

# Electroweak radiative corrections to Dark Matter relic density - effective couplings approach

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# Outline

- 1 Some basics
- 2 Concept of effective couplings
- 3 Application to DM relic density
- 4 Results
- 5 Comparisons to full one loop
- 6 Conclusions

# DARK MATTER

THE ULTIMATE POST-WORKOUT  
MUSCLE GROWTH ACCELERATOR!



# Neutralino LSP as DM

## DM - cosmology

- Observational evidence for Dark matter exists!
- Major aim - Accurate relic density determinations
- Precision cosmology
- Percent accuracy with Plank

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- CDM scenario  
 $\Rightarrow \Omega h^2 \propto \frac{1}{\sigma}$
- Loop corrections will change calculated  $\Omega h^2$
- Electroweak contributions should be taken into account

# Neutralino LSP as DM

## DM - cosmology

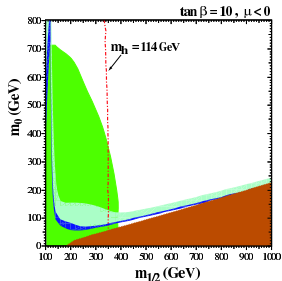
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## DM - collider

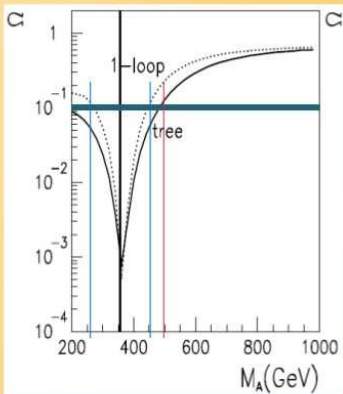
- Within framework of MSSM - Neutralino LSP
- Well studied at tree level



hep-ph/0404175v2

# Neutralino LSP as DM

Playing with WMAP exclusions : the one-loop gap



Points allowed :

One-Loop

- 260 & 450 GeV

Tree-level

- 500 GeV

*Shamelessly stolen from a talk by F. Boudjema*

- Full one loop calculations in progress SLOOPS team, Annecy

# Neutralino LSP as DM

After  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$ :

$$\begin{array}{ll} \tilde{W}^\pm, \tilde{H}^\pm & \xrightarrow{\text{MIX}} \tilde{\chi}_{i=1,2}^\pm \quad \text{Charginos} \\ \tilde{B}^0, \tilde{W}^0, \tilde{h}^0, \tilde{H}^0 & \xrightarrow{\text{MIX}} \tilde{\chi}_{i=1,2,3,4}^0 \quad \text{Neutralinos} \end{array}$$

$$\mathcal{M}_D^0 = N^* \mathcal{M} N^\dagger$$

Neutralino mass matrix

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

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Bino

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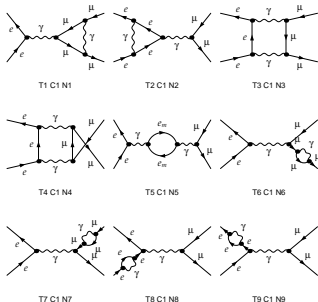
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Higgsino

# Back to the (grad)school

All possible one loop diagrams

$$e e \rightarrow \mu \mu$$



Lagrangian  $\mathcal{L}(e, m, g)$

$$m^{(1)} \rightarrow m^{(0)} + \Delta m$$

$$g^{(1)} \rightarrow g^{(0)} + \Delta g$$

$$e^{(1)} \rightarrow e^{(0)} + \Delta e$$

$$\mathcal{L}^{(1)} \rightarrow \mathcal{L}^{(0)} + \Delta \mathcal{L}$$

$\Rightarrow$  Amplitude:

$$|\mathcal{M}^{(1)}|^2 = |\mathcal{M}^{(0)}|^2 + |\Delta \mathcal{M}|^2 + 2|\Delta \mathcal{M}||\mathcal{M}^{(0)}|$$

