

# Helical magnetic fields from inflation and their evolution

Lukas Hollenstein

DPT & CAP, Université de Genève

reference:

R Durrer, LH, RK Jain, JCAP 03 (2011) 037



PONT Avignon 2011



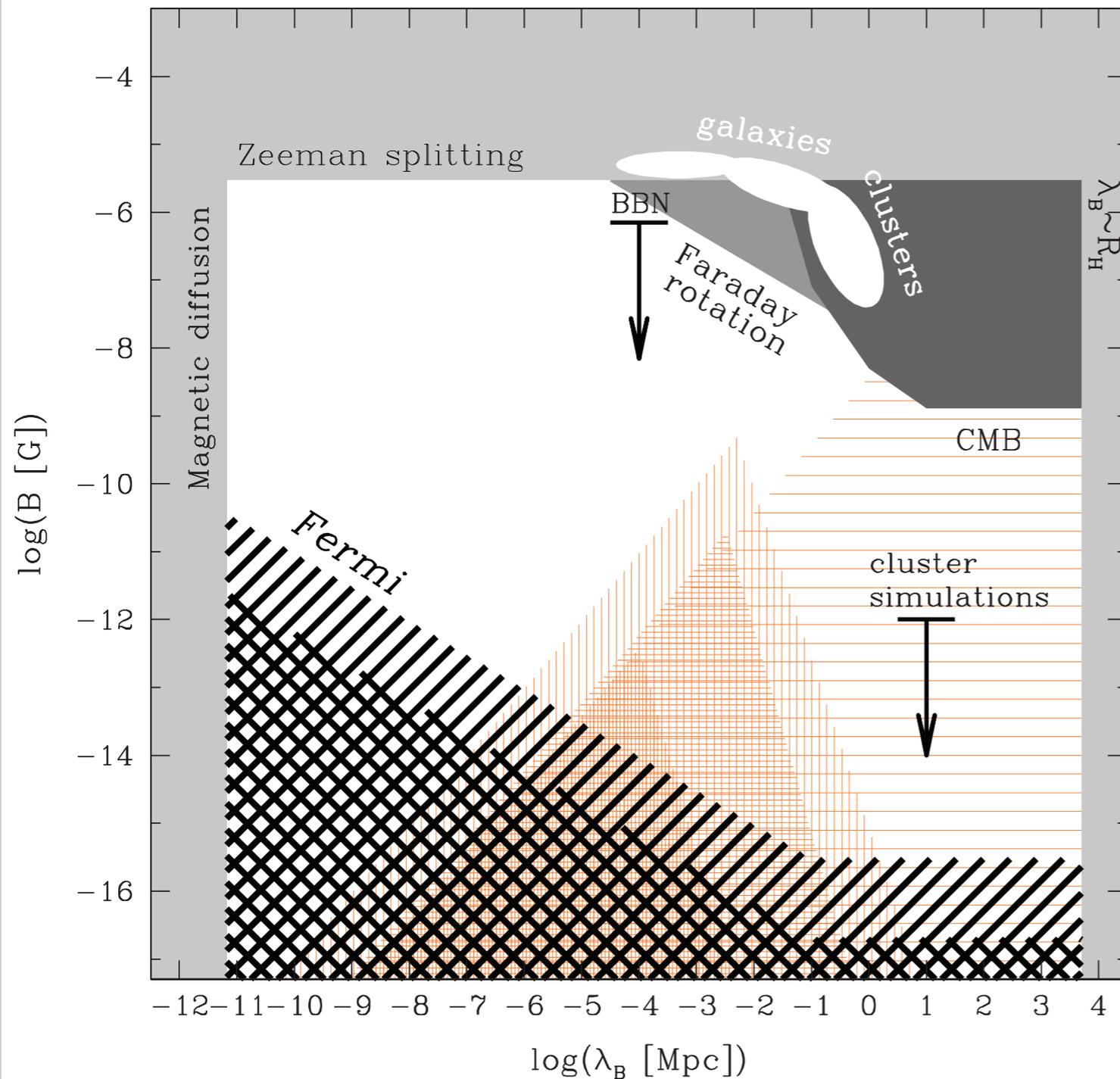
# Outline

---

- ▶ Motivation for primordial magnetic fields
- ▶ Helical magnetic field generation during inflation
- ▶ Slow roll and beyond
- ▶ Further evolution: inverse cascade
- ▶ Conclusions

Motivation for primordial magnetic fields

# Observations



- ▶ Magnetic fields of  $\sim \mu\text{Gauss}$  observed in galaxies, clusters & regions around high- $z$  quasars
- ▶ CMB bounds:  $B < 10^{-9} \text{ G}$
- ▶ Constrained cluster simulations:  $B < 10^{-10} \text{ G}$
- ▶ Missing secondary GeV gamma-rays from TeV blazars:  $B > 10^{-15} \text{ G}$

# Why (helical) primordial magnetic fields?

---

- ▶ Primordial magnetic fields (from inflation)

  - ... can produce the observed weak fields on very large scales, even in the intergalactic medium.

  - ... would explain why there are magnetic fields everywhere.

- ▶ Helicity is a conserved quantity and leads to

  - ... an inverse cascade in the turbulent plasma era, that can move power from small to large scales.

  - ... distinct signatures in CMB, such as TE- and EB-cross-correlations.

- ▶ In many inflation scenarios helical magnetic fields are generated naturally (see Neil Barnaby and Lorenzo Sorbo's talks)

Helical magnetic field generation during inflation

# Electromagnetism & inflation

---

- ▶ Standard EM is conformally invariant  $\rightarrow$  fluctuations do not grow in conformally flat space-times such as Friedmann-Lemaître.
- ▶ Modifications could be introduced to break conformal invariance:

$$\mathcal{L} \supset -\frac{1}{4} I^2(\phi, R) F_{\mu\nu} F^{\mu\nu} + M^2(\phi, R) A_\mu A^\mu + \frac{1}{4} f(\phi, R) F_{\mu\nu} \tilde{F}^{\mu\nu}$$

where  $I(\dots)$ ,  $M(\dots)$ ,  $f(\dots)$  are possible couplings to (pseudo-)scalar fields (inflaton or not) or curvature invariants.

