(Non-Relativistic) Quantum Gravity

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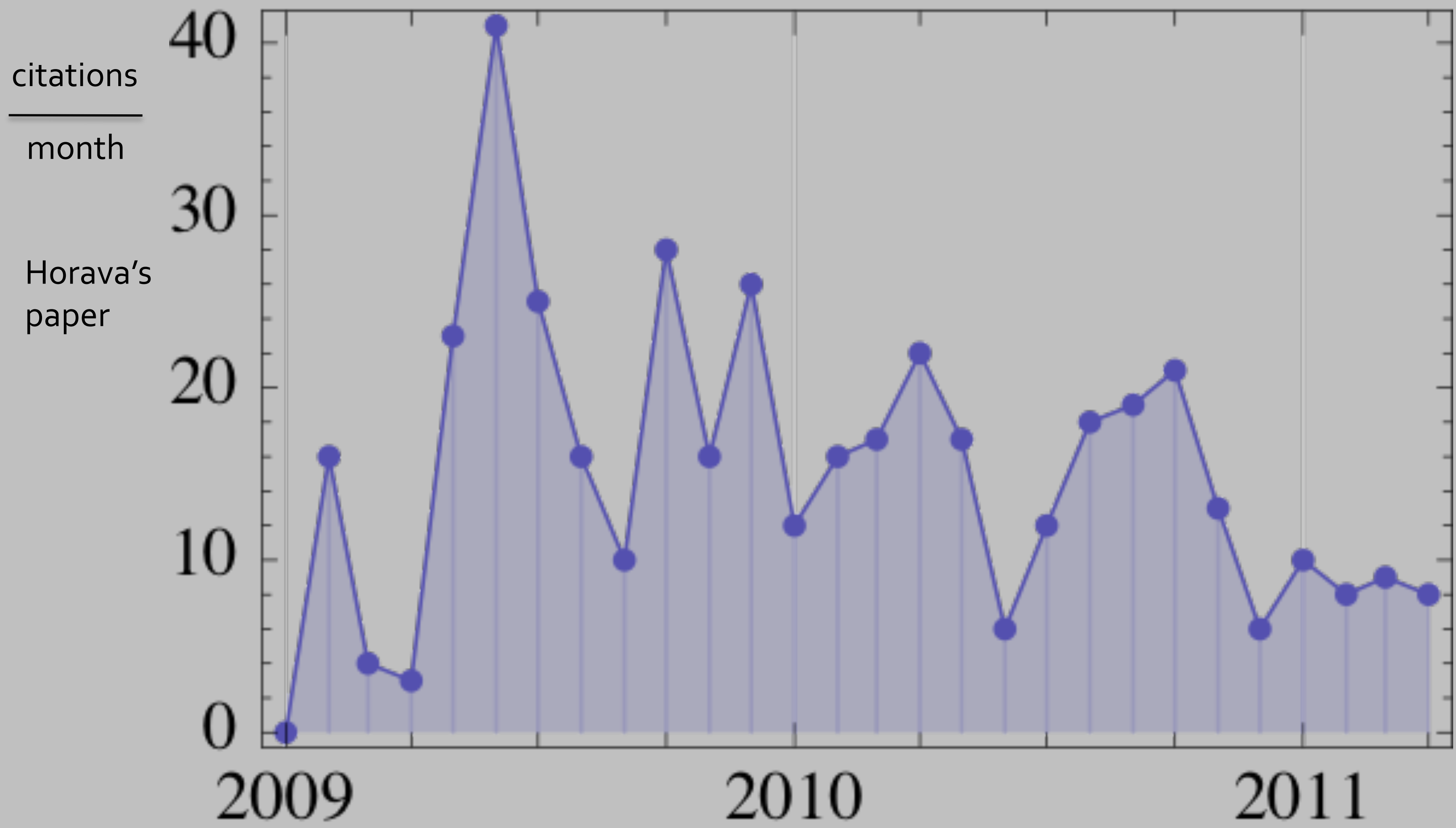
The convulse story of Horava Gravity...

