

# Supercurvation

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based on a work with **Andrei Linde** and **Viatcheslav Mukhanov**

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# Motivations

**Single field inflation** predicts fluctuations which are almost scale-invariant and nearly Gaussian

Seven-year WMAP data:

E. Komatsu et al. (2010)

$$n_s = 0.968 \pm 0.012$$

$$-10 < f_{NL}^{obs} < 74$$

$$\mathcal{P}_s \sim k^{n_s - 1}$$

$$\frac{6}{5} f_{NL} = -\frac{B_\zeta(k_1, k_2, k_3)}{[P_\zeta(k_1)P_\zeta(k_2) + \text{cyclic}]}$$

**Curvaton scenario:**

model of inflation with two scalar fields

it allows a level of non-Gaussianity bigger than in single field inflation

# Why Supercurvaton?

It would be nice to have a curvaton model as simple as the basic chaotic inflation scenario.

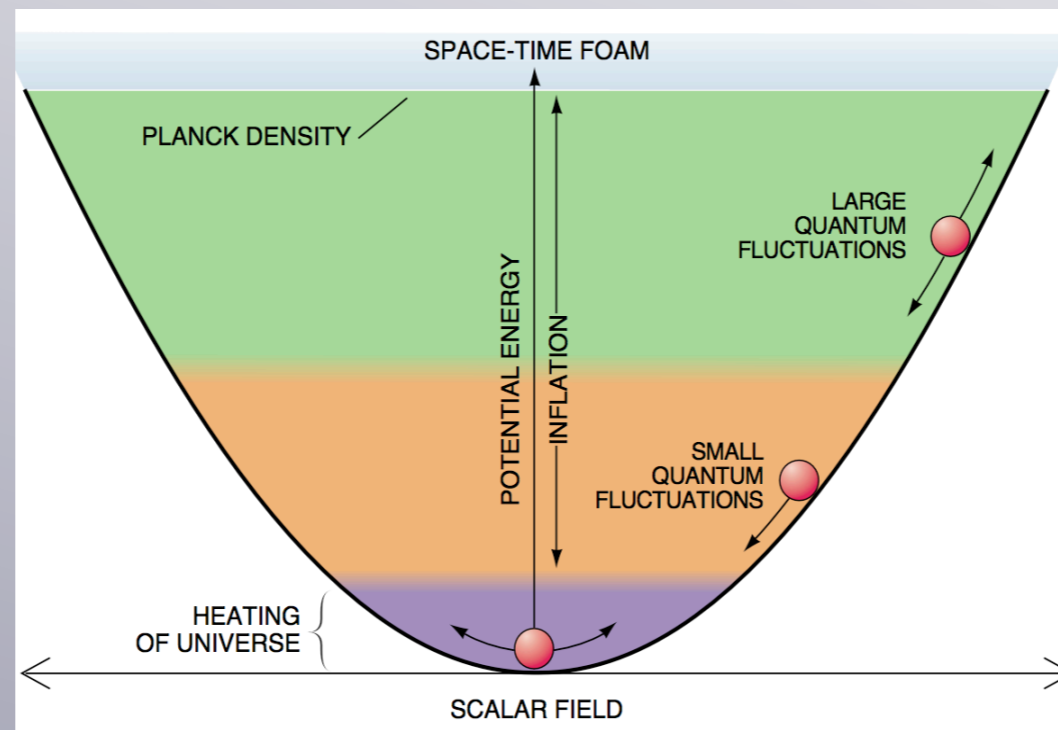
It would be good also to naturally implement this scenario in the context of supergravity.