- ▶ As a consequence the evolution of the Fourier modes of quantum fluctuations in the photon field is modified.

# Electromagnetism & inflation

---

- ▶ For an axial coupling to scalar inflaton:

$$\nabla_{\alpha} F^{\mu\alpha} = f'(\phi) (\partial_{\alpha} \phi) \tilde{F}^{\mu\alpha}$$

“scalar current”

$$\nabla^{\alpha} \partial_{\alpha} \phi - V'(\phi) = \frac{1}{4} f'(\phi) F_{\alpha\beta} \tilde{F}^{\alpha\beta}$$

feedback

# Electromagnetism & inflation

---

- ▶ For an axial coupling to scalar inflaton:

$$\nabla_{\alpha} F^{\mu\alpha} = f'(\phi) (\partial_{\alpha} \phi) \tilde{F}^{\mu\alpha}$$

“scalar current”

$$\nabla^{\alpha} \partial_{\alpha} \phi - V'(\phi) = \frac{1}{4} f'(\phi) F_{\alpha\beta} \tilde{F}^{\alpha\beta}$$

feedback

- ▶ Some notation:

$$\langle \tilde{B}_i(t, \mathbf{k}) \tilde{B}_j^*(t, \mathbf{q}) \rangle = \frac{(2\pi)^3}{2} \delta(\mathbf{k} - \mathbf{q}) \left\{ (\delta_{ij} - \hat{k}_i \hat{k}_j) P_S(t, k) - i \epsilon_{ijn} \hat{k}_n P_A(t, k) \right\}$$

$$\frac{d\tilde{\rho}_B}{d \ln k}(t, k) = \frac{k^3}{(2\pi)^2} P_S(t, k)$$

magnetic energy density

$$\frac{d\tilde{\mathfrak{H}}}{d \ln k}(t, k) = \frac{k^2}{2\pi^2} P_A(t, k)$$

helicity density

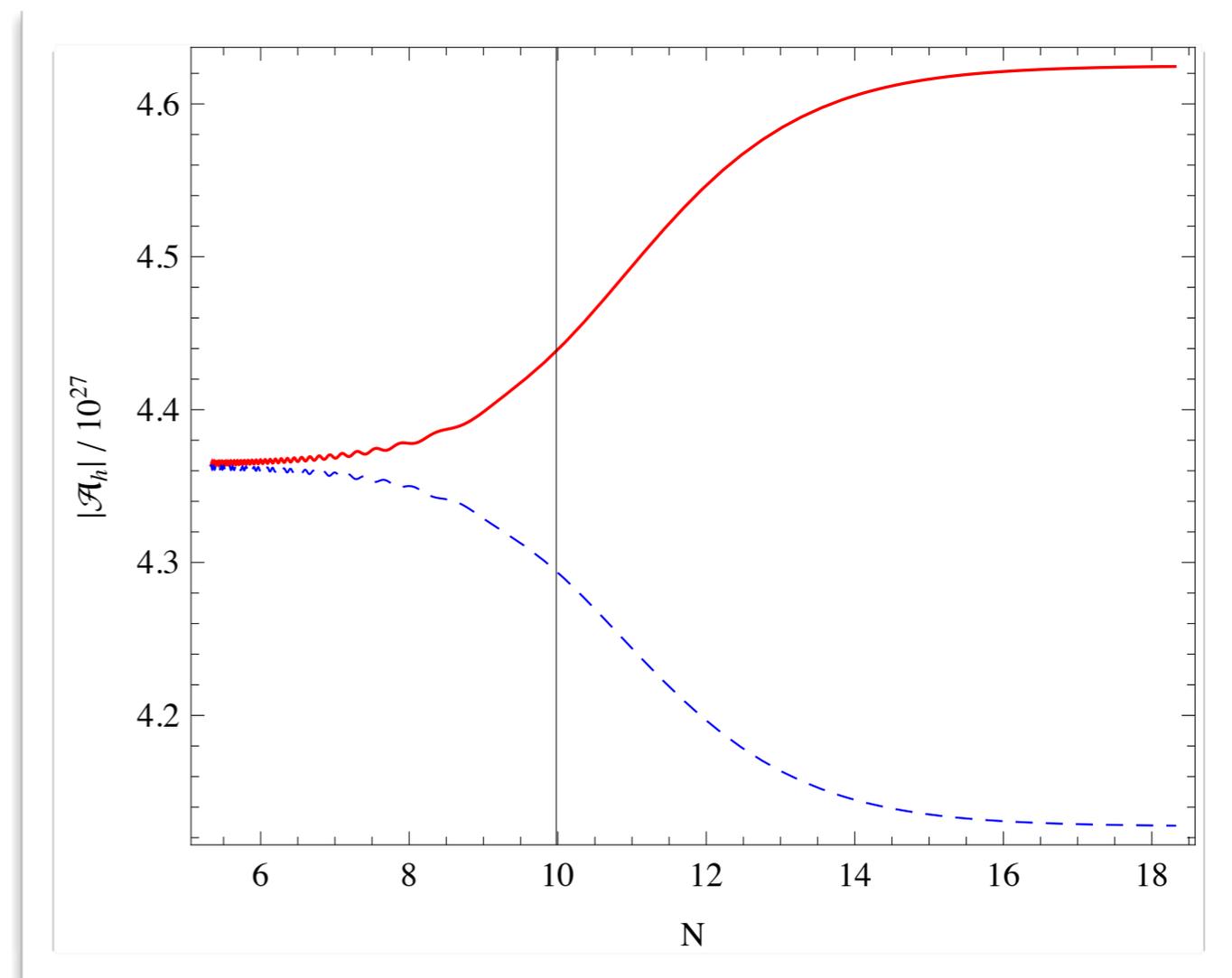
# Helical vs. non-helical

**Non-helical case:**  $\ddot{\bar{\mathcal{A}}}_{\pm} + (k^2 - \ddot{I}/I)\bar{\mathcal{A}}_{\pm} = 0, \quad \bar{\mathcal{A}}_{\pm} = I\mathcal{A}_{\pm}$