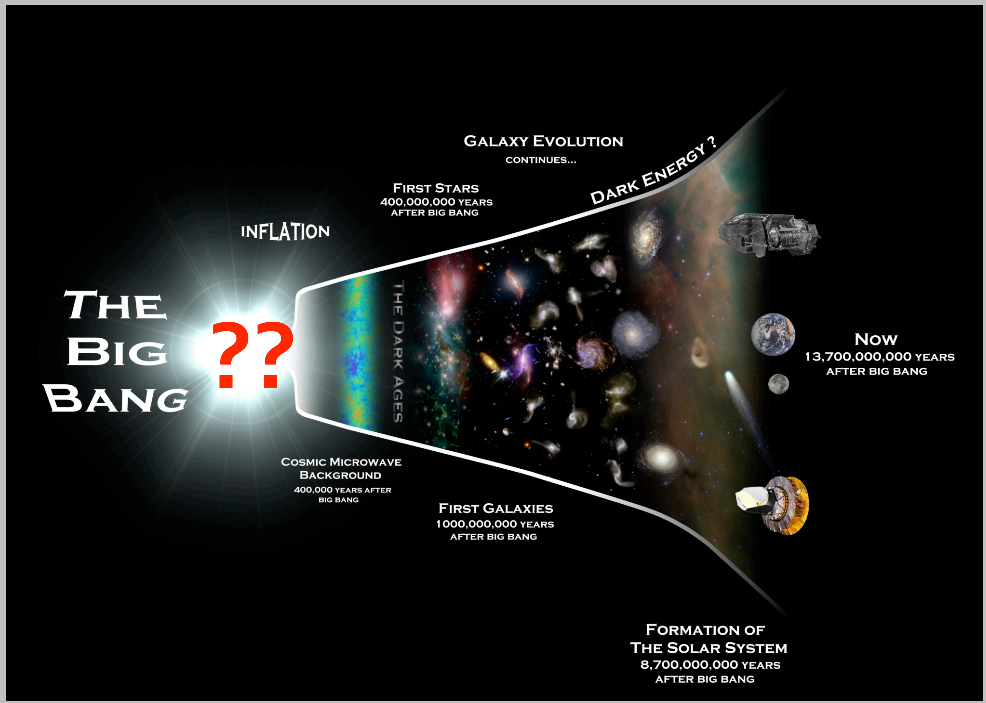


Motivation

Motivation

Constructing theories of QG is a fundamental problem in physics.

Relevant e.g. for BH physics, singularities, cosmology etc.



Motivation

Interest of Hořava's proposal: ***explicitness***

a (renormalizable) QFT of gravity can give very well defined answers!

In this talk, focus on application as QG for our 3+1 world:

can any version of NR QG be phenomenologically viable?

- no ghosts, no instabilities
- weakly coupled, if possible
- pass observational tests
- recovery of Lorentz invariance in IR

Plan

- Anisotropic Scaling
- Non-Relativistic Gravity
- Stückelberg Formalism
- Phenomenology & bounds
- Cosmology

Anisotropic Scaling

Anisotropic Scaling

Hořava's proposal exploits 'Anisotropic Scaling'

Dispersion relation in UV $w \sim k^z$ $z > 1$

Propagators $\sim \dfrac{1}{w^2 - a\,k^{2z}}$

=> Loops are less divergent

and no ghosts!

2 ways to see how Anisotropic Scaling assists renormalizability

Anisotropic Scaling

1) Dimensional analysis

Free Kinetic term: $\int dt\, d^3x \left\{ (\dot{\phi})^2 + \frac{\phi \Delta^z \phi}{M^{2(z-1)}} \right\}$ invariant under

$$x \mapsto b^{-1} x$$
$$t \mapsto b^{-z} t$$
$$\phi \mapsto b^{\frac{3-z}{2}} \phi$$

Interactions:

For $z = 2$, $[\phi] = 1/2 \Rightarrow \phi^{10}$ is marginal (renormalizable)

Anisotropic Scaling

1) Dimensional analysis

Free Kinetic term: $\int dt\, d^3x \left\{ (\dot{\phi})^2 + \frac{\phi \Delta^z \phi}{M^{2(z-1)}} \right\}$ invariant under $x \mapsto b^{-1} x$, $t \mapsto b^{-z} t$, $\phi \mapsto b^{\frac{3-z}{2}} \phi$

Interactions:

For $z = 3$, ϕ^n relevant (super-renormalizable)

$\phi^n \Delta^3 \phi$ marginal (renormalizable)

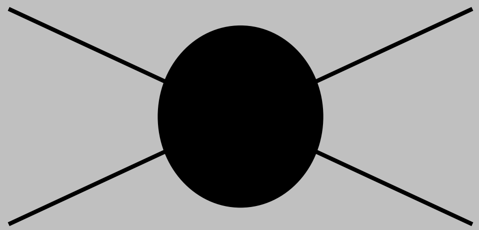
$\Rightarrow$ e.g., $L = (\dot{\phi})^2 + c^2(\phi)\, \phi \Delta \phi + d^2(\phi)\, \frac{\phi \Delta^2 \phi}{M^2} + e^2(\phi)\, \frac{\phi \Delta^3 \phi}{M^4}$

is P.C. renormalizable.

