Cosmological Magnetic Fields

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Outline

Introduction: Cosmology is a search for relics

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- or the electroweak transition at $t \simeq 10^{-10}$ sec which may have led to the observed baryon asymmetry in the Universe.
- It has been proposed that confinement and, especially the electroweak phase transition but also inflation might generate primordial magnetic fields which represent seeds for the magnetic fields observed in galaxies and clusters.

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- Estimates by equi-partition (e.g. of magnetic field and thermal or turbulent energy).

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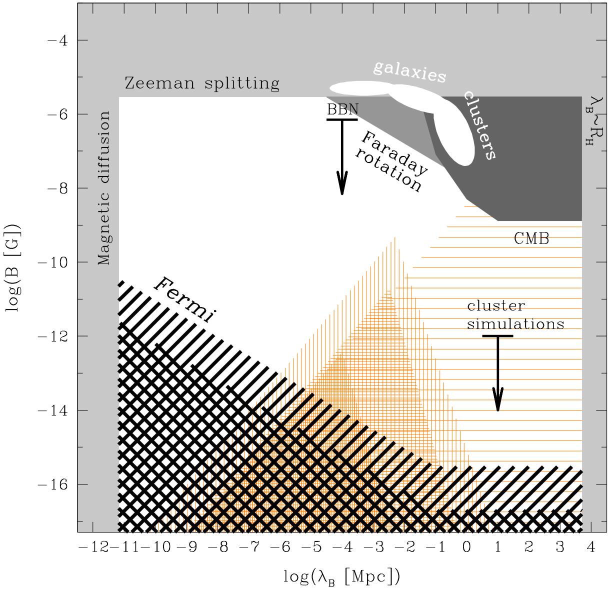
- Intergalactic space, voids: The fact that certain blazars do emit TeV γ-radiation but not GeV, means that lower energy electrons which are produced by scattering with intergalactic background light and which then generate a cascade of GeV photons by inverse Compton scattering must be deflected out of the beam. This requires intergalactic fields of $B \gtrsim 10^{-16}$Gauss with coherence scales of 1Mpc (Neronov & Vovk, 2010), or, more robust limits of $B \gtrsim 10^{-19}$Gauss (Takahashi et al. 2011).

Observations



from
Neronov & Vovk, 2006

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In this talk I concentrate of the possibilities to generate primordial magnetic fields and on their limitations.

A constant magnetic field

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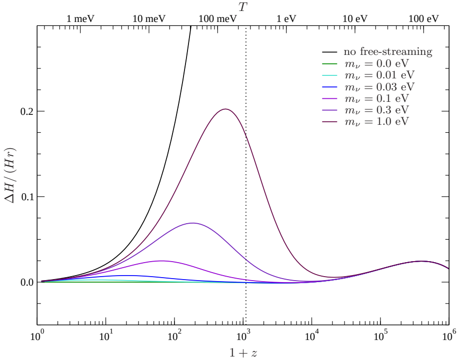

From Adamek, RD, Fenu & Vonlanthen, '11

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A constant magnetic field still interacts with the electrons in the cosmic plasma and leads to Faraday rotation of the CMB photons.
Since a constant magnetic field breaks parity, its Faraday rotation leads to parity odd correlations between B-polarization and temperature anisotropies (and E- and B-polarization) in the CMB (Scannapieco & Ferreira, '97). This leads to limits of the order of $B < 10^{-8}$Gauss.

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CMB limits on magnetic fields are all of this order. This is not surprising since

$$\Omega_B = 10^{-5}\Omega_\gamma \left(\frac{B}{10^{-8}\mathrm{Gauss}}\right)^2$$

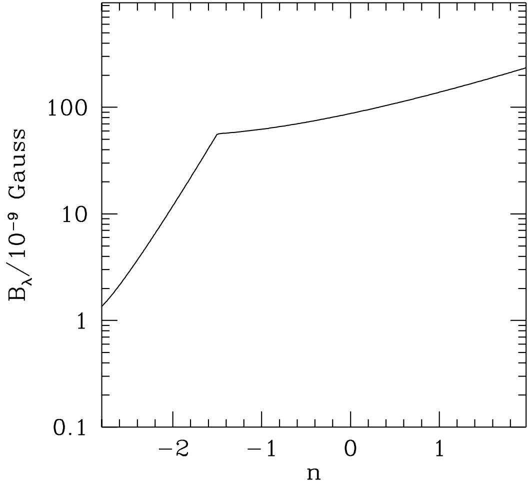
Magnetic fields of the order 3×10^{-9}Gauss (on CMB scales) will leave 10% effects on the CMB anisotropies while 10^{-9}Gauss will leave 1% effects. It is thus clear that we can never detect magnetic fields of the order of 10^{-16} or even 10^{-20}Gauss (on galactic scales) in the CMB.