**Helical case:**  $\ddot{\mathcal{A}}_{\pm} + (k^2 \pm kf)\mathcal{A}_{\pm} = 0$

- ▶ The two helicity modes evolve differently.
- ▶ The coupling term is scale-dependent & only active around horizon-crossing. On large scales:

$$\left[ 1 + \frac{k^2}{\mathcal{H}^2} \pm \frac{kf}{\mathcal{H}^2} \right] \mathcal{A}_{\pm} \simeq 0$$



Slow roll and beyond

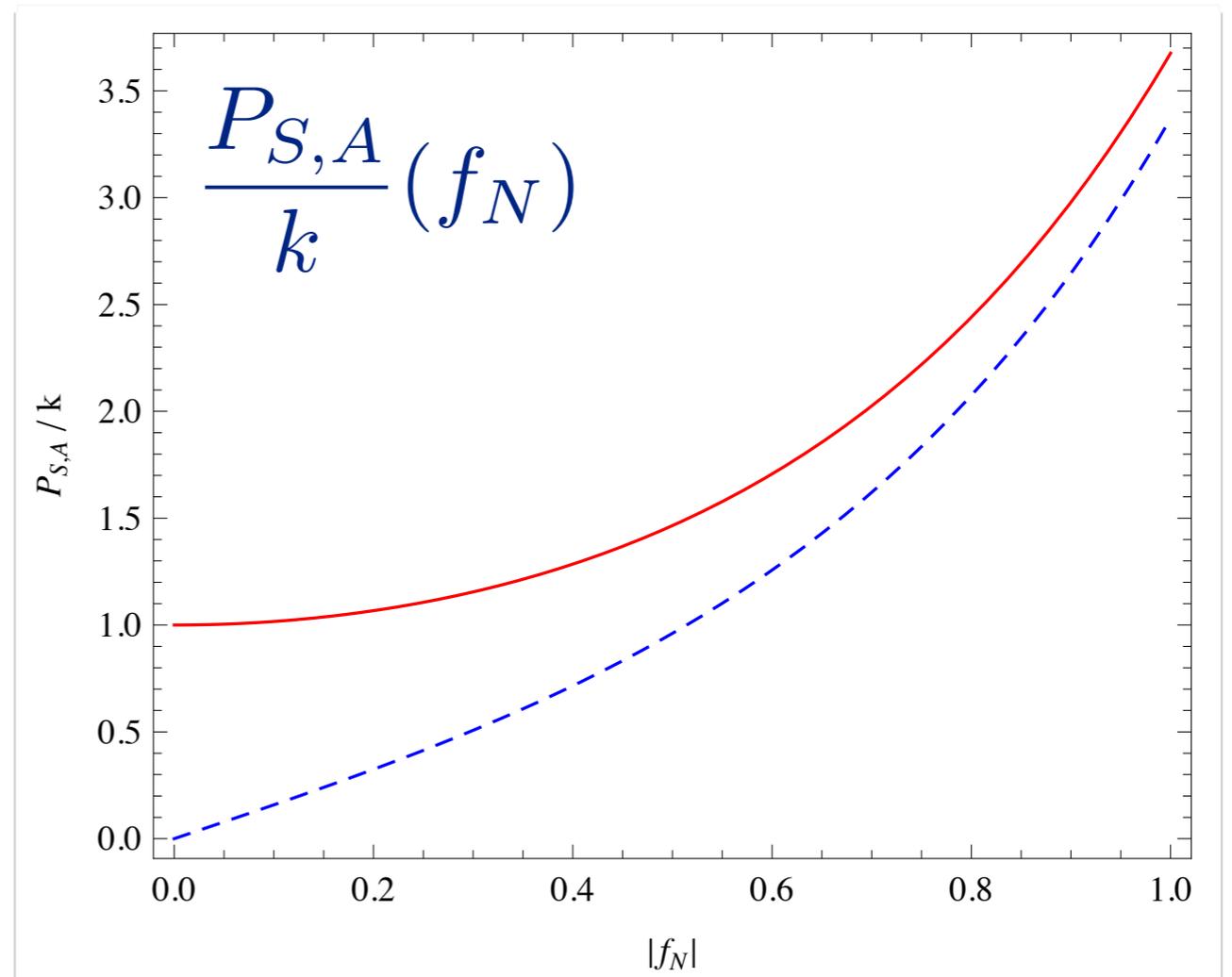
# Slow roll

- ▶ Slowly rolling inflaton:  $\epsilon \equiv \frac{m_P^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1 \Rightarrow \dot{\varphi} \simeq \sqrt{2\epsilon} m_p \mathcal{H}$   
 $\mathcal{H} \simeq -1/t \quad \varphi \simeq \varphi_{in} + \sqrt{2\epsilon} m_P \ln(t/t_{in})$

- ▶ For **constant**  $f_N \equiv \dot{f}/\mathcal{H}$  the evolution of the modes can be solved analytically:

$$P_S(k) = k \frac{\sinh(\pi f_N)}{\pi f_N}$$

$$P_A(k) = k \frac{\cosh(\pi f_N) - 1}{\pi f_N}$$



# Conditions on the coupling function $f(\varphi)F_{\alpha\beta}\tilde{F}^{\alpha\beta}$

---

- ▶ Is constant  $f_N$  a good approximation? (reminder:  $f_N \equiv \dot{f}/\mathcal{H} = 2\xi_{\text{Sorbo}}$ )
  - ▶ The coupling is only active around horizon crossing (a few e-foldings):  
→  $f_N$  can be taken constant for each mode, at least.
  - ▶ The inflaton only varies slowly during inflation:  $f_N \simeq f'(\varphi)\sqrt{2\epsilon}m_P$  if  $f(\phi)$  is not an extremely rapidly varying function (double exponential), then  $f_N$  constant throughout is a good approximation.
- ▶ Naïve bound from perturbativity of the modified electromagnetism:

$$\left| \frac{S_I[\phi, A_\mu]}{S_{\text{em}}[A_\mu]} \right| < 1 \quad \rightarrow \quad |f_N| < 1$$