Anisotropic Scaling

2) Power-counting argument

Count superficial degree of divergence of 1PI diagrams



$$\sim \left[\int dw\, d^3k \right]^L G(w,k)^I \, (k^{2z})^V$$

$$\delta \leq (z+3)L - 2zI + 2zV = \boxed{(3-z)L + 2z}$$

$$(V - I + L = 1)$$

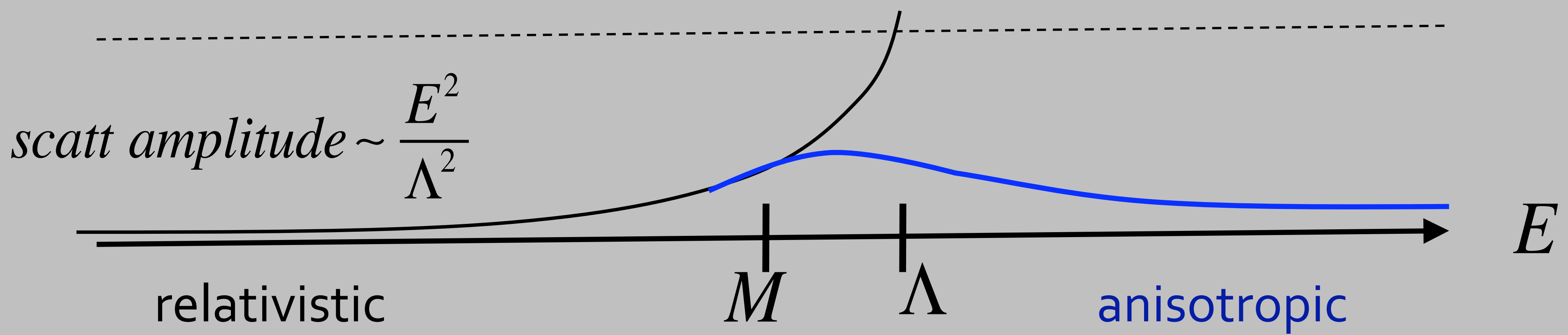
With $z=3$, all loop orders diverge equally ~> Renormalizable

Anisotropic Scaling

So, $L = \Lambda^2 \left[(\dot{\phi})^2 + c^2(\phi)\,\phi\Delta\phi + d^2(\phi)\,\frac{\phi\Delta^2\phi}{M^2} + e^2(\phi)\,\frac{\phi\Delta^3\phi}{M^4} \right]$ is P.C. renor-malizable…

But not only that!…

It *may be* weakly coupled at all energy scales, provided $M \leq \Lambda$



Anisotropic Scaling

The trick also works for gauge theories

(Anselmi & Halat '08)
(Iengo & Serone '10)

The big question:

Does the same trick work for gravity ??