- On-shell scheme: external particle masses are known, ideal for dark matter case
- number of neutralino annihilation diagrams for SUSY scenarios easily reach thousands

## Beauty of broken SUSY - Non-decoupling effect

- Unbroken SUSY - equal couplings for the standard and SUSY particles

$$g(e\tilde{e}\tilde{\gamma}) = g(e e \gamma)$$

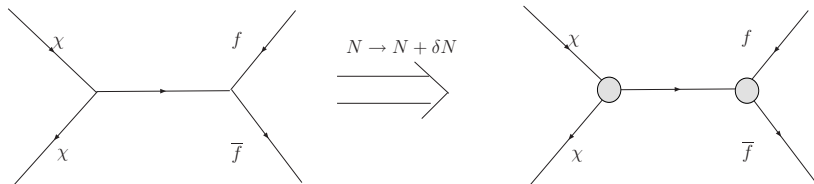
- SUSY breaking  $\Rightarrow$  difference between SUSY and SM couplings grows with the SUSY breaking scale
- Same RGE above SUSY breaking scale, SUSY and SM particle decoupling at different scales due to mass hierarchy
- At one loop comparison between two couplings ( $Q < m_{\tilde{q}}$ )

$$\frac{\tilde{\alpha}(Q)}{\alpha(Q)} - 1 = \frac{\alpha(m_{\tilde{q}})}{\alpha(Q)} - 1 = \beta \log \frac{m_{\tilde{q}}}{m_q}, \quad \text{On-shell}$$

Can we use this to include the most dominant radiative corrections to  
Neutralino DM relic density?

## Effective couplings - a closer look

- Analogous to Oblique corrections in the Standard Model
- Effective couplings - extracting a set of dominant flavor independent corrections to gaugino-fermion-sfermion couplings
- Does not work for higgsinos because the Yukawa couplings are not flavor blind
- Such corrections can be expressed as a finite shift in the neutralino mixing matrices (with only matter sector in the loops)



- No vertex corrections in this case
- Include the fermion-sfermion non-decoupling effects

# Effective couplings - a closer look

- chargino/neutralino -fermion -sfermion process renormalized

Input parameters	
SM	$M_w, M_z$ $\alpha_{em}$
-ino	$M_{\chi_1^-}, M_{\chi_2^-}$ $M_{\chi_i^0}$
Other	$\tan \beta$ $M_{\tilde{f}_i}$

- Requires consistent on-shell renormalization scheme
- Renormalization of  $\tan \beta$  - simplest not unique, nor the best
- The most bino-like neutralino on-shell, not necessarily LSP

$$\Delta N_{\alpha 1} \equiv N_{\alpha 1} \left( \frac{\delta g}{g} + \frac{\delta Z_R^\alpha}{2} + \frac{\delta t_W}{t_W} \right) + \sum_{\beta \neq \alpha} N_{\beta 1} Z_R^{\alpha \beta}$$

$$\Delta N_{\alpha 2} \equiv N_{\alpha 2} \left( \frac{\delta g}{g} + \frac{\delta Z_R^\alpha}{2} \right) + \sum_{\beta \neq \alpha} N_{\beta 2} Z_R^{\alpha \beta}$$

$$\Delta N_{\alpha 3} \equiv N_{\alpha 3} \left( \frac{\delta g}{g} + \frac{\delta Z_R^\alpha}{2} + \frac{1}{2} \frac{\delta M_w^2}{M_w^2} - \frac{\delta \cos \beta}{\cos \beta} \right) + \sum_{\beta \neq \alpha} N_{\beta 3} Z_R^{\alpha \beta}$$

$$\Delta N_{\alpha 4} \equiv N_{\alpha 4} \left( \frac{\delta g}{g} + \frac{\delta Z_R^\alpha}{2} + \frac{1}{2} \frac{\delta M_w^2}{M_w^2} - \frac{\delta \sin \beta}{\sin \beta} \right) + \sum_{\beta \neq \alpha} N_{\beta 4} Z_R^{\alpha \beta}$$



