Novel mechanism of generation of primordial magnetic fields

Oleg Ruchayskiy

In collaboration with:

- Alexey Boyarsky (EPFL & Leiden University)
- Mikhail Shaposhnikov (EPFL)
- . . .

papers to appear soon

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Reminder : massless fermions

- Massless fermions can be left and right-chiral (left and right moving):

$$(i\gamma^\mu\partial_\mu - \cancel{m})\psi = \begin{pmatrix} -\cancel{m}^{\,0} & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & -\cancel{m}^{\,0} \end{pmatrix}\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix} = 0$$

  where $\gamma_5\psi_{R,L} = \pm\psi_{R,L}$ and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$

- Number of left $N_L = \int d^3x\,\psi_L^\dagger\psi_L$ and right $N_R = \int d^3x\,\psi_R^\dagger\psi_R$ particles is conserved **independently**

  $N_L + N_R$ and $N_L - N_R$ are conserved independently in the free theory

- Introduce a **chemical potential** $\mu_{L,R}$ for each conserved quantity. The **density matrix** is $\hat{\varrho} = \exp\left(-\frac{\hat{\mathcal{H}}}{T} + \mu_R\hat{N}_R + \mu_L\hat{N}_L\right)$ Equilibrium value of *any* quantity is determined by these numbers (T, μ_L, μ_R)

Massless fermions with gauge fields

- The situation changes if gauge fields are present. Gauge interactions respects chirality ($D_\mu = \partial_\mu + eA_\mu$)...

$$\begin{pmatrix} 0 & i(D_t + \vec{\sigma}\cdot\vec{D}) \\ i(D_t - \vec{\sigma}\cdot\vec{D}) & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

- ...but the difference of left and right-movers is still **not conserved**:

$$\frac{d(N_L - N_R)}{dt} = \int d^3\vec{x} \left(\partial_\mu j^5_\mu\right) \propto \frac{e^2}{32\pi^2} \int d^3\vec{x}\vec{E}\cdot\vec{B}$$

- This effect is known as "**chiral anomaly**" – symmetry of the classical theory is spoiled by the quantum corrections

- Physical consequences? Fast decay of $\pi^0 \to \gamma\gamma$. Baryon number non-conservation in the presence of sphalerons

Maxwell equations

- The presence of different number of left and right fermions leads to additional terms in the effective Lagrangian for gauge fields

- As a result Maxwell equations get term current, proportional to $\Delta\mu$:

Redlich & Wijewardhana (1985);

Fröhlich et al. (2000–2001)

$$\begin{aligned}
\mathrm{curl}\,\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\
\mathrm{curl}\,\vec{B} &= \sigma\vec{E} + \frac{e^2}{4\pi}\Delta\mu\vec{B} \\
\frac{\partial\Delta\mu}{\partial t} &\propto \frac{e^2}{16\pi^2}\vec{E}\cdot\vec{B}
\end{aligned}$$

- If $\Delta\mu$ is a function of t **only** Maxwell equations have a solution for two circular polarizations:

$$\vec{B}_{k,\pm} \propto e^{\lambda_{k,\pm}(t)}$$

Instability

- Solution of the Maxwell equations with the $\Delta\mu$ term is given by $\vec{B}_{k,\pm} \propto e^{\lambda_{k,\pm}(t)}$, where

Instability for one of the circular polarizations

$$\lambda_{k,\pm}(t) = \left(-\frac{k^2 t}{\sigma}\right) \pm \frac{kt}{\sigma} \underbrace{\frac{1}{t}\int^t d\tau\, \Delta\mu(\tau)}_{\bar{\mu}\text{ — time average}}$$

magnetic diffusion

- Exponential growth while $\bar{\mu}(t) > k$ for one of the circular polarizations (depending on the sign of $\Delta\mu$ – **generation of maximally helical magnetic fields**

- Maximal growth is achieved for $k = \frac{\bar{\mu}}{2}$:

$$\lambda_k^{max} = \frac{\bar{\mu}^2 t}{4\sigma}$$

Can this mechanism be used to generate primordial magnetic fields?

Chirality flipping rates in the SM

All charged fermions are massive in the Standard Model. But $T \gg m$?

Above electroweak transition:

- Chirality flipping **above** 100 GeV: $\Gamma = \frac{T_R T}{M_*}$
  — in equilibrium at $T < T_R \sim 80$ TeV

Cambell et al. (1992)

Below electroweak transition:

- Chirality flipping (electro-magnetic processes): $\Gamma_{\mathbf{EM}} \propto \alpha^2 T \left(\frac{m}{T}\right)^2$
  — in equilibrium for $T \lesssim 80 - 100$ GeV
- Chirality flipping (weak processes): $\Gamma_{\mathbf{W}} \propto G_F^2 T^5 \left(\frac{m}{3T}\right)^2$
  — in equilibrium for $T \gtrsim 400$ MeV

Recall: reactions are in equilibrium if $\Gamma \gtrsim H(T)$

Chirality flipping rates in the SM

- The mechanism can be efficient at temperatures above 80 TeV (if primordial left-right asymmetry in the electron sector existed)

Joyce & Shaposhnikov (1993)

- No primordial left-right asymmetries can survive in plasma by $T \sim 100$ GeV

- What if we had a **source of chiral asymmetry** in plasma?

Source of asymmetry?