Other effects of Magnetic Fields on the CMB

Magnetic fields effect the CMB via

- their energy-momentum tensor which leads to metric perturbations $\Rightarrow$ perturbed photon geodesics
- magnetosonic waves affect the acoustic peaks in the CMB spectrum
- Alvèn waves (vector perturbations)
- Faraday rotation can turn E-mode polarization into B-modes

All these lead to magnetic field limits on the order of 10^{-9}Gauss on CMB scales. Depending on the spectral index this leads to different limits on galactic scales $\lambda \sim 0.1$Mpc.



(from: RD, Ferreira & Kahniashvili '98)

Generation of primordial magnetic fields

There are three main ideas how magnetic fields may have formed in the early Universe:

- Second order perturbations: To generate magnetic fields in the cosmic fluid one needs vorticity and a charge and current density. The first can be obtained only in second order perturbation theory (first order vector perturbations decay) and the second only in second order in the tight coupling limit. Estimates have shown that typical fields on cluster scales are of the order of 10^{-28}Gauss (Matarrese et al. '04, Ichiki et al. '07, Fenu et al. '10). This is far too small to be consistent with the Neronov-Vovk bound or with the minimal amplitude needed for dynamo amplification.

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- Inflation: The electromagnetic field is conformally coupled to gravity and is therefore usually not generated during inflation. However, if conformal symmetry is explicitly broken or if the electromagnetic field is coupled to the inflation, it can also be generated during inflation (Turner & Widrow '88, Ratra '92, Anber & Sorbo '06, RD, Hollenstain & Jain '10).

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In the remainder of this talk I restrict to the 2nd and 3rd possibility.

The spectrum

We assume that the process leading to a magnetic field is statistically homogeneous and isotropic. A magnetic field spectrum generated by such a process is of the form

$$\langle B_i(\mathbf{k}) B_j^*(\eta, \mathbf{q})\rangle = \frac{(2\pi)^3}{2}\delta(\mathbf{k}-\mathbf{q})\Big\{(\delta_{ij} - \hat{k}_i\hat{k}_j)P_S(k) - i\epsilon_{ijn}\hat{k}_n P_A(k)\Big\}$$

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$P_A \propto |B_+|^2 - |B_-|^2$ determines the helicity of the magnetic field. Its integral is the helicity density while the integral of $P_S \propto |B_+|^2 + |B_-|^2$ determines the energy density in magnetic fields.

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On small scales the magnetic field is damped by the viscosity of the cosmic plasma, $P_S = P_A = 0$ for $k > k_d(t)$. Here $k_d(t)$ is a time-dependent damping scale.

Magnetic field evolution

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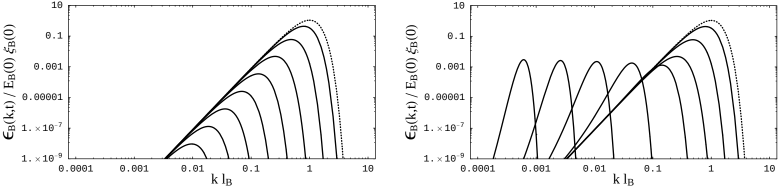
However, if the field is helical, helicity conservation leads to an inverse cascade which moves the correlation scale to larger and larger scales (numerical simulations by Jedamzik et al. 2000-2005, Campanelli, 2007)



Campanelli, 2007

Causality

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For the energy density per log-k-interval this implies

$$\frac{d\rho_B}{d\log(k)} \propto k^5$$

Limits on magnetic fields from phase transitions

As we now show this already implies very stringent limits on magnetic fields from phase transitions. Be $\epsilon = \Omega_B^*/\Omega_r^*$ the ratio of the magnetic field to the radiation energy density at the moment of formation and k_* the cutoff scale. Since radiation and magnetic fields scale the same way, at later times and scales larger than the cutoff, the magnetic field to radiation density is given by

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For the electroweak phase transition with $k_* > \mathcal{H}_* \simeq 10^{-3}$Hz and $k_1 = 1/(0.1\text{Mpc}) \simeq 10^{-13}Hz$ this yields the following limit for the field at scale k_1:

$$\left(\frac{B(k_1)}{10^{-6}\text{Gauss}}\right)^2 \simeq \Omega_r^{-1}\frac{d\Omega_B}{d\log(k_1)} < \epsilon \times 10^{-50}$$

Requiring $\epsilon < 1$ this implies $B(k_1) < 10^{-31}$Gauss. Using slightly more model dependent but also more realistic numbers (e.g. $k_* \simeq 100\mathcal{H}_*$ one arrives at $B(k_1) < 10^{-36}$Gauss.

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The limits from the QCD phase transition are somewhat less stringent but still discouraging, $B(k_1) < 10^{-30}$Gauss.

