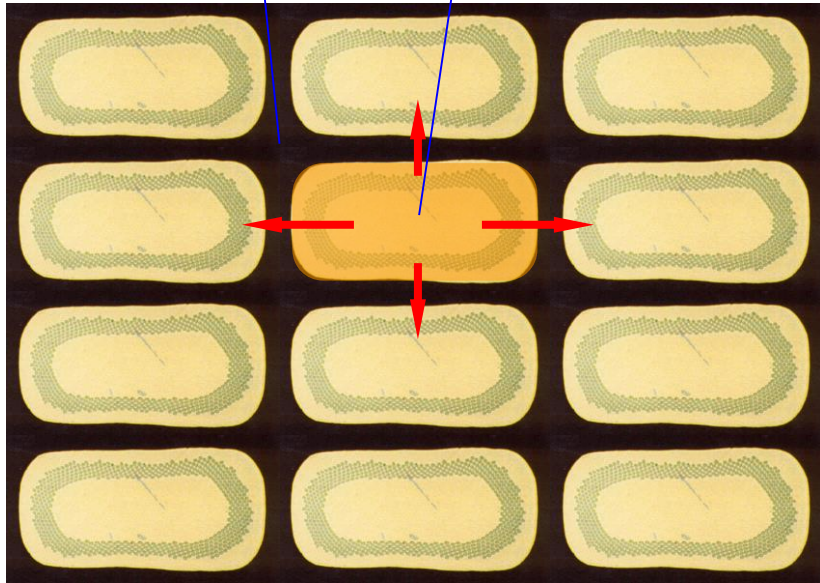
moving reference frame

$$k \frac{\partial^2 T}{\partial X^2} + v_{quench} C \frac{\partial T}{\partial X} + q''' = 0$$

Turn-to-turn propagation

insulation

conductor in normal state



Heat conduction spreads the quench from turn to turn as it plods happily along a conductor at speed $v_{longitudinal}$. The $v_{transverse}$ is approximated as:

insulation conductivity

$$\frac{v_{transverse}}{v_{longitudinal}} \gg \sqrt{\frac{k_{in}}{k_{st}}}$$

(large) correction factors for geometry, heat capacity, non-linear material properties apply to the scaling !



The $Z(T_{max})$ function

- the function $Z(T_{max})$ is a *cable property*:

$$Z(T_{max}) = \int_{T_{op}}^{T_{max}} \frac{C}{h_{st}} dT$$

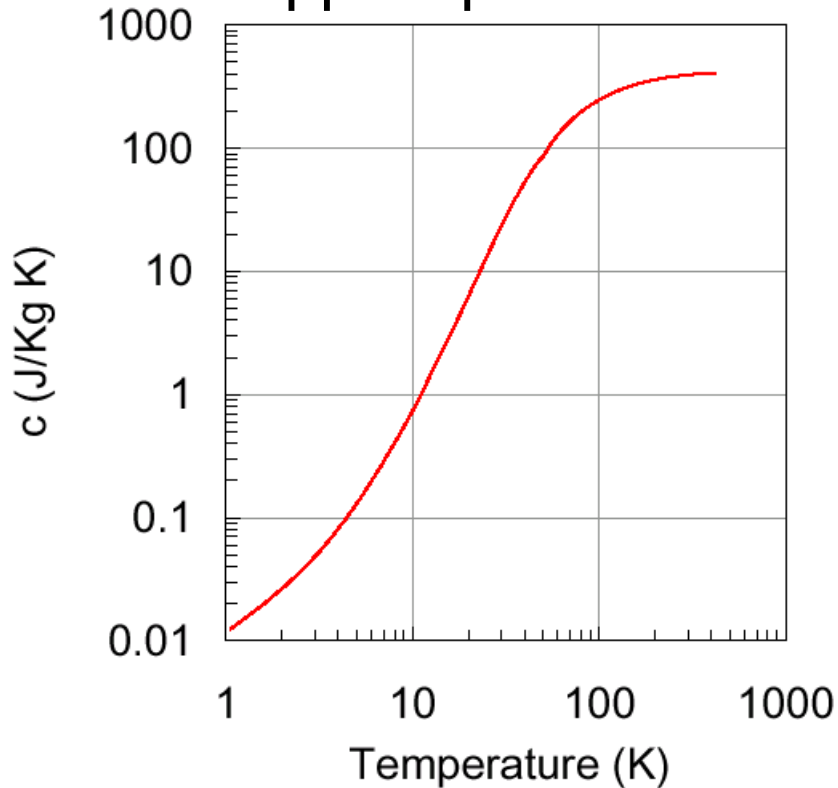
- the volumetric heat capacity C is defined using the material fractions f_i :