- Evolution of the **difference** of chemical potential:

$$\frac{\partial \Delta\mu}{\partial t} = -\Gamma\,\Delta\mu + \mathcal{S}(t) + \frac{e^2}{16\pi^2}\vec{E}\cdot\vec{H}$$

Chirality flipping rate

Source of left-right asymmetry

- The situation when $\Gamma \gg H$ implies **source-tracking solution** (system forgets initial conditions)

$$\Delta\mu \approx \frac{\mathcal{S}(t)}{\Gamma}$$

works if $\frac{\partial \log \mathcal{S}(t)}{\partial t} \ll \Gamma$ – the faster chirality flipping the better

Any known examples?

Entire history of the Universe

Neutrino Minimal Standard Model (νMSM) solves several *beyond the Standard Model problems*

Asaka, Shaposhnikov, Laine, **O.R.**, Boyarsky et al (2005-2011)

✓ . . . explains neutrino oscillations

✓ . . . matter-antimatter asymmetry of the Universe

✓ . . . provides a viable dark matter candidate that can be cold, **warm** or **mixed** (cold+warm)

- The νMSM is self-consistent and does not require any other particles $\Rightarrow$ we have a **complete description of the Universe** from the time of reheating

- Coupled with Higgs inflation the νMSM is a complete and self-consistent theory up to the Planck scale

Bezrukov & Shaposhnikov (2008)

- The νMSM **predicts** CMF

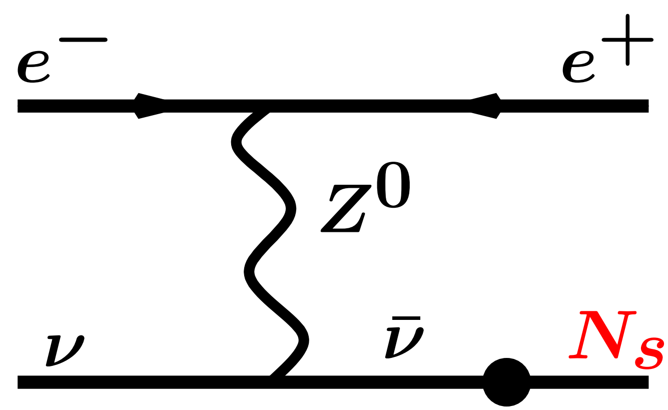
Some general properties of sterile neutrino

Sterile neutrinos behave as **superweakly interacting** heavy neutrinos

$M_I < 1$ MeV	$M_I > 1$ MeV	$M_I > 150$ MeV	$\dots$
$N_I \to \nu\nu\bar{\nu}$	$N_I \to \nu e^+ e^-$	$N_I \to \pi^\pm e^\mp$	
$N_I \to \nu\gamma$		$N_I \to \pi^0 \nu$	

Mixing angle with usual neutrinos θ_I:

$$\theta_I^2 = \sum_{\alpha=e,\mu,\tau} \frac{M^2_{\text{Dirac},\alpha I}}{M^2_{\text{Majorana},I}} \ll 1$$



Fermi constant: $G_F \to \boldsymbol{\theta} G_F$

Lifetime $\tau \propto \theta_I^{-2} M_I^{-5}$. Can be cosmologically long

Mixing angle $\theta \ll 1$ means that sterile neutrinos can be out of equilibrium in the early Universe

Sakharov conditions in the SM

Quick reminded: necessary conditions for generation of baryon asymmetry of the Universe (**Sakharov conditions**):

Sakharov (1967)

⊕ B-number violation $\longrightarrow$ sphalerons

Kuzmin, Rubakov, Shaposhnikov (1985)

? CP (and C) non-conservation $\longrightarrow$ phase of the CKM matrix

Farrar & Shaposhnikov (1994)

⊖ Out-of-equilibrium processes $\longrightarrow$ no phase transition in the SM for $m_H > 72$ GeV!

Kajantie et al. (1996)

What changes in the νMSM?

Sakharov conditions in the νMSM

Sakharov (1967)

Necessary conditions for generation of baryon asymmetry of the Universe (**Sakharov conditions**):

Kuzmin, Rubakov, Shaposhnikov (1985)

(+) B-number violation $\longrightarrow$ sphalerons

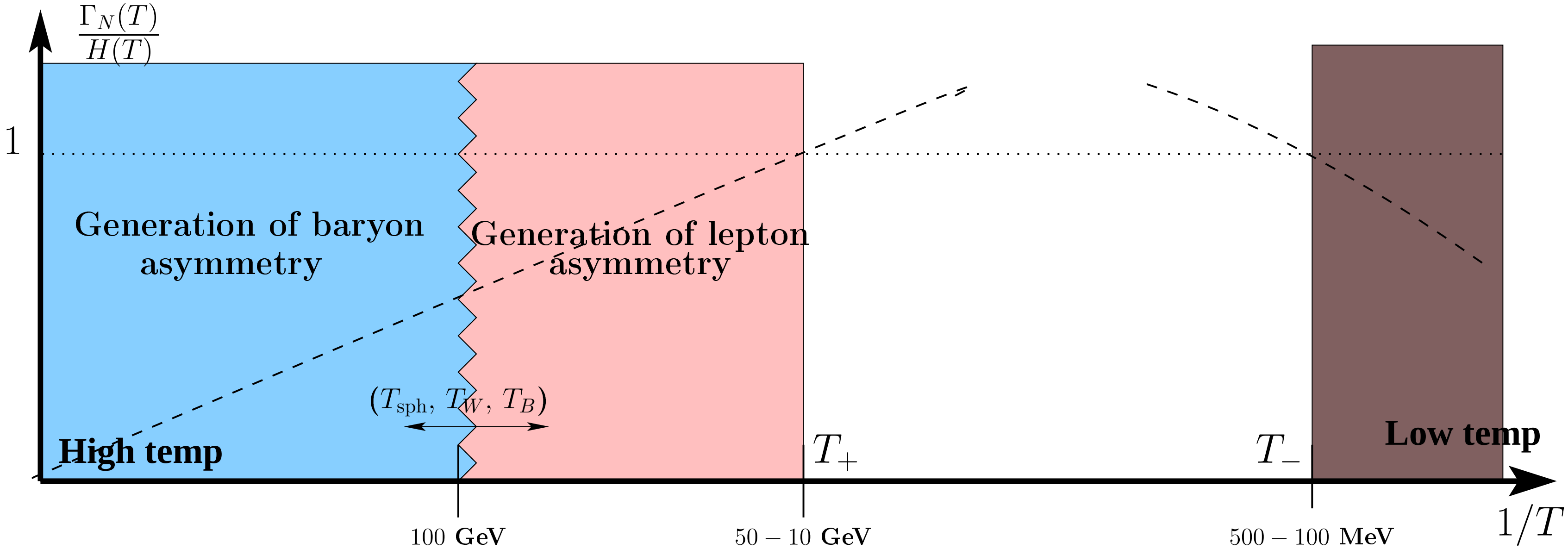
Farrar & Shaposhnikov (1994)

