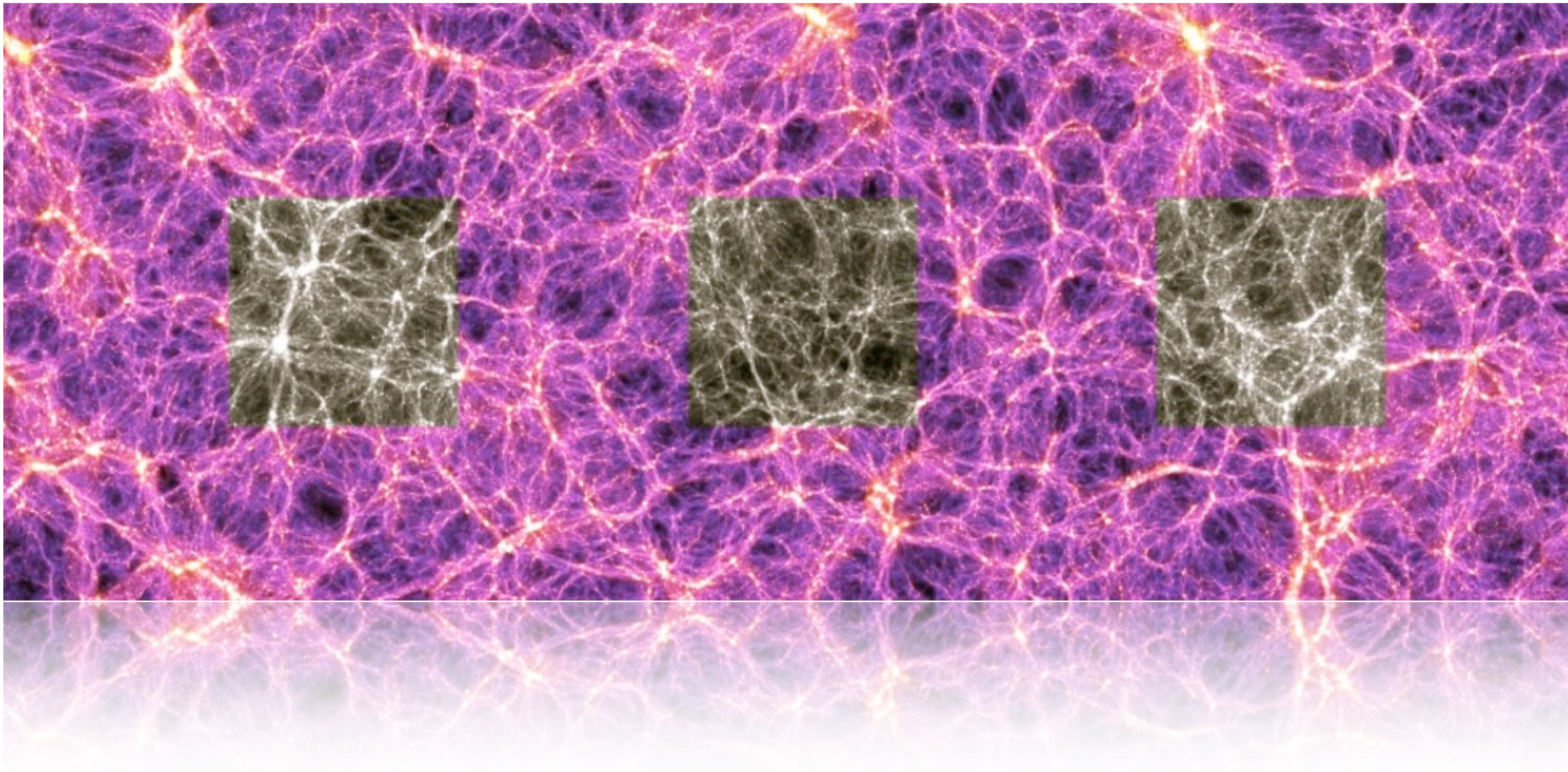
Is backreaction really small within concordance cosmology?

