# Anisotropic stress and (background) stability in modified gravity models

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# Goal & Motivation

- Modified gravity models provide a *well motivated explanation* to the late time acceleration of the universe → No need for cosmological constant. (*Though the coincidence problem is still there!*)
- Their higher order nature provides us with *enough freedom to reproduce any expansion history* → Indistinguishable from GR at the background level!

[2] S. F. Daniel and E. V. Linder (2010), arXiv: 1008.0397

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- Their higher order nature provides us with *enough freedom to reproduce any expansion history* $\rightarrow$ Indistinguishable from GR at the background level!
- Crucial test at the perturbation level: Modified gravity models predict a *non-zero anisotropic stress* $\phi/\psi \neq 1$. So far, *no signature of departure from GR has been observed.* $(\phi/\psi \sim 1)$ [2]
- **Question**: *Is it possible to build a viable modified gravity scenario with a signature sufficiently close to that of GR?*

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# What is the significance of anisotropic stress?

- Scalar, linear perturbations around flat FRW metric:

$$\boxed{ds^2 = -(1+2\psi)dt^2 + a(t)^2(1+2\phi)d\mathbf{x}^2}$$

- Anisotropy equation defines the *scalar anisotropic stress* $\Pi$:

$$\boxed{\phi - \psi \equiv \Pi(\mathbf{x}, t)}$$

- The ratio $\phi/\psi$ *is a signature of any gravity model at (linear) perturbation level.* $\rightarrow$ *Future surveys, ex. Euclid, Planck: Weak lensing + Galaxy power spectrum + CMB.*

- In GR, *at all times* $\phi/\psi = 1$, unless relativistic species present (negligible at late times).

# Anisotropic stress in modified gravity

- The higher order nature of modified gravity models contributes an *effective anisotropic stress* contribution *of geometrical origin*.

$$\boxed{\phi - \psi = \Pi^{(\mathrm{eff})}(g)}$$

- As a result, $\phi/\psi \neq 1 \leftrightarrow$ non zero (effective) anisotropic stress.
- Typical behavior for all higher order gravity models, $f(R)$, $f(R, G)$, DGP, e.t.c.

So, can we have a modified gravity model with a *sufficiently small effective anisotropic stress contribution*, such that it could escape observations?

$\rightarrow$ *Try to attempt a study for* $f(R, G)$*models.*

## Special case I: Anisotropic stress in $f(R)$ gravity

The most straightforward generalization of GR:

$$S = \int d^4x\sqrt{-g}f(R)$$

- Exhibits an extra scalar degree of freedom ("scalaron") with

$$m^2_{eff} \propto \frac{f_R}{f_{RR}}.$$

- Anisotropy equation:

$$\boxed{\phi - \psi = \Pi^{(\mathrm{eff})}_R \equiv \frac{f_{RR}}{f_R}\delta R}$$

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- Anisotropy equation:
$$\boxed{\phi - \psi = \Pi_R^{(\mathrm{eff})} \equiv \frac{f_{RR}}{f_R}\delta R}$$
- If we would like $\Pi_R^{(\mathrm{eff})} = 0$ at all times we should require that $f_{RR} = 0$.
$\rightarrow$ GR is the only solution, i.e $f(R) = R - 2\Lambda$!

## Special case II: Anisotropic stress in $f(G)$ gravity

$$S = \int d^4x \sqrt{-g}\,(R + f(G))$$

- The Gauss–Bonnet term: $G \equiv R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\kappa\lambda}R_{\mu\nu\kappa\lambda}$. [3]
- Gauss–Bonnet term is a curvature correction to the Einstein–Hilbert term motivated by string theory. [4]
- Extra scalar degree of freedom with $m^2_{eff} \propto \frac{1}{H^4 f_{GG}}$.
- The anisotropy equation is similar to the $f(R)$ case:

$$\boxed{\phi - \psi \equiv \Pi_G^{(\mathrm{eff})} = 4H^2 f_{GG}\delta G}$$

[3] D. Lovelock (1971) *J. Math. Phys.* **12** 3 498–501

[4] B. Zwiebach *Phys. Lett.* B **156** 315 (1985)

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- $\Pi_G^{(\text{eff})} = 0$ at all times $\leftrightarrow S = \int d^4x\sqrt{-g}\,(R + G - 2\Lambda)$ .