(?) CP (and C) non-conservation $\longrightarrow$ phase of the CKM matrix **plus additional CP phases in the Dirac mass matrix of sterile neutrinos**

Kajantie et al. (1996)

(−) Out-of-equilibrium processes $\longrightarrow$ no phase transition in the νMSM for $m_H > 72$ GeV! **but Yukawa couplings of sterile neutrinos are small enough to keep them out of thermal equilibrium at $T \sim 100$ GeV**

Baryo- and lepto-genesis in the νMSM



Asaka & Shaposhnikov (2005);

Shaposhnikov (2008);

Canetti & Shaposhnikov (2010)

- At $T \gg M_N$ the total lepton number in the νMSM is conserved.
- At $T \geq T_{\rm sph}$ sterile neutrino interaction rate $\Gamma_N \ll H(T) \Rightarrow$ sterile neutrinos are **not in thermal equilibrium**
- Lepton number distributes between left neutrinos (SU(2) lepton doublets) and right (sterile) neutrinos $\Rightarrow$ Lepton asymmetry effectively appears in the SM sector

Lepton asymmetry in the νMSM

- Lepton asymmetry continues to be generated while $T > T_+ \sim 10 - 50$ GeV
- Production rate is very fast (can go as T^{15})
- Lepton asymmetry generated by T_+ can be very large:

$$1 \leq L_6 \equiv 10^6 \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{s} \leq 700$$

Laine, Shaposhnikov (2008);

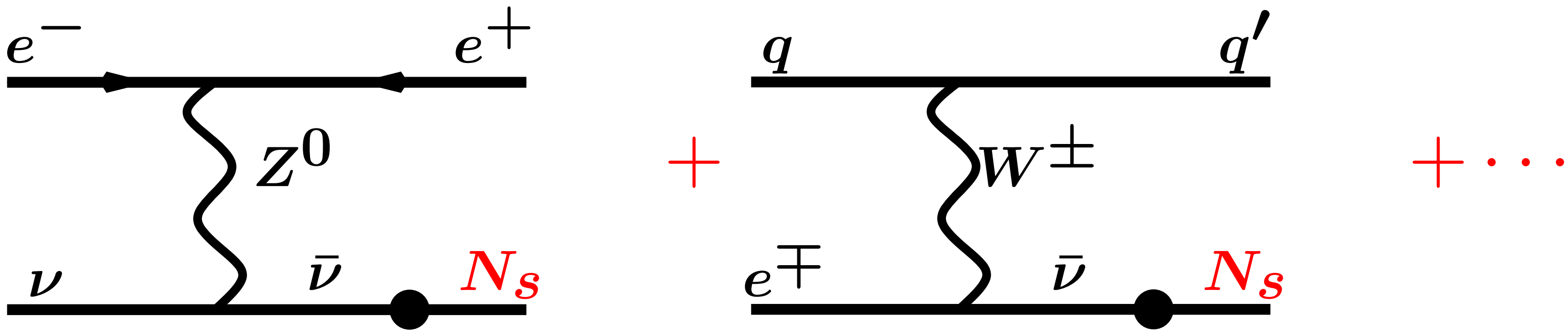
Boyarsky, **O.R.**, Shaposhnikov (2009)

- At sphaleron freeze-out $L_6(T_{\rm sph}) \sim 10^{-4}$
- Current BBN bound on lepton asymmetry is $L_6^{\rm BBN} \leq 2500$

