Relic Gravitational waves from light primordial black holes

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Primordial black holes formation

- Initially predicted by Zeldovich and Novikov (1966) and after by Hawking and Carr (1971).
- They are formed when the Schwartzchild radius of perturbation was of order of horizon scale.
- The mass of a PBHs formed at cosmological time, t, would be

$$\mathbf{M(t)} = \mathbf{4} \cdot \mathbf{10^{38}} \left(\frac{\mathbf{t}}{\mathrm{sec}} \right) \mathrm{g}$$

- The CMB data shows us an almost flat Harrison-Zeldovich spectrum of the primordial density perturbations $\Rightarrow$ low probability of PBHs formations, $\Omega_p \ll 1$.
- Inflation predicts flat spectrum on all scales, but there are scenarios with large deviations from flatness at small scales A. D. Dolgov, J. Silk

Intrinsic parameters

- **A black holes emit thermal particles with a black-body temperature according to the law**

$$\mathbf{T_{BH}} = \frac{\mathbf{m_{Pl}^2}}{8\pi \mathbf{M}}$$

- **Black hole evaporates on a timescale**

$$\tau_{\mathbf{BH}} = \frac{\mathbf{N_{eff}}}{32170} \frac{\mathbf{M^3}}{\mathbf{m_{Pl}^4}}$$

- **Particle production of PBHs can have impact on BBN $\Rightarrow$ their number density at production is bounded. However, PBHs that evaporated before $t \approx 10^{-2}$ s or $M < 10^8$ g are not constrained by any astrophysical observations.**
- **We assume that PBHs are produced by some mechanisms and their number density at production, Ω_p, would be a free parameter of the model.**

Proposed Model: A. D. Dolgov, D. Ejlli

- **PBHs are formed at standard radiation domination regime (RD) and their density parameter goes like**

$$\Omega_{BH}(t) = \Omega_p \left(\frac{a(t)}{a_p}\right)$$

- **If they lived long enough, they would DOMINATE the cosmological energy density and the Universe would be matter dominated, at time,** $t > t_{eq}$

$$t_{eq} = \frac{M}{m_{Pl}^2\, \Omega_p}$$

- **At (RD) stage their density parameter goes like** $\Omega_{BH} \sim t^{1/2}$**, after onset of PBHs domination** $\Omega_{BH} \sim constant$ **till PBHs evaporation.**
- **To survive till equilibration their evaporation time,** $t_{ev} > t_{eq}$ **so,**

$$M > \left(\frac{N_{eff}}{3.2\cdot 10^4}\right)^{1/2} m_{Pl} \left(\frac{1}{\Omega_p^2} - 1\right)^{1/2} \simeq 5.6\cdot 10^{-2}\,\mathrm{g} \left(\frac{N_{eff}}{100}\right)^{1/2} \frac{m_{Pl}}{\Omega_p}$$

Building the model

- **The evolution of density perturbations depends on the moment time when they cross the horizon.**
- **Let λ be some wavelength of a density perturbations for a Harrison-Zeldovich spectrum which crossed the horizon at time moment, $t_{in} > t_{eq}$.**
- **The mass inside horizon at this moment is**

$$\mathbf{M_c(t_{in}) = m_{Pl}^2 t_{in}}$$

- **For flat spectrum of perturbations, Δ, at horizon crossing is the same for all, λ.**
- **After horizon crossing, $\Delta(t) = \Delta_{in}(t/t_{in})^{2/3}$**
- **Perturbations continued rising till moment, $t_1(t_i)$**

$$\mathbf{\Delta[t_1(t_{in})] = \Delta_{in}[t_1(t_{in})/t_{in}]^{2/3} = 1} \text{ or } \mathrm{t_1(t_{in}) = t_{in}\Delta_{in}^{-3/2}}$$

Building the model

- **The radius of PBH cluster rose as** $R_c \sim a(t)$
- **After that** $\Delta(t_1) \sim 1$**, the cluster would decouple from cosmic expansion** $\Rightarrow$ **it started to shrink.**
- **At** $t = t_1$ **the size of the cluster,** R_c **drop down,** n^c_{BH} **would rise and** $\Delta_c = \rho^c_{BH}/\rho_c \gg 1$
- $\Delta = 10^5 - 10^6$**, as in contemporary galaxies.**
- **In order to survive till the rise of density perturbations,** $t_{ev} > t_1$ **which imply that**

$$\mathbf{M} > \mathbf{M_{low}} = \mathbf{1.2} \cdot \mathbf{10^3}\,\mathrm{g} \left(\frac{\mathbf{10^{-6}}}{\mathbf{\Omega_p}}\right) \left(\frac{\mathbf{10^{-4}}}{\mathbf{\Delta_{in}}}\right)^{\mathbf{3/4}} \left(\frac{\mathbf{N_{eff}}}{\mathbf{100}}\right)^{\mathbf{1/2}}$$