Chris Clarkson
Astrophysics, Cosmology & Gravitation Centre
University of Cape Town

with Obinna Umeh

How does structure affect the background?



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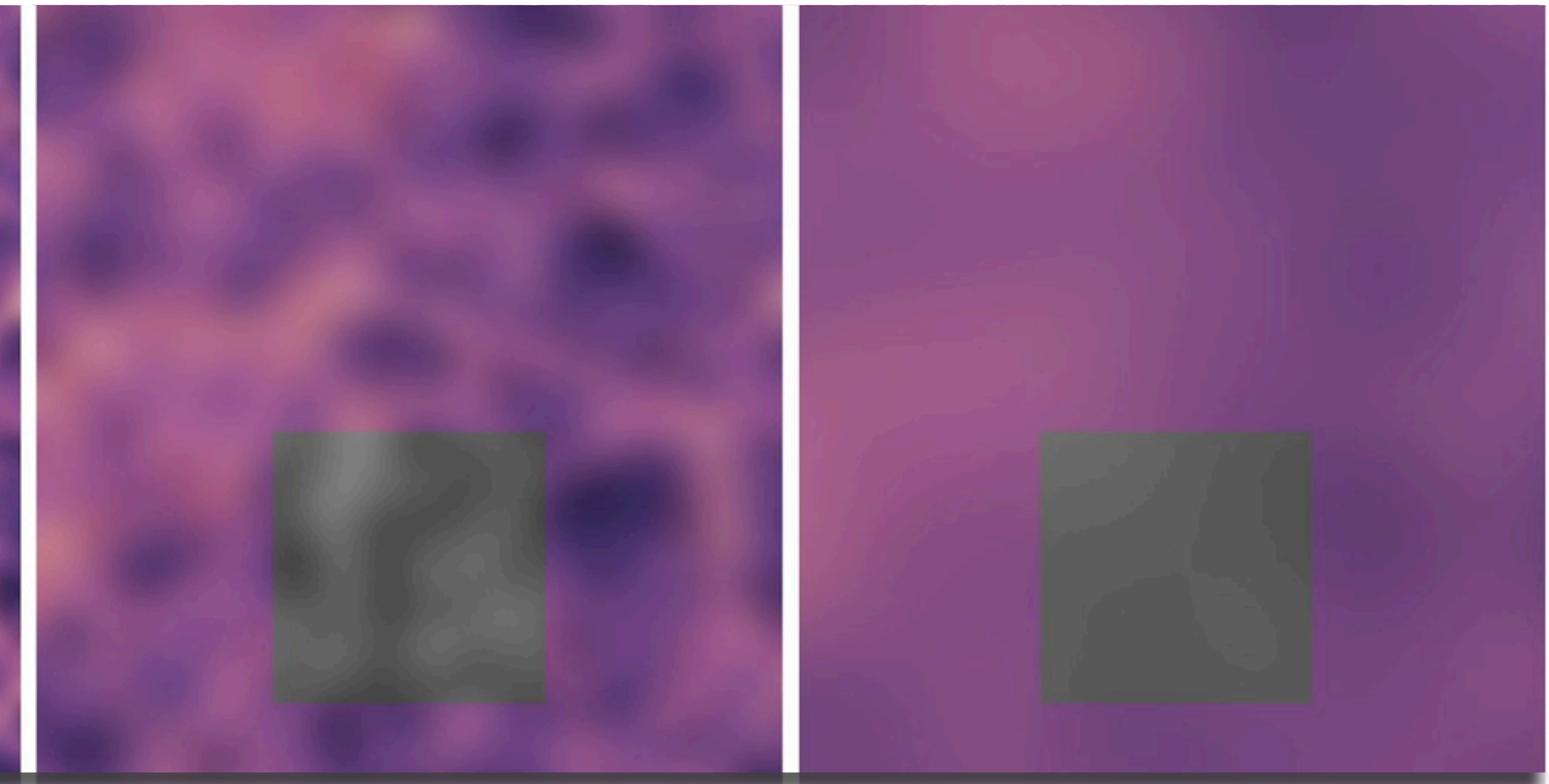

