The Strained State Cosmology

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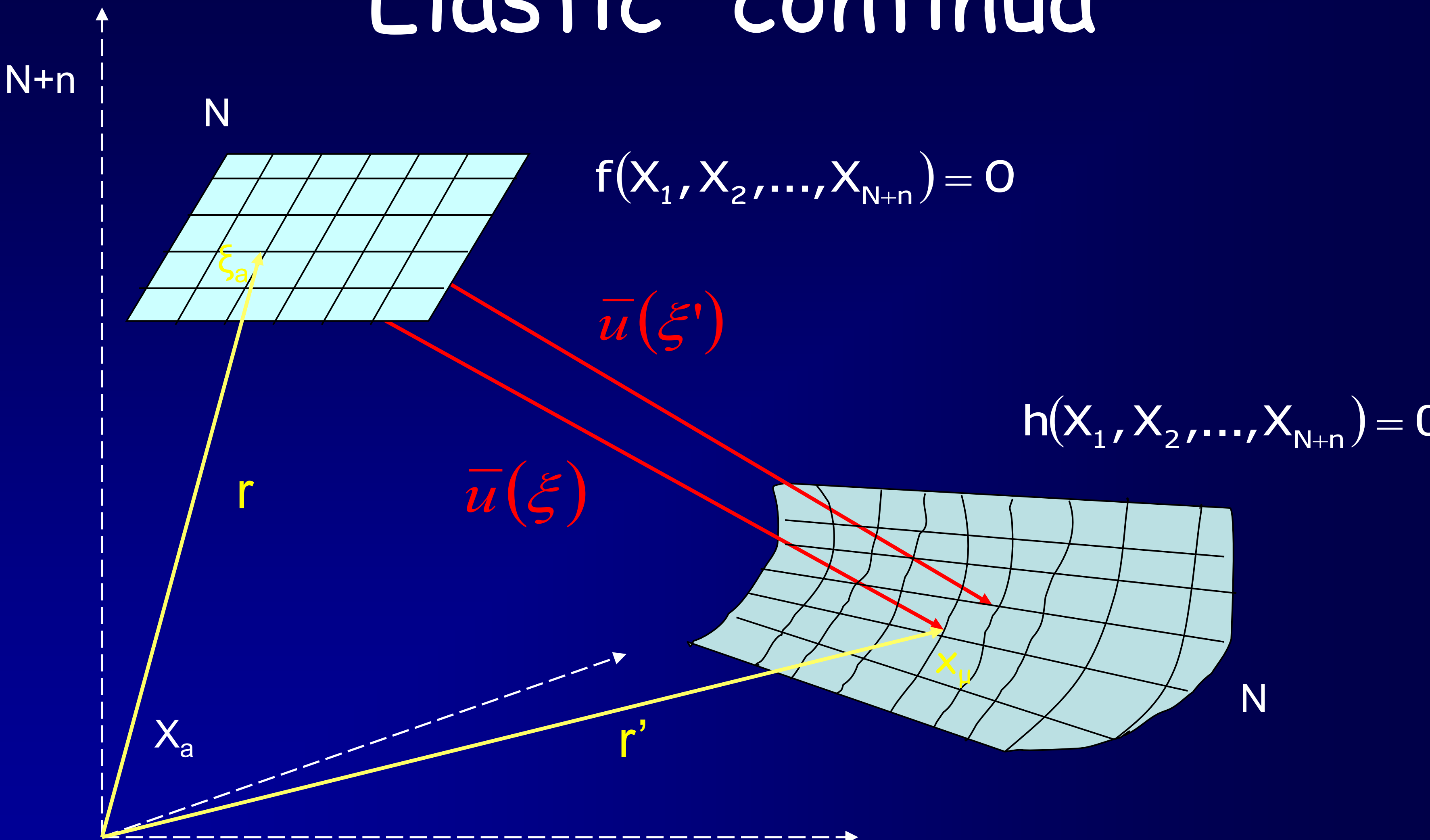