~~Lorentz Invariance~~ => (part of the) gauge group broken

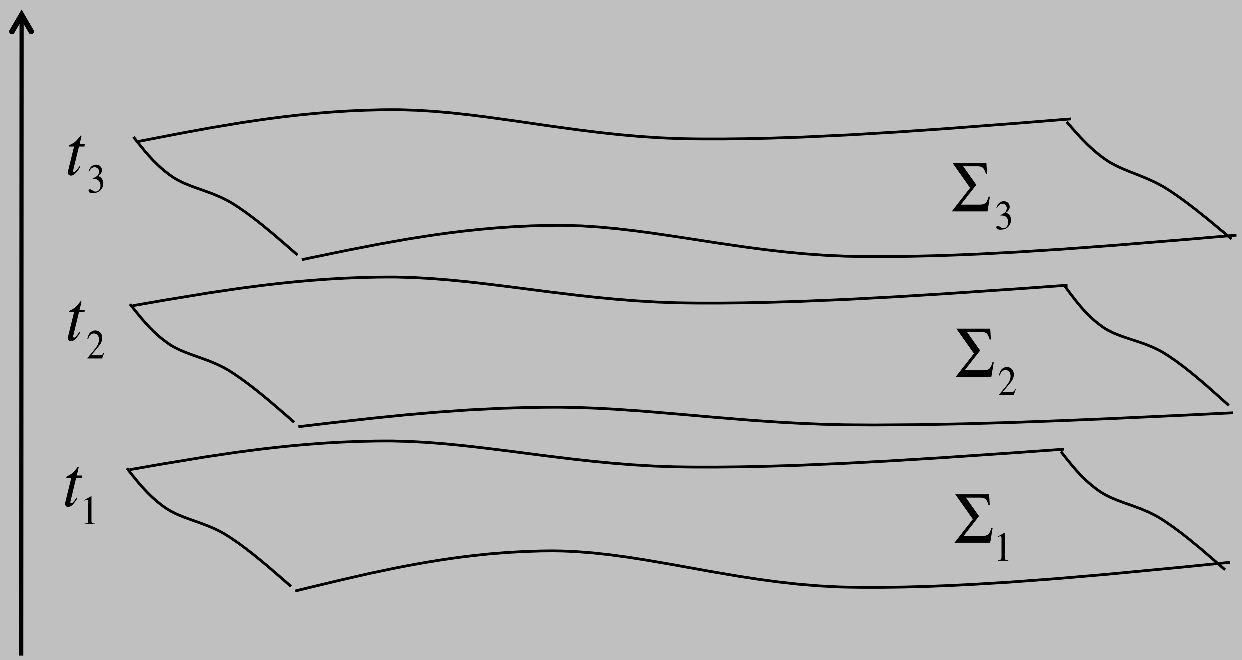
=> additional degrees of freedom

One needs to be careful, or else the extra d.o.f. are pathological!

Non-Relativistic Gravity

Introduce preferred time coordinate t (back to "Absolute time")

Globally defined *Foliation* by spatial 3D surfaces:



Non-Relativistic Gravity

Horava '09

3+1 split:
(ADM)

$$g_{\mu\nu} = \left(\begin{array}{c|c} N^2 & N^i \\ \hline N^j & \gamma_{ij} \end{array}\right)$$

Specify symmetry:

Foliation-preserving diffs

$$t \mapsto \hat{t}(t)$$

$$x \mapsto \hat{x}(t,x)$$

covariant objects:

$$\begin{cases} K_{ij} \equiv \dfrac{1}{2N}\left(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i\right) \\ a_i \equiv \dfrac{\partial_i N}{N}, \qquad R^{(3)}_{ijkl}[\gamma_{ij}] \ \ldots \end{cases}$$

Non-Relativistic Gravity

Horava '09

Blas Pujolas Sibiryakov '09

Action: $S = M_P^2 \int d^3x\, dt\, N\sqrt{\gamma}\left[K_{ij}K^{ij} - \lambda (K_i^i)^2 - V\left(\gamma_{ij}, N\right)\right]$

$(z = 3)$ $V\left(\gamma_{ij}, N\right) = R_{(3)} + \alpha\, a_i a^i + \dfrac{R_{(3)}^2 + a^4 + \ldots}{M_P^2} + \dfrac{R_{(3)}^3 + a^6 + \ldots}{M_P^4}.$

covariant objects:

$$\begin{cases} K_{ij} \equiv \dfrac{1}{2N}\left(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i\right) \\ a_i \equiv \dfrac{\partial_i N}{N}\,, \qquad R^{(3)}_{ijkl}[\gamma_{ij}] \;\ldots \end{cases}$$

Non-Relativistic Gravity

On top of helicity 2 graviton,

the model propagates a scalar:

$$L_0^{(2)} = M_P^2 \frac{3\lambda - 1}{\lambda - 1}\left[(\dot{\psi})^2 - c_0^2 (\partial_i \psi)^2 \right]$$

sound speed: $c_0^2 = \frac{2-\alpha}{\alpha} \frac{\lambda - 1}{3\lambda - 1} > 0$

Original proposals	
$\alpha = 0$	$\alpha \to \infty$ Projectable, $N(t)$

$\Rightarrow 0 < \alpha < 2$

Non-Relativistic Gravity

In short,

the problems with original proposals came by attempting to get rid of the scalar.