# Backreaction & other bounds

---

- ▶ Bound from backreaction: demanding the generated electromagnetic fields not to affect the inflaton background evolution, yields

$$\mathcal{S}(f_N) \lesssim \frac{m_P}{H_{end}} \simeq \frac{m_P^2}{T_\star^2} \simeq 1.5 \times 10^{10} \left( \frac{10^{14} \text{ GeV}}{T_\star} \right)^2$$

reheating  
temperature

where

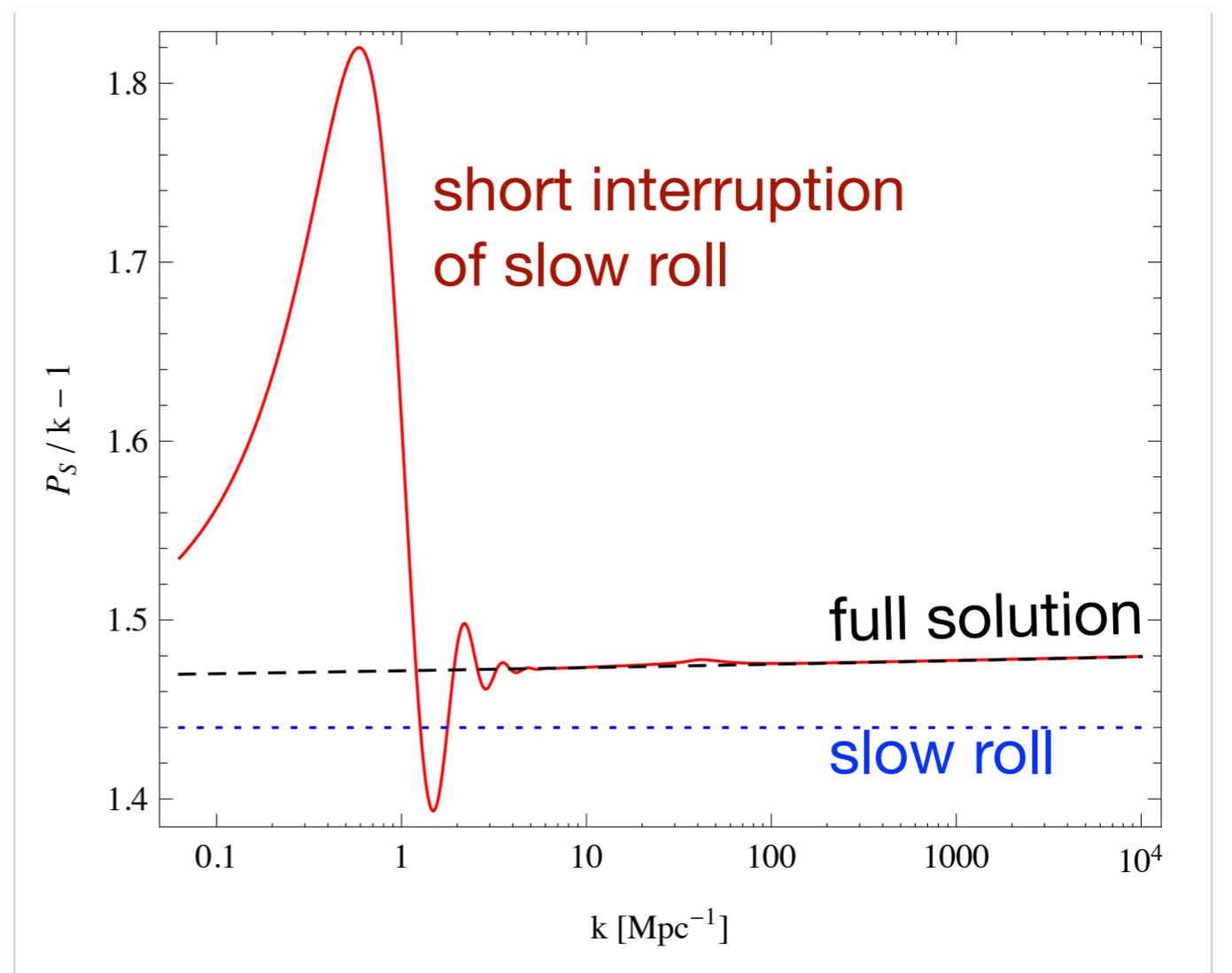
$$\mathcal{S}(f_N) \equiv \sqrt{\frac{\sinh(\pi f_N)}{4\pi^3 f_N}}$$

$$\rightarrow f_N \lesssim 17 \frac{10^{14} \text{ GeV}}{T_\star}$$

- ▶ For  $|f_N| \gtrsim \mathcal{O}(1)$  feedback on the evolution of inflaton perturbations leads to detectable non-Gaussianity in the primordial curvature perturbation or to a “large” scalar-to-tensor ratio. See Barnaby & Peloso 2010, Barnaby et al 2011 and Sorbo 2011 for helical GW from this scenario. Bound:  $\xi \lesssim 2.5 \Rightarrow |f_N| < 5$

# Deviations from slow roll

- ▶ Numerical comparison of the slow roll result with real model
- ▶ A short interruption of slow roll inflation is allowed by the CMB data, though already strongly constrained.



$$V_\beta(\varphi) = \frac{1}{2} m^2 \varphi^2 \left[ 1 + \beta \tanh \left( \frac{\varphi - \varphi_0}{\Delta\varphi} \right) \right]$$