# Outline

- Chaotic inflation in supergravity
- Supercurvaton
- Level of non-Gaussianity:  $f_{NL}$
- Conclusions

# Chaotic inflation

$$V(\varphi) = \frac{1}{2}m^2\varphi^2$$



A. Linde

The inflaton can take any initial value provided that  $(\partial_\mu\varphi)^2 \lesssim V(\varphi) \lesssim 1$

To cause inflation the inflaton must have a value larger than 1 :  $\varphi > 1$

# How can we implement chaotic inflation in supergravity?

M. Kawasaki, M. Yamaguchi and T. Yanagida (2000)

## Problem:

the generic supergravity potential has an exponential factor  $\exp(\Phi^* \Phi + \dots)$  which prevents any scalar field  $\Phi$  from having values larger than 1

## Solution:

to introduce a **shift symmetry of the inflaton**, namely to assume that the Kähler potential is invariant under the shift symmetry of the inflaton

$$\mathcal{K}(\Phi, \Phi^*) = \mathcal{K}(\Phi - \Phi^*) e^{\mathcal{K}(\Phi - \Phi^*)}$$

However, as long as the shift symmetry is exact, the inflaton never has a potential and hence **it never causes inflation**



small **breaking term** of the shift symmetry


$$W = mS\Phi$$

# Supercurvaton

Let us consider a very simple model with two fields  $S$  and  $\Phi$

$$W = mS\Phi \quad \mathcal{K} = SS^* - \frac{1}{2}(\Phi - \Phi^*)^2$$

Note that the Kähler potential does not depend on the phase of  $S$  and on the real part of  $\Phi$

  $S = \frac{\sigma}{\sqrt{2}}e^{i\theta} \quad \Phi = \frac{1}{\sqrt{2}}(\varphi + i\chi)$

$\varphi$  plays the role of the **inflaton** with  $V(\varphi) = \frac{m^2}{2}\varphi^2$

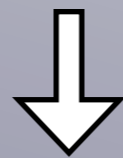
$m_\sigma^2 = m^2$   $\sigma$  is the **curvaton field**

Taking  $\chi = 0 \quad \rightarrow \quad V(\varphi, \sigma) = \frac{m^2}{2} e^{\sigma^2/2} \left[ \varphi^2 + \sigma^2 + \frac{\varphi^2}{4} \sigma^2 (\sigma^2 - 2) \right]$

Expanding with respect to  $\sigma^2$  and using the Friedmann equation

$$\left\{ \begin{array}{ll} \varphi\sigma \ll 1 & m_\sigma^2 = m^2 \\ \varphi\sigma \gtrsim 1 & m_\sigma^2 \sim \frac{3}{4} m^2 \varphi^2 \sigma^2 \simeq \frac{9}{2} H^2 \sigma^2 > m^2 \end{array} \right. \text{ supergravity corrections are important!}$$

Note:  $\sigma \gtrsim 1$  the potential becomes exponentially steep



$$\sigma \lesssim 1$$

For the rest of the talk we concentrate on the simplest model:

$$W = mS\Phi$$

$$\mathcal{K} = SS^* - \frac{1}{2}(\Phi - \Phi^*)^2 - \frac{\alpha}{12}(SS^*)^2$$

$$m_\sigma^2 = \alpha H^2 + m^2$$

R. Kallosh and A. Linde (2010)  
R. Kallosh, A. Linde and T. Rube (2010)

# Curvaton perturbations and non-Gaussianity

During inflation, the curvaton perturbations are produced. Then these fluctuations are stretched, overlap with each other and eventually produce a classical curvaton field  $\sigma$  which looks relatively homogeneous in the observable part of the Universe, but may take different values in other parts.

These perturbations will match the COBE normalization of the spectrum for

$$r \frac{\delta\sigma}{\sigma} \sim 7 \times 10^{-5}$$

The prediction for the curvaton scenario is

$$f_{NL} = \frac{5}{4r}$$

$f_{NL}$

where  $r \approx \left( \frac{\rho_\sigma}{\rho} \right)_{decay}$

$$-10 < f_{NL}^{obs} < 74$$

Seven-year WMAP data

E. Komatsu et al. (2010)

During inflation the long-wavelength distribution behaves as a **nearly homogeneous classical field** in slow roll regime with:

$$3H\dot{\sigma} + m_{\sigma}^2\sigma = 0$$

For the moment we assume a quadratic potential.