Averaging

- Define Riemannian averaging operator on arbitrary domain D

$$\psi_{\mathcal{D}} = \langle \psi \rangle_{\mathcal{D}} \equiv \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} \psi(t, x^i) J d^3x$$

Riemannian volume element

$$J \equiv \sqrt{\det(h_{ij})}$$

spatial average implies wrt
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to specify average energy density need full solution of the field equations

Riemannian volume element

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spatial average implies wrt some foliation of spacetime

Canonical Cosmology

- compute everything as power series in small parameter ε

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \epsilon\delta^{(1)}g_{\mu\nu} + \epsilon^2\delta^{(2)}g_{\mu\nu} + \cdots$$

‘real’ spacetime

first-order perturbation

second-order correction

‘background’ spacetime
FLRW

fit to ‘background’ observables - SNIa etc

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- Corrections from averaging enter Friedmann and Raychaudhuri equations
  - is this degenerate with 'dark energy'?
  - can we separate the effects [if there are any]?
  - or ... is it dark energy? neat solution to the coincidence problem

Perturbation theory

- metric to second-order

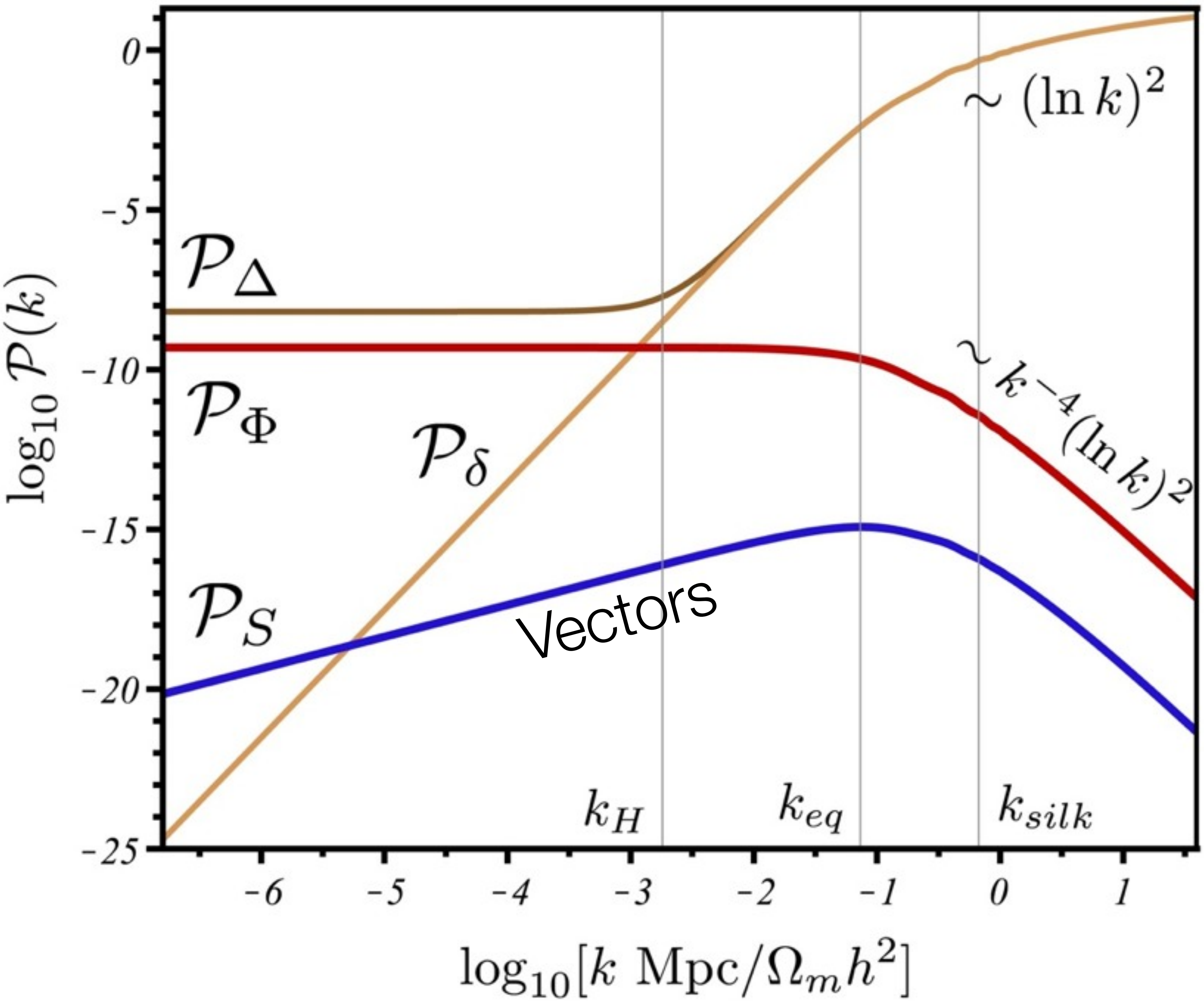
$$ds^2 = -\left[1 + 2\Phi + \Phi^{(2)}\right] dt^2 - aV_i dx^i dt + a^2 \left[(1 - 2\Phi - \Psi^{(2)})\gamma_{ij} + h_{ij}\right] dx^i dx^j$$