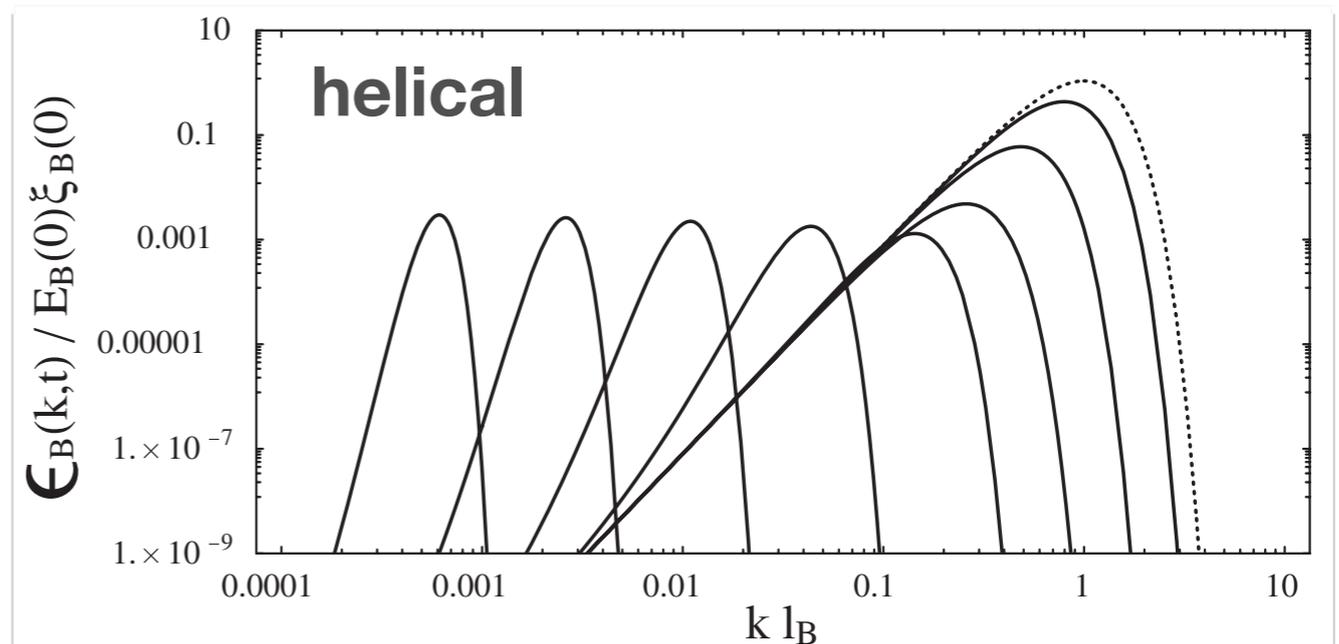
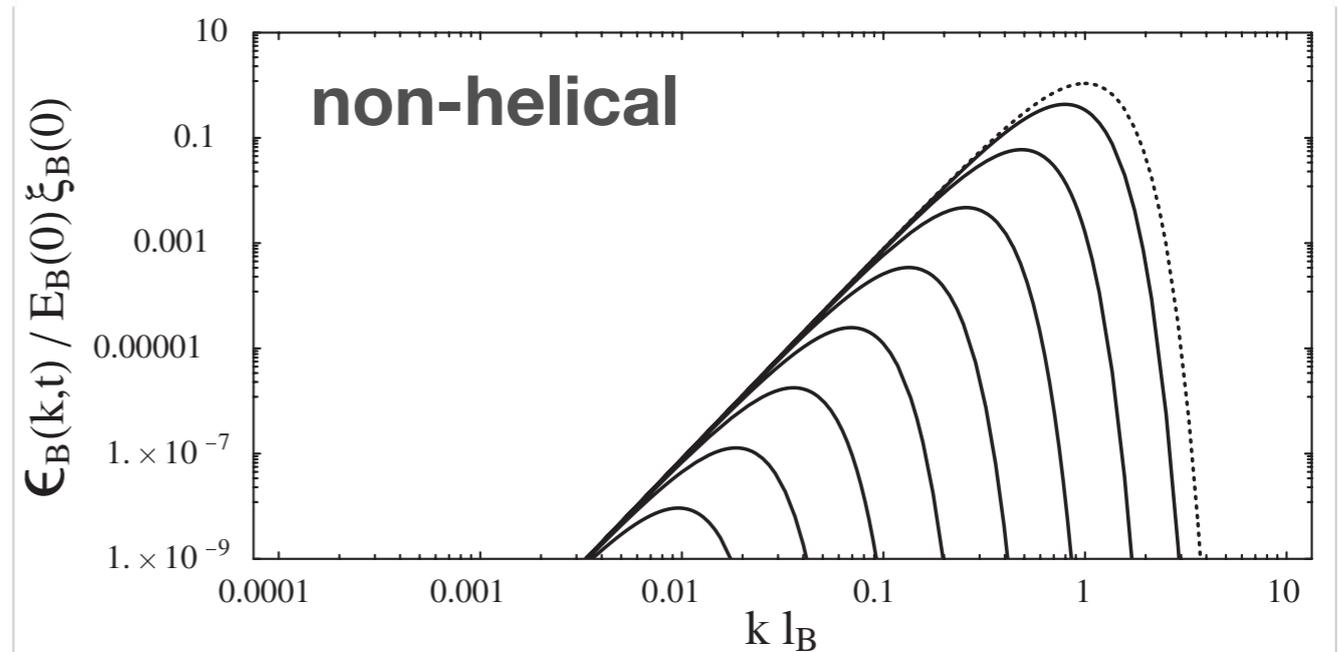
$$f(\phi) \propto \phi^2$$

Further evolution: inverse cascade

# Inverse cascade

Evolution in a dense turbulent plasma (after inflation):

- ▶ **Viscosity damping** on small scales
- ▶ **Selective decay** of non-helical magnetic field spectrum
- ▶ **Inverse cascade** of helical magnetic field spectra due to **helicity conservation**