Let's forget about 'recovering GR' and accept a scalar d.o.f.

Require only that the model is
- phenomenologically viable
- weakly coupled

Stückelberg formalism

Stückelberg formalism

Blas Pujolas Sibiryakov '09

Isolating the scalar mode: $S = S_{GR} + S[\phi;\alpha,\lambda\ldots]$

ϕ defines the preferred frame: $\langle\phi\rangle = t$

unbroken $t \to \hat{t}(t)$ symmetry implies

$\phi \to f(\phi)$ internal symmetry

Invariants: $u_\mu \equiv \frac{\partial_\mu \phi}{\sqrt{(\partial\phi)^2}}$ and its derivatives

$$S[\phi] = M_P^2 \int d^4x \sqrt{-g}\,\left\{(\lambda - 1)\left(\nabla_\mu u^\mu\right)^2 + \alpha\,(u^\nu \nabla_\nu u_\mu)^2 + \ldots\right\}$$

In the *unitary gauge* $(\phi = t)$ the action coincides with previous form

Stückelberg formalism

Around flat space and $\phi = t$, let us study the scalar mode $\phi = t + \chi(t,x)$

$$S[\chi] = M_P^2 \int d^4x \left[\alpha (\partial_i \dot{\chi})^2 - (\lambda - 1)(\Delta\chi)^2 \right]$$

Disp. relation at low energies $w^2 = \frac{\lambda - 1}{\alpha} k^2 + \ldots$

Stückelberg formalism

Around flat space and $\phi = t$, let us study the scalar mode $\phi = t + \chi(t,x)$

$$S[\chi] = M_P^2 \int d^4x \left[\alpha (\partial_i \dot{\chi})^2 - (\lambda - 1)(\Delta\chi)^2 - (\lambda - 1)\dot{\chi}(\Delta\chi)^2 + \ldots \right]$$

Naive strong coupling scale $\Lambda \approx \sqrt{|\lambda - 1|}\, M_P$

If High. deriv. terms suppressed by $M_* \leq \Lambda$ $\Rightarrow$ NO strong coupling!

Projectable version of the model inevitably strongly coupled

Phenomenological Bounds

Phenomenological Bounds

The model reduces to a Scalar-tensor theory

$$S = \int d^4x \sqrt{-g} \left(M_P^2 R + \Lambda^2 L^{(\phi)}\left[\partial_\mu \phi \ , \ldots\right] + L^{(matter)}\left[\psi, g_{\mu\nu}, \partial_\mu \phi\right]\right)$$

E.g. ϕ mediates velocity-dependent forces:

$$V = \frac{\beta^2}{\lambda - 1} \frac{M_1 M_2}{r_{12}} \left[v_1 \cdot v_2 - (v_1 \cdot \hat{r}_{12})(v_2 \cdot \hat{r}_{12}) \right]$$

Phenomenology very similar to

- ghost condensation
- Einstein-aether

Blas OP Sibiryakov '09

Jacobson '10

Phenomenological Bounds

Coupling to ϕ gives different effective G_N

Newton's law $\phi_N = -G_N \frac{M}{r}$; $G_N = \frac{1}{8\pi M_P^2} \frac{1}{1-\alpha/2}$

Cosmology $H^2 = \frac{8\pi}{3} G_{\rm cosmo}\, \rho$; $G_{\rm cosmo} = \frac{1}{8\pi M_P^2} \frac{1}{1-3(\lambda-1)/2}$

Observational bound (BBN):

$$\left| \frac{G_{\rm cosmo}}{G_N} - 1 \right| < 0.13 \quad \Rightarrow \quad \boxed{\alpha, (\lambda - 1) \lesssim 0.1}$$

Phenomenological Bounds

Blas Pujolas Sibiryakov '10

PPN parameterization

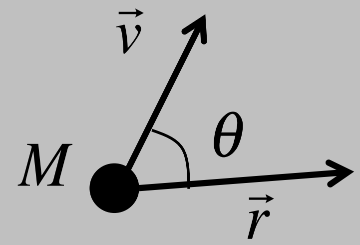
$$\beta^{PPN} = \gamma^{PPN} = 1 \quad \xi^{PPN} = 0$$

$$\alpha_1^{PPN},\ \alpha_2^{PPN} \neq 0$$ --> preferred frame effects

$$\alpha_1^{PPN} = -4(\alpha + 2\beta)$$

$$\alpha_2^{PPN} = \frac{(\alpha + 2\beta)(\alpha + 2\beta - \lambda + 1)}{2(\lambda - 1)}$$