- first-order potential: $\ddot{\Phi} + 4H\dot{\Phi} + \Lambda\Phi = 0$

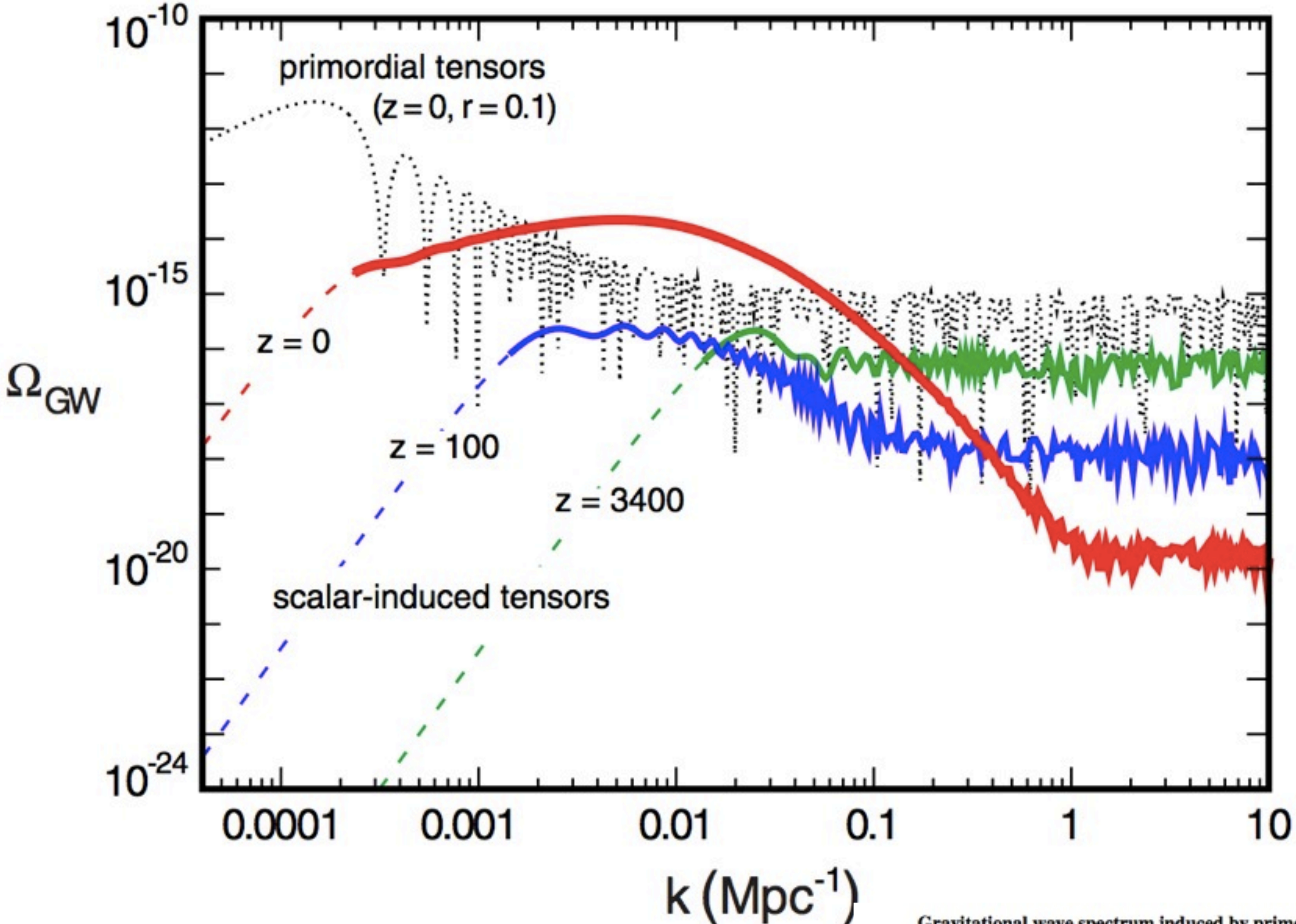
- second-order potentials:

$$\Phi^{(2)} \simeq \Psi^{(2)} \sim (\partial\Phi)^2 \qquad V_i \sim \Phi\partial_i\Phi \qquad h_{ij} \sim \Phi\partial_i\partial_j\Phi$$

- backreaction is concerned with the homogeneous, average contributions



0
-5
-10
-15
-20
-25
-6
-5
-4
-3
-2
-1
0
1
$\log_{10} \mathcal{P}(k)$
$\log_{10}[k \text{ Mpc}/\Omega_m h^2]$
$\mathcal{P}_\Delta$
$\mathcal{P}_\Phi$
$\mathcal{P}_\delta$
$\mathcal{P}_S$
Vectors
$\sim (\ln k)^2$
$\sim k^{-4}(\ln k)^2$
k_H
k_{eq}
k_{silk}



Gravitational wave spectrum induced by primordial scalar perturbations

Daniel Baumann,[1,*] Paul Steinhardt,[1,2,†] and Keitaro Takahashi[1,‡]

[1]*Department of Physics, Jadwin Hall, Princeton University, Princeton, New Jersey 08544, USA*

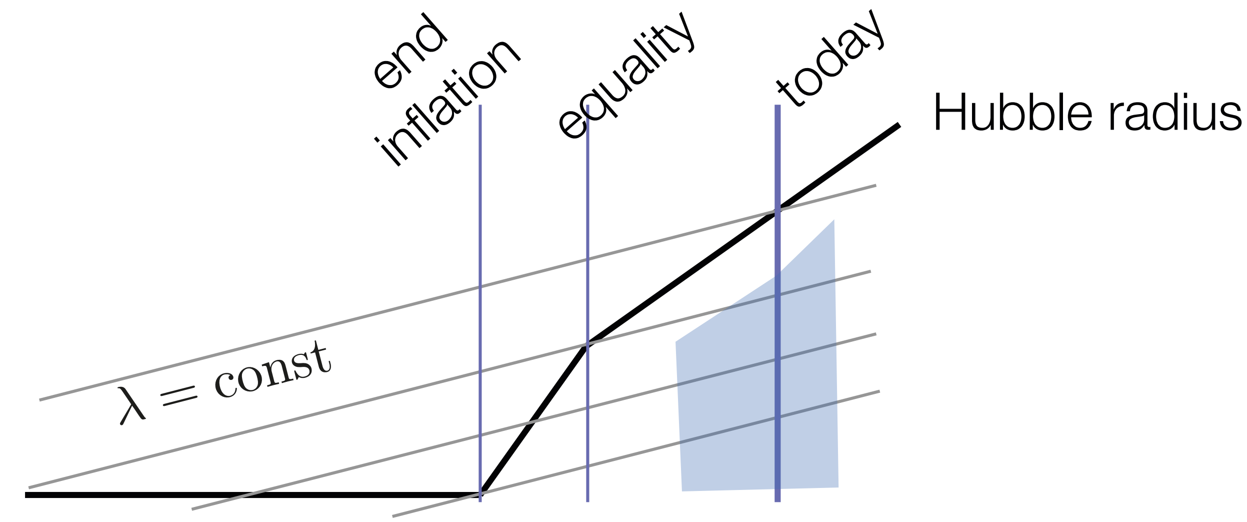
[2]*Princeton Center for Theoretical Physics, Jadwin Hall, Princeton University, Princeton, New Jersey 08544, USA*

Kiyotomo Ichiki[§]

Research Center for the Early Universe, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

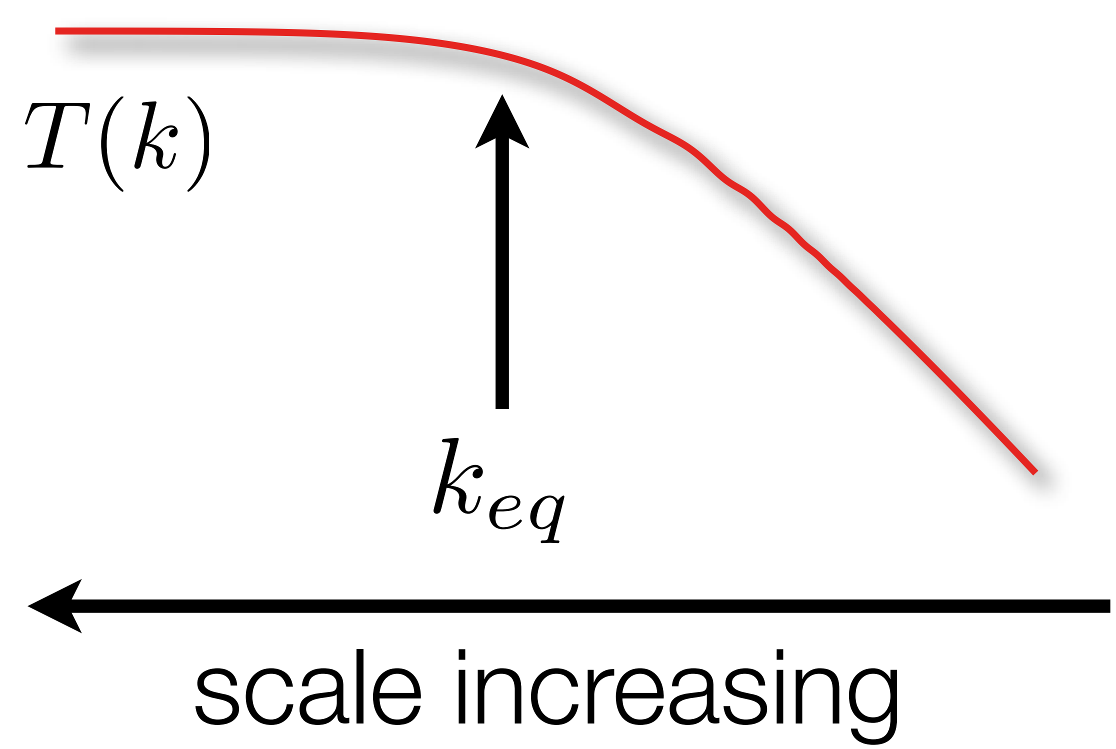
(Received 9 June 2007; published 17 October 2007)