Limits on magnetic fields from phase transitions

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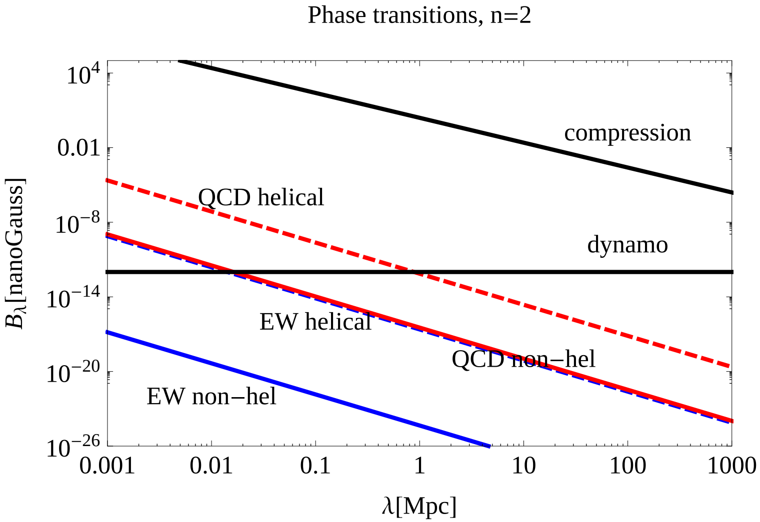
$B(k_1) < 10^{-21}$Gauss for the QCD transition. This result could be marginally sufficient dynamo amplification, but is still several orders of magnitude below the Neronov-Vovk-bound.

Limits on magnetic fields from phase transitions



(from: Caprini 2011)

Magnetic fields from inflation

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This is a wave equation with a time-dependent mass term. We know how to calculate the generation of its modes out of the vacuum. This case has been discussed for the first time in (Ratra '92).

Magnetic fields from inflation: example

For example if $f \propto a^\gamma$ is a simple power law, we can compute the resulting magnetic fields spectrum to

$$P_S \propto k^n \quad \text{with} \quad n = \begin{cases} 1 - 2\gamma & \text{if } \gamma \geq -1/2 \\ 3 + 2\gamma & \text{if } \gamma \leq -1/2 \end{cases}$$

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Since there is no infrared cutoff, the spectral index should not be less than -3 otherwise $\frac{d\rho_B}{d\log(k)} \propto k^3 P_S \propto k^{3+n}$ or $\frac{d\rho_E}{d\log(k)} \propto k^3 P_E \propto k^{3+m}$ diverges. This limits

$$-2 \lesssim \gamma \lesssim 2.$$

Problems

On the other hand, if the spectrum is too blue, the fact that magnetic fields should not dominate the energy density of the Universe leads to very stringent constraints on small scales. Since the Hubble scale at the end of inflation is so small, the spectrum needs not be very blue for this to happen.

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Hence during inflation the electron charge was much larger than 1. In this regime we cannot trust perturbation theory and our calculation does actually not apply... (Demozzi et al. '09).

Note that choosing $f_i = 1$ does not help since then $f_0 \gg 1$ and the presently measured electron charge is $e_N = e/\sqrt{f_0}$, again we need $e/\sqrt{f_i} \gg 1$.

These problems motivated us to see what can be gained from a helical coupling of the inflaton to the magnetic field. The idea was that this does not mix with the electron charge and because of the inverse cascade invoked by helicity conservation in the evolution of the field after inflation, a somewhat bluer spectral index might be admissible. (See talk by Lukas Hollenstein this afternoon)

Generation of helical magnetic fields during inflation

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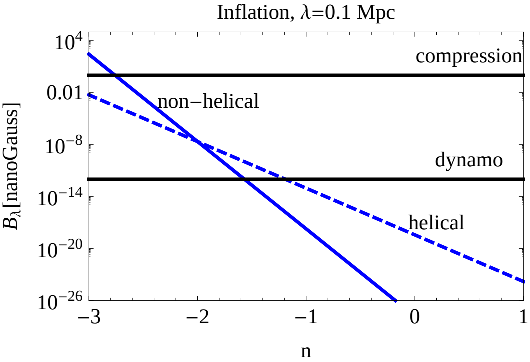

From Caprini, 2011

Conclusions

- Magnetic fields are observed on all cosmological scales (galaxies, clusters, filaments and probably even voids) with significant amplitudes. Intergalactic fields with coherence length of about 1Mpc and amplitudes of 10^{-20}Gauss (for dynamo amplification) or even 10^{-16}Gauss (Neronov-Vovk-bound) are required.

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- Fields from phase transition are too blue, they do not have enough power on large scales.
- Inverse cascade of helical magnetic fields can mitigate this problem but seems not quite sufficient to solve it.

- Magnetic fields from inflation can have many different spectra. They can actually be scale invariant leading to sufficient fields on large scales, but in this case, the effective electron charge during inflation must have been much larger than today and perturbation theory cannot be trusted.

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- Is there a loop-hole in the argumentation and the the observed fields are nevertheless a window to the early Universe...