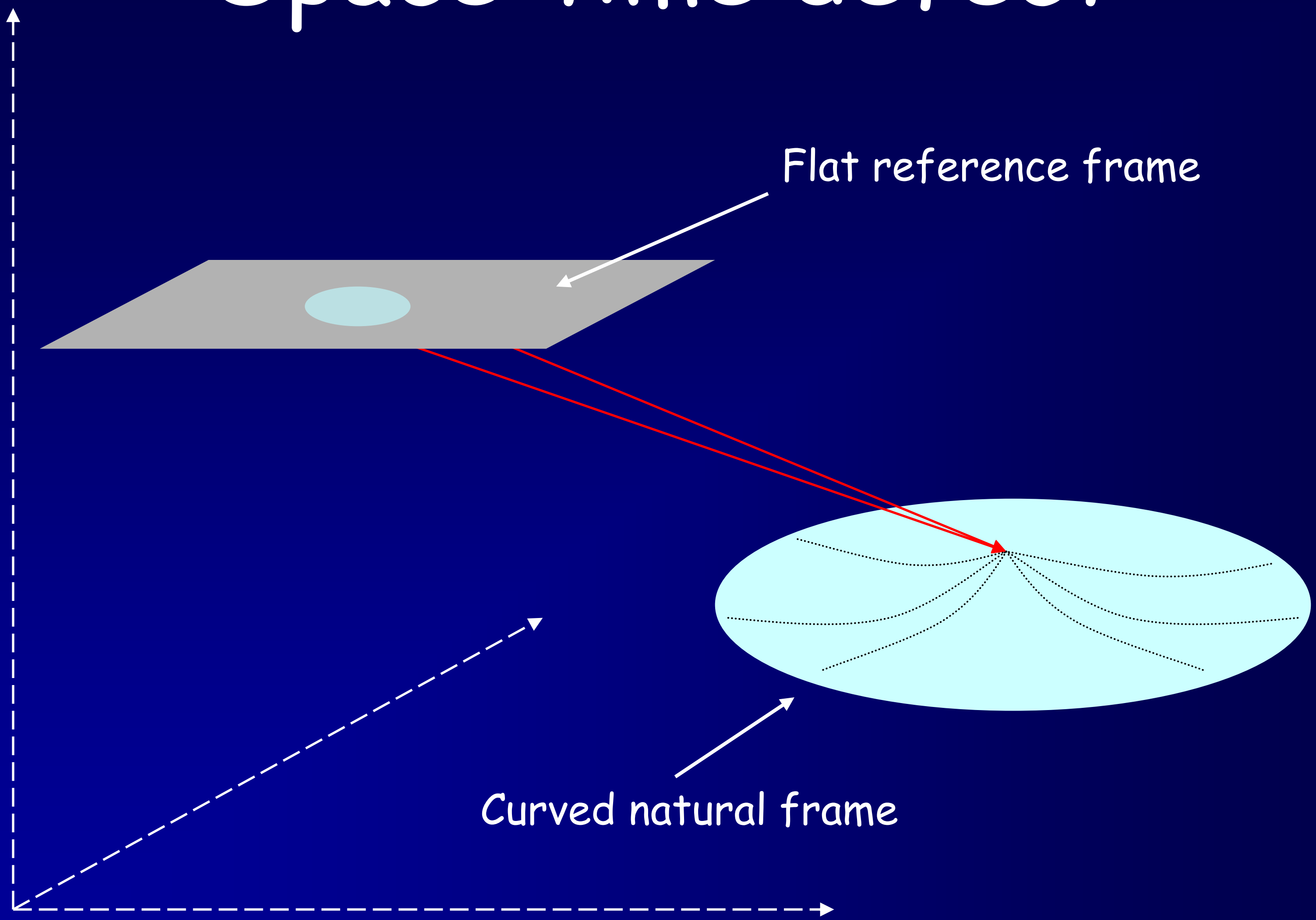
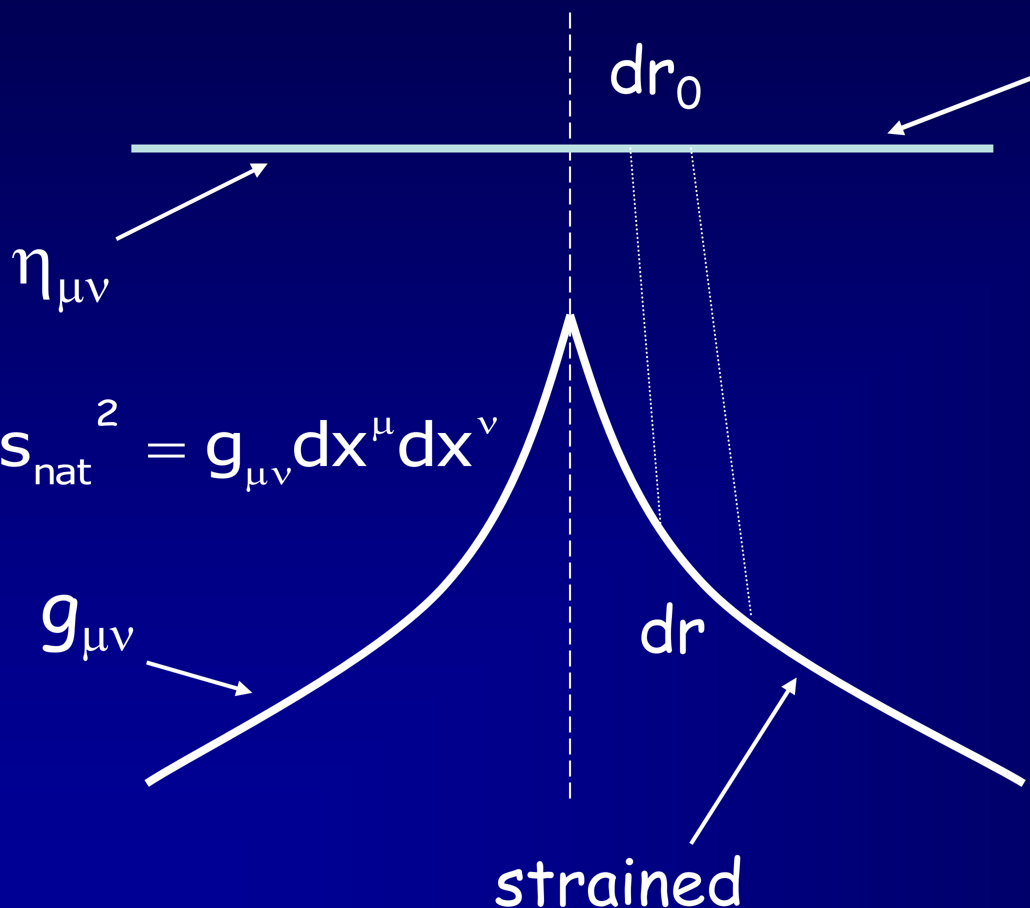
“Elastic” continua