scaling behaviour at first-order



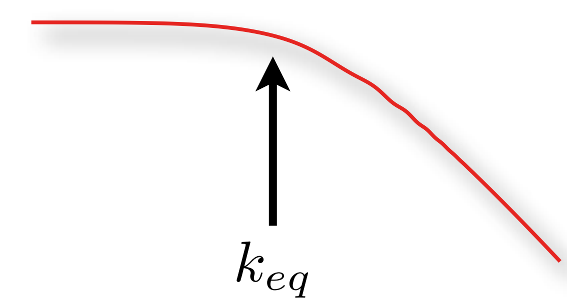
$$\mathcal{P}_\Phi \sim \Delta^2_{\mathcal{R}} T(k)^2$$

$$2.4 \times 10^{-9}$$



amplitude of second-order contributions

$$\overline{\partial^m \Phi \, \partial^n \Phi} \sim \int_0^\infty \frac{\mathrm{d}k}{k} \, k^{m+n} \mathcal{P}_\Phi(k)$$



$$\sim \Delta^2_{\mathcal{R}} \left(\frac{k_{eq}}{k_H} \right)^{m+n} \underbrace{\int_0^\infty \mathrm{d}\kappa \, \kappa^{m+n-1} T(\kappa)^2}$$

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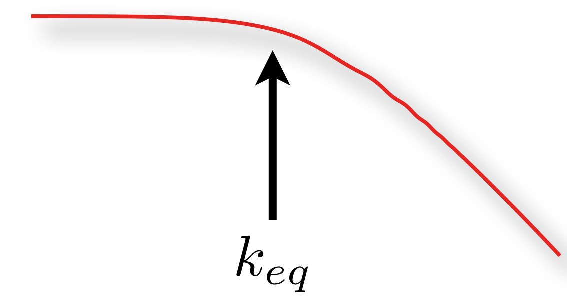

$$\sim \Delta_{\mathcal{R}}^2 \left(\frac{k_{eq}}{k_H}\right)^{m+n} \underbrace{\int_0^\infty \mathrm{d}\kappa \, \kappa^{m+n-1} T(\kappa)^2}$$

$$\approx \begin{array}{lll} -\ln(\kappa_{\mathrm{IR}}) & \text{for} & m+n=0 \\ 3.9 & \text{for} & m+n=2 \\ 53 \ln^3(\kappa_{\mathrm{UV}}) & \text{for} & m+n=4 \end{array}$$

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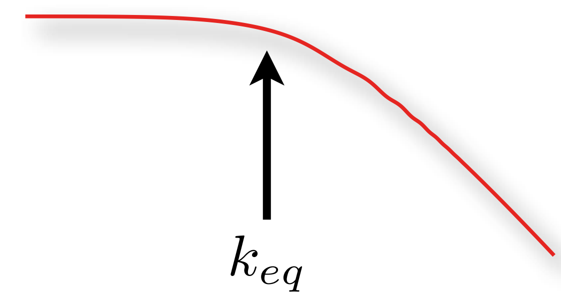

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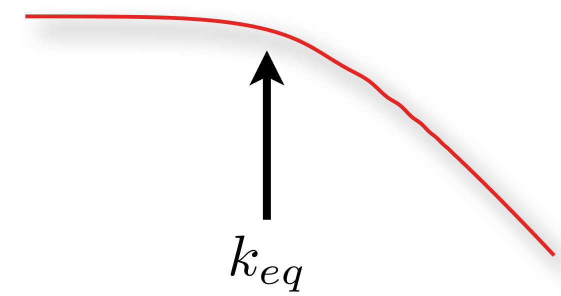