Each interval  $H^{-1}$  new fluctuations are generated:

$$\langle \delta\sigma^2 \rangle = \frac{H^2}{4\pi^2}$$

This effect increases the average squared of the classical field:

$$\frac{d\langle\sigma^2\rangle}{dt} = -\frac{2m_{\sigma}^2\langle\sigma^2\rangle}{3H} + \frac{H^3}{4\pi^2}$$

$$\frac{d\varphi}{dt} \simeq -\frac{V_{,\varphi}}{3H}$$

$$H^2 \simeq \frac{V}{3}$$

$\Rightarrow$

$$\frac{d\langle\sigma^2\rangle}{d\varphi} = \frac{2m_{\sigma}^2\langle\sigma^2\rangle}{V_{,\varphi}} - \frac{V^2}{12\pi^2 V_{,\varphi}}$$

Note that in different parts of the Universe  $\sigma$  might be smaller or bigger than  $\langle\sigma^2(\varphi)\rangle$

# Curvaton with $\alpha = 0$ : $m_\sigma^2 = m^2$

$$\boxed{\varphi\sigma \lesssim 1} \rightarrow \varphi_i \lesssim m^{-1/3}$$

$$\langle \sigma^2(\varphi) \rangle \simeq \frac{m^2 \varphi^2 \varphi_i^2}{96\pi^2}$$
$$\delta\sigma \sim \frac{H}{2\pi} = \frac{m\varphi}{2\pi\sqrt{6}}$$
$$\Rightarrow \boxed{\frac{\delta\sigma}{\sigma} = \frac{2}{\varphi_i}}$$

Upper bound on  $\varphi_i$  to remain inside the region:  $\varphi_i \simeq m^{-1/3}$

$$\rightarrow \frac{\delta\sigma}{\sigma} \sim m^{1/3}$$

$$\text{For } m \sim 10^{-7} \rightarrow \boxed{f_{NL} \sim 80}$$

# Curvaton with $\alpha = 0$ : $m_{\sigma}^2 = m^2 + \frac{9}{2}H^2\sigma^2$

$$\varphi\sigma \gtrsim 1$$

Solve the equation for  $V = \frac{m^2\varphi^2}{2} + \frac{m^2\sigma^2}{2} + \frac{m^2\varphi^2\sigma^4}{16}$

$$\rightarrow \frac{\delta\sigma}{\sigma} \sim 0.4m^{1/3}$$

$$m \sim 10^{-7} \quad \boxed{f_{NL} \sim 30}$$

Curvaton with  $m_\sigma^2 = \alpha H^2 + m^2$  and  $0 < \alpha \ll 1$

$$\rightarrow \frac{\delta\sigma}{\sigma} \simeq \sqrt{\frac{\alpha}{3}}$$

For  $\alpha = 10^{-5} \rightarrow f_{NL} \sim 30$

For  $\alpha = 10^{-2} \rightarrow f_{NL} \sim 10^3$  !!!

Why is the result so sensitive to the parameter  $\alpha$ ?

This parameter makes the mass of the curvaton field bigger than the mass of the inflaton:

$$m_\sigma^2 = \alpha H^2 + m^2$$

So the distribution of the curvaton shrinks fast while the inflaton rolls down.