Space-time defect



Strained space-time



$$ds_{ref}^{\;2} = \eta_{\mu\nu} dx^{\mu} dx^{\nu}$$

$$ds_{nat}^{\;2} = g_{\mu\nu} dx^{\mu} dx^{\nu}$$

Strain tensor

$$\varepsilon_{\mu\nu} = \frac{g_{\mu\nu} - \eta_{\mu\nu}}{2}$$

Physical relevance of the strain

- In a continuum, strain is associated with stress
- Strain and stress correspond to a deformation energy
- Defects can fix the global symmetry
- Strain is originated by the presence of matter/energy or by defects

Isotropic medium

$$C_{\alpha\beta\mu\nu} = \lambda \eta_{\alpha\beta}\eta_{\mu\nu} + \mu\left(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu}\right)$$

Lamé coefficients

$$\sigma^{\mu\nu} = C^{\mu\nu\alpha\beta}\varepsilon_{\alpha\beta} = \lambda\eta^{\mu\nu}\varepsilon + 2\mu\varepsilon^{\mu\nu}$$

Elastic energy $\quad V = \frac{1}{2}\sigma^{\mu\nu}\varepsilon_{\mu\nu} = \frac{1}{2}\lambda\varepsilon^2 + \mu\left(\varepsilon_{\mu\nu}\varepsilon^{\mu\nu}\right)$