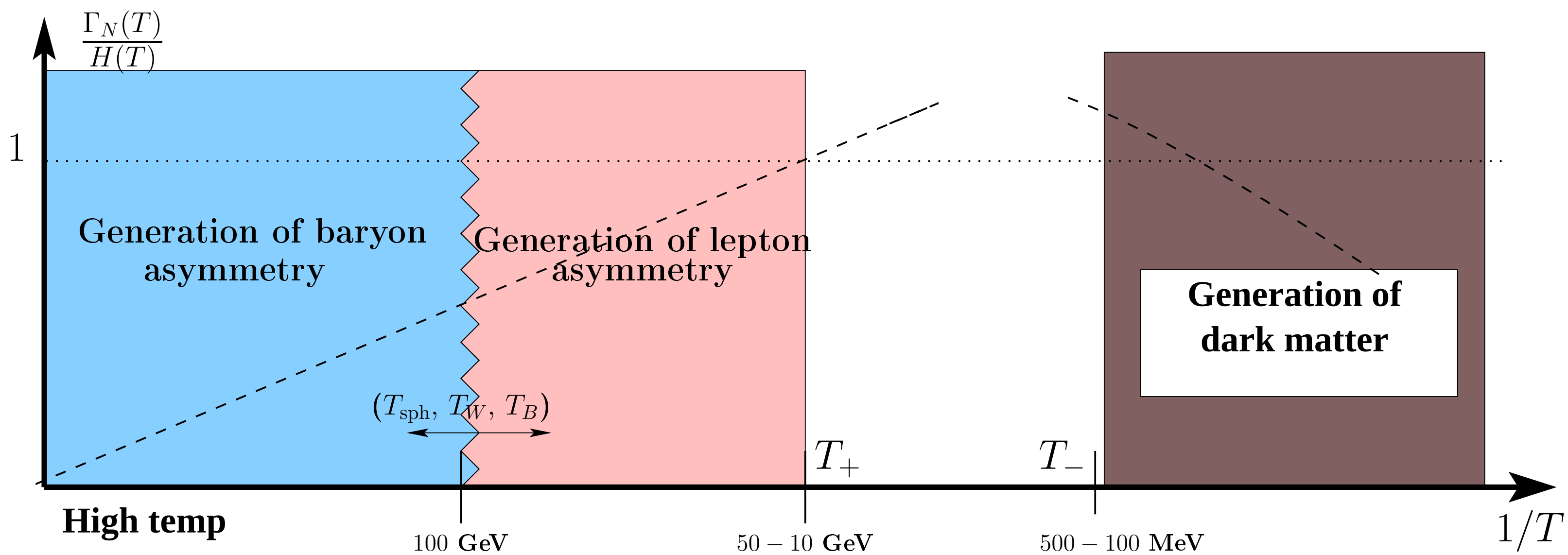
Serpico & Raffelt (2005)

- **Present experimental bounds put $L_6 \gtrsim 1$ in the νMSM**

Sterile neutrino dark matter



Dodelson & Widrow'93

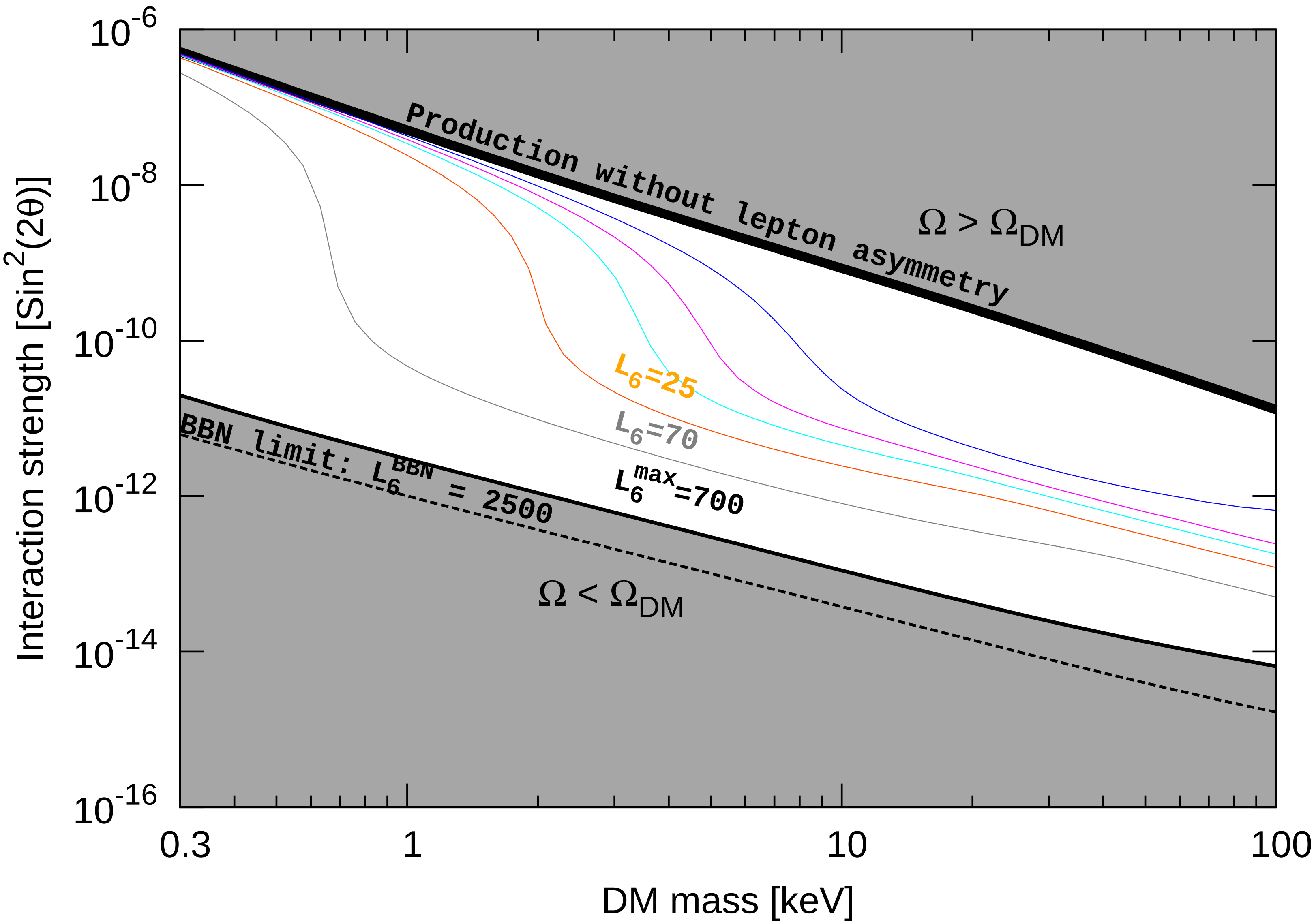


Asaka, Laine, Shaposhnikov (2006)