# Reynolds number & correlation scale

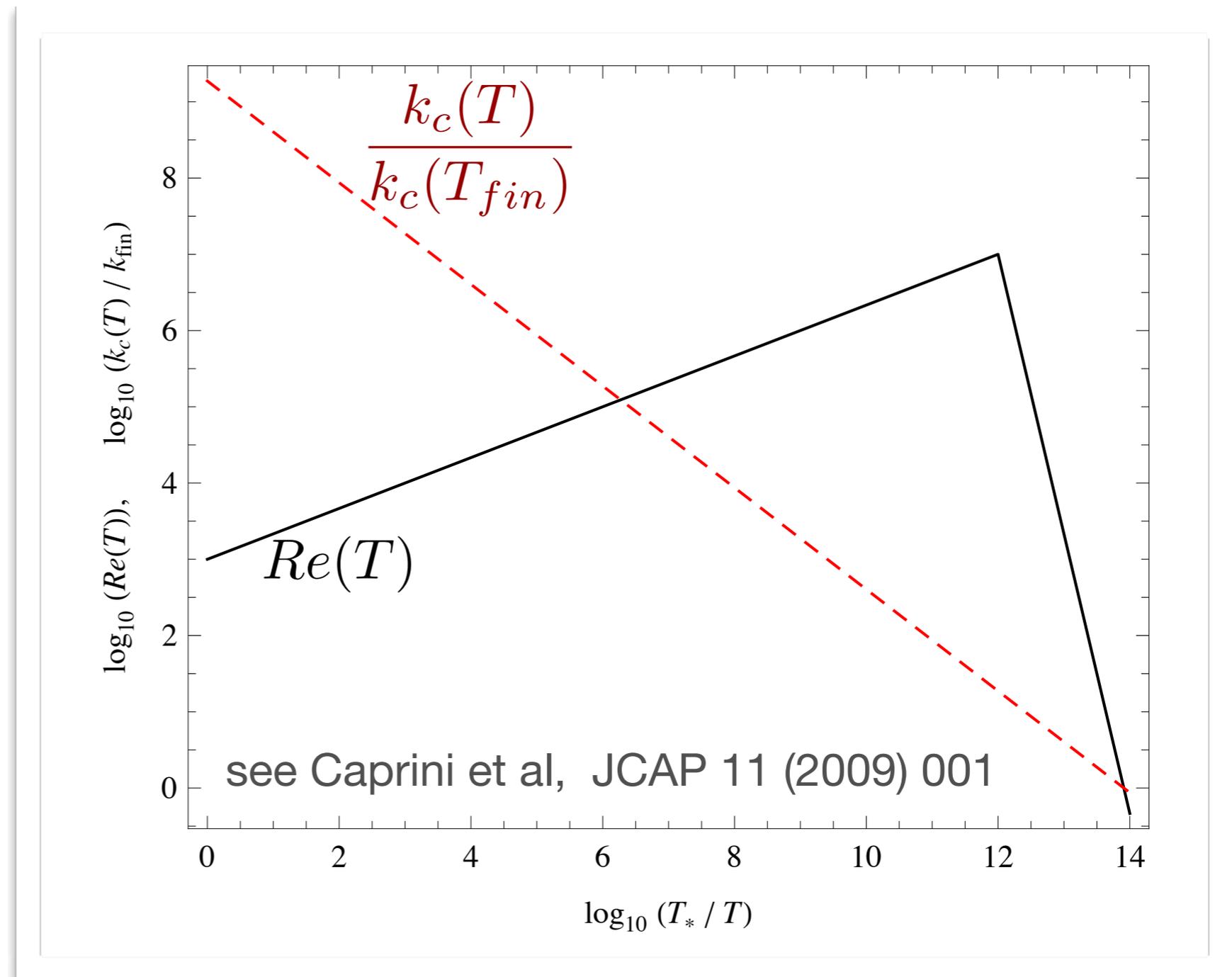
- ▶ After inflation we deal with “ideal MHD”.