The Lagrangian density

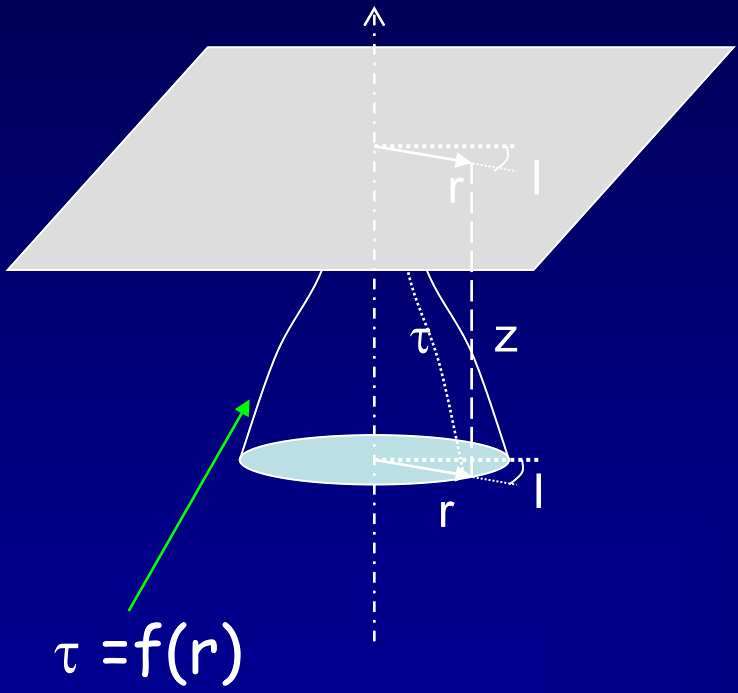
$$S = \int \left[R + \frac{1}{2}\left(\lambda\varepsilon^2 + 2\mu\varepsilon_{\mu\nu}\varepsilon^{\mu\nu}\right) + \kappa\mathcal{L}_{\text{matter}} \right]\sqrt{-g}\,d^4x$$