Shi Fuller'98
Laine, Shaposhnikov

The presence of lepton asymmetry in primordial plasma makes **active-sterile mixing** much more effective (as in MSW effect) – resonant production

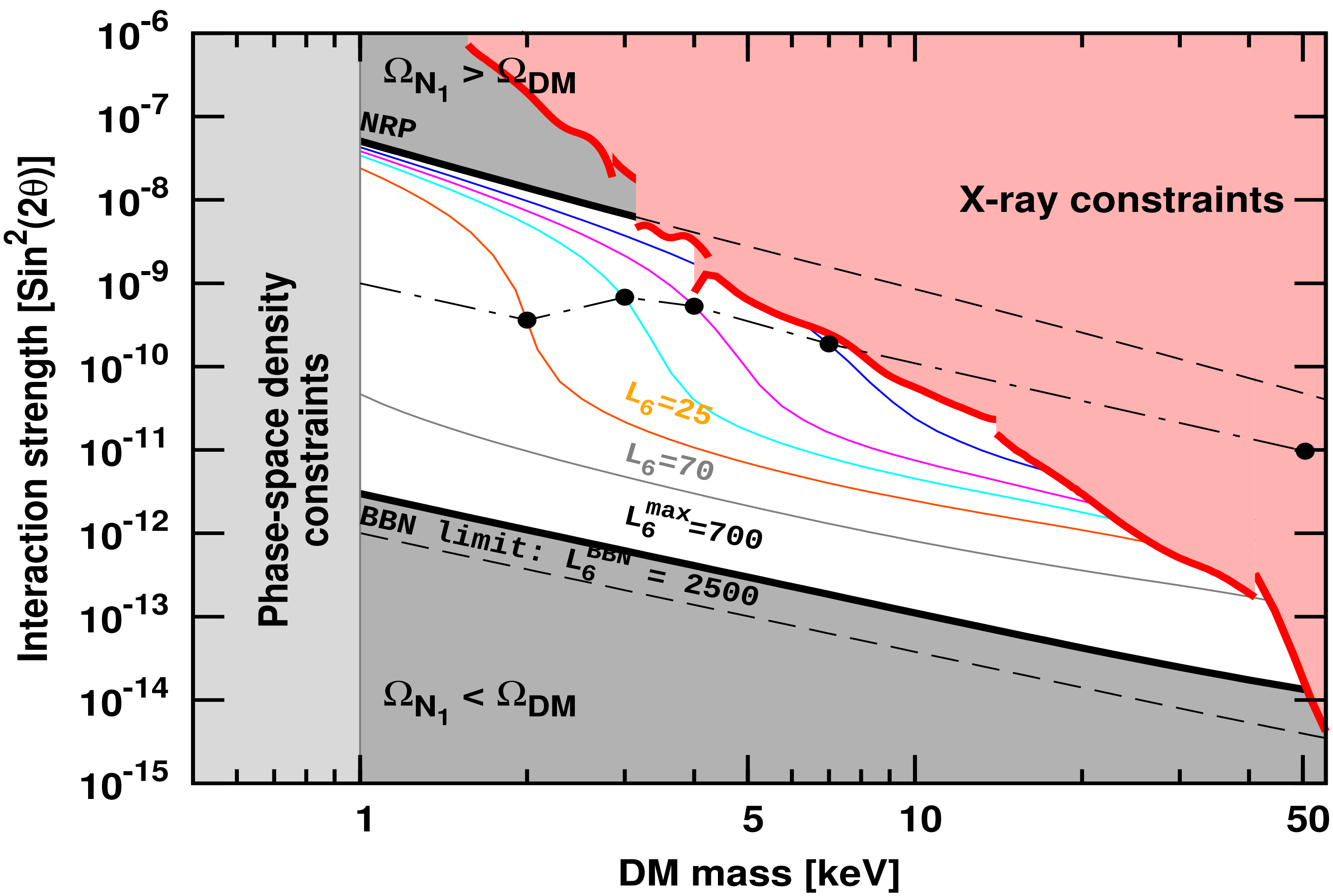
Window of parameters of sterile neutrino DM



Asaka, Laine, Shaposhnikov

Laine, Shaposhnikov

Sterile neutrino DM in the νMSM



Boyarsky, **O.R.**, Lesgourgues, Viel PRL (2009)

Boyarsky, **O.R.**, Shaposhnikov Ann. Rev. Nucl. Part. Sci. (2009)

Cosmological and astrophysical bounds imply large lepton asymmetry $L_6 \gtrsim 1$ *within the νMSM*

CMF instability in the νMSM

- At $T \geq T_+$ lepton asymmetry in the left neutrino + left-electron sector $\Rightarrow$ $\Delta\mu$ between left and right electrons appears

$$\frac{\partial \Delta\mu}{\partial t} = -\Gamma\,\Delta\mu + \mathcal{S}(t) \Rightarrow \Delta\mu \approx \frac{\mathcal{S}}{\Gamma}$$