- **We have also a stronger restriction on,** Ω_p

$$\mathbf{\Omega_p} > \mathbf{0.7} \cdot \mathbf{10^{-11}} \left(\frac{\mathbf{10^{-4}}}{\mathbf{\Delta_{in}}}\right)^{\mathbf{3/4}} \left(\frac{\mathbf{N_{eff}}}{\mathbf{100}}\right)^{\mathbf{1/6}}$$

Building the model

- **After the size of the cluster stabilized, n_{BH}^c would be constant but Δ_c would continue to rise as $(t_1/t)^2$**
- **From cluster formation till BH evaporation the density contrast would additionally rise by the factor**

$$\mathbf{\Delta}(\tau_{\mathbf{BH}}) = \mathbf{\Delta}(\mathbf{t_1}) \left(\frac{\tau_{\mathbf{BH}}}{\mathbf{t_1}} \right)^{\mathbf{2}}$$

- **The size of high density clusters would be**

$$\mathbf{R_c} = \mathbf{\Delta_b^{-1/3} t_1^{2/3} t_{in}^{1/3}}$$

- **The average distance between PBHs in the cluster is estimated:**

$$< \mathbf{d_c} >= (\mathbf{M}/\mathbf{M_c})^{\mathbf{1/3}} \mathbf{R_c} = \mathbf{\Delta_c^{-1/3} t_1^{2/3} r_g^{1/3}}$$

- **The virial velocity inside the cluster would be**

$$\mathbf{v} = \sqrt{\frac{\mathbf{2M_c}}{\mathbf{m_{Pl}^2 R_c}}} = \mathbf{2^{1/2} \Delta_c^{1/6} \Delta_{in}^{1/2}} \approx \mathbf{0.14} \left(\frac{\mathbf{\Delta_c}}{\mathbf{10^6}} \right)^{\mathbf{1/6}} \left(\frac{\mathbf{\Delta_{in}}}{\mathbf{10^{-4}}} \right)^{\mathbf{1/2}}$$

Building the model

Gravitational waves (GWs) production

- From their production time, t_p, till PBHs evaporation, τ_{BH} are produced gravitational by different mechanisms
- **Scattering: Quantum and Classical**
- **Binary formation inside the clusters**
- **Evaporation of gravitons**

- At cosmological time, $t = \tau_{BH}$ the Universe return to RD stage and the previous RD regime would be lost
- Moreover, the stochastic background of GWs coming from **INFLATION would be noticeably diluted by a factor** $(t_{eq}/\tau_{BH})^{2/3}$.

Binary formation in high density clusters

- **PBHs loose their energy due to scattering and dynamical friction**
- **Binary system of PBH are formed with high probability in high density clusters**
- **The fraction of PBHs that went into binary systems is ϵ**
- **We assume that all binaries are in circular orbits $\Rightarrow$ as a result we obtain a lower bound on the energy density of GWs**
- **The emitted GWs energy per unit time would be**

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{32 M_1^2 M_2^2 (M_1 + M_2)}{5 R^5 m_{Pl}^8} = \frac{32}{5} m_{Pl}^2 \left(\frac{M_c\, \omega_{orb}}{m_{Pl}^2} \right)^{10/3}$$

- **Two different regimes could had realized: stationary regime and in-spiral regime**

Stationary approximation

- **This situation is realized when,** $\tau_{co} \gg \tau_{BH}$
- **The system has no time to evolve in the in-spiral phase, because PBHs evaporate and disappear.**
- **The energy spectrum of GWs is**

$$\frac{\mathrm{d}E}{\mathrm{d}\ln\omega} = \frac{2^{1/3}\omega^{2/3}}{3}\frac{M_1 M_2}{m_{Pl}^{4/3}(M_1+M_2)^{1/3}}$$

- **In order to calculate the density parameter of GWs today,** $h_0^2\Omega_{GW}$**, we need to take redshift into account**
- **The redshift is different for different frequencies** $\Rightarrow$ **spectrum distortion**
- **The energy density of GWs emitted at,** $t = \tau_{BH}$

$$\frac{\mathrm{d}\rho_*}{\mathrm{d}\ln\omega_*} = \frac{2^{10/3} n_{BH}^c(\tau_{BH})}{15 n_{BH}^b}\frac{M_1^2 M_2^2 F(R)}{(M_1+M_2)^{1/3} m_{Pl}^{16/3}}\omega_*^{8/3}\int_{t_0}^{t_p+\tau_{BH}} \mathrm{d}t\left(\frac{t}{t_p+\tau_{BH}}\right)^{8/9}$$