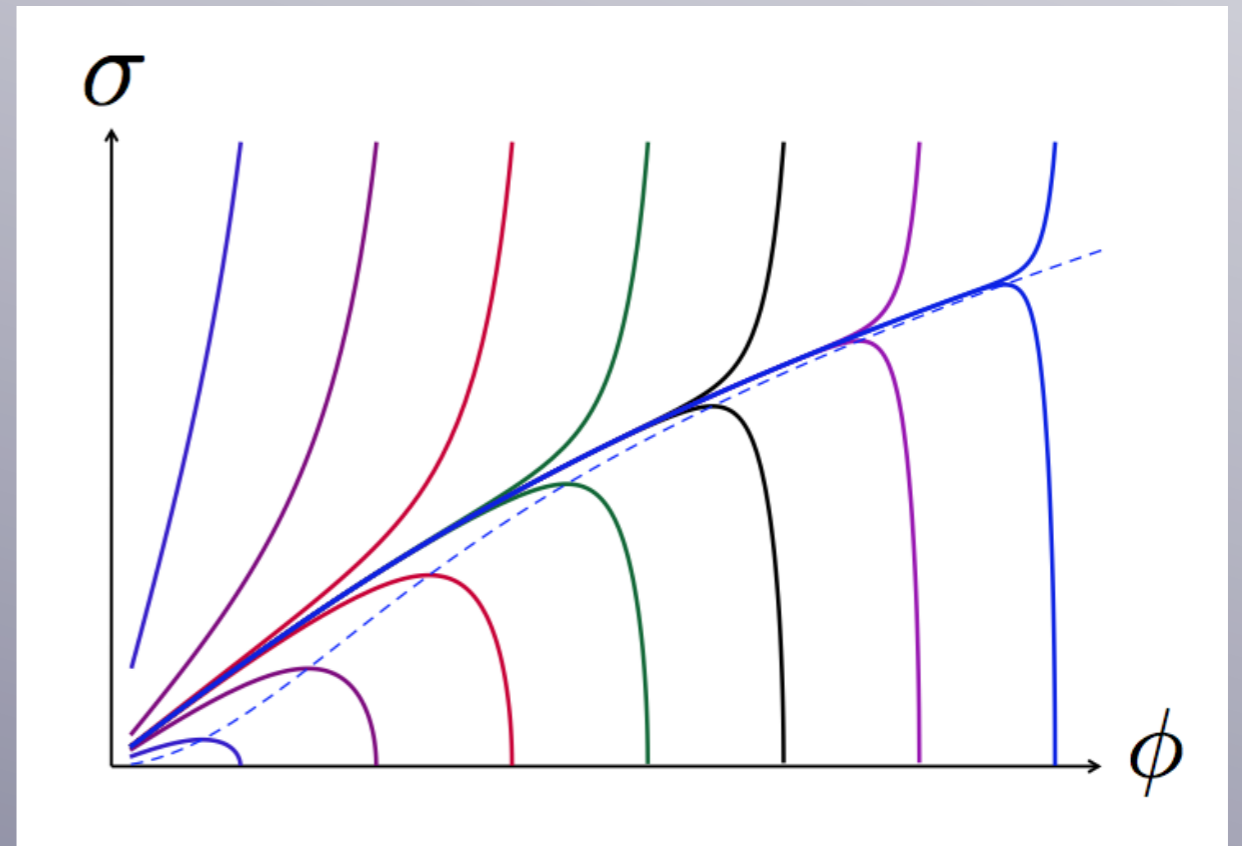
Curvaton with  $m_\sigma^2 = \alpha H^2 + m^2 + \frac{9}{2}H^2\sigma^2$  and  $\alpha > 0$

$$V = \frac{m^2\varphi^2}{2} + \frac{m^2\sigma^2}{2} + \frac{m^2\varphi^2\sigma^4}{16} + \alpha\frac{m^2\varphi^2\sigma^2}{6}$$

There is no exact analytical solution.

If inflation lasts long enough, all solutions, **independently of the initial conditions**, converge at a certain **attractor trajectory**.

If inflation is long enough, the final results do not depend on the choice of the initial conditions for the curvaton field.



# Conclusions

- ★ Curvaton scenario implemented in the simplest supergravity realization of chaotic inflation.
- ★ Discussion of observational consequences: non-Gaussianity
  - $f_{NL}$  can be in the observationally interesting range
- ★ If inflation is long enough, the average value of the non-Gaussianity parameter does not depend on initial conditions for the curvaton field.