The COINCIDENCE PROBLEM in a PHANTOM CYCLIC MODEL of the UNIVERSE

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Outline

- Composition of the universe and governing principles
- Phantom models
- Coincidence problem
- Phantom model with future singularity
- Phantom cyclic model
- Toy model
- Coincidental fraction
- Conclusion

- Equation of State

$$p_\omega = \omega \rho_\omega$$

- Energy density

$$\rho_\omega = \rho_{\omega 0} \left(\frac{R}{R_0} \right)^{-3(1+\omega)}$$

- Composition of universe:
  - 30% non-relativistic matter
  - 70% dark energy
- Equation of state for
  - ρ_M is $\omega = 0$
  - ρ_{DE} evolves differently according to different models

In phantom models, for ρ_{DE}, $\omega < -1$

- Therefore, at epochs much earlier than the present, $\rho_M \gg \rho_{DE}$
- while at epochs much later than the present, $\rho_{DE} \gg \rho_M$

$$r = \frac{\rho_{DE}}{\rho_M} = \frac{\rho_{DE0}}{\rho_{M0}} \left(\frac{R}{R_0} \right)^{-3\omega}$$

where $r \approx 2$ according to current observations.

Period of Coincidence

$$\frac{1}{r_0} < r < r_0 ,$$

where r_0 is accepted to be 10.

Phantom Dark Energy Model with future singularity

- Finite lifetime
- Possible to calculate coincidental fraction *f*

- For $\frac{1}{10} < r < 10$,
  - $f < \frac{1}{3}$ for $\omega = -1.5$
  - $f \approx \frac{1}{5}$ for $\omega = -1.2$
  - $f \approx \frac{1}{8}$ for $\omega = -1.1$ (Scherrer)
- Coincidence problem significantly ameliorated

Ilie et al.'s Phantom Cyclic Model

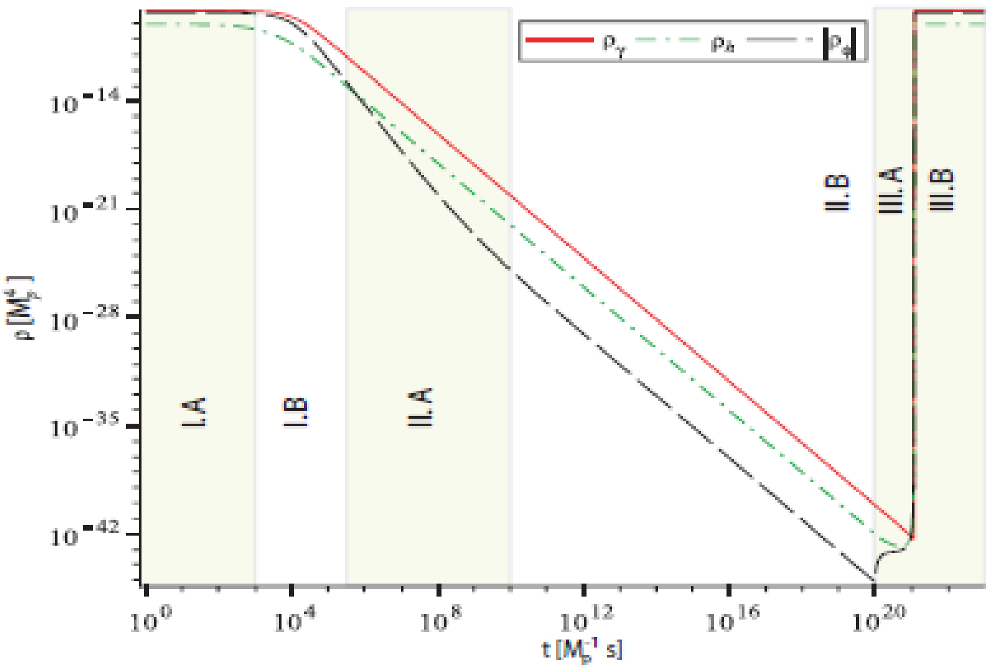


FIG. 1: Numerical solutions for the energy densities as we complete a cycle from a deStiter phase back to it. Here we have chosen: $\omega = \frac{1}{3}$, $\mu_p = 5$, $\mu_r = .1$, $m = 10^{-2} M_p$ and we have set M_p to one. Note the six distinct phases: I.A and I.B corresponding to reheating; II.A and II.B corresponding to a radiation dominated universe; III.A and III.B corresponding to the phantom and dS phase respectively. In order to make all phases clearly distinct we chose the transition set the minimum energy density at around 10^{-45} instead of the realistic meV^4

Toy model

- ρ_{DE} tracks ρ_M
- ρ_{DE} decreases to m^4
- ρ_{DE} behaves like phantom fluid, enters inflationary phase
- ρ_{DE} increases to M^4

Friedmann equation in terms of ρ_M and ρ_{DE} components:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8}{3}\pi G\left[\rho_{M_0}\left(\frac{R}{R_0}\right)^{-3} + \rho_{DE_0}\left(\frac{R}{R_0}\right)^{-3(1+\omega)}\right]$$

Assume ω is constant.