In-spiral regime

- **This regime is realized when ,$\tau_{BH} > t_{co}$**
- **The system goes into in-spiral phase and coalesce producing a burst of GWs**
- **An important quantity for GWs detectors is**

$$h_0^2\Omega_{GW}(f;t_0) \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{GW}}{\mathrm{d}\ln f}$$

- **The density parameter of GWs from binary system today is**

$$h_0^2\Omega_{GW}(f) \approx 5.84{\cdot}10^{-9}\epsilon_{co}\left[\frac{100}{g_S(T_{BH})}\right]^{5/18}\left[\frac{N_{eff}}{100}\right]^{1/3}\left[\frac{f}{10^{12}\mathbf{Hz}}\right]^{2/3}\left[\frac{10^5\,\mathbf{g}}{M}\right]^{1/3}$$

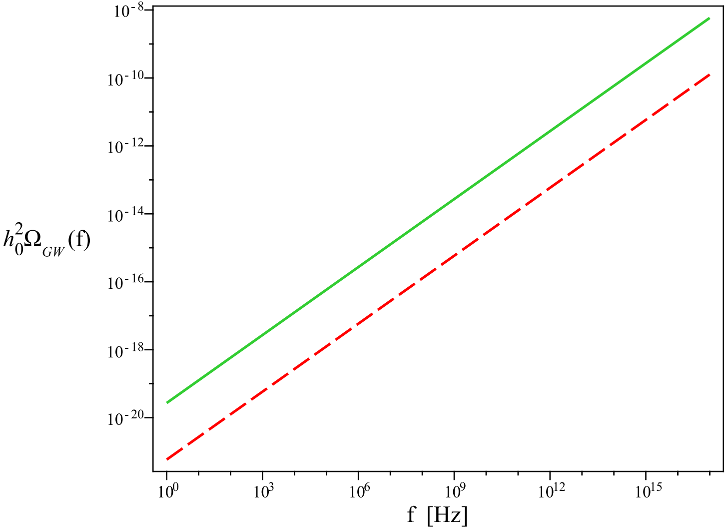


Figure: $h_0^2\Omega_{GW}$ as a function of expected frequency today for PBHs binaries for $\epsilon \sim 10^{-5}$, PBH mass $M \sim 1$ g (solid line) and $M \sim 10^5$ g (dashed line).

PBHs scattering

- **PBHs from t_{eq} till PBHs evaporation, scattered with each other emitting burst of GWs**
- **PBHs with masses $M < 10^{10}$ g have gravitational radii, $r_g < 10^{-18}$ cm $\Rightarrow$ BHs can be considered as point-like quantum particles**
- **The energy density of GWs per unit time is**

$$\frac{\mathrm{d}\rho_{GW}}{\mathrm{d}t} = < \mathrm{d}\sigma v_{rel} > \omega n_{BH}^2$$

- **In the case of quantum scattering the cross-section was calculated by B. M. Barker, S. N. Gupta, J. Kaskas, Phys. Rev. 182 (1969) 1391-1396**

$$\mathrm{d}\sigma = \frac{64 M^2 m^2}{15 m_{pl}^6} \frac{\mathrm{d}\xi}{\xi} \left[5\sqrt{1-\xi} + \frac{3}{2}(2-\xi) \ln \frac{1+\sqrt{1-\xi}}{1-\sqrt{1-\xi}} \right]$$

PBHs scattering

- **For classical bremsstrahlung the energy of GW emitted for a single collision (Peters, 1970)**

$$\delta E_{GW}(\omega) \approx \frac{M^4}{m_{Pl}^6}\,\omega^3$$

- **The cross section in the non-relativistic regime**

$$\mathrm{d}\sigma = \frac{M^2}{m_{Pl}^2}\frac{\mathrm{d}q^2}{q^4} = \frac{2M^2}{m_{Pl}^2} b\mathrm{d}b$$

- **We are interested in the lower part of the spectrum**

$$b > b_{min} = \sqrt{\frac{37\pi}{15}}\frac{M^2}{m_{Pl}^3} = \sqrt{\frac{37\pi}{15}}\left(\frac{M}{m_{Pl}}\right) r_g$$

- **The density parameter at present for the case of classical scattering in the non-relativistic regime**

$$h_0^2\Omega_{GW}(f;t_0) \approx 7.75\cdot 10^{-13}\alpha' \left(\frac{f}{\mathrm{GHz}}\right)\left(\frac{10^5\,\mathrm{g}}{M}\right)^{1/2}$$