"Kinetic" term

Potential term

Geometry

Robertson-Walker symmetry



$$ds_{ref}^{\ 2} = dr^2 - dl^2 = b^2(\tau)d\tau^2 - dl^2$$

$$ds_{nat}^{\ 2} = d\tau^2 - a^2(\tau)dl^2$$

$$\varepsilon_{\mu\nu} = \frac{1}{2}\left(g_{\mu\nu} - \eta_{\mu\nu}\right)$$

$$\varepsilon_{00} = \frac{1-b^2}{2} \qquad \varepsilon_{ii} = \frac{1-a^2}{2}$$

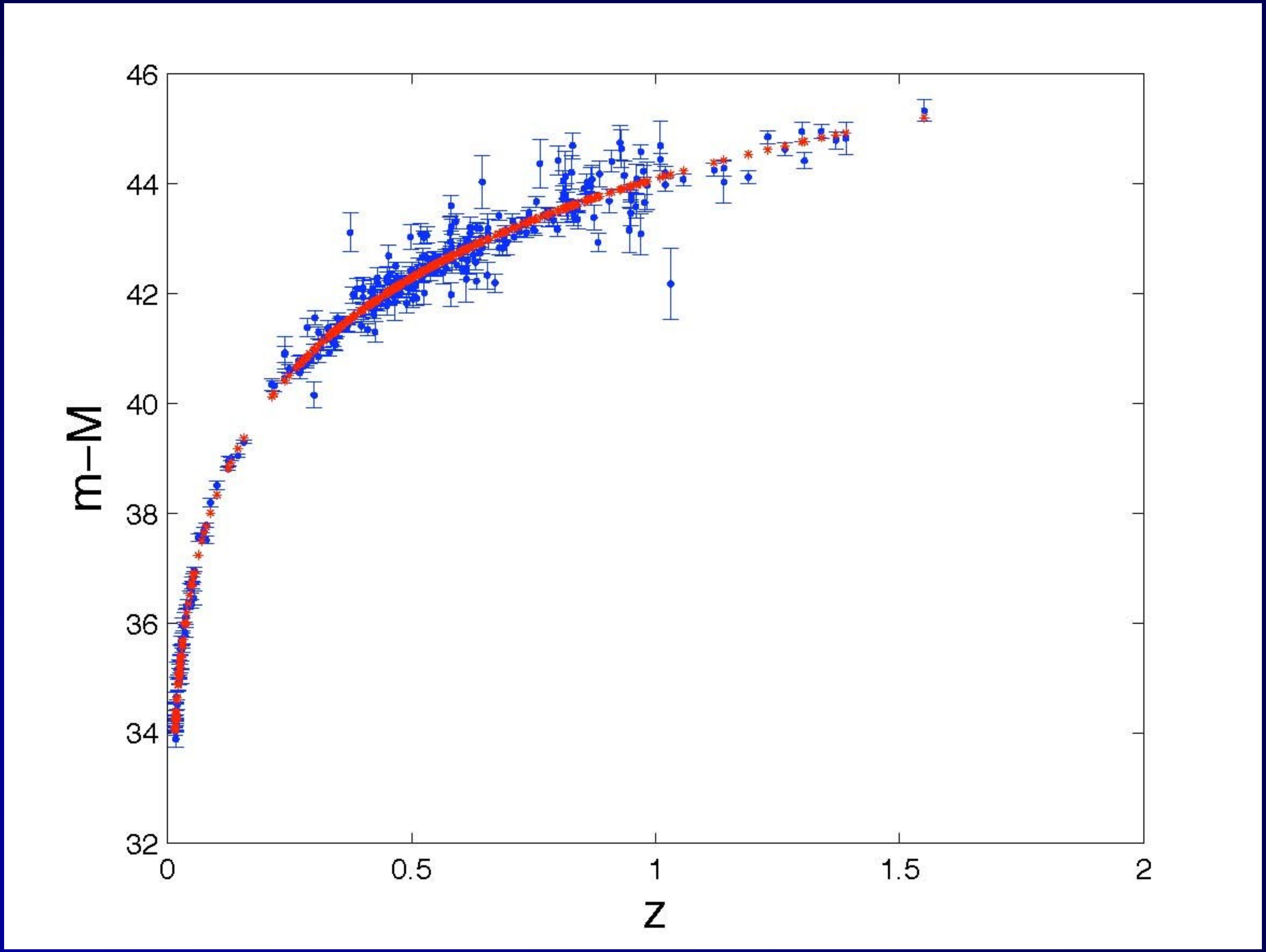
The Hubble parameter

$$H = \frac{\dot{a}}{a} = c\left\{3\frac{B}{16}\left(1 - \frac{(1+z)^2}{a_0^{\ 2}}\right)^2 + \frac{\kappa}{6}(1+z)^3\left[\rho_{m0} + \rho_{r0}(1+z)\right]\right\}^{1/2}$$

$$\kappa = \frac{16\pi}{c^2}G \qquad B = \frac{\mu}{4}\frac{2\lambda + \mu}{\lambda + 2\mu}$$

A. Tartaglia and N. Radicella, *CQG*, **27**, 035001 (2010)

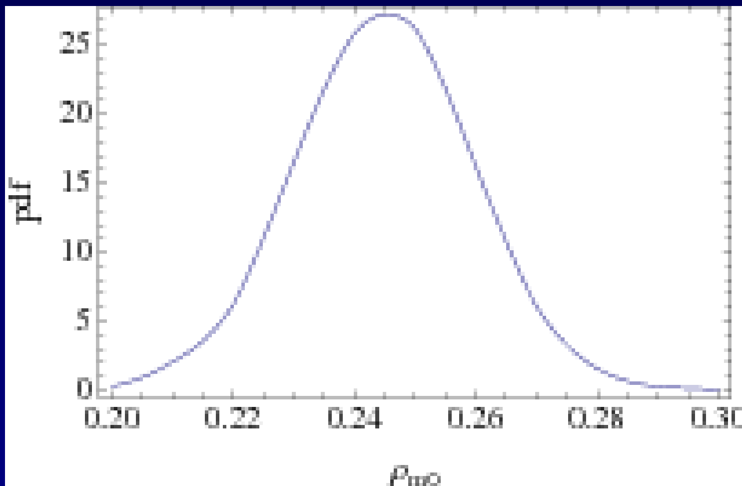
Fitting the data (307 SnIa)

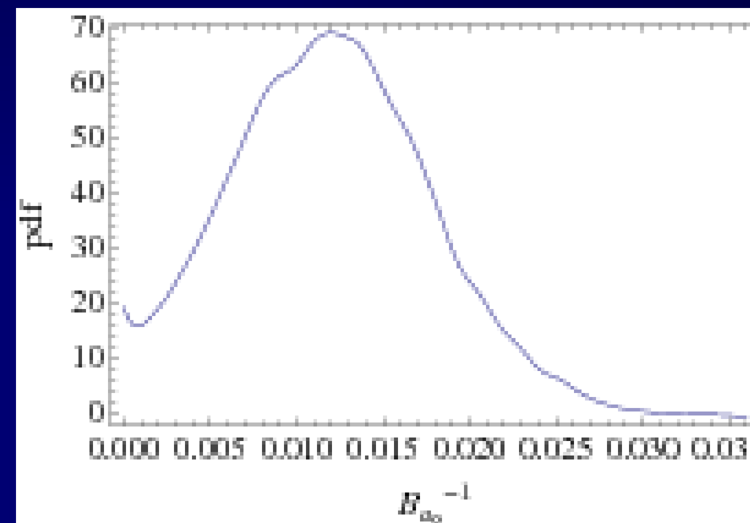


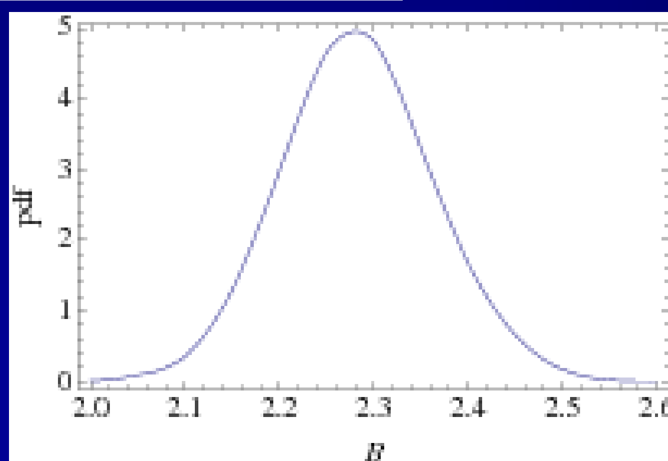
Other cosmological tests

- Primordial nucleosynthesis (correct proportion between He, D and hydrogen)
- CMB spectrum
- Structure formation after the recombination era.

Bayesian posterior probability







Optimal value of the parameters

$$B=(2.28\pm 0.08)\times 10^{-52}\ m^{-2}$$

$$\rho_{m0}=(2.45\pm 0.15)\times 10^{-27}\ kg/m^{3}$$

$$B_{a_0}{}^{-1}=(0.012\pm 0.06)\times 10^{52}\ m^{2}$$

$$B_{a_0}=\frac{8}{9}\kappa\rho_{r0}a_0{}^{4}$$

N. Radicella, M. Sereno, A. Tartaglia, IJMPD, **20**, n. 3 (2011)

Schwarzschild symmetry

Natural frame

$$ds^2 = f d\tau^2 - h dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

Reference frame (Minkowski)

$$ds^2 = d\tau^2 - \left(\frac{dw}{dr}\right)^2 dr^2 - w^2 d\theta^2 - w^2 \sin^2\theta d\phi^2$$

Gauge function

The strain tensor

$$\begin{cases} \varepsilon_{00} = \dfrac{f-1}{2} \\ \varepsilon_{rr} = \dfrac{w'^2 - h}{2} \\ \varepsilon_{\theta\theta} = \dfrac{w^2 - r^2}{2} \\ \varepsilon_{\phi\phi} = \dfrac{w^2 - r^2}{2}\sin^2\theta \end{cases}$$

$$w' = \frac{dw}{dr}$$

Weak strain

$$\begin{cases} f = f_o + f_1 \\ h = h_o + h_1 \\ w = w_o + w_1 \end{cases}$$

$$f_o = \frac{1}{h_o} = 1 - 2\frac{m}{r}$$

$$\frac{m}{r} < 1;\ f_1, h_1, \frac{w_1}{r} \approx \lambda r^2, \mu r^2 << 1$$

Approximate solutions

$$\begin{cases} g_{00} = f \cong 1 - 2\dfrac{m}{r} + \Psi r^2 \\ g_{rr} = -h \cong -\left(\dfrac{1}{1 - 2\dfrac{m}{r}} + \Lambda r^2 \right) \end{cases}$$

Ψ, Λ = functions of $\lambda,\mu \sim \lambda,\mu$

Post-Keplerian circular orbits

$$\omega^2 = G\frac{M}{R^3} + c^2\Psi = G\frac{M}{R^3} + c^2\frac{2\mu^3 + 23\lambda\mu^2 + 37\lambda^2\mu + 16\lambda^3}{2(\lambda + 2\mu)^2}$$

Looks like the effect of dark matter

Light rays

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^2}{b^2}\left(1 - 2\frac{M}{r}\right)\left(r^2 - b^2\left(1 - 2\frac{M}{r}\right)\right) - \frac{r^6}{b^2}\Psi$$

Conclusion

- The strained space-time theory introduces a strain energy of vacuum depending on curvature
- The theory accounts for the accelerated expansion of the universe and is consistent with BBN, structure formation and SnIa's
- The strain affects free fall also in spherically symmetric stationary space-times

References

- A. Tartaglia and N. Radicella, CQG, **27**, 035001 (2010)
- N. Radicella, M. Sereno, A. Tartaglia, IJMPD, **20**, n. 3 (in press) 2011