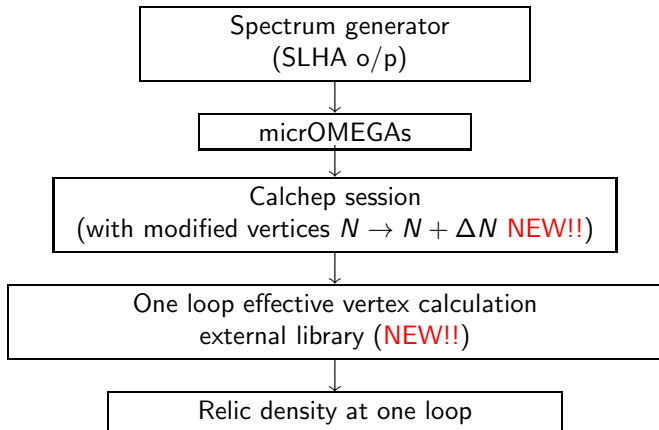
# How exactly - technical details



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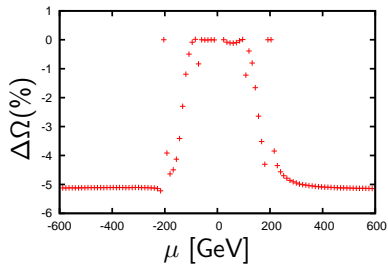
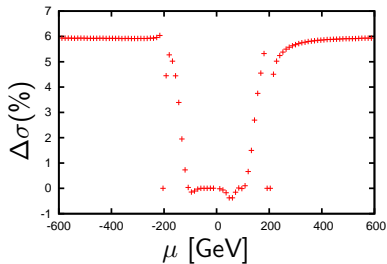
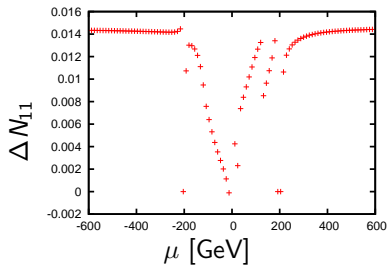
- Including dominant radiative corrections essentially reduces to calculating two point functions
- Total number of diagrams need to be calculated  $\approx 400$  ( $\approx 6000$  in full one loop)
- Work the best for a gaugino -fermion -sfermion couplings
- In most of the mSUGRA parameter space observationally compatible Neutralino DM is binolike - we expect the effective couplings to work

## How exactly - technical details



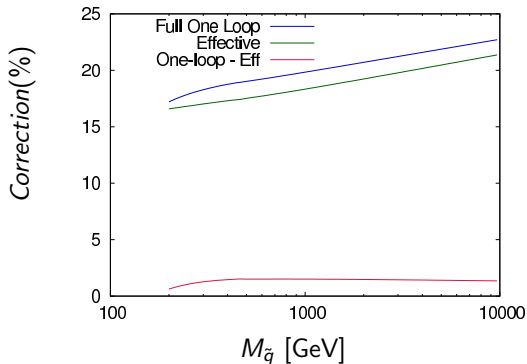
- Used FeynArts, FormCalc, LoopTools for loop calculations

# Results



- EWSB scenario  
 $M_2 = 200$ ,  $M_1 = 90$ ,  
 $\tan\beta = 5$ ,  $A_f = 0$
- Dominant leptonic  
channel annihilation

# Comparisons to full one loop - squark mass variation



- Bino case  $M_1 = 90$ ;  $M_2, \mu \gg M_1$
- Binolike neutralino 99%
- Sizable improvement
- Non-effective part stays small and flat

# Conclusions

If Neutralino is Dark Matter . . .

- Electroweak corrections to Neutralino Dark Matter relic density should be taken into account
- Relic density gets corrected upto  $\approx 4 - 5\%$  in some regions of parameter space
- Effective couplings work the best for a binolike neutralino
- Comparisons with full one-loop calculations for a binolike neutralino show a percent level match
- Formalism needs to be extended to capture the Yukawa corrections as well