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## The more general case: Anisotropic stress in $f(R,G)$ gravity

$$S = \int d^4x\sqrt{-g}f(R,G)$$

- $f(R,G)$ models are characterized by *two different contributions*: $R$- and $G$- contribution.
- The anisotropy equation can be written as:

$$\phi - \psi = \Pi^{(\text{eff})}_{\text{total}} \equiv \Pi^{(\text{eff})}_{R} + \Pi^{(\text{eff})}_{G}$$

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- In order for $\Pi^{(\mathrm{eff})}_{\mathrm{total}} = 0$ we demand, in the sub horizon limit, that $\phi = \psi$:

$$\boxed{f_{RR} - 2H^2(5 + 9w_{\mathrm{eff}})f_{RG} + 8H^4\left(2 + 9w_{\mathrm{eff}}(1 + w_{\mathrm{eff}})\right)f_{GG} = 0}$$

Condition in the $f(R, G)$ model space for models with a zero anisotropic stress for $k >> H$.

## A model with zero anisotropic stress in a matter background

For a matter background we choose $w_{\rm eff} = 0$:

$$\boxed{f_{RR} - 10H^2 f_{RG} + 16H^4 f_{GG} = 0 = 0}$$

- Hard to to find a general solution, but we can do an ansatz instead and find:

$$\boxed{f(R, G) = R + c_1 G^{n_0} R^{m_0} + c_2 G^{-n_0} R^{-m_0}}$$

- There is **at least one model** in the context of $f(R, G)$ gravity that gives $\phi = \psi$.
- However, $\phi = \psi$ **only during matter domination** and $k >> H$.
- Model highly fine tuned.

## Model(s) with zero anisotropic stress in de Sitter background

The case of a de Sitter background corresponds to $w_{\text{eff}} = -1$:

$$\boxed{f_{RR} + 8H_0^2 f_{RG} + 16H_0^4 f_{GG} = 0}$$

- This time it is easy to find a general solution:

$$\boxed{f(R,G) = f_1\left(R - \frac{G}{4M^2}\right) + Rf_2\left(R - \frac{G}{4M^2}\right)}$$

  with $M = H_0$.

- A general class of models able to give very small (or exactly zero) anisotropic stress in de Sitter space as $M \to H_0$.

- Value $M \to H_0$ corresponds to the limit where the R- and G- contributions in the effective anisotropic stress cancel each other out.

## Problem(s) with zero anisotropic stress in de Sitter background

However, in the limit $M \to H_0$ one has to deal with the following problems:

- The square of the effective mass of the extra scalar d.o.f blows up, $\Longrightarrow m^2_{eff} \to \infty$: **Scalar d.o.f infinitely suppressed!**
- For an infinitesimally perturbed trajectory around de Sitter, i.e $H(t) = H_0 + \delta H(t)$,
  $\Longrightarrow \delta \ddot{H} \to \infty$: **Background singularity!**
- For the curvature perturbation, $\delta R(t) \propto m^2_{eff} \Phi(t)$, as $m^2_{eff} \to \infty$,
  $\Longrightarrow$ **Large curvature oscillations.** (Violation of stability conditions? [5])
- Propagation of the perturbations becomes superluminal, $c_s^2 \to \infty$, when spacetime not exactly Sitter.

[5] A. A. Starobinsky JETP Lett. **86**, 157-163 (2007)

# What can we learn?

- Very hard (if possible at all) to suppress the effective anisotropic stress in (4-dimensional) higher order gravity theories **in a consistent way**.
  → *Lower bound of $\phi/\psi$ in this context will be crucial for ruling out modified gravity observationally.*

- Even if successful, high amount of **fine tuning** required or (background) **stability** of the model is at high risk!
  → *Close link between anisotropic stress and the extra (scalar) degrees of freedom of the theory.*

- One expects **similar behavior** for other higher order models like higher dimensional, stringy ones (e.g DGP).
  → *Anisotropic stress is a* **key observation** *for testing gravity at large scales.*

Thank you !