$\vec{v}$ = velocity wrt preferred frame



$$\phi_N = -\frac{G_N M}{r}\left(1 + \frac{\alpha_2^{PPN}}{2} v^2 \sin^2\theta\right)$$

Observational bounds:

$$\alpha_2^{PPN} \leq 10^{-7}$$

$$\alpha_1^{PPN} \leq 10^{-4}$$

$$\Rightarrow \boxed{\Lambda \lesssim 10^{15} GeV}$$

Phenomenological Bounds

Upper bounds on scale of NR physics M_* (=> on Λ)

Within gravitational sector: $\Lambda > 0.1 eV$

Assuming that UV scale M_* is *the same* as in the Matter sector:

$$w^2 = c^2 k^2 + \cancel{\frac{k^3}{M_{*3}}} + \frac{k^4}{M_{*4}^2} + \ldots$$

From GRB and AGN observations, $M_* \geq 10^{10} \div 10^{11}\, GeV$

$$\Rightarrow \quad \boxed{10^{10 \div 11} GeV \lesssim \Lambda \lesssim 10^{15} GeV}$$

in the simplest model

Cosmology

1) Early Cosmology

Can the scalar graviton drive inflation? No (symmetries forbid)

However, it can drive bouncing and cyclic cosmologies...

$$H^2 = \rho + c_1 \frac{\kappa}{a^2} + c_2 \frac{\kappa^2}{a^4} + c_3 \frac{\kappa^3}{a^6}$$

Brandenberger 09 .
Kiritsis 09, Calcagni 09

Superluminal modes abound -> enough to solve horizon problem?

Scale-invariant perturbations generated more easily

Mukohyama 09

$$Dim[\Phi] = 0 \qquad \frac{d^2 a^z}{dt^2} > 0$$

Cosmology

2) Late Cosmology

Armendariz-Picon, Garriga Farina 10

Carruthers Jacobson 10

Kobayashi Urakawa Yamaguchi 10

Cerioni Brandenberger 10

(Mis)alignment between `CMB' and preferred frames

LV Dark Energy:

"θCDM" exploits relevant coupling of NG-boson to aether.
Technically natural DE sector with large cutoff!

Blas Sibiryakov '11

Mechanisms to relax CC?

CC could become an irrelevant operator
provided z>1 and nonlocality in deep IR??

Porto Zee '09

Recovery of Lorentz Invariance

Would be trivial in a single species theory

$$L = (\dot{\phi})^2 + c^2 \phi \Delta \phi + \frac{\phi \Delta^2 \phi}{M^2} + \ldots$$

relevant operator

with more species "Lorentz Fine-Tuning Problem":

c is species- dependent

Generically, RG flow generates $c_i - c_j \neq 0$

But $c_i - c_j$ experimentally constrained $\leq 10^{-20}$

Severe fine-tuning

Collins Perez Sudarsky Urrutia Vucetich 04

Iengo Russo Serone 09

Recovery of Lorentz Invariance

Possible Way out: SUSY

(~~Lorentz~~ SUSY = SUSY / boosts)

$$\{Q,\bar{Q}\} = \sigma^0 E + c\,\sigma^i P_i$$

LV operators in LV-MSSM are Dim 5 or higher!

Groot-Nibbelink
Pospelov '04

If CPT respected =>
first LV operator is Dim 6

$$\Rightarrow c_i - c_j \propto \frac{M_{SUSY}^2}{M_*^2}$$

$\Rightarrow$ SUSY at $< 10^2 TeV$!!

Conclusions

There exists a consistent and phenomenologiaclly viable power-counting renormalizable non-relativistic model of Quantum Gravity

At low energies:

Lorentz-breaking scalar-tensor theory

Deviations from GR are small, the model is weakly coupled

Acceptable window for UV scale $10^{10\div 11} GeV \lesssim \Lambda \lesssim 10^{15} GeV$

Conclusions

Many open questions :

- Recovery of Lorentz Invariance (matter sector): fine tuning avoidance

- Is it really renormalizable / UV complete ??

- Instantaneous Interactions

- Black Hole physics

- 'Resolution' of singularities

- Anisotropic Condensed matter (via holography)

Thank you!