$$C = \frac{\sum_i A_i r_i c_i}{\sum_i A_i} = \sum_i f_i r_i c_i$$

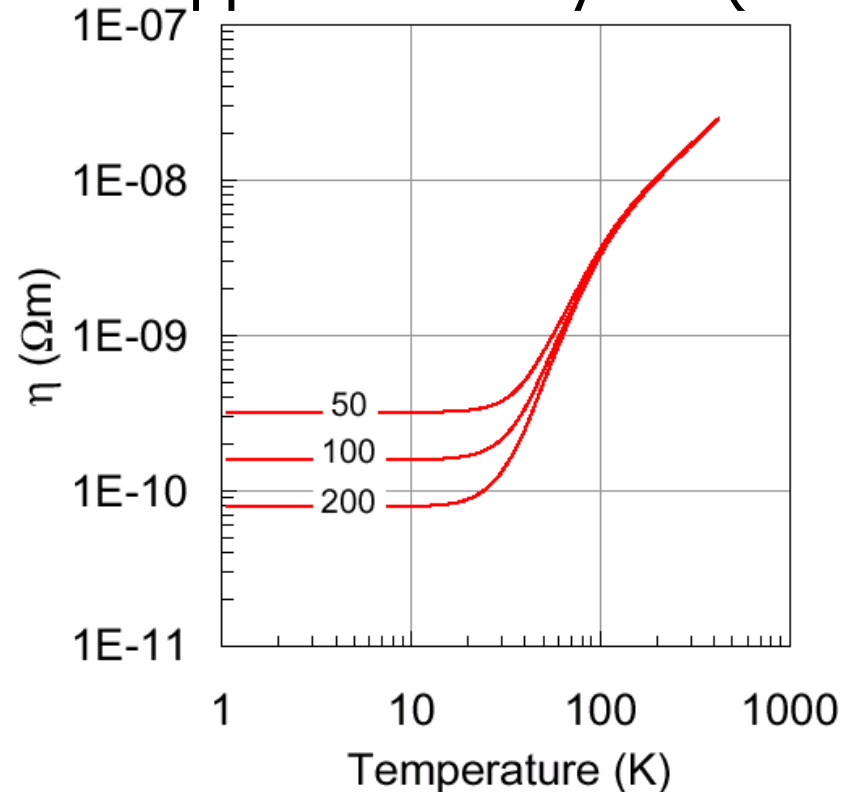
- $Z(T_{max})$ can be computed (universal function) for a given cable design (i.e. f_i fixed) !