- ▶ Reynolds number characterises the turbulence

$$Re = \frac{v_L L}{\nu}$$

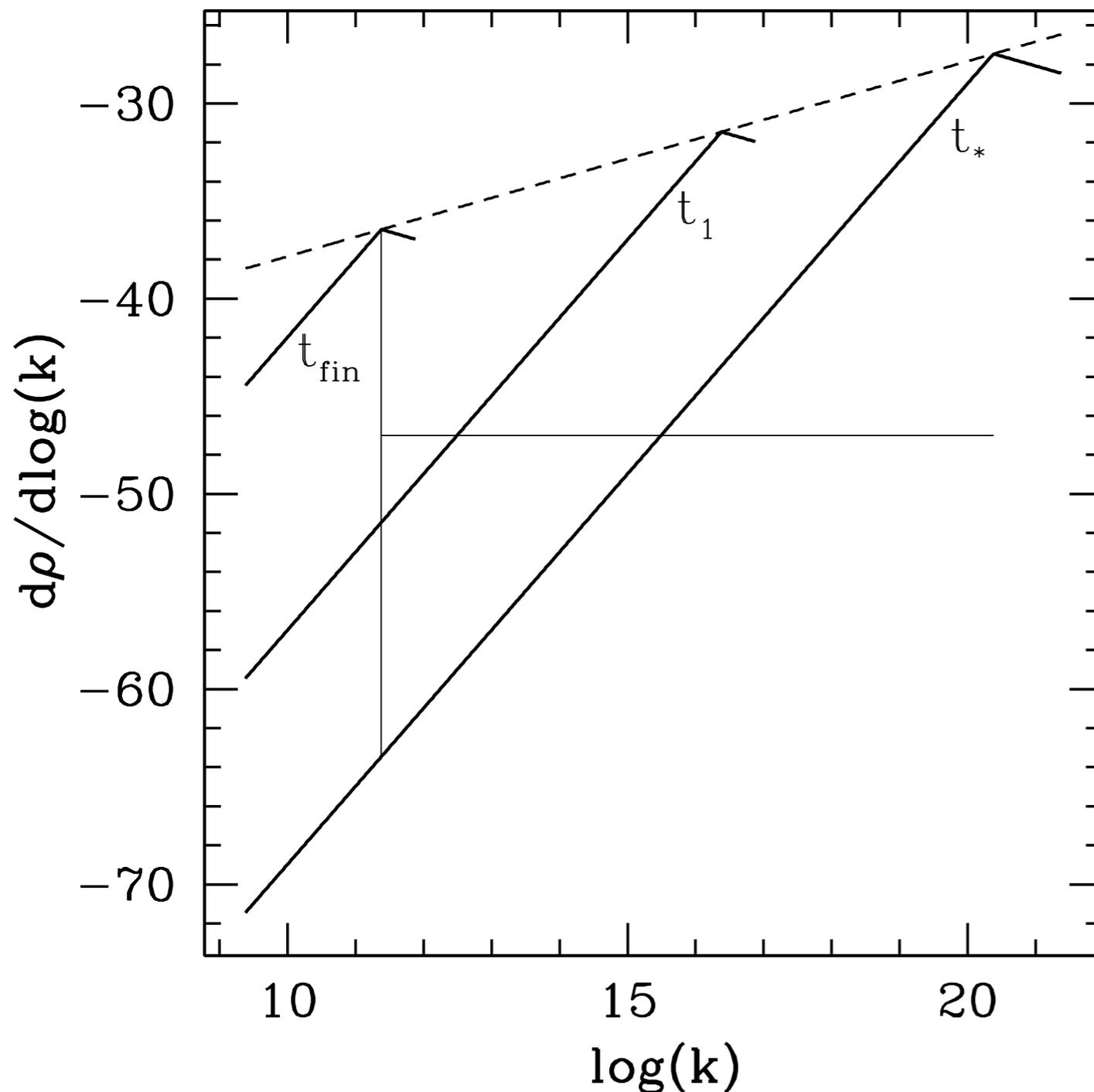
- ▶ Correlation scale evolves as

$$k_c \propto \left( \frac{T_\star}{T} \right)^{2/3}$$



as found numerically by Campanelli, PRL 98 (2007) 251302.

# Magnetic power spectrum after turbulence



- ▶ On large scales the shape of the spectrum stays unchanged.
- ▶ The amplitude increases because the correlation scale grows and the total helicity is conserved

$$\mathfrak{H} \propto \rho_B / k_c$$

# Final magnetic field strength

---

- ▶ In terms of the final correlation scale:

$$\tilde{B}(k) \simeq 3 \times 10^{-19} \text{ Gauss } \mathcal{S}(f_N) \left( \frac{k}{k_{\text{fin}}} \right)^2 \left( \frac{T_*}{10^{14} \text{ GeV}} \right)^{19/11}$$

- ▶ And at scales of 0.1 Mpc:

$$\tilde{B}(k = 10/\text{Mpc}) \simeq 3 \times 10^{-41} \text{ Gauss } \mathcal{S}(f_N) \left( \frac{T_*}{10^{14} \text{ GeV}} \right)^{9/11}$$

- ▶ With the bound from **backreaction**, we get at most

$$\tilde{B}(k = 10/\text{Mpc}) \leq 10^{-32} \text{ Gauss } \left( \frac{10^{14} \text{ GeV}}{T_*} \right)^{13/11}$$