- ... and instability quickly develops: $B \propto e^{\lambda_k(t)}$ with

$$\lambda_k(T) \sim \frac{k(\alpha\bar{\mu} - k)}{\sigma}\frac{M_*}{T^2}$$

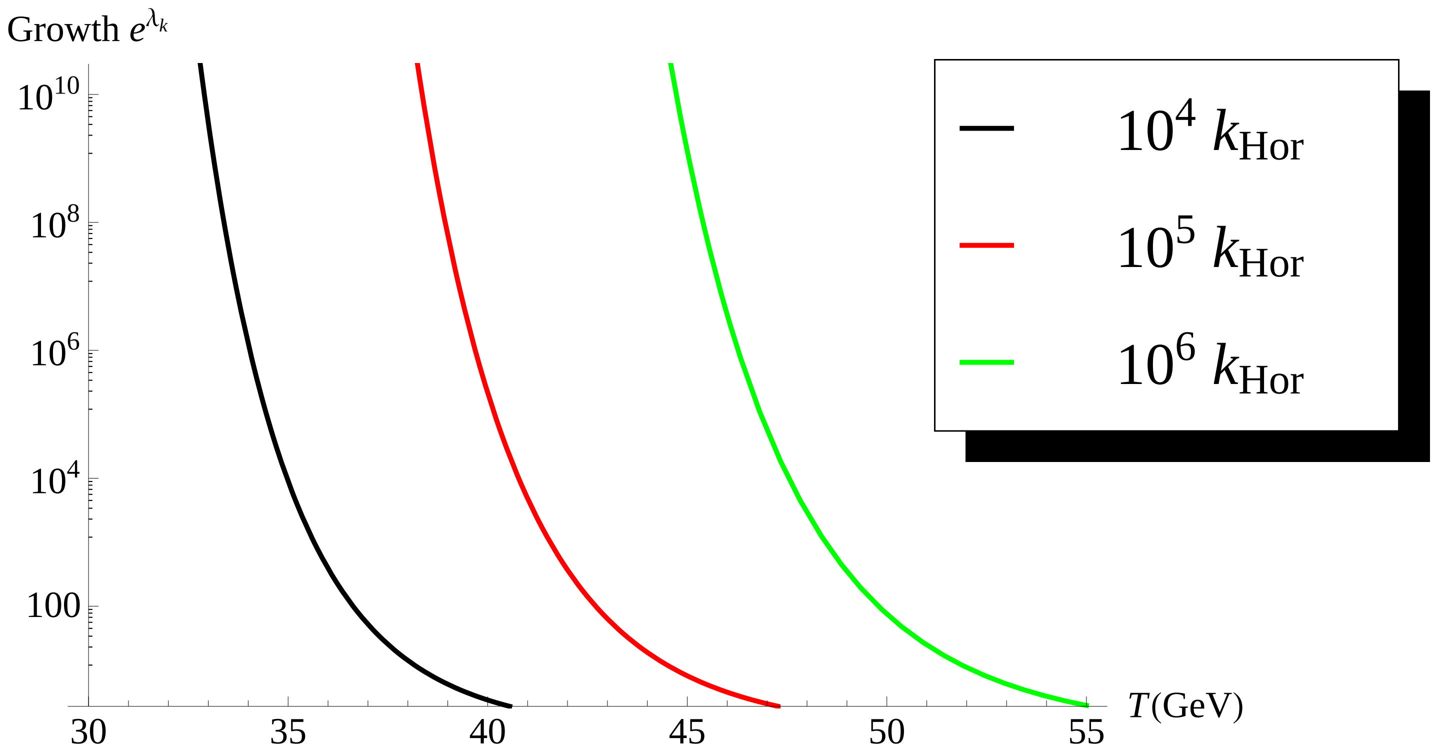
- The fastest growing scale: $\boxed{k_{max} = \alpha\bar{\mu}(T)}$. Notice that $k_{max} \gg H(T)$