Material properties

copper specific heat

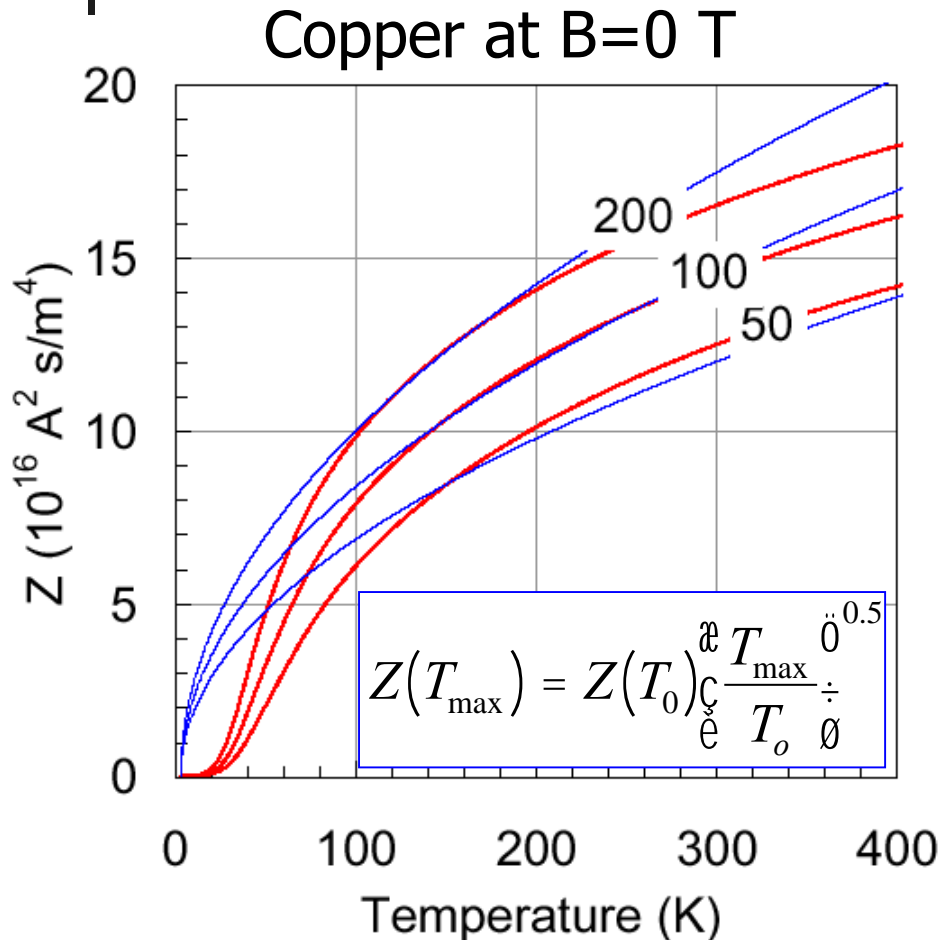


copper resistivity as f(RRR)



large variation over the range of interest !

$Z(T_{max})$ for pure materials

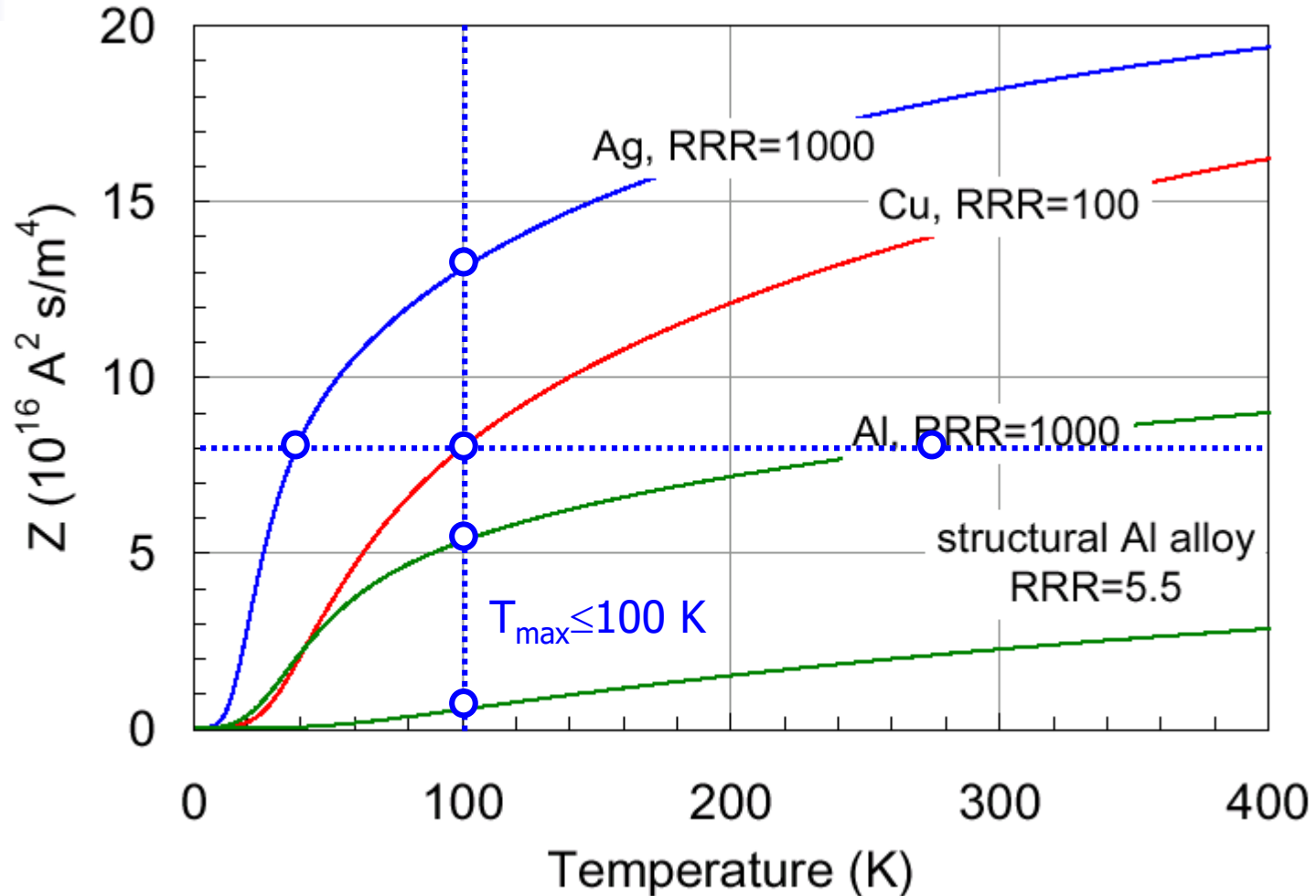


- assuming the cable as being made of stabilizer (good approximation):
 - $f_{st} = 1,$
 - $C = \rho_{st} c_{st}$
- $Z(T_{max})$ is a *material property* that can be tabulated:

$$Z(T_{max}) = \int_{T_{op}}^{T_{max}} \frac{r_{st} c_{st}}{h_{st}} dT$$

$$Z(T_{\max}) \gg \frac{1}{f_{st}} J_{op}^2 t_{decay}$$

$Z(T_{\max})$ for typical stabilizers



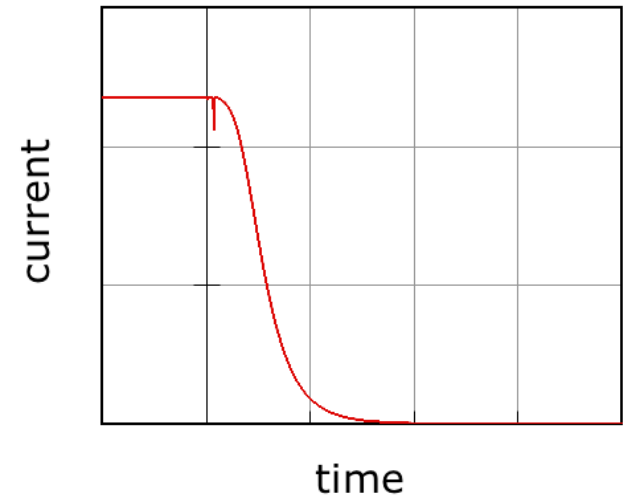
- sometimes (HEP accelerator and detector magnets) the energy balance is written as follows:

$$\int_{T_{op}}^{T_{max}} \frac{C}{h_{st}} dT = \frac{1}{f_{st}} \int_0^{\infty} J^2 dt$$



$$f_{st} A^2 \int_{T_{op}}^{T_{max}} \frac{C}{h_{st}} dT = \int_0^{\infty} I^2 dt$$

$$\gg I_0^2 \left(t_{detection} + t_{delay} + t_{switch} + \frac{t_{dump}}{2} \right)$$



- the r.h.s is measured in: **Mega I × I x Time (MIITs)**
- however, now the l.h.s. is **no longer a material property**

Dump time constant

- magnetic energy:

$$E_m = \frac{1}{2} L I_{op}^2$$

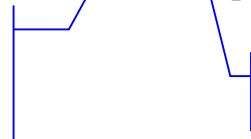
- maximum terminal voltage:

$$V_{max} = R_{dump} I_{op}$$

- dump time constant:

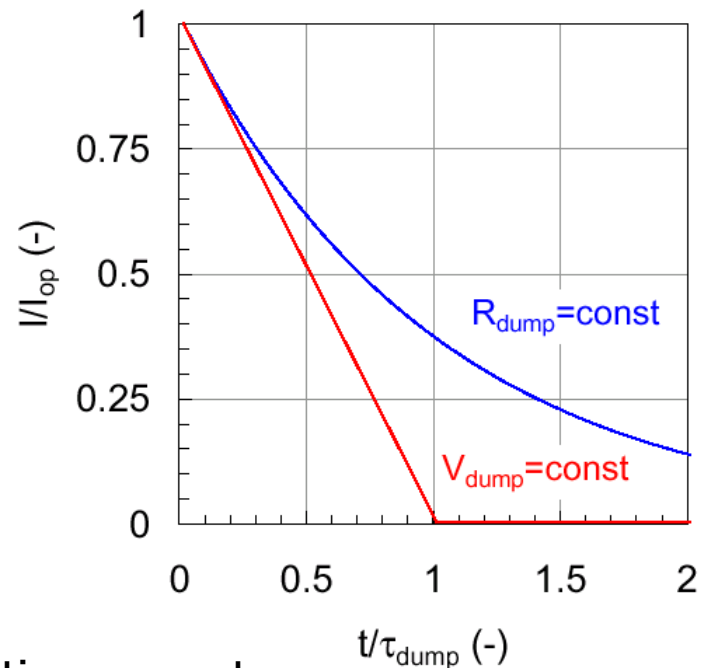
$$t_{dump} = \frac{L}{R_{dump}} = \frac{2E_m}{V_{max} I_{op}}$$

maximum terminal
voltage



operating current

interesting alternative:
non-linear R_{dump} or voltage source



increase V_{max} and I_{op} to achieve fast dump time