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large equality scale suppresses backreaction - but overcomes factors of Delta

backreaction

- second-order modes give non-trivial backreaction

- Hubble rate depends on

$$\mathrm{H}^{(2)} \sim [\cdots]\Phi^2 + [\cdots](\partial\Phi)^2 + [\cdots]\Phi^{(2)}$$

  - UV divergent terms don't contribute on average

  - well defined and well behaved backreaction

- this is *only* well behaved because of the long radiation era

  - what would we do if the equality scale were smaller?

backreaction

- other quantities are much stranger
- time derivative of the Hubble rate represented in the deceleration parameter

$$q^{(2)} \sim [\cdots]\Phi^2 + [\cdots](\partial\Phi)^2 + [\cdots]\Phi^{(2)} + \cdots + [\cdots](\partial^2\Phi)^2$$

  - same types of things appear in q defined via distance-redshift relation
  - UV divergent terms do not cancel out

divergent terms

$$\overline{\partial^2\Phi\,\partial^2\Phi} \quad \sim \quad \Delta^2_{\mathcal{R}} \left(\frac{k_{eq}}{k_H}\right)^4 \ln^3 \frac{k_{UV}}{k_{eq}}$$

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k_{UV} should be $< \mathcal{O}(\mathrm{pc})$ from end of inflation which makes this huge

wtf?

ignore them - probably gauge or unphysical ?

where else could they appear?

Hubble rate at fourth-order

$$\frac{H^{(4)}}{H} \sim -\frac{16}{729\Omega_m^4}\left(1+3\hat{g}+3\hat{g}^2+\hat{g}^3\right)\,\tilde{\partial}_k\Phi\tilde{\partial}^k\Phi\tilde{\partial}_j\tilde{\partial}^2\Phi\tilde{\partial}^j\Phi$$

$$+\frac{8}{243\Omega_m^4}\left[3(1-\hat{g}^2-2\hat{g}^3)+\Omega_m(27-67\hat{g}+30\hat{g}^2)\right]\,\tilde{\partial}_k\Phi\tilde{\partial}^k\Phi\tilde{\partial}^2\Phi\tilde{\partial}^2\Phi$$

$$-\frac{16}{243\Omega_m^4}\left[2(1+3\hat{g}+3\hat{g}^2+\hat{g}^3)+7\Omega_m(1+\hat{g})\right]\,\tilde{\partial}_k\Phi\tilde{\partial}_k\tilde{\partial}_j\Phi\tilde{\partial}^j\Phi\tilde{\partial}^2\Phi$$

$$+\frac{1}{81\Omega_m^3}\left(\tilde{\partial}_k\tilde{\partial}^2\Psi^{(2)}-\tilde{\partial}_k\tilde{\partial}^2\Phi^{(2)}\right)\left(H\tilde{\partial}^k\Phi^{(2)}+\tilde{\partial}^k\dot{\Psi}^{(2)}\right)$$

$$+\text{ terms of the form: }\;\Phi^{(4)},\;\partial_k v^k_{(4)},\;\partial_k\Phi\partial^k\Phi\partial^2\Phi^{(2)},$$

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plus contributions from induced vectors and tensors

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fourth-order contribution could give change to background value

~~Conclusions~~ Confusions

- Why are second-order perturbations so large?
- why is $\Delta_{\mathcal{R}} \left(\frac{k_{eq}}{k_H}\right)^2 \sim 1$?
    - yet another coincidence problem?
- tells us that perturbation theory must be relativistic, not Newtonian
- role of UV divergence must be understood to decide whether backreaction is small - higher order or resummation methods needed? must include tensors!
- do we need relativistic N-body replacement?