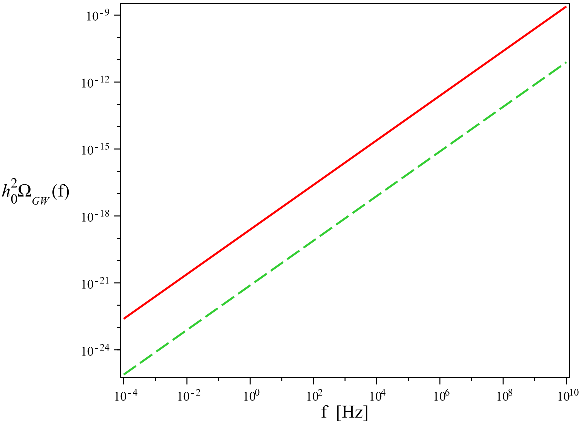


Figure: The density parameter today $h_0^2\Omega_{GW}$ as a function of expected frequency today in classical approximation for $N_{eff} \sim 100$, $g_S(T_{BH}) \sim 100$, $\Delta \sim 10^5$, and $v_{rel} \sim 0.1$ for different values of PBH mass $M \sim 1$ g (solid line) and $M \sim 10^5$ g (dashed line).

GWs from PBHs evaporation

- **A black hole emits all kind of particles with masses $m < T_{BH}$ and, in particular gravitons.**
- **The graviton emission is independent on the structure formation that took place during PBHs domination.**
- **The total energy per unit time and frequency.**

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t\mathrm{d}\omega}\right) = \frac{2N_{eff}}{\pi}\frac{M^2}{m_{Pl}^4}\frac{\omega^3}{e^{\omega/T_{BH}} - 1}$$

- **The spectrum is not thermal but rather similar to it.**
- **At the PBHs evaporation time the density parameter is.**

$$\Omega_{GW}^{peak}(\omega_*^{peak}; \tau_{BH}) \approx 3.8 \cdot 10^{-3}$$

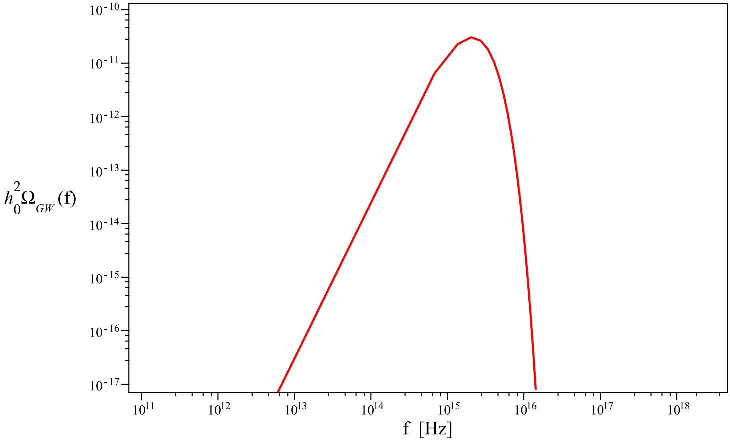
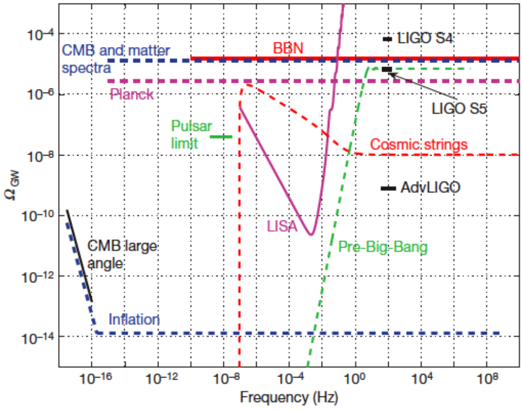


Figure: The density parameter per logarithmic frequency $h_0^2\Omega_{GW}(f;t_0)$ as a function of frequency today f for the case of $T_0 = 3.36 \cdot 10^{15}$ Hz, $g_S(T_{BH}) \sim 100$, $N_{eff} \sim 100$ and black hole mass $M = 10^5$ g.

Current and Planned GWs detectors

- **Ultimate DECIGO will reach a strain sensitivity $h_{rms} \sim 10^{-27}$ Hz$^{-1/2}$ and $h_0^2\Omega_{GW} \sim 10^{-20}$ at $f \sim 0.1 - 10$ Hz after 10 years of data correlations N. Seto, S. Kawamura, T. Nakamura, Phys. Rev. Lett. 87 (2001) 221103.**



Conclusions

- **PBHs could had dominated the Universe for a very short time.**
- **Structure formation took place during PBHs domination (Clusters of PBHs)**
- **The previous RD regime would be lost and the stochastic background of GWs from inflation would be noticeably diluted.**
- **Interaction between PBHs produced a substantial amount of GWs by various mechanisms.**
- **The intensity of GWs produced would be maximal in the GHz or higher frequency band of the spectrum.**
- **However, the lower frequency part of the spectrum in the range $f \sim 10^{-4} - 10^{-2}$ may be detectable by DECIGO.**