- The time the universe takes to complete one cycle:

$$t_{M^4} = \int_0^{R=R_0\left(\frac{\rho_{M_0} M^4}{\rho_{DE_0}\rho_M}\right)^{-\frac{1}{3\omega}}} R^{-1} \left\{\frac{8}{3}\pi G\left[\rho_{M_0}\left(\frac{R}{R_0}\right)^{-3} + \rho_{DE_0}\left(\frac{R}{R_0}\right)^{-3(1+\omega)}\right]\right\}^{-\frac{1}{2}} dR$$

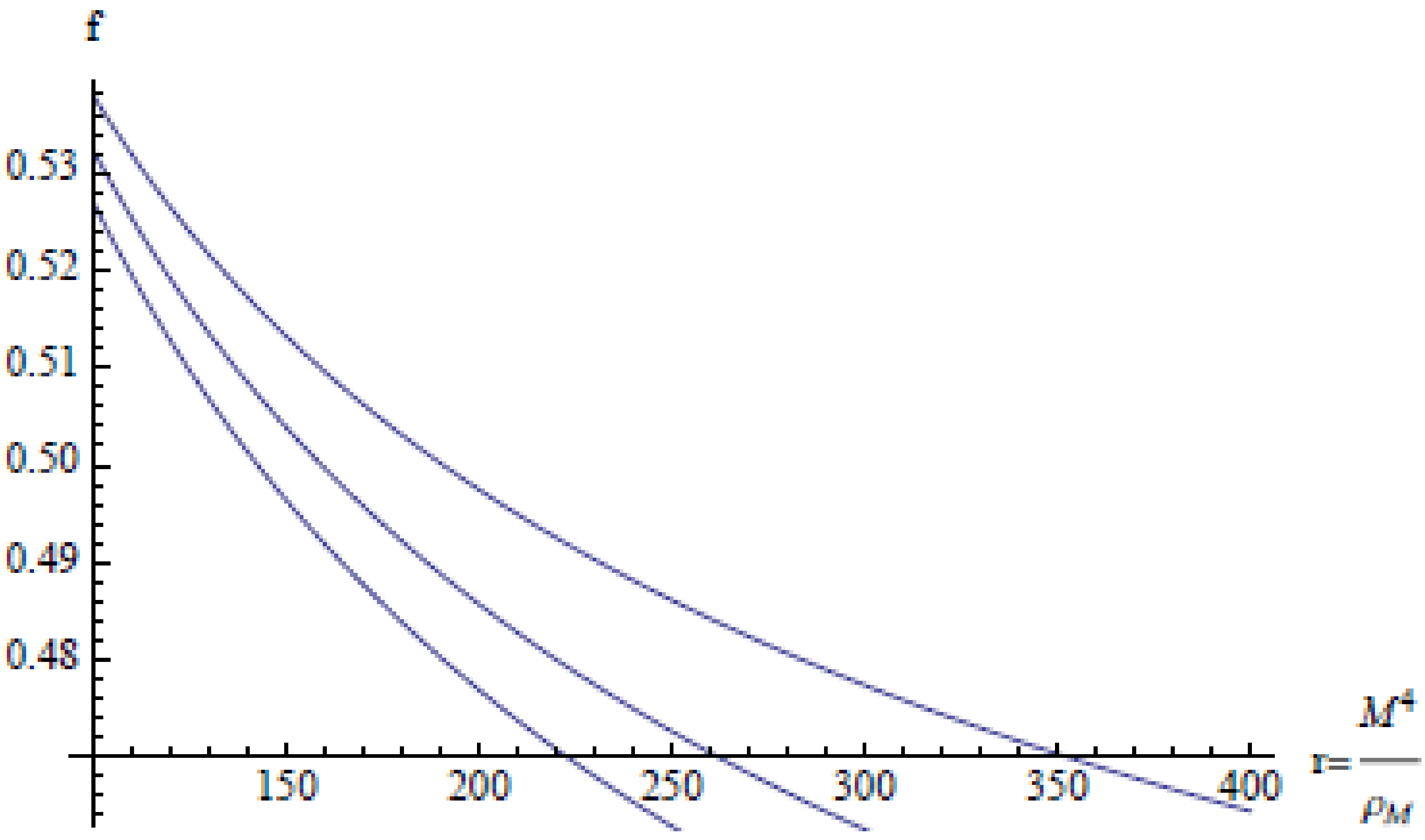
- The time the universe takes to expand from R_1 to R_2:

$$t_{12} = \int_{R_1}^{R_2} R^{-1} \left\{\frac{8}{3}\pi G\left[\rho_{M_0}\left(\frac{R}{R_0}\right)^{-3} + \rho_{DE_0}\left(\frac{R}{R_0}\right)^{-3(1+\omega)}\right]\right\}^{-\frac{1}{2}} dR$$

In each cycle, the fraction of time that the universe spends in a coincidental state:

$$f = \frac{t_{12}}{t_{M^4}} = \frac{\int_{r_1}^{r_2} \frac{r^{-\frac{2\omega+1}{2\omega}}}{\sqrt{1+r}} dr}{\int_0^{r=\frac{M^4}{\rho_M}} \frac{r^{-\frac{2\omega+1}{2\omega}}}{\sqrt{1+r}} dr},$$

which is also that of entire universe’s lifetime.



f
0.53
0.52
0.51
0.50
0.49
0.48
150
200
250
300
350
400
$r = \frac{M^4}{\rho_M}$

This ameliorates the coincidence problem.

Reference:

Preprint: H.-Y. Chang and R.J. Scherrer, *The Coincidence Problem in a Phantom Cyclic Model of the Universe.*

R.J. Scherrer, *Phantom Dark Energy, Cosmic Doomsday, and the Coincidence Problem*, Phys. Rev. D **71**, 063519 (2005).

C. Ilie, T. Biswas and K. Freese, *Are we seeing the beginnings of inflation?*, Phys. Rev. D **80**, 103521 (2009).