- Growth is really fast: $\lambda_{max} \sim 10 - 10^4$

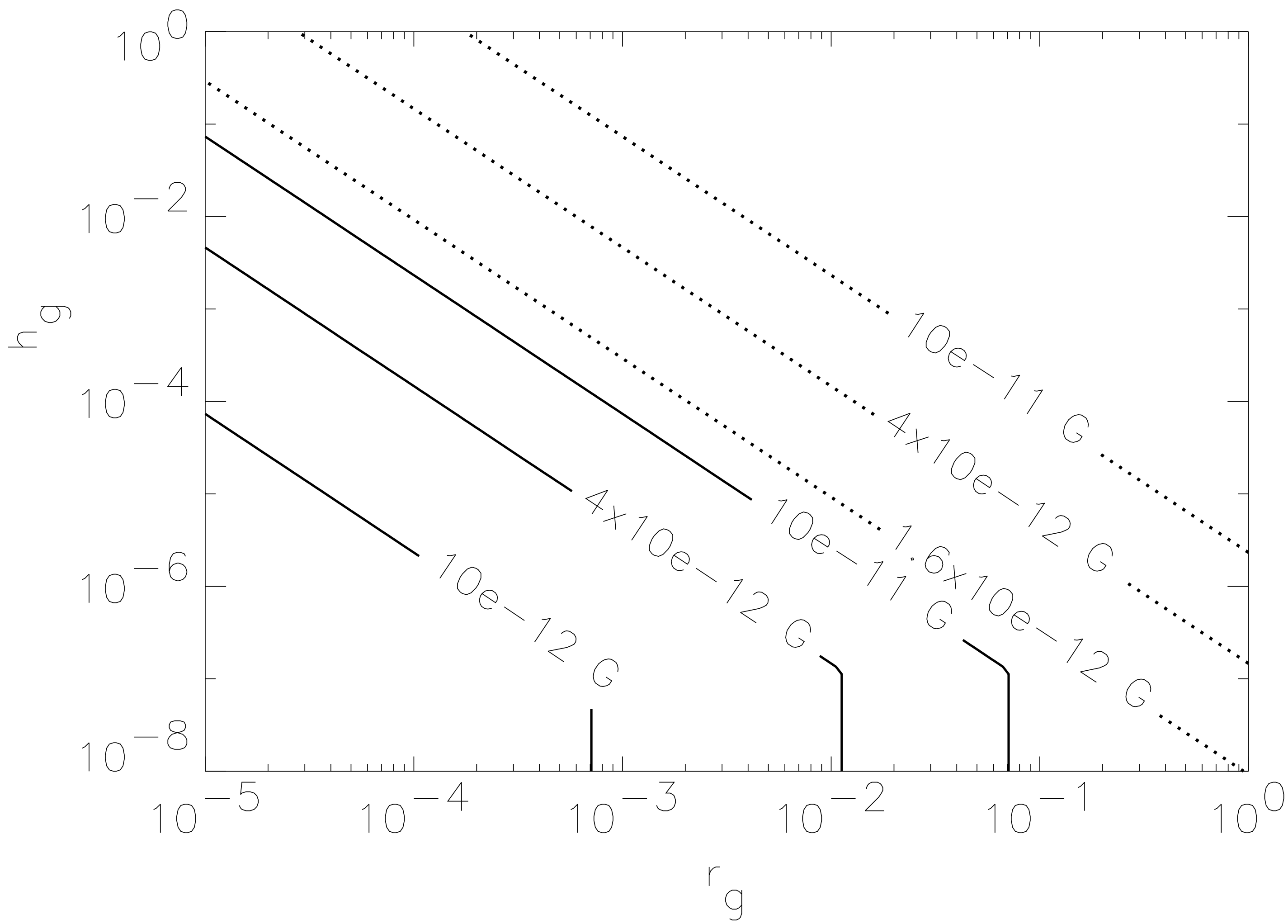
CMF instability in the νMSM

The longest wave-length that has experienced instability at temperature T is

$$k_{min} = H(T)\frac{\sigma}{\alpha\bar{\mu}} \sim \frac{H(T)}{\alpha^2 \log(1/\alpha)\bar{\mu}(T)}$$



MHD evolution of magnetic fields



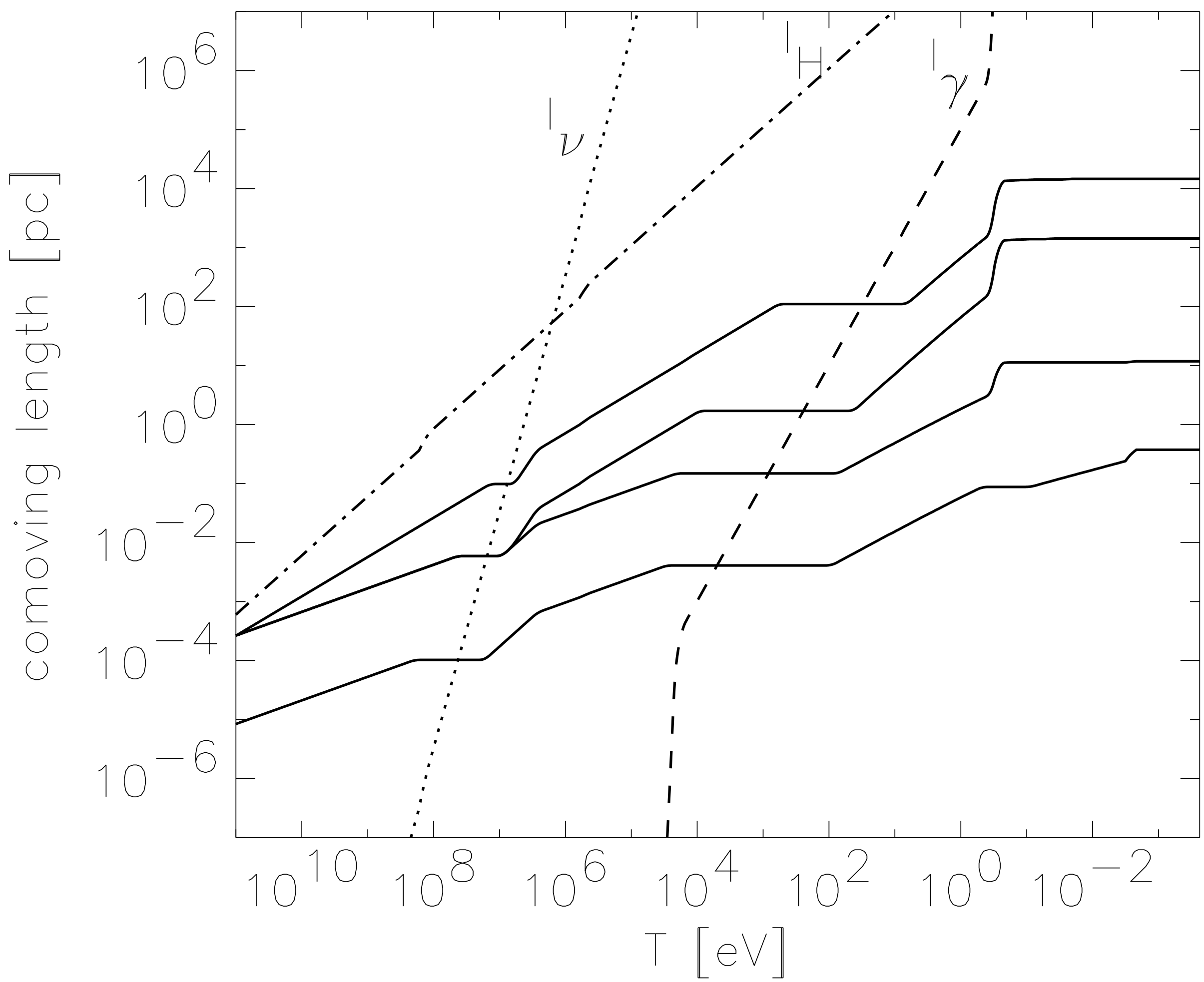
JKO (1998);