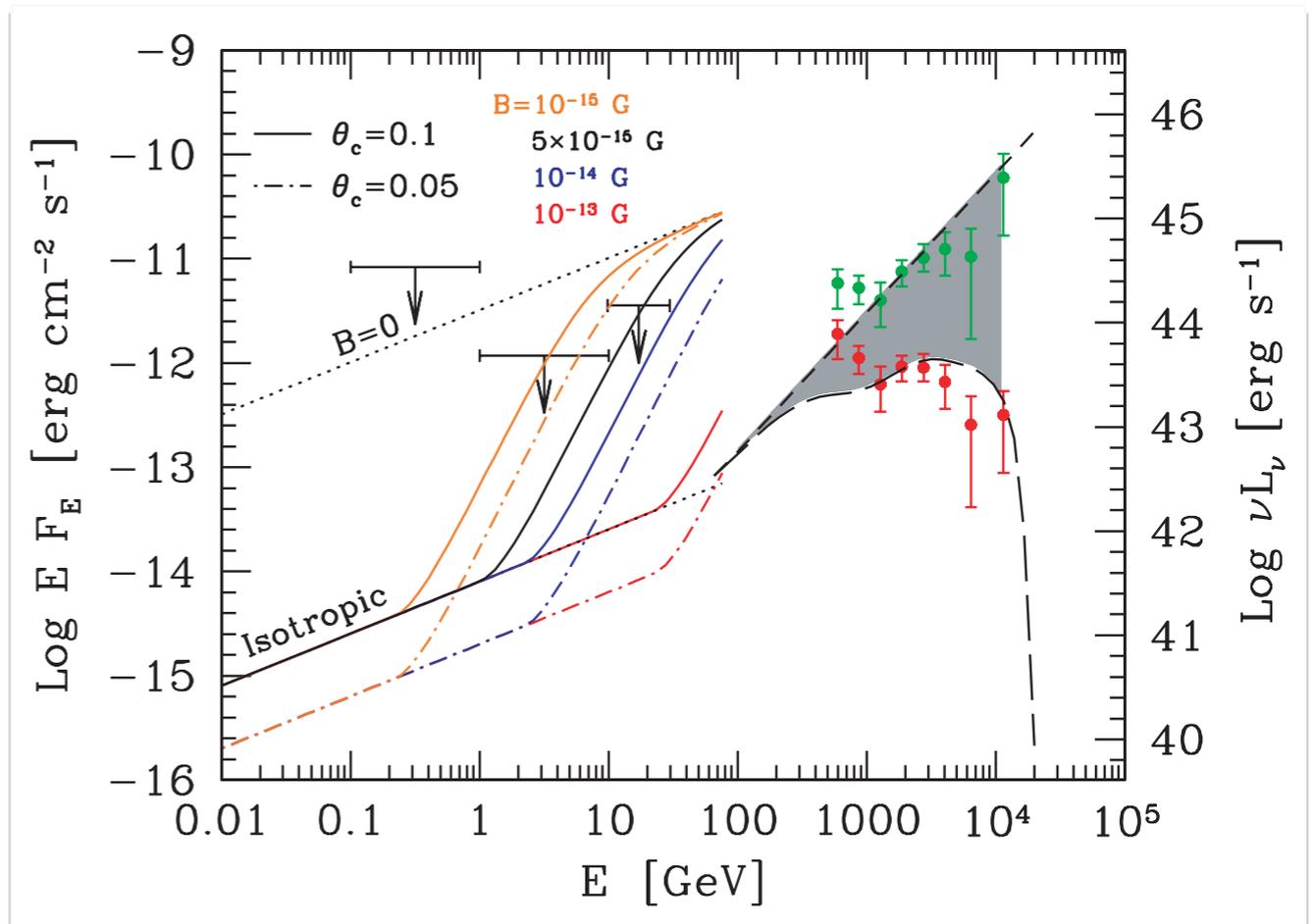
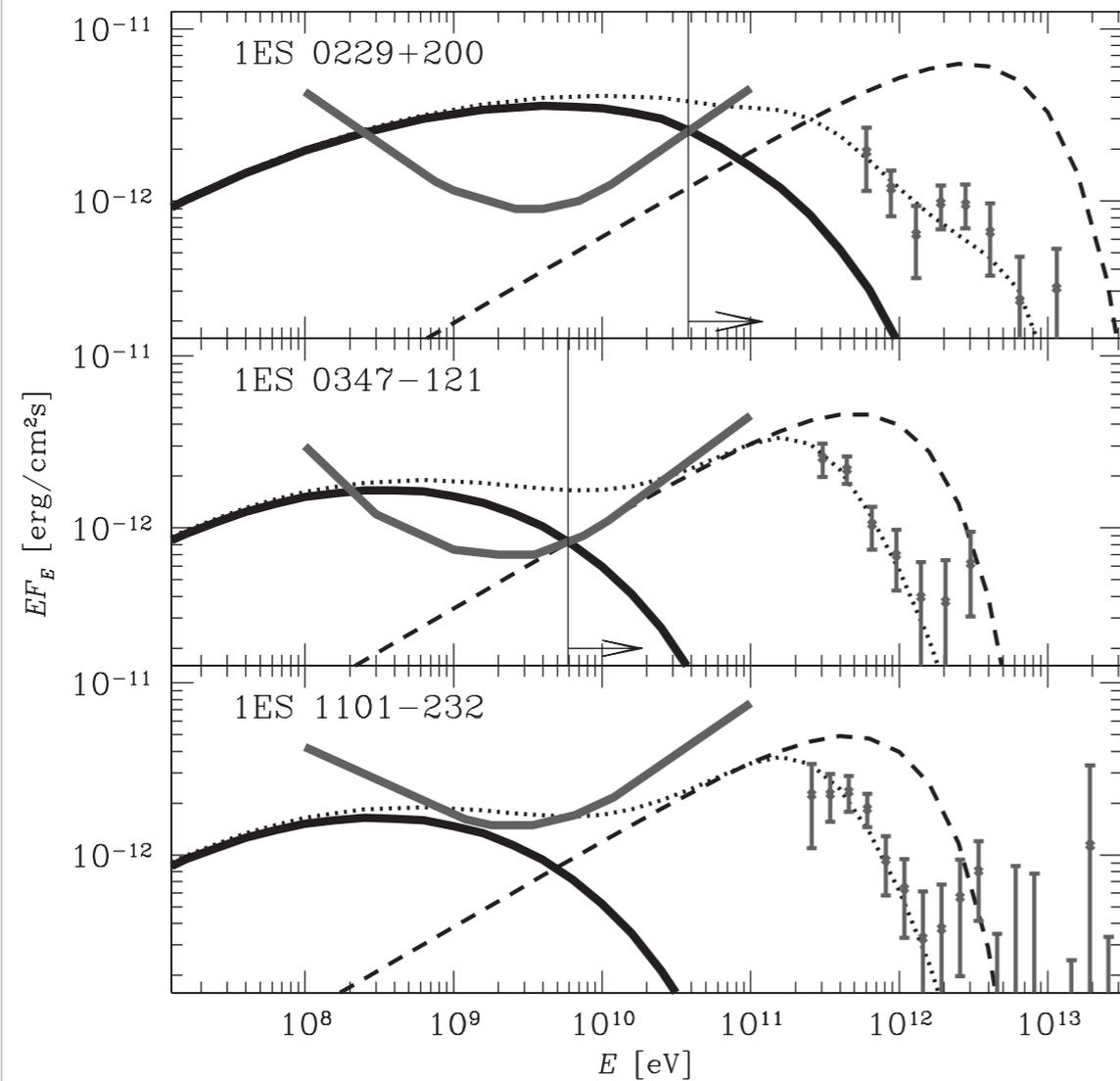
- ▶ **NOT ENOUGH** to seed subsequent astrophysical amplification.

# Conclusions

---

- ▶ Helical magnetic field generation is typical for inflation scenarios with pseudo-scalars -> Neil and Lorenzo's talks
- ▶ Typically, helical magnetic field are only produced with a **blue spectrum**  $\sim k$ . (CMB data seems to prefer white or red spectra slightly.)
- ▶ Difficult to produce large amplitude without **backreaction**, large **non-Gaussianity**, and other complications.
- ▶ Although the inverse cascade can move power from small to large scales, this is **insufficient** to account for the observed fields on large scales.

# Observations: lower limit from gamma-rays



from: **Neronov & Vovk**, Science 328 (2010) 73

from: **Tavecchio et al.**, MNRAS 406 (2010) L70