Banerjee & Jedamzik (2003, 2004);

Jedamzik & Sigl (2010)

Assuming $n = 3$ spectral index

MHD evolution of magnetic fields



JKO (1998);

Banerjee & Jedamzik (2003, 2004);

Jedamzik & Sigl (2010)

Turbulent flows develop **during** the magnetogenesis epoch

Summary

- Instability in Maxwell's equations caused by chiral anomaly may lead to generation of magnetic fields at temperatures below 100 GeV **provided that there is a source of left-right asymmetry**

- In the νMSM generation of lepton asymmetry at temperatures below EW scale provides such a source

- Generation takes place in the epoch 10 GeV $\lesssim T \lesssim 100$ GeV.

- Generated fields are quite large, always maximally helical.

- Detailed predictions for the spectra and subsequent evolution (MHD) – work in progress

- Magnetic fields become one more predictions of the νMSM, with their properties intertwined with those of dark matter and other sterile neutrino particles.

Thank you for your attention!

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TOC

Additional slides

Backreaction

- Magnetic fields quickly grow from thermal fluctuations and significant $\vec{E}\cdot\vec{B}\neq 0$ appears

- **Back-reaction** of magnetic fields on chemical potential becomes important:[1]

$$\frac{\partial\Delta\mu}{\partial t} = -\Gamma\,\Delta\mu + \mathcal{S}(t) + \boxed{\vec{E}\cdot\vec{B}}$$

- Back-reaction becomes important (last two terms of the same order) when magnetic energy $\dfrac{\vec{B}^2}{8\pi}$ becomes of the order of $\dfrac{1}{6}\Delta\mu^2T^2$. In the νMSM scenario this can be **quite significant**: $\Delta\mu \leq 0.15T$

[1] For simplicity we omit here coefficients like $\frac{e^2}{4\pi}, \ldots$

Backreaction

- According to Maxwell equations: $\vec{E} = \frac{1}{\sigma}\Big(\mathrm{curl}\,\vec{B} - \Delta\mu\vec{B}\Big)$

- Evolution of $\Delta\mu$ becomes:

Work in progress

Giovannini & Shaposhnikov (1998)

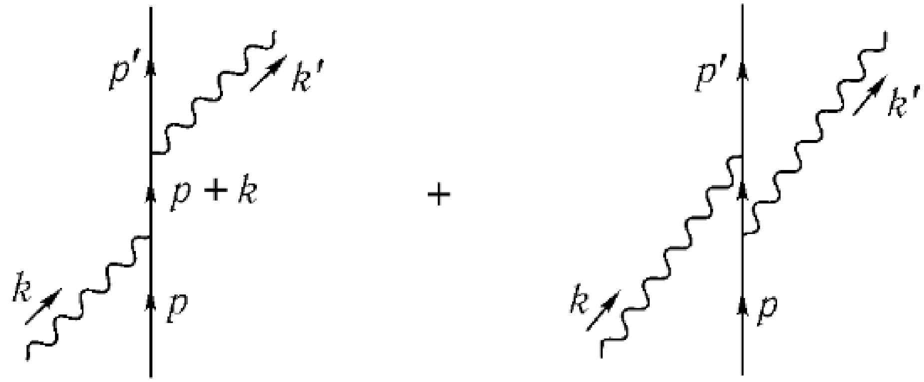
$$\frac{\partial \Delta\mu}{\partial t} = -\Big(\Gamma + \frac{\vec{B}^2}{\sigma}\Big)\Delta\mu + \Big(\mathcal{S}(t) + \frac{\vec{B}\cdot\mathrm{curl}\,\vec{B}}{\sigma}\Big)$$

- In the regime of large fields: $\Delta\mu \approx \frac{\vec{B}\cdot\mathrm{curl}\,\vec{B} + \sigma\mathcal{S}(t)}{\vec{B}^2 + \sigma\Gamma} \approx \frac{\vec{B}\cdot\mathrm{curl}\,\vec{B}}{\vec{B}^2}$

- Given chemical potential $\Delta\mu(\vec{B})$ we can find self-consistent solution of the (non-linear) Maxwell equations

Chirality flipping Compton scattering

Chirality flipping Compton scattering $e_L + \gamma \to e_R + \gamma$ in the limit $E \ggg m$:



- Differential cross-section is:

$$\frac{d\sigma}{d\cos\theta} \propto \frac{m^2 (p\cdot p')}{(p\cdot k')^2} = \frac{E}{\omega'} \frac{(1-\cos\theta) + \mathcal{O}(\frac{m^2}{E^2})}{\left((1+\cos\theta) + \frac{m^2}{2E^2}\right)^2}$$

- Singularity $\frac{d\sigma}{d\cos\theta} \propto \frac{1}{m^2}$ for $\theta \to \pi$ makes the resulting **total** cross-section **Independent on** m in the leading order!

- This discontinuity is another exhibition